Efficiency of Reversible vs. Irreversible Cycles

A Thermodynamic Demonstration (Expanded)

Generated by AI

May 6, 2025

Introduction: What is Thermal Efficiency?

The thermal efficiency (η) of a heat engine measures how effectively it converts heat into work. It's defined as the ratio of the net work done

(W) by the engine to the heat absorbed (Q_H) from the high-temperature reservoir:

$$\eta = \frac{W}{Q_H}$$

Understanding Efficiency

A higher efficiency (η) means that a larger fraction of the absorbed heat energy is converted into useful work. Our goal is to demonstrate a

fundamental principle: A reversible engine (R) is always more efficient

than an irreversible engine (I) when both operate between the same two temperature reservoirs.

The Scenario: Two Heat Engines

We consider two types of heat engines:

- Engine R: A Reversible engine.
- Engine I: An Irreversible engine.

Both engines operate between the same two thermal reservoirs:

- A hot reservoir at temperature T_H.
- A cold reservoir at temperature T_C.

Visualizing the Setup

The diagram shows Engine I and Engine R. Initially, imagine both operating as standard heat engines.

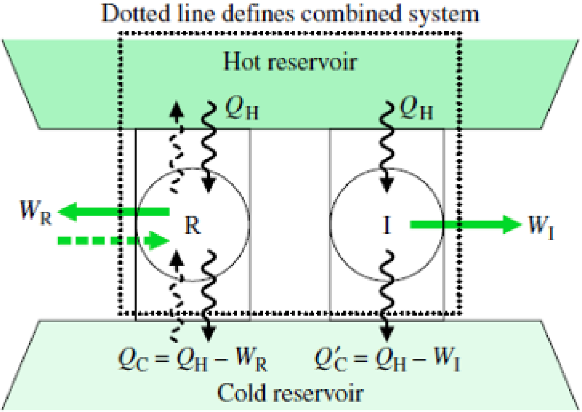


Figure: Engine I (Irreversible) and Engine R (Reversible) operating between two reservoirs.

Engine Operation (Forward Mode)

Let's assume both engines absorb the same amount of heat Q_H from the hot reservoir when running forwards. **Engine I (Irreversible):**

- Absorbs heat Q_H from hot reservoir.
- Produces work W_I.
- Rejects heat $Q'_C = Q_H - W_I$ to cold reservoir.
- Efficiency: $\eta_I = W_I / Q_H$.

Engine R (Reversible):

- Absorbs heat Q_H from hot reservoir.
- Produces work W_R.
- Rejects heat $Q_C = Q_H - W_R$ to cold reservoir.
- Efficiency: $\eta_R = W_R / Q_H$.

The Core Assumption (for Contradiction)

To prove our point, we start with an assumption that we will later show is false. Let's assume that the **irreversible engine (I) is more efficient**

than the reversible engine (R):

$$\eta_I > \eta_R$$

Implication of the Assumption

If $\eta_I > \eta_R$, and both engines absorb the same heat Q_H:

$$\frac{W_I}{Q_H} > \frac{W_R}{Q_H}$$

This directly implies that the work produced by Engine I is greater than the work produced by Engine R:

$$W_I > W_R$$

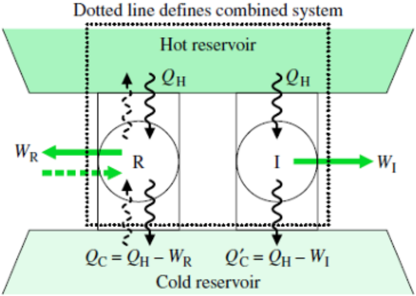
Strategy: Constructing a Combined System

Our goal is to show that the assumption $W_I > W_R$ (and thus $\eta_I > \eta_R$) leads to a violation of a fundamental law of physics. We will do this by

creating a special combined system using Engine I and Engine R.

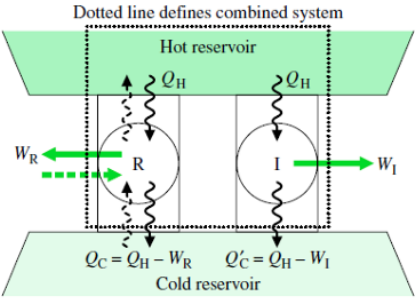
Step 1: Operate Engine I as a normal heat engine.

- It absorbs Q_H from the hot reservoir.
- It produces work W_I.
- It rejects heat $Q'_C = Q_H - W_I$ to the cold reservoir.

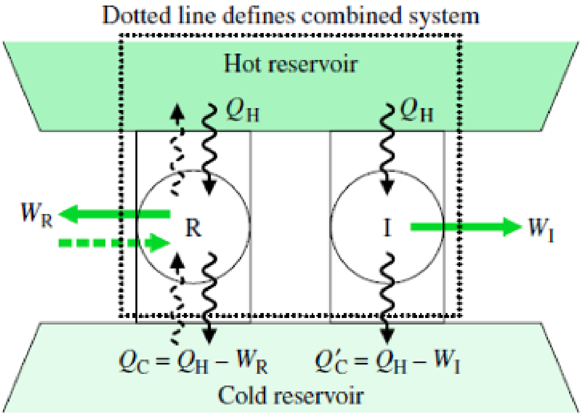


Step 2: Operate Engine R in reverse (as a refrigerator).

- Since it's reversible, it can be run backwards.
- It will absorb heat $Q_C = Q_H - W_R$ from the cold reservoir.
- It will require a work input of W_R.
- It will reject heat Q_H to the hot reservoir.



Now, we couple these two engines: The work W_I produced by Engine I is used to drive the reversed Engine R, which requires work W_R. The "dotted line" in the figure below defines our combined system.



