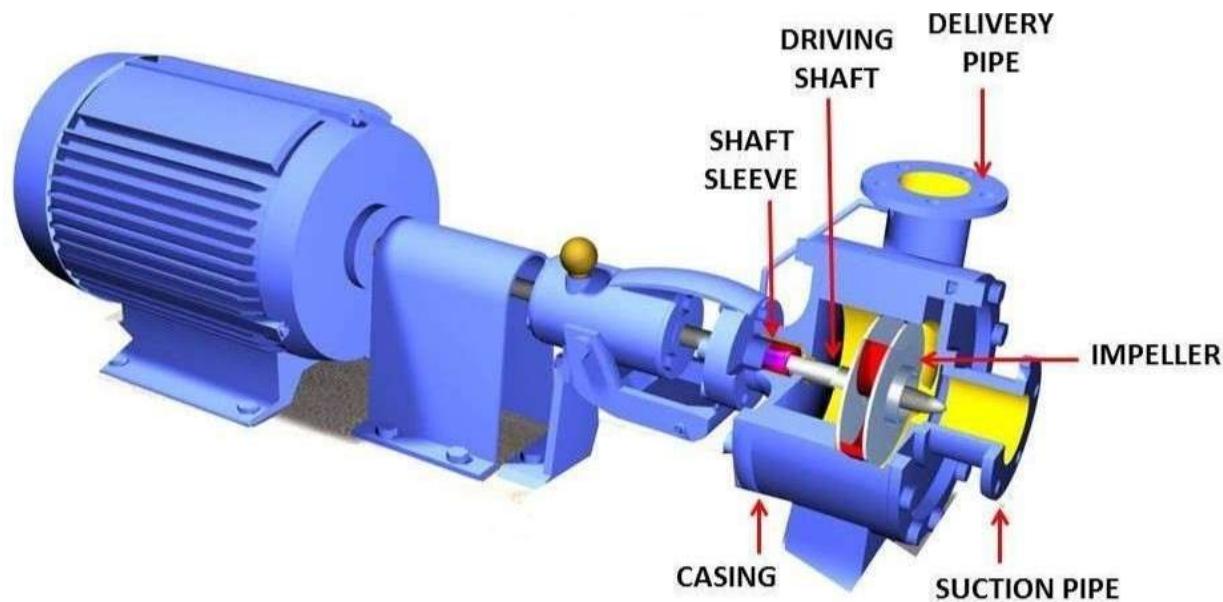


Turbomachinery - Pumps



Instructor: Joaquín Valencia

ME 3140

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2. Pumps
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 - 2.2. Pump Cavitation
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 - 2.4. Positive-Displacement Pumps
 - 2.5. Dynamic Pumps
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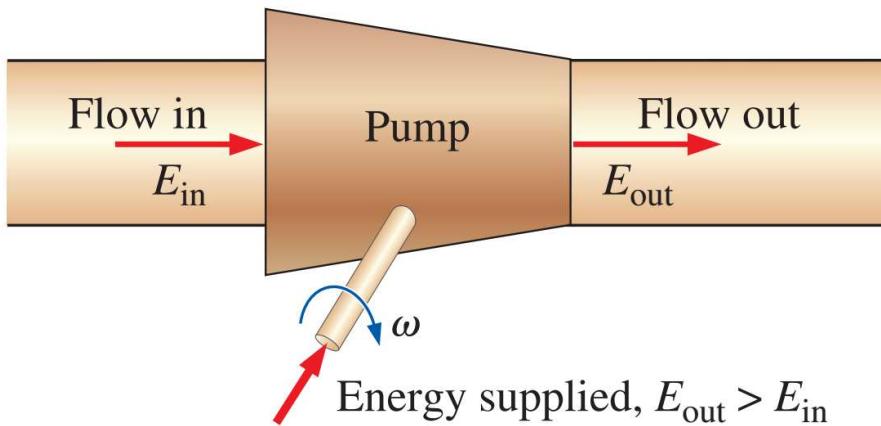
- **Classifications and Terminology**

Classifications and Terminology

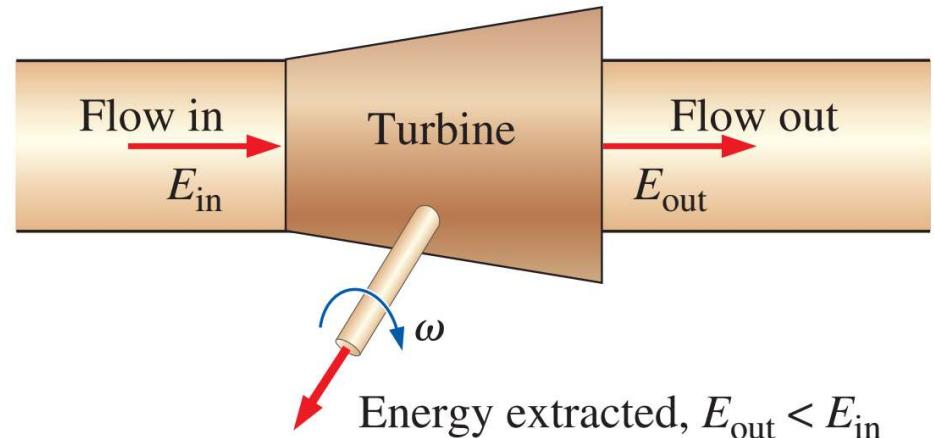
Turbomachinery is a class of machines in which a rotating shaft supplies energy to, or extracts energy from, a fluid. The Latin prefix **turbo** means “to spin.”

Classification of turbomachinery

- (a) Pumps
- (b) Turbines



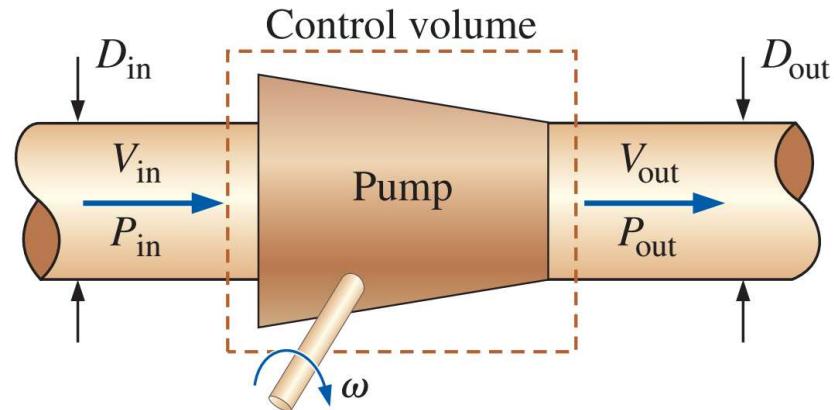
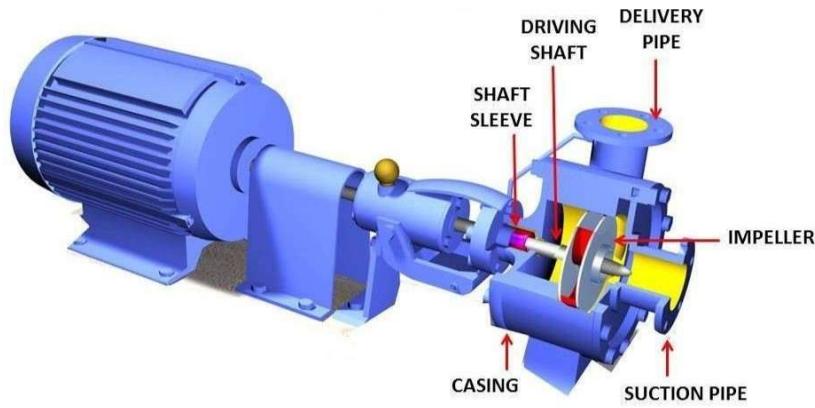
(a) A pump supplies energy to a fluid



(b) A turbine extracts energy from a fluid

Classifications and Terminology

Pump. The purpose of a pump is to add energy to a fluid, resulting in an increase in fluid pressure, not necessarily an increase in fluid speed across the pump.



For the case of **steady flow**, conservation of mass requires that the mass flow rate out of a pump must equal the mass flow rate into the pump.

$$\dot{m}_{\text{out}} = \dot{m}_{\text{in}}$$

For **incompressible flow** with equal inlet and outlet cross-sectional areas ($D_{\text{out}} = D_{\text{in}}$), the average velocities are:

$$V_{\text{out}} = V_{\text{in}} \quad \text{but} \quad P_{\text{out}} > P_{\text{in}}$$

Classifications and Terminology

When used with **gases**, pumps are called **fans, blowers, or compressors**, depending on the relative values of pressure rise and volume flow rate.

	Fan	Blower	Compressor
ΔP	Low	Medium	High
Q	High	Medium	Low



Examples

Squirrel cage blowers

Fan. Ceiling fans, house fans, and propellers.

Blower. Centrifugal blowers and squirrel cage blowers are used in automobile ventilation systems, furnaces, and leaf blowers.

Compressor. Air compressors that run pneumatic tools and inflate tires at automobile service stations, and refrigerant compressors used in heat pumps, refrigerators, and air conditioners.

Content

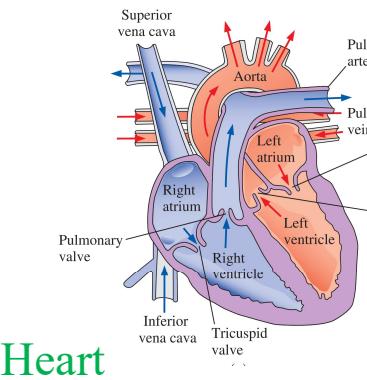
1. Classifications and Terminology

2. Pumps

Pumps

General Classification of Pumps

Positive-displacement Pumps. Fluid enters a closed chamber, and energy is transferred by motion of the chamber boundary—expanding to draw fluid in and contracting to discharge it. **Example:** heart, piston pump, peristaltic pump, gear pump, etc.



Heart



Peristaltic pump



Piston pump

Dynamic pumps use rotating blades, also called impeller blades, to supply energy to the fluid.

Example:

- Enclosed/ducted pumps (with casings around the blades, e.g., a car's engine water pump)
- Open rotors (without casings, e.g., a ceiling fan, an airplane propeller, or a helicopter rotor).

Pumps: Performance Parameters

Mass flow rate, \dot{m}

$$\dot{m} = \rho A V$$

Volume flow rate (or capacity), Q

$$Q = \frac{\dot{m}}{\rho} \quad (\text{For incompressible flow})$$

Net head, H

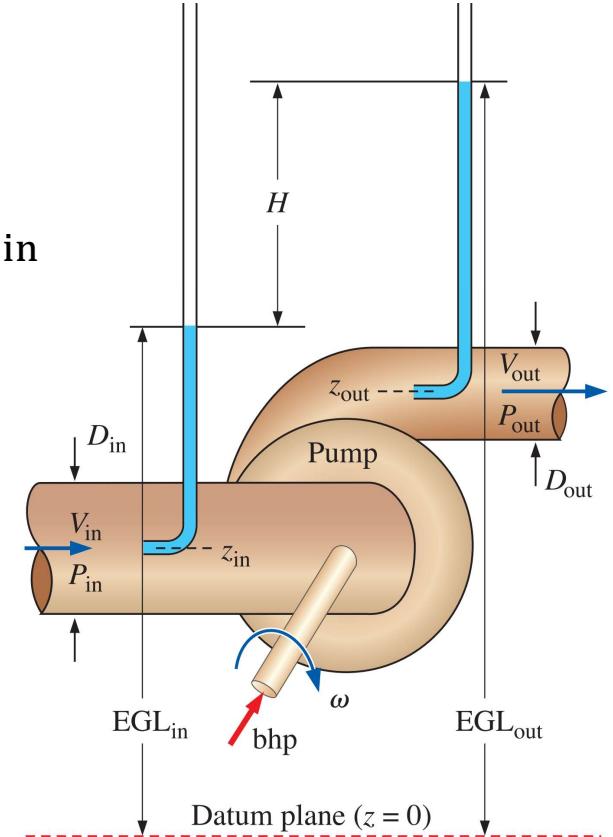
$$H = \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z \right)_{\text{out}} - \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z \right)_{\text{in}}$$

or $H = EGL_{\text{out}} - EGL_{\text{in}}$

For incompressible flow

Special case with $D_{\text{out}} = D_{\text{in}}$ and $z_{\text{out}} = z_{\text{in}}$

$$H = \frac{P_{\text{out}} - P_{\text{in}}}{\rho g}$$



Pumps: Performance Parameters

Water horsepower, $\dot{W}_{\text{water horsepower}}$

$$\dot{W}_{\text{water horsepower}} = \dot{m}gH = \rho g Q H \quad \text{H: neat head}$$

Brake horsepower, bhp $bhp = \dot{W}_{\text{shaft}} = \omega T_{\text{shaft}}$

Pump efficiency, η_{pump}

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{water horsepower}}}{\dot{W}_{\text{shaft}}} = \frac{\dot{W}_{\text{water horsepower}}}{bhp} = \frac{\rho g Q H}{\omega T_{\text{shaft}}}$$

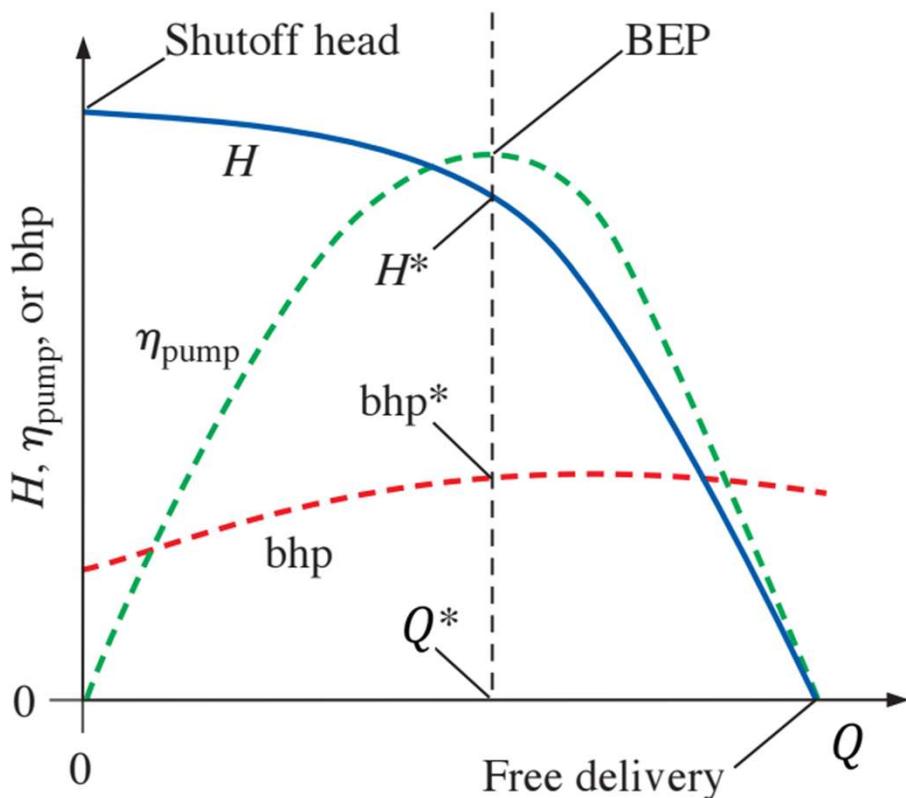
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1. Classifications and Terminology
2. Pumps

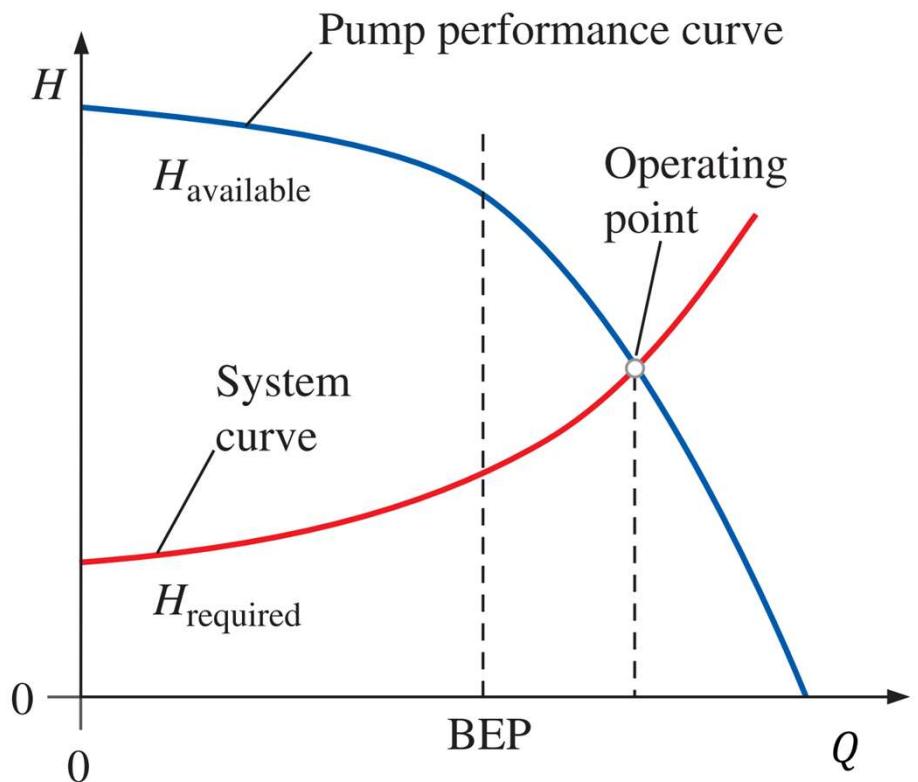
2.1. Pump Performance Curves

Pump Performance Curves

BEP: Best Efficiency Point



Typical pump performance curves for a centrifugal pump with backward-inclined blades



$$H_{\text{required}} = H_{\text{available}}$$

The operating point of a piping system

Pump Performance Curves

Energy equation for the Required net head, H_{required}

$$H_{\text{required}} = h_{\text{pump,u}}$$

$$= \frac{P_2 - P_1}{\rho g} + \frac{\alpha_2 V_2^2 - \alpha_1 V_1^2}{2g} + (z_2 - z_1) + h_{L,\text{total}}$$

