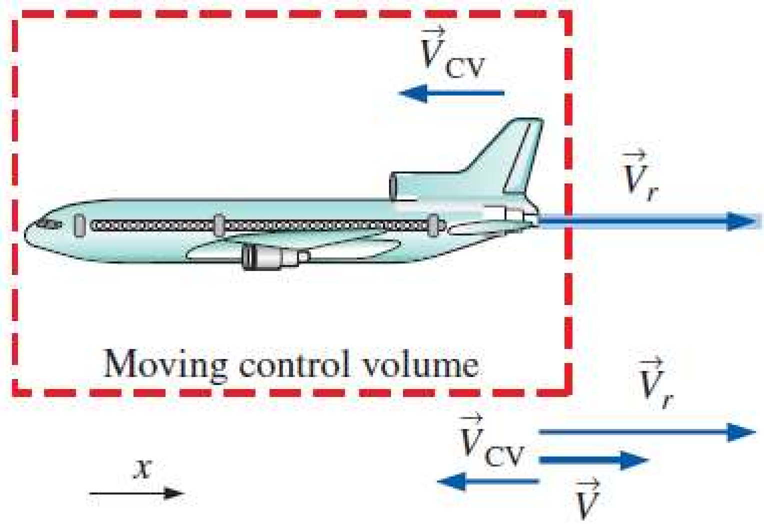
Finite Control Volume Analysis



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ENGI 2420

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- Conservation of Mass – Continuity Equation
- Newton’s Second Law – Momentum Equation

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- **Conservation of Mass – Continuity Equation**

Conservation of Mass – Continuity Equation

Conservation of mass

The conservation of mass principle for a system is given by

$$\frac{Dm_{sys}}{Dt} = 0 \qquad \text{where} \qquad m_{sys} = \int_{sys} \rho d\forall$$

From the Reynolds Transport Theorem :

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b d\forall + \int_{cs} \rho b \boldsymbol{V} \cdot \hat{\boldsymbol{n}} dA$$

$$B = m \qquad b = 1$$

$$\frac{Dm_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \boldsymbol{V} \cdot \hat{\boldsymbol{n}} dA$$

Continuity Equation for a fixed, nondeforming control volume

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathrm{V} \cdot \hat{\mathrm{n}} dA = 0$$

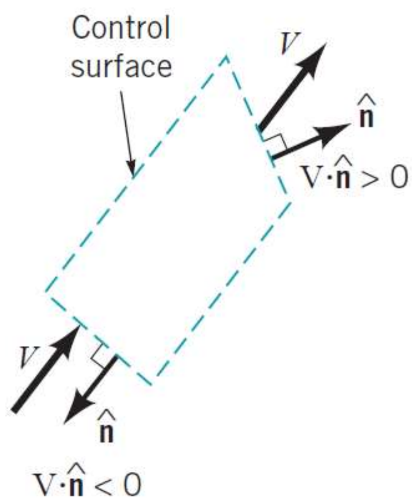
It states that the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is zero.

Conservation of Mass – Continuity Equation

Steady mass flow rate

$$\int_{cs} \rho \mathbf{V} \cdot \mathbf{n} dA = \sum_{\text{out}} \dot{m} - \sum_{\text{in}} \dot{m} = 0$$

$$\rightarrow \boxed{\sum_{\text{out}} \dot{m} = \sum_{\text{in}} \dot{m}}$$



Mass flow rate through a section of control surface having area A

$$\dot{m} = \rho Q = \rho A V$$

where

- ρ **:** fluid density.
- Q **:** is the volume flow rate.
- V **:** component of fluid velocity perpendicular to area A.

Equation of state (Ideal gases)

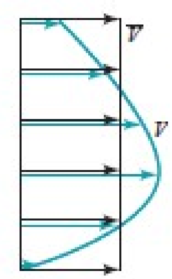
$$\rho = \frac{p}{RT}$$

- T **:** temperature.
- p **:** pressure.
- R **:** specific gas constant, for air
  R = 287 J/(kg K) or
  R = 53.32 lbf ft/ (lbm R).

