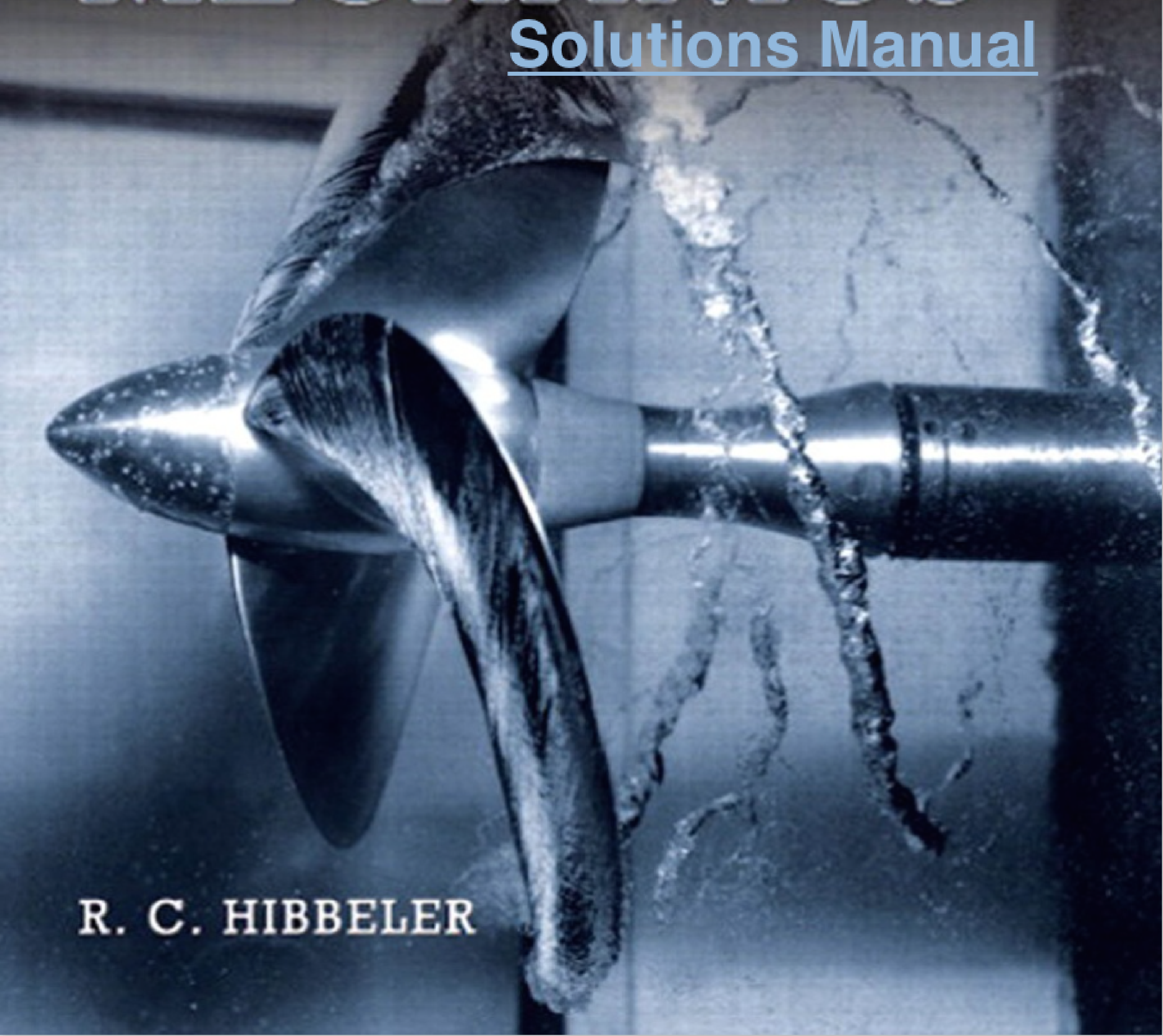
FLUID MECHANICS

Solutions Manual



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1–1. Represent each of the following quantities with combinations of units in the correct SI form, using an appropriate prefix: (a) $\mathrm{GN \cdot \mu m}$, (b) $\mathrm{kg/\mu m}$, (c) $\mathrm{N/ks^2}$, (d) $\mathrm{kN/\mu s}$.

SOLUTION

a) $\mathrm{GN \cdot \mu m} = (10^9)\mathrm{N}(10^{-6})\mathrm{m} = 10^3\ \mathrm{N \cdot m} = \mathrm{kN \cdot m}$ **Ans.**

b) $\mathrm{kg/\mu m} = (10^3)\mathrm{g}/(10^{-6})\mathrm{m} = 10^9\ \mathrm{g/m} = \mathrm{Gg/m}$ **Ans.**

c) $\mathrm{N/ks^2} = \mathrm{N}/(10^3\ \mathrm{s})^2 = 10^{-6}\ \mathrm{N/s^2} = \mathrm{\mu N/s^2}$ **Ans.**

d) $\mathrm{kN/\mu s} = (10^3)\mathrm{N}/(10^{-6})\mathrm{s} = 10^9\ \mathrm{N/s} = \mathrm{GN/s}$ **Ans.**

Ans:
a) $\mathrm{kN \cdot m}$
b) $\mathrm{Gg/m}$
c) $\mathrm{\mu N/s^2}$
d) $\mathrm{GN/s}$

1–2. Evaluate each of the following to three significant figures, and express each answer in SI units using an appropriate prefix: (a) $(425\ \text{mN})^2$, (b) $(67\,300\ \text{ms})^2$, (c) $\left[723(10^6)\right]^{1/2}\ \text{mm}$.

SOLUTION

a) $(425\ \text{mN})^2 = \left[425(10^{-3})\ \text{N}\right]^2 = 0.181\ \text{N}^2$ **Ans.**

b) $(67\,300\ \text{ms})^2 = \left[67.3(10^3)(10^{-3})\ \text{s}\right]^2 = 4.53(10^3)\ \text{s}^2$ **Ans.**

c) $\left[723(10^6)\right]^{1/2}\ \text{mm} = \left[723(10^6)\right]^{1/2}(10^{-3})\ \text{m} = 26.9\ \text{m}$ **Ans.**

Ans:
a) $0.181\ \text{N}^2$
b) $4.53(10^3)\ \text{s}^2$
c) $26.9\ \text{m}$

1–3. Evaluate each of the following to three significant figures, and express each answer in SI units using an appropriate prefix: (a) 749 μm/63 ms, (b) (34 mm)(0.0763 Ms)/263 mg, (c) (4.78 mm)(263 Mg).

SOLUTION

a) $749\ \mu\text{m}/63\ \text{ms} = 749(10^{-6})\ \text{m}/63(10^{-3})\ \text{s} = 11.88(10^{-3})\ \text{m/s}$

$= 11.9\ \text{mm/s}$ **Ans.**

b) $(34\ \text{mm})(0.0763\ \text{Ms})/263\ \text{mg} = \left[34(10^{-3})\ \text{m}\right]\left[0.0763(10^{6})\ \text{s}\right]/\left[263(10^{-6})(10^{3})\ \text{g}\right]$

$= 9.86(10^{6})\ \text{m}\cdot\text{s/kg} = 9.86\ \text{Mm}\cdot\text{s/kg}$ **Ans.**

c) $(4.78\ \text{mm})(263\ \text{Mg}) = \left[4.78(10^{-3})\ \text{m}\right]\left[263(10^{6})\ \text{g}\right]$

$= 1.257(10^{6})\ \text{g}\cdot\text{m} = 1.26\ \text{Mg}\cdot\text{m}$ **Ans.**

Ans:
a) 11.9 mm/**s**
b) 9.86 Mm · s/kg
c) 1.26 Mg · m

***1–4.** Convert the following temperatures: (a) 20°C to degrees Fahrenheit, (b) 500 K to degrees Celsius, (c) 125°F to degrees Rankine, (d) 215°F to degrees Celsius.

SOLUTION

a) $T_C = \frac{5}{9}(T_F - 32)$

$20°\text{C} = \frac{5}{9}(T_F - 32)$

$T_F = 68.0°\text{F}$ **Ans.**

b) $T_K = T_C + 273$

$500\text{ K} = T_C + 273$

$T_C = 227°\text{C}$ **Ans.**

c) $T_R = T_F + 460$

$T_R = 125°\text{F} + 460 = 585°\text{R}$ **Ans.**

d) $T_C = \frac{5}{9}(T_F - 32)$

$T_C = \frac{5}{9}(215°\text{F} - 32) = 102°\text{C}$ **Ans.**

1–5. Mercury has a specific weight of 133 kN/m^3 when the temperature is 20°C. Determine its density and specific gravity at this temperature.