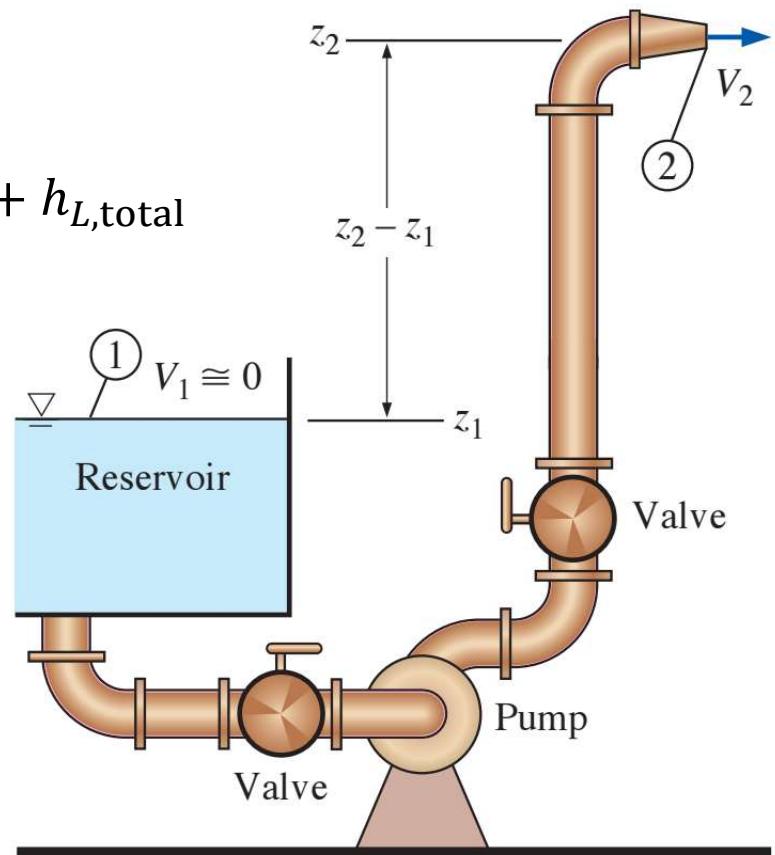
$$h_{L,\text{total}} = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

$h_{\text{pump,u}}$: useful pump head

α_2, α_1 : kinetic energy correction fraction

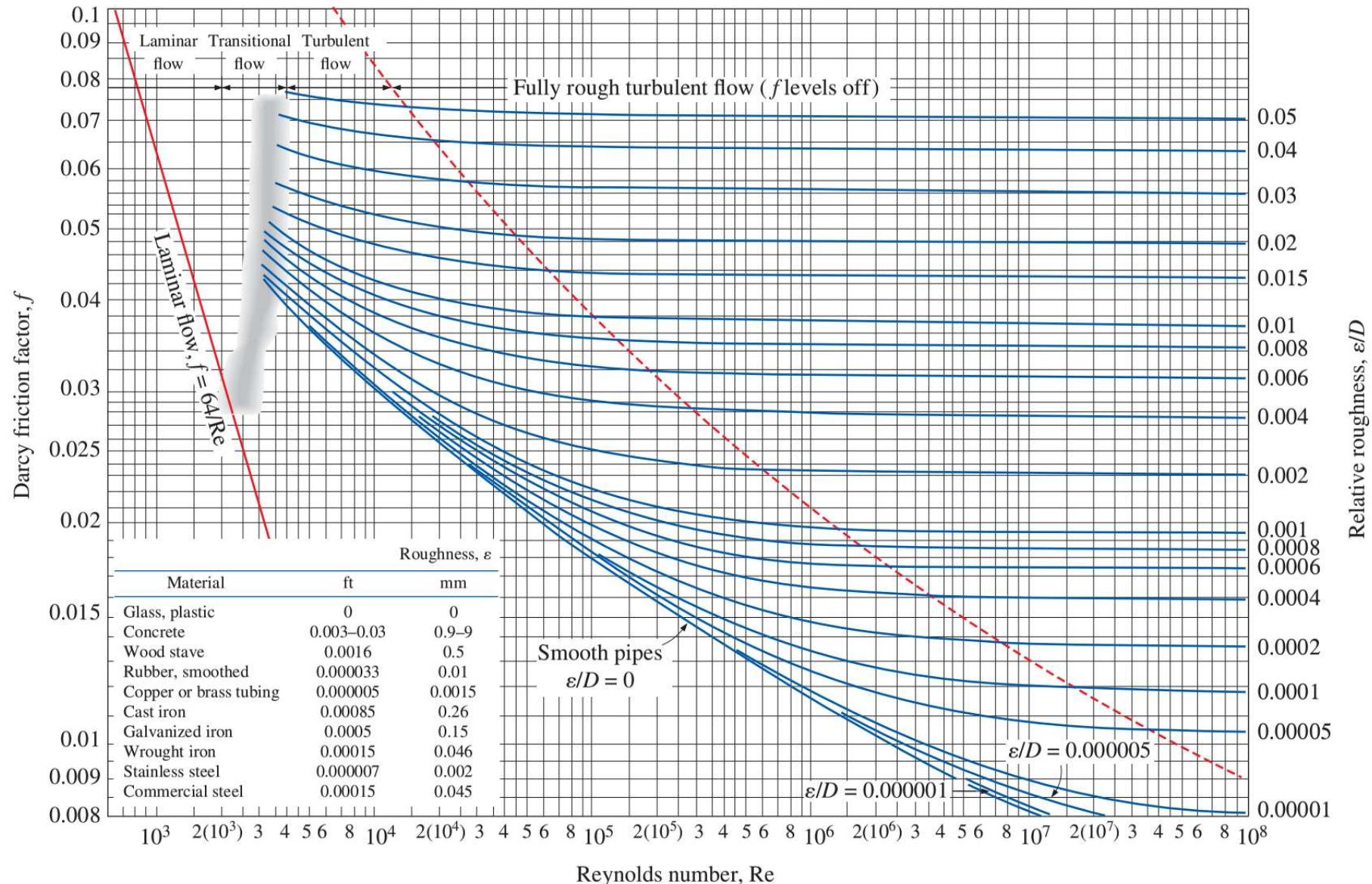
$h_{L,\text{total}}$: irreversible head losses

f : friction factor



Pump Performance Curves:

Moody chart



Pump Performance Curves

Example 1

A local ventilation system (hood and exhaust duct) is used to remove air and contaminants produced by a dry-cleaning operation (see [Figure E-1](#)). The duct is round and is constructed of galvanized steel with longitudinal seams and with joints every 30 in (0.76 m). The inner diameter (ID) of the duct is $D = 9.06$ in (0.230 m), and its total length is $L = 44.0$ ft (13.4 m). There are five CD3-9 elbows along the duct. The equivalent roughness height of this duct is 0.15 mm, and each elbow has a minor (local) loss coefficient of $K_L = C_0 = 0.21$. Note the notation C_0 for minor loss coefficient, commonly used in the ventilation industry (ASHRAE, 2001). To ensure adequate ventilation, the minimum required volume flow rate through the duct is $Q = 600$ cfm (cubic feet per minute), or 0.283 m^3/s at 25°C . Literature from the hood manufacturer lists the hood entry loss coefficient as 1.3 based on duct velocity. When the damper is fully open, its loss coefficient is 1.8. A centrifugal fan with 9.0-in inlet and outlet diameters is available. If the flow is turbulent, assume that $\alpha = 1.05$. Its performance data are shown in Table E-1, as listed by the manufacturer. Predict the operating point of this local ventilation system, and draw a plot of required and available fan pressure rise as functions of volume flow rate. Is the chosen fan adequate?

Answer: 0.709 in of water

Pump Performance Curves

Example 1 ...continuation

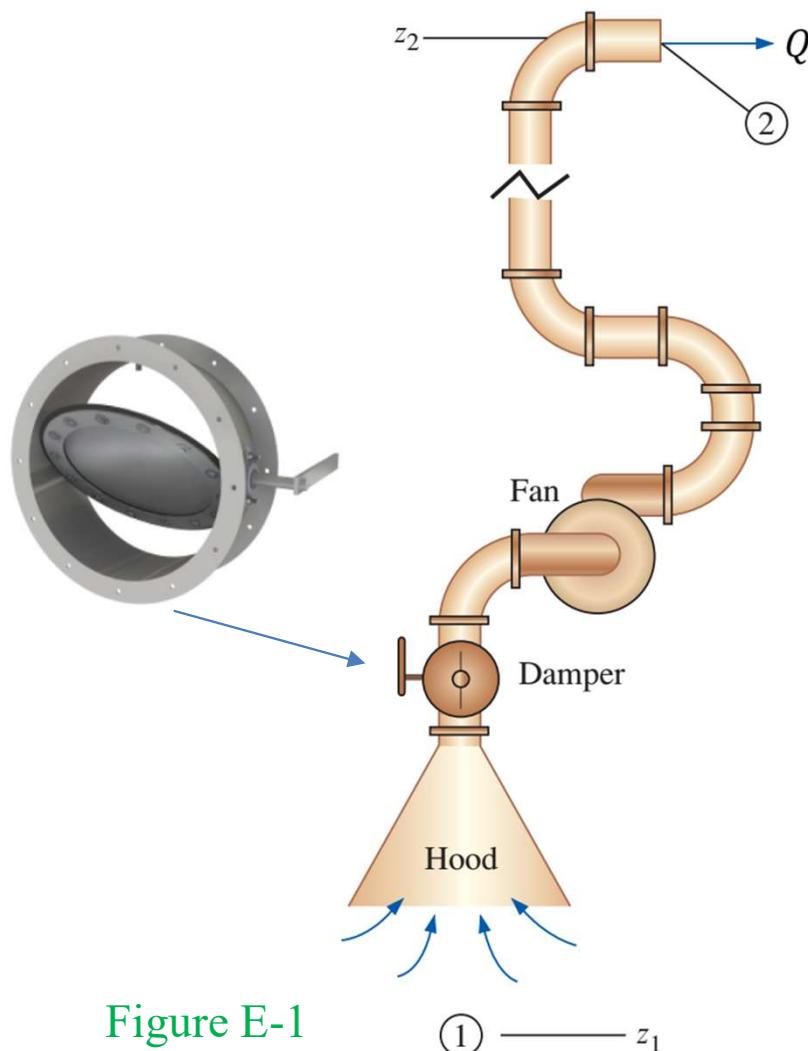


Table E-1

Manufacturer's performance data for the fan of Example 1*

Q , cfm	$H_{\text{available}}$, inches H_2O
0	0.90
250	0.95
500	0.90
750	0.75
1000	0.40
1200	0.0

Figure E-1

Pump Performance Curves

TABLE A-9

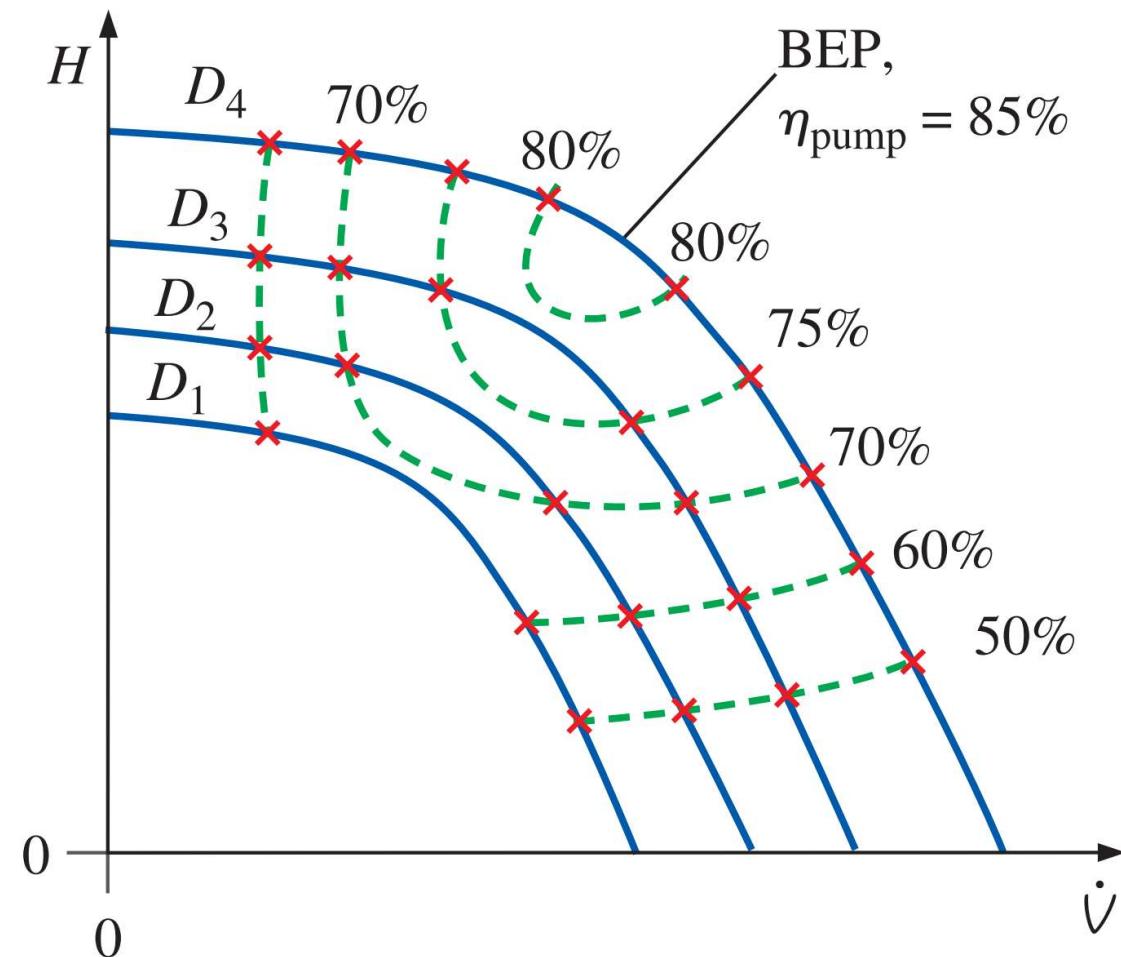
Properties of air at 1 atm pressure

Temp. <i>T</i> , °C	Density <i>ρ</i> , kg/m ³	Specific Heat <i>c_p</i> , J/kg·K	Thermal Conductivity <i>k</i> , W/m·K	Thermal Diffusivity <i>α</i> , m ² /s	Dynamic Viscosity <i>μ</i> , kg/m·s	Kinematic Viscosity <i>ν</i> , m ² /s	Prandtl Number <i>Pr</i>
-150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
-100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-6}	5.837×10^{-6}	0.7263
-50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
-40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
-30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
-20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
-10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177

