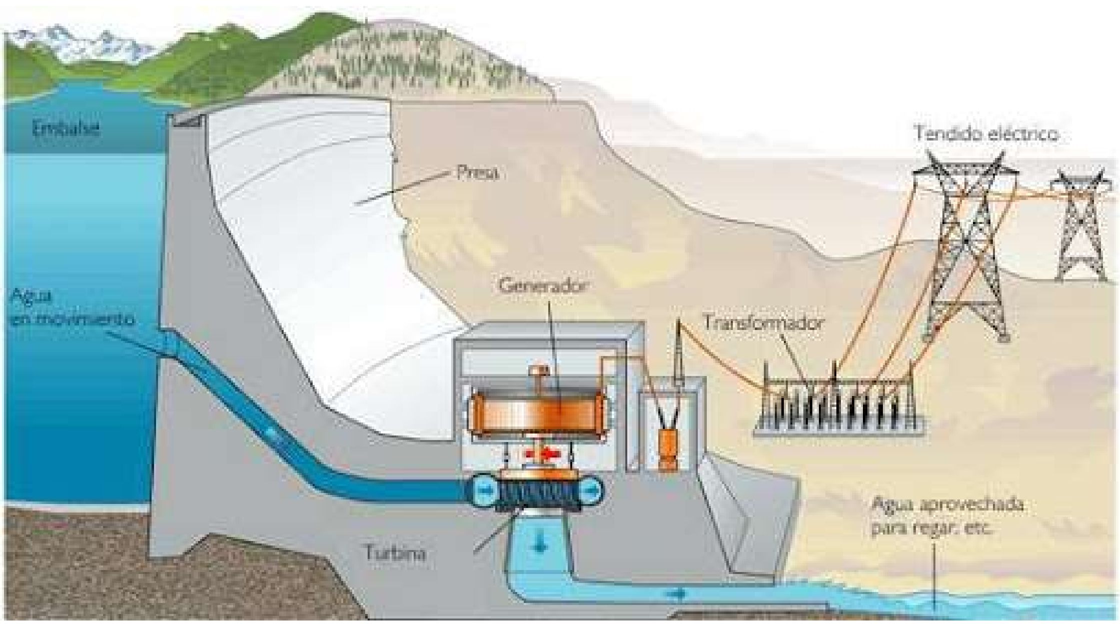
Turbomachinery - Turbines



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ME 3140

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1. Turbines
2. Dynamic Turbines
3. Impulse Turbines
4. Reaction Turbines
5. Wind Turbines
6. Turbine Scaling Laws

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1. Turbines

Turbines

Turbines

Turbines are energy-producing devices that extract energy from fluid flows (rivers and wind) and convert it into mechanical work, typically via a rotating shaft.

Types of Turbines According to the Working Fluid Used

- When the working fluid is water, the turbomachines are called **hydraulic turbines**, or hydroturbines.

- When air is the working fluid and energy is extracted from the wind, the machine is called a **wind turbine**.

- Coal/nuclear plants use steam; turbines converting its energy to shaft work are **steam turbines**.

- Turbines using compressible gas are **gas turbines** (e.g., jet-engine turbines).

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1. Turbines
2. **Dynamic Turbines**

Dynamic Turbines

Types of Dynamic Turbines that Produce Electricity

- **Impulse turbines** require a higher head but can operate with a smaller volumetric flow rate.

- **Reaction turbines** operate with much less head but require a higher volumetric flow rate.

Content

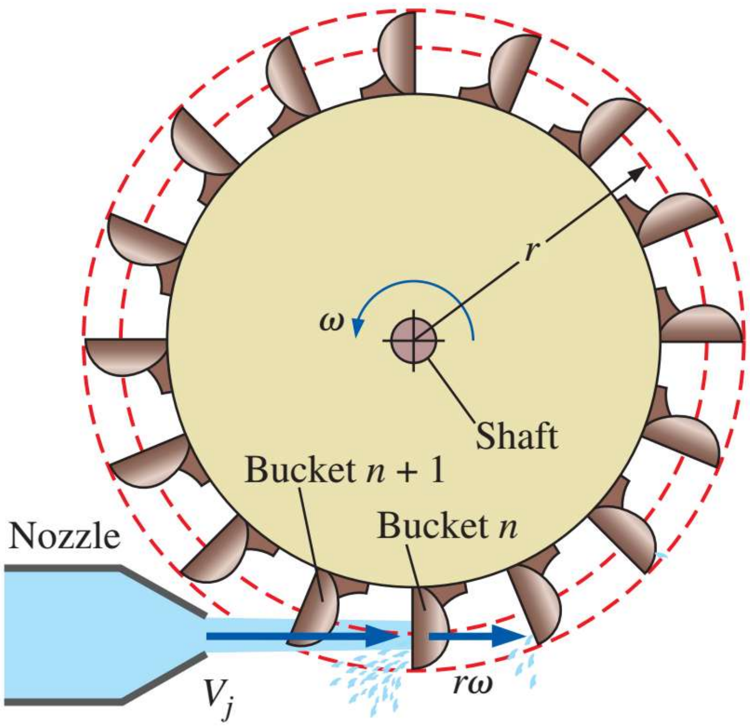
1. Turbines
2. Dynamic Turbines
3. **Impulse Turbines**

Impulse Turbines

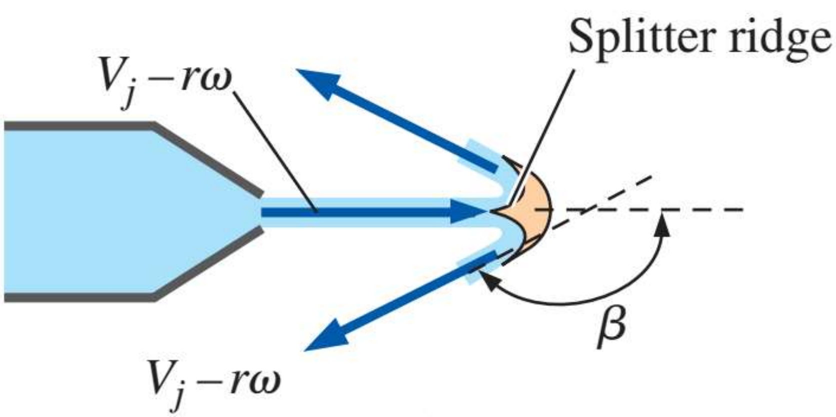
In an **impulse turbine**, the fluid is accelerated through a nozzle, converting most of its available mechanical energy into kinetic energy. The resulting high-speed jet impinges on bucket-shaped rotor blades, transferring energy to the turbine shaft.