Average velocity ($\bar{V}$)

$$\dot{m} = \int_A \rho \mathbf{V} \cdot \mathbf{n} dA$$

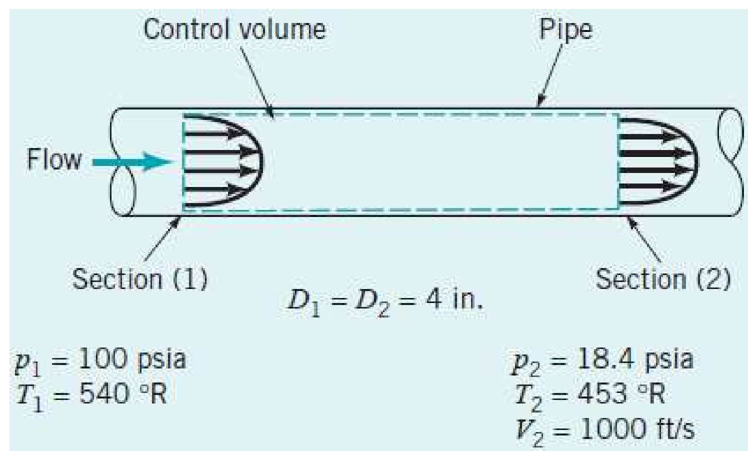
$$\Rightarrow \bar{V} = \frac{\int_A \rho \mathbf{V} \cdot \mathbf{n} dA}{\rho A}$$



Conservation of Mass – Continuity Equation

Example 5.1

Air flows steadily between two sections in a long, straight portion of 4-in. inside diameter pipe. The uniformly distributed temperature and pressure at each section are given. The average air velocity (nonuniform velocity distribution) at section (2) is 1000 ft/s. Calculate the average air velocity at section (1). Answer: 219 ft/s



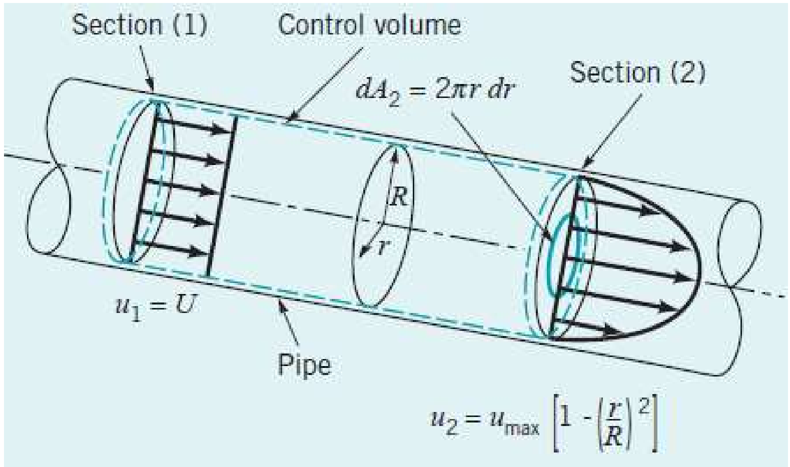
Conservation of Mass – Continuity Equation

Example 5.2

Incompressible, laminar water flow develops in a straight pipe having radius R. At section (1), the velocity profile is uniform; the velocity is equal to a constant value U and is parallel to the pipe axis everywhere. At section (2), the velocity profile is axisymmetric and parabolic, with zero velocity at the pipe wall and a maximum value of u_{max} at the centerline.

(a) How are U and u_{max} related?

(b) How are the average velocity at section (2), and u_{max} related?