Pump Performance Curves

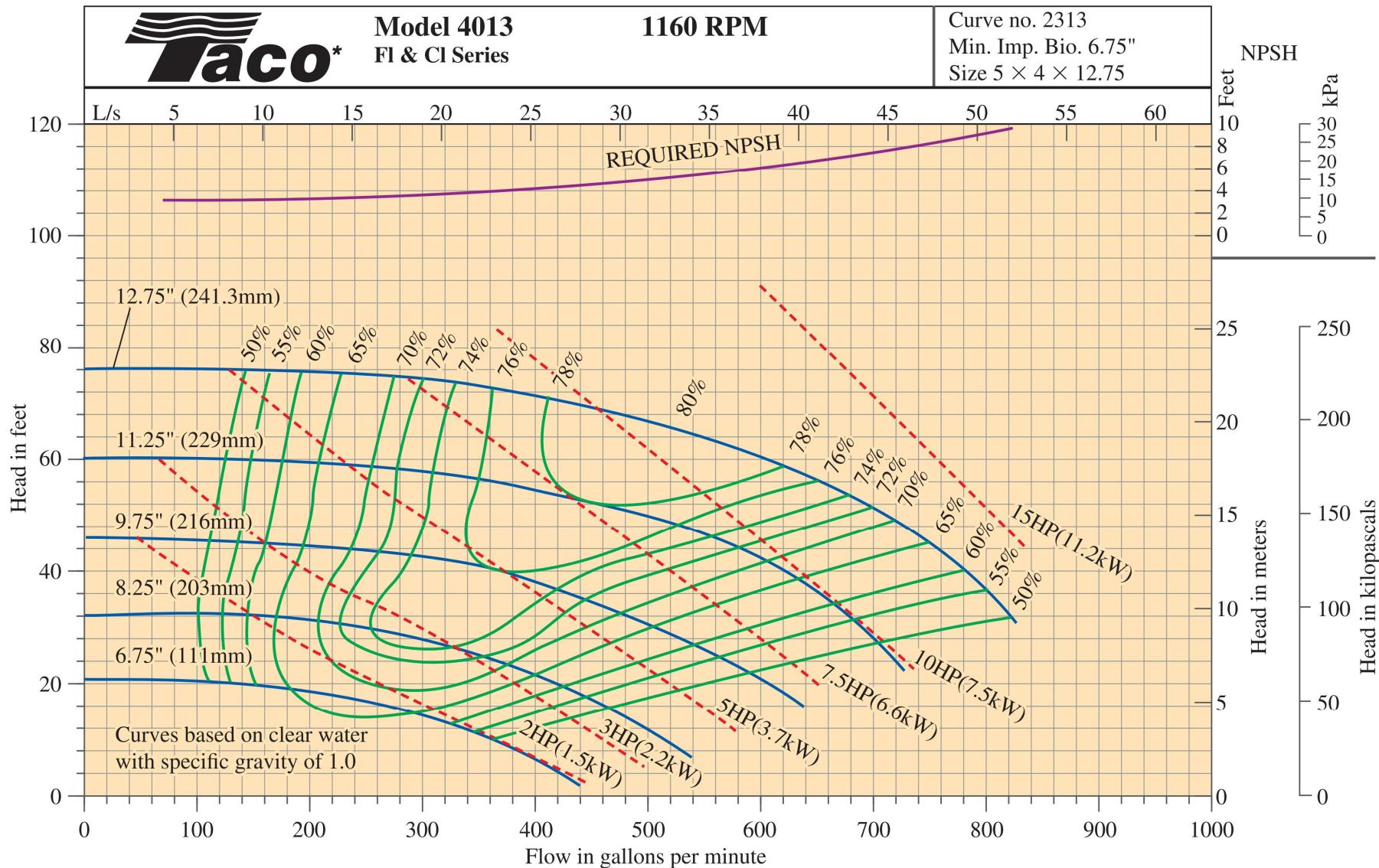
Typical pump performance curves for a family of centrifugal pumps of the same casing diameter but different impeller diameters.

BEP: Best efficiency point



Pump Performance Curves

Example of a manufacturer's performance plot for a family of centrifugal pumps. Each pump has the same casing, but a different impeller diameter.



Pump Performance Curves

Required NPSH for Model 4013

Flow (gpm)	Required NPSH (ft)
79	2.85
159	2.94
238	3.11
317	3.38
396	3.82
476	4.34
555	5.04
634	5.92
713	6.97
793	8.20

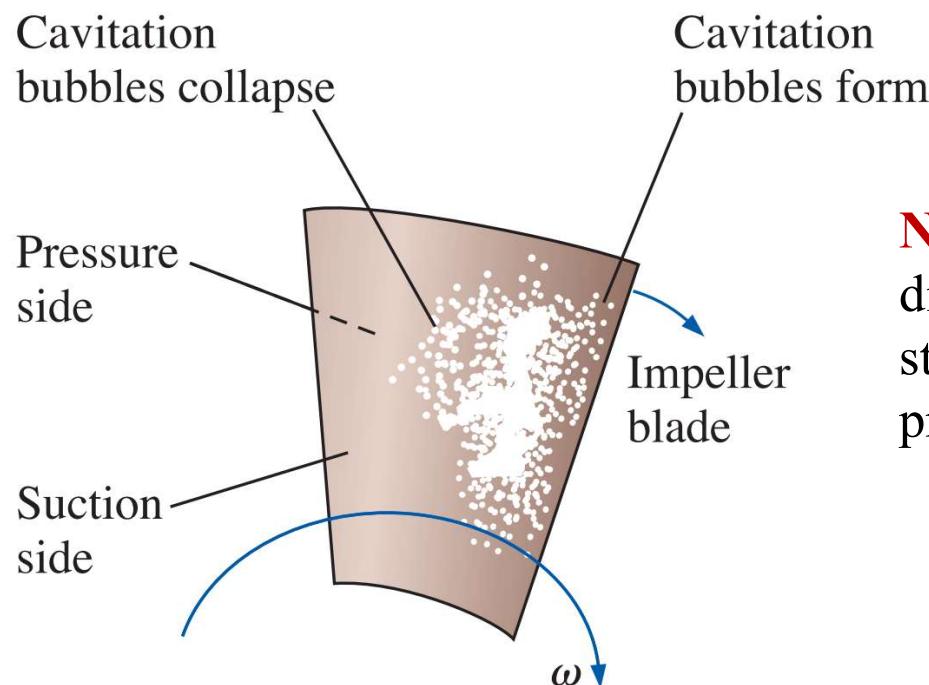
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 - 2.1. Pump Performance Curves

2.2. Pump Cavitation

Pump Cavitation

When $P < P_v$, vapor-filled bubbles called **cavitation bubbles** appear.



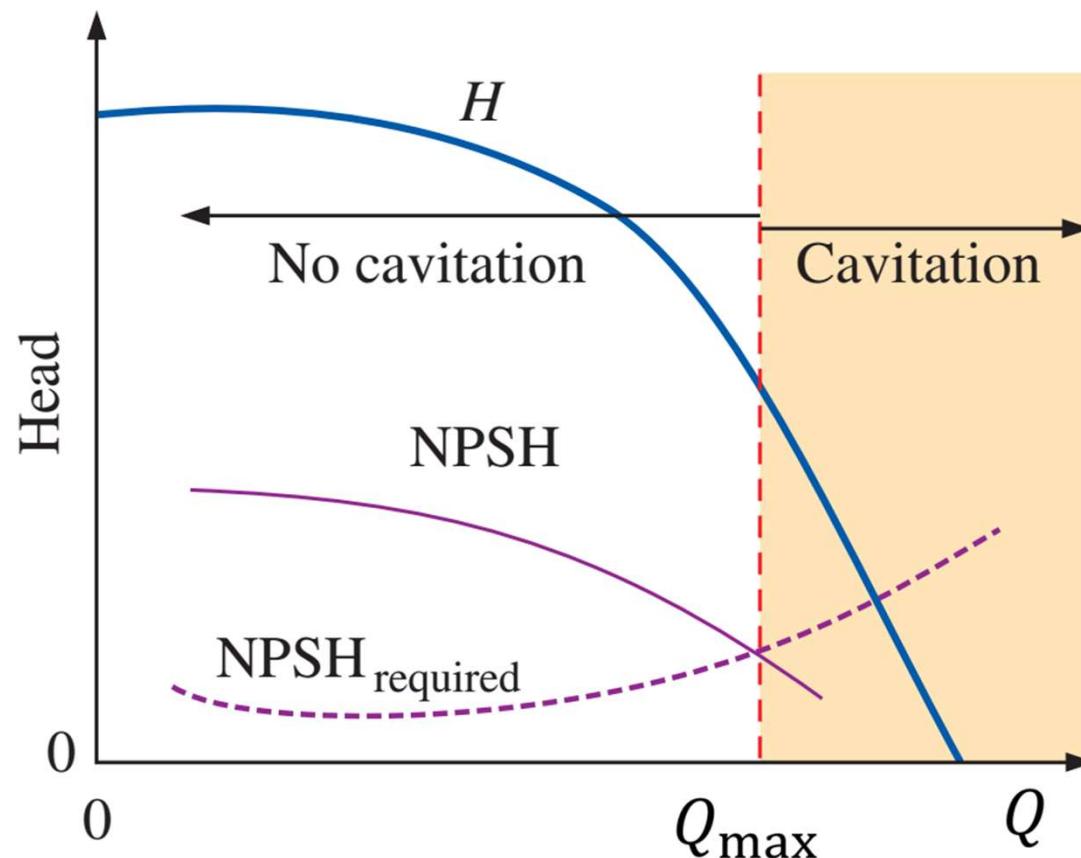
P_v : vapor pressure of the liquid (also called the saturation pressure P_{sat})

Net positive suction head: defined as the difference between the pump's inlet stagnation pressure head and the vapor pressure head.

$$NPSH = \left(\frac{P}{\rho g} + \frac{V^2}{2g} \right)_{\text{pump inlet}} - \frac{P_v}{\rho g}$$

Pump Cavitation

The **volume flow rate (Q)** at which the **actual NPSH** and the **required NPSH** intersect represents the maximum flow rate that can be delivered by the pump without the occurrence of **cavitation**.



Pump Cavitation

Example 2

The 11.25-in impeller option of the Taco Model 4013 FI Series centrifugal pump of Figure E-2 is used to pump water at 25°C from a reservoir whose surface is 4.0 ft above the centerline of the pump inlet (Figure E-3). The piping system from the reservoir to the pump consists of 10.5 ft of cast iron pipe with an ID of 4.0 in and an average inner roughness height of 0.02 in. There are several minor losses: a sharp-edged inlet ($K_L = 0.5$), three flanged smooth 90° regular elbows ($K_L = 0.3$ each), and a fully open flanged globe valve ($K_L = 6.0$). Estimate the maximum volume flow rate (in units of gpm) that can be pumped without cavitation. If the water were warmer, would this maximum flow rate increase or decrease? Why? Discuss how you might increase the maximum flow rate while still avoiding cavitation.

Answer: NPSH = 23.5 ft

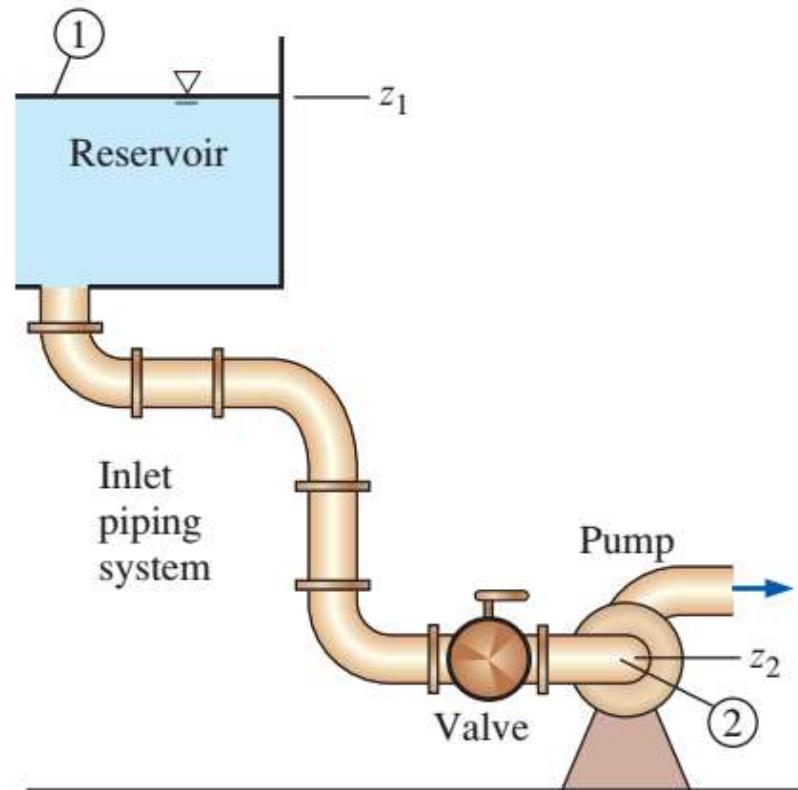


Figure E-3

Pump Cavitation

TABLE A-3

Properties of saturated water

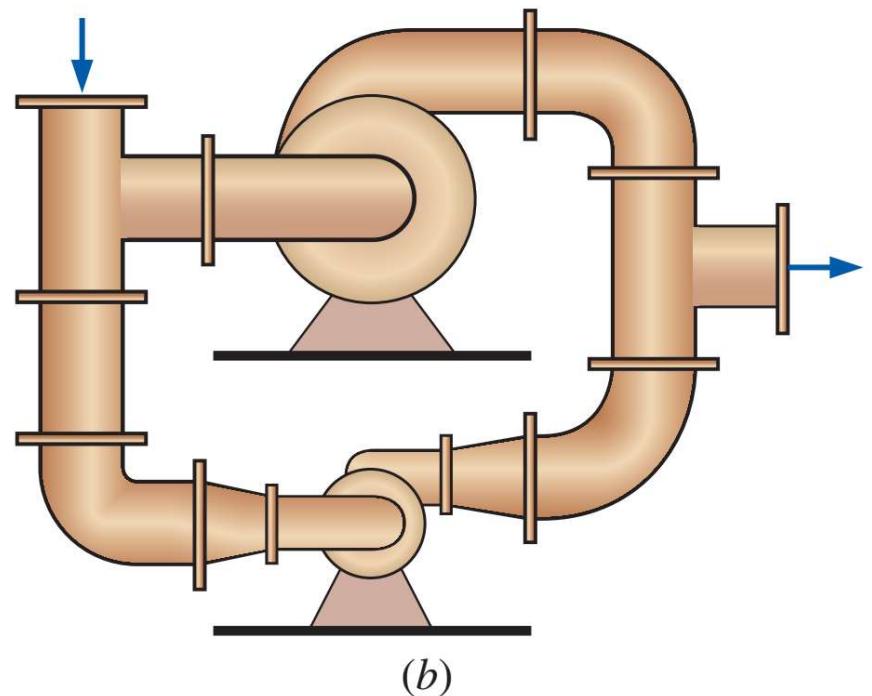
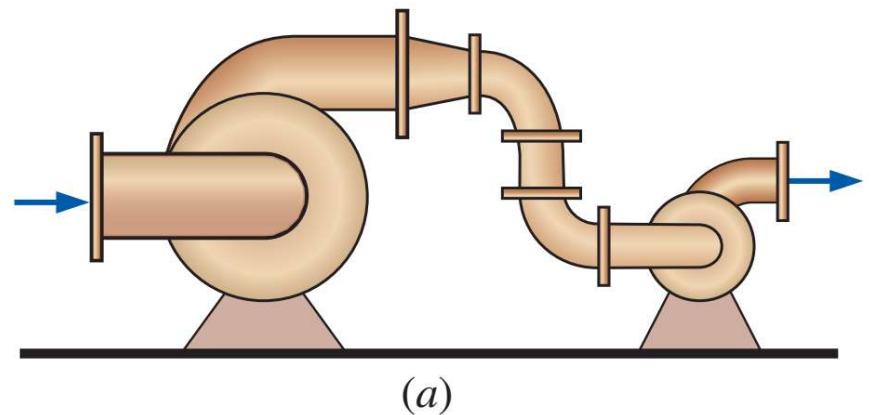
Temp. <i>T, °C</i>	Saturation Pressure <i>P_{sat}, kPa</i>	Density <i>ρ, kg/m³</i>		Enthalpy of Vaporization <i>h_{fg}, kJ/kg</i>		Specific Heat <i>c_p, J/kg·K</i>		Thermal Conductivity <i>k, W/m·K</i>		Dynamic Viscosity <i>μ, kg/m·s</i>		Prandtl Number <i>Pr</i>		Volume Expansion Coefficient <i>β, 1/K</i>	Surface Tension <i>N/m</i>
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Liquid
0.01	0.6113	999.8	0.0048	2501	4217	1854	0.561	0.0171	1.792×10^{-3}	0.922×10^{-5}	13.5	1.00	-0.068×10^{-3}	0.0756	
5	0.8721	999.9	0.0068	2490	4205	1857	0.571	0.0173	1.519×10^{-3}	0.934×10^{-5}	11.2	1.00	0.015×10^{-3}	0.0749	
10	1.2276	999.7	0.0094	2478	4194	1862	0.580	0.0176	1.307×10^{-3}	0.946×10^{-5}	9.45	1.00	0.733×10^{-3}	0.0742	
15	1.7051	999.1	0.0128	2466	4186	1863	0.589	0.0179	1.138×10^{-3}	0.959×10^{-5}	8.09	1.00	0.138×10^{-3}	0.0735	
20	2.339	998.0	0.0173	2454	4182	1867	0.598	0.0182	1.002×10^{-3}	0.973×10^{-5}	7.01	1.00	0.195×10^{-3}	0.0727	
25	3.169	997.0	0.0231	2442	4180	1870	0.607	0.0186	0.891×10^{-3}	0.987×10^{-5}	6.14	1.00	0.247×10^{-3}	0.0720	
30	4.246	996.0	0.0304	2431	4178	1875	0.615	0.0189	0.798×10^{-3}	1.001×10^{-5}	5.42	1.00	0.294×10^{-3}	0.0712	
35	5.628	994.0	0.0397	2419	4178	1880	0.623	0.0192	0.720×10^{-3}	1.016×10^{-5}	4.83	1.00	0.337×10^{-3}	0.0704	
40	7.384	992.1	0.0512	2407	4179	1885	0.631	0.0196	0.653×10^{-3}	1.031×10^{-5}	4.32	1.00	0.377×10^{-3}	0.0696	
45	9.593	990.1	0.0655	2395	4180	1892	0.637	0.0200	0.596×10^{-3}	1.046×10^{-5}	3.91	1.00	0.415×10^{-3}	0.0688	
50	12.35	988.1	0.0831	2383	4181	1900	0.644	0.0204	0.547×10^{-3}	1.062×10^{-5}	3.55	1.00	0.451×10^{-3}	0.0679	
55	15.76	985.2	0.1045	2371	4183	1908	0.649	0.0208	0.504×10^{-3}	1.077×10^{-5}	3.25	1.00	0.484×10^{-3}	0.0671	
60	19.94	983.3	0.1304	2359	4185	1916	0.654	0.0212	0.467×10^{-3}	1.093×10^{-5}	2.99	1.00	0.517×10^{-3}	0.0662	
65	25.03	980.4	0.1614	2346	4187	1926	0.659	0.0216	0.433×10^{-3}	1.110×10^{-5}	2.75	1.00	0.548×10^{-3}	0.0654	
70	31.19	977.5	0.1983	2334	4190	1936	0.663	0.0221	0.404×10^{-3}	1.126×10^{-5}	2.55	1.00	0.578×10^{-3}	0.0645	
75	38.58	974.7	0.2421	2321	4193	1948	0.667	0.0225	0.378×10^{-3}	1.142×10^{-5}	2.38	1.00	0.607×10^{-3}	0.0636	
80	47.39	971.8	0.2935	2309	4197	1962	0.670	0.0230	0.355×10^{-3}	1.159×10^{-5}	2.22	1.00	0.653×10^{-3}	0.0627	
85	57.83	968.1	0.3536	2296	4201	1977	0.673	0.0235	0.333×10^{-3}	1.176×10^{-5}	2.08	1.00	0.670×10^{-3}	0.0617	
90	70.14	965.3	0.4235	2283	4206	1993	0.675	0.0240	0.315×10^{-3}	1.193×10^{-5}	1.96	1.00	0.702×10^{-3}	0.0608	
95	84.55	961.5	0.5045	2270	4212	2010	0.677	0.0246	0.297×10^{-3}	1.210×10^{-5}	1.85	1.00	0.716×10^{-3}	0.0599	
100	101.33	957.9	0.5978	2257	4217	2029	0.679	0.0251	0.282×10^{-3}	1.227×10^{-5}	1.75	1.00	0.750×10^{-3}	0.0589	
110	143.27	950.6	0.8263	2230	4229	2071	0.682	0.0262	0.255×10^{-3}	1.261×10^{-5}	1.58	1.00	0.798×10^{-3}	0.0570	
120	198.53	943.4	1.121	2203	4244	2120	0.683	0.0275	0.232×10^{-3}	1.296×10^{-5}	1.44	1.00	0.858×10^{-3}	0.0550	
130	270.1	934.6	1.496	2174	4263	2177	0.684	0.0288	0.213×10^{-3}	1.330×10^{-5}	1.33	1.01	0.913×10^{-3}	0.0529	
140	361.3	921.7	1.965	2145	4286	2244	0.683	0.0301	0.197×10^{-3}	1.365×10^{-5}	1.24	1.02	0.970×10^{-3}	0.0509	