Type: Pelton wheel

Schematic diagram of a Pelton-type impulse turbine:

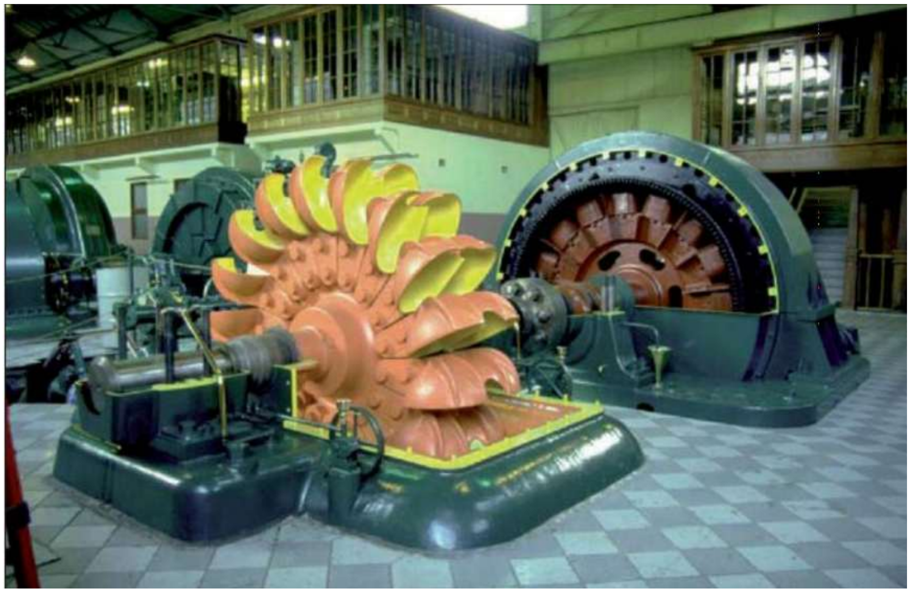


(a) Side view

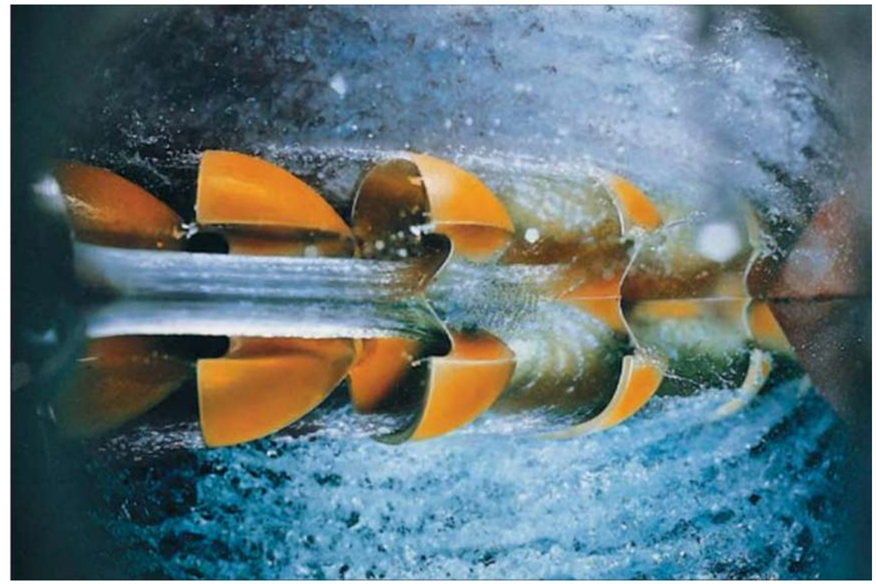


(b) Bottom view

Impulse Turbines



A close-up view of a Pelton wheel



A view from the bottom of an operating Pelton wheel

Euler turbomachine equation for a turbine:

$$\dot{W}_{\text{shaft}} = \omega T_{\text{shaft}} = \rho \omega Q \left(r_2 V_{2,t} - r_1 V_{1,t} \right)$$

$\dot{W}_{\text{shaft}}$: shaft power transferred to or from the fluid

ω: angular velocity of the runner

T_{shaft}: torque applied at the shaft

ρ: fluid density

Q: volumetric flow rate

r_1, r_2: inlet and outlet radii

$V_{1,t}, V_{2,t}$: tangential (swirl) components of the absolute velocity

Impulse Turbines

Shaft power, $\dot{W}_{shaft}$, for impulse turbines

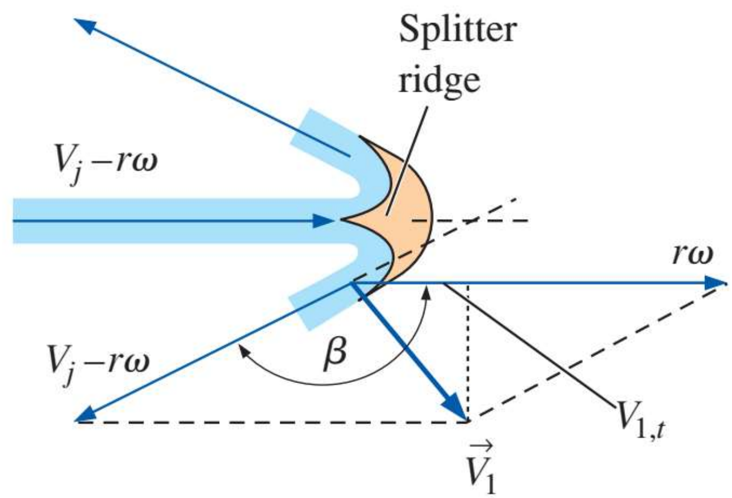
$$\dot{W}_{\text{shaft}} = \rho r\omega Q(V_j - r\omega)(1 - \cos\beta)$$

V_j: absolute velocity at the nozzle exit, entering the runner.

β: deflection angle of the relative flow through the runner.

r: runner radius at the interaction location.

$\vec{V}_1$: absolute inlet flow velocity vector after the splitter.

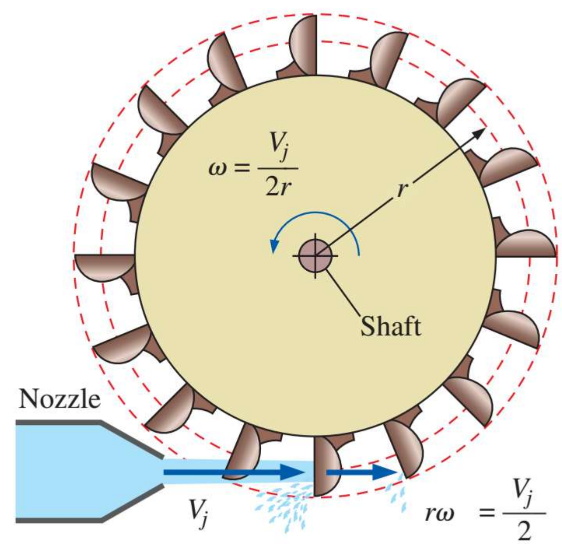


Tangential velocity, U : $U = r\omega$

For maximum power: $U = \dfrac{V_j}{2}$

Efficiency factor due to β:

$$\eta_\beta = \frac{\dot{W}_{\text{shaft,actual}}}{\dot{W}_{\text{shaft,ideal}}} = \frac{1 - \cos\beta}{1 - \cos 180°}$$



Impulse Turbines

Example 1

Water to drive a Pelton wheel is supplied through a pipe from a lake as indicated in Fig. E-1. The head loss due to friction in the pipe is important, but minor losses can be neglected.
(a) Determine the nozzle diameter, D_1, that will give the maximum power output.
(b) Determine the maximum power (in hp) and the rotational speed of the rotor (in rpm) at the conditions found in part (a).

Use: $1hp = 550\dfrac{lbf \cdot ft}{s}$

$$1lbf = 32.17\frac{lbm \cdot ft}{s^2}$$

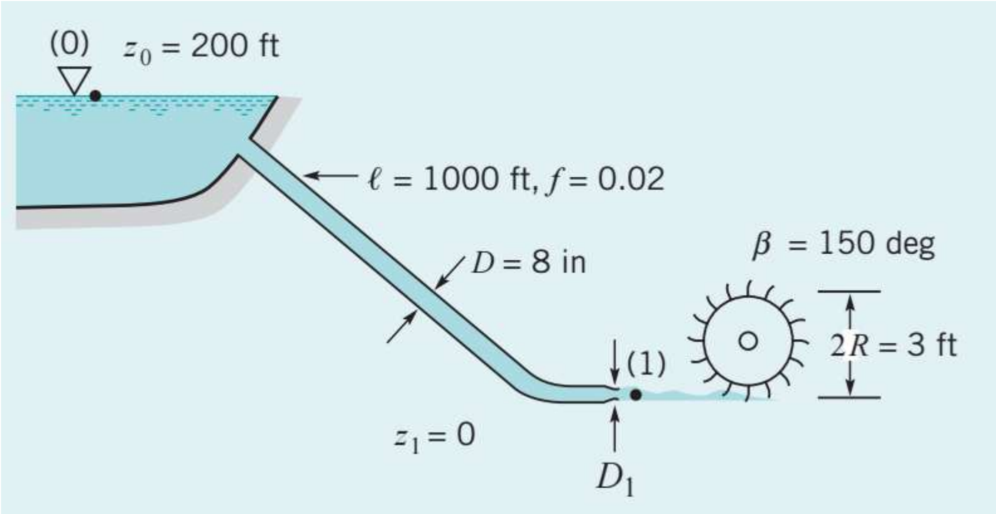


Figure E-1.