Analyzing the Combined System: Net Work

Let's find the net work output (W_{net}) of this combined system. Engine I produces work W_I. Reversed Engine R consumes work W_R. So,

$W_{net} = W_I - W_R$.

Analyzing the Combined System: Net Work (Continued)

Recall our assumption: $W_I > W_R$. Therefore, the net work output of the combined system is positive:

$$W_{net} = W_I - W_R > 0$$

This means the combined system, as a whole, produces useful work.

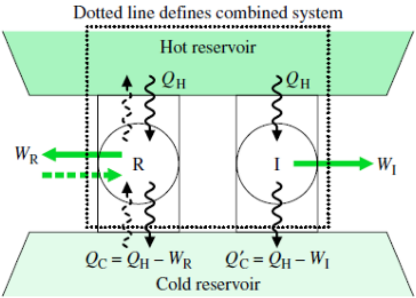
Analyzing the Combined System: Hot Reservoir

What is the net heat exchange with the **hot reservoir**?

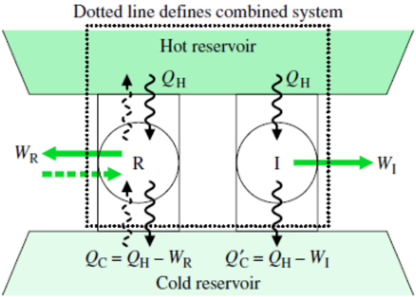
- Engine I **absorbs** Q_H from the hot reservoir.
- Reversed Engine R **rejects** Q_H to the hot reservoir.

Net heat from hot reservoir: $Q_{H,net} = Q_H(\text{in}) - Q_H(\text{out}) = 0$. There is **no net heat exchange** with the hot reservoir.

What is the net heat exchange with the **cold reservoir**? Engine I **rejects** heat $Q'_C = Q_H - W_I$ to the cold reservoir.



Reversed Engine R **absorbs** heat $Q_C = Q_H - W_R$ from the cold reservoir.



Net heat absorbed from cold reservoir:

$$Q_{cold,net} = Q_C(\text{absorbed by R}) - Q'_C(\text{rejected by I})$$

Analyzing the Combined System: Cold Reservoir - Part 3

Calculating the net heat from the cold reservoir:

$$Q_{cold,net} = (Q_H - W_R) - (Q_H - W_I)$$

$$Q_{cold,net} = Q_H - W_R - Q_H + W_I$$

$$Q_{cold,net} = W_I - W_R$$

Since we assumed $W_I > W_R$, this means $Q_{cold,net}$ is positive. The combined system **absorbs a net amount of heat** from the cold reservoir.

Summary: Behavior of the Combined System

Let's summarize what our combined system (within the dotted line) does:

1. Produces net useful work: $W_{net} = W_I - W_R > 0$.
2. Has no net heat exchange with the hot reservoir ($Q_{H,net} = 0$).
3. Absorbs net heat from the cold reservoir: $Q_{cold,net} = W_I - W_R > 0$.

The Critical Outcome

Notice that $W_{net} = Q_{cold,net}$. This means our combined system takes a certain amount of heat ($Q_{cold,net}$) from the **cold reservoir** and converts it **entirely into work** (W_{net}). It does this with no other effects (like rejecting heat to another reservoir, as the net heat with the hot reservoir is zero).

The Contradiction: Kelvin-Planck Statement

This behavior directly violates the **Kelvin-Planck statement** of the Second Law of Thermodynamics. The Kelvin-Planck statement says: *"It is*

impossible to construct a device which operates in a cycle and produces no effect other than the raising of a weight (i.e., doing work) and the exchange of heat with a single reservoir."

Our combined system:

- Operates in a cycle (as its components do).
- Produces net work ($W_{net} > 0$).
- Exchanges heat with effectively only a **single reservoir** (it only takes net heat from the cold reservoir; no net exchange with the hot one).

This is precisely what the Kelvin-Planck statement declares impossible!

Revisiting the Assumption

Our initial assumption was $\eta_I > \eta_R$. This assumption led us to construct a hypothetical combined system that violates a fundamental law of nature (the Second Law of Thermodynamics). Therefore, the initial assumption must be **false**.

The Correct Conclusion (Carnot's Theorem - Part 1)

Since the assumption $\eta_I > \eta_R$ is false, the reality must be:

$$\eta_I \leq \eta_R$$

The efficiency of an irreversible engine cannot be greater than the

efficiency of a reversible engine operating between the same two reservoirs. This is a key part of Carnot's Theorem.

We've established $\eta_I \leq \eta_R$.

- If Engine I is indeed **irreversible** (as stated) and Engine R is reversible, it can be rigorously shown that the inequality is strict:

$$\eta_I < \eta_R$$

An irreversible engine is *always less efficient* than a reversible one.

What if we compared two *reversible* engines, R1 and R2, operating between the same reservoirs?

- Using the same logic, we could show $\eta_{R1} \leq \eta_{R2}$.
- By swapping their roles, we could show $\eta_{R2} \leq \eta_{R1}$.

The only way both are true is if $\eta_{R1} = \eta_{R2}$. This means: **All reversible engines operating between the same two temperature reservoirs have the same maximum possible efficiency** (known as the Carnot efficiency).

Final Summary: The Supremacy of Reversible Cycles

Combining these insights:

1. No engine can be more efficient than a reversible engine ($\eta_I \leq \eta_R$).
2. Any irreversible engine is strictly less efficient than a reversible engine ($\eta_I < \eta_R$).
3. All reversible engines between the same two reservoirs share the same maximum efficiency.

Therefore, a **reversible cycle (R) represents the most efficient thermodynamic cycle possible** when operating between any two given thermal reservoirs.