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 - 2.1. Pump Performance Curves
 - 2.2. Pump Cavitation
 - 2.3. Pumps in Series and Parallel**

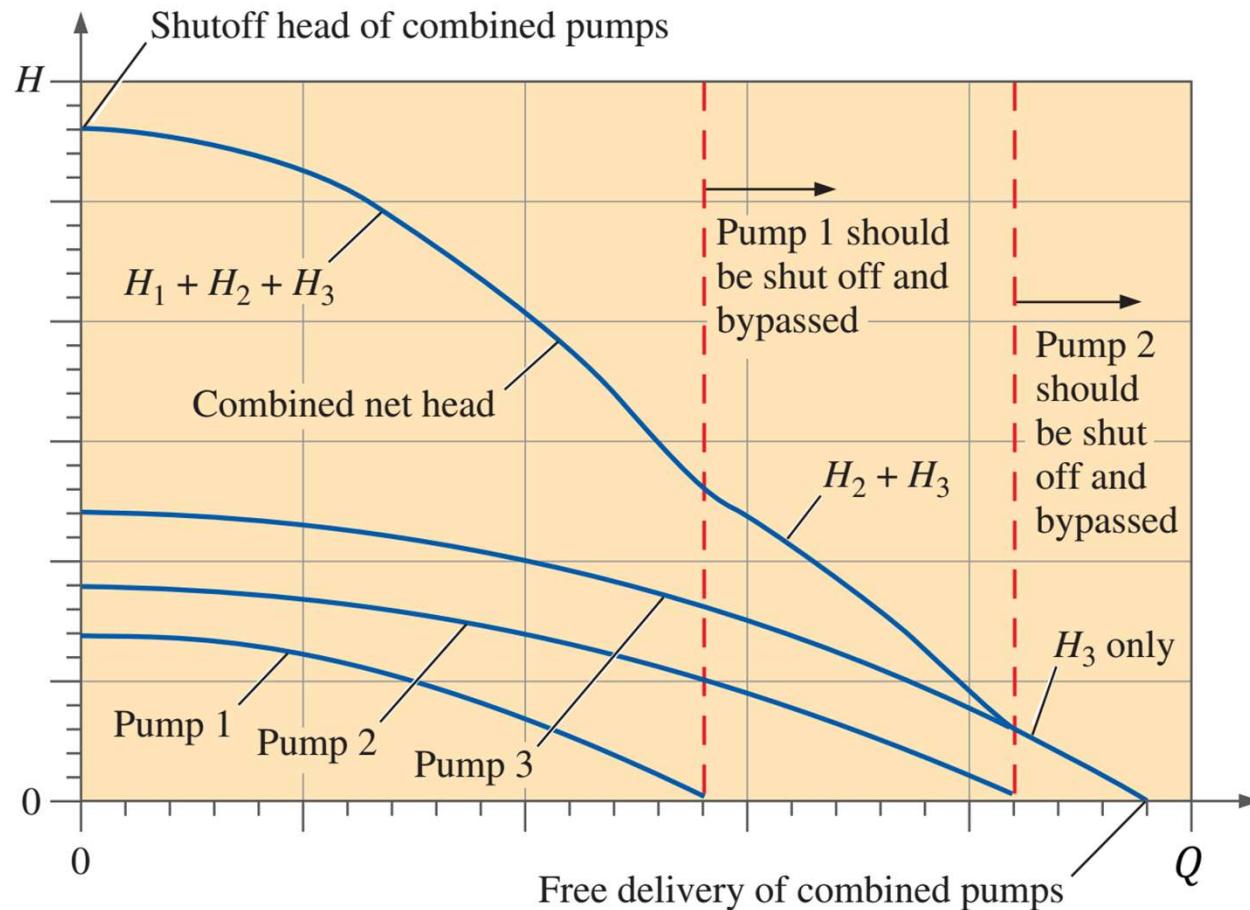
Pumps in Series and Parallel

Arranging two very dissimilar pumps in (a) series or (b) parallel can sometimes lead to problems.



Pumps in Series and Parallel

Pump performance curve (dark blue) for three **dissimilar pumps in series**.

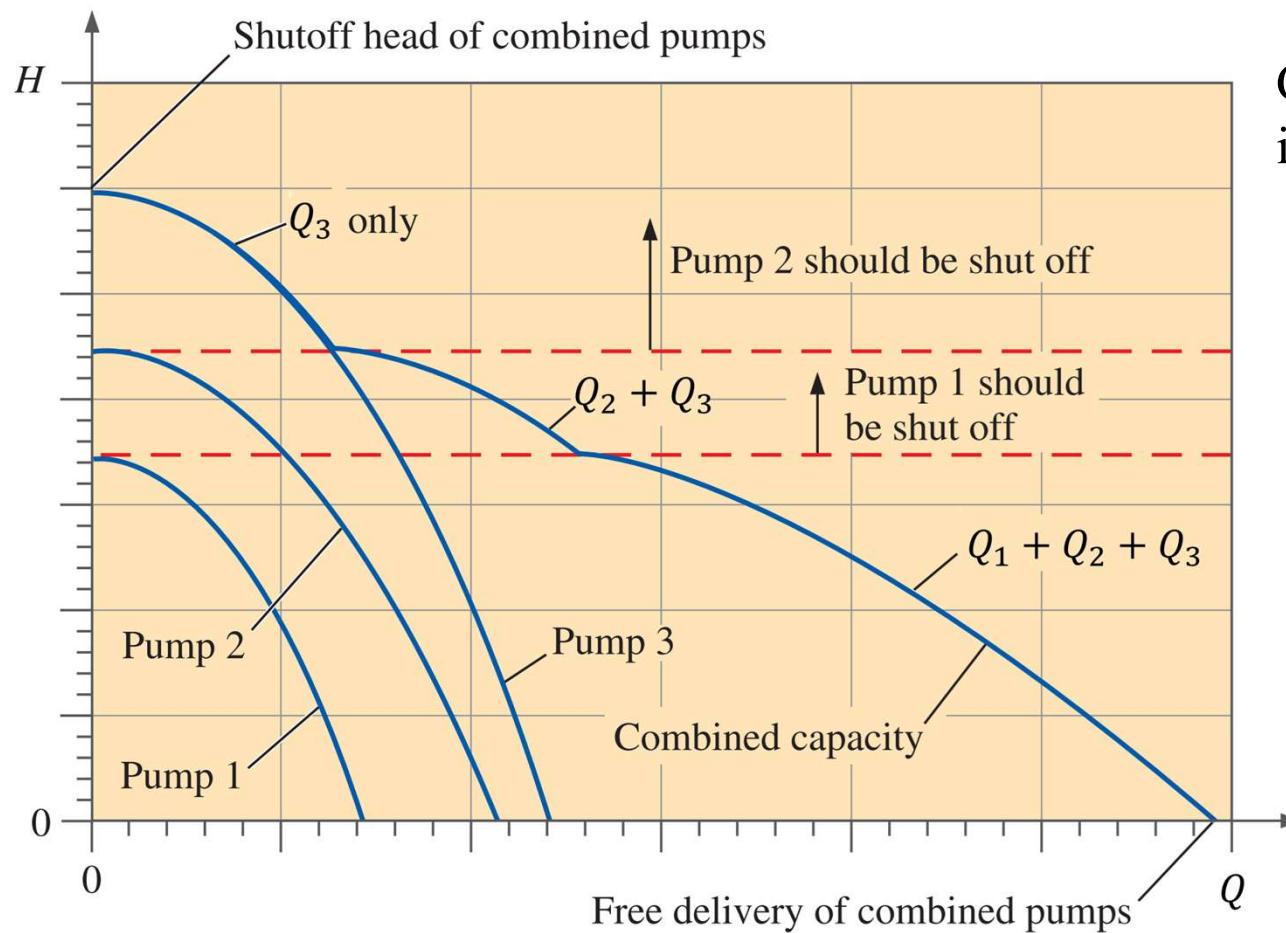


Combined net head for n pumps in series:

$$H_{\text{combined}} = \sum_{i=1}^n H_i$$

Pumps in Series and Parallel

Pump performance curve (dark blue) for three pumps in parallel.



Combined capacity for n pumps in parallel:

$$Q_{\text{combined}} = \sum_{i=1}^n Q_i$$

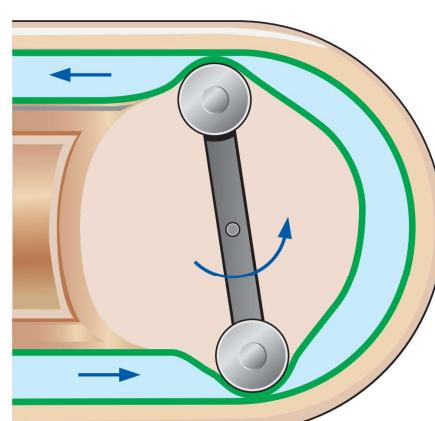
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 - 2.3. Pumps in Series and Parallel
- 2.4. Positive-Displacement Pumps**

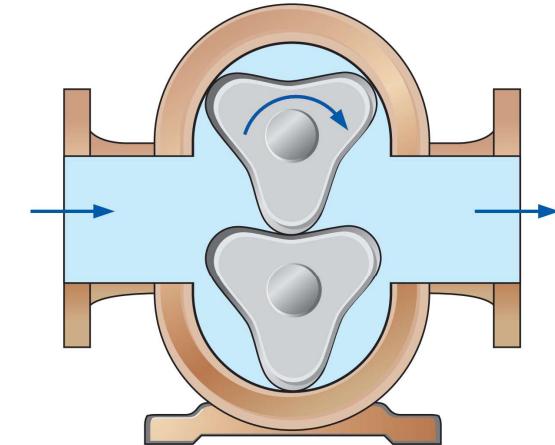
Positive-Displacement Pumps

Examples of positive-displacement pumps:

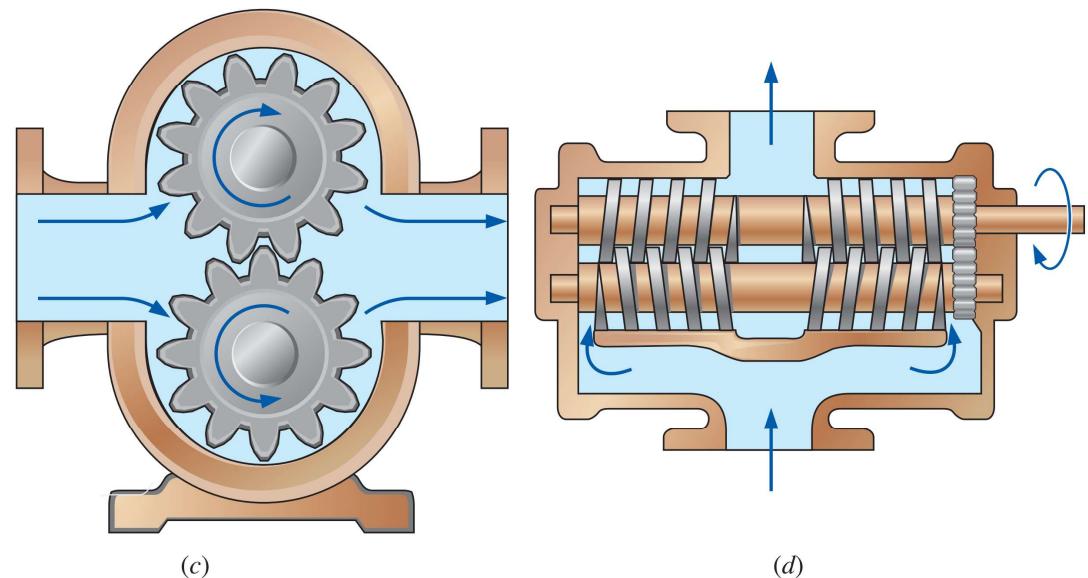
- (a) Flexible-tube peristaltic pump
- (b) Three-lobe rotary pump
- (c) Gear pump
- (d) Double screw pump



(a)



(b)

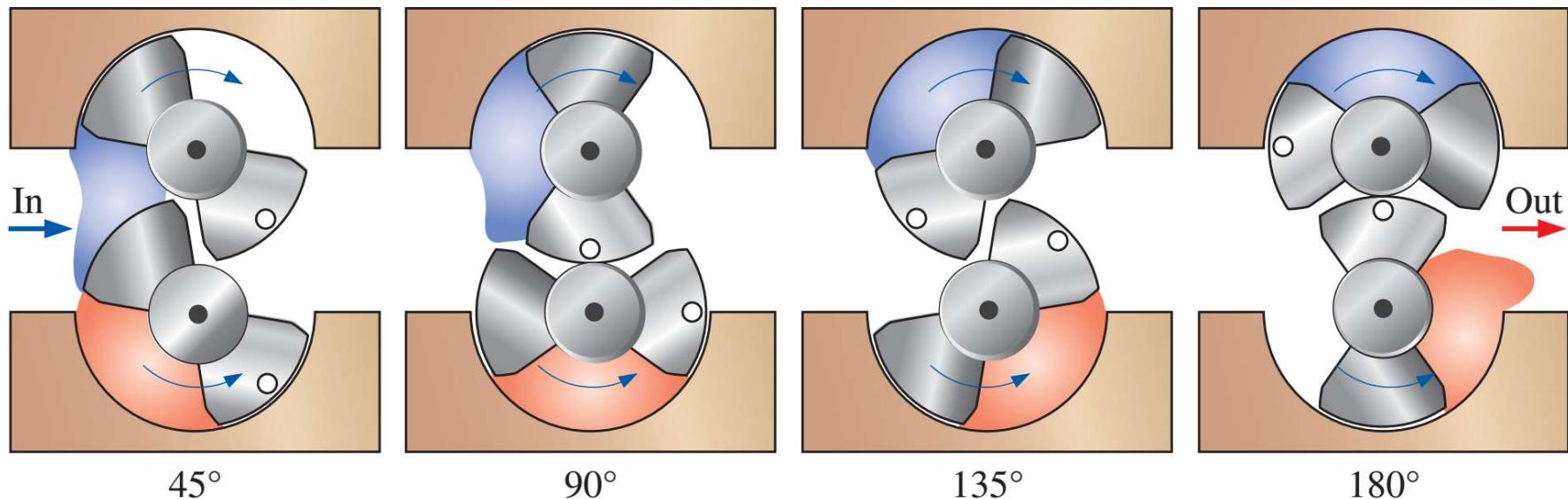


(c)

(d)

Positive-Displacement Pumps

Four phases (one-eighth of a turn apart) in the operation of a two-lobe rotary pump, a type of positive-displacement pump.



$$Q = (\text{revolutions per time}) \times (\text{volume displaced per revolution})$$

$$Q = \dot{n} \frac{V_{\text{closed}}}{n}$$

V_{closed} : displaced fluid volume in the closed chamber.