SOLUTION

$$\gamma = \rho g$$

$$133(10^3)\ \text{N/m}^3 = \rho_{\text{Hg}}(9.81\ \text{m/s}^2)$$

$$\rho_{\text{Hg}} = 13\,558\ \text{kg/m}^3 = 13.6\ \text{Mg/m}^3$$ **Ans.**

$$S_{\text{Hg}} = \frac{\rho_{\text{Hg}}}{\rho_w} = \frac{13\,558\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = 13.6$$ **Ans.**

Ans:
$\rho_{\text{Hg}} = 13.6\ \text{Mg/m}^3$
$S_{\text{Hg}} = 13.6$

1–6. The fuel for a jet engine has a density of $1.32\,\text{slug/ft}^3$. If the total volume of fuel tanks A is $50\ \text{ft}^3$, determine the weight of the fuel when the tanks are completely full.

SOLUTION

The specific weight of the fuel is

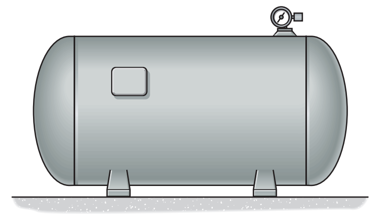
$$\gamma = \rho g = \left(1.32\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right) = 42.504\ \text{lb/ft}^3$$

Then, the weight of the fuel is

$$W = \gamma V\!\!\!/ = \left(42.504\ \text{lb/ft}^3\right)\left(50\ \text{ft}^3\right) = 2.13\left(10^3\right)\ \text{lb} = 2.13\ \text{kip}$$ **Ans.**

Ans:
$\gamma = 42.5\ \text{lb/ft}^3$
$W = 2.13\ \text{kip}$

1–7. If air within the tank is at an absolute pressure of 680 kPa and a temperature of 70°C, determine the weight of the air inside the tank. The tank has an interior volume of 1.35 m^3.



SOLUTION

From the table in Appendix A, the gas constant for air is $R = 286.9\ \text{J/kg}\cdot\text{K}$.

$$p = \rho R T$$

$$680\left(10^3\right)\ \text{N/m}^2 = \rho(286.9\ \text{J/kg}\cdot\text{K})(70° + 273)\ \text{K}$$

$$\rho = 6.910\ \text{kg/m}^3$$

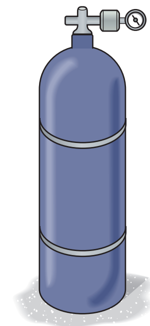
The weight of the air in the tank is

$$W = \rho g \forall = \left(6.910\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(1.35\ \text{m}^3\right)$$

$$= 91.5\ \text{N} \qquad \textbf{Ans.}$$

Ans:
91.5 N

***1–8.** The bottle tank has a volume of 1.12 m^3 and contains oxygen at an absolute pressure of 12 MPa and a temperature of 30°C. Determine the mass of oxygen in the tank.



SOLUTION

From the table in Appendix A, the gas constant for oxygen is $R = 259.8\ \text{J/kg}\cdot\text{K}$.

$$p = \rho RT$$

$$12(10^6)\ \text{N/m}^2 = \rho(259.8\ \text{J/kg}\cdot\text{K})(30^\circ + 273)\ \text{K}$$

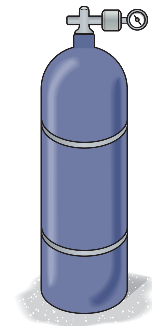
$$\rho = 152.44\ \text{kg/m}^3$$

The mass of oxygen in the tank is

$$m = \rho \forall = (152.44\ \text{kg/m}^3)(0.12\ \text{m}^3)$$

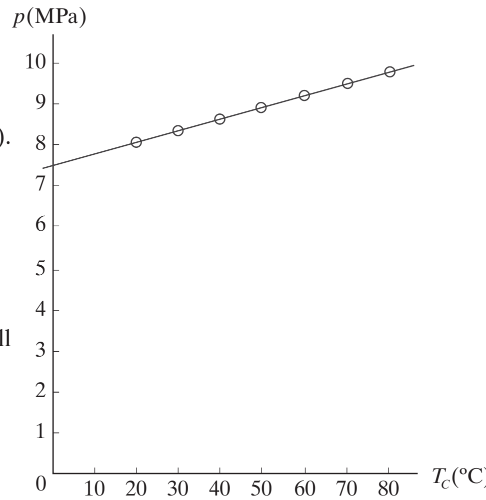
$$= 18.3\ \text{kg} \qquad \textbf{Ans.}$$

1–9. The bottle tank has a volume of $0.12\ m^3$ and contains oxygen at an absolute pressure of 8 MPa and temperature of 20°C. Plot the variation of the temperature in the tank (horizontal axis) versus the pressure for $20°C \le T \le 80°C$. Report values in increments of $\Delta T = 10°C$.



SOLUTION

T_C(°C)	20	30	40	50	60	70	80
p(MPa)	8.00	8.27	8.55	8.82	9.09	9.37	9.64



(a)

From the table in Appendix A, the gas constant for oxygen is $R = 259.8\ \mathrm{J/(kg \cdot K)}$. For $T = (20°C + 273)\ K = 293\ K$,

$$p = \rho RT$$
$$8(10^6)\ \mathrm{N/m^2} = \rho[259.8\ \mathrm{J/(kg \cdot K)}](293\ \mathrm{K})$$
$$\rho = 105.10\ \mathrm{kg/m^3}$$

Since the mass and volume of the oxygen in the tank remain constant, its density will also be constant.

$$p = \rho RT$$
$$p = (105.10\ \mathrm{kg/m^3})[259.8\ \mathrm{J/(kg \cdot K)}](T_C + 273)$$
$$p = (0.02730\ T_C + 7.4539)(10^6)\ \mathrm{Pa}$$
$$p = (0.02730 T_C + 7.4539)\ \mathrm{MPa} \text{ where } T_C \text{ is in °C.}$$

The plot of p vs T_C is shown in Fig. *a*.

Ans:
$p = (0.0273\ T_c + 7.45)$ MPa, where T_c is in C°

1–10. Determine the specific weight of carbon dioxide when the temperature is 100°C and the absolute pressure is 400 kPa.

SOLUTION

From the table in Appendix A, the gas constant for carbon dioxide is $R = 188.9\ \text{J/kg}\cdot\text{K}$.

$$p = \rho RT$$

$$400\left(10^3\right)\ \text{N/m}^2 = \rho(188.9\ \text{J/kg}\cdot\text{K})(100° + 273)\ \text{K}$$

$$\rho = 5.677\ \text{kg/m}^3$$

The specific weight of carbon dioxide is

$$\gamma = \rho g = \left(5.677\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)$$

$$= 55.7\ \text{N/m}^3 \qquad \textbf{Ans.}$$

Ans:
$55.7\ \text{N/m}^3$

1–11. Determine the specific weight of air when the temperature is 100°F and the absolute pressure is 80 psi.

SOLUTION

From the table in Appendix A, the gas constant for the air is $R = 1716 \text{ ft} \cdot \text{lb/slug} \cdot \text{R}$.

$$p = \rho R T$$

$$80 \text{ lb/in}^2 \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 = \rho(1716 \text{ ft} \cdot \text{lb/slug} \cdot \text{R})(100° + 460) \text{ R}$$

$$\rho = 0.01200 \text{ slug/ft}^3$$

The specific weight of the air is

$$\gamma = \rho g = \left(0.01200 \text{ slug/ft}^3\right)\left(32.2 \text{ ft/s}^2\right)$$

$$= 0.386 \text{ lb/ft}^3 \qquad \textbf{Ans.}$$

Ans:
0.386 lb/ft^3

***1–12.** Dry air at 25°C has a density of 1.23 kg/m^3. But if it has 100% humidity at the same pressure, its density is 0.65% less. At what temperature would dry air produce this same density?

SOLUTION

For both cases, the pressures are the same. Applying the ideal gas law with $\rho_1 = 1.23\ \text{kg/m}^3$, $\rho_2 = \left(1.23\ \text{kg/m}^3\right)(1 - 0.0065) = 1.222005\ \text{kg/m}^3$ and $T_1 = (25°\text{C} + 273) = 298\ \text{K}$,

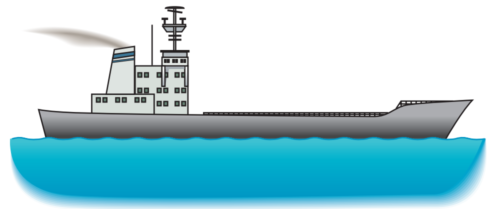
$$p = \rho_1 R T_1 = \left(1.23\ \text{kg/m}^3\right) R\,(298\ \text{K}) = 366.54\ \text{R}$$

Then

$$p = \rho_2 R T_2; \quad 366.54\,R = \left(1.222005\ \text{kg/m}^3\right) R(T_C + 273)$$

$$T_C = 26.9°\text{C} \qquad \textbf{Ans.}$$

1–13. The tanker carries $1.5(10^6)$ barrels of crude oil in its hold. Determine the weight of the oil if its specific gravity is 0.940. Each barrel contains 42 gallons, and there are 7.48 gal/ft^3.



