Fluid Kinematics & The Bernoulli Equation

Instructor: Joaquín Valencia

ENGI 2420

Content

- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - Control Volume and System Representations
  - Reynolds Transport Theorem
- Fluid Dynamics - The Bernoulli Equation
  - Newton’s Second Law
  - F=ma Along a Streamline
  - Static, Stagnation, Dynamic, and Total Pressure
  - Example of use of the Bernoulli Equation
  - The Energy Line and the Hydraulic Grade Line

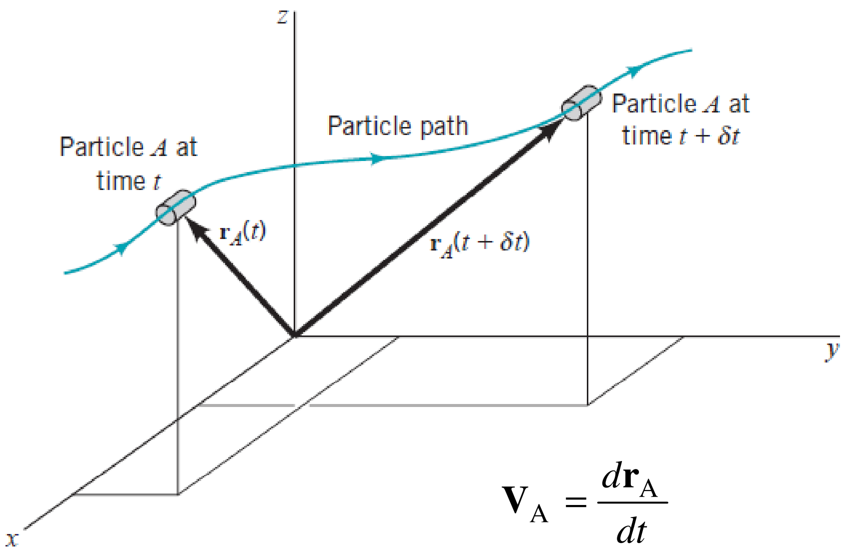
Content

- Fluid Kinematics
  - **The Velocity Field**

The Velocity Field

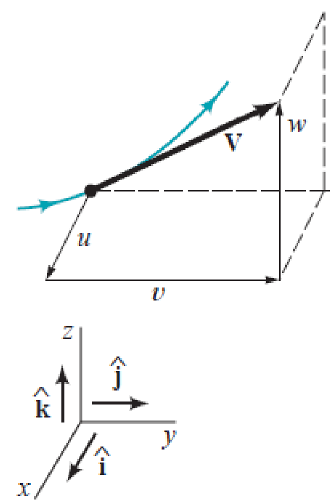
Velocity Field

By definition, the velocity of a particle is the time rate of change of the position vector for that particle.



$$\mathbf{V}_A = \frac{d\mathbf{r}_A}{dt}$$

Particle location in terms of its position vector.



$$\mathbf{V} = u(x, y, z, t)\hat{\mathbf{i}} + v(x, y, z, t)\hat{\mathbf{j}} + w(x, y, z, t)\hat{\mathbf{k}}$$

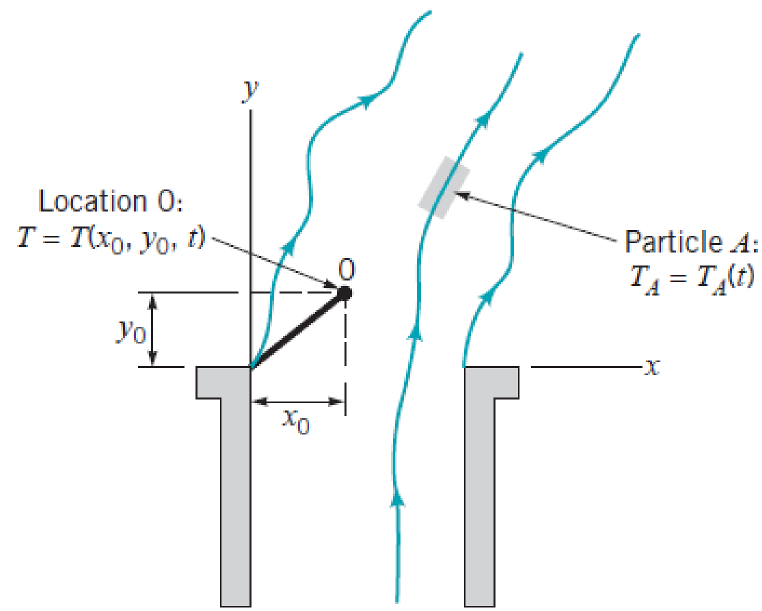
The Velocity Field: Eulerian and Lagrangian Flow Descriptions

Eulerian method

From this method we obtain information about the flow in terms of what happens at fixed points in space as the fluid flows through those points.

Lagrangian method

Involves following individual fluid particles as they move about and determining how the fluid properties associated with these particles change as a function of time.



Eulerian and Lagrangian descriptions of temperature of a flowing fluid.

The Velocity Field: One-, Two-, and Three–Dimensional Flows

Three-dimensional flow

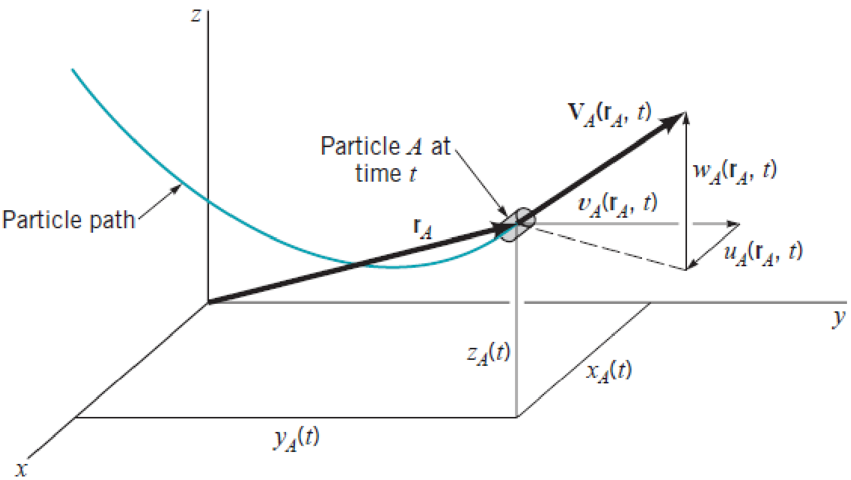
$$\mathbf{V} = \mathbf{V}(x, y, z, t) = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$$