Conservation of Mass – Continuity Equation

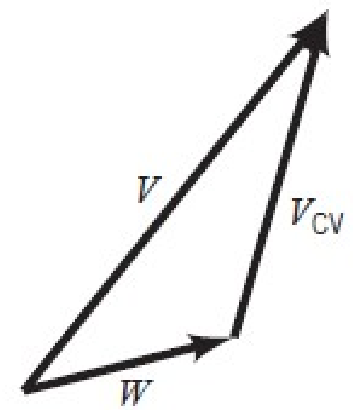
Moving, Nondeforming Control Volume

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cv}$$

V : is the fluid velocity seen by a stationary observer.

W : is the fluid velocity seen by an observer moving with the control volume..

$\mathbf{V}_{cv}$: is the velocity of the control volume as seen from a fixed coordinate system.



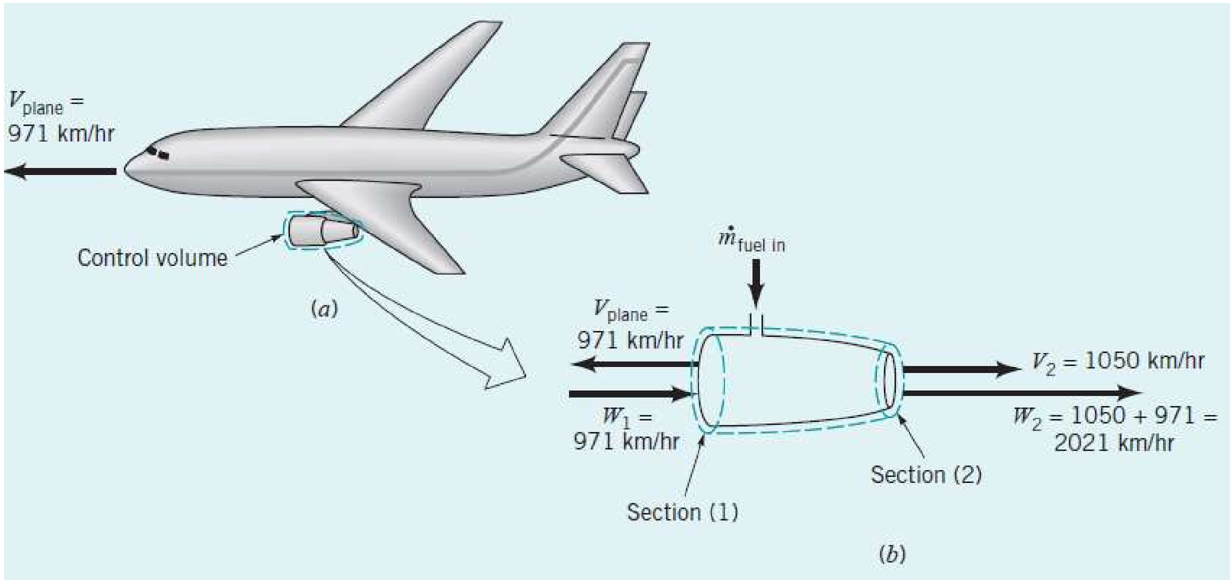
The conservation of mass for a moving, nondeforming control volume,

$$\frac{Dm_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho dV\!\!\!\!- + \int_{cs} \rho \mathbf{W} \cdot \boldsymbol{n} dA = 0$$

Conservation of Mass – Continuity Equation

Example 5.3

An airplane moves forward at a speed of 971 km/hr. The frontal intake area of the jet engine is 0.80 m^2 and the entering air density is 0.736 kg/m^3. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2, and the exhaust gas density is 0.515 kg/m^3. Estimate the mass flowrate of fuel into the engine in kg/hr. Answer: 9050 kg/hr



Conservation of Mass – Continuity Equation

Deforming Control Volume

$$\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \, \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$$