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1. Turbines
2. Dynamic Turbines
3. Impulse Turbines

4. Reaction Turbines

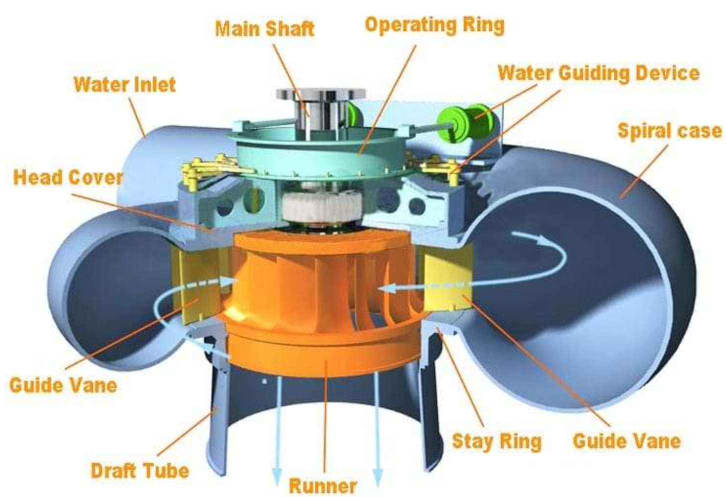
Reaction Turbines

A **reaction turbine** differs significantly from an impulse turbine; instead of using water jets, a volute is filled with swirling water that drives the runner.

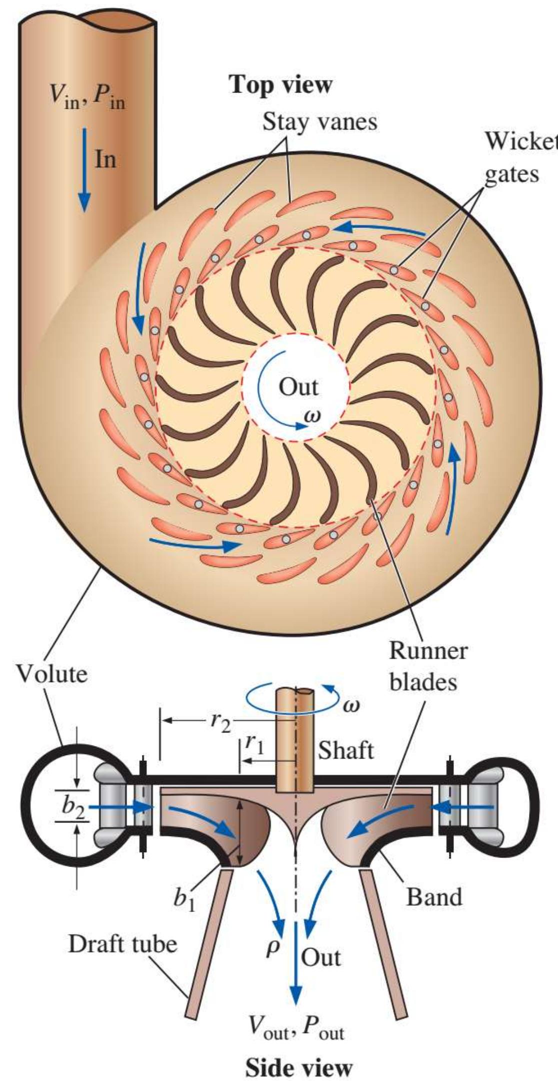
For **hydroturbine** applications, the axis is typically vertical. Top and side views are shown, including the fixed stay vanes and adjustable wicket gates.

Two main types of reaction turbine

- Francis turbine
- Kaplan turbine

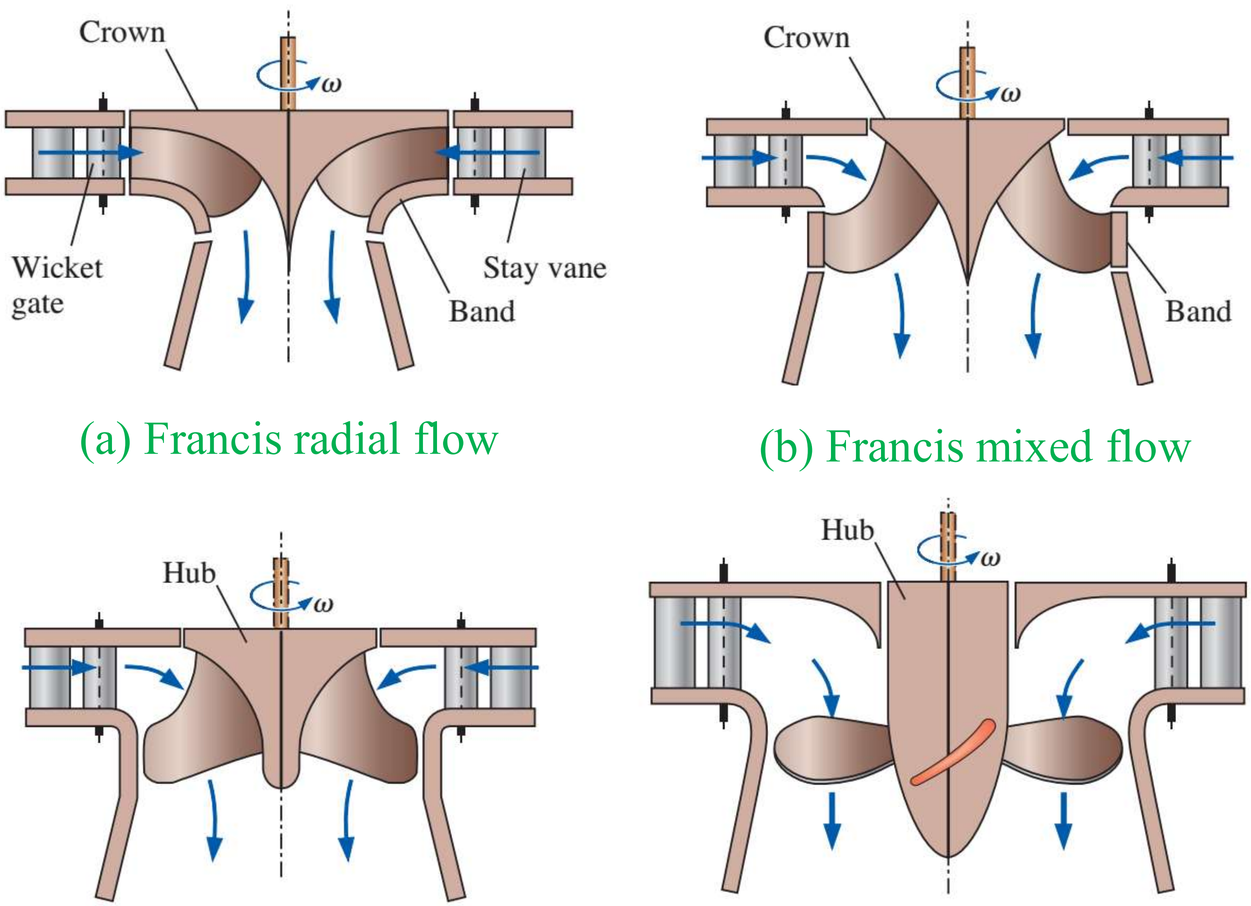


Francis Turbine



Reaction Turbines

The distinguishing characteristics of the four subcategories of reaction turbines:



