Forces on Surfaces and Buoyancy

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ENGI 2420

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- Hydrostatic Forces on Submerged Plane Surfaces
- Hydrostatic Forces on Submerged Curved Surfaces
- Buoyancy and Stability

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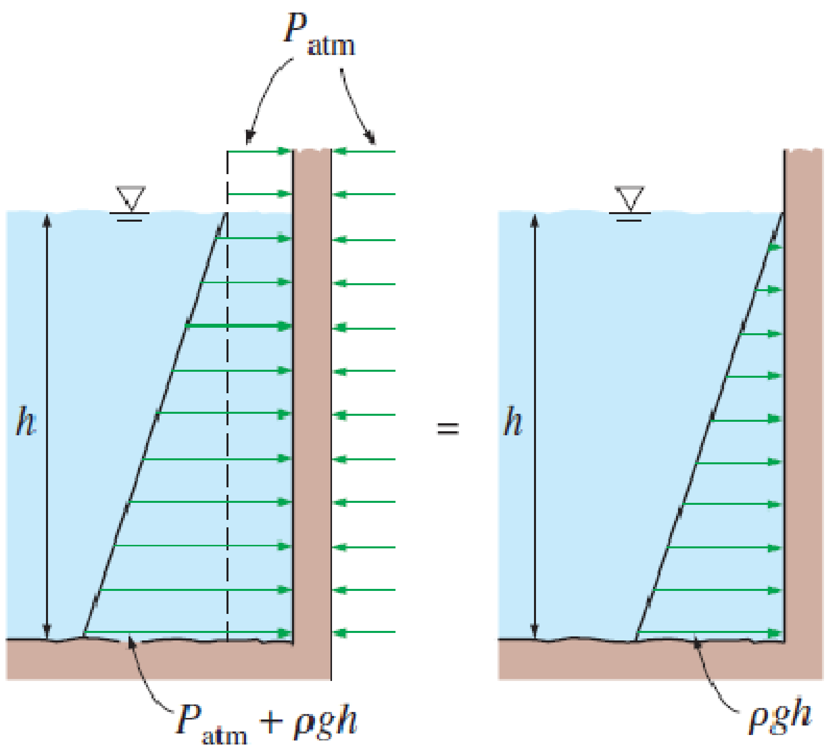
- **Hydrostatc Forces on Submerged Plane Surfaces**

Hydrostatic Forces on Submerged Plane Surfaces



(*a*) P_{atm} considered (*b*) P_{atm} subtracted

Hydrostatic Forces on Submerged Plane Surfaces

Absolute pressure at any point on the plate

$$P = P_0 + \rho g h$$

h : Is the vertical distance of the point from the free surface.

P_0 **:** Usually atmospheric pressure.

Resultant hydrostatic force (F_R)

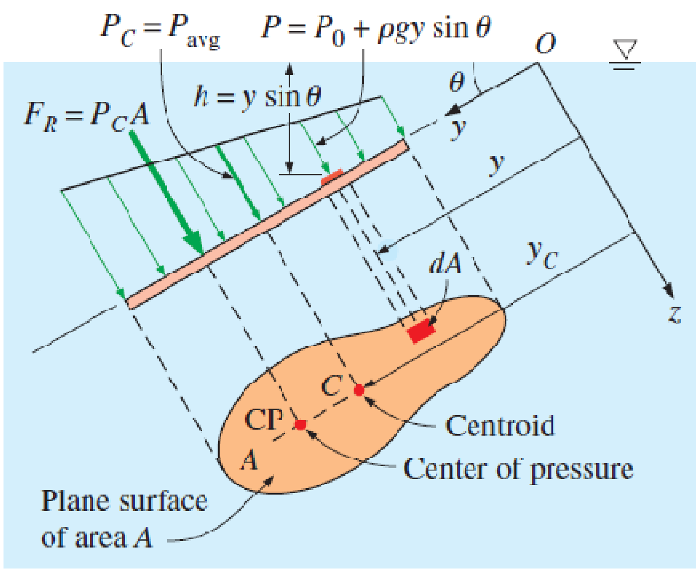
$$F_R = \left(P_0 + \rho g\, y_C \sin\theta\right)A$$

Where $y_C = \frac{1}{A}\int_A y\,dA$

$\int_A y\,dA$ **:** First moment of the area.

y_C **:** y-coordinate of the centroid.

Center of Pressure (CP) is the point of application of the resultant force.



Hydrostatic force on an inclined plane surface.

Hydrostatic Forces on Submerged Plane Surfaces

Magnitude of the resultant force (F_R)

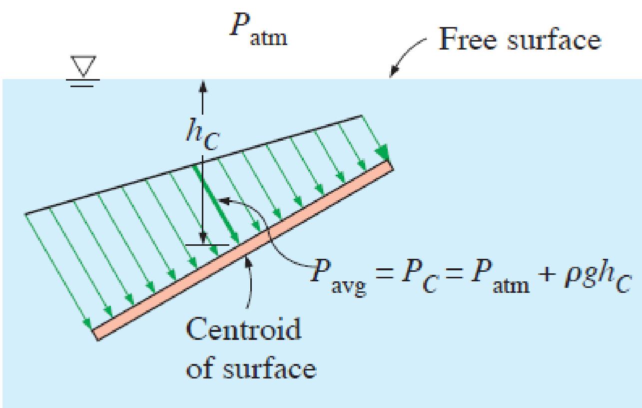
$$F_R = P_C A$$

where

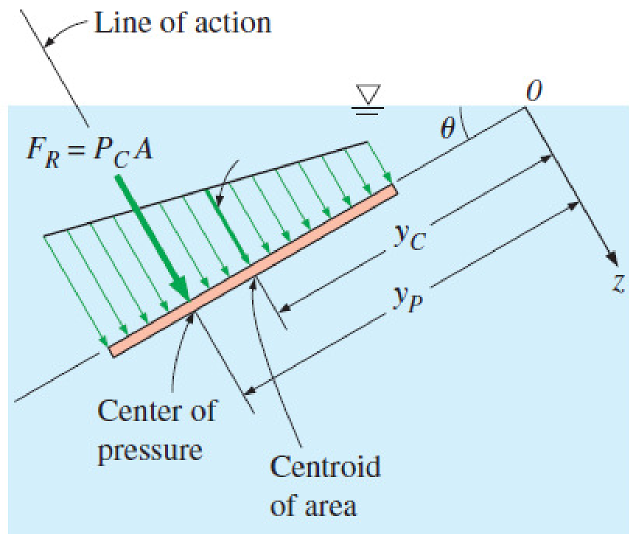
$$P_C = P_0 + \rho g y_C \sin\theta$$
$$= P_0 + \rho g h_C$$

P_C : pressure at the centroid.

