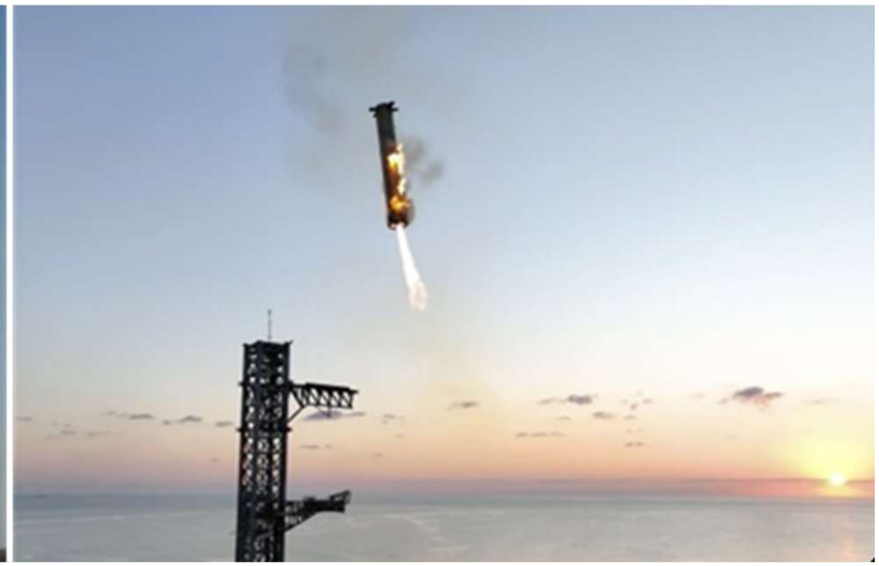
# Compressible Flow



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ME 3140

# Content

1. Stagnation Properties
2. One-Dimensional Isentropic Flow
3. Isentropic Flow Through Nozzles
4. Shock Waves and Expansion Waves
5. Duct Flow with Heat Transfer and Negligible Friction
6. Adiabatic Duct Flow with Friction

# Content

1. **Stagnation Properties**

# Stagnation Properties

**Compressible flows** involve significant changes in density. They are frequently encountered in devices that handle high-speed gas flow.

**Total Enthalpy** represents the total energy of a fluid: $H = U + P\mathcal{V}$

$\mathcal{V}$: Volume

**Specific Enthalpy (or static enthalpy), *h***

$$h = \frac{H}{m} = u + \frac{P}{\rho}$$

$m$: mass

$u$: specific internal energy

$P$: absolute thermodynamic pressure

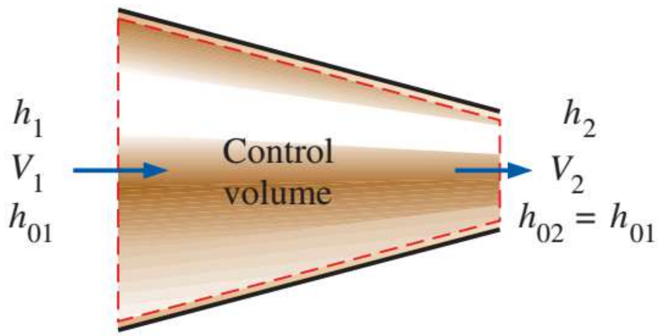
$\rho$: mass density

$P/\rho$: flow energy

**Specific Stagnation Enthalpy, $h_0$,** represents the enthalpy a fluid attains when it is brought to rest isentropically.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_{01} = h_{02}$$




For an ideal gas with constant specific heat: $h = c_p T$

# Stagnation Properties

**Stagnation (or Total) Temperature, $T_0$**, represents the temperature an ideal gas attains when it is brought to rest adiabatically.

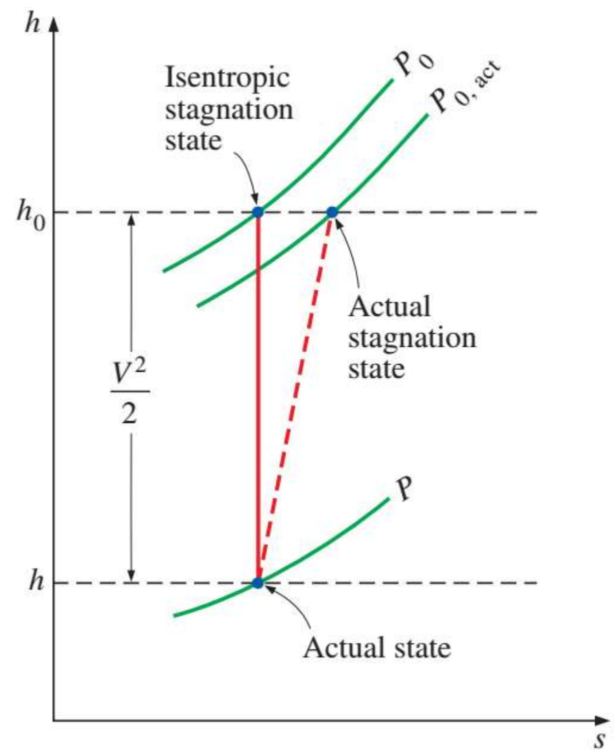
$$T_0 = T + \frac{V^2}{2c_p}$$

**Stagnation pressure, $P_0$**, represents the pressure a fluid attains when brought to rest isentropically:

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)}$$

Isentropic relation: $Pv^k = P_0 v_0^k$

**Stagnation density, $\rho_0$** $\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}$




**Energy Balance:** $\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$ ➡ $q_{\text{in}} + w_{\text{in}} + (h_{01} + gz_1) = q_{\text{out}} + w_{\text{out}} + (h_{02} + gz_2)$

# Stagnation Properties

**Example 1**

An aircraft is flying at a cruising speed of 250 m/s at an altitude of 5000 m where the atmospheric pressure is 54.05 kPa and the ambient air temperature is 255.7 K. The ambient air is first decelerated in a diffuser before it enters the compressor (Figure E-1). Approximating both the diffuser and the compressor to be isentropic, determine (a) the stagnation pressure at the compressor inlet and (b) the required compressor work per unit mass if the stagnation pressure ratio of the compressor is 8.

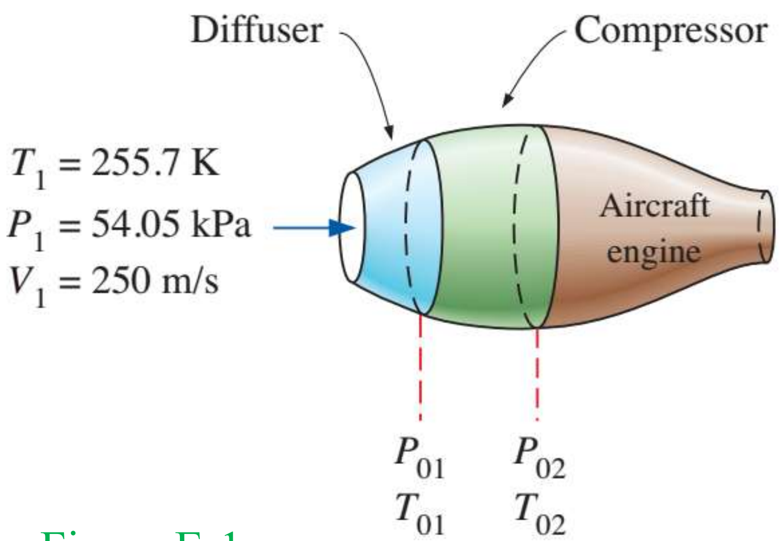



Figure E-1

# Content

1. Stagnation Properties
2. **One-Dimensional Isentropic Flow**

# One-Dimensional Isentropic Flow

**Speed of Sound, *c***

$$c = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} \qquad \text{or} \qquad c = \sqrt{k\left(\frac{\partial P}{\partial \rho}\right)_T}$$

For an ideal gas:

$$c = \sqrt{kRT}$$

$k$: specific heat ratio of the gas

$R$: specific gas constant

**Mach Number, *M***

$$M = \frac{V}{c}$$

$V$: fluid velocity

# One-Dimensional Isentropic Flow

**Example 2**

Carbon dioxide flows steadily through a varying cross-sectional area duct such as a nozzle shown in Figure E-2 at a mass flow rate of 3.00 kg/s. The carbon dioxide enters the duct at a pressure of 1400 kPa and 200°C with a low velocity, and it expands in the nozzle to an exit pressure of 200 kPa. The duct is designed so that the flow can be approximated as isentropic. Determine the density, velocity, flow area, and Mach number at each location along the duct that corresponds to an overall pressure drop of 200 kPa.

