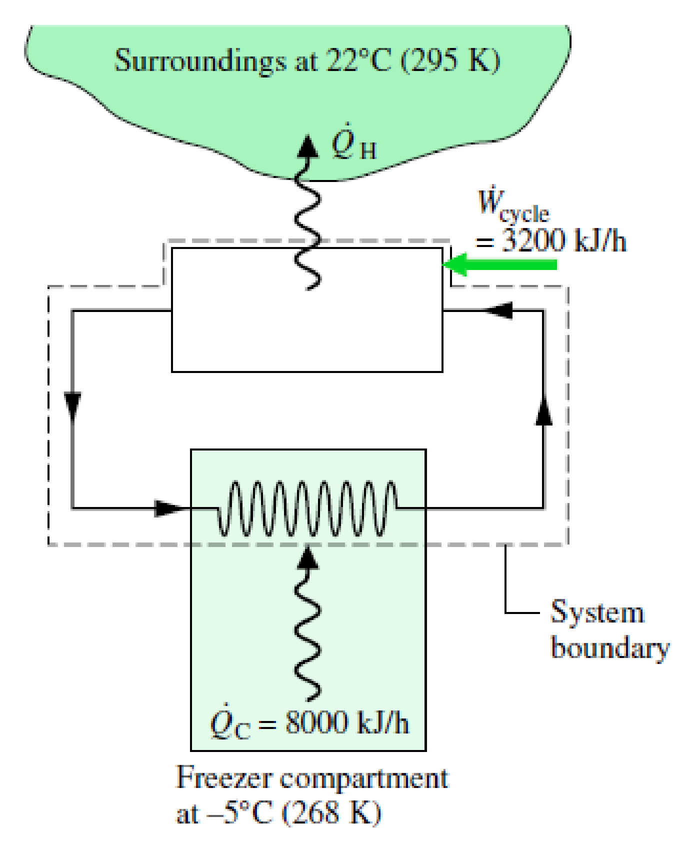
By steadily circulating a refrigerant at low temperature through passages in the walls of the freezer compartment, a refrigerator maintains the freezer compartment at −5°C when the air surrounding the refrigerator is at 22°C. The rate of heat transfer from the freezer compartment to the refrigerant is 8000 kJ/h and the power input required to operate the refrigerator is 3200 kJ/h. Determine the coefficient of performance of the refrigerator and compare with the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at the same two temperatures.

Schematic and Given Data:



Assumptions:

1. The system shown on the accompanying figure is at steady state.
2. The freezer compartment and the surrounding air play the roles of cold and hot reservoirs, respectively.

SOLUTION

Known: A refrigerator maintains a freezer compartment at a specified temperature. The rate of heat transfer from the refrigerated space, the power input to operate the refrigerator, and the ambient temperature are known.

Find: Determine the coefficient of performance and compare with that of a reversible refrigerator operating between reservoirs at the same two temperatures.

Analysis: Inserting the given operating data into Eq. 5.3, the coefficient of performance of the refrigerator is

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{8000 \text{ kJ/h}}{3200 \text{ kJ/h}} = 2.5$$

Substituting values into Eq. 5.9 gives the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at $T_C = 268$ K and $T_H = 295$ K