A : surface area.



Pressure at the centroid of a plane Surface.



Resultant force acting on a plane surface.

Hydrostatic Forces on Submerged Plane Surfaces

Location of the line of action

$$\sum M_O = 0 \quad \Rightarrow \quad y_P F_R = P_0 y_C A + \rho g \sin\theta \, I_{xx,O}$$

y_p : is the distance of the center of pressure from the x-axis.

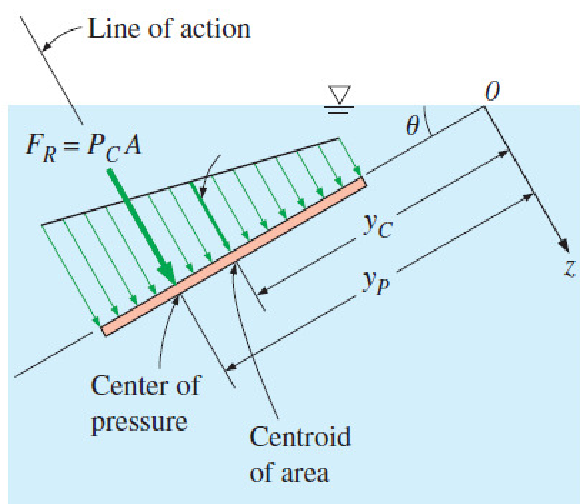
$I_{xx,O} = \int_A y^2 dA$: The second moment of area about point O (or Area moment of inertia).

The second moments of area about two parallel axes are related to each other by the parallel axis theorem.

$$I_{xx,O} = I_{xx,C} + y_C^2 A$$

$I_{xx,C}$: The second moment of area about the centroid.

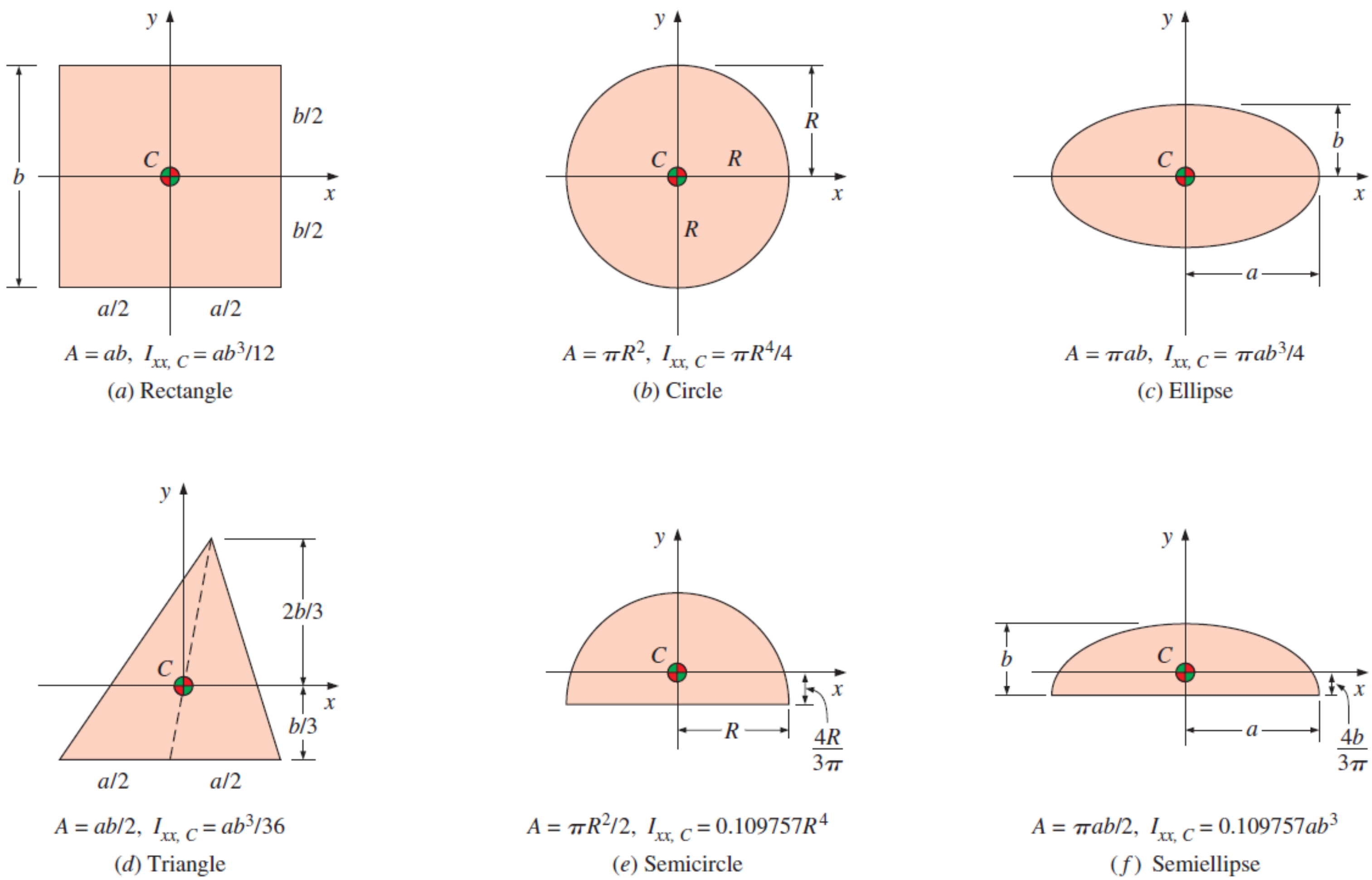
y_C : Distance between two parallel axis (y-coordinate of the centroid).