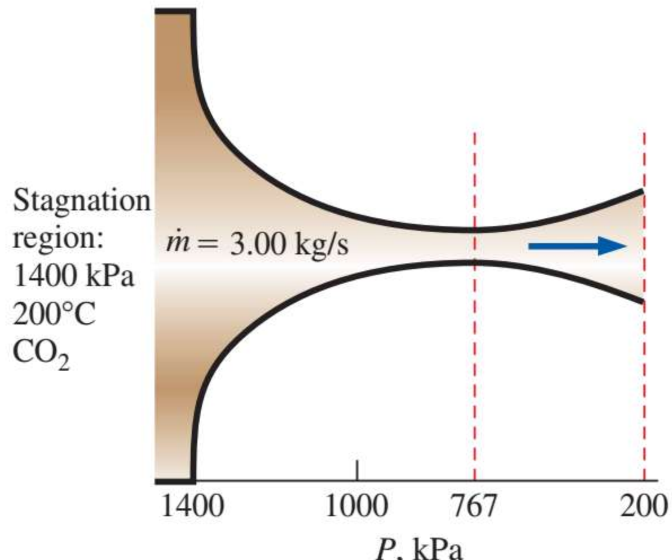



Figure E-2

# One-Dimensional Isentropic Flow

**TABLE A–1**

Molar mass, gas constant, and ideal-gas specfic heats of some substances

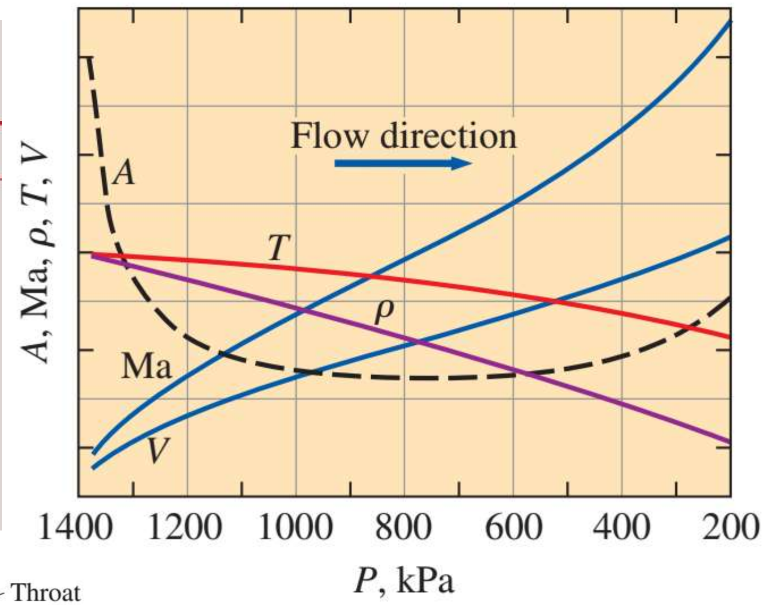
| Substance | Molar Mass $M$, kg/kmol | Gas Constant $R$, kJ/kg·K* | Specific Heat Data at 25°C $c_p$, kJ/kg·K | $c_v$, kJ/kg·K | $k = c_p/c_v$ |
|---|---|---|---|---|---|
| Air | 28.97 | 0.2870 | 1.005 | 0.7180 | 1.400 |
| Ammonia, $NH_3$ | 17.03 | 0.4882 | 2.093 | 1.605 | 1.304 |
| Argon, Ar | 39.95 | 0.2081 | 0.5203 | 0.3122 | 1.667 |
| Bromine, $Br_2$ | 159.81 | 0.05202 | 0.2253 | 0.1732 | 1.300 |
| Isobutane, $C_4H_{10}$ | 58.12 | 0.1430 | 1.663 | 1.520 | 1.094 |
| *n*-Butane, $C_4H_{10}$ | 58.12 | 0.1430 | 1.694 | 1.551 | 1.092 |
| Carbon dioxide, $CO_2$ | 44.01 | 0.1889 | 0.8439 | 0.6550 | 1.288 |
| Carbon monoxide, CO | 28.01 | 0.2968 | 1.039 | 0.7417 | 1.400 |
| Chlorine, $Cl_2$ | 70.905 | 0.1173 | 0.4781 | 0.3608 | 1.325 |
| Chlorodifluoromethane (R-22), $CHClF_2$ | 86.47 | 0.09615 | 0.6496 | 0.5535 | 1.174 |
| Ethane, $C_2H_6$ | 30.070 | 0.2765 | 1.744 | 1.468 | 1.188 |
| Ethylene, $C_2H_4$ | 28.054 | 0.2964 | 1.527 | 1.231 | 1.241 |
| Fluorine, $F_2$ | 38.00 | 0.2187 | 0.8237 | 0.6050 | 1.362 |
| Helium, He | 4.003 | 2.077 | 5.193 | 3.116 | 1.667 |
| *n*-Heptane, $C_7H_{16}$ | 100.20 | 0.08297 | 1.649 | 1.566 | 1.053 |
| *n*-Hexane, $C_6H_{14}$ | 86.18 | 0.09647 | 1.654 | 1.558 | 1.062 |
| Hydrogen, $H_2$ | 2.016 | 4.124 | 14.30 | 10.18 | 1.405 |
| Krypton, Kr | 83.80 | 0.09921 | 0.2480 | 0.1488 | 1.667 |
| Methane, $CH_4$ | 16.04 | 0.5182 | 2.226 | 1.708 | 1.303 |
| Neon, Ne | 20.183 | 0.4119 | 1.030 | 0.6180 | 1.667 |
| Nitrogen, $N_2$ | 28.01 | 0.2968 | 1.040 | 0.7429 | 1.400 |
| Nitric oxide, NO | 30.006 | 0.2771 | 0.9992 | 0.7221 | 1.384 |
| Nitrogen dioxide, $NO_2$ | 46.006 | 0.1889 | 0.8060 | 0.6171 | 1.306 |
| Oxygen, $O_2$ | 32.00 | 0.2598 | 0.9180 | 0.6582 | 1.395 |
| *n*-Pentane, $C_5H_{12}$ | 72.15 | 0.1152 | 1.664 | 1.549 | 1.074 |
| Propane, $C_3H_8$ | 44.097 | 0.1885 | 1.669 | 1.480 | 1.127 |
| Propylene, $C_3H_6$ | 42.08 | 0.1976 | 1.531 | 1.333 | 1.148 |
| Steam, $H_2O$ | 18.015 | 0.4615 | 1.865 | 1.403 | 1.329 |
| Sulfur dioxide, $SO_2$ | 64.06 | 0.1298 | 0.6228 | 0.4930 | 1.263 |
| Tetrachloromethane, $CCl_4$ | 153.82 | 0.05405 | 0.5415 | 0.4875 | 1.111 |
| Tetrafluoroethane (R-134a), $C_2H_2F_4$ | 102.03 | 0.08149 | 0.8334 | 0.7519 | 1.108 |
| Trifluoroethane (R-143a), $C_2H_3F_3$ | 84.04 | 0.09893 | 0.9291 | 0.8302 | 1.119 |
| Xenon, Xe | 131.30 | 0.06332 | 0.1583 | 0.09499 | 1.667 |

# One-Dimensional Isentropic Flow

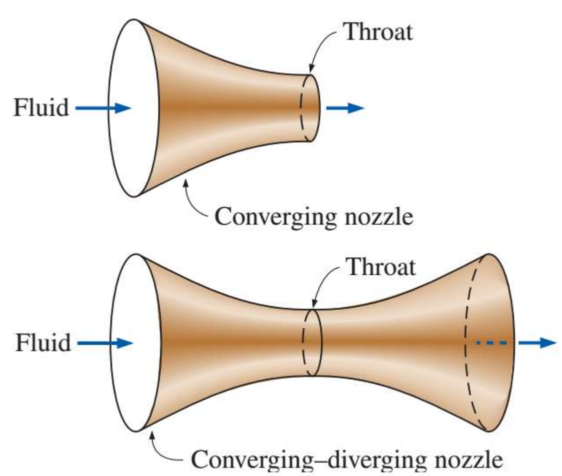
## Results for Example 2

Variation of fluid properties in flow direction in the duct described in Example 12–2 for $\dot{m}$ = 3 kg/s = constant

| $P$, kPa | $T$, K | $V$, m/s | $\rho$, kg/m$^3$ | $c$, m/s | $A$, cm$^2$ | Ma |
|---|---|---|---|---|---|---|
| 1400 | 473 | 0 | 15.7 | 339.4 | ∞ | 0 |
| 1200 | 457 | 164.5 | 13.9 | 333.6 | 13.1 | 0.493 |
| 1000 | 439 | 240.7 | 12.1 | 326.9 | 10.3 | 0.736 |
| 800 | 417 | 306.6 | 10.1 | 318.8 | 9.64 | 0.962 |
| 767* | 413 | 317.2 | 9.82 | 317.2 | 9.63 | 1.000 |
| 600 | 391 | 371.4 | 8.12 | 308.7 | 10.0 | 1.203 |
| 400 | 357 | 441.9 | 5.93 | 295.0 | 11.5 | 1.498 |
| 200 | 306 | 530.9 | 3.46 | 272.9 | 16.3 | 1.946 |




The cross-section of a nozzle at the smallest flow area is called the **throat**.



# One-Dimensional Isentropic Flow

## Variation of Fluid Velocity with Flow Area

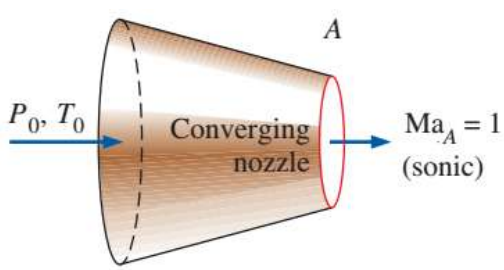
Mass balance for a steady flow process: $\dot{m} = \rho A V$ =constant