Two-dimensional flow

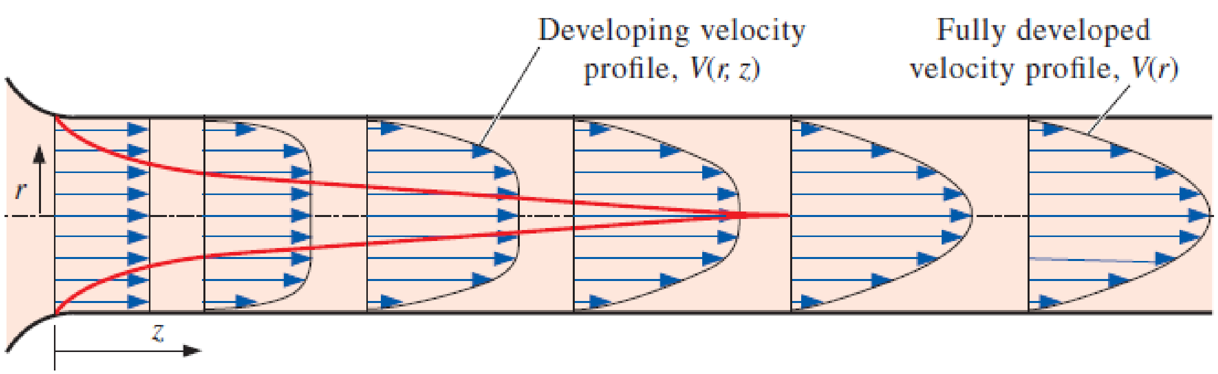
$$\mathbf{V} = \mathbf{V}(x, y, t) = u\hat{\mathbf{i}} + v\hat{\mathbf{j}}$$

One-dimensional flow

$$\mathbf{V} = \mathbf{V}(x, t) = u\hat{\mathbf{i}}$$



Velocity and position of particle *A* at time *t*.



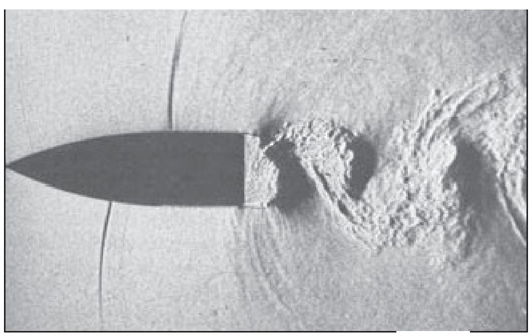
The development of the velocity profile in a circular pipe.

The Velocity Field: Steady and Unsteady Flows

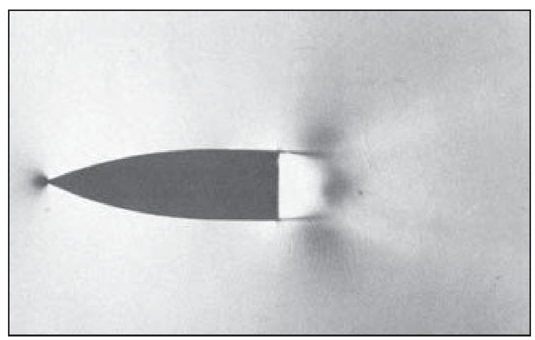
Steady flow — the velocity at a given point in space does not vary with time.

$$\frac{\partial \mathbf{V}}{\partial t} = 0$$

Unsteady flow —the velocity does vary with time.



(a)



(b)

(a) is an instantaneous image.

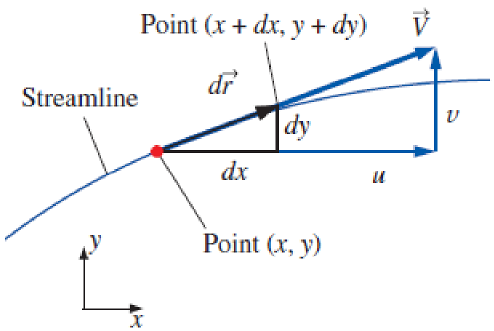
(b) is a long-exposure (time-averaged) image.

The Velocity Field: Streamlines, Streaklines, and Pathlines

Streamlines

A streamline is a curve that is everywhere tangent to the instantaneous local velocity vector. It is often used in analytical work.

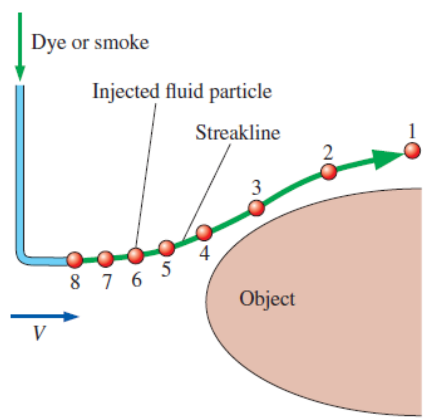
$$\frac{dy}{dx} = \frac{v}{u}$$



Streamline for two-dimensional flow in the *xy*-plane,

Streaklines

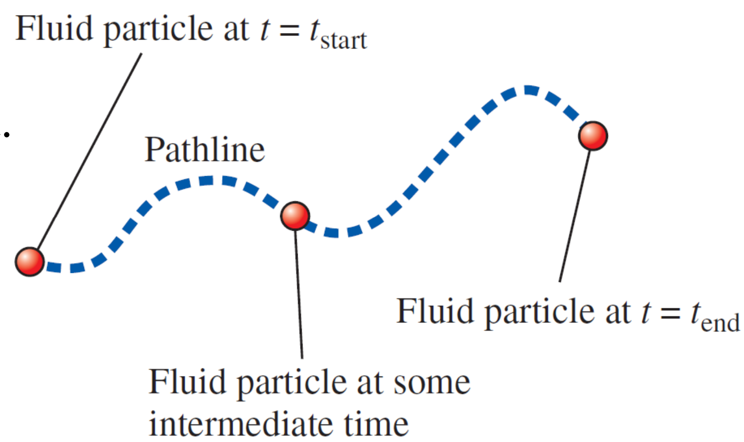
A streakline is the locus of fluid particles that have passed sequentially through a prescribed point in the flow. Streakline is often used in experimental work.



The Velocity Field: Streamlines, Streaklines, and Pathlines

Pathlines

A pathline is the actual path traveled by an individual fluid particle over some time period. A pathline is often used in experimental work



Note : While the three flow patterns are identical in steady flow, they can be quite different in unsteady flow.

The Velocity Field: Streamlines, Streaklines, and Pathlines

Example 4.1

Consider the two-dimensional steady flow given by,

$$\mathbf{V} = \frac{V_0}{l}\left(-x\hat{\mathbf{i}} + y\hat{\mathbf{j}}\right)$$

Determine the streamlines for this flow. Answer: $xy = C$

Plot for C = 1 and C = -1

Content

- Fluid Kinematics
  - The Velocity Field
  - **The Acceleration Field**

The Acceleration Field

The acceleration of a particle is the time rate of change of its velocity.

Material acceleration

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}$$

Where:

∇ : Gradient operator or del operator.

$\mathbf{V}$: Velocity vector.