SOLUTION

The specific weight of the oil is

$$\gamma_o = S_o\gamma_w = 0.940\left(62.4 \text{ lb/ft}^3\right) = 58.656 \text{ lb/ft}^3$$

Weight of one barrel of oil:

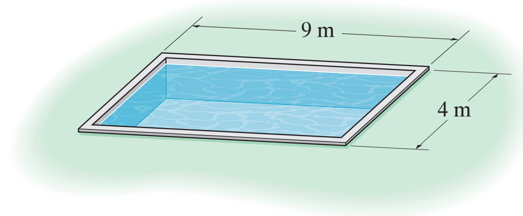
$$W_b = \gamma_o V = \left(58.656 \text{ lb/ft}^3\right)(42 \text{ gal/bl})\left(\frac{1 \text{ ft}^3}{7.48 \text{ gal}}\right)$$
$$= 329.4 \text{ lb/bl}$$

Total weight:

$$W = 1.5\left(10^6\right) \text{ bl}(329.4 \text{ lb/bl})$$
$$= 494\left(10^6\right) \text{ lb}$$ **Ans.**

Ans:
$494\left(10^6\right)$ lb

1–14. Water in the swimming pool has a measured depth of 3.03 m when the temperature is 5°C. Determine its approximate depth when the temperature becomes 35°C. Neglect losses due to evaporation.



SOLUTION

From Appendix A, at $T_1 = 5°\text{C}$, $(\rho_w)_1 = 1000.0\ \text{kg/m}^3$. The volume of the water is $\forall = Ah$. Thus, $\forall_1 = (9\ \text{m})(4\ \text{m})(3.03\ \text{m})$. Then

$$(\rho_w)_1 = \frac{m}{\forall_1}; \qquad 1000.0\ \text{kg/m}^3 = \frac{m}{36\ \text{m}^2(3.03\ \text{m})}$$

$$m = 109.08\left(10^3\right)\ \text{kg}$$

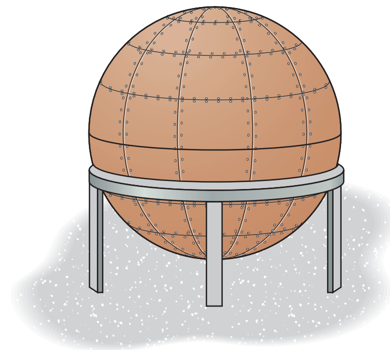
At $T_2 = 35°\text{C}$, $(\rho_w)_2 = 994.0\ \text{kg/m}^3$. Then

$$(\rho_w)_2 = \frac{m}{\forall_2}; \qquad 994.0\ \text{kg/m}^3 = \frac{109.08\left(10^3\right)}{\left(36\ \text{m}^2\right)h}$$

$$h = 3.048\ \text{m} = 3.05\ \text{m} \qquad \textbf{Ans.}$$

Ans:
3.05 m

1–15. The tank contains air at a temperature of 15°C and an absolute pressure of 210 kPa. If the volume of the tank is 5 m^3 and the temperature rises to 30°C, determine the mass of air that must be removed from the tank to maintain the same pressure.



SOLUTION

For $T_1 = (15 + 273)\text{ K} = 288\text{ K}$ and $R = 286.9\text{ J/kg}\cdot\text{K}$ for air, the ideal gas law gives

$$p_1 = \rho_1 R T_1; \qquad 210\left(10^3\right)\text{ N/m}^2 = \rho_1(286.9\text{ J/kg}\cdot\text{K})(288\text{ K})$$
$$\rho_1 = 2.5415\text{ kg/m}^3$$

Thus, the mass of air at T_1 is

$$m_1 = \rho_1 \mathcal{V} = \left(2.5415\text{ kg/m}^3\right)\left(5\text{ m}^3\right) = 12.70768\text{ kg}$$

For $T_2 = (273 + 30)\text{ K} = 303\text{ K}$ and $R = 286.9\text{ J/kg}\cdot\text{K}$

$$p_2 = \rho_1 R T_2; \qquad 210(10^3)\text{ N/m}^2 = \rho_2(286.9\text{ J/kg}\cdot\text{K})(303\text{ K})$$
$$\rho_2 = 2.4157\text{ kg/m}^3$$

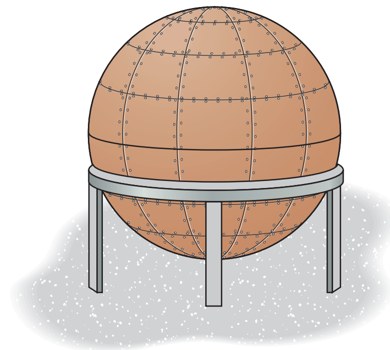
Thus, the mass of air at T_2 is $m_2 = \rho_2 \mathcal{V} = \left(2.4157\text{ kg/m}^3\right)\left(5\text{ m}^3\right) = 12.07886\text{ kg}$

Finally, the mass of air that must be removed is

$$\Delta m = m_1 - m_2 = 12.70768\text{ kg} - 12.07886\text{ kg} = 0.629\text{ kg} \qquad \textbf{Ans.}$$

Ans:
0.629 kg

***1–16.** The tank contains 2 kg of air at an absolute pressure of 400 kPa and a temperature of 20°C. If 0.6 kg of air is added to the tank and the temperature rises to 32°C, determine the pressure in the tank.



SOLUTION

For $T_1 = 20 + 273 = 293$ K, $p_1 = 400$ kPa and $R = 286.9$ J/kg·K for air, the ideal gas law gives

$$p_1 = \rho_1 R T_1; \qquad 400(10^3)\text{ N/m}^2 = \rho_1(286.9\text{ J/kg}\cdot\text{K})(293\text{ K})$$
$$\rho_1 = 4.7584\text{ kg/m}^3$$

Since the volume is constant. Then

$$\forall = \frac{m_1}{\rho_1} = \frac{m_2}{\rho_2};\ \rho_2 = \frac{m_2}{m_1}\rho_1$$

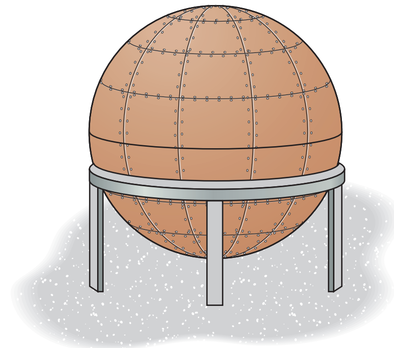
Here $m_1 = 2$ kg and $m_2 = (2 + 0.6)$ kg $= 2.6$ kg

$$\rho_2 = \left(\frac{2.6\text{ kg}}{2\text{ kg}}\right)\left(4.7584\text{ kg/m}^3\right) = 6.1859\text{ kg/m}^3$$

Again applying the ideal gas law with $T_2 = (32 + 273)$ K $= 305$ K

$$p_2 = \rho_2 R T_2 = \left(6.1859\text{ kg/m}^3\right)(286.9\text{ J/kg}\cdot\text{k})(305\text{ K}) = 541.30\left(10^3\right)\text{ Pa}$$
$$= 541\text{ kPa} \qquad \textbf{Ans.}$$

1–17. The tank initially contains carbon dioxide at an absolute pressure of 200 kPa and temperature of 50°C. As more carbon dioxide is added, the pressure is increasing at 25 kPa/min. Plot the variation of the pressure in the tank (vertical axis) versus the temperature for the first 10 minutes. Report the values in increments of two minutes.



SOLUTION

p(kPa)	200	225	250	275	300	325
T_C(°C)	50.00	90.38	130.75	171.12	211.50	251.88

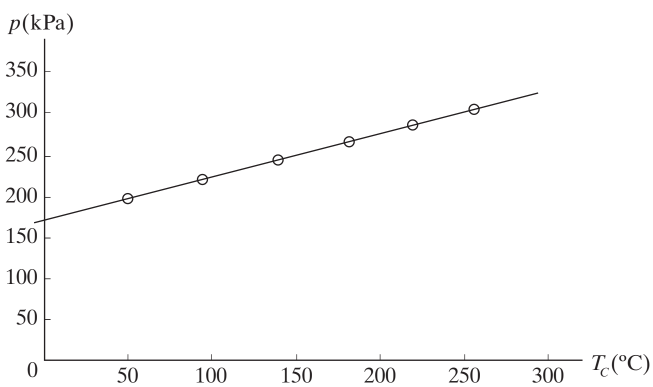
From the table in Appendix A, the gas constant for carbon dioxide is $R = 188.9\ \text{J}/(\text{kg}\cdot\text{K})$. For $T = (50°\text{C} + 273)\ \text{K} = 323\ \text{K}$,

$$p = \rho R T$$
$$200(10^3)\ \text{N/m}^2 = \rho[188.9\ \text{J}/(\text{kg}\cdot\text{K})](323\ \text{K})$$
$$\rho = 3.2779\ \text{kg/m}^3$$

Since the mass and the volume of carbon dioxide in the tank remain constant, its density will also be constant.

$$p = \rho R T$$
$$p = (3.2779\ \text{kg/m}^3)[188.9\ \text{J}/(\text{kg}\cdot\text{K})](T_C + 273)\ \text{K}$$
$$p = (0.6192\ T_C + 169.04)(10^3)\ \text{Pa}$$
$$p = (0.6192\ T_C + 169.04)\ \text{kPa where } T_C \text{ is in °C}$$

The plot of p vs T_C is shown in Fig. *a*



(a)

Ans:
$p = (0.619\ T_c + 169)$ kPa, where T_c is in C°

1–18. Kerosene has a specific weight of $\gamma_k = 50.5\ \text{lb/ft}^3$ and benzene has a specific weight of $\gamma_b = 56.2\ \text{lb/ft}^3$. Determine the amount of kerosene that should be mixed with 8 lb of benzene so that the combined mixture has a specific weight of $\gamma = 52.0\ \text{lb/ft}^3$.