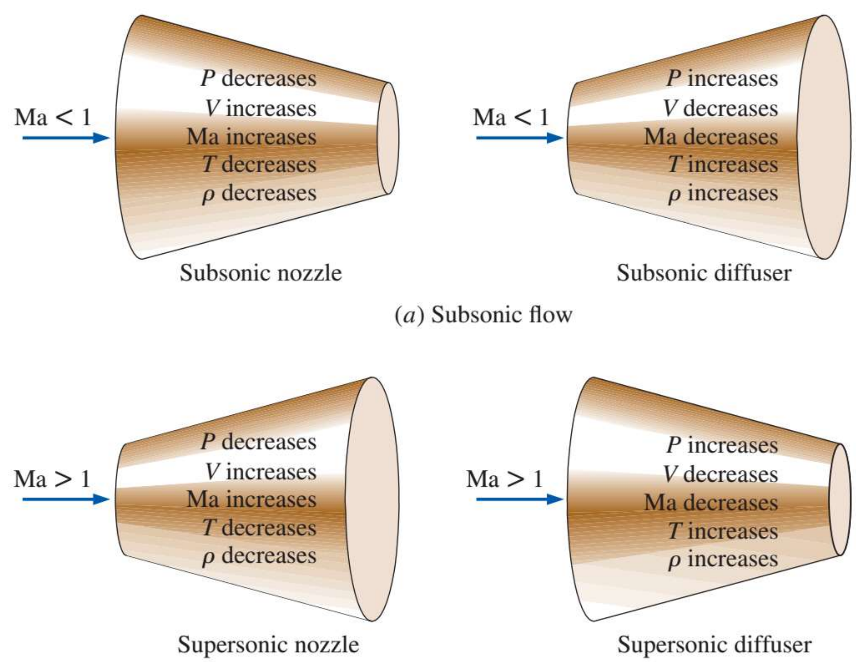
Differential form of Energy balance: $$\frac{dP}{\rho} + V dV = 0$$

Relations for isentropic flow in ducts:

$$\frac{dA}{A} = \frac{dP}{\rho V^2}(1 - M^2)$$

and $$\frac{dA}{A} = -\frac{dV}{V}(1 - M^2)$$




(a) Subsonic flow

(b) Supersonic flow

# One-Dimensional Isentropic Flow

## Property Relations for isentropic Flow of Ideal Gases

Relation between $T_0$ and $T$: $$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)M^2$$

Ratio between $P_0$ and $P$: $$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{k/(k-1)}$$

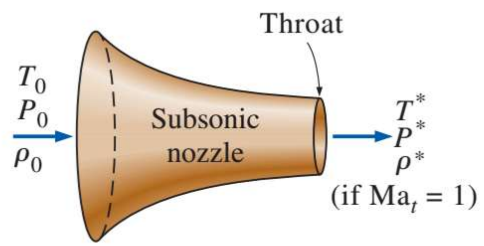
Ratio between $\rho_0$ and $\rho$: $$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{1/(k-1)}$$

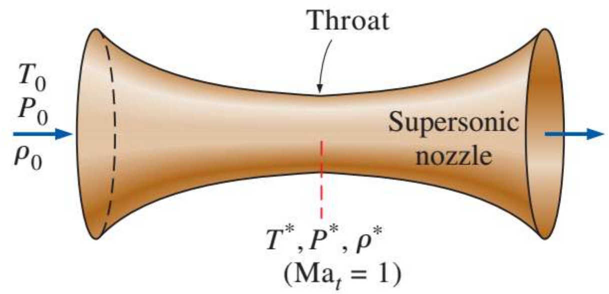
Critical Ratios:

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

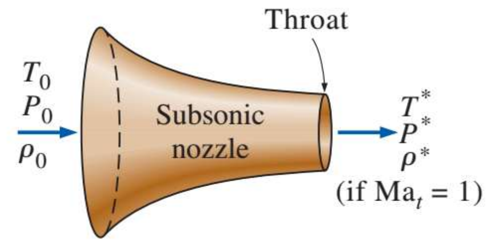


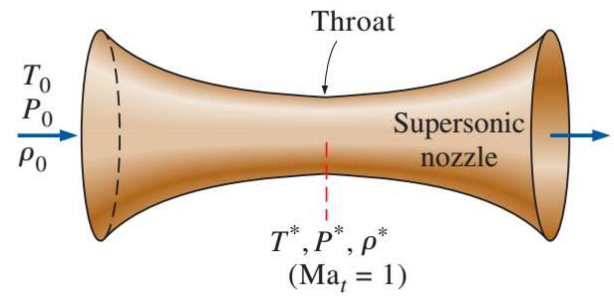



# One-Dimensional Isentropic Flow

The critical-pressure, critical-temperature, and critical-density ratios for isentropic flow of some ideal gases

| | Superheated steam, $k = 1.3$ | Hot products of combustion, $k = 1.33$ | Air, $k = 1.4$ | Monatomic gases, $k = 1.667$ |
|---|---|---|---|---|
| $\frac{P^*}{P_0}$ | 0.5457 | 0.5404 | 0.5283 | 0.4871 |
| $\frac{T^*}{T_0}$ | 0.8696 | 0.8584 | 0.8333 | 0.7499 |
| $\frac{\rho^*}{\rho_0}$ | 0.6276 | 0.6295 | 0.6340 | 0.6495 |



# One-Dimensional Isentropic Flow

**Example 3**

Calculate the critical pressure and temperature of carbon dioxide for the flow conditions described in Example 2 (Figure E-3).

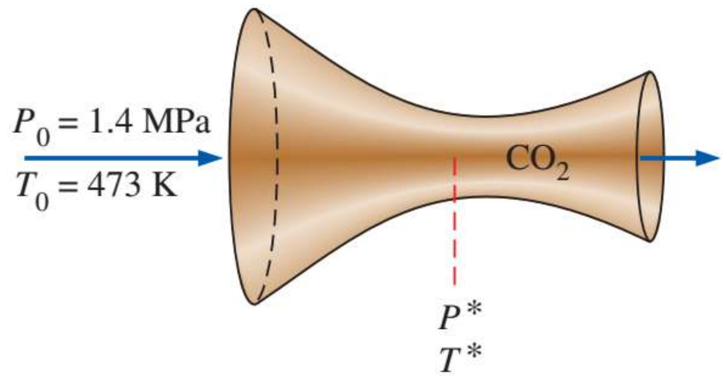



Figure E-3

# Content

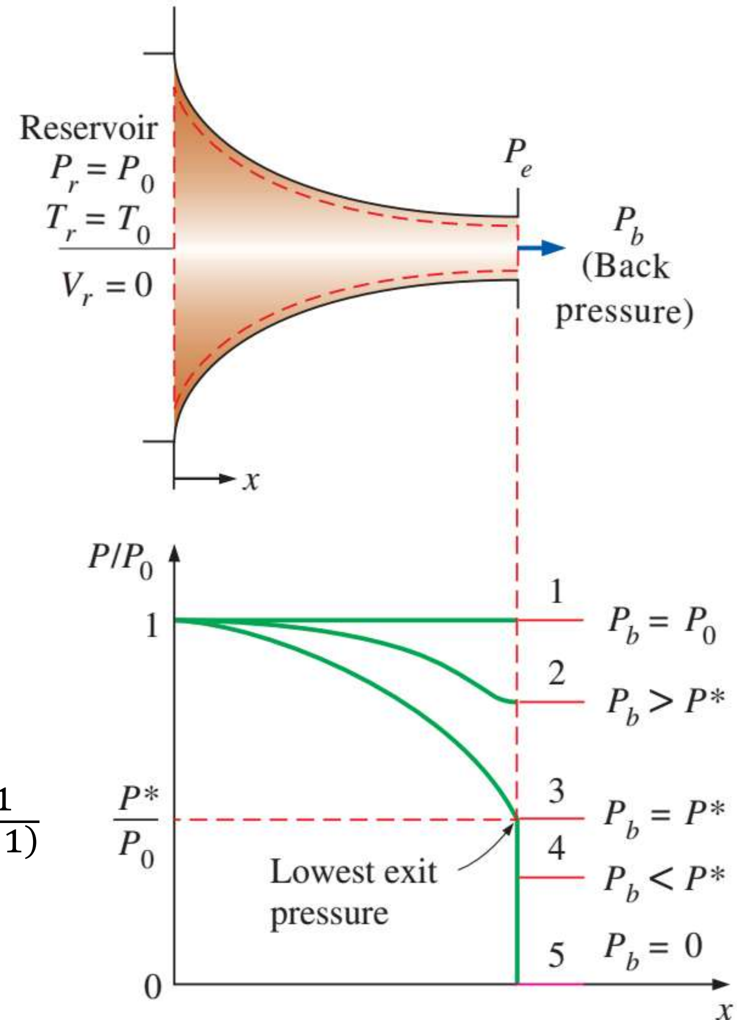
1. Stagnation Properties
2. One-Dimensional Isentropic Flow
3. **Isentropic Flow Through Nozzles**

# Isentropic Flow Through Nozzles

**Converging and converging–diverging nozzles** are shaped ducts used to speed up and direct fluids.

They are used in:

- Turbines (steam/gas) to turn heat into high-speed flow.
- Aircraft and rocket engines to create thrust.
- Industrial nozzles (blasting, torch) to produce fast, focused jets for cutting, cleaning, or heating.




## Converging Nozzles

The mass flow rate of a particular fluid through a nozzle:

$$\dot{m} = \frac{A\ M\ P_0\sqrt{\frac{k}{RT_0}}}{\left[1 + (k-1)\frac{M^2}{2}\right]^{\frac{k+1}{2(k-1)}}}$$

Maximum mass flow rate:

$$\dot{m} = A * P_0\sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

# Isentropic Flow Through Nozzles

The effect of **back pressure $P_b$** on the mass flow rate $\dot{m}$. and the **exit pressure $P_e$** of a converging nozzle.