For the deforming control volume,

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cs}$$

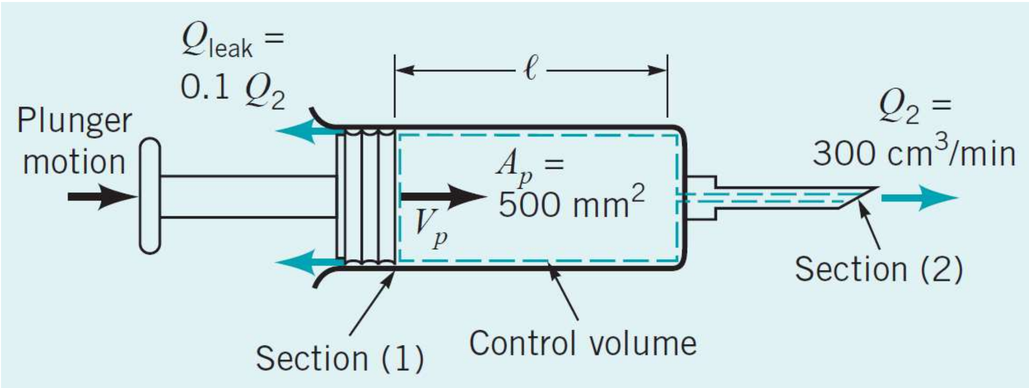
$\mathbf{V}_{cs}$ **:** is the velocity of the control surface as seen by a fixed observer.

NOTE: The velocity of the surface of a deforming control volume is not the same at all points on the surface.

Conservation of Mass – Continuity Equation

Example 5.4

A syringe (see Figure) is used to inoculate a cow. The plunger has a face area of 500 mm^2. The liquid in the syringe is to be injected steadily at a rate of 300 cm^3/min. The leakage rate past the plunger is 0.10 times the volume flowrate out of the Needle. With what speed should the plunger be advanced? Answer = 0.011 m/s



Content

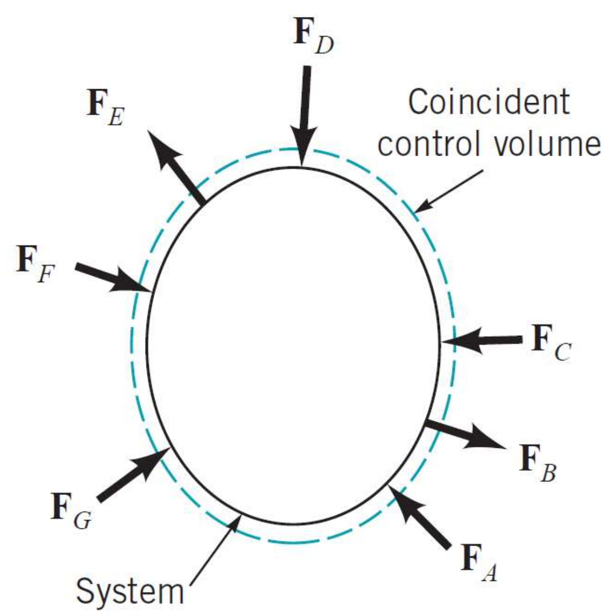
- Conservation of Mass – Continuity Equation
- **Newton's Second Law – Momentum Equation**

Newton's Second Law – Momentum Equation

Linear Momentum Equation

Newton's second law can be stated as the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system.

$$\sum \mathbf{F} = \frac{D}{Dt}\int_{sys} \mathbf{V}\rho \, d\mathcal{V}$$



Using the Reynolds transport theorem

$$B = m\mathbf{V} \qquad b = \mathbf{V}$$

$$\frac{D}{Dt}\int_{sys} \mathbf{V}\rho \, d\mathcal{V} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho \, d\mathcal{V} + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot\hat{\mathbf{n}} \, dA$$

Linear Momentum Equation

$$\sum \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho \, d\mathcal{V} + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot\hat{\mathbf{n}} \, dA$$

(The sum of all external forces acting on a CV) = (The time rate of change of the linear momentum of the contents of the CV) + (The net flow rate of linear momentum out of the control surface by mass flow)