(a) Francis radial flow

(b) Francis mixed flow

(c) Propeller mixed flow

(d) Propeller axial flow

Reaction Turbines

Ideal power production, $\dot{W}_{\text{ideal}}$ (no irreversible losses anywhere in the system)

$$\dot{W}_{\text{ideal}} = \rho g Q H_{\text{gross}}$$

$$H_{\text{gross}} = z_A - z_E$$

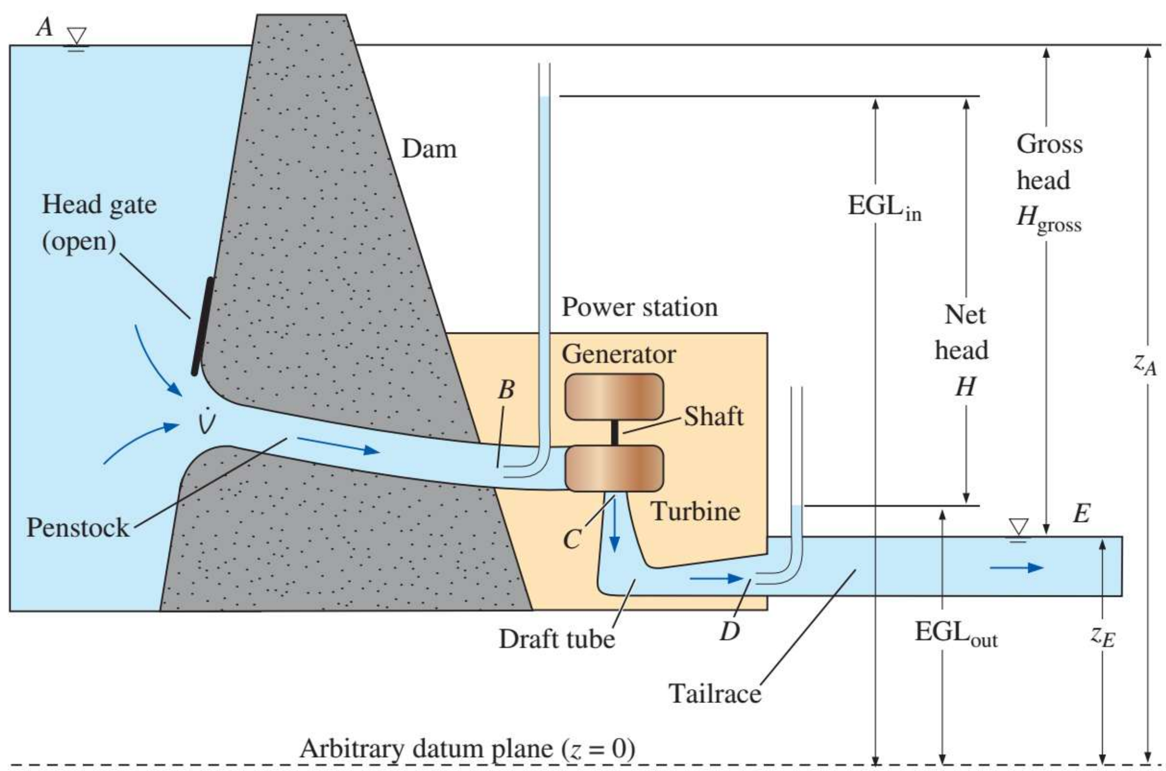
Net head for a hydraulic turbine, H

$$H = EGL_{\text{in}} - EGL_{\text{out}}$$

EGL: energy grade line

Turbine efficiency, η_{turbine}

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{water hp}}} = \frac{\text{bhp}}{\rho g H Q}$$

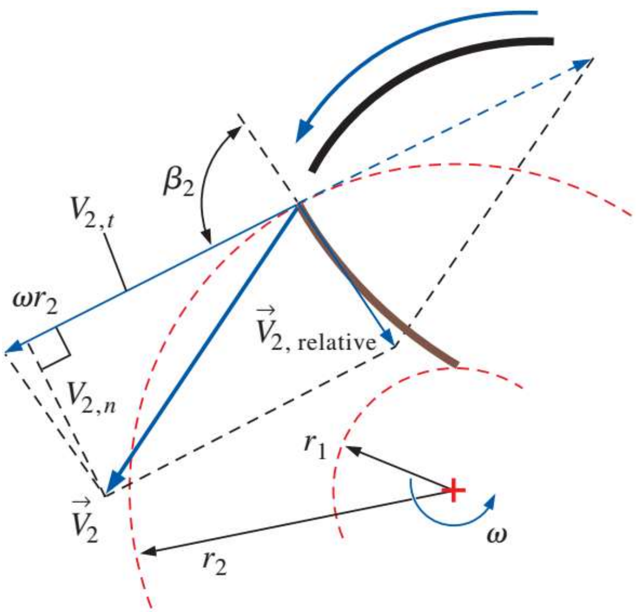


Typical setup and terminology for a hydroelectric plant that utilizes a Francis turbine to generate electricity

Reaction Turbines

Runner leading edge:

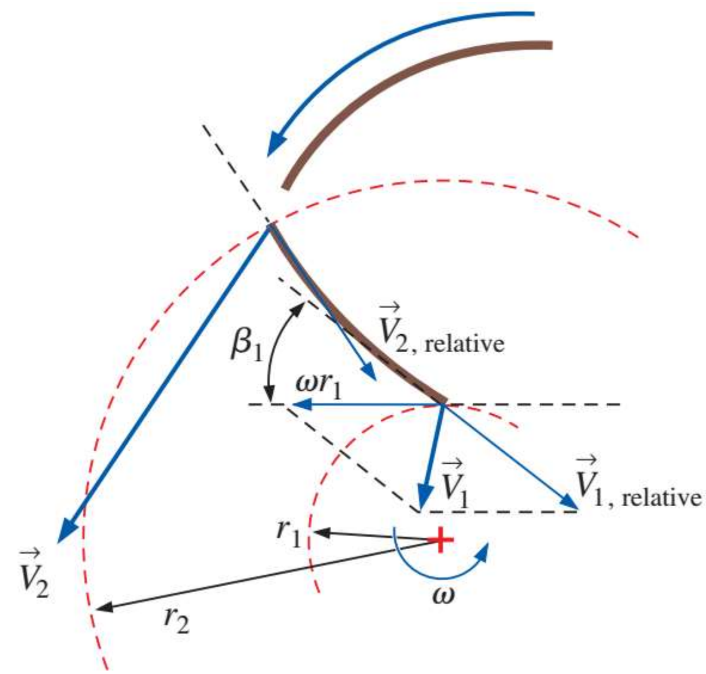
$$V_{2,\mathrm{t}} = \omega r_2 - \frac{V_{2,n}}{\tan\beta_2}$$



Relative and absolute velocity vectors and geometry for the outer radius of the runner of a Francis turbine.

Runner trailing edge:

$$V_{1,\mathrm{t}} = \omega r_1 - \frac{V_{1,n}}{\tan\beta_1}$$



Relative and absolute velocity vectors and geometry for the inner radius of the runner of a Francis turbine.