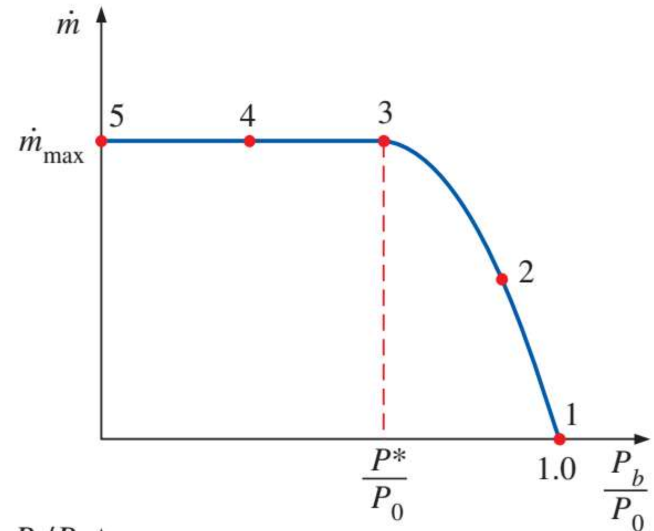
$$P_e = \begin{cases} P_b & \text{for} \quad P_b \geq P^* \\ P^* & \text{for} \quad P_b < P^* \end{cases}$$

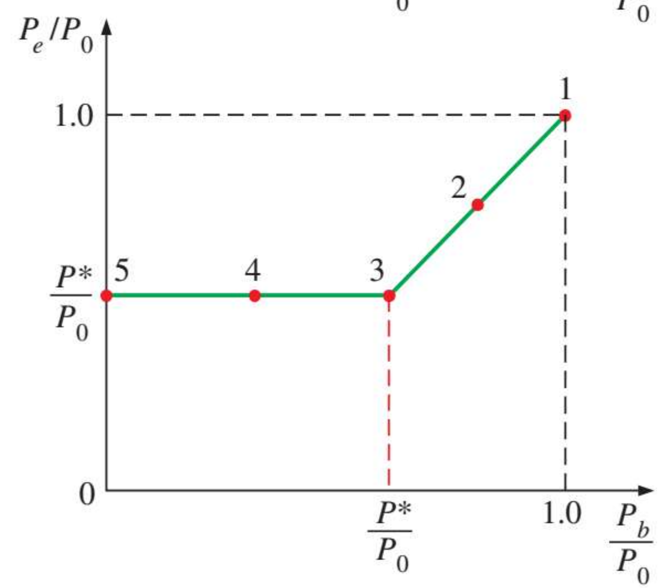
Relation for the variation of **flow area $A$** through the nozzle relative to **throat area $A^*$** :

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}M^2\right)\right]^{\frac{k+1}{2(k-1)}}$$

**Critical Mach number $M^*$:** is the local velocity nondimensionalized with respect to the sonic velocity at the throat.

$$M^* = M\sqrt{\frac{k+1}{2+(k-1)M^2}}$$



# Isentropic Flow Through Nozzles

**Example 4**

Air at 1 MPa and 600°C enters a converging nozzle, shown in Figure E-4, with a velocity of 150 m/s. Determine the mass flow rate through the nozzle for a nozzle throat area of 50 cm$^2$ when the back pressure is (a) 0.7 MPa and (b) 0.4 MPa.

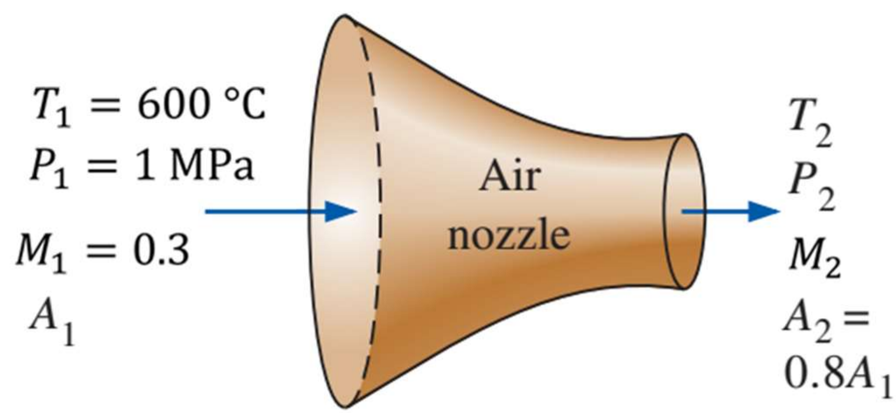



Figure E-4

# Isentropic Flow Through Nozzles

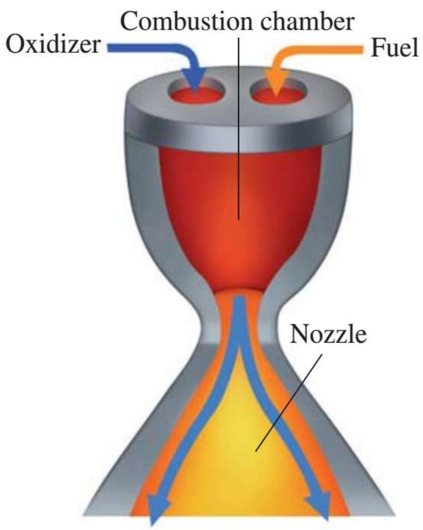
**TABLE A–13**

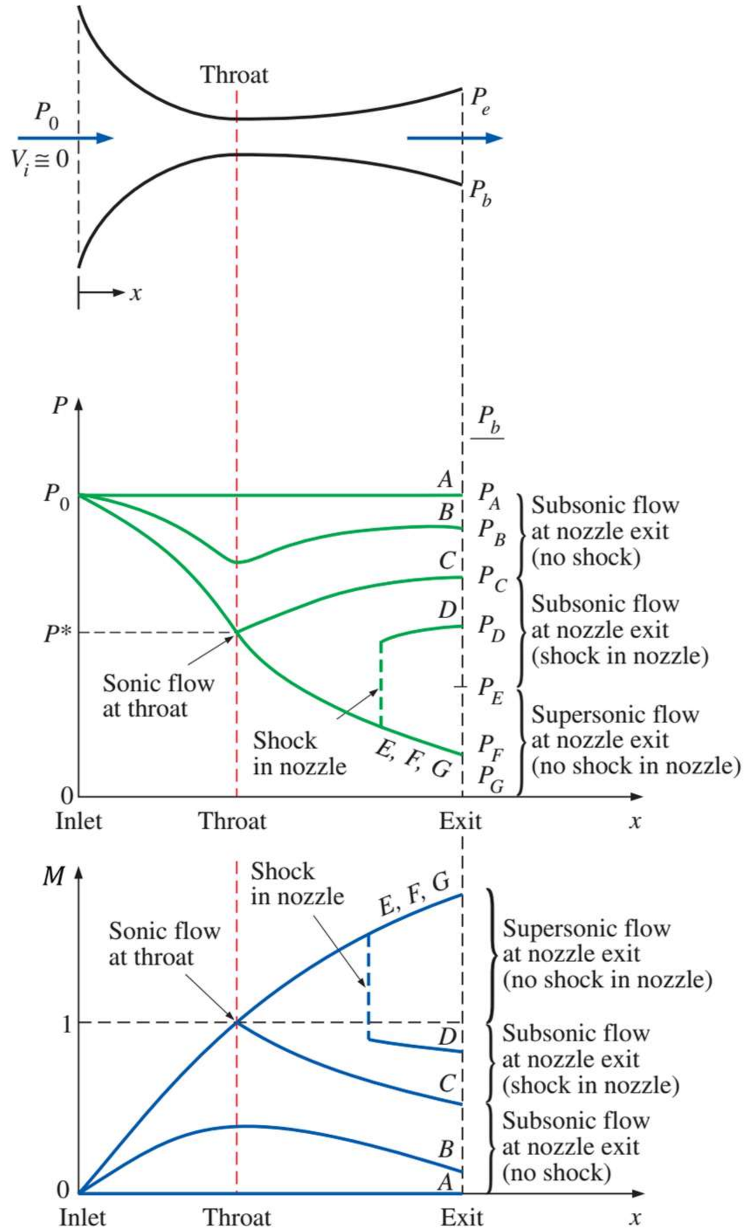
One-dimensional isentropic compressible flow functions for an ideal gas with $k = 1.4$

| Ma | Ma* | $A/A^*$ | $P/P_0$ | $\rho/\rho_0$ | $T/T_0$ |
|---|---|---|---|---|---|
| 0 | 0 | ∞ | 1.0000 | 1.0000 | 1.0000 |
| 0.1 | 0.1094 | 5.8218 | 0.9930 | 0.9950 | 0.9980 |
| 0.2 | 0.2182 | 2.9635 | 0.9725 | 0.9803 | 0.9921 |
| 0.3 | 0.3257 | 2.0351 | 0.9395 | 0.9564 | 0.9823 |
| 0.4 | 0.4313 | 1.5901 | 0.8956 | 0.9243 | 0.9690 |
| 0.5 | 0.5345 | 1.3398 | 0.8430 | 0.8852 | 0.9524 |
| 0.6 | 0.6348 | 1.1882 | 0.7840 | 0.8405 | 0.9328 |
| 0.7 | 0.7318 | 1.0944 | 0.7209 | 0.7916 | 0.9107 |
| 0.8 | 0.8251 | 1.0382 | 0.6560 | 0.7400 | 0.8865 |
| 0.9 | 0.9146 | 1.0089 | 0.5913 | 0.6870 | 0.8606 |
| 1.0 | 1.0000 | 1.0000 | 0.5283 | 0.6339 | 0.8333 |
| 1.2 | 1.1583 | 1.0304 | 0.4124 | 0.5311 | 0.7764 |
| 1.4 | 1.2999 | 1.1149 | 0.3142 | 0.4374 | 0.7184 |
| 1.6 | 1.4254 | 1.2502 | 0.2353 | 0.3557 | 0.6614 |
| 1.8 | 1.5360 | 1.4390 | 0.1740 | 0.2868 | 0.6068 |
| 2.0 | 1.6330 | 1.6875 | 0.1278 | 0.2300 | 0.5556 |
| 2.2 | 1.7179 | 2.0050 | 0.0935 | 0.1841 | 0.5081 |
| 2.4 | 1.7922 | 2.4031 | 0.0684 | 0.1472 | 0.4647 |
| 2.6 | 1.8571 | 2.8960 | 0.0501 | 0.1179 | 0.4252 |
| 2.8 | 1.9140 | 3.5001 | 0.0368 | 0.0946 | 0.3894 |
| 3.0 | 1.9640 | 4.2346 | 0.0272 | 0.0760 | 0.3571 |
| 5.0 | 2.2361 | 25.000 | 0.0019 | 0.0113 | 0.1667 |
| ∝ | 2.2495 | ∝ | 0 | 0 | 0 |

# Isentropic Flow Through Nozzles

## Converging-Diverging Nozzles



# Isentropic Flow Through Nozzles

**Example 5**

Air enters a converging–diverging nozzle, shown in Figure E-5, at 1.0 MPa and 800 K with negligible velocity. The flow is steady, one-dimensional, and isentropic with $k = 1.4$. For an exit Mach number of $M = 2$ and a throat area of 20 $cm^2$, determine (a) the throat conditions, (b) the exit plane conditions, including the exit area, and (c) the mass flow rate through the nozzle.