Acceleration of any particle

Vector form

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

Scalar form

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Steady flow

$$\frac{\partial \mathbf{V}}{\partial t} = 0$$

Unsteady flow

$$\frac{\partial \mathbf{V}}{\partial t} \neq 0$$

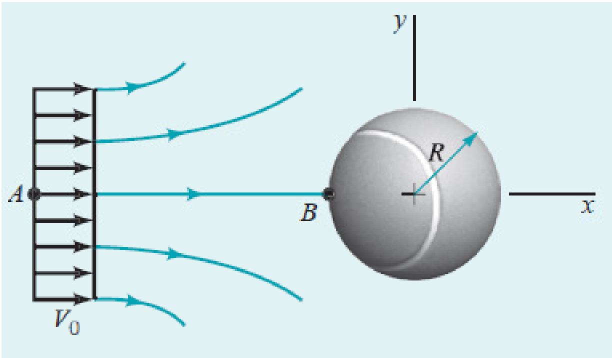
The Acceleration Field

Example 4.2

An incompressible, inviscid fluid flows steadily past a ball of radius *R*, as shown in Fig. According to a more advanced analysis of the flow, the fluid velocity along streamline *A–B* is given by

$$\mathbf{V} = u(x)\hat{\mathbf{i}} = V_0\left(1+\frac{R^3}{x^3}\right)\hat{\mathbf{i}}$$

where V_0 is the upstream velocity far ahead of the sphere. Determine the acceleration experienced by fluid particles as they flow along this streamline.



Answer:

$$a_x = -3\left(\frac{V_0^2}{R}\right)\frac{1+\left(\frac{R}{x}\right)^3}{\left(\frac{x}{R}\right)^4}$$

Content

- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - **Control Volume and System Representations**

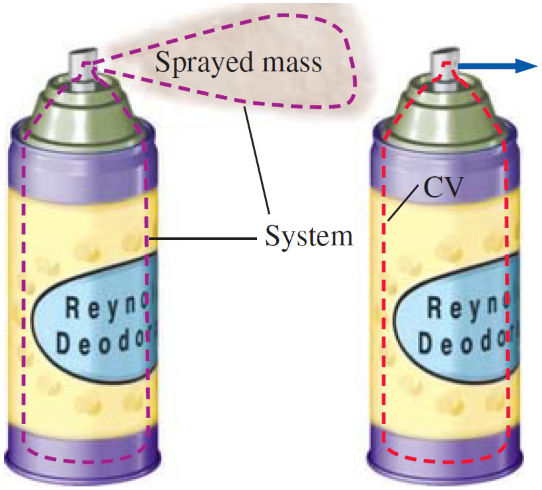
Control Volume and System Representation

System

A system (also called a closed system), is defined as a quantity of matter of fixed identity. The size and shape of a system may change during a process, but no mass crosses its boundaries.

Control Volume

A control volume (also called an open system), defined as a region in space chosen for study. A control volume allows mass to flow in or out across its boundaries, which are called the **control surface**.



(*a*) (*b*)

(a) We follow the fluid as it moves and deforms. This is the system approach.

(b) We consider a fixed interior volume of the can. This is the control volume approach.

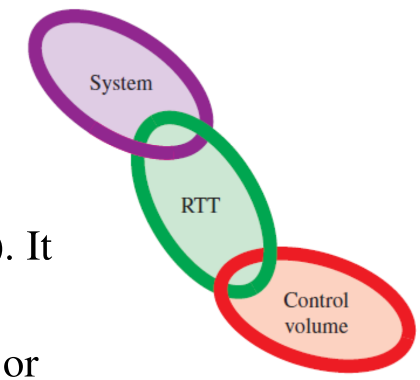
Content

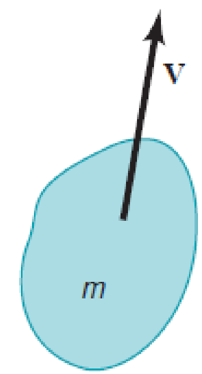
- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - Control Volume and System Representations
  - **Reynolds Transport Theorem**

Reynolds Transport Theorem

Reynolds Transport Theorem: The Reynolds transport theorem (RTT) provides a link between the system approach and the control volume approach.

$$B = mb$$





m : Mass of the portion of fluid.

B : Any physical parameter (scalar or vector). It represents an extensive property.

b : Physical parameter per unit mass (scalar or vector). It represents an intensive property.

B	$b = B/m$
m	1
$m\mathbf{V}$	$\mathbf{V}$
$\frac{1}{2}mV^2$	$\frac{1}{2}V^2$

Physical parameters

- Velocity,
- Acceleration,
- Mass,
- Temperature,
- Momentum, etc.

Reynolds Transport Theorem

$$B_{sys,t} = B_{CV,t} \qquad \text{(At time } t\text{)}$$

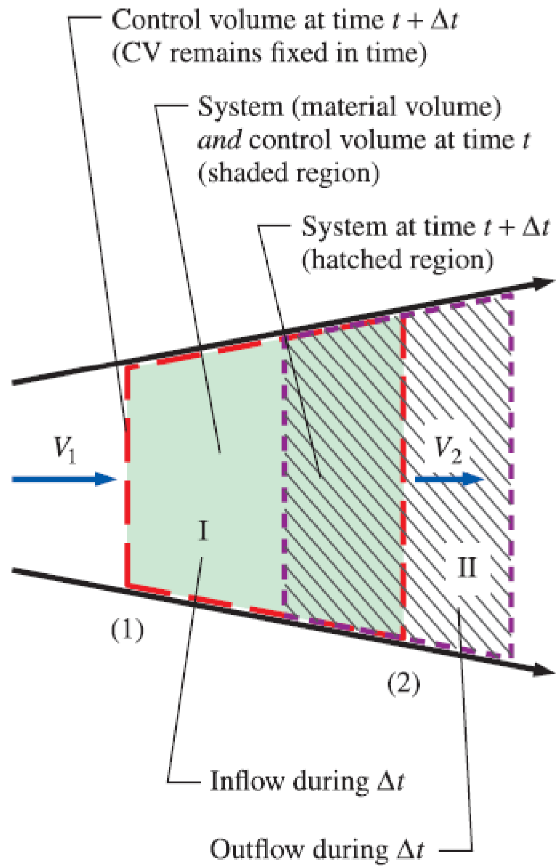
$$B_{sys,t+\Delta t} = B_{CV,t+\Delta t} - B_{I,t+\Delta t} + B_{II,t+\Delta t} \qquad \text{(At time } t+\Delta t\text{)}$$

$$\frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} - \dot{B}_{in} + \dot{B}_{out}$$

or

$$\frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} - b_1 \rho_1 V_1 A_1 + b_2 \rho_2 V_2 A_2$$

States that the time rate of change of the property *B* of the system is equal to the time rate of change of *B* of the control volume plus the net flux of *B* out of the control volume by mass crossing the control surface.

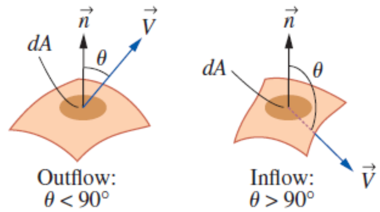


At time t: Sys = CV
At time $t + \Delta t$: Sys = CV − I + II

Reynolds Transport Theorem (RTT)

Reynolds Transport Theorem (RTT) for a fixed control volume.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b d\forall + \int_{CS} \rho b \mathbf{V} \cdot \mathbf{n} dA$$



where: $\frac{dB_{sys}}{dt}$ The time rate of change of the property B of the system.

$\int_{CS} \rho b \mathbf{V} \cdot \mathbf{n} dA = \dot{B}_{net} = \dot{B}_{out} - \dot{B}_{in}$ Net rate of **outflow** through the entire control surface.

$\frac{d}{dt}\int_{CV} \rho b d\forall$ Time rate of change of the property *B* content within the control volume.