Reaction Turbines

Example 2

A retrofit Francis radial-flow hydroturbine is being designed to replace an old turbine in a hydroelectric dam. The new turbine must meet the following design restrictions in order to properly couple with the existing setup: The runner inlet radius is $r_2 = 8.20$ ft (2.50 m) and its outlet radius is $r_1 = 5.80$ ft (1.77 m). The runner blade widths are $b_2 = 3.00$ ft (0.914 m) and $b_1 = 8.60$ ft (2.62 m) at the inlet and outlet, respectively. The runner must rotate at $\dot{n}$= 120 rpm (ω =12.57 rad/s) to turn the 60-Hz electric generator. The wicket gates turn the flow by angle $\alpha_2 = 33°$ from radial at the runner inlet, and the flow at the runner outlet is to have angle α_1 between -10° and 10° from radial (Figure E-2) for proper flow through the draft tube. The volume flow rate at design conditions is 9.50×10^6 gpm (599 m^3/s), and the gross head provided by the dam is H_{gross} = 303 ft (92.4 m). (a) Calculate the inlet and outlet runner blade angles β_2 and β_1, respectively, and predict the power output and required net head if irreversible losses are neglected for the case with α_1=10° from radial (with-rotation swirl). (b) Repeat the calculations for the case with α_1=0° from radial (no swirl). (c) Repeat the calculations for the case with $\alpha_1 = -10°$ from radial (reverse swirl). Assume: $\eta_{turbine} = 100\%$

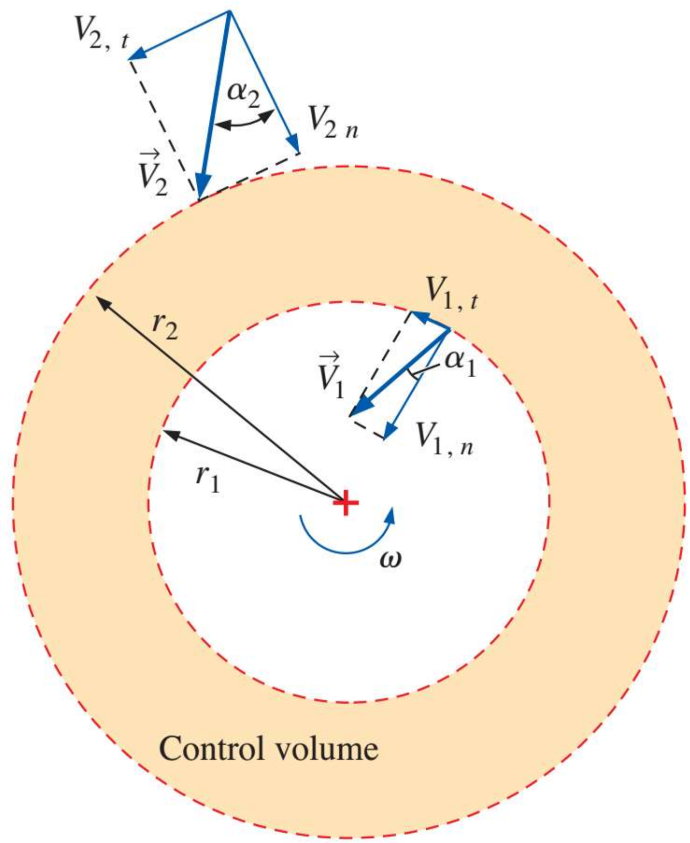


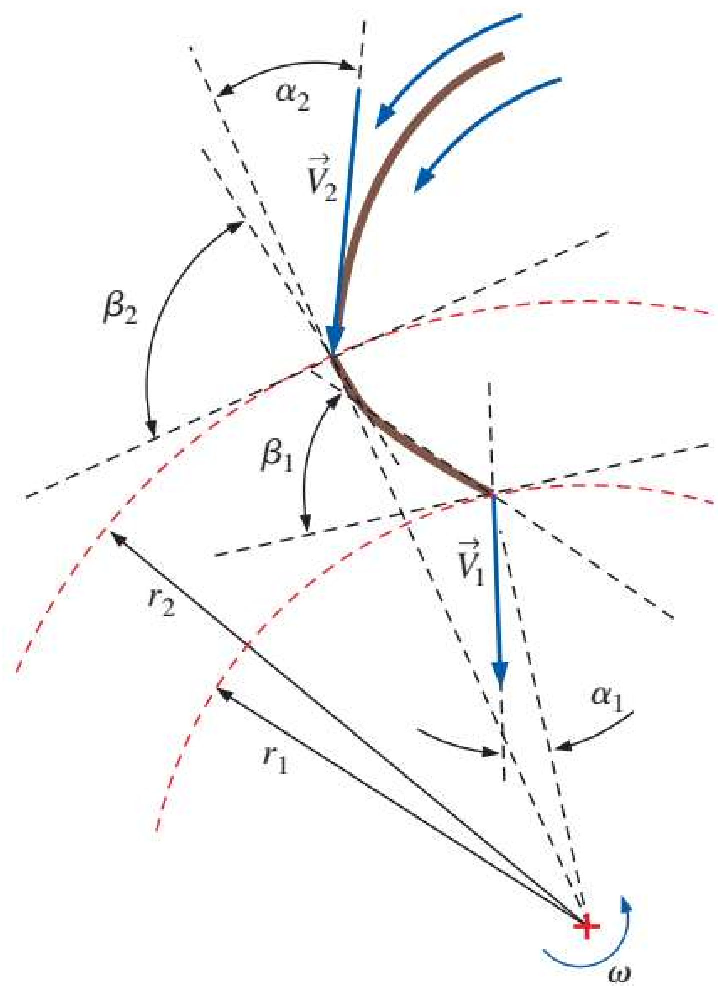
Figure E-2

Reaction Turbines

Hint:

$$Q = 2\pi r_1 b_1 V_{1,n} = 2\pi r_2 b_2 V_{2,n}$$

$$\dot{W}_{\text{shaft}} = \omega T_{\text{shaft}} = \rho\omega Q\left(r_2 V_{2,t} - r_1 V_{1,t}\right)$$



Sketch of the runner blade design of Example 2

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3. Impulse Turbines
4. Reaction Turbines
5. **Wind Turbines**

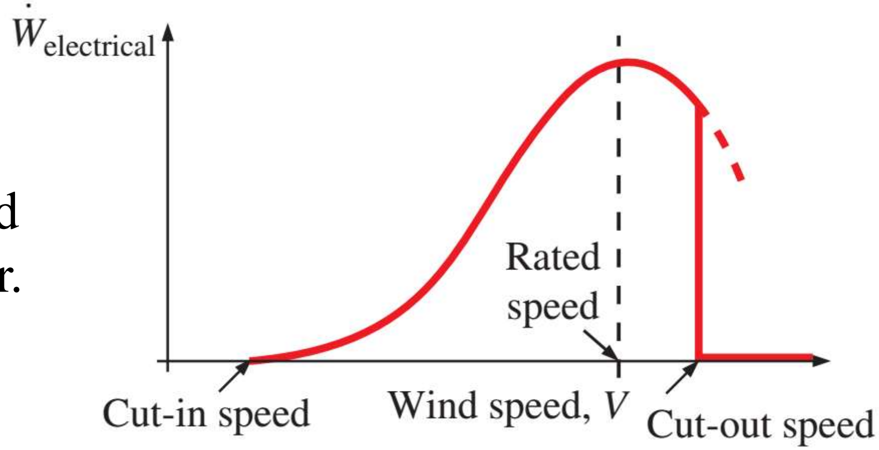
Wind Turbines

Three key locations on the wind-speed scale

Cut-in speed is the minimum wind speed at which useful power can be generated.

