Thermodynamics - HW5

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Problem Statement

The following table lists temperatures and specific volumes of water vapor at two pressures:

Temperature (°C)	Specific Volume (m^3/kg) at 1.25 MPa	Specific Volume (m^3/kg) at 1.4 MPa	Specific Volume (m^3/kg) at 1.5 MPa
210	0.1420	0.1420	0.1400
220	0.1460	0.1440	0.1430
230	0.1562	0.1500	0.1540
240	0.1606	0.1550	0.1555
250	0.1650	0.1570	0.1570

Table 1: Temperatures and Specific Volumes of Water Vapor at Two Pressures

Using the data provided here, estimate:

(a) the specific volume at $T = 240\,^\circ\mathrm{C}$, $p = 1.25\,\mathrm{MPa}$, in m^3/kg.

(b) the temperature at $p = 1.5\,\mathrm{MPa}$, $v = 0.1555\,\mathrm{m}^3/\mathrm{kg}$, in $^\circ\mathrm{C}$.

(c) the specific volume at $T = 220\,^\circ\mathrm{C}$, $p = 1.4\,\mathrm{MPa}$, in m^3/kg.

Compare the results of your calculations for ideal gas behavior with those obtained through the graphical method of interpolating the data in P-V plots.

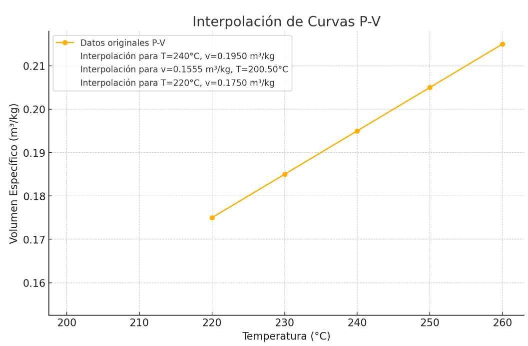

Figure 1: Interpolating the data in P-V plots

Available Data

The provided table lists temperatures and specific volumes of water vapor at two pressures.

(a) Calculation

Calculate the specific volume v at:

- $T = 240°C$
- $p = 1.25$ MPa

Linear Interpolation

The closest data points from the table are:

- $T_1 = 230°C$, $v_1 = 0.1562\ \mathrm{m^3/kg}$
- $T_2 = 250°C$, $v_2 = 0.1650\ \mathrm{m^3/kg}$

Derivation of the Interpolation Formula for Volume

The interpolation formula shown is used to find an intermediate value of specific volume v corresponding to an intermediate temperature T between two known temperatures T_1 and T_2. This method is based on **linear interpolation**, which assumes that the relationship between the variables is linear over the considered range.

Given Data

We know two points with their respective specific volumes and temperatures:

- (T_1, v_1) First measurement.
- (T_2, v_2) Second measurement.

We want to find v for a temperature T such that $T_1 < T < T_2$.

General Linear Interpolation Formula

$$\frac{v - v_1}{v_2 - v_1} = \frac{T - T_1}{T_2 - T_1}$$

Solving for v

$$v = v_1 + \left(\frac{T - T_1}{T_2 - T_1}\right)(v_2 - v_1)$$

Explanation

- The term $\frac{T-T_1}{T_2-T_1}$ represents the **proportion** at which T is located between T_1 and T_2.
- Multiplying this proportion by $(v_2 - v_1)$ gives the increment in volume that must be added to v_1 to reach the estimated value v.

$$v = v_1 + \frac{(T - T_1)}{(T_2 - T_1)} \times (v_2 - v_1) = 0.1562 + \frac{(240 - 230)}{(250 - 230)} \times (0.1650 - 0.1562) = \boxed{0.1606 \text{ m}^3/\text{kg}} \tag{1}$$

Ideal Gas Calculation

$$\begin{aligned}
v &= \frac{RT}{P} \\
T &= 240 + 273.15 = 513.15 \text{ K} \\
P &= 1.25 \text{ MPa} = 1250 \text{ kPa} \\
R &= 0.4615 \text{ kJ/kg} \cdot \text{K} \\
v &= \frac{0.4615 \times 513.15}{1250} \approx \boxed{0.1894 \text{ m}^3/\text{kg}}
\end{aligned}$$