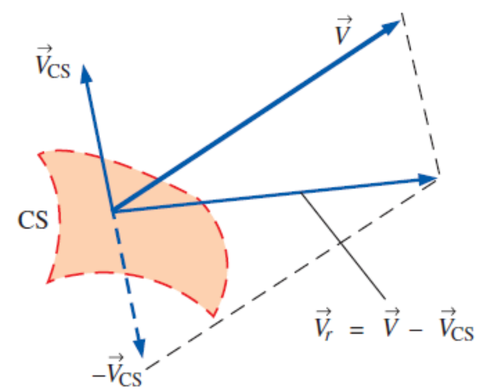
Reynolds Transport Theorem (RTT) for the control volume that moves and/or deforms.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b d\forall + \int_{CS} \rho b \mathbf{V}_r \cdot \mathbf{n} dA$$

Where: $\mathbf{V}_r$: Relative velocity.

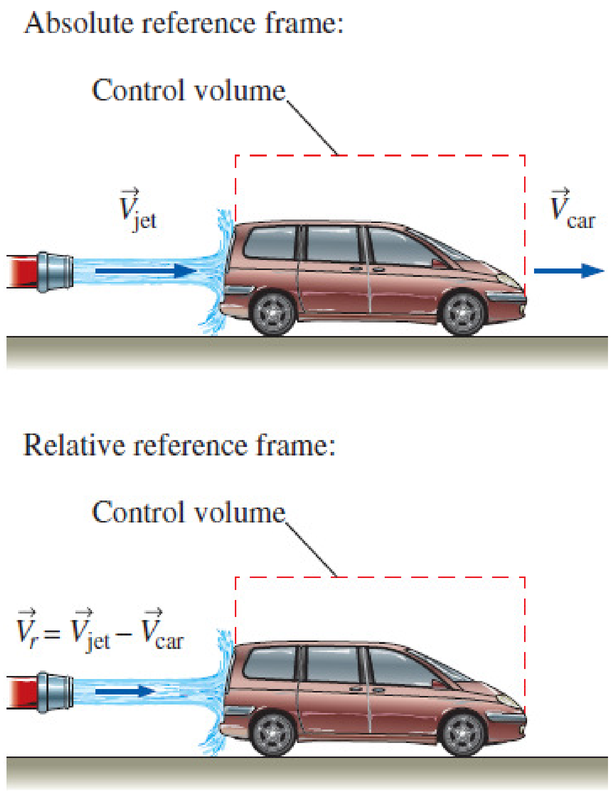
$\mathbf{V}$: Absolute velocity.

$\mathbf{V}_{CS}$: Local velocity of the control surface.



Reynolds Transport Theorem (RTT)

Reynolds transport theorem applied to a control volume moving at constant velocity.



Reynolds Transport Theorem (RTT)

RTT in terms of average values of fluid properties crossing the control surface.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b d\forall + \sum_{out} \dot{m}_r b_{avg} - \sum_{in} \dot{m}_r b_{avg}$$

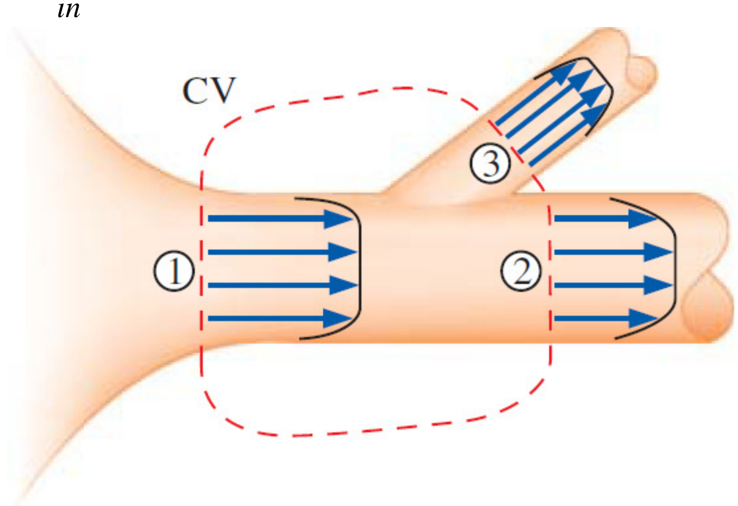
or

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b d\forall + \sum_{out} \rho_{avg} b_{avg} V_{r,avg} A - \sum_{in} \rho_{avg} b_{avg} V_{r,avg} A$$

Where:

$\dot{m}_r$ Is the mass flow rate through the inlet or outlet.

$$\dot{m}_r \approx \rho_{avg} \dot{\forall}_r = \rho_{avg} V_{r,avg} A$$



Content

- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - Control Volume and System Representations
  - Reynolds Transport Theorem
- **Fluid Dynamics - The Bernoulli Equation**
  - **Newton’s Second Law**

Newton's Second Law

Newton's second law of motion

$$\mathbf{F} = m\mathbf{a} \qquad \text{(Vectorial form)}$$

F : net force acting on a fluid particle (Vector form).

m **:** mass.

a : acceleration (Vector form).

$$\mathbf{a} = \frac{d\mathbf{V}}{dt}$$

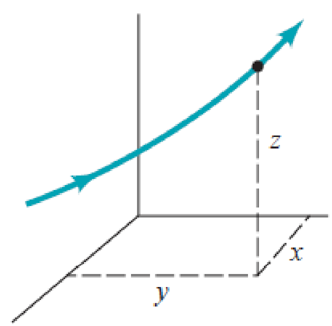
For inviscid flows : viscous forces << net pressure forces
viscous forces << gravity forces

$$\Rightarrow \mu = 0 \qquad \mu : \text{ dynamic viscosity.}$$

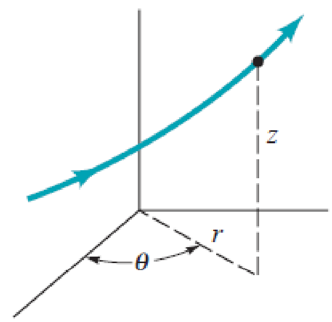
Net pressure force **+** net gravity force **=** particle mass **×** particle acceleration

Newton's Second Law

Coordinate systems



Rectangular



Cylindrical

s **:** distance along the streamline, $s = s(t)$.

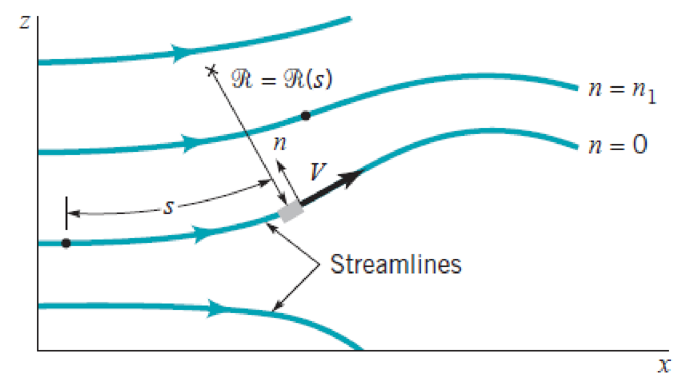
n **:** coordinate normal to the streamline.

$\mathcal{R}$ **:** radius of curvature of the streamline.