Rated speed is the wind speed at which the rated power is delivered, usually the maximum power.

Cut-out speed is the maximum wind speed at which the wind turbine is designed to produce power



Available wind power, $\dot{W}_{\text{available}}$

$$\dot{W}_{\text{available}} = \frac{d\left(\frac{1}{2}mV^2\right)}{dt} = \frac{1}{2}\rho V^3 A$$

***A*:** The disk (rotor) area of a wind turbine is the area perpendicular to the wind direction that is swept by the blades as they rotate.

Wind Turbines

Wind power density: $$\frac{\dot{W}_{\text{available}}}{A} = \frac{1}{2}\rho V^3$$

Average wind power density: $$\overline{\frac{\dot{W}_{\text{available}}}{A}} = \frac{1}{2}\rho_{\text{avg}} V^3 K_e$$

K_e: correction factor called the energy pattern factor.

N: number of hours in a year, $N = 8760$.

$$K_e = \frac{1}{N\bar{V}^3}\sum_{i=1}^{N} V_i^3$$

Power coefficient, C_p $$C_p = \frac{\dot{W}_{\text{rotor shaft output}}}{\dot{W}_{\text{available}}} = \frac{\dot{W}_{\text{rotor shaft output}}}{\frac{1}{2}\rho V^3 A}$$

Wind Turbines

Momentum equation for steady flow

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

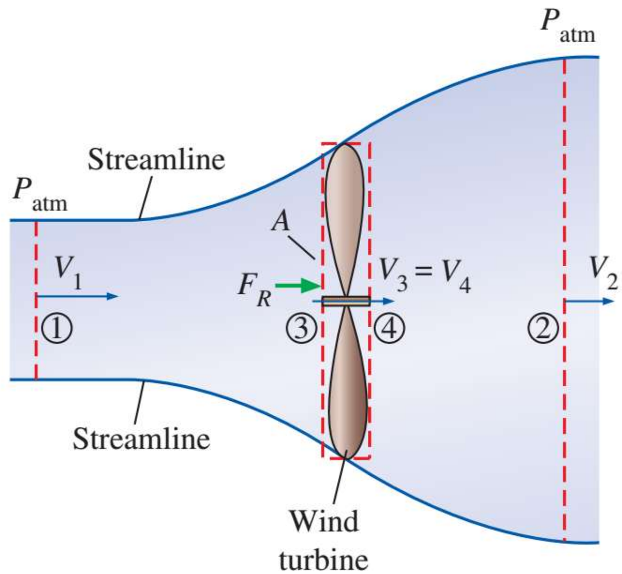
β: momentum flux correction factor
$\dot{m}$: mass flow rate
$\vec{V}$: vector velocity

$$F_R = \dot{m}(V_2 - V_1) \quad \text{where:} \quad F_R = A(P_4 - P_3)$$

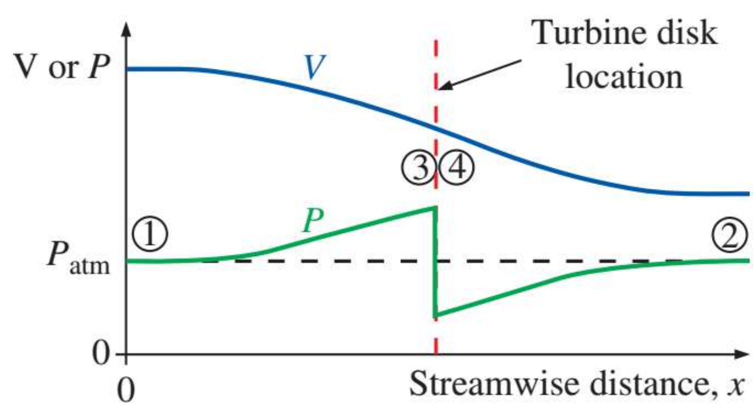
$$V_3 = \frac{V_1 + V_2}{2}$$

Fractional loss of velocity from far upstream to the turbine disk

$$a = \frac{V_1 - V_3}{V_1} \quad \Longrightarrow \quad V_2 = V_1(1 - 2a)$$



The axisymmetric stream tube



Wind Turbines

Ideal Power

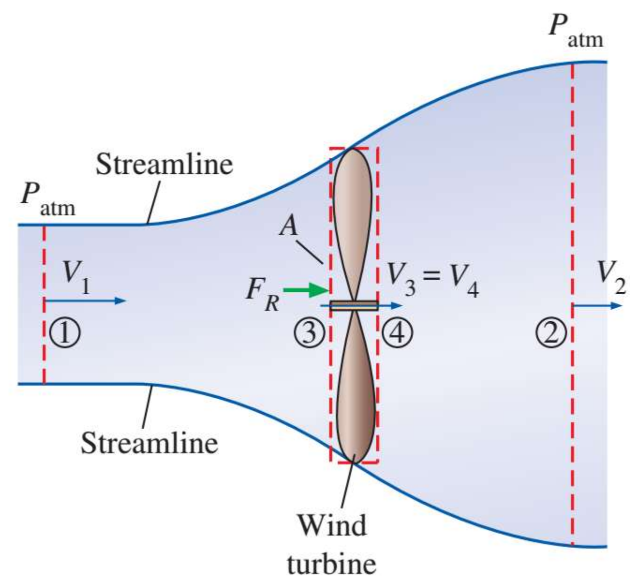
$$\dot{W}_{\text{ideal}} = \dot{m}\frac{V_1^2 - V_2^2}{2} = 2\rho A V_1^3 a(1-a)^2$$

Power coefficient, C_P

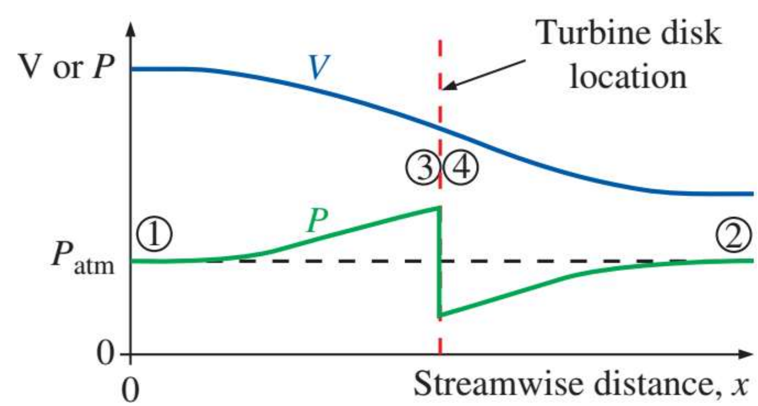
$$C_P = \frac{\dot{W}_{\text{rotor shaft output}}}{\frac{1}{2}\rho V_1^3 A} = 4a(1-a)^2$$