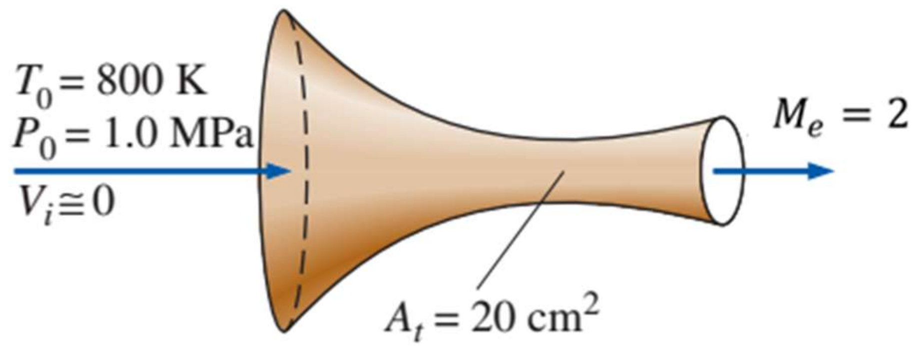



Figure E-5

# Content

1. Stagnation Properties
2. One-Dimensional Isentropic Flow
3. Isentropic Flow Through Nozzles
4. **Shock Waves and Expansion Waves**

# Shock Waves and Expansion

**Shock Waves** are very thin regions in a supersonic flow where pressure, temperature, density, and velocity change abruptly.

**Normal shock waves** are shock waves that stand in a plane perpendicular to the flow direction, causing abrupt changes in a supersonic stream.

Conservation of mass:

$$\rho_1 \, V_1 = \rho_2 \, V_2$$

Conservation of energy:

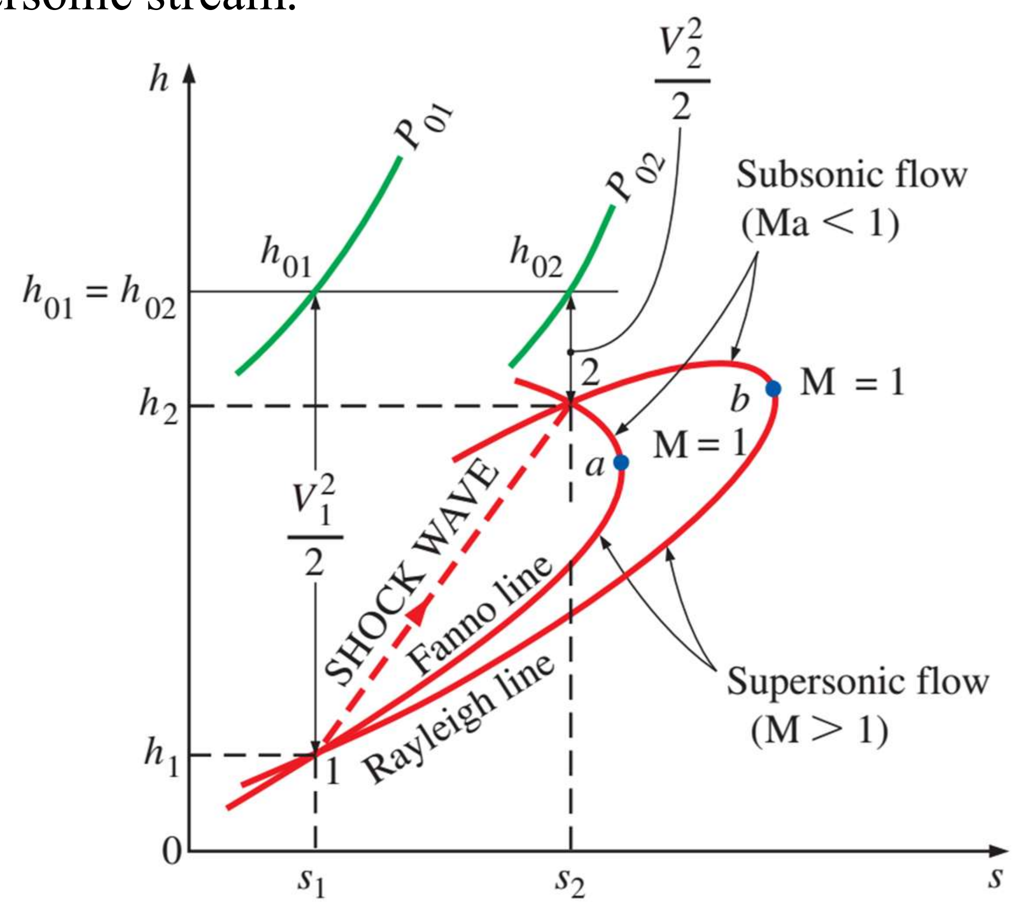
$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad \text{or} \quad h_{01} = h_{02}$$

Linear momentum equation:

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1)$$

Increase of entropy:

$$s_2 - s_1 \geq 0$$



# Shock Waves and Expansion

**Variation of flow properties across a normal shock in an ideal gas.**

Conservation of energy: $T_{01} = T_{02}$

Ratio of the static temperatures $T_2 / T_1$ :

$$\frac{T_2}{T_1} = \frac{1 + \frac{M_1^2(k-1)}{2}}{1 + \frac{M_2^2(k-1)}{2}} = \left(\frac{P_2}{P_1}\right)^2 \left(\frac{M_2}{M_1}\right)^2$$

Fanno Line:

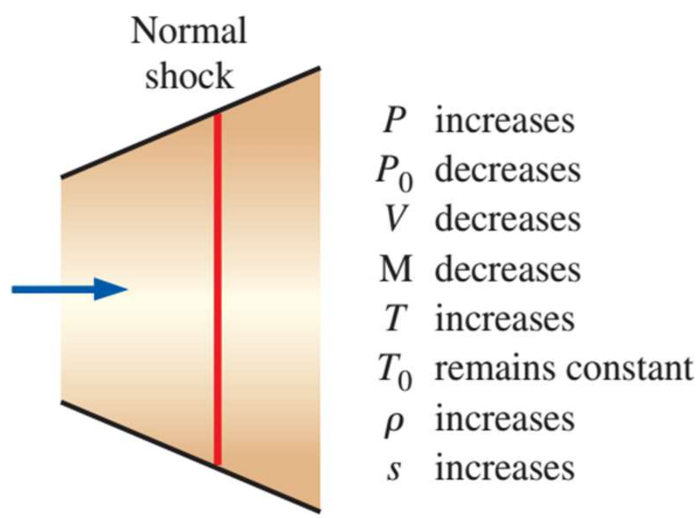
$$\frac{P_2}{P_1} = \frac{M_1\sqrt{1 + \frac{M_1^2(k-1)}{2}}}{M_2\sqrt{1 + \frac{M_2^2(k-1)}{2}}}$$

Rayleigh Line:

$$\frac{P_2}{P_1} = \frac{1 + kM_1^2}{1 + kM_2^2}$$

Normal shock Mach number relation

$$M_2^2 = \frac{M_1^2 + \frac{2}{k-1}}{\frac{2M_1^2 k}{k-1} - 1}$$



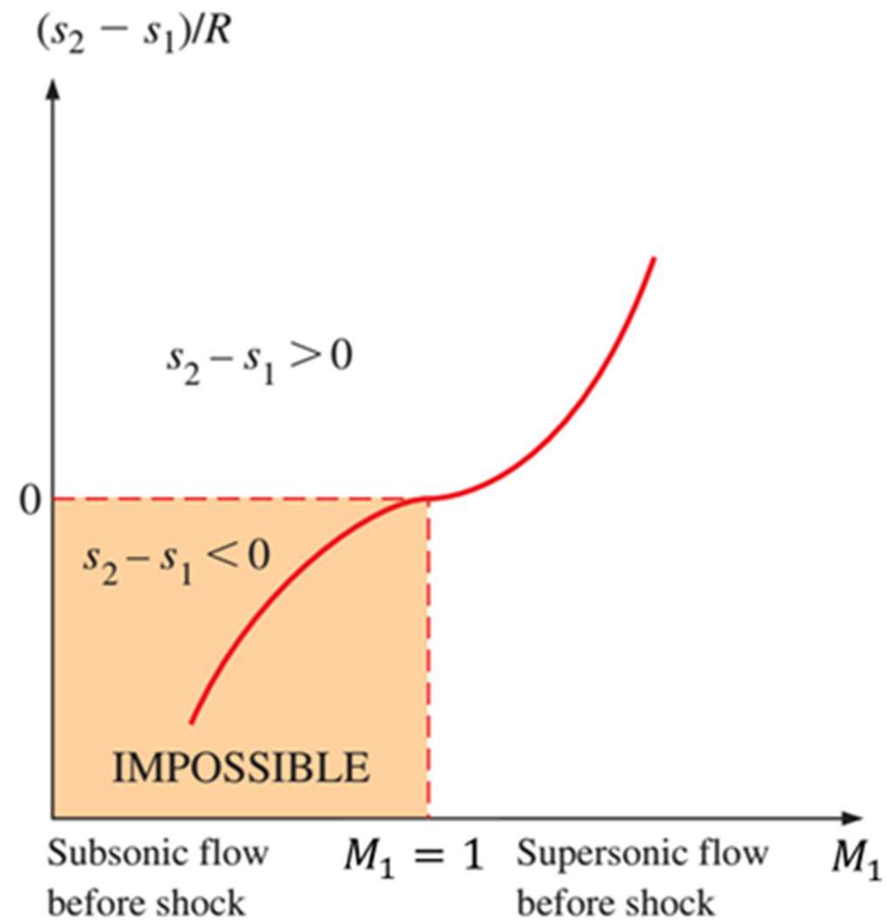
# Shock Waves and Expansion

**Variation of flow properties across a normal shock in an ideal gas.**

Entropy change across the shock

$$ds = c_p \frac{dT}{T} - R\frac{dP}{P}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$