\dot{n} : rotational speed.

n : number of revolutions associated with the displaced volume.

Positive-Displacement Pumps

Example 3

A two-lobe rotary positive-displacement pump moves 0.45 cm^3 of SAE 30 motor oil in each lobe volume V_{lobe} , as sketched in Figure E-4. Calculate the volume flow rate of oil for the case where $\dot{n} = 900 \text{ rpm}$.

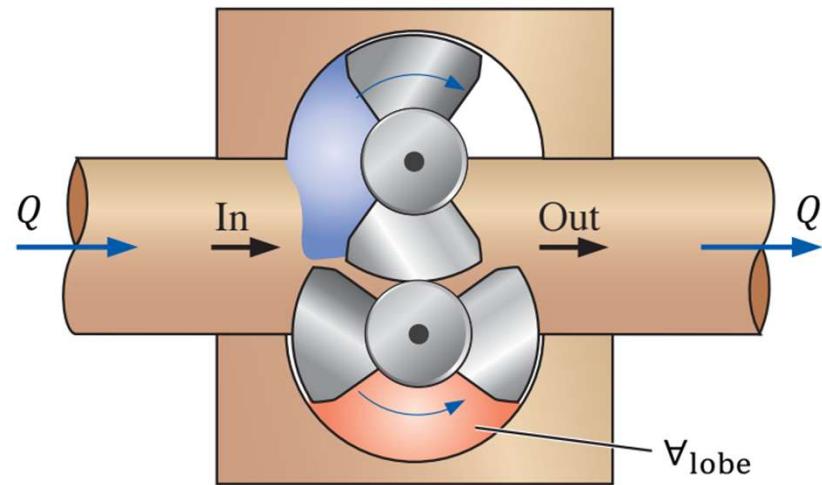


Figure E-4

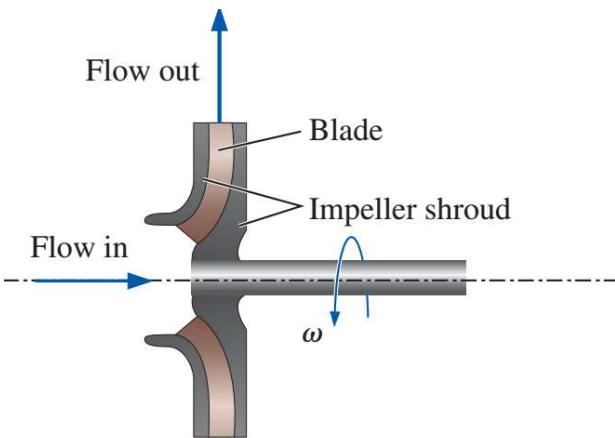
Content

1. Classifications and Terminology
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 - 2.5. Dynamic Pumps**

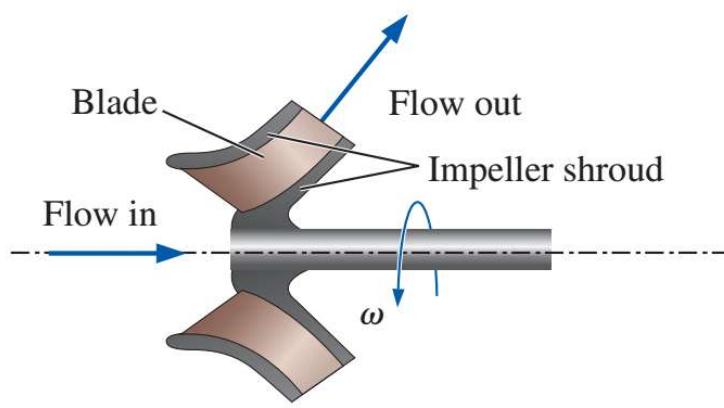
Dynamic Pumps

Dynamic pumps use rotating blades, also called impeller blades, to supply energy to the fluid.

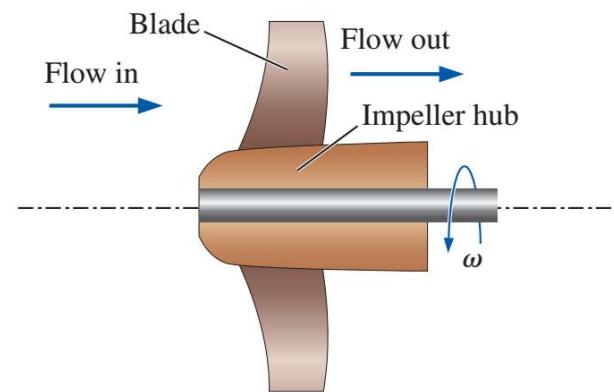
The impeller (rotating portion) of the three main categories of dynamic pumps:



(a) Centrifugal flow



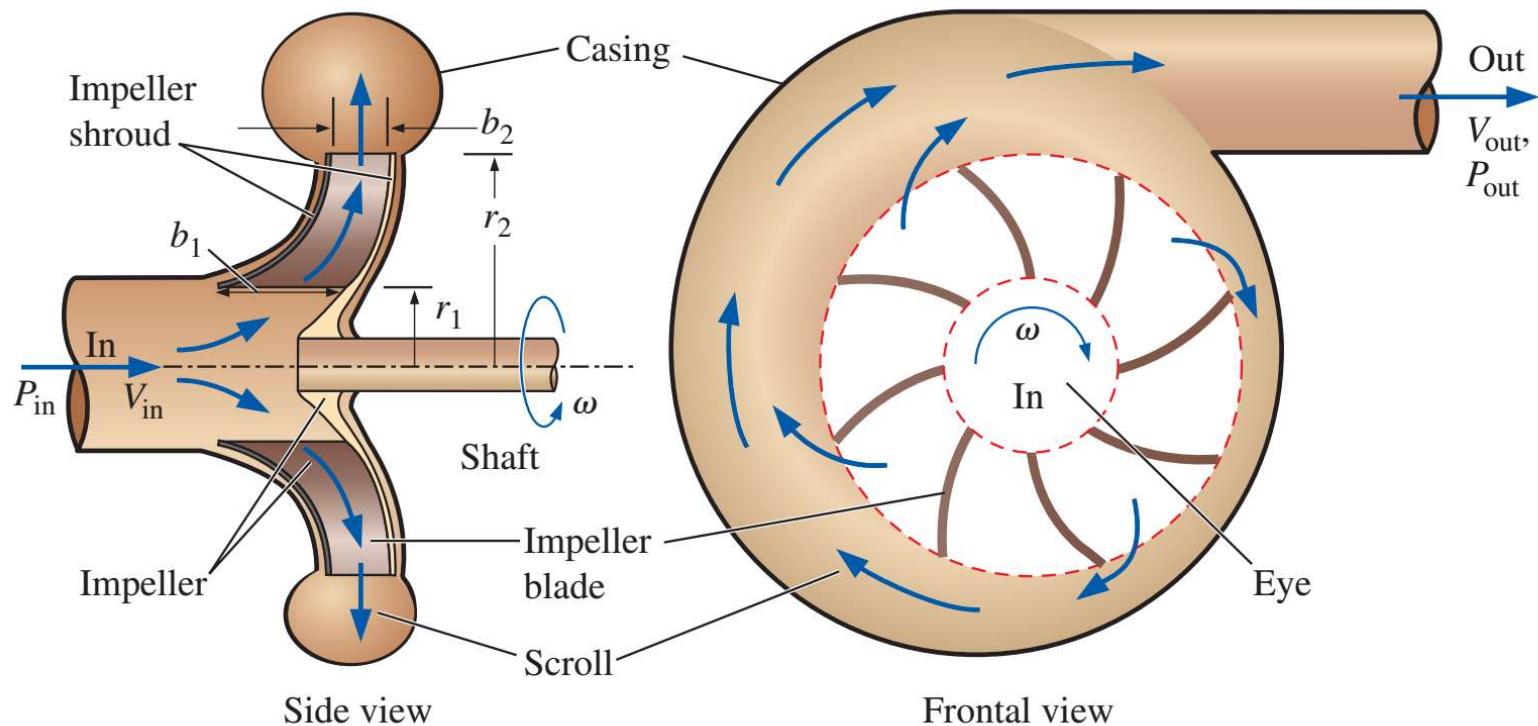
(b) Mixed flow



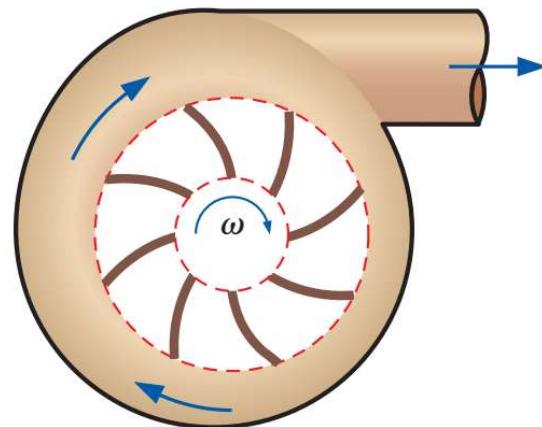
(c) Axial flow

Dynamic Pumps: Centrifugal Flow Pumps

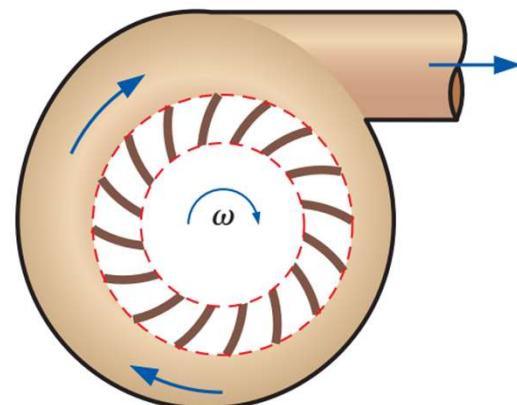
Side view and frontal view of a typical centrifugal pump.



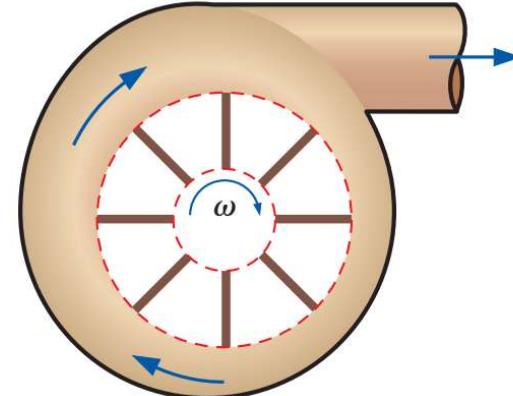
Dynamic Pumps: Centrifugal Flow Pumps



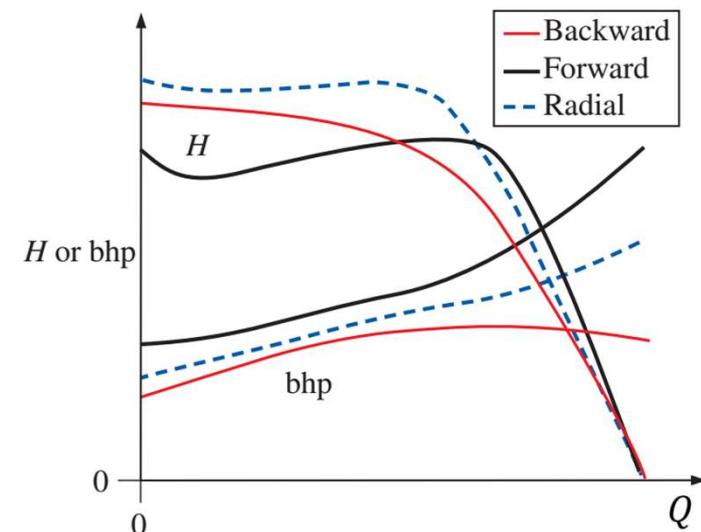
(a) Backward-inclined blades



(c) Forward-inclined blades



(b) Radial blades

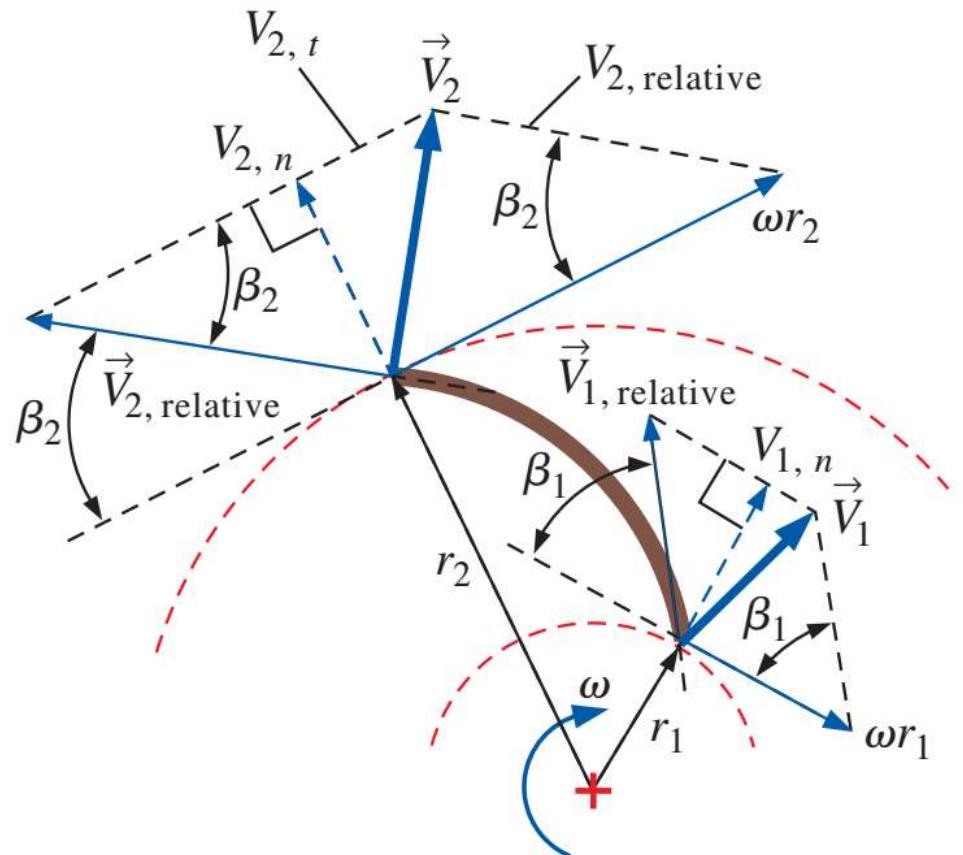
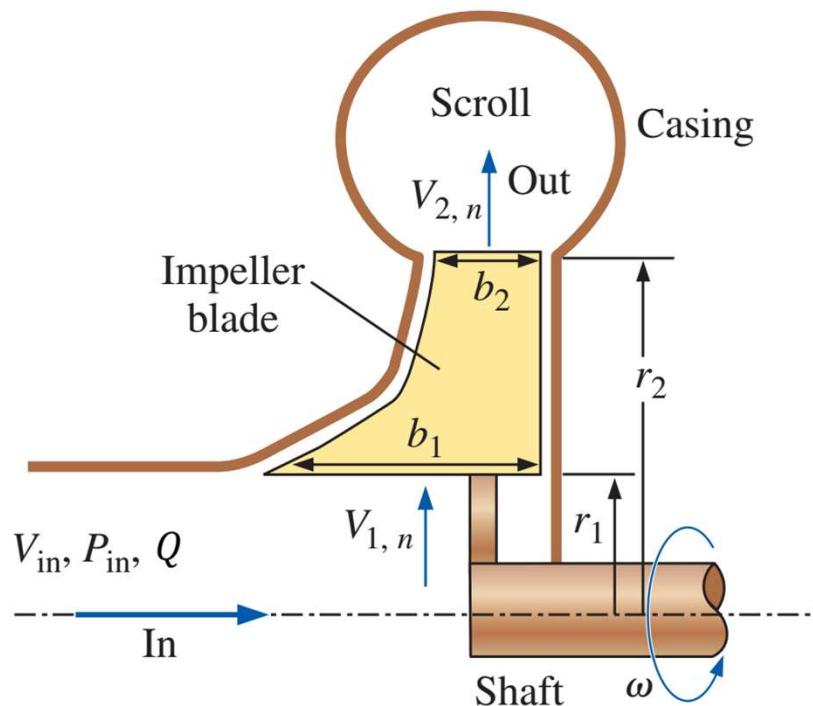