SOLUTION

The volumes of benzene and kerosene are given by

$$\gamma_b = \frac{W_b}{V_b}; \qquad 56.2\ \text{lb/ft}^3 = \frac{8\ \text{lb}}{V_b} \qquad V_b = 0.1423\ \text{ft}^3$$

$$\gamma_k = \frac{W_k}{V_k}; \qquad 50.5\ \text{lb/ft}^3 = \frac{W_k}{V_k} \qquad V_k = 0.019802\ W_k$$

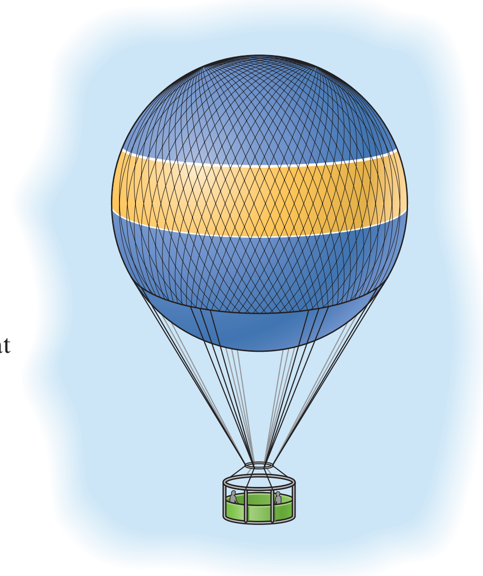
The specific weight of mixture is

$$\gamma = \frac{W_m}{V_m}; \qquad 52.0\ \text{lb/ft}^3 = \frac{W_k + 8\ \text{lb}}{0.1423\ \text{ft}^3 + 0.019802\ W_k}$$

$$W_k = 20.13\ \text{lb} = 20.1\ \text{lb} \qquad \textbf{Ans.}$$

Ans:
20.1 lb

1–19. The 8-m-diameter spherical balloon is filled with helium that is at a temperature of 28°C and a pressure of 106 kPa. Determine the weight of the helium contained in the balloon. The volume of a sphere is $\forall = \frac{4}{3}\pi r^3$.



SOLUTION

For Helium, the gas constant is $R = 2077\ \mathrm{J/kg \cdot K}$. Applying the ideal gas law at $T = (28 + 273)\ \mathrm{K} = 301\ \mathrm{K}$,

$$p = \rho RT; \qquad 106(10^3)\ \mathrm{N/m^2} = \rho(2077\ \mathrm{J/kg \cdot K})(301\ \mathrm{K})$$
$$\rho = 0.1696\ \mathrm{kg/m^3}$$

Here

$$\forall = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4\ \mathrm{m})^3 = \frac{256}{3}\pi\ \mathrm{m^3}$$

Then, the mass of the helium is

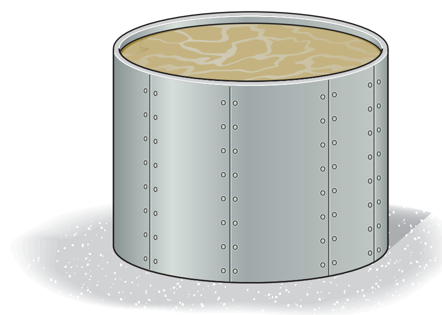
$$M = \rho\forall = (0.1696\ \mathrm{kg/m^3})\left(\frac{256}{3}\pi\ \mathrm{m^3}\right) = 45.45\ \mathrm{kg}$$

Thus,

$$W = mg = (45.45\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) = 445.90\ \mathrm{N} = 446\ \mathrm{N} \qquad \textbf{Ans.}$$

Ans:
446 N

***1–20.** Kerosene is mixed with 10 ft^3 of ethyl alcohol so that the volume of the mixture in the tank becomes 14 ft^3. Determine the specific weight and the specific gravity of the mixture.



SOLUTION

From Appendix A,

$$\rho_k = 1.58\ \text{slug/ft}^3$$
$$\rho_{ea} = 1.53\ \text{slug/ft}^3$$

The volume of kerosene is

$$V_k = 14\ \text{ft}^3 - 10\ \text{ft}^3 = 4\ \text{ft}^3$$

Then the total weight of the mixture is therefore

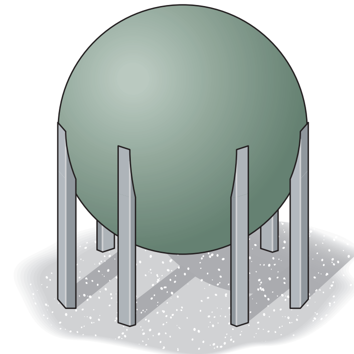
$$\begin{aligned} W &= \rho_k g V k + \rho_{ea} g V_{ea} \\ &= (1.58\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(4\ \text{ft}^3) + (1.53\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(10\ \text{ft}^3) \\ &= 696.16\ \text{lb} \end{aligned}$$

The specific weight and specific gravity of the mixture are

$$\gamma_m = \frac{W}{V} = \frac{696.16\ \text{lb}}{14\ \text{ft}^3} = 49.73\ \text{lb/ft}^3 = 49.7\ \text{lb/ft}^3 \qquad \textbf{Ans.}$$

$$S_m = \frac{\gamma_m}{\gamma_w} = \frac{49.73\ \text{lb/ft}^3}{62.4\ \text{lb/ft}^3} = 0.797 \qquad \textbf{Ans.}$$

1–21. The tank is fabricated from steel that is 20 mm thick. If it contains carbon dioxide at an absolute pressure of 1.35 MPa and a temperature of 20°C, determine the total weight of the tank. The density of steel is 7.85 Mg/m^3, and the inner diameter of the tank is 3 m. *Hint*: The volume of a sphere is $\forall = \left(\frac{4}{3}\right)\pi r^3$.



SOLUTION

From the table in Appendix A, the gas constant for carbon dioxide is $R = 188.9\ \text{J/kg}\cdot\text{K}$.

$$p = \rho R T$$

$$1.35\left(10^6\right)\ \text{N/m}^2 = \rho_{co}(188.9\ \text{J/kg}\cdot\text{K})(20° + 273)\ \text{K}$$

$$\rho_{co} = 24.39\ \text{kg/m}^3$$

Then, the total weight of the tank is

$$W = \rho_{st}\, g\, \forall_{st} + \rho_{co}\, g\, \forall_{co}$$

$$W = \left[7.85\left(10^3\right)\ \text{kg/m}^3\right]\left(9.81\ \text{m/s}^2\right)\left(\frac{4}{3}\right)(\pi)\left[\left(\frac{3.04}{2}\text{m}\right)^3 - \left(\frac{3.00}{2}\text{m}\right)^3\right]$$

$$+ \left(24.39\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(\frac{4}{3}\right)(\pi)\left(\frac{3.00}{2}\text{m}\right)^3$$

$$W = 47.5\ \text{kN}$$ **Ans.**

Ans:
47.5 kN

1–22. What is the increase in the density of helium when the pressure changes from 230 kPa to 450 kPa while the temperature *remains constant* at 20°C? This is called an *isothermal process*.