Vertical distance of the center of pressure

if $P_0 = 0 \quad \Rightarrow \quad \boxed{h_p = y_p \sin\theta}$ where $\boxed{y_p = y_C + \frac{I_{xx,C}}{y_C A}}$

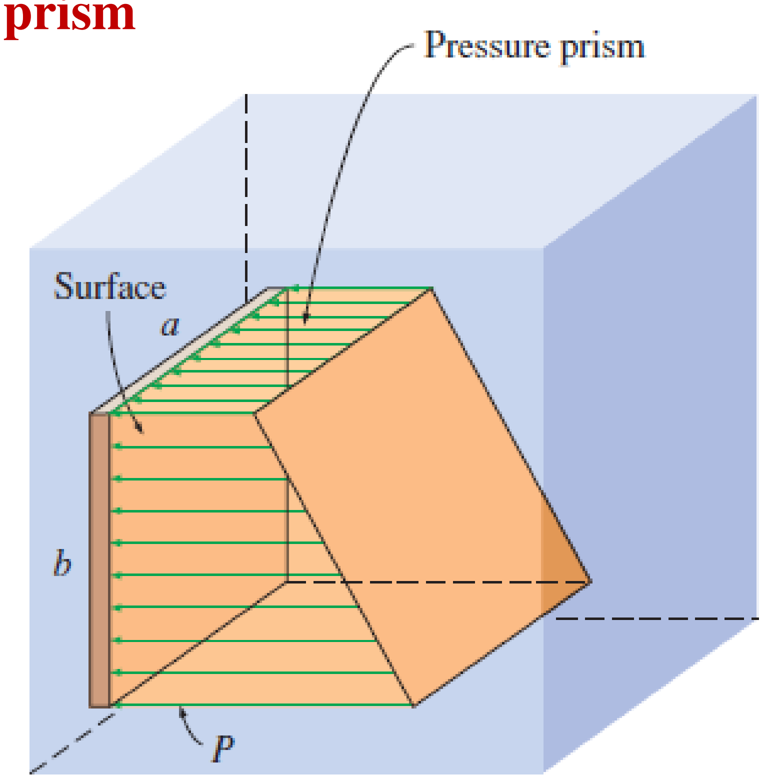
Hydrostatic Forces on Submerged Plane Surfaces



The centroid and the centroidal moments of inertia for some common geometries.

Hydrostatic Forces on Submerged Plane Surfaces

Pressure prism



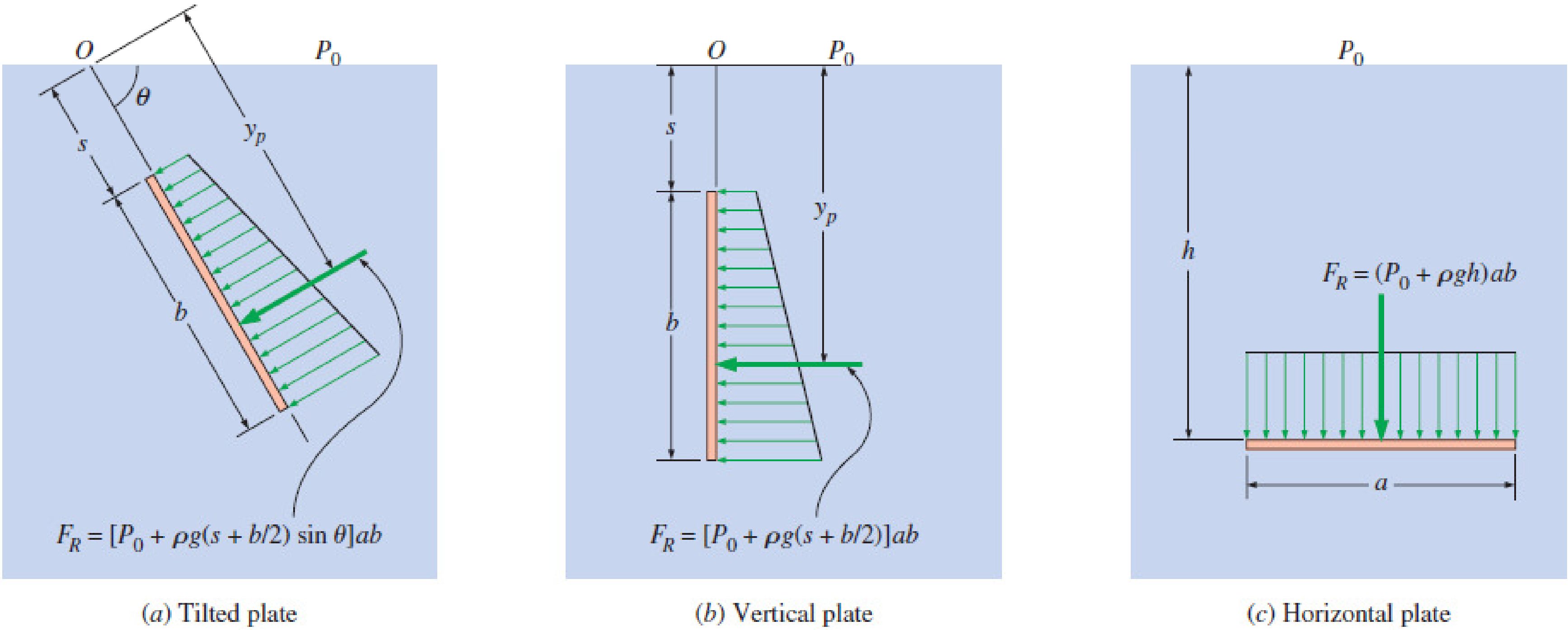
$$F_R = \text{V}\!\!\!/$$
$$= \int P dA$$

V̶ : Volume of pressure prism.