V **:** velocity, $V = ds/dt$.

a_s **:** streamwise acceleration, $a_s = \dfrac{dV}{dt} = V\dfrac{dV}{ds}$.

a_n **:** normal acceleration, $a_n = \dfrac{V^2}{R}$.



Flow in terms of streamline and normal coordinates.

Content

- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - Control Volume and System Representations
  - Reynolds Transport Theorem
- Fluid Dynamics - The Bernoulli Equation
  - Newton's Second Law
  - **F=ma Along a Streamline**

F=ma Along a Streamline

Steady flow along a streamline

$$\sum dF_s = dm\, a_s$$

Equation of motion along the streamline direction

$$-\gamma \sin\theta - \frac{dP}{ds} = \rho V \frac{dV}{ds}$$

General

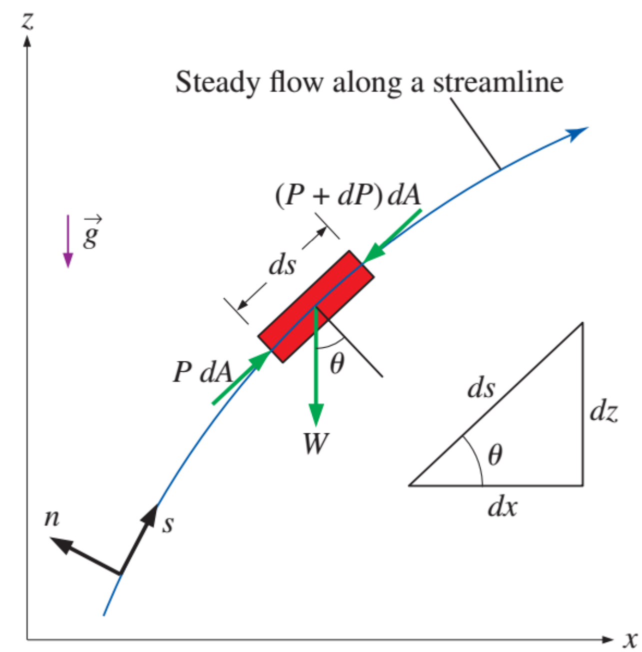
$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Bernoulli equation (Incompressible flow)

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

The Bernoulli equation between any two points

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$



Freebody diagram of a fluid particle.

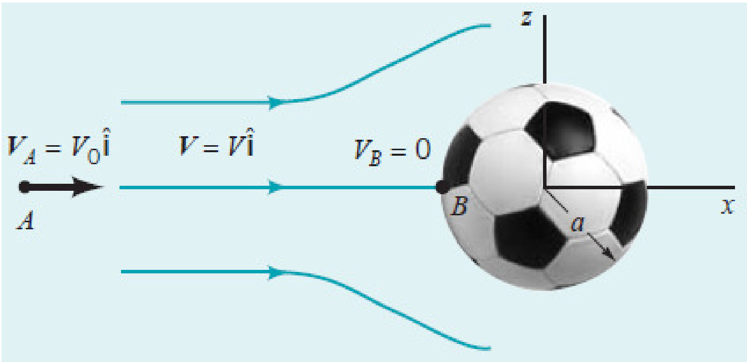
F=ma Along a Streamline

Example 4.3

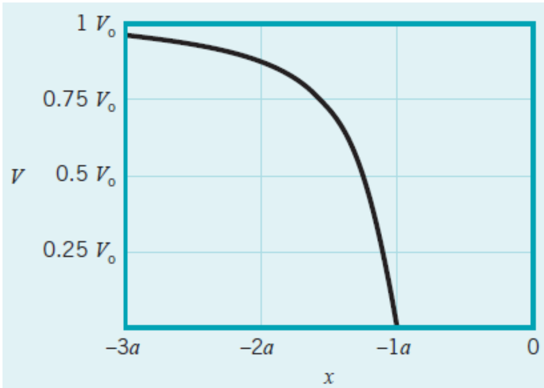
Consider the inviscid, incompressible, steady flow along the horizontal streamline *A–B* in front of the sphere of radius *a*, as shown in Figure *a*. From a more advanced theory of flow past a sphere, the fluid velocity along this streamline is

$$V = V_0\left(1 + \frac{a^3}{x^3}\right)$$

as shown in Figure *b*. Determine the pressure variation along the streamline from point *A* far in front of the sphere (x_A = -∞ and $V_A = V_0$) to point *B* on the sphere ($x_B = -a$ and $V_B = 0$).



(a)



(b)

Content

- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - Control Volume and System Representations
  - Reynolds Transport Theorem
- Fluid Dynamics - The Bernoulli Equation
  - Newton's Second Law
  - F=ma Along a Streamline
  - **Static, Stagnation, Dynamic, and Total Pressure**

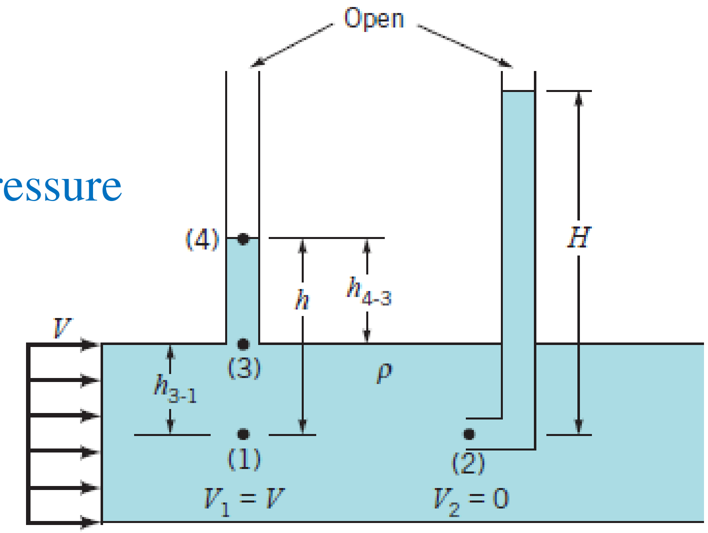
Static, Stagnation, Dynamic, and Total Pressure

$$p_2 = p_1 + \frac{1}{2}\rho V_1^{\,2}$$

Stagnation pressure

Static pressure

Dynamic pressure



Measurement of static and stagnation pressures.

Static, Stagnation, Dynamic, and Total Pressure

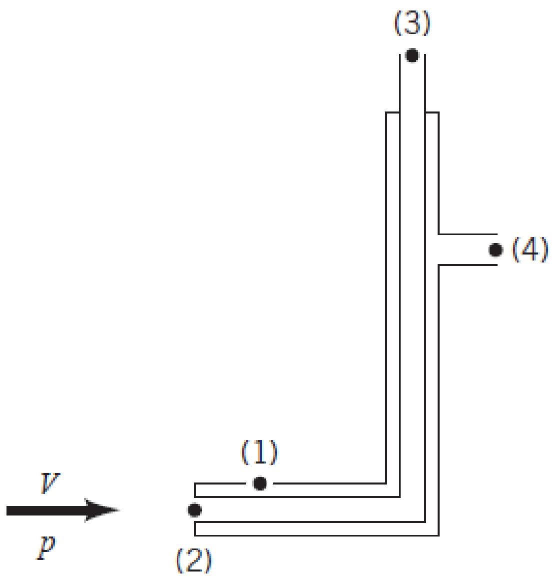
Pitot-static tube

Pitot-static tubes measure fluid velocity by converting velocity into pressure.

$$p_3 = p + \frac{1}{2}\rho V^2$$

$$p_4 = p_1 = p$$

$$V = \sqrt{\frac{2(p_3 - p_4)}{\rho}}$$



Measurement of static and stagnation pressures.