SOLUTION

Applying the ideal gas law with $T_1 = (20 + 273)\text{ K} = 293\text{ K}$, $p_1 = 230\text{ kPa}$ and $R = 2077\text{ J/(kg}\cdot\text{k)}$,

$$p_1 = \rho_1 R T_1; \qquad 230\left(10^3\right)\text{ N/m}^2 = \rho_1(2077\text{ J/(kg}\cdot\text{K))}(293\text{ K})$$
$$\rho_1 = 0.3779\text{ kg/m}^3$$

For $p_2 = 450\text{ kPa}$ and $T_2 = (20 + 273)\text{ K} = 293\text{ K}$,

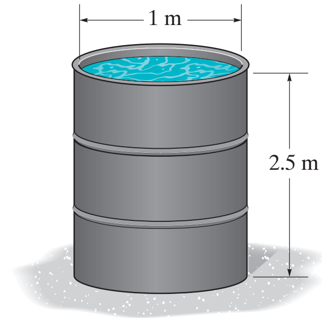
$$p_2 = \rho_2 R T_2; \qquad 450\left(10^3\right)\text{ N/m}^2 = \rho_2(2077\text{ J/(kg}\cdot\text{k))}(293\text{ K})$$
$$\rho_2 = 0.7394\text{ kg/m}^3$$

Thus, the change in density is

$$\Delta\rho = \rho_2 - \rho_1 = 0.7394\text{ kg/m}^3 - 0.3779\text{ kg/m}^3 = 0.3615\text{ kg/m}^3$$
$$= 0.362\text{ kg/m}^3 \qquad \textbf{Ans.}$$

Ans:
0.362 kg/m^3

1–23. The container is filled with water at a temperature of 25°C and a depth of 2.5 m. If the container has a mass of 30 kg, determine the combined weight of the container and the water.



SOLUTION

From Appendix A, $\rho_w = 997.1\ \text{kg/m}^3$ at $T = 25°\text{C}$. Here the volume of water is

$$\forall = \pi r^2 h = \pi(0.5\ \text{m})^2(2.5\ \text{m}) = 0.625\pi\ \text{m}^3$$

Thus, the mass of water is

$$M_w = \rho_w \forall = 997.1\ \text{kg/m}^3\left(0.625\pi\ \text{m}^3\right) = 1957.80\ \text{kg}$$

The total mass is

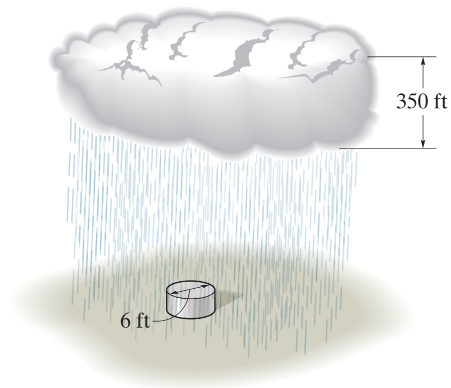
$$M_T = M_w + M_c = (1957.80 + 30)\ \text{kg} = 1987.80\ \text{kg}$$

Then the total weight is

$$W_T = M_T g = (1987.80\ \text{kg})\left(9.81\ \text{m/s}^2\right) = 19500\ \text{N} = 19.5\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
19.5 kN

***1–24.** The rain cloud has an approximate volume of 6.50 mile^3 and an average height, top to bottom, of 350 ft. If a cylindrical container 6 ft in diameter collects 2 in. of water after the rain falls out of the cloud, estimate the total weight of rain that fell from the cloud. 1 mile = 5280 ft.



SOLUTION

The volume of rain water collected is $\forall_w = \pi(3\text{ ft})^2\left(\frac{2}{12}\text{ ft}\right) = 1.5\pi\text{ ft}^3$. Then, the weight of the rain water is $W_w = \gamma_w \forall_w = \left(62.4\text{ lb/ft}^3\right)\left(1.5\pi\text{ ft}^3\right) = 93.6\pi\text{ lb}$. Here, the volume of the overhead cloud that produced this amount of rain is

$$\forall_c' = \pi(3\text{ ft})^2(350\text{ ft}) = 3150\pi\text{ ft}^3$$

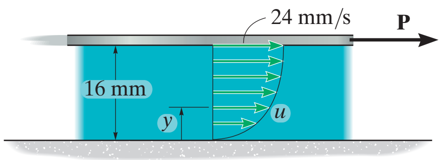
Thus,

$$\gamma_c = \frac{W}{\forall_c'} = \frac{93.6\pi\text{ lb}}{3150\pi\text{ ft}^3} = 0.02971\text{ lb/ft}^3$$

Then

$$W_c = \gamma_c \forall_c = \left(0.02971\,\frac{\text{lb}}{\text{ft}^3}\right)\left[(6.50)\left(\frac{5280^3\text{ ft}^3}{1}\right)\right]$$
$$= 28.4\left(10^9\right)\text{ lb} \qquad \textbf{Ans.}$$

1–38. When the force **P** is applied to the plate, the velocity profile for a Newtonian fluid that is confined under the plate is approximated by $u = (12y^{1/4})$ mm/s, where y is in mm. Determine the minimum shear stress within the fluid. Take $\mu = 0.5(10^{-3})\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = 12y^{1/4}$$

$$\frac{du}{dy} = 3y^{-3/4}$$

The velocity gradient is smallest when $y = 16$ mm. Thus,

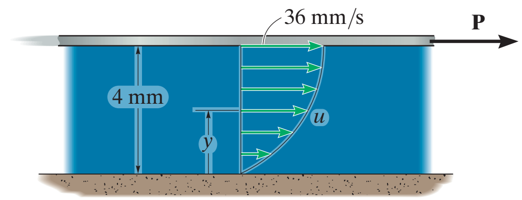
$$\tau_{\min} = \mu\frac{du}{dy} = \left[0.5(10^{-3})\ \mathrm{N \cdot s/m^2}\right]\left[3(16\ \mathrm{mm})^{-3/4}\ \mathrm{s^{-1}}\right]$$

$$\tau_{\min} = 0.1875\ \mathrm{mPa}$$ **Ans.**

Note: When $y = 0$, $\frac{du}{dy} \rightarrow \infty$, so, that $\tau \rightarrow \infty$. Hence the equation can not be used at this point.

Ans:
0.1875 mPa

1–39. The velocity profile for a thin film of a Newtonian fluid that is confined between a plate and a fixed surface is defined by $u = (10y - 0.25y^2)$ mm/s, where y is in mm. Determine the shear stress that the fluid exerts on the plate and on the fixed surface. Take $\mu = 0.532\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = \left(10y - 0.25y^2\right)\ \mathrm{mm/s}$$

$$\frac{du}{dy} = (10 - 0.5y)\ \mathrm{s^{-1}}$$

At the plate

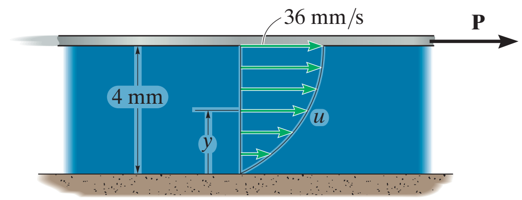
$$\tau_p = \mu\frac{du}{dy} = \left(0.532\ \mathrm{N \cdot s/m^2}\right)\left[10 - 0.5(4\ \mathrm{mm})\ \mathrm{s^{-1}}\right] = 4.26\ \mathrm{Pa} \qquad \textbf{Ans.}$$

At the fixed surface

$$\tau_{fs} = \mu\frac{du}{dy} = \left(0.532\ \mathrm{N \cdot s/m^2}\right)\left[(10 - 0)\ \mathrm{s^{-1}}\right] = 5.32\ \mathrm{Pa} \qquad \textbf{Ans.}$$

Ans:
$\tau_p = 4.26$ Pa, $\tau_{fs} = 5.32$ Pa

***1–40.** The velocity profile for a thin film of a Newtonian fluid that is confined between the plate and a fixed surface is defined by $u = (10y - 0.25y^2)$ mm/s, where y is in mm. Determine the force **P** that must be applied to the plate to cause this motion. The plate has a surface area of 5000 mm^2 in contact with the fluid. Take $\mu = 0.532\ \text{N}\cdot\text{s/m}^2$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = \left(10y - 0.25y^2\right)\ \text{mm/s}$$

$$\frac{du}{dy} = (10 - 0.5y)\ \text{s}^{-1}$$

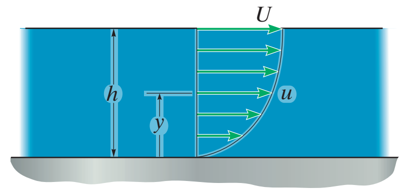
At the plate

$$\tau_p = \mu\frac{du}{dy} = \left(0.532\ \text{N}\cdot\text{s/m}^2\right)[10 - 0.5(4\ \text{mm})]\ \text{s}^{-1} = 4.256\ \text{Pa}$$

$$P = \tau_p A = \left[(4.256)\ \text{N/m}^2\right]\left[5000\left(10^{-6}\right)\ \text{m}^2\right]$$

$$= 21.3\ \text{mN}$$ **Ans.**

1–41. The velocity profile of a Newtonian fluid flowing over a fixed surface is approximated by $u = U\sin\left(\frac{\pi}{2h}y\right)$. Determine the shear stress in the fluid at $y = h$ and at $y = h/2$. The viscosity of the fluid is μ.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = U\sin\left(\frac{\pi}{2h}y\right)$$

$$\frac{du}{dy} = U\left(\frac{\pi}{2h}\right)\cos\left(\frac{\pi}{2h}y\right)$$

At $y = h$,

$$\tau = \mu\frac{du}{dy} = \mu U\left(\frac{\pi}{2h}\right)\cos\frac{\pi}{2h}(h)$$