$a = \frac{1}{3}$ for the maximum possible power coefficient of any wind turbine (known as the **Betz limit**), thus:

$$C_{\text{P,max}} = 0.5926$$



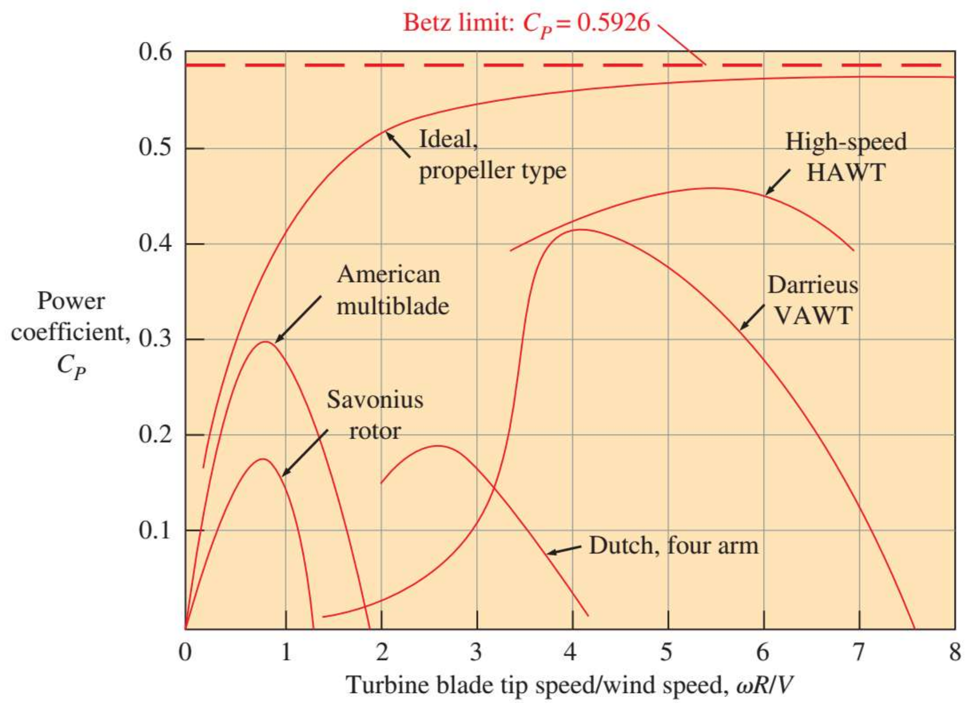
The axisymmetric stream tube



Wind Turbines

Example 3

To save money, a school plans to generate some of their own electricity using a HAWT wind turbine on top of a hill where it is fairly windy. As a conservative estimate based on the data of Figure E-2, they hope to achieve a power coefficient of 40 percent. The combined efficiency of the gearbox and generator is estimated to be 85 percent. If the diameter of the wind turbine disk is 12.5 m, estimate the electrical power production when the wind blows at 10.0 m/s.



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Turbine Scaling Laws

Dimensionless Turbine Parameters

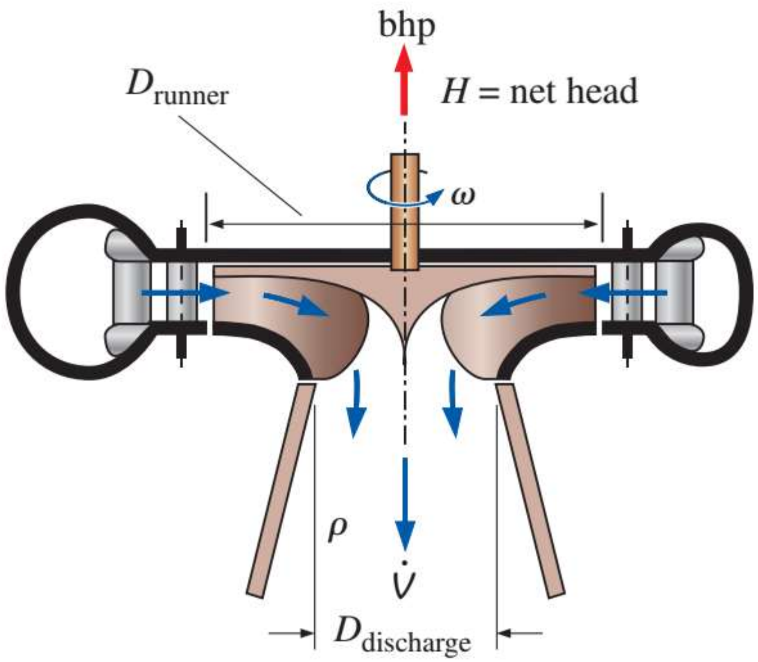
$$C_{\text{H}} = \text{Head coefficient} = \frac{gH}{\omega^2 D^2}$$

$$C_{\text{Q}} = \text{Capacity coefficient} = \frac{Q}{\omega D^3}$$

$$C_{\text{P}} = \text{Power coefficient} = \frac{\text{bhp}}{\rho \omega^3 D^5}$$

$$\eta_{\text{turbine}} = \text{Turbine efficiency} = \frac{\text{bhp}}{\rho g H Q}$$

$$\eta_{\text{turbine}} = \frac{C_P}{C_Q C_H}$$



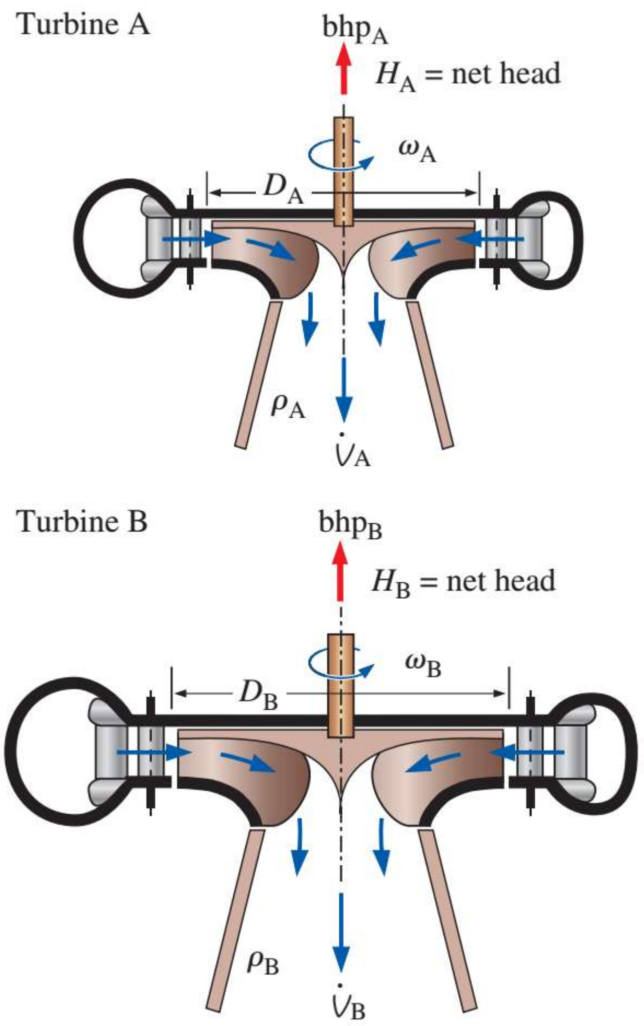
The axisymmetric stream tube

Turbine Scaling Laws

Dimensional analysis is useful for scaling two **geometrically similar** turbines. If all the dimensionless turbine parameters of turbine A are equivalent to those of turbine B, the two turbines are **dynamically similar**.

Moody efficiency correction equation for turbines:

$$\eta_{\text{turbine, p}} \cong 1 - (1 - \eta_{\text{turbine, m}})\left(\frac{D_\text{m}}{D_\text{p}}\right)^{1/5}$$



Turbine Scaling Laws

Example 4

A Francis turbine is being designed for a hydroelectric dam. Instead of starting from scratch, the engineers decide to geometrically scale up a previously designed hydroturbine that has an excellent performance history. The existing turbine (turbine A) has diameter $D_A = 2.05$ m, and spins at $\dot{n}_A = 120$ rpm ($\omega_A = 12.57\ \text{rad/s}$). At its best efficiency point, $Q_A = 350\ m^3/s$, $H_A = 75.0\ m$ of water, and $\text{bhp}_A = 242$ MW. The new turbine (turbine B) is for a larger facility. Its generator will spin at the same speed (120 rpm), but its net head will be higher ($H_B = 104$ m). Calculate the diameter of the new turbine such that it operates most efficiently, and calculate Q_B, bhp_B, and $\eta_{\text{turbine,B}}$.