(d) Comparison of net head and brake horsepower performance curves

Dynamic Pumps: Centrifugal Flow Pumps

Volume flow rate:

$$Q = 2\pi r_1 b_1 V_{1,n} = 2\pi r_2 b_2 V_{2,n}$$



$\omega r_1, \omega r_2$: tangential speeds

V_1, V_2 : absolute velocities

$\vec{V}_{1,\text{relative}}, \vec{V}_{2,\text{relative}}$: relative velocities

Close-up frontal view

Dynamic Pumps: Centrifugal Flow Pumps

Euler turbomachine equation:

$$T_{\text{shaft}} = \rho Q (r_2 V_{2,t} - r_1 V_{1,t})$$

$$bhp = \omega T_{\text{shaft}} = \rho g Q H$$

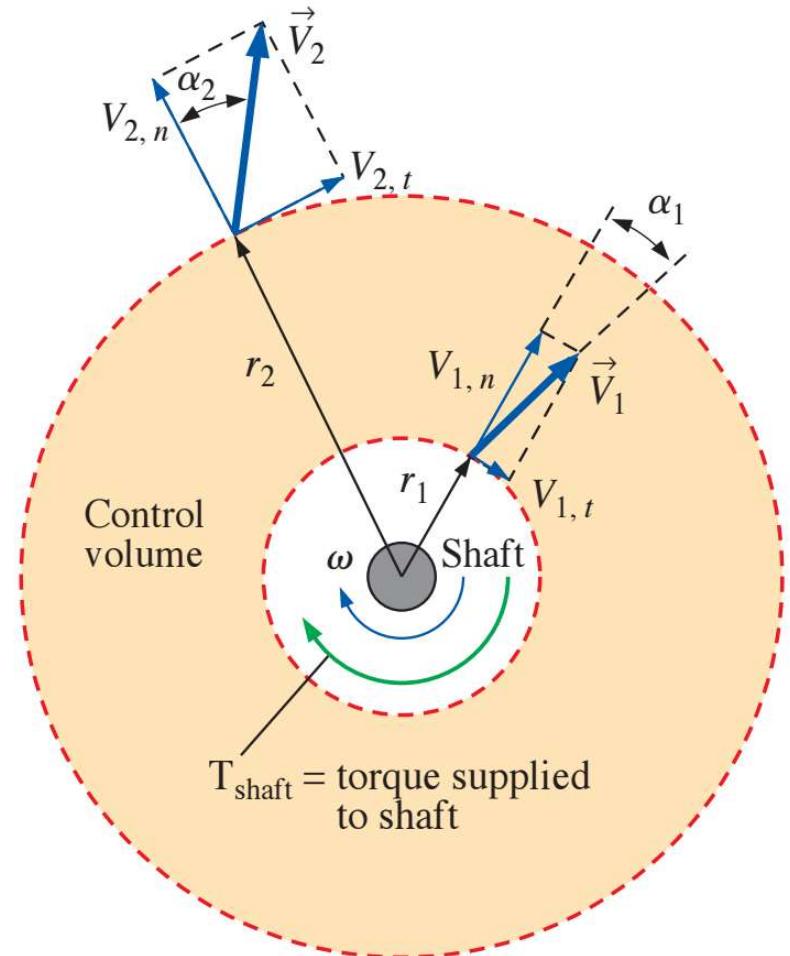
For $\eta_{\text{pump}} = 1 \rightarrow \dot{W}_{\text{water horsepower}} = \rho g Q H$

$V_{1,n}, V_{2,n}$: Normal components of the absolute velocities

$V_{1,t}, V_{2,t}$: tangential components of the absolute velocities

Neat Head:

$$H = \frac{1}{g} (\omega r_2 V_{2,t} - \omega r_1 V_{1,t})$$



Control volume (shaded) used for angular momentum analysis of a centrifugal pump

Dynamic Pumps: Centrifugal Flow Pumps

Example 4

A centrifugal blower rotates at $\dot{n} = 1750$ rpm (183.3 rad/s). Air enters the impeller normal to the blades ($\alpha_1 = 0^\circ$) and exits at an angle of 40° from radial ($\alpha_2 = 40^\circ$) as sketched in Figure E-5. The inlet radius is $r_1 = 4.0$ cm, and the inlet blade width $b_1 = 5.2$ cm. The outlet radius is $r_2 = 8.0$ cm, and the outlet blade width $b_2 = 2.3$ cm. The volume flow rate is $0.13 \text{ m}^3/\text{s}$. For the idealized case, i.e., 100 percent efficiency, calculate the net head produced by this blower in equivalent millimeters of water column height. Also calculate the required brake horsepower in watts.

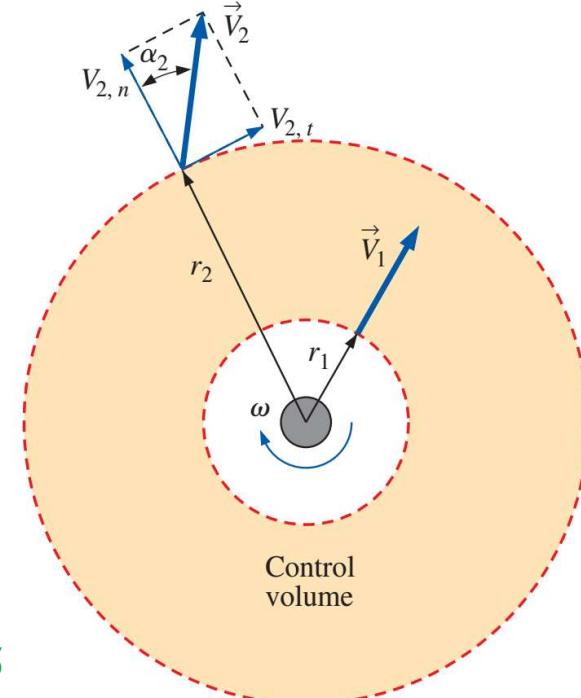
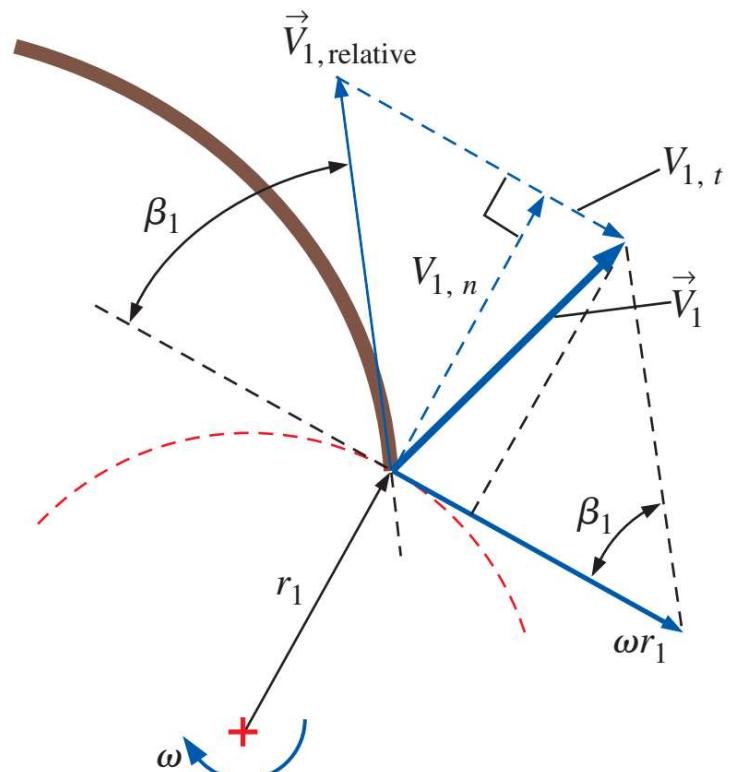
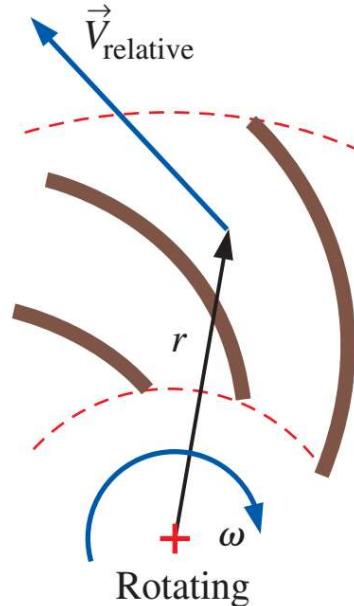
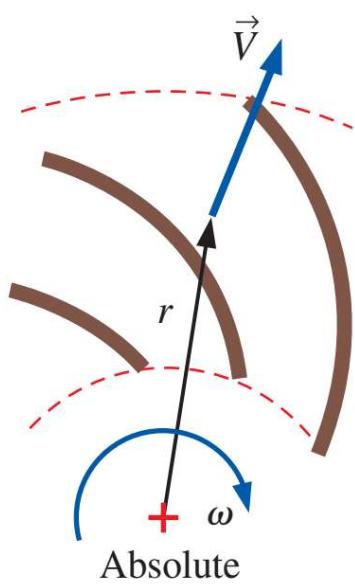


Figure E-5

Dynamic Pumps: Centrifugal Flow Pumps

Bernoulli equation in a rotating reference frame

$$\frac{P}{\rho g} + \frac{V_{\text{relative}}^2}{2g} - \frac{\omega^2 r^2}{2g} + z = \text{constant}$$



Volume flow rate

$$\text{For } V_{1,t} = 0 \text{ and } V_{1,n} = V_1 \rightarrow Q = 2\pi b_1 \omega r_1^2 \tan \beta_1$$

Dynamic Pumps: Centrifugal Flow Pumps

Example 5

A centrifugal pump is being designed to pump liquid refrigerant R-134a at room temperature and atmospheric pressure. The impeller inlet and outlet radii are $r_1 = 100$ and $r_2 = 180$ mm, respectively (Figure E-5). The impeller inlet and outlet widths are $b_1 = 50$ and $b_2 = 30$ mm (into the page of Figure E-5). The pump is to deliver $0.25 \text{ m}^3/\text{s}$ of the liquid at a net head of 14.5 m when the impeller rotates at 1720 rpm. Design the blade shape for the case in which these operating conditions are the design conditions of the pump ($V_{1,t} = 0$, as sketched in the figure); specifically, calculate angles β_1 and β_2 . Also predict the horsepower required by the pump.

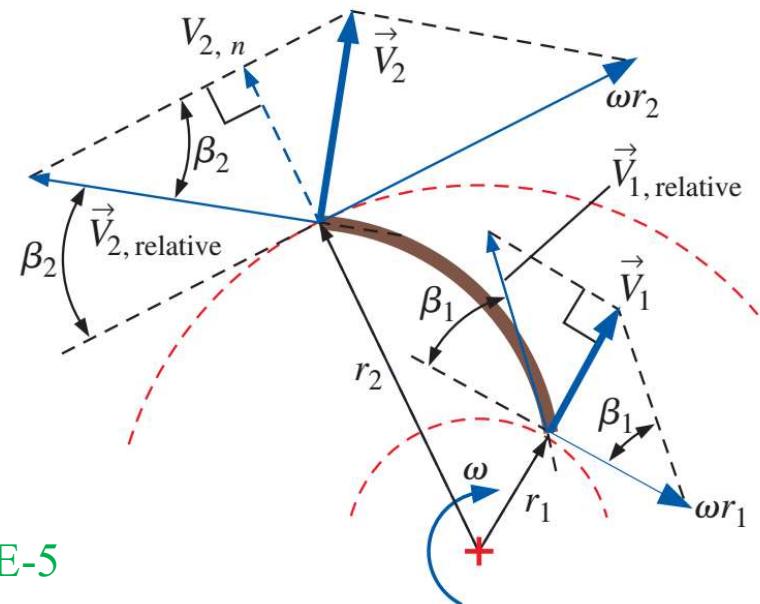


Figure E-5

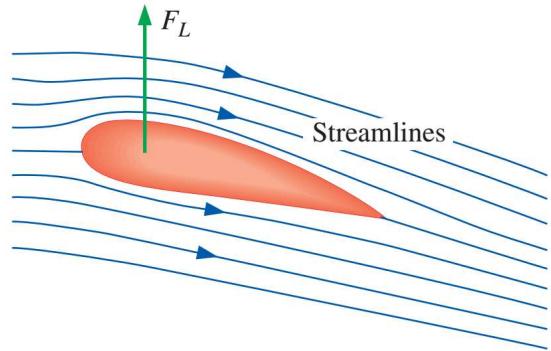
Dynamic Pumps: Centrifugal Flow Pumps

TABLE A-4

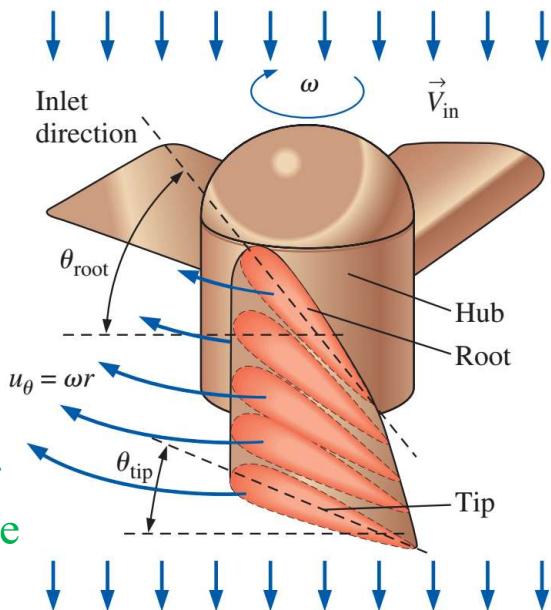
Properties of saturated refrigerant-134a