$$\tau = 0; \qquad \textbf{Ans.}$$

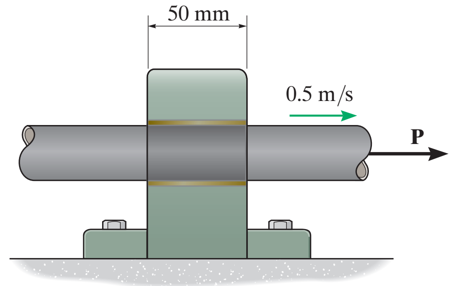
At $y = h/2$,

$$\tau = \mu\frac{du}{dy} = \mu U\left(\frac{\pi}{2h}\right)\cos\frac{\pi}{2h}\left(\frac{h}{2}\right)$$

$$\tau = \frac{0.354\pi\mu U}{h} \qquad \textbf{Ans.}$$

Ans:
At $y = h, \tau = 0$;
At $y = h/2, \quad \tau = \dfrac{0.354\pi\mu U}{h}$

1–42. If a force of $P = 2\text{ N}$ causes the 30-mm-diameter shaft to slide along the lubricated bearing with a constant speed of 0.5 m/s, determine the viscosity of the lubricant and the constant speed of the shaft when $P = 8\text{ N}$. Assume the lubricant is a Newtonian fluid and the velocity profile between the shaft and the bearing is linear. The gap between the bearing and the shaft is 1 mm.



SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{2\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = \mu\left(\frac{0.5\text{ m/s}}{0.001\text{ m}}\right)$$

$$\mu = 0.8498\text{ N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Thus,

$$\frac{8\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = \left(0.8488\text{ N}\cdot\text{s/m}^2\right)\left(\frac{v}{0.001\text{ m}}\right)$$

$$v = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

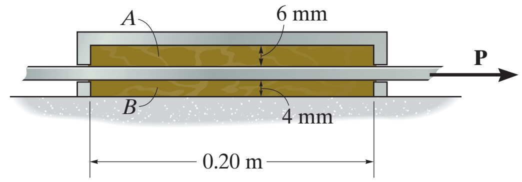
Also, by proportion,

$$\frac{\left(\frac{2\text{ N}}{A}\right)}{\left(\frac{8\text{ N}}{A}\right)} = \frac{\mu\left(\frac{0.5\text{ m/s}}{t}\right)}{\mu\left(\frac{v}{t}\right)}$$

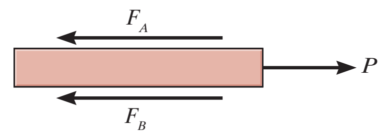
$$v = \frac{4}{2}\text{ m/s} = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\mu = 0.849\text{ N}\cdot\text{s/m}^2$
$v = 2.00\text{ m/s}$

1–43. The 0.15-m-wide plate passes between two layers, A and B, of oil that has a viscosity of $\mu = 0.04\ \mathrm{N \cdot s/m^2}$. Determine the force **P** required to move the plate at a constant speed of 6 mm/s. Neglect any friction at the end supports, and assume the velocity profile through each layer is linear.



SOLUTION



(a)

The oil is a Newtonian fluid.

Considering the force equilibrium along the x axis, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_A - F_B = 0$$

$$P = F_A + F_B$$

Since the velocity distribution is linear, the velocity gradient will be constant.

$$\tau_A = \mu \frac{du}{dy} = \left(0.04\ \mathrm{N \cdot s/m^2}\right)\left(\frac{6\ \mathrm{mm/s}}{6\ \mathrm{mm}}\right) = 0.04\ \mathrm{Pa}$$

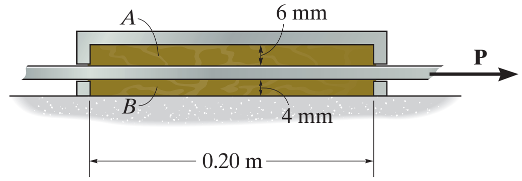
$$\tau_B = \mu \frac{du}{dy} = \left(0.04\ \mathrm{N \cdot s/m^2}\right)\left(\frac{6\ \mathrm{mm/s}}{4\ \mathrm{mm}}\right) = 0.06\ \mathrm{Pa}$$

$$P = \left(0.04\ \mathrm{N/m^2}\right)(0.2\ \mathrm{m})(0.15\ \mathrm{m}) + \left(0.06\ \mathrm{N/m^2}\right)(0.2\ \mathrm{m})(0.15\ \mathrm{m})$$

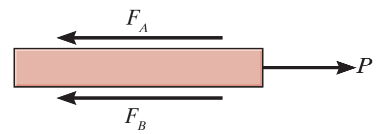
$$= 3.00\ \mathrm{mN}$$ **Ans.**

Ans:
3.00 mN

***1–44.** The 0.15-m-wide plate passes between two layers of oil, A and B, having viscosities of $\mu_A = 0.03\ \text{N}\cdot\text{s/m}^2$ and $\mu_B = 0.01\ \text{N}\cdot\text{s/m}^2$. Determine the force P required to move the plate at a constant speed of 6 mm/s. Neglect any friction at the end supports, and assume the velocity profile through each layer is linear.



SOLUTION



(a)

The oil is a Newtonian fluid.

Considering the force equilibrium along the x axis, Fig. a,

$$\Sigma F_x = 0; \qquad P - F_A - F_B = 0$$
$$P = F_A + F_B$$

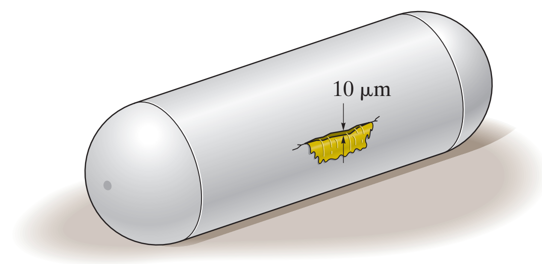
Since the velocity distribution is linear, the velocity gradient will be constant.

$$\tau_A = \mu\frac{du}{dy} = \left(0.03\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{6\ \text{mm/s}}{6\ \text{mm}}\right) = 0.03\ \text{Pa}$$

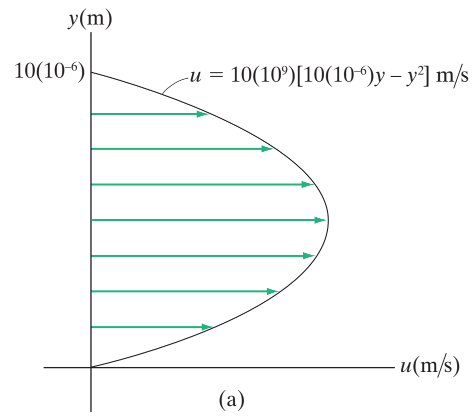
$$\tau_B = \mu\frac{du}{dy} = \left(0.01\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{6\ \text{mm/s}}{4\ \text{mm}}\right) = 0.015\ \text{Pa}$$

$$P = \left(0.03\ \text{N/m}^2\right)(0.2\ \text{m})(0.15\ \text{m}) + \left(0.015\ \text{N/m}^2\right)(0.2\ \text{m})(0.15\ \text{m})$$
$$= 1.35\ \text{mN}$$ **Ans.**

1–45. The tank containing gasoline has a long crack on its side that has an average opening of 10 μm. The velocity through the crack is approximated by the equation $u = 10(10^9)\ [10(10^{-6}y - y^2)]$ m/s, where y is in meters, measured upward from the bottom of the crack. Find the shear stress at the bottom, at $y = 0$ and the location y within the crack where the shear stress in the gasoline is zero. Take $\mu_g = 0.317(10^{-3})\ \mathrm{N \cdot s/m^2}$.



SOLUTION



(a)

Gasoline is a Newtonian fluid.