Turbine Scaling Laws

Turbine Specific Speed, N_{St}: Turbine specific speed is used to characterize the operation of a turbine at its optimum conditions (best efficiency point) and is useful for preliminary turbine selection.

$$N_{St} = \frac{C_P^{1/2}}{C_H^{5/4}} = \frac{\omega(\text{bhp})^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

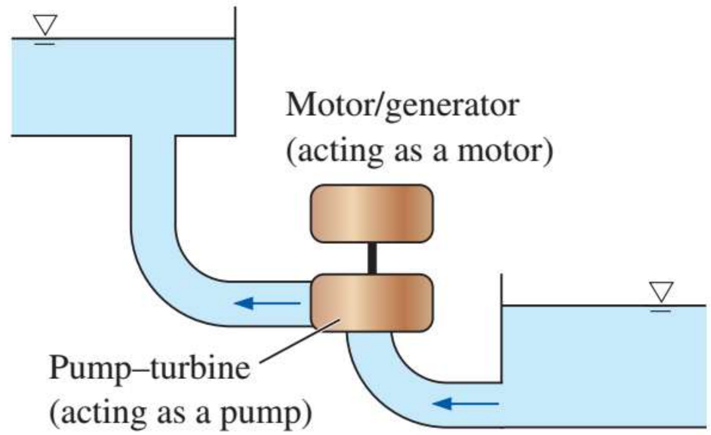
Turbine Specific Speed, N_{St}

$$N_{St} = N_{Sp}\sqrt{\eta_t}$$

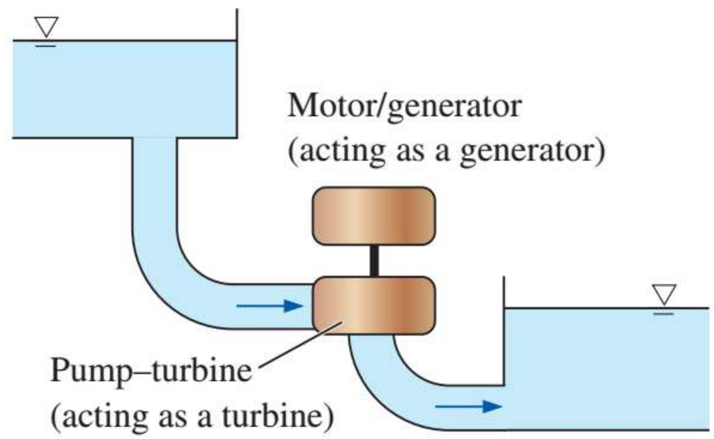
Pump-turbine specific speed relationship at the same flow rate and rpm:

$$N_{St} = N_{Sp}\sqrt{\eta_t}\left(\frac{H_p}{H_t}\right)^{3/4}$$

$$= N_{Sp}(\eta_t)^{5/4}(\eta_p)^{3/4}\left(\frac{bhp_p}{bhp_t}\right)^{3/4}$$



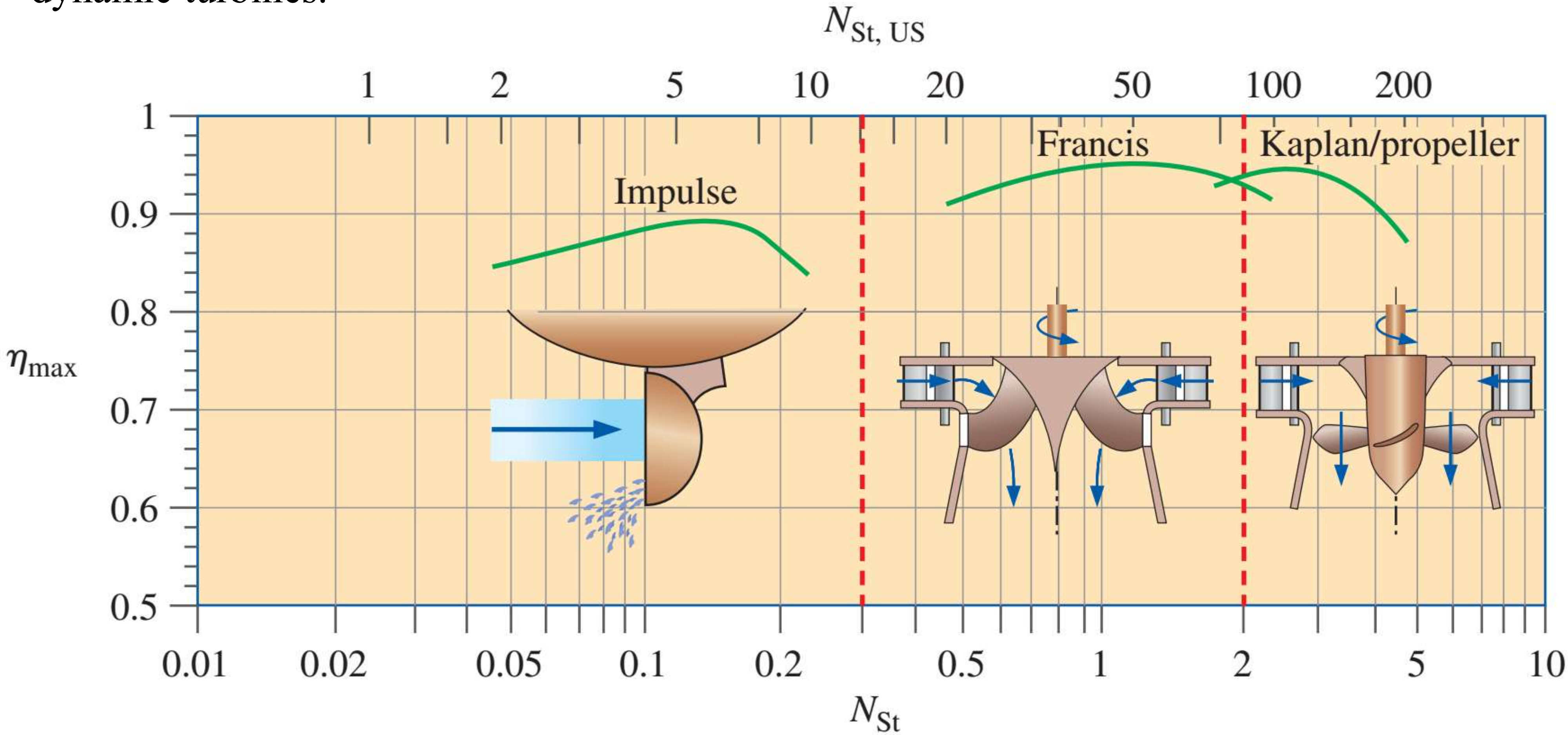
(a) Water is pumped by the pump-turbine during periods of low demand for power



(b) Electricity is generated by the pump-turbine during periods of high demand for power

Turbine Scaling Laws

Maximum efficiency as a function of **turbine specific speed** for the three main types of dynamic turbines.



Turbine Scaling Laws

Example 5

Calculate and compare the turbine specific speed for both the small (A) and large (B) turbines of Example 5.

References

[1] Cengel Y., Cimbala, J. (2014). Fluid Mechanics: Fundamentals and Applications (3th Edition). New York: NY: McGraw-Hill Co.

[2] Munson, B.R., Young, D.F., Okiishi, T.H., and Huebsch, W.W. (2016). Fundamentals of Fluid Mechanics (8th Edition). John Wiley & Sons. ISBN 1119080703.