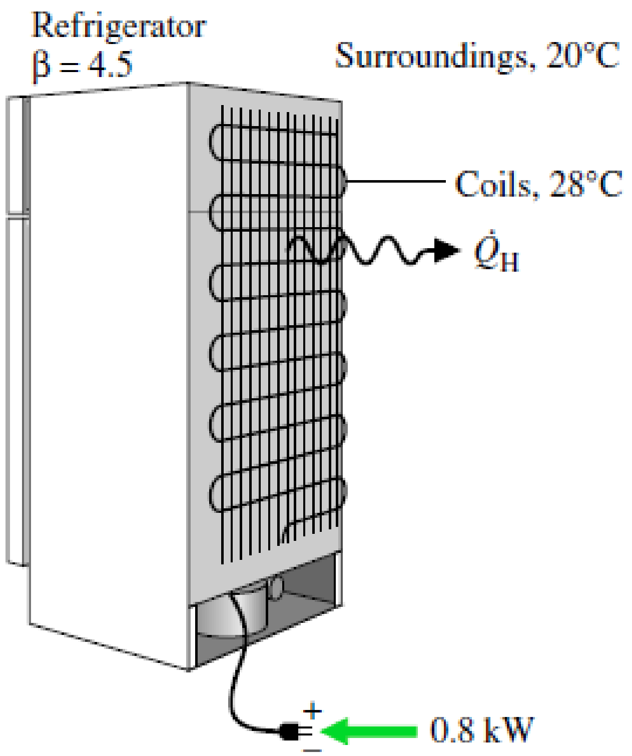
$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{268 \text{ K}}{295 \text{ K} - 268 \text{ K}} = 9.9$$

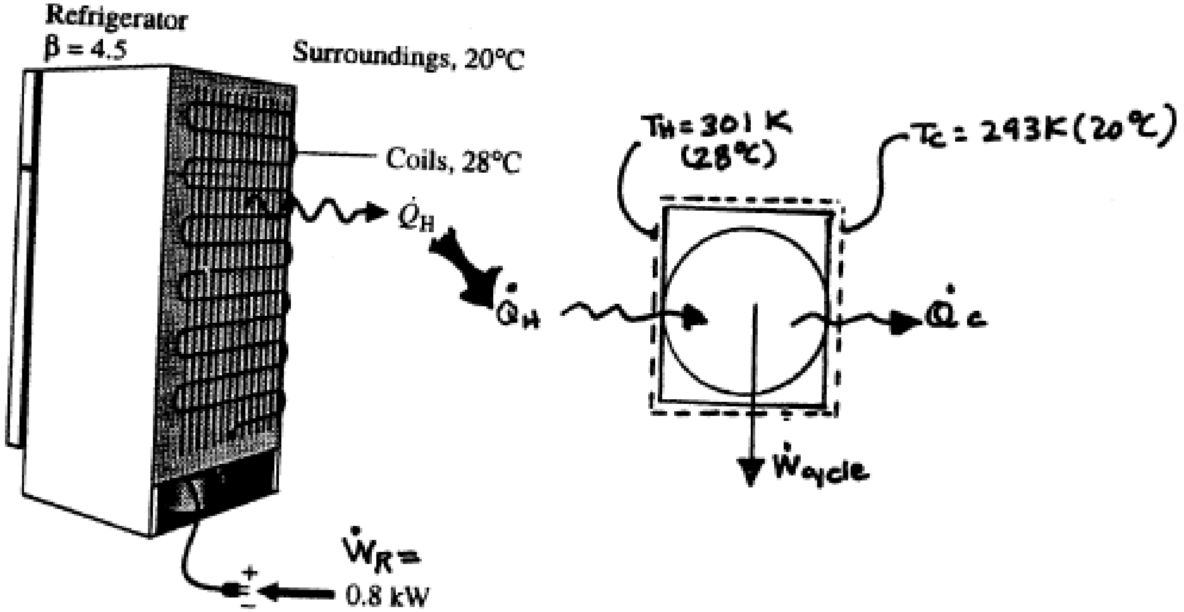
5.35 The refrigerator shown in Fig. P5.35 operates at steady state with a coefficient of performance of 4.5 and a power input of 0.8 kW. Energy is rejected from the refrigerator to the surroundings at 20°C by heat transfer from metal coils whose average surface temperature is 28°C. Determine

(a) the rate energy is rejected, in kW.

(b) the lowest theoretical temperature *inside* the refrigerator, in K.