The hydrostatic forces acting on a plane surface form a pressure prism.

Hydrostatic Forces on Submerged Plane Surfaces

Special Case: Submerged Rectangular Plate



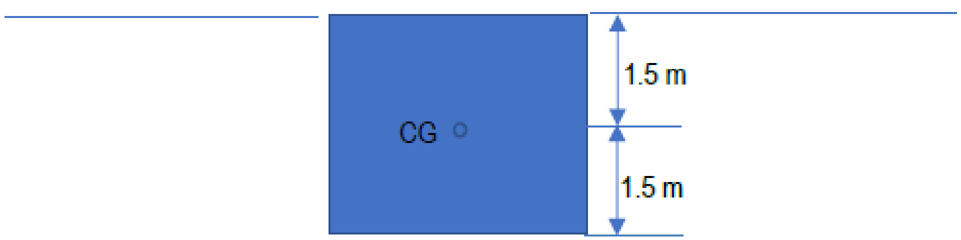
Hydrostatic force acting on the top surface of a submerged rectangular plate.

Hydrostatic Forces on Submerged Plane Surfaces

Example 3.1

A square gate 3 m x 3 m lies in a vertical plane. Determine the Hydrostatic force on the gate and the location of the pressure center when the upper edge of the gate is at the water surface. Compare with values when the upper edge is 15 m below the water surface. Answer: a) F_R=0.13 MN; $h_p = y_p$ = 2 m

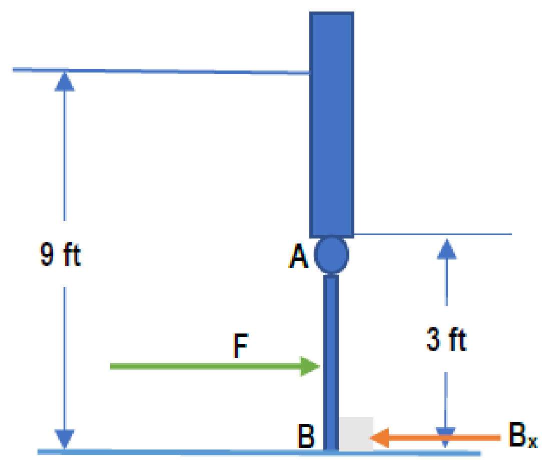
b) F_R=1.46 MN; $h_p = y_p$ = 16.76 m



Hydrostatic Forces on Submerged Plane Surfaces

Example 3.2

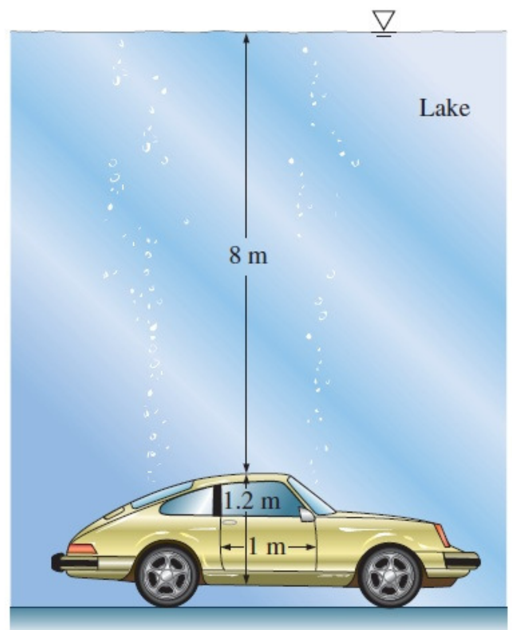
Gate AB in Figure is 5 ft wide, hinged at point A, and restrained by a stop at point B. Compute the hydrostatic force on the gate and the center of pressure if water depth *h* is 9 ft. What is the magnitude of reaction B_x? Answer = 3.74 klbf



Hydrostatic Forces on Submerged Plane Surfaces

Example 3.4

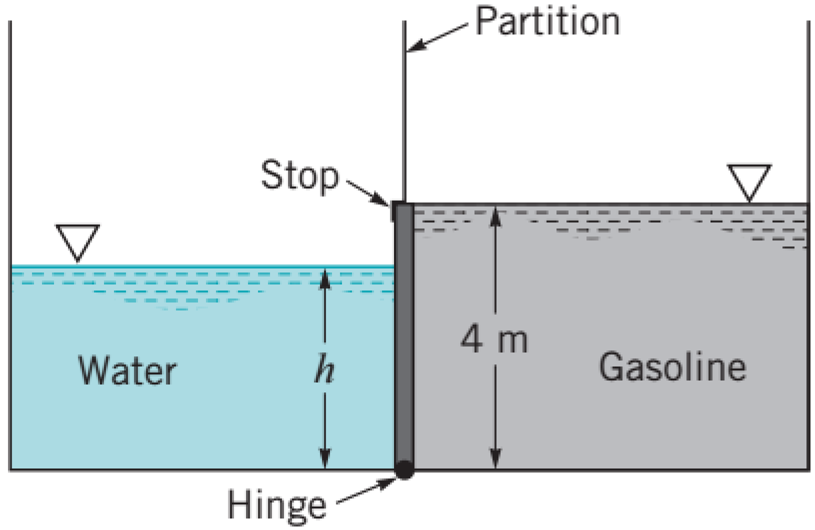
A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels. The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door. Answer: F_R = 101.3 kN, h_p = 8.61m