Temp. <i>T</i> , °C	Saturation Pressure <i>P</i> , kPa	Density <i>ρ</i> , kg/m ³		Enthalpy of Vaporization <i>h_{fg}</i> , kJ/kg		Specific Heat <i>c_p</i> , J/kg·K		Thermal Conductivity <i>k</i> , W/m·K		Dynamic Viscosity <i>μ</i> , kg/m·s		Prandtl Number Pr		Volume Expansion Coefficient <i>β</i> , 1/K	Surface Tension, N/m
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Liquid
-40	51.2	1418	2.773	225.9	1254	748.6	0.1101	0.00811	4.878×10^{-4}	2.550×10^{-6}	5.558	0.235	0.00205	0.01760	
-35	66.2	1403	3.524	222.7	1264	764.1	0.1084	0.00862	4.509×10^{-4}	3.003×10^{-6}	5.257	0.266	0.00209	0.01682	
-30	84.4	1389	4.429	219.5	1273	780.2	0.1066	0.00913	4.178×10^{-4}	3.504×10^{-6}	4.992	0.299	0.00215	0.01604	
-25	106.5	1374	5.509	216.3	1283	797.2	0.1047	0.00963	3.882×10^{-4}	4.054×10^{-6}	4.757	0.335	0.00220	0.01527	
-20	132.8	1359	6.787	213.0	1294	814.9	0.1028	0.01013	3.614×10^{-4}	4.651×10^{-6}	4.548	0.374	0.00227	0.01451	
-15	164.0	1343	8.288	209.5	1306	833.5	0.1009	0.01063	3.371×10^{-4}	5.295×10^{-6}	4.363	0.415	0.00233	0.01376	
-10	200.7	1327	10.04	206.0	1318	853.1	0.0989	0.01112	3.150×10^{-4}	5.982×10^{-6}	4.198	0.459	0.00241	0.01302	
-5	243.5	1311	12.07	202.4	1330	873.8	0.0968	0.01161	2.947×10^{-4}	6.709×10^{-6}	4.051	0.505	0.00249	0.01229	
0	293.0	1295	14.42	198.7	1344	895.6	0.0947	0.01210	2.761×10^{-4}	7.471×10^{-6}	3.919	0.553	0.00258	0.01156	
5	349.9	1278	17.12	194.8	1358	918.7	0.0925	0.01259	2.589×10^{-4}	8.264×10^{-6}	3.802	0.603	0.00269	0.01084	
10	414.9	1261	20.22	190.8	1374	943.2	0.0903	0.01308	2.430×10^{-4}	9.081×10^{-6}	3.697	0.655	0.00280	0.01014	
15	488.7	1244	23.75	186.6	1390	969.4	0.0880	0.01357	2.281×10^{-4}	9.915×10^{-6}	3.604	0.708	0.00293	0.00944	
20	572.1	1226	27.77	182.3	1408	997.6	0.0856	0.01406	2.142×10^{-4}	1.075×10^{-5}	3.521	0.763	0.00307	0.00876	
25	665.8	1207	32.34	177.8	1427	1028	0.0833	0.01456	2.012×10^{-4}	1.160×10^{-5}	3.448	0.819	0.00324	0.00808	
30	770.6	1188	37.53	173.1	1448	1061	0.0808	0.01507	1.888×10^{-4}	1.244×10^{-5}	3.383	0.877	0.00342	0.00742	
35	887.5	1168	43.41	168.2	1471	1098	0.0783	0.01558	1.772×10^{-4}	1.327×10^{-5}	3.328	0.935	0.00364	0.00677	
40	1017.1	1147	50.08	163.0	1498	1138	0.0757	0.01610	1.660×10^{-4}	1.408×10^{-5}	3.285	0.995	0.00390	0.00613	
45	1160.5	1125	57.66	157.6	1529	1184	0.0731	0.01664	1.554×10^{-4}	1.486×10^{-5}	3.253	1.058	0.00420	0.00550	
50	1318.6	1102	66.27	151.8	1566	1237	0.0704	0.01720	1.453×10^{-4}	1.562×10^{-5}	3.231	1.123	0.00456	0.00489	
55	1492.3	1078	76.11	145.7	1608	1298	0.0676	0.01777	1.355×10^{-4}	1.634×10^{-5}	3.223	1.193	0.00500	0.00429	
60	1682.8	1053	87.38	139.1	1659	1372	0.0647	0.01838	1.260×10^{-4}	1.704×10^{-5}	3.229	1.272	0.00554	0.00372	
65	1891.0	1026	100.4	132.1	1722	1462	0.0618	0.01902	1.167×10^{-4}	1.771×10^{-5}	3.255	1.362	0.00624	0.00315	
70	2118.2	996.2	115.6	124.4	1801	1577	0.0587	0.01972	1.077×10^{-4}	1.839×10^{-5}	3.307	1.471	0.00716	0.00261	
75	2365.8	964	133.6	115.9	1907	1731	0.0555	0.02048	9.891×10^{-5}	1.908×10^{-5}	3.400	1.612	0.00843	0.00209	
80	2635.2	928.2	155.3	106.4	2056	1948	0.0521	0.02133	9.011×10^{-5}	1.982×10^{-5}	3.558	1.810	0.01031	0.00160	

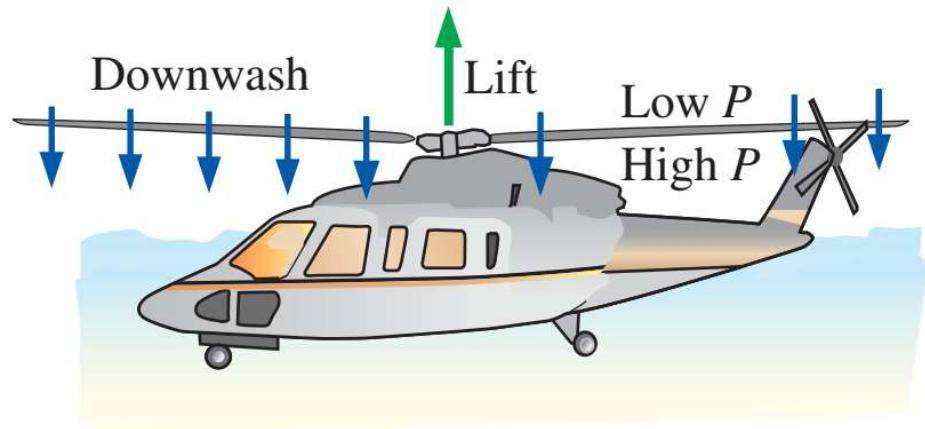
Dynamic Pumps: Axial Flow Pumps



The blades of an axial-flow pump behave like the wing of an airplane.



A well-designed rotor blade or propeller blade has twist



Downwash and pressure rise across the rotor plane of a helicopter, which is a type of axial-flow pump.

Tangential speed of the blade:

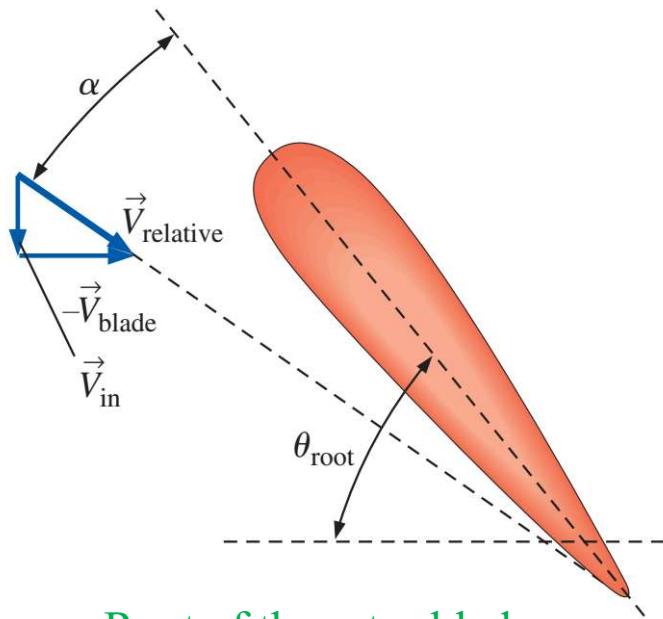
$$u_\theta = \omega r$$

$$\theta_{\text{root}} > \theta_{\text{tip}}$$

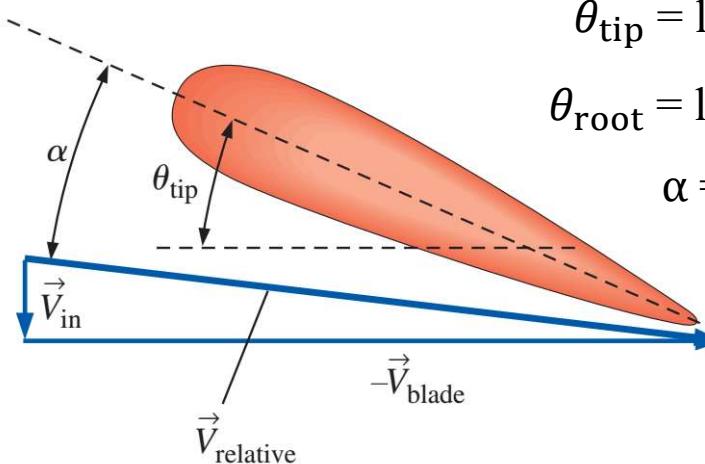
θ : pitch angle

Dynamic Pumps: Axial Flow Pumps

Graphical computation of a vector $\vec{V}_{\text{relative}}$

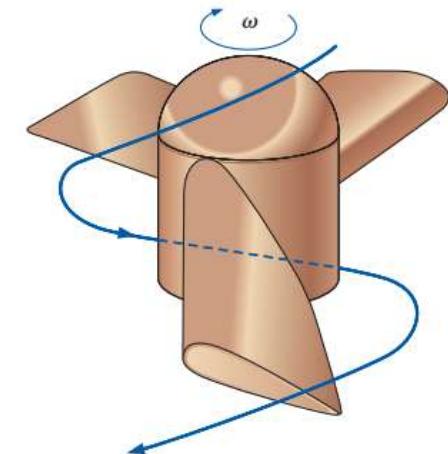


Root of the rotor blade



Tip of the rotor blade

θ_{tip} = local pitch angle at the tip
 θ_{root} = local pitch angle at the root
 α = Angle of attack



\vec{V}_{in} = inflow (or Induced) velocity
 \vec{V}_{blade} = blade element tangential speed
 $\vec{V}_{\text{relative}}$ = velocity of the air relative to the air

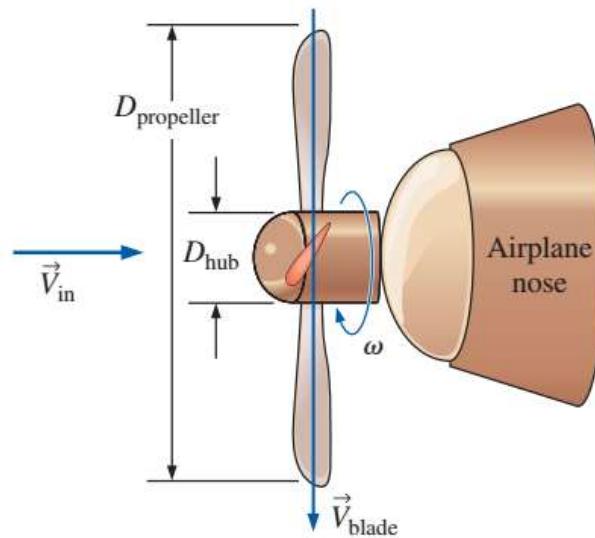
$$\vec{V}_{\text{relative}} \cong \vec{V}_{\text{in}} + (-\vec{V}_{\text{blade}})$$

The rotating blades of a rotor or propeller induce swirl in the surrounding fluid.

Dynamic Pumps: Axial Flow Pumps

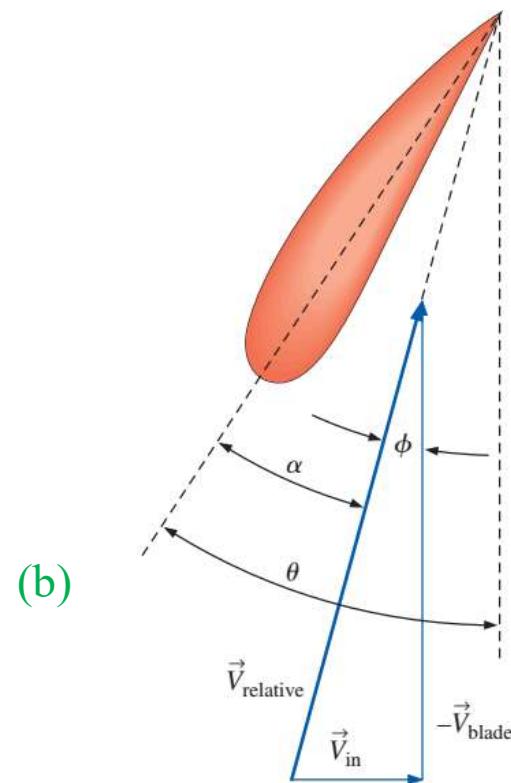
Example 6

Suppose you are designing the propeller of a radio-controlled model airplane. The overall diameter of the propeller is 34.0 cm, and the hub assembly diameter is 5.5 cm (Figure E-6a). The propeller rotates at 1700 rpm, and the airfoil chosen for the propeller cross-section achieves its maximum efficiency at an angle of attack of 14° . When the airplane flies at 30 mi/h (13.4 m/s), calculate the blade pitch angle from the root to the tip of the blade such that $\alpha = 14^\circ$ everywhere along the propeller blade.



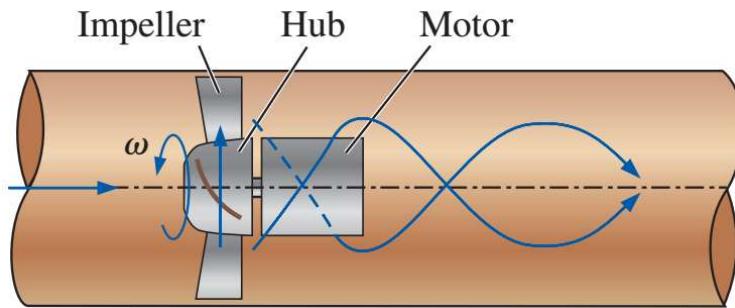
(a)

Figure E-6

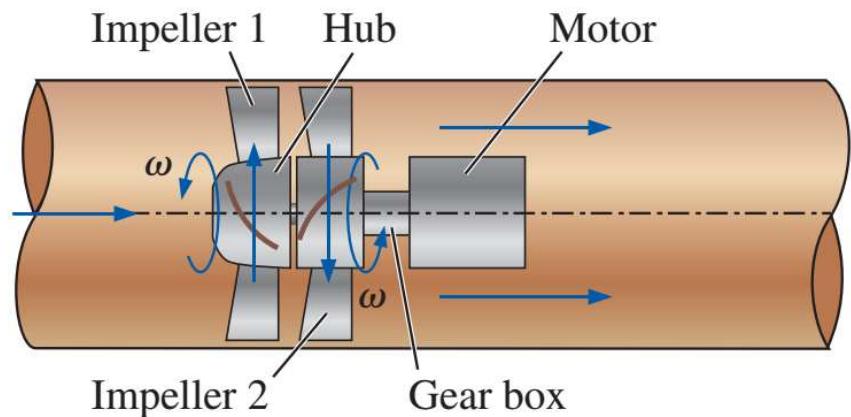


Dynamic Pumps: Axial Flow Pumps

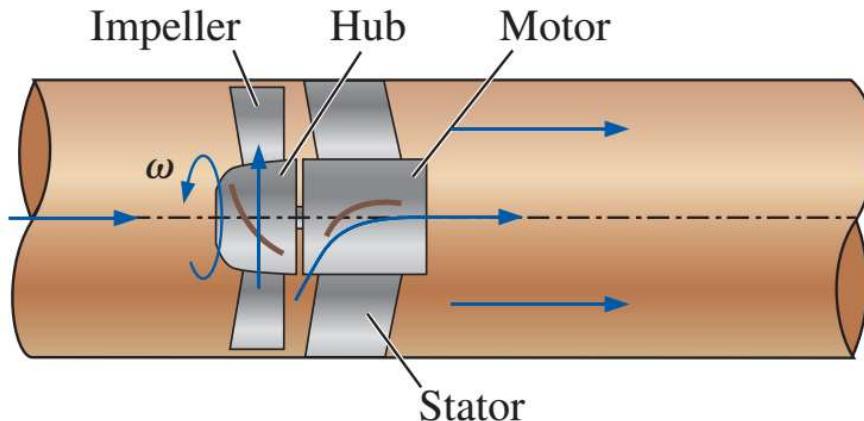
Tube-axial fan



(a) Impart swirl to the exiting fluid



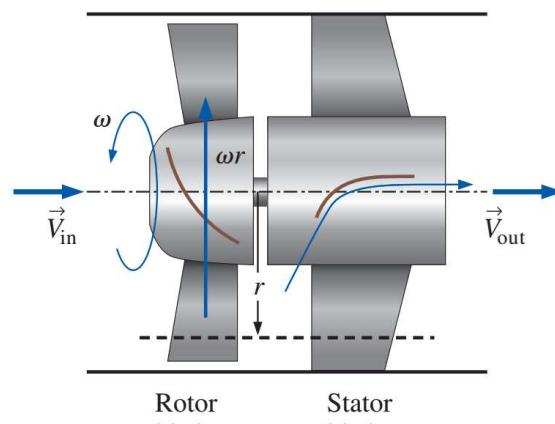
(b) A counter-rotating axial-flow fan



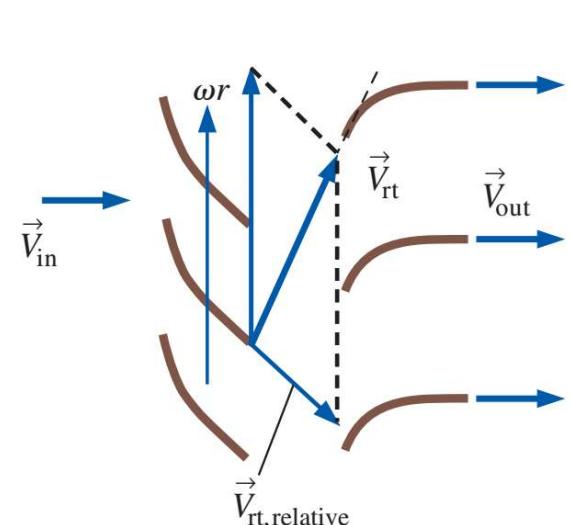
(c) A vane-axial fan

Dynamic Pumps: Axial Flow Pumps

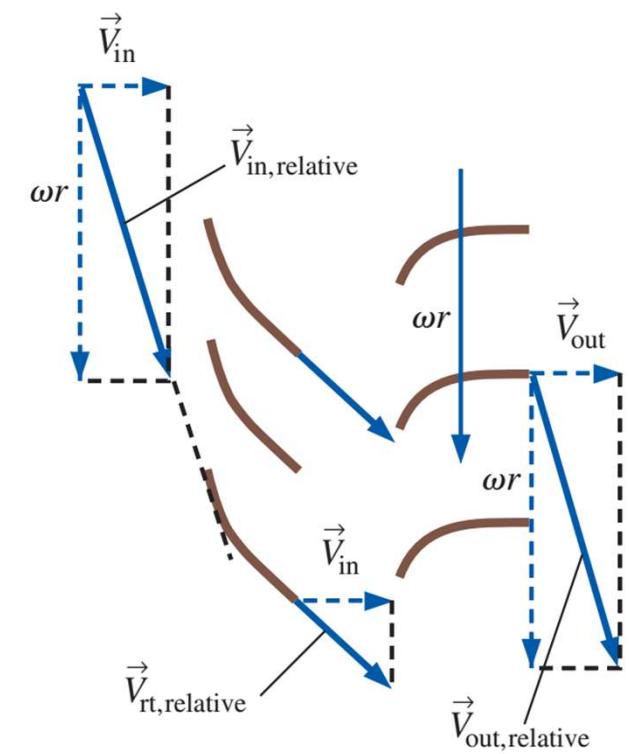
Analysis of a **vane-axial flow fan** at radius r using the two-dimensional blade row approximation