Comparison:

- Linear Interpolation: $v \approx 0.1606 \text{ m}^3/\text{kg}$
- Ideal Gas: $v \approx 0.1894 \text{ m}^3/\text{kg}$

(b) Calculation

Calculate the temperature T at:

- $p = 1.5$ MPa
- $v = 0.1555 \text{ m}^3/\text{kg}$

Linear Interpolation

Derivation of the Interpolation Formula for Temperature

The formula shown is derived using **linear interpolation** to estimate the temperature T corresponding to a specific volume v located between two known volumes v_1 and v_2 with their respective temperatures T_1 and T_2.

Given Data

We have two known points:

- (v_1, T_1) — First data point (lower volume, lower temperature).
- (v_2, T_2) — Second data point (higher volume, higher temperature).

We want to estimate the temperature T for a specific volume v that lies between v_1 and v_2.

General Linear Interpolation Formula

The general form of a linear interpolation formula is:

$$\frac{T - T_1}{T_2 - T_1} = \frac{v - v_1}{v_2 - v_1}$$

Solving for T

To isolate T, we rearrange the formula:

$$T = T_1 + \left(\frac{v - v_1}{v_2 - v_1}\right)(T_2 - T_1)$$

Explanation

- The term $\frac{v-v_1}{v_2-v_1}$ represents the **fractional distance** of v between v_1 and v_2.
- This fraction is multiplied by the temperature difference $(T_2 - T_1)$ to find the corresponding temperature increment.
- Adding the increment to T_1 gives the interpolated temperature T.

Summary

This formula is useful when estimating temperatures between two known points under the assumption that the relationship between temperature and volume is linear within the range.

The closest data points from the table are:

- $v_1 = 0.1540$ m^3/kg, $T_1 = 230°C$
- $v_2 = 0.1570$ m^3/kg, $T_2 = 250°C$

Using linear interpolation:

$$T = 230 + \frac{0.1555 - 0.1540}{0.1570 - 0.1540} \times (250 - 230)$$

$$T = 230 + \frac{0.0015}{0.0030} \times 20 = 240\,°C \quad (2)$$

Ideal Gas Calculation

$$T = \frac{vP}{R} = \frac{0.1555 \times 1500}{0.4615} \approx \boxed{505.5\,\mathrm{K} = 232.35\,^{\circ}\mathrm{C}}$$

Comparison:

- Linear Interpolation: $T \approx 240°C$
- Ideal Gas: $T \approx 232.35°C$

(c) Calculation

Calculate the specific volume v at:

- $T = 220°C$
- $p = 1.4$ MPa

Linear Interpolation

- $T_1 = 210°C$, $v_1 = 0.1420$ $\mathrm{m^3/kg}$
- $T_2 = 230°C$, $v_2 = 0.1500$ $\mathrm{m^3/kg}$

$$v = v_1 + \frac{(T - T_1)}{(T_2 - T_1)} \times (v_2 - v_1)$$

$$v = 0.1420 + \frac{(220 - 210)}{(230 - 210)} \times (0.1500 - 0.1420)$$

$$v = 0.1420 + \frac{10}{20} \times 0.0080 = \boxed{0.1460\,\mathrm{m^3/kg}} \tag{3}$$

Ideal Gas Calculation

$$v = \frac{0.4615 \times 493.15}{1400} \approx \boxed{0.1624\,\mathrm{m^3/kg}}$$

Comparison:

- Linear Interpolation: $v \approx 0.1460$ $\mathrm{m^3/kg}$
- Ideal Gas: $v \approx 0.1624$ $\mathrm{m^3/kg}$

Explanation of Differences

The differences between the results obtained by linear interpolation and the ideal gas equation are mainly due to:

1. **Non-Ideality of Water Vapor:** At high pressures, the vapor behaves differently than an ideal gas. Real gas behavior involves intermolecular forces that are neglected in the ideal gas law.

2. **Calculation Method Differences:** Linear interpolation uses tabulated experimental data which accurately reflects real properties of water vapor, whereas the ideal gas law is a simplified approximation.

3. **Ideal Gas Law Error:** The difference is more pronounced at higher pressures or lower temperatures where real gas effects are significant.