Hydrostatic Forces on Submerged Plane Surfaces

Example 3.3

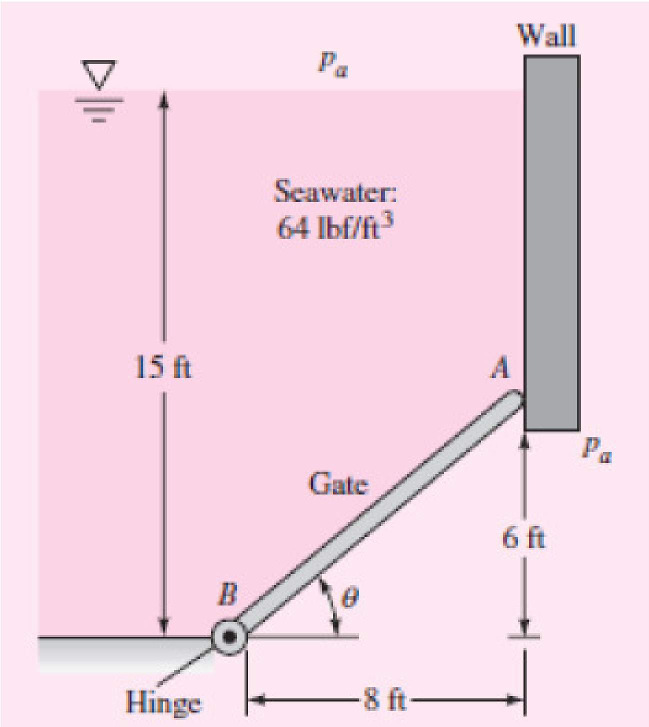
An open tank has a vertical partition and on one side contains gasoline with a density ρ =700 kg/m^3 at a depth of 4 m, as shown in the Fig. below. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?



Hydrostatic Forces on Submerged Plane Surfaces

Example 3.5

The gate in the Figure below is 5 ft wide, is hinged at point B and rests against a smooth wall at point A. Compute a) the force on the gate due to seawater pressure; b) the horizonal force P exerted by the wall at point A. Answer: F_R =38400lbf, P = 29,334 lbf



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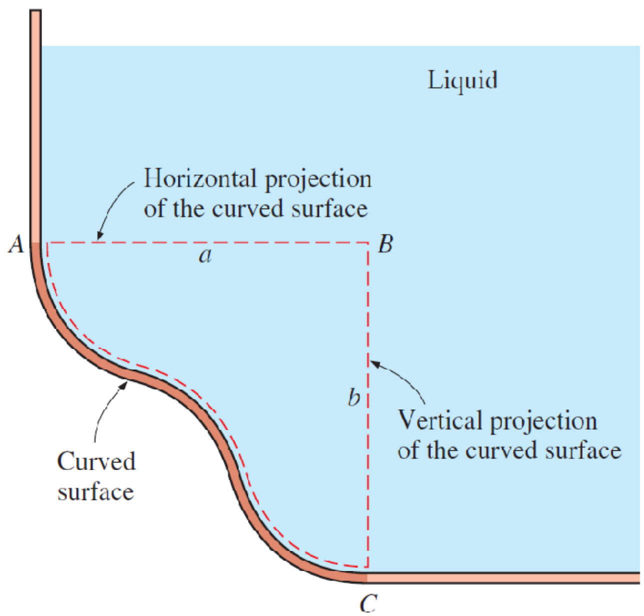
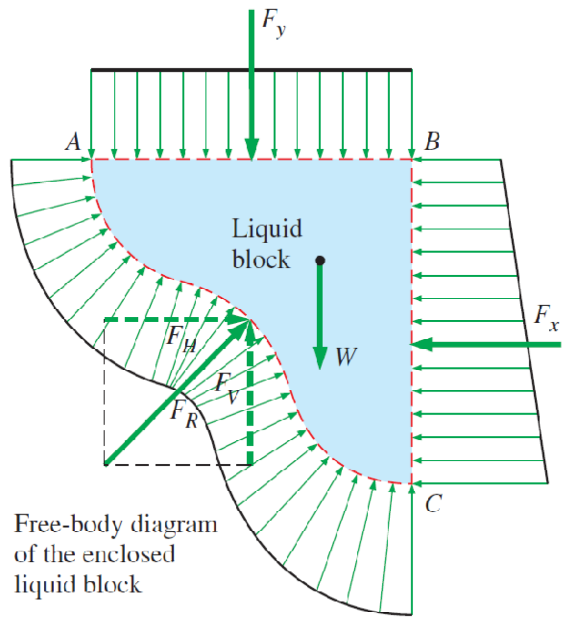
- Hydrostatic Forces on Submerged Plane Surfaces
- **Hydrostatic Forces on Submerged Curved Surfaces**

Hydrostatic Forces on Submerged Curved Surfaces



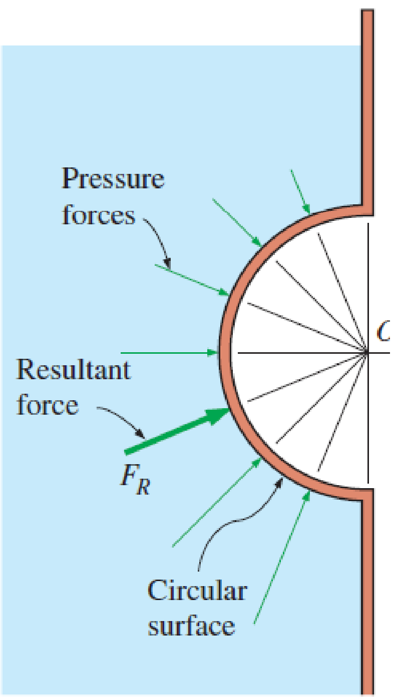


Determination of the hydrostatic force acting on a submerged curved surface.