Static, Stagnation, Dynamic, and Total Pressure

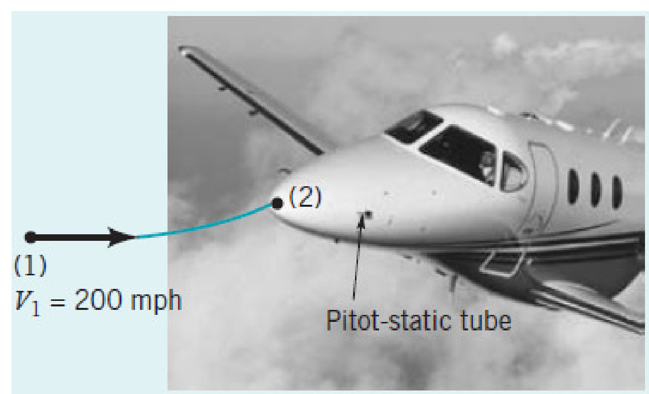
Example 4.4

An airplane flies 200 mi/hr at an elevation of 10,000 ft in a standard atmosphere as shown in Fig. Determine the pressure at point (1) far ahead of the airplane, the pressure at the stagnation point on the nose of the airplane, point (2), and the pressure difference indicated by a Pitot static probe attached to the fuselage.

(a) 10.1 psia

(b) 10.63 psia

(c) 0.524 psi



Content

- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - Control Volume and System Representations
  - Reynolds Transport Theorem
- Fluid Dynamics - The Bernoulli Equation
  - Newton's Second Law
  - F=ma Along a Streamline
  - Static, Stagnation, Dynamic, and Total Pressure
  - **Example of use of the Bernoulli Equation**

Example of use of the Bernoulli Equation: Free Jets

Vertical flow from a tank.

Bernoulli equation $$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

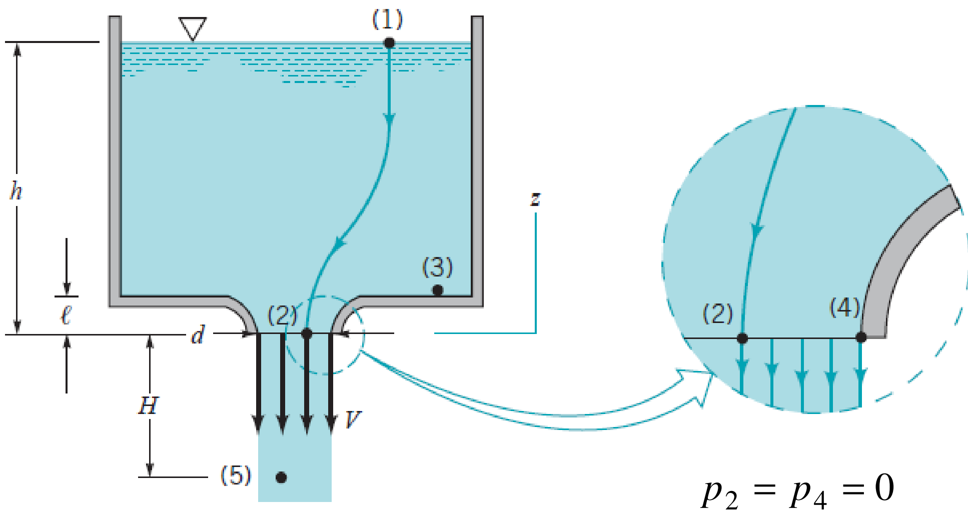
Applying to the flow from a tank

$$\gamma h = \frac{1}{2}\rho V_2^2$$

$$\therefore \; V_2 = \sqrt{2gh}$$

and

$$V_5 = \sqrt{2g(h+H)}$$



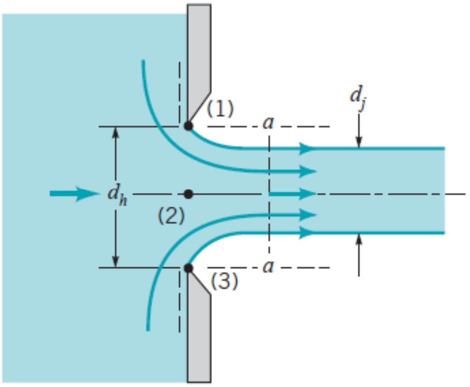
$$p_5 = 0$$

$$p_2 = p_4 = 0$$

$$V_2 = V_4$$

Example of use of the Bernoulli Equation: Confined Flows

Vena contracta effect for a sharp-edged orifice.



$$C_c = \frac{A_j}{A_h}$$

A_j **:** area of the jet at the vena contracta.

A_h **:** area of the hole.

Confined Flows

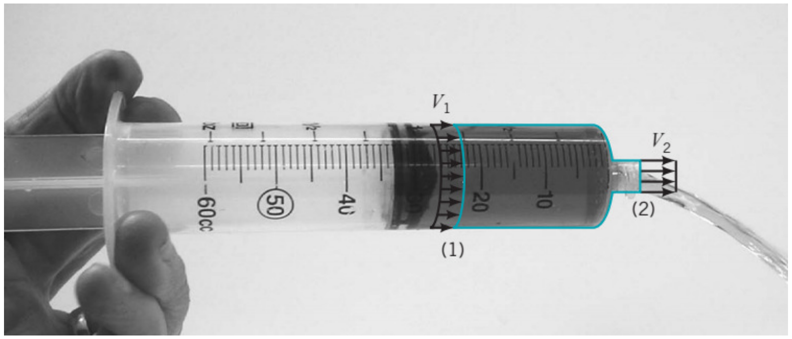
Conservation of mass

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For incompressible flow ($\rho_1 = \rho_2$)

$$A_1 V_1 = A_2 V_2 \;\Rightarrow\; Q_1 = Q_2$$



Steady flow into and out of a volume - syringe.

Example of use of the Bernoulli Equation: Flowrate Measurement

Flowrate Measurement

Ideal flow meters

- Steady.
- Inviscid.
- Incompressible.