The rate of change of shear strain as a function of y is

$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right]\ \mathrm{s^{-1}}$$

At the surface of crack, $y = 0$ and $y = 10(10^{-6})$ m. Then

$$\left.\frac{du}{dy}\right|_{y=0} = 10(10^9)\left[10(10^{-6}) - 2(0)\right] = 100(10^3)\ \mathrm{s^{-1}}$$

or

$$\left.\frac{du}{dy}\right|_{y=10\,(10^{-6})\ \mathrm{m}} = 10(10^9)\left\{10(10^{-6}) - 2\left[10(10^{-6})\right]\right\} = -100(10^3)\ \mathrm{s^{-1}}$$

Applying Newton's law of viscosity,

$$\tau_{y=0} = \mu_g \left.\frac{du}{dy}\right|_{y=0} = \left[0.317(10^{-3})\ \mathrm{N \cdot s/m^2}\right]\left[100(10^3)\ \mathrm{s^{-1}}\right] = 31.7\ \mathrm{N/m^2} \qquad \textbf{Ans.}$$

$\tau = 0$ when $\dfrac{du}{dy} = 0$. Thus

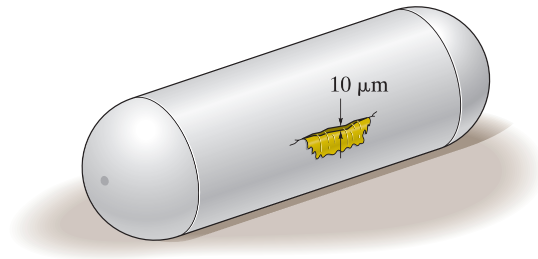
$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right] = 0$$

$$10(10^{-6}) - 2y = 0$$

$$y = 5(10^{-6})\ \mathrm{m} = 5\ \mu\mathrm{m} \qquad \textbf{Ans.}$$

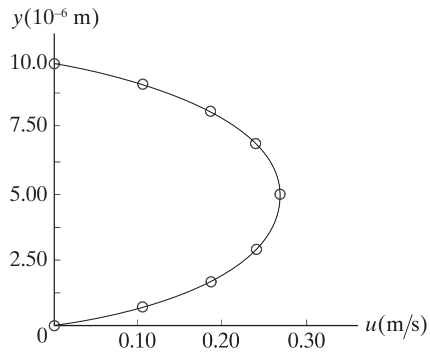
Ans:
$\tau_{y=0} = 31.7\ \mathrm{N/m^2}$
$\tau = 0$ when $y = 5\ \mu\mathrm{m}$

1–46. The tank containing gasoline has a long crack on its side that has an average opening of 10 μm. If the velocity profile through the crack is approximated by the equation $u = 10(10^9)[10(10^{-6}y - y^2)]$ m/s, where y is in meters, plot both the velocity profile and the shear stress distribution for the gasoline as it flows through the crack. Take $\mu_g = 0.317(10^{-3})\ \text{N}\cdot\text{s/m}^2$.



SOLUTION

$y(10^{-6}\ \text{m})$	0	1.25	2.50	3.75	5.00
$u(\text{m/s})$	0	0.1094	0.1875	0.2344	0.250
	6.25	7.50	8.75	10.0	
	0.2344	0.1875	0.1094	0	



(a)

Gasoline is a Newtonian fluid. The rate of change of shear strain as a function of y is

$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right]\ \text{s}^{-1}$$

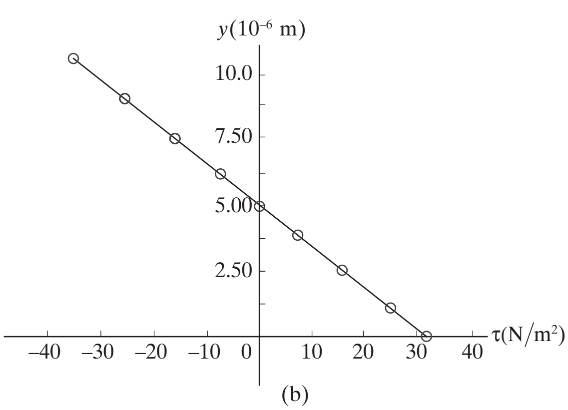
Applying Newton's law of viscoscity,

$$\tau = \mu\frac{du}{dy} = \left[0.317(10^{-3})\ \text{N}\cdot\text{s/m}^2\right]\left\{10(10^9)\left[10(10^{-6}) - 2y\right]\ \text{s}^{-1}\right\}$$

$$\tau = 3.17(10^6)\left[10(10^{-6}) - 2y\right]\ \text{N/m}^2$$

The plots of the velocity profile and the shear stress distribution are shown in Fig. *a* and *b* respectively.

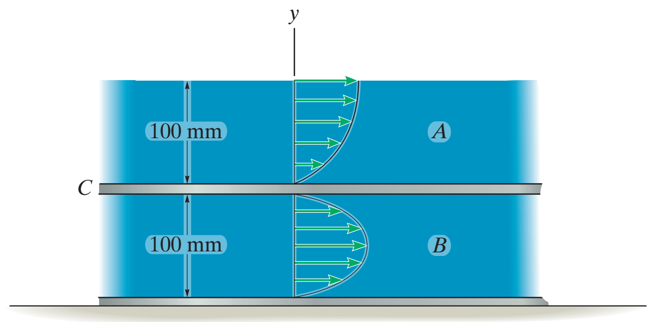
$y(10^{-6}\ \text{m})$	0	1.25	2.50	3.75	5.00
$\tau(\text{N/m}^2)$	31.70	23.78	15.85	7.925	0
	6.25	7.50	8.75	10.0	
	−7.925	−15.85	−23.78	−31.70	



(b)

Ans:
$y = 1.25\left(10^{-6}\right)$ m, $u = 0.109$ m/s, $\tau = 23.8\ \text{N/m}^2$

1–47. Water at A has a temperature of 15°C and flows along the top surface of the plate C. The velocity profile is approximated as $\mu_A = 10\sin(2.5\pi y)$ m/s, where y is in meters. Below the plate the water at B has a temperature of 60°C and a velocity profile of $u_B = 4(10^3)(0.1y - y^2)$, where y is in meters. Determine the resultant force per unit length of plate C the flow exerts due to viscous friction. The plate is 3 m wide.



SOLUTION

Water is a Newtonian fluid.

Water at A, $T = 15$°C. From Appendix A $\mu = 1.15(10^{-3})\ \text{N}\cdot\text{s/m}^2$. Here

$$\frac{du_A}{dy} = 10\left(\frac{5\pi}{2}\right)\cos\left(\frac{5\pi}{2}y\right) = \left(25\pi\cos\frac{5\pi}{2}y\right)\text{s}^{-1}$$

At surface of plate C, $y = 0$. Then

$$\left.\frac{du_A}{dy}\right|_{y=0} = 25\pi\cos\left[\frac{5\pi}{2}(0)\right] = 25\pi\ \text{s}^{-1}$$

Applying Newton's law of viscosity

$$\tau_A\big|_{y=0} = \mu\left.\frac{du_A}{dy}\right|_{y=0} = \left[1.15(10^{-3})\ \text{N}\cdot\text{s/m}^2\right](25\pi\ \text{s}^{-1}) = 0.02875\pi\ \text{N/m}^2$$

Water at B, $T = 60$°C. From Appendix A $\mu = 0.470(10^{-3})\ \text{N}\cdot\text{s/m}^2$. Here

$$\frac{du_B}{dy} = \left[4(10^3)(0.1 - 2y)\right]\text{s}^{-1}$$

At the surface of plate C, $y = 0.1$ m. Then

$$\left.\frac{du_B}{dy}\right|_{y=0.1\text{ m}} = 4(10^3)[0.1 - 2(0.1)] = -400\ \text{s}^{-1}$$

Applying Newton's law of viscosity,

$$\tau_B\big|_{y=0.1\text{ m}} = \mu\left.\frac{du_B}{dy}\right|_{y=0.1\text{ m}} = \left[0.470(10^{-3})\ \text{N}\cdot\text{s/m}^2\right](400\ \text{s}^{-1}) = 0.188\ \text{N/m}^2$$

Here, the area per unit length of plate is $A = 3$ m. Thus

$$F = (\tau_A + \tau_B)A = (0.02875\pi\ \text{N/m}^2 + 0.188\ \text{N/m}^2)(3\text{ m})$$
$$= 0.835\ \text{N/m}$$ **Ans.**

Ans:
0.835 N/m

***1–48.** Determine the constants B and C in Andrade's equation for water if it has been experimentally determined that $\mu = 1.00(10^{-3})\ \mathrm{N \cdot s/m^2}$ at a temperature of 20°C and that $\mu = 0.554(10^{-3})\ \mathrm{N \cdot s/m^2}$ at 50°C.

SOLUTION

The Andrade's equation is

$$\mu = Be^{C/T}$$

At $T = (20 + 273)\ \mathrm{K} = 293\ \mathrm{K}$, $\mu = 1.00(10^{-3})\ \mathrm{N \cdot s/m^2}$. Thus

$$\begin{aligned}
1.00(10^{-3})\ \mathrm{N \cdot s/m^2} &= Be^{C/293\ \mathrm{K}} \\
\ln\left[1.00(10^{-3})\right] &= \ln\left(Be^{C/293}\right) \\
-6.9078 &= \ln B + \ln e^{C/293} \\
-6.9078 &= \ln B + C/293 \\
\ln B &= -6.9078 - C/293 \qquad \textbf{(1)}
\end{aligned}$$

At $T = (50 + 273)\ \mathrm{K} = 323\ \mathrm{K}$, $\mu = 0.554(10^{-3})\ \mathrm{N \cdot s/m^2}$. Thus,

$$\begin{aligned}
0.554(10^{-3})\ \mathrm{N \cdot s/m^2} &= Be^{C/323} \\
\ln\left[0.554(10^{-3})\right] &= \ln\left(Be^{C/323}\right) \\
-7.4983 &= \ln B + \ln e^{C/323} \\
-7.4983 &= \ln B + \frac{C}{323} \\
\ln B &= -7.4983 - \frac{C}{323} \qquad \textbf{(2)}
\end{aligned}$$

Equating Eqs. (1) and (2)

$$\begin{aligned}
-6.9078 - \frac{C}{293} &= -7.4983 - \frac{C}{323} \\
0.5906 &= 0.31699(10^{-3})\,C \\
C &= 1863.10 = 1863\ \mathrm{K} \qquad \textbf{Ans.}
\end{aligned}$$

Substitute this result into Eq. (1)

$$\begin{aligned}
B &= 1.7316(10^{-6})\ \mathrm{N \cdot s/m^2} \\
&= 1.73(10^{-6})\ \mathrm{N \cdot s/m^2} \qquad \textbf{Ans.}
\end{aligned}$$

1–49. The viscosity of water can be determined using the empirical Andrade's equation with the constants $B = 1.732(10^{-6})\ \text{N}\cdot\text{s/m}^2$ and $C = 1863\ \text{K}$. With these constants, compare the results of using this equation with those tabulated in Appendix A for temperatures of $T = 10°\text{C}$ and $T = 80°\text{C}$.