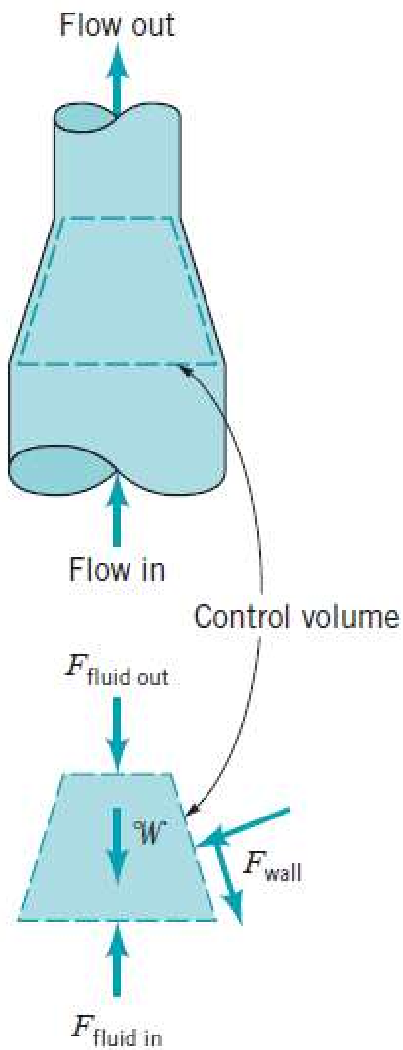
Newton's Second Law – Momentum Equation

External forces acting on a CV

$$\sum \mathbf{F} = \text{body forces} + \text{surface forces}$$

Steady linear momentum equation

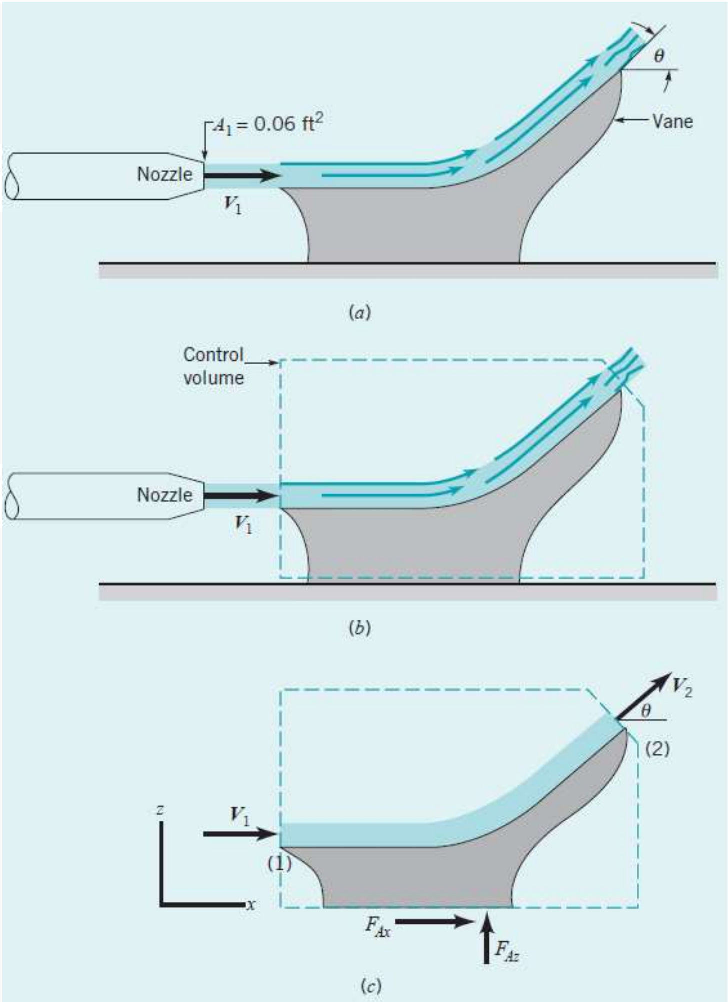
$$\sum \mathbf{F} = \sum_{out} \dot{m}\boldsymbol{V} - \sum_{in} \dot{m}\boldsymbol{V}$$