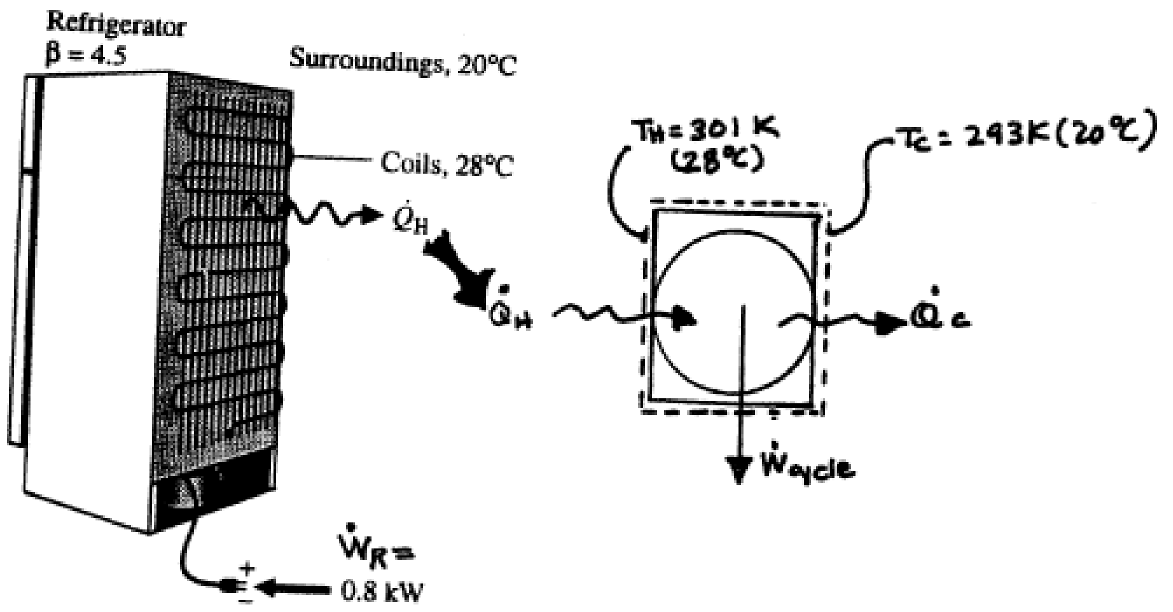
(c) the maximum theoretical power, in kW, that could be developed by a power cycle operating between the coils and





Assumptions for the Analysis:

1. The refrigerator operates at a steady state.
2. For part (b), the interior of the refrigerator and the surroundings act as the cold and hot reservoirs, respectively.
3. For part (c), the coils and surroundings act as the hot and cold bodies, respectively.



(a) Rate of Heat Rejection to the Surroundings ($\dot{Q}_H$)

1. **Coefficient of Performance (COP):** The COP (β) of a refrigerator is defined as the ratio of the rate of heat absorbed from the refrigerated space ($\dot{Q}_{in}$) to the power input ($\dot{W}_R$).

   - Formula: $\beta = \frac{\dot{Q}_{in}}{\dot{W}_R}$

2. **Energy Balance:** An energy balance for the refrigerator states that the rate of heat rejected to the surroundings ($\dot{Q}_H$) is the sum of the rate of heat absorbed from the refrigerated space ($\dot{Q}_{in}$) and the power input ($\dot{W}_R$).

   - Formula: $\dot{Q}_H = \dot{Q}_{in} + \dot{W}_R$

3. **Combining Formulas:** By substituting $\dot{Q}_{in}$ from the COP formula into the energy balance equation, we can solve for $\dot{Q}_H$:

   - From COP: $\dot{Q}_{in} = \beta \cdot \dot{W}_R$
   - Substituting: $\dot{Q}_H = (\beta \cdot \dot{W}_R) + \dot{W}_R = (1 + \beta)\dot{W}_R$

4. **Calculation:** Given $\beta = 4.5$ and $\dot{W}_R = 0.8$ kW:

   - $\dot{Q}_H = (1 + 4.5) \cdot 0.8 \text{ kW} = 5.5 \cdot 0.8 \text{ kW} = 4.4 \text{ kW}$
   - Therefore, the rate at which energy is rejected from the refrigerator to the surroundings by heat transfer is 4.4 kW.