Bernoulli equation

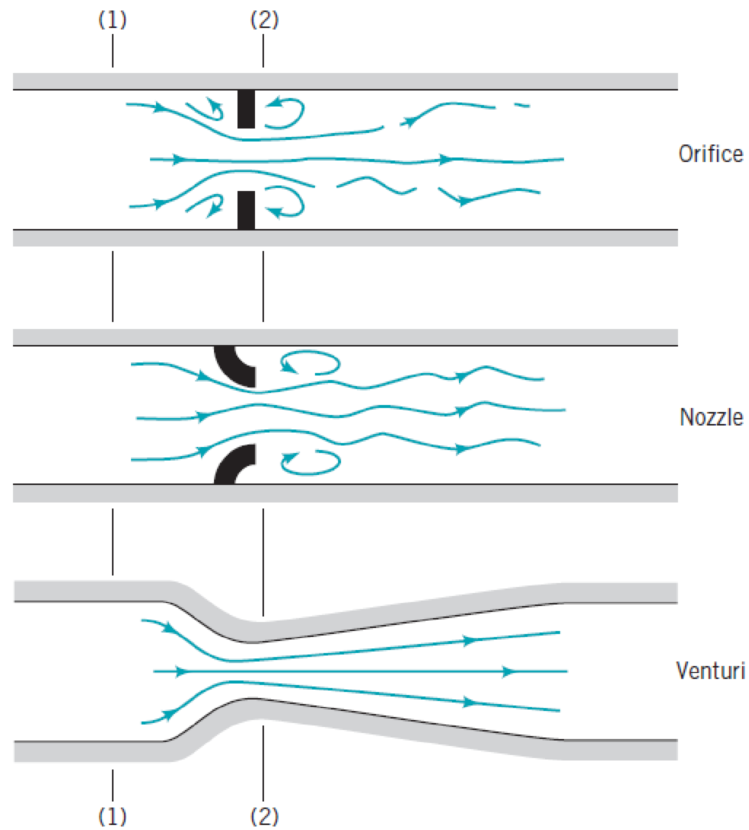
$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

Continuity equation

$$Q = A_1 V_1 = A_2 V_2$$

Flowrate Q

$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho\left[1 - (A_2/A_1)^2\right]}}$$



Typical devices for measuring flowrate in pipes.

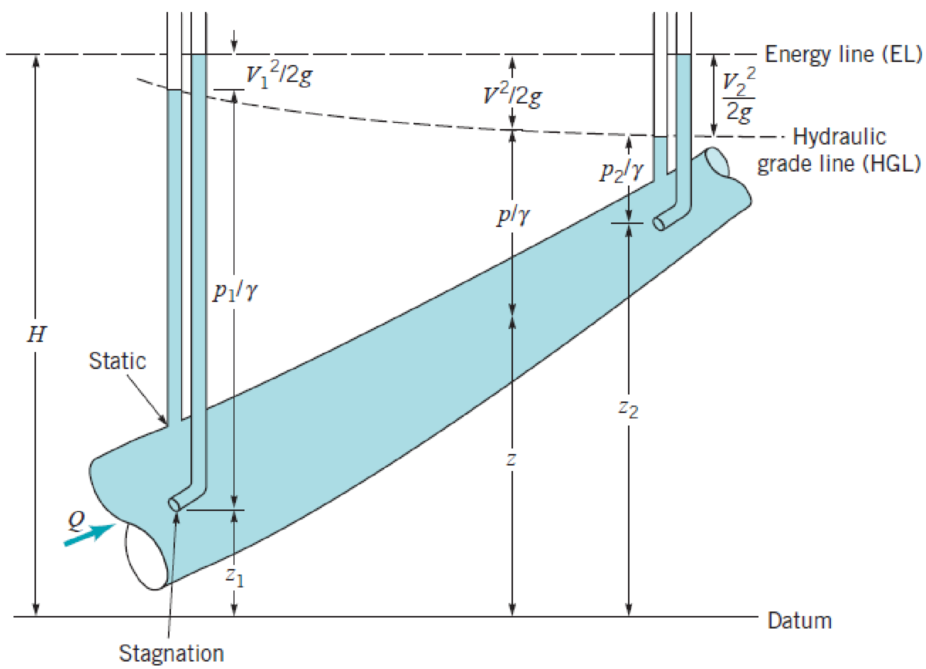
Content

- Fluid Kinematics
  - The Velocity Field
  - The Acceleration Field
  - Control Volume and System Representations
  - Reynolds Transport Theorem
- Fluid Dynamics - The Bernoulli Equation
  - Newton's Second Law
  - F=ma Along a Streamline
  - Static, Stagnation, Dynamic, and Total Pressure
  - Example of use of the Bernoulli Equation
  - **The Energy Line and the Hydraulic Grade Line**

The Energy Line and the Hydraulic Grade Line

The Bernoulli equation is actually an energy equation representing the partitioning of energy for an **inviscid, incompressible, steady flow**.

$$\frac{p}{\gamma}+\frac{V^2}{2g}+z=\text{constant on a streamline}=H$$



EGL : Energy grade line.

HGL : Hydraulic grade line.

Examples – Bernoulli Equation

Example 4.5

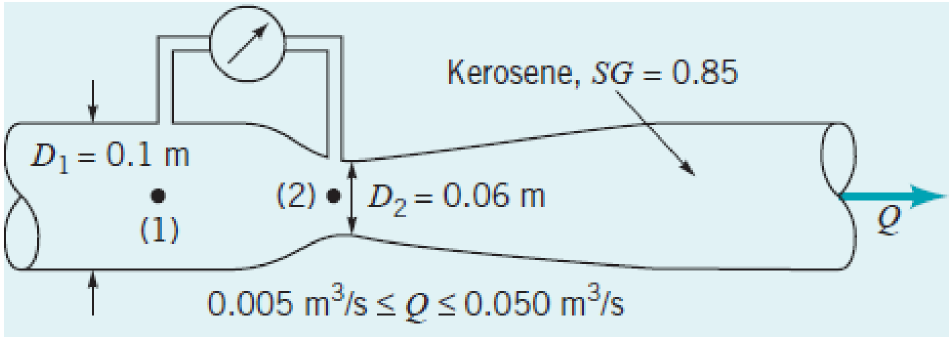
A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 *s* to fill the bucket with water, determine a) the volume and mass flow rates of water through the hose, and b) the average velocity of water at the nozzle exit.

Answer: a) $7.56\text{x}10^{-4}$ m^3/s
0.756 kg/s
b) 15.04 m/s

Examples – Bernoulli Equation

Example 4.6

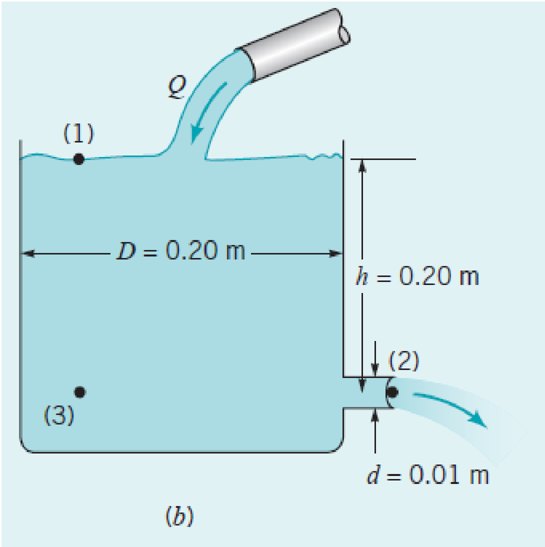
Kerosene (SG = 0.85) flows through the Venturi meter with flowrates between 0.005 and 0.05 m^3/s. Determine the range in pressure difference, p_1 - p_2, needed to measure these flowrates.