# Shock Waves and Expansion

**Example 6**

If the air flowing through the converging–diverging nozzle of Example 5 experiences a normal shock wave at the nozzle exit plane (Figure E-6), determine the following after the shock: (a) the stagnation pressure, static pressure, static temperature, and static density; (b) the entropy change across the shock; (c) the exit velocity; and (d ) the mass flow rate through the nozzle. Approximate the flow as steady, one-dimensional, and isentropic with $k = 1.4$ from the nozzle inlet to the shock location.

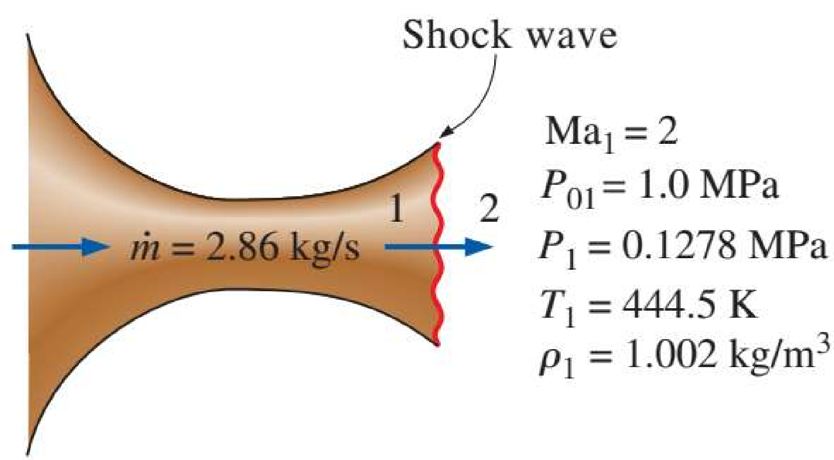



Figure E-6

# Shock Waves and Expansion

**TABLE A–14**

One-dimensional normal shock functions for an ideal gas with $k = 1.4$

| $Ma_1$ | $Ma_2$ | $P_2/P_1$ | $\rho_2/\rho_1$ | $T_2/T_1$ | $P_{02}/P_{01}$ | $P_{02}/P_1$ |
|---|---|---|---|---|---|---|
| 1.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.8929 |
| 1.1 | 0.9118 | 1.2450 | 1.1691 | 1.0649 | 0.9989 | 2.1328 |
| 1.2 | 0.8422 | 1.5133 | 1.3416 | 1.1280 | 0.9928 | 2.4075 |
| 1.3 | 0.7860 | 1.8050 | 1.5157 | 1.1909 | 0.9794 | 2.7136 |
| 1.4 | 0.7397 | 2.1200 | 1.6897 | 1.2547 | 0.9582 | 3.0492 |
| 1.5 | 0.7011 | 2.4583 | 1.8621 | 1.3202 | 0.9298 | 3.4133 |
| 1.6 | 0.6684 | 2.8200 | 2.0317 | 1.3880 | 0.8952 | 3.8050 |
| 1.7 | 0.6405 | 3.2050 | 2.1977 | 1.4583 | 0.8557 | 4.2238 |
| 1.8 | 0.6165 | 3.6133 | 2.3592 | 1.5316 | 0.8127 | 4.6695 |
| 1.9 | 0.5956 | 4.0450 | 2.5157 | 1.6079 | 0.7674 | 5.1418 |
| 2.0 | 0.5774 | 4.5000 | 2.6667 | 1.6875 | 0.7209 | 5.6404 |
| 2.1 | 0.5613 | 4.9783 | 2.8119 | 1.7705 | 0.6742 | 6.1654 |
| 2.2 | 0.5471 | 5.4800 | 2.9512 | 1.8569 | 0.6281 | 6.7165 |
| 2.3 | 0.5344 | 6.0050 | 3.0845 | 1.9468 | 0.5833 | 7.2937 |
| 2.4 | 0.5231 | 6.5533 | 3.2119 | 2.0403 | 0.5401 | 7.8969 |
| 2.5 | 0.5130 | 7.1250 | 3.3333 | 2.1375 | 0.4990 | 8.5261 |
| 2.6 | 0.5039 | 7.7200 | 3.4490 | 2.2383 | 0.4601 | 9.1813 |
| 2.7 | 0.4956 | 8.3383 | 3.5590 | 2.3429 | 0.4236 | 9.8624 |
| 2.8 | 0.4882 | 8.9800 | 3.6636 | 2.4512 | 0.3895 | 10.5694 |
| 2.9 | 0.4814 | 9.6450 | 3.7629 | 2.5632 | 0.3577 | 11.3022 |
| 3.0 | 0.4752 | 10.3333 | 3.8571 | 2.6790 | 0.3283 | 12.0610 |
| 4.0 | 0.4350 | 18.5000 | 4.5714 | 4.0469 | 0.1388 | 21.0681 |
| 5.0 | 0.4152 | 29.000 | 5.0000 | 5.8000 | 0.0617 | 32.6335 |
| $\infty$ | 0.3780 | $\infty$ | 6.0000 | $\infty$ | 0 | $\infty$ |

# Shock Waves and Expansion: Oblique Shocks

**Oblique Shocks**

Conservation of mass:

$$\rho_1 \ V_{1,n} = \rho_2 \ V_{2,n}$$

Conservation of energy:

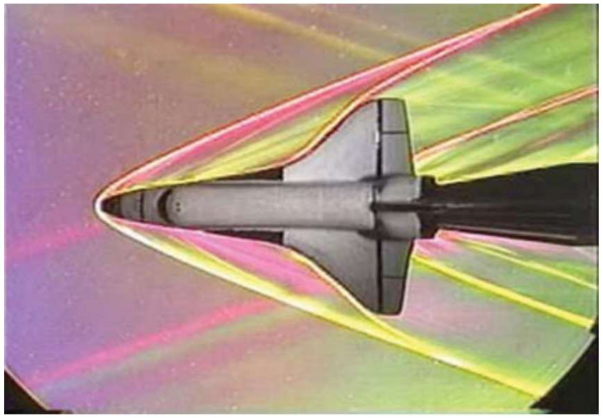
$$h_1 + \frac{V_{1,n}^2}{2} = h_2 + \frac{V_{2,n}^2}{2}$$

Linear momentum equation:

$$P_1 - P_2 = \rho_2 V_{2,n}^2 - \rho_1 V_{1,n}^2$$

Increase of entropy:

$$s_2 - s_1 \ \geq 0$$



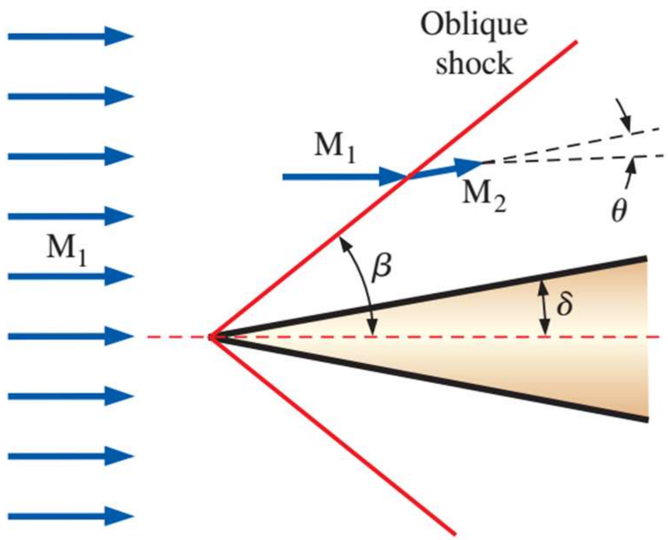
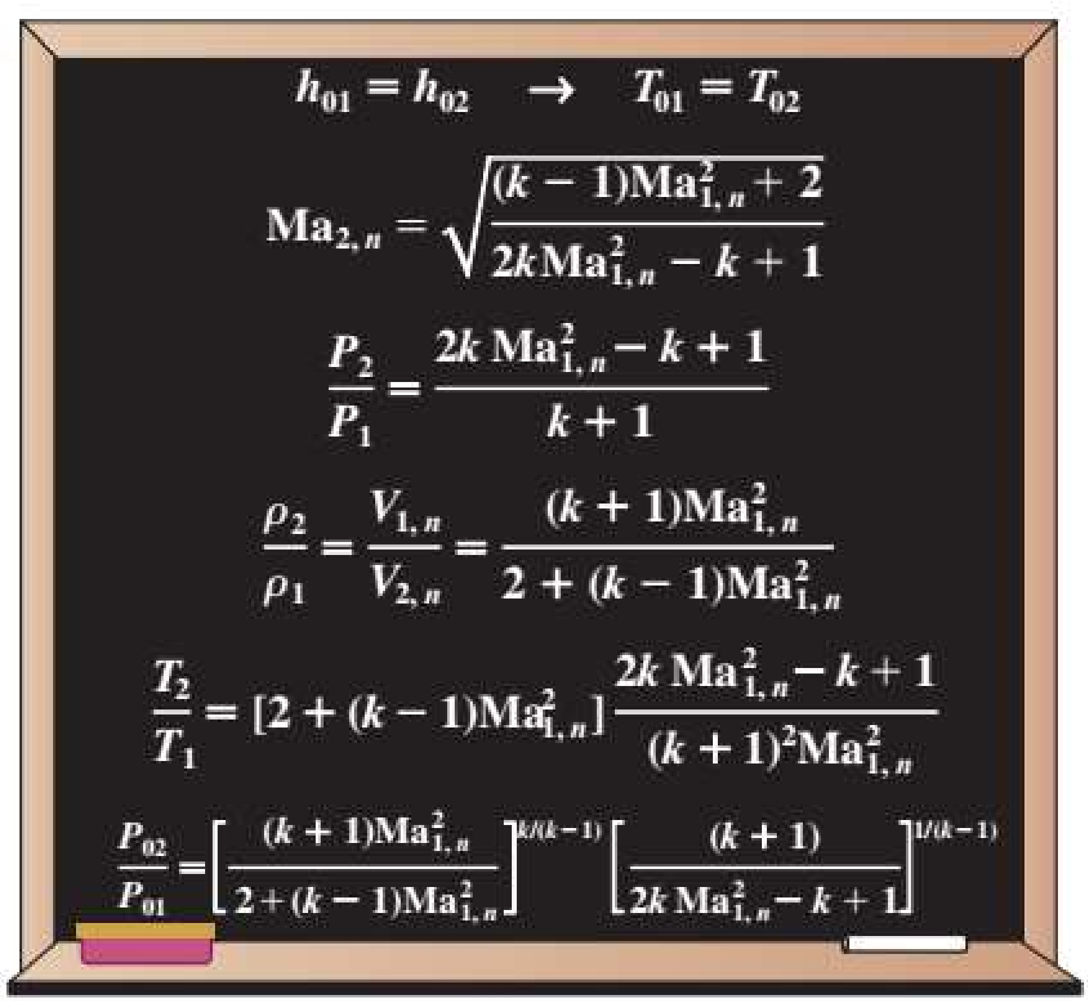



Figure E-6

# Shock Waves and Expansion:

# Oblique Shocks

Relationships across an **oblique shock** for an ideal gas in terms of the normal component of upstream Mach number $M_{1,n}$.



# Shock Waves and Expansion: Oblique Shocks

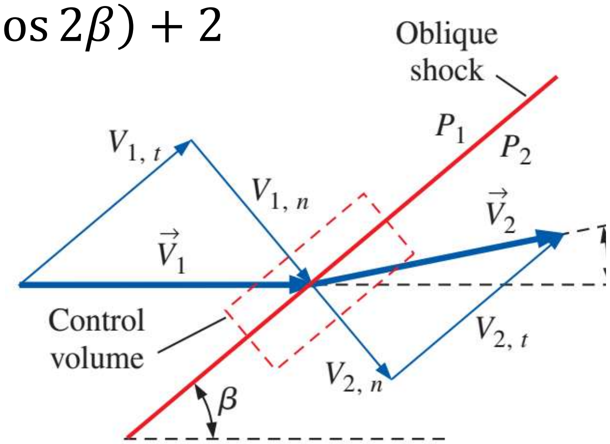
## Oblique Shocks

$$M_{1,n} = M_1 \sin\beta \qquad M_{2,n} = M_2 \sin(\beta - \theta)$$

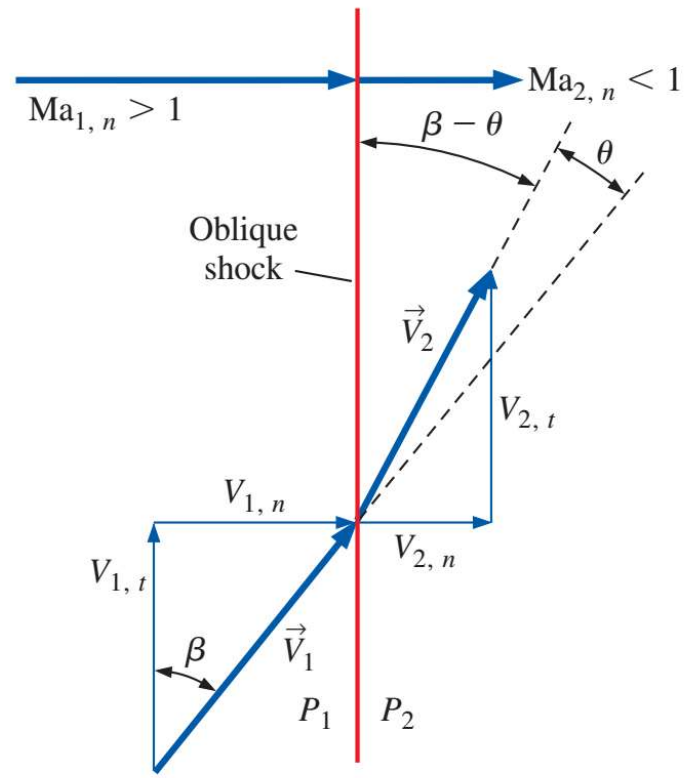
The $\theta - \beta - M$ relationship:

$$\tan\theta = \frac{2\cot\beta\,(M_1^2 \sin^2\beta - 1)}{M_1^2(k + \cos 2\beta) + 2}$$

NOTE: All the equations, shock tables, etc., for normal shocks apply to oblique shocks as well, provided that we use only the normal components of the Mach number.