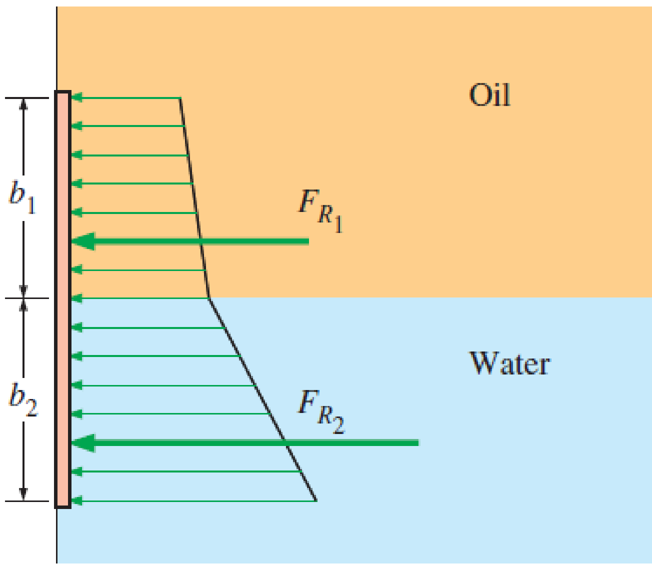
Horizontal force component on curved surface: $F_H = F_x$

Vertical force component on curved surface: $F_V = F_y + W$

Hydrostatic Forces on Submerged Curved Surfaces



The hydrostatic force acting on a circular surface.



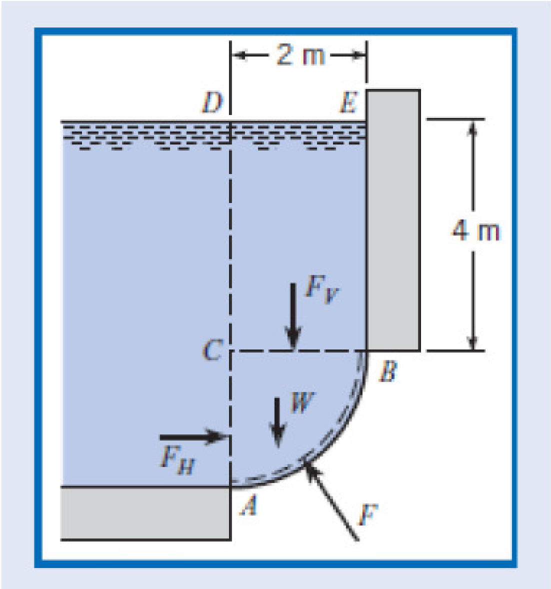
The hydrostatic force on a surface submerged in a multilayered fluid

$$F_R = \sum F_{R,i} = \sum P_{C,i} A$$

Forces on Submerged Surfaces in Static Fluids

Example 3.6

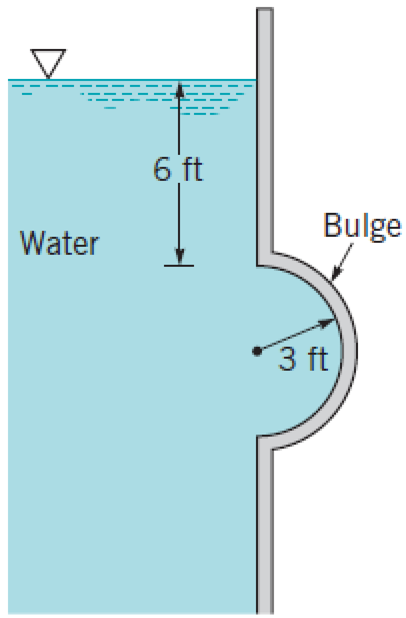
Surface AB is a circular arc with a radius of 2 m and a width of 1 m. The distance EB is 4 m. The fluid above surface AB is water, and atmospheric pressure prevails on the free surface of the water and on the bottom side of surface AB. Find the magnitude and line of action of the hydrostatic force acting on surface AB. Answer: 146.9 kN, θ=48°



Forces on Submerged Surfaces in Static Fluids

Example 3.7

An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in the Figure below. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge. Answer: 3370 lbf, 882 lbf



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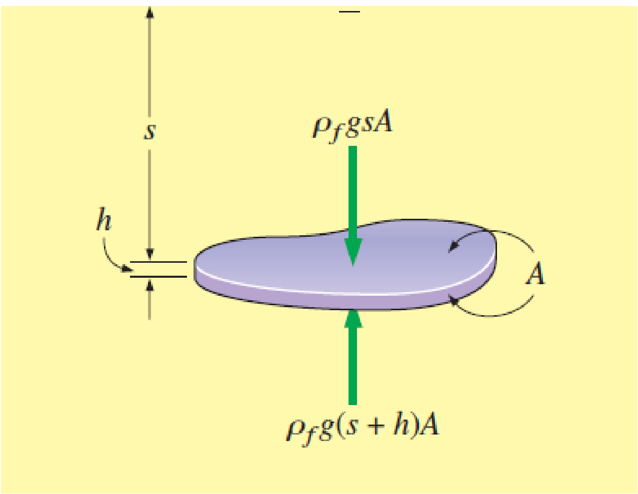
- Hydrostatic Forces on Submerged Curved Surfaces
- **Buoyancy and Stability**

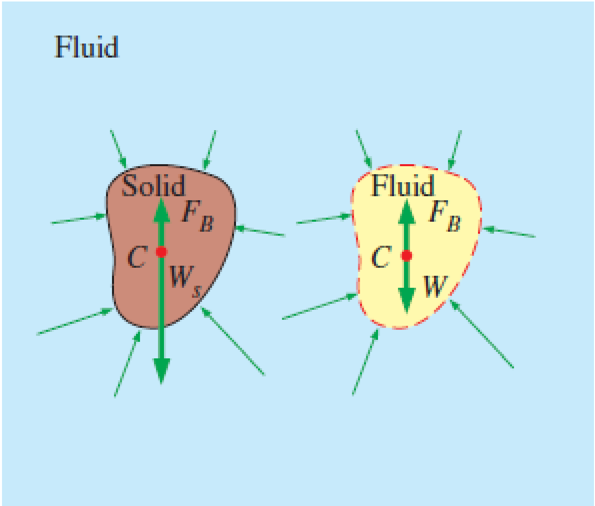
Buoyancy and Stability

The buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.