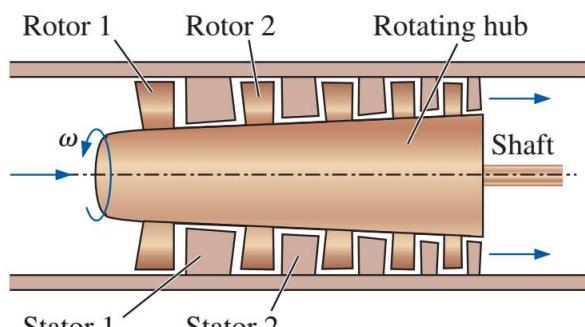
(a) Overall view



(b) Absolute reference frame



(c) Reference frame relative to the rotating rotor blades (impeller)



(d) A multistage axial-flow pump

Dynamic Pumps: Axial Flow Pumps

Example 7

A vane-axial flow fan is being designed to power a wind tunnel. There must not be any swirl in the flow downstream of the fan. It is decided that the stator blades should be upstream of the rotor blades (Figure E-7) to protect the impeller blades from damage by objects that might accidentally get blown into the fan. To reduce expenses, both the stator and rotor blades are to be constructed of sheet metal. The leading edge of each stator blade is aligned axially ($\beta_{sl} = 0.0^\circ$) and its trailing edge is at angle $\beta_{st} = 60.0^\circ$ from the axis as shown in the sketch. (The subscript notation “sl” indicates stator leading edge and “st” indicates stator trailing edge.) There are 16 stator blades. At design conditions, the axial-flow speed through the blades is 47.1 m/s, and the impeller rotates at 1750 rpm. At radius $r = 0.40$ m, calculate the leading and trailing edge angles of the rotor blade, and sketch the shape of the blade. How many rotor blades should there be?

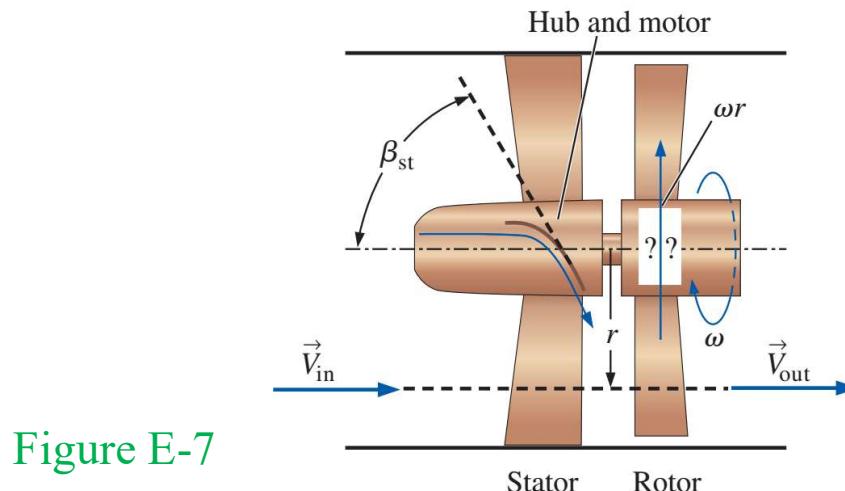


Figure E-7

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 - 2.5. Dynamic Pumps
 - 2.6. Centrifugal Pumps
 - 2.7. Axial Pumps

3. Pump Scaling Laws

Pump Scaling Laws

Dimensionless pump parameter:

$$C_H = f(Q, D, \varepsilon, \omega, \rho, \mu)$$

Π : Dimensional parameter (or group)

$$k = n - j = 7 - 3 = 4 \text{ } \Pi's \text{ expected}$$

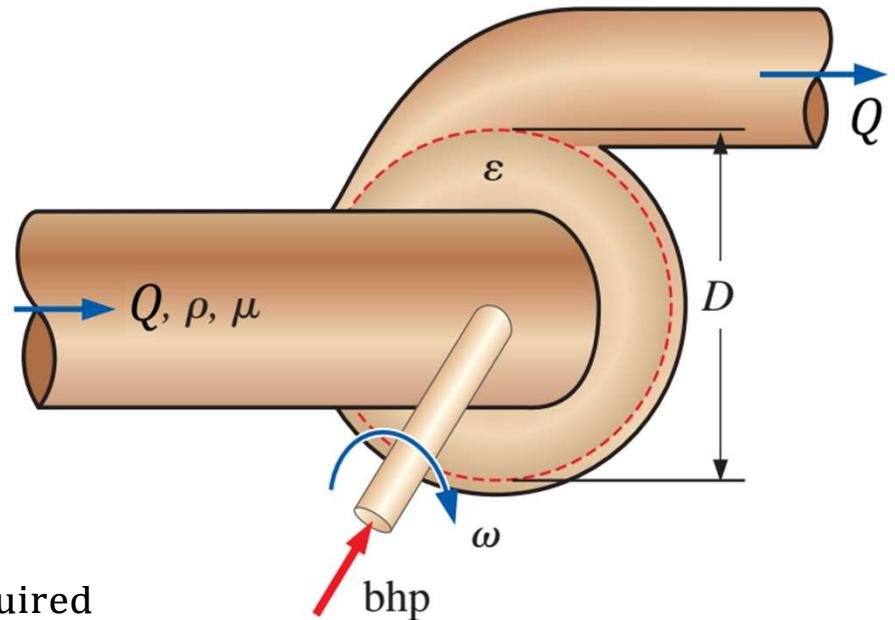
$$C_H = \text{Head coefficient} = \frac{gH}{\omega^2 D^2}$$

$$C_Q = \text{Capacity coefficient} = \frac{Q}{\omega D^3}$$

$$C_P = \text{Power coefficient} = \frac{\text{bhp}}{\rho \omega^3 D^5}$$

$$C_{NPSH} = \text{Suction head coefficient} = \frac{g \text{ NPSH}_{\text{required}}}{\omega^2 D^2}$$

Pump Efficiency: $\eta_{\text{pump}} = \frac{C_Q C_H}{C_P}$



Dimensional analysis of a pump.

Pump Scaling Laws

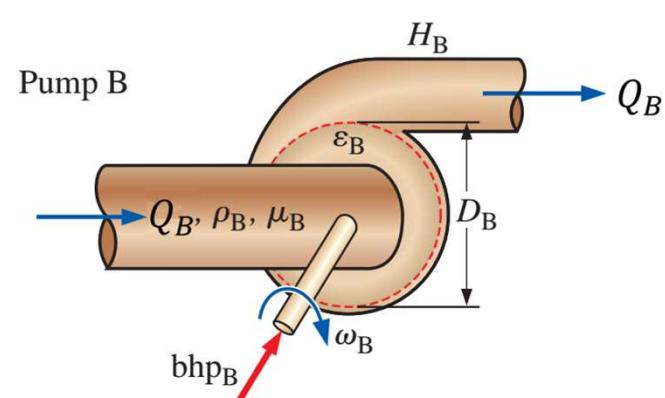
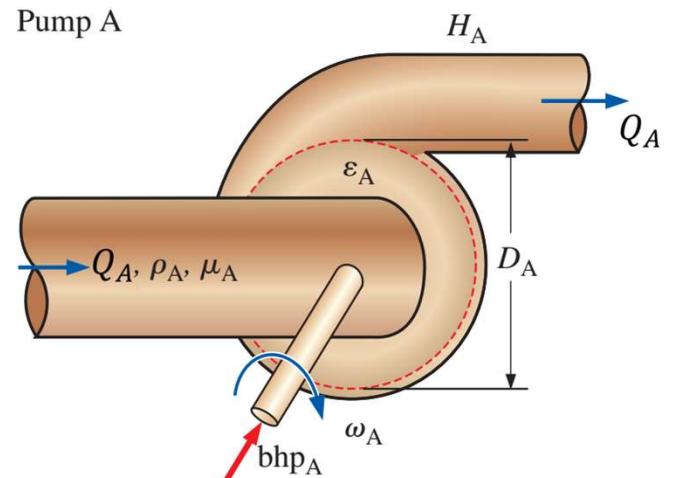
Dimensional analysis is useful for scaling two **geometrically similar** pumps. If all the dimensionless pump parameters of pump A are equivalent to those of pump B, the two pumps are **dynamically similar**.

Similarity rules:

$$\frac{Q_B}{Q_A} = \frac{\omega_B}{\omega_A} \left(\frac{D_B}{D_A} \right)^3$$

$$\frac{H_B}{H_A} = \left(\frac{\omega_B}{\omega_A} \right)^2 \left(\frac{D_B}{D_A} \right)^2$$

$$\frac{\text{bhp}_B}{\text{bhp}_A} = \frac{\rho_B}{\rho_A} \left(\frac{\omega_B}{\omega_A} \right)^3 \left(\frac{D_B}{D_A} \right)^5$$



Pump Scaling Laws

Moody efficiency correction equation for pumps

$$\eta_{\text{pump,p}} \approx 1 - (1 - \eta_{\text{pump,m}}) \left(\frac{D_m}{D_p} \right)^{1/5}$$

Pump Specific Speed

$$N_{\text{Sp}} = \frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{\omega Q^{1/2}}{(gH)^{3/4}}$$

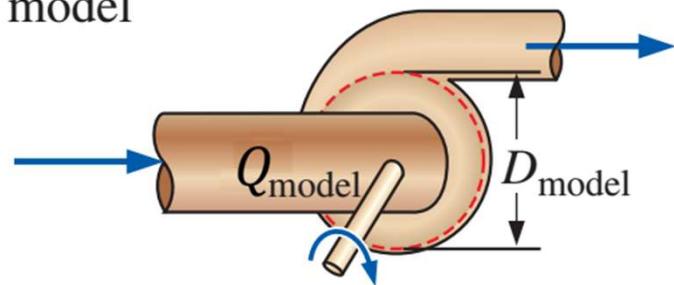
Customary U.S. units:

$$N_{\text{Sp,US}} = \frac{(\dot{n}, \text{rpm})(Q, \text{gpm})^{1/2}}{(H, \text{ft})^{3/4}}$$

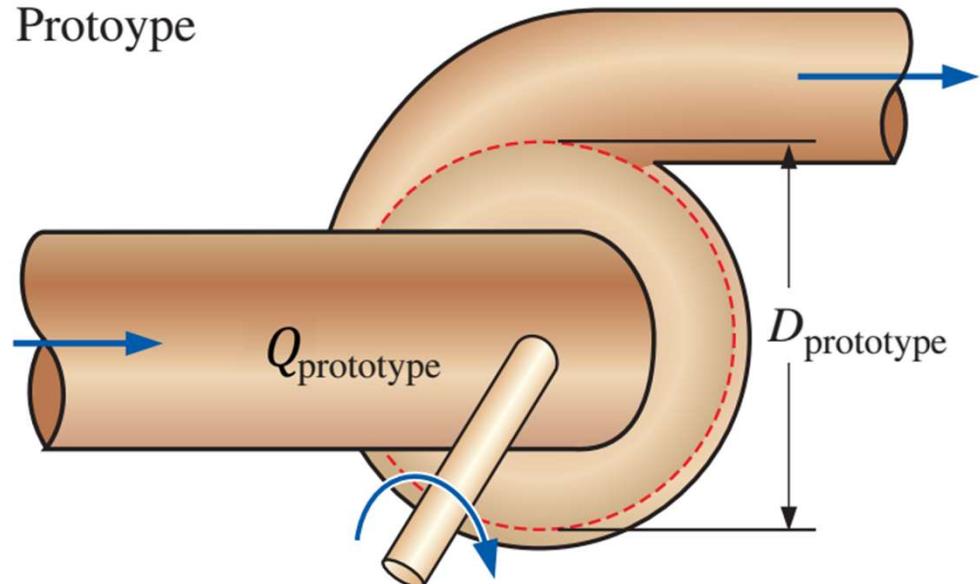
Customary European units:

$$N_{\text{Sp,US}} = \frac{(\dot{n}, \text{Hz})(Q, \text{m}^3/\text{s})^{1/2}}{(gH, \text{m}^2/\text{s}^2)^{3/4}}$$

Scale model



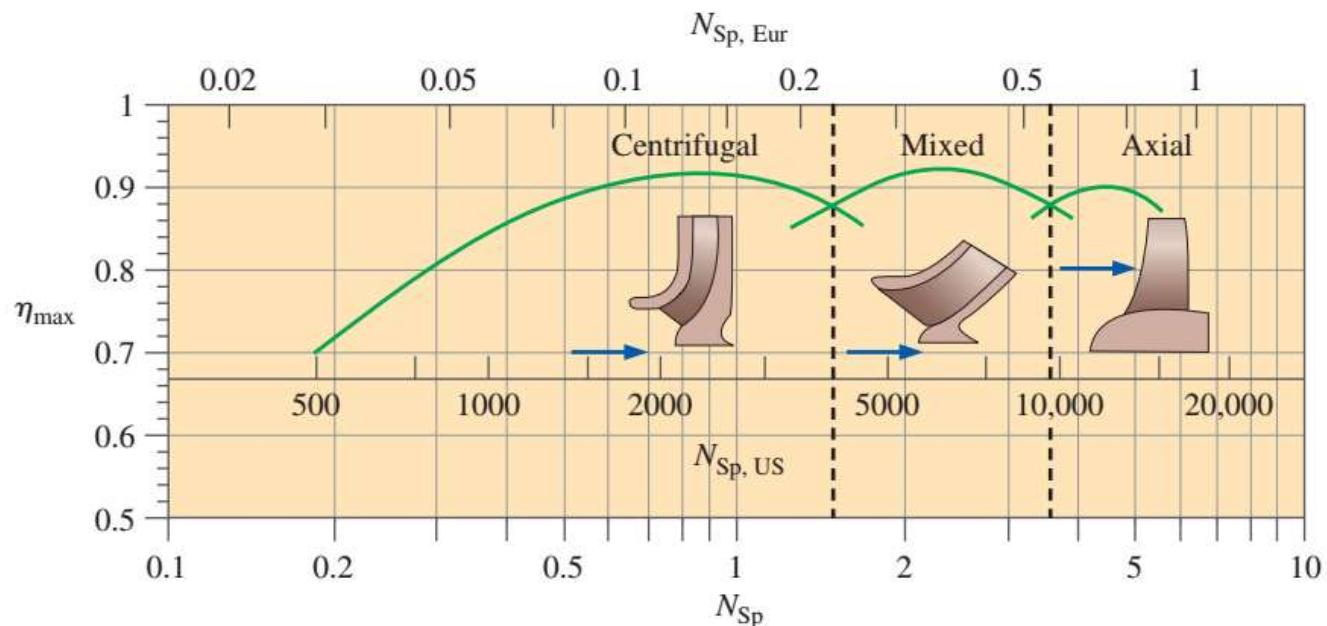
Prototype



Dynamic Pumps: Pump Scaling Laws

Example 8

A pump is being designed to deliver 320 gpm of gasoline at room temperature. The required net head is 23.5 ft (of gasoline). It has already been determined that the pump shaft is to rotate at 1170 rpm. Calculate the pump specific speed in both nondimensional form and customary U.S. form. Based on your result, decide which kind of dynamic pump would be most suitable for this application.



References

- [1] Cengel Y., Cimbala, J. (2014). Fluid Mechanics: Fundamentals and Applications (3th Edition). New York: NY: McGraw-Hill Co.
- [2] Munson, B.R., Young, D.F., Okiishi, T.H., and Huebsch, W.W. (2016). Fundamentals of Fluid Mechanics (8th Edition). John Wiley & Sons. ISBN 1119080703.