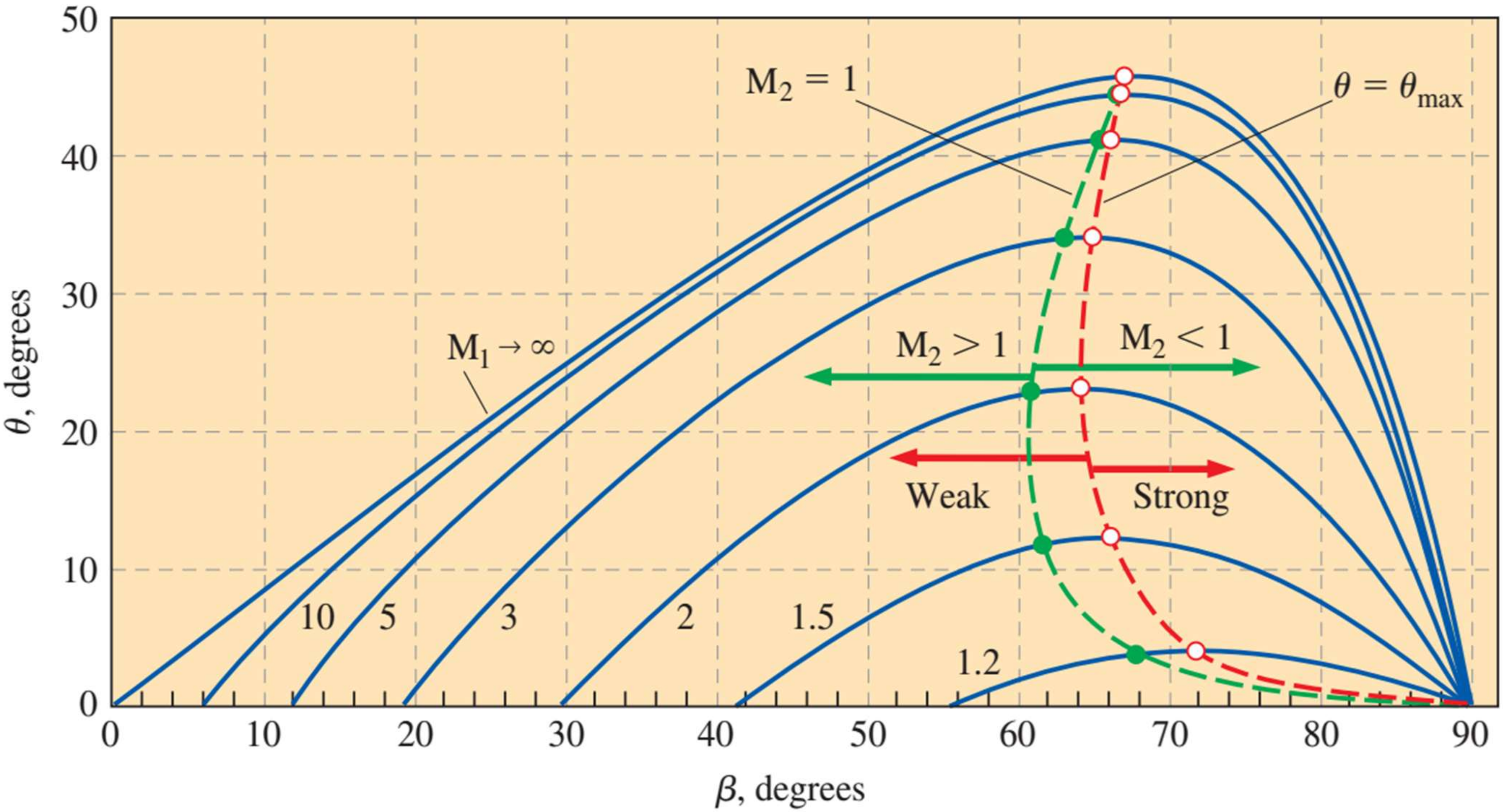

Velocity vectors through an oblique shock




Velocity vectors rotated by angle $\pi/2 - \beta$

# Shock Waves and Expansion: Oblique Shocks

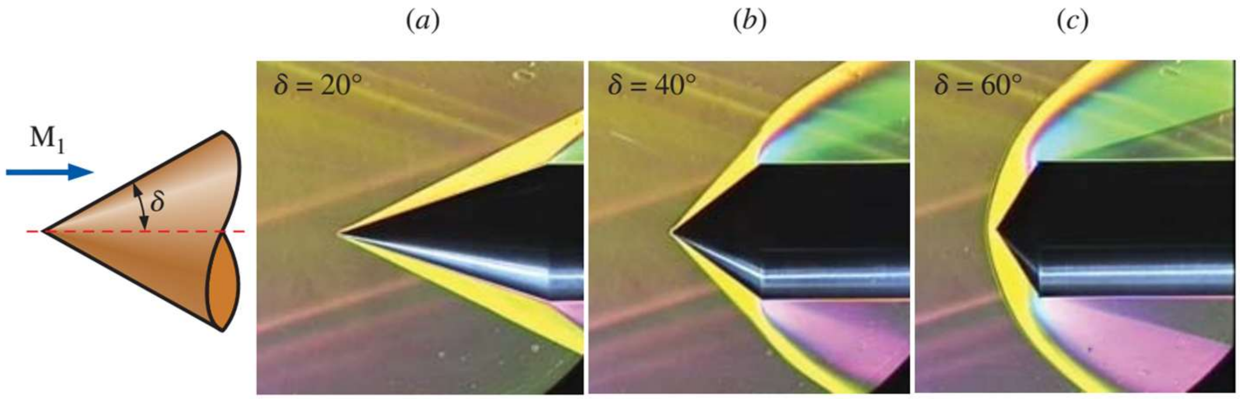
The dependence of straight oblique shock deflection angle $\theta$ on shock angle $\beta$ for several values of upstream Mach number $M_1$.



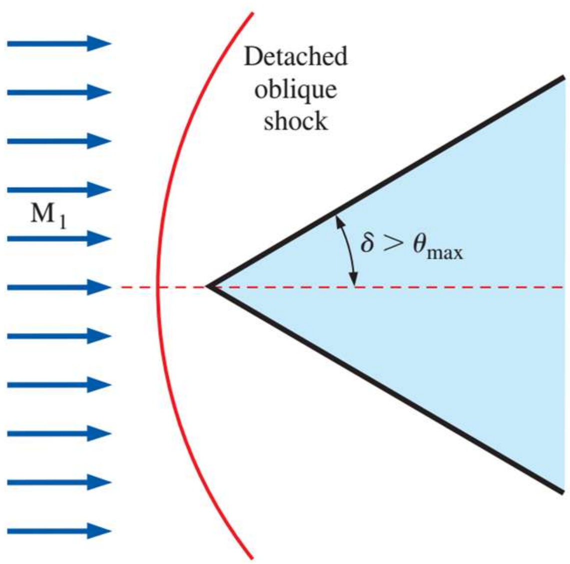
# Shock Waves and Expansion:
# Oblique Shocks

Mach angle:

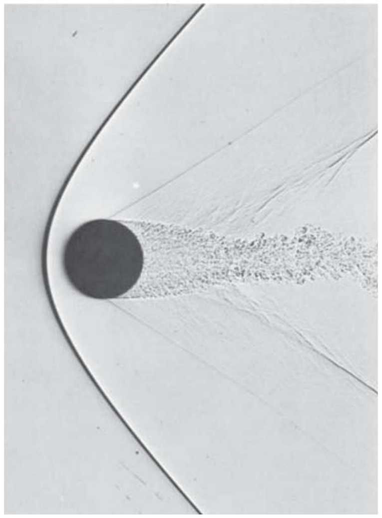
$$\mu = \sin^{-1}\left(\frac{1}{M_1}\right)$$




Schlieren videography illustrating shock Detachment from a Cone at Mach 3




Shock Detachment over a Wedge

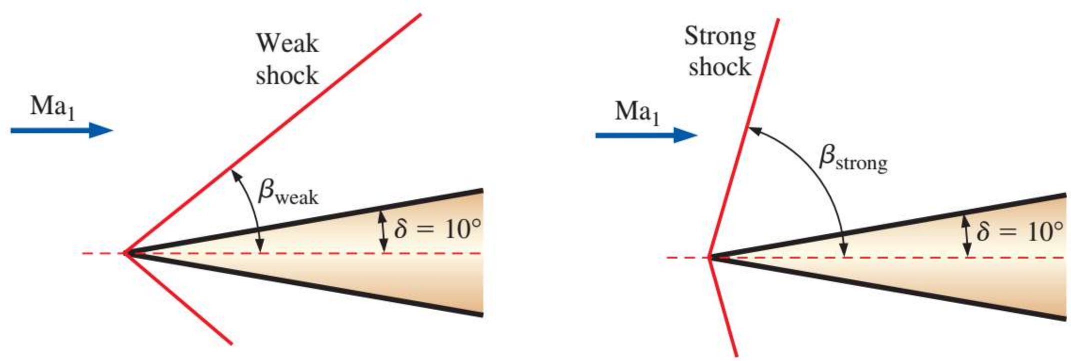


Sphere Shadowgram at Mach 1.53

# Shock Waves and Expansion: Oblique Shocks

**Example 7**

Supersonic air at $M_1 = 2.0$ and 75.0 kPa impinges on a two-dimensional wedge of half-angle $\delta = 10°$ (Figure E-7). Calculate the two possible oblique shock angles, $\beta_{\text{weak}}$ and $\beta_{\text{strong}}$, that could be formed by this wedge. For each case, calculate the pressure and Mach number downstream of the oblique shock, compare, and discuss.



(a) $\beta_{\text{weak}}$ (b) $\beta_{\text{strong}}$

Figure E-7

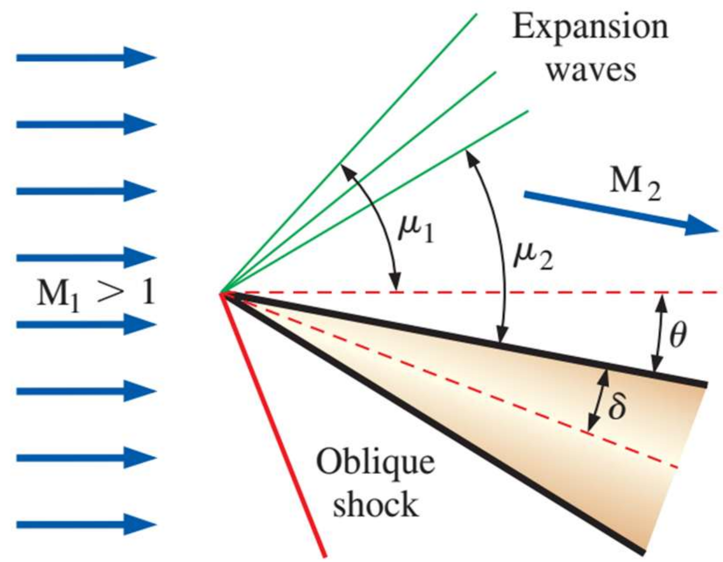
# Shock Waves and Expansion: Expansion Waves

**Prandtl-Mayer Expansion Waves**

**Turning angle across an expansion fan:**

$$\theta = \nu(M_2) - \nu(M_1)$$

$\nu(M)$: is an angle called Prandtl-Meyer function




$$\nu(M) = \sqrt{\frac{k+1}{k-1}} \tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right)$$

$\nu(M)$ is the angle through which the flow must expand, starting with $\nu = 0$ at $M = 1$, in order to reach a supersonic Mach number, $M > 1$.

# Shock Waves and Expansion:
# Expansion Waves

**Example 8**

Supersonic air at $M_1 = 2.0$ and 230 kPa flows parallel to a flat wall that suddenly expands by $\delta = 10°$ (Figure E-8). Ignoring any effects caused by the boundary layer along the wall, calculate downstream Mach number $M_2$ and pressure $P_2$.

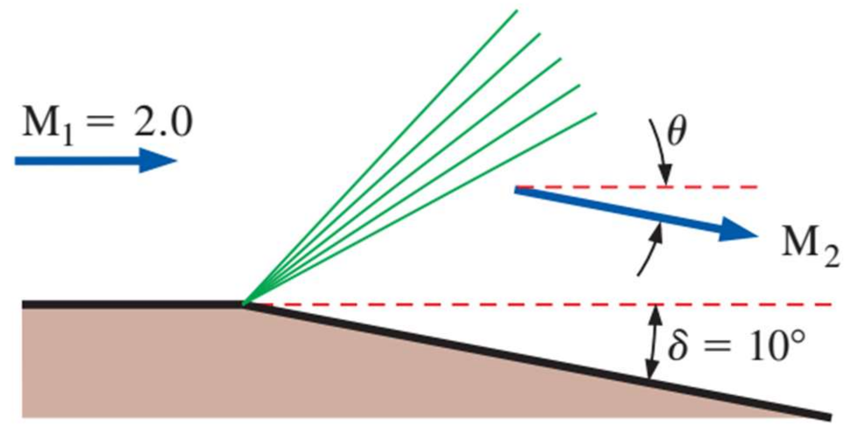



Figure E-8

# References

[1] Cengel Y., Cimbala, J. (2014). Fluid Mechanics: Fundamentals and Applications (3th Edition). New York: NY: McGraw-Hill Co.

[2] Munson, B.R., Young, D.F., Okiishi, T.H., and Huebsch, W.W. (2016). Fundamentals of Fluid Mechanics (8th Edition). John Wiley & Sons. ISBN 1119080703.