Newton's Second Law – Momentum Equation

Example 5.5

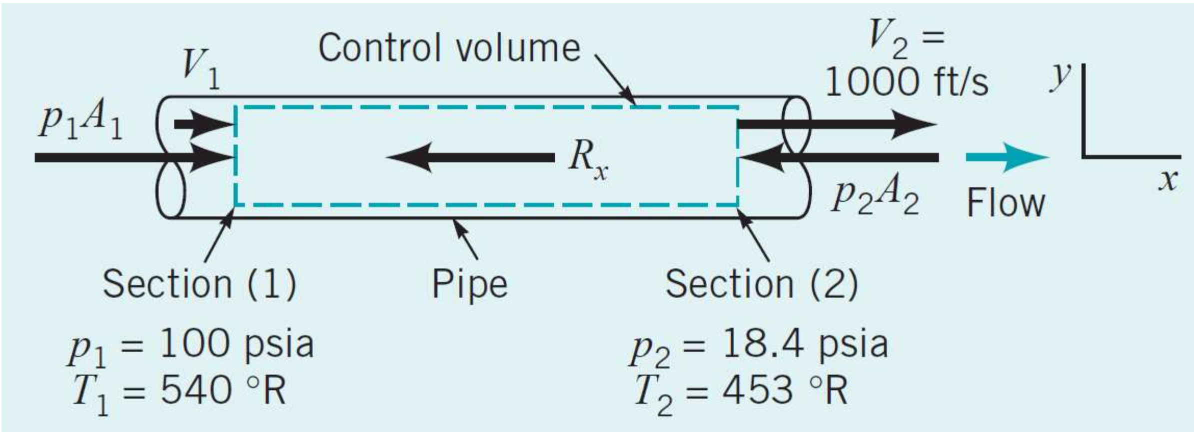
As shown in the Figure, a horizontal jet of water exits a nozzle with a uniform speed of $V_1 = 10$ ft/s, strikes a vane, and is turned through an angle $\theta = 45°$. Determine the anchoring force needed to hold the vane stationary if gravity and viscous effects are negligible. Answer: $F_{Ax} = -3.41$ lbf, $F_{Az} = 8.23$ lbf



Newton's Second Law – Momentum Equation

Example 5.7

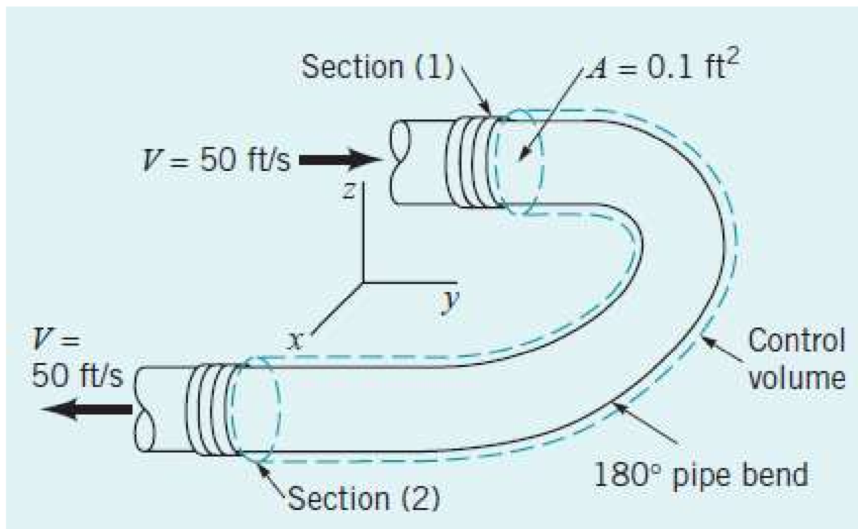
Air flows steadily between two cross sections in a long, straight portion of 4-in. inside diameter pipe, where the uniformly distributed temperature and pressure at each cross section are given. If the average air velocity at section (2) is 1000 ft/s, the average air velocity at section (1) must be 219 ft/s. Assume uniform velocity distributions at sections (1) and (2). Determine the frictional force exerted by the pipe wall on the air flow between sections (1) and (2). Answer: 793 lbf