Examples – Bernoulli Equation

Example 4.7

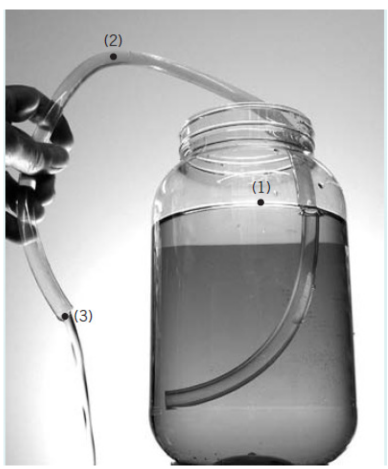
A stream of refreshing beverage of diameter d = 0.01 m flows steadily from the cooler of diameter D = 0.20 m. Determine the flowrate, Q, from the bottle into the cooler if the depth of beverage in the cooler is to remain constant at h = 0.20 m. Answer : $1.56\text{x}10^{-4}$ m^3/s



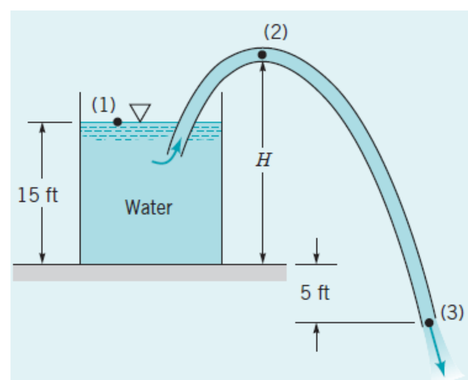
Examples – Bernoulli Equation

Example 4.8

A liquid can be siphoned from a container as shown in Fig. *a* provided the end of the tube, point (3), is below the free surface in the container, point (1), and the maximum elevation of the tube, point (2), is "not too great." Consider water at 60°F being siphoned from a large tank through a constant diameter hose as shown in Fig. *b*. The end of the siphon is 5 ft below the bottom of the tank, and the atmospheric pressure is 14.7 psia. Determine the maximum height of the hill, H, over which the water can be siphoned without cavitation occurring. Answer : 28.2 ft



(*a*)



(*b*)

Appendix

Properties of the U.S. Standard Atmosphere (BG Units)[a]

Altitude (ft)	Temperature (°F)	Acceleration of Gravity, g (ft/s^2)	Pressure, p [lb/in.^{2}(abs)]	Density, ρ (slugs/ft^3)	Dynamic Viscosity, μ (lb·s/ft^2)
−5,000	76.84	32.189	17.554	2.745 E − 3	3.836 E − 7
0	59.00	32.174	14.696	2.377 E − 3	3.737 E − 7
5,000	41.17	32.159	12.228	2.048 E − 3	3.637 E − 7
10,000	23.36	32.143	10.108	1.756 E − 3	3.534 E − 7
15,000	5.55	32.128	8.297	1.496 E − 3	3.430 E − 7
20,000	−12.26	32.112	6.759	1.267 E − 3	3.324 E − 7
25,000	−30.05	32.097	5.461	1.066 E − 3	3.217 E − 7
30,000	−47.83	32.082	4.373	8.907 E − 4	3.107 E − 7
35,000	−65.61	32.066	3.468	7.382 E − 4	2.995 E − 7
40,000	−69.70	32.051	2.730	5.873 E − 4	2.969 E − 7
45,000	−69.70	32.036	2.149	4.623 E − 4	2.969 E − 7
50,000	−69.70	32.020	1.692	3.639 E − 4	2.969 E − 7
60,000	−69.70	31.990	1.049	2.256 E − 4	2.969 E − 7
70,000	−67.42	31.959	0.651	1.392 E − 4	2.984 E − 7
80,000	−61.98	31.929	0.406	8.571 E − 5	3.018 E − 7
90,000	−56.54	31.897	0.255	5.610 E − 5	3.052 E − 7
100,000	−51.10	31.868	0.162	3.318 E − 5	3.087 E − 7
150,000	19.40	31.717	0.020	3.658 E − 6	3.511 E − 7
200,000	−19.78	31.566	0.003	5.328 E − 7	3.279 E − 7
250,000	−88.77	31.415	0.000	6.458 E − 8	2.846 E − 7

[a]Data abridged from *U.S. Standard Atmosphere*, 1976, U.S. Government Printing Office, Washington, D.C.

$$1\,slug = 32.17\,lbm$$

Appendix

Physical Properties of Water (BG Units)[a]

Temperature (°F)	Density, ρ (slugs/ft^3)	Specific Weight[b], γ (lb/ft^3)	Dynamic Viscosity, μ (lb·s/ft^2)	Kinematic Viscosity, ν (ft^2/s)	Surface Tension[c], σ (lb/ft)	Vapor Pressure, p_v [lb/in.^{2}(abs)]	Speed of Sound[d], c (ft/s)
32	1.940	62.42	3.732 E − 5	1.924 E − 5	5.18 E − 3	8.854 E − 2	4603
40	1.940	62.43	3.228 E − 5	1.664 E − 5	5.13 E − 3	1.217 E − 1	4672
50	1.940	62.41	2.730 E − 5	1.407 E − 5	5.09 E − 3	1.781 E − 1	4748
60	1.938	62.37	2.344 E − 5	1.210 E − 5	5.03 E − 3	2.563 E − 1	4814
70	1.936	62.30	2.037 E − 5	1.052 E − 5	4.97 E − 3	3.631 E − 1	4871
80	1.934	62.22	1.791 E − 5	9.262 E − 6	4.91 E − 3	5.069 E − 1	4819
90	1.931	62.11	1.500 E − 5	8.233 E − 6	4.86 E − 3	6.979 E − 1	4960
100	1.927	62.00	1.423 E − 5	7.383 E − 6	4.79 E − 3	9.493 E − 1	4995
120	1.918	61.71	1.164 E − 5	6.067 E − 6	4.67 E − 3	1.692 E + 0	5049
140	1.908	61.38	9.743 E − 6	5.106 E − 6	4.53 E − 3	2.888 E + 0	5091
160	1.896	61.00	8.315 E − 6	4.385 E − 6	4.40 E − 3	4.736 E + 0	5101
180	1.883	60.58	7.207 E − 6	3.827 E − 6	4.26 E − 3	7.507 E + 0	5195
200	1.869	60.12	6.342 E − 6	3.393 E − 6	4.12 E − 3	1.152 E + 1	5089
212	1.860	59.83	5.886 E − 6	3.165 E − 6	4.04 E − 3	1.469 E + 1	5062

[a]Based on data from *Handbook of Chemistry and Physics*, 69th Ed., CRC Press, 1988. Where necessary, values obtained by interpolation.
[b]Density and specific weight are related through the equation $\gamma = \rho g$. For this table, $g = 32.174$ ft/s^2.
[c]In contact with air.
[d]From R. D. Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Co., Inc., New York, 1984.

$1\,slug = 32.17\,lbm$

References

[1] Cengel Y., Cimbala, J. (2014). Fluid Mechanics: Fundamentals and Applications (3th Edition). New York: NY: McGraw-Hill Co.

[2] Munson, B.R., Young, D.F., Okiishi, T.H., and Huebsch, W.W. (2016). Fundamentals of Fluid Mechanics (8th Edition). John Wiley & Sons. ISBN 1119080703.