The buoyant forces acting on a solid body submerged in a fluid

$$F_B = F_{bottom} - F_{top}$$
$$= \rho_f g V\!\!\!/$$



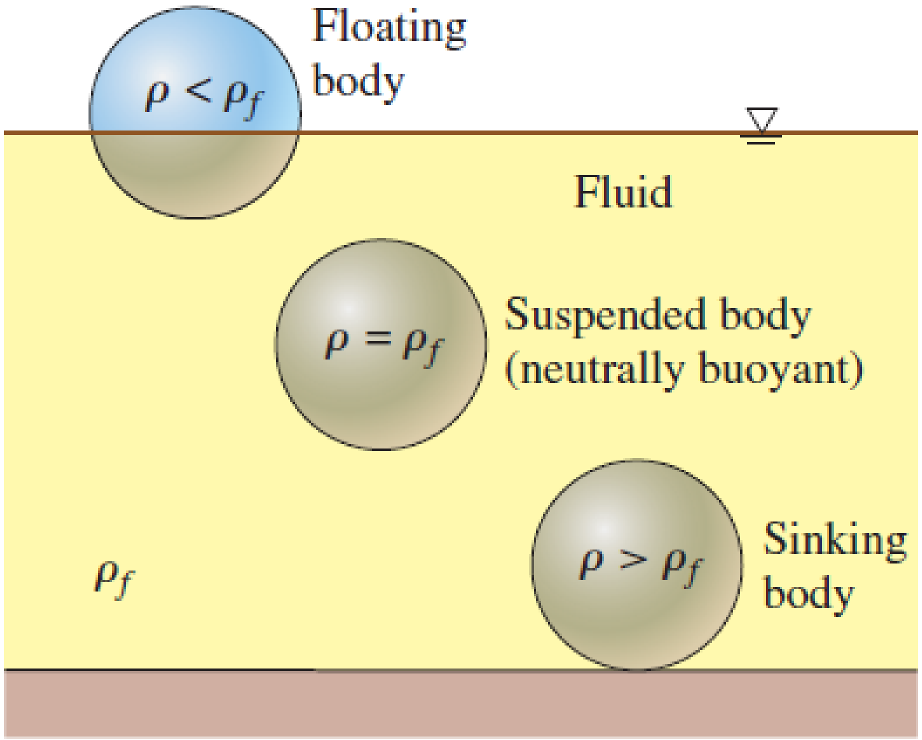
The buoyant forces acting on a solid body submerged in a fluid

$$F_B = W$$

$$W_s > W \quad \Rightarrow \quad W_s > F_B$$

Buoyancy and Stability

Note : A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.



Floating bodies

$$F_B = W$$

$$\rightarrow \quad \rho_f \, g V_{sub} = \rho_{avg,body} \, g V_{total}$$

Buoyancy and Stability

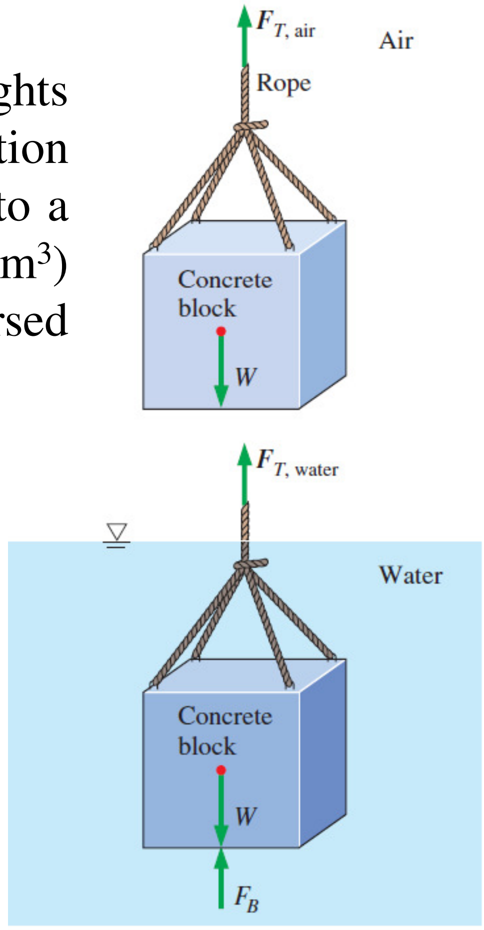
Example 3.9

A solid wooden cube (0.5 m x 0.5 m x 0.5 m) is floating in water and 0.2 m of the cube extends above the water surface. Determine the specific weight of the wood. For equilibrium, $W = F_B$. Answer: 5886 N/m^3

Buoyancy and Stability

Example 3.10

The crane (see the Figure on right side) is used to lower weights into the sea (γ = 10.1 KN/m^3) for an underwater construction project. Determine the tension in the rope of the crane due to a rectangular 0.4 m x 0.4 m x 3 m concrete block (γ = 22.6 KN/m^3) when it is (a) suspended in the air and (b) completely immersed in water.



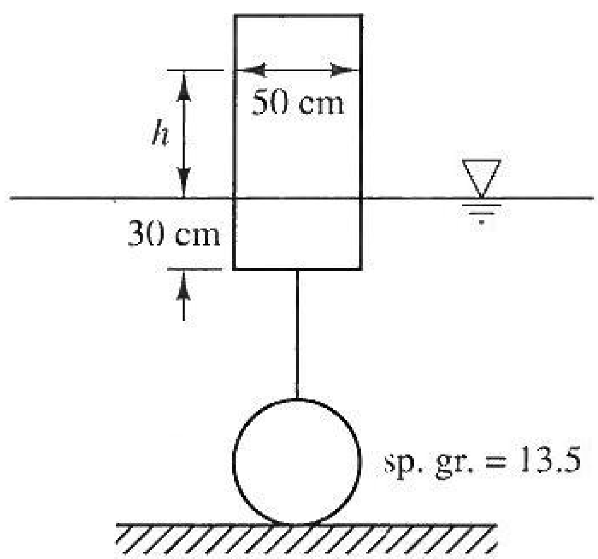
Assumptions

1. The buoyant force in air is negligible.
2. The weight of the ropes is negligible.

Buoyancy and Stability

Example 3.12

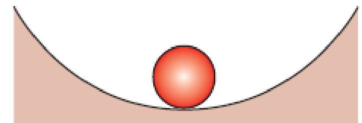
A solid brass sphere (SG = 13.5) of 30-cm diameter is used to weight down a cylindrical buoy in sea water (SG = 1.03). The buoy has a height of 2 m and a SG of 0.45 and is tied to the sphere at one end. What rise in tide, h, will be required to lift the anchor off the bottom? $\forall_{sphere} = \frac{4}{3}\pi r^3$ Answer: 1.2 m