SOLUTION

The Andrade's equation for water is

$$\mu = 1.732(10^{-6})e^{1863/T}$$

At $T = (10 + 273)\ \text{K} = 283\ \text{K}$,

$$\mu = 1.732(10^{-6})e^{1863\ \text{K}/283\ \text{K}} = 1.25(10^{-3})\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

From the Appendix at $T = 10°\text{C}$,

$$\mu = 1.31(10^{-3})\ \text{N}\cdot\text{s/m}^2$$

At $T = (80 + 273)\ \text{K} = 353\ \text{K}$,

$$\mu = 1.732(10^{-6})e^{1863\ \text{K}/353\ \text{K}} = 0.339(10^{-3})\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

From the Appendix at $T = 80°\text{C}$,

$$\mu = 0.356(10^{-3})\ \text{N}\cdot\text{s/m}^2$$

Ans:
At $T = 283\ \text{K}$, $\mu = 1.25(10^{-3})\ \text{N}\cdot\text{s/m}^2$
At $T = 353\ \text{K}$, $\mu = 0.339(10^{-3})\ \text{N}\cdot\text{s/m}^2$

1–50. Determine the constants B and C in the Sutherland equation for air if it has been experimentally determined that at standard atmospheric pressure and a temperature of 20°C, $\mu = 18.3(10^{-6})\ \text{N}\cdot\text{s/m}^2$, and at 50°C, $\mu = 19.6(10^{-6})\ \text{N}\cdot\text{s/m}^2$.

SOLUTION

The Sutherland equation is

$$\mu = \frac{BT^{3/2}}{T + C}$$

At $T = (20 + 273)\ \text{K} = 293\ \text{K}$, $\mu = 18.3(10^{-6})\ \text{N}\cdot\text{s/m}^2$. Thus,

$$18.3(10^{-6})\ \text{N}\cdot\text{s/m}^2 = \frac{B(293^{3/2})}{293\ \text{K} + C}$$

$$B = 3.6489(10^{-9})(293 + C) \qquad \textbf{(1)}$$

At $T = (50 + 273)\ \text{K} = 323\ \text{K}$, $\mu = 19.6(10^{-6})\ \text{N}\cdot\text{s/m}^2$. Thus

$$19.6(10^{-6})\ \text{N}\cdot\text{s/m}^2 = \frac{B(323^{3/2})}{323\ \text{K} + C}$$

$$B = 3.3764(10^{-9})(323 + C) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$B = 1.36(10^{-6})\ \text{N}\cdot\text{s/}(\text{m}^2\,\text{K}^{\frac{1}{2}})$ $\qquad C = 78.8\ \text{K}$ **Ans.**

Ans:
$B = 1.36\ (10^{-6})\ \text{N}\cdot\text{s/}(\text{m}^2\cdot\text{K}^{\frac{1}{2}})$, $C = 78.8\ \text{K}$

1–51. The constants $B = 1.357(10^{-6})\ \text{N}\cdot\text{s}/(\text{m}^2\cdot K^{1/2})$ and $C = 78.84\ \text{K}$ have been used in the empirical Sutherland equation to determine the viscosity of air at standard atmospheric pressure. With these constants, compare the results of using this equation with those tabulated in Appendix A for temperatures of $T = 10°\text{C}$ and $T = 80°\text{C}$.

SOLUTION

The Sutherland Equation for air at standard atmospheric pressure is

$$\mu = \frac{1.357(10^{-6})T^{3/2}}{T + 78.84}$$

At $T = (10 + 273)\ \text{K} = 283\ \text{K}$,

$$\mu = \frac{1.357(10^{-6})(283^{3/2})}{283 + 78.84} = 17.9(10^{-6})\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

From Appendix A at $T = 10°\text{C}$,

$$\mu = 17.6(10^{-6})\ \text{N}\cdot\text{s/m}^2$$

At $T = (80 + 273)\ \text{K} = 353\ \text{K}$,

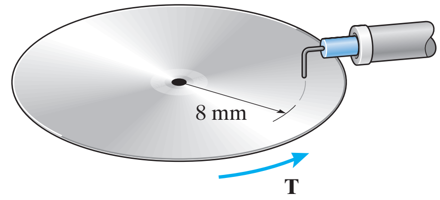
$$\mu = \frac{1.357(10^{-6})(353^{3/2})}{353 + 78.84} = 20.8(10^{-6})\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

From Appendix A at $T = 80°\text{C}$,

$$\mu = 20.9(10^{-6})\ \text{N}\cdot\text{s/m}^2$$

Ans:
Using the Sutherland equation,
at $T = 283\ \text{K}$, $\mu = 17.9\,(10^{-6})\ \text{N}\cdot\text{s/m}^2$
at $T = 353\ \text{K}$, $\mu = 20.8(10^{-6})\ \text{N}\cdot\text{s/m}^2$

***1–52.** The read–write head for a hand-held music player has a surface area of 0.04 mm^2. The head is held 0.04 μm above the disk, which is rotating at a constant rate of 1800 rpm. Determine the torque **T** that must be applied to the disk to overcome the frictional shear resistance of the air between the head and the disk. The surrounding air is at standard atmospheric pressure and a temperature of 20°C. Assume the velocity profile is linear.



SOLUTION

Here Air is a Newtonian fluid.

$$\omega = \left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 60\pi\text{ rad/s}.$$

Thus, the velocity of the air on the disk is $U = \omega r = (60\pi)(0.008) = 0.48\pi$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. *a*,

$$\frac{du}{dy} = \frac{U}{t} = \frac{0.48\pi\text{ m/s}}{0.04\left(10^{-6}\right)\text{m}} = 12\left(10^{6}\right)\pi\text{ s}^{-1}$$

For air at $T = 20°\text{C}$ and standard atmospheric pressure, $\mu = 18.1\left(10^{-6}\right)\text{ N}\cdot\text{s/m}^2$ (Appendix A). Applying Newton's law of viscosity,

$$\tau = \mu\frac{du}{dy} = \left[18.1\left(10^{-6}\right)\text{ N}\cdot\text{s/m}^2\right]\left[12\left(10^{6}\right)\pi\text{ s}^{-1}\right] = 217.2\pi\text{ N/m}^2$$

Then, the drag force produced is

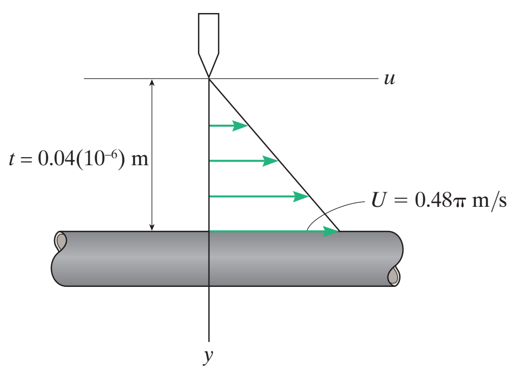
$$F_D = \tau A = \left(217.2\pi\text{ N/m}^2\right)\left(\frac{0.04}{1000^2}\text{ m}^2\right) = 8.688\left(10^{-6}\right)\pi\text{ N}$$

The moment equilibrium about point O requires

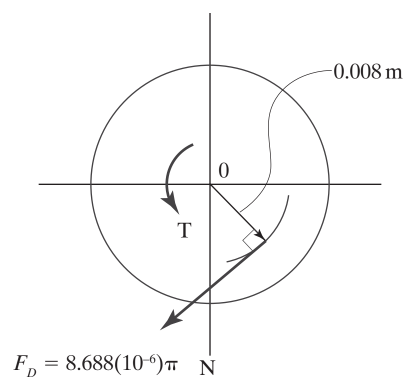
$$\circlearrowleft{+}\Sigma M_O = 0; \qquad T - \left[8.688\left(10^{-6}\right)\pi\text{ N}\right](0.008\text{ m}) = 0$$

$$T