Newton's Second Law – Momentum Equation

Example 5.6

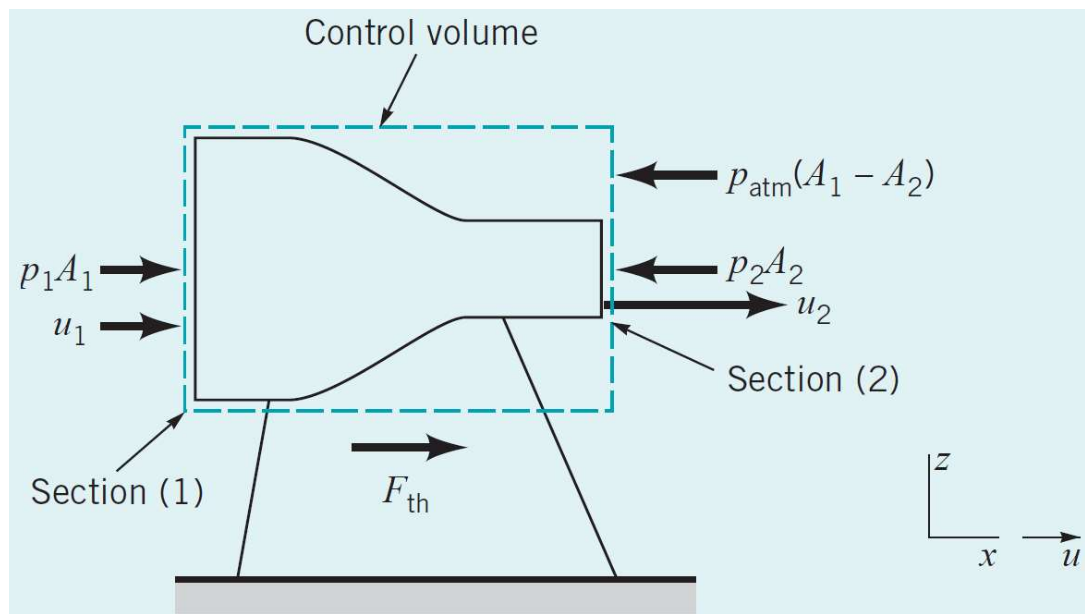
Water flows through a horizontal, 180° pipe bend as illustrated in Figure. The flow cross-sectional area is constant at a value of 0.1 ft^2 through the bend. The magnitude of the flow velocity everywhere in the bend is axial and 50 ft/s. The absolute pressures at the entrance and exit of the bend are 30 psia and 24 psia, respectively. Calculate the horizontal (x and y) components of the anchoring force required to hold the bend in place. Answer: F_y = 1324 lbf



Newton's Second Law – Momentum Equation

Example 5.8

A static thrust stand as sketched in Figure is to be designed for testing a jet engine. The following conditions are known for a typical test: Intake air velocity = 200 m/s; exhaust gas velocity = 500 m/s; intake cross-sectional area = 1 m^2; intake static pressure = -22.5 kPa = 78.5 kPa (abs); intake static temperature = 268 K; exhaust static pressure 0 kPa = 101 kPa (abs). Estimate the nominal anchoring force for which to design. Answer: F_{th} = 83.7 kN



References

[1] Cengel Y., Cimbala, J. (2014). Fluid Mechanics: Fundamentals and Applications (3th Edition). New York: NY: McGraw-Hill Co.

[2] Munson, B.R., Young, D.F., Okiishi, T.H., and Huebsch, W.W. (2016). Fundamentals of Fluid Mechanics (8th Edition). John Wiley & Sons. ISBN 1119080703.