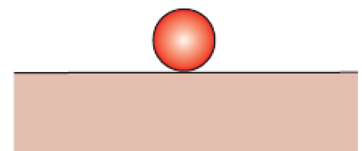
Buoyancy and Stability:

Stability of Immersed and Floating Bodies

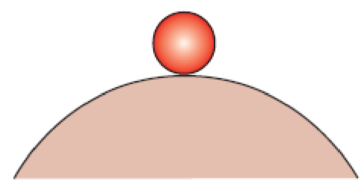
Ship



(*a*) Stable



(*b*) Neutrally stable

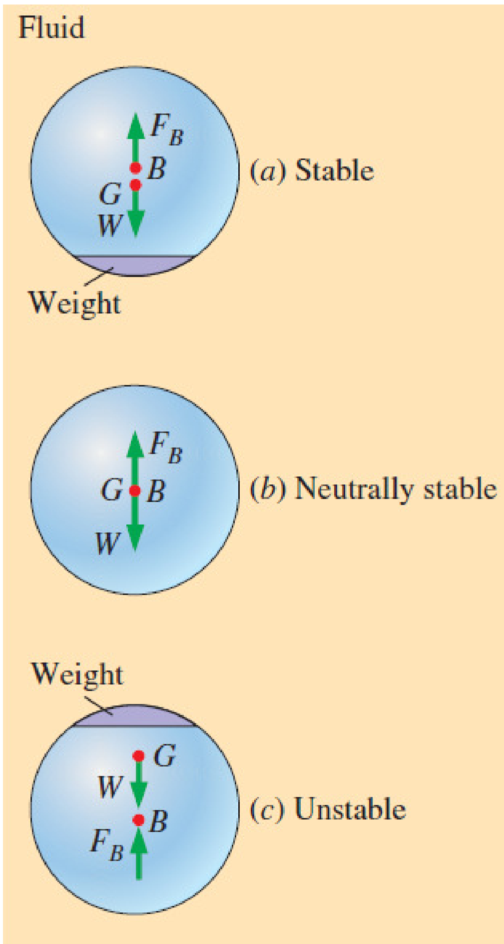


(*c*) Unstable

Stability is easily understood by analyzing a ball on the floor.

Buoyancy and Stability:

Stability of Immersed and Floating Bodies



An immersed neutrally buoyant body is

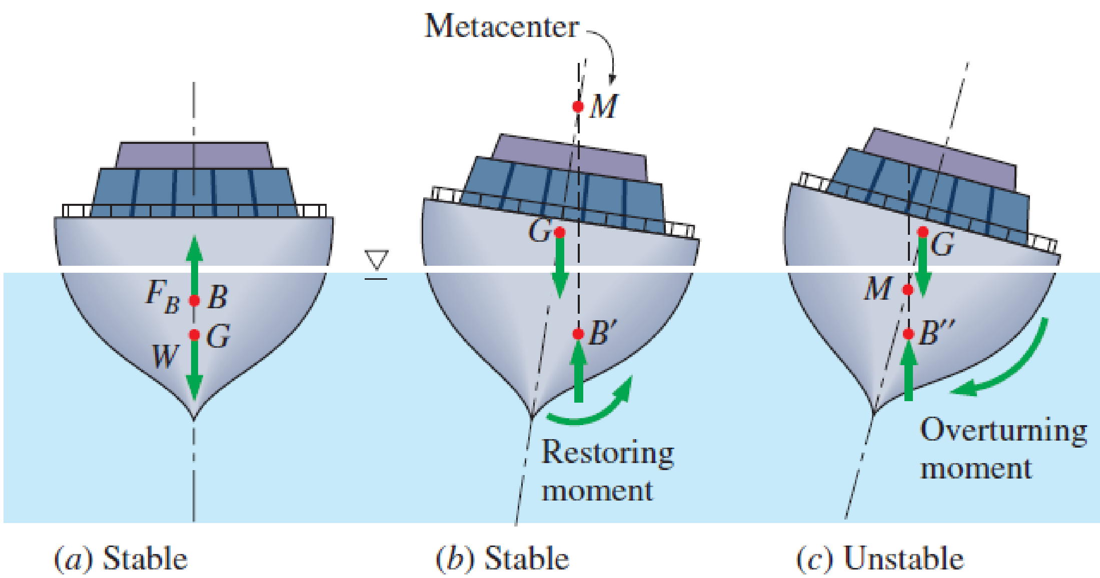
(a) stable if the center of gravity G is directly below the center of buoyancy B of the body,

(b) neutrally stable if G and B are coincident.

(c) Unstable if G is directly above B.

Buoyancy and Stability:

Stability of Immersed and Floating Bodies



Note:
If the **displaced body** has a uniform density, the center of buoyancy will coincide with the centroid (or geometric center).

A floating body is stable if the body is

(a) bottom-heavy and thus the center of gravity G is below the centroid B of the body, or

(b) if the metacenter M is above point G.

(c) However, the body is unstable if point M is below point G.

References

[1] Cengel Y., Cimbala, J. (2014). Fluid Mechanics: Fundamentals and Applications (3th Edition). New York: NY: McGraw-Hill Co.

[2] Munson, B.R., Young, D.F., Okiishi, T.H., and Huebsch, W.W. (2016). Fundamentals of Fluid Mechanics (8th Edition). John Wiley & Sons. ISBN 1119080703.