(b) Lowest Theoretical Temperature Inside the Refrigerator (T_C)

1. **Maximum COP (β_{MAX}):** The actual COP (β) must be less than or equal to the maximum theoretical COP (β_{MAX}) for a reversible refrigeration cycle operating between two temperatures. The β_{MAX} is given by the formula involving the absolute temperatures of the hot reservoir (T_H) and the cold reservoir (T_C). In this case, the surroundings are the hot reservoir ($T_H = 20°\text{C} = 293$ K), and T_C is the temperature inside the refrigerator.

   - Formula: $\beta \leq \beta_{MAX} = \frac{T_C}{T_H - T_C}$

2. **Solving for T_C:**

   - Given $\beta = 4.5$ and $T_H = 293$ K (20°C).
   - $4.5 \leq \frac{T_C}{293 \text{ K} - T_C}$
   - $4.5 \cdot (293 \text{ K} - T_C) \leq T_C$
   - $4.5 \cdot 293 \text{ K} - 4.5 \cdot T_C \leq T_C$
   - $1318.5 \text{ K} \leq 5.5 \cdot T_C$
   - $T_C \geq \frac{1318.5 \text{ K}}{5.5}$
   - $T_C \geq 239.7$ K

3. **Result:** The lowest theoretical temperature inside the refrigerator is 239.7 K.

   - It is noted that the surroundings at 20°C (293 K) act as the natural hot reservoir, not the coil temperature.

(c) Maximum Theoretical Power Developed by a Power Cycle ($\dot{W}_{cycle}$)

1. **Thermal Efficiency (η):** A power cycle operating between the hot coils (source of rejected heat, $T_H = 28°\text{C} = 301$ K) and the surroundings (cold body, $T_C = 20°\text{C} = 293$ K) would have a thermal efficiency η.
   - Formula: $\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H}$ (where $\dot{Q}_H$ is the heat input to this power cycle, which is the heat rejected by the refrigerator, 4.4 kW)
2. **Maximum Thermal Efficiency (η_{MAX}):** The actual thermal efficiency (η) must be less than or equal to the maximum theoretical efficiency (η_{MAX}) for a reversible power cycle (Carnot efficiency).
   - Formula: $\eta \leq \eta_{MAX} = 1 - \frac{T_C}{T_H}$
3. **Solving for $\dot{W}_{cycle}$:**
   - $\frac{\dot{W}_{cycle}}{\dot{Q}_H} \leq 1 - \frac{T_C}{T_H}$
   - $\dot{W}_{cycle} \leq \left(1 - \frac{T_C}{T_H}\right) \dot{Q}_H$
   - Given $T_C = 293$ K (surroundings) and $T_H = 301$ K (coils), and $\dot{Q}_H = 4.4$ kW.
   - $\dot{W}_{cycle} \leq (1 - \frac{293 \text{ K}}{301 \text{ K}}) \cdot 4.4$ kW
   - $\dot{W}_{cycle} \leq (1 - 0.97342) \cdot 4.4$ kW
   - $\dot{W}_{cycle} \leq 0.02658 \cdot 4.4$ kW
   - $\dot{W}_{cycle} \leq 0.116952$ kW (approximately 0.12 kW as per the document)

4. **Result:** The maximum theoretical power that could be developed by a power cycle using the energy rejected from the coils is 0.12 kW.

   - The analysis notes that although the refrigerator rejects a significant amount of energy (4.4 kW), the thermodynamic value of this energy, in terms of power that could be developed, is relatively small (0.12 kW). Therefore, there is little incentive to try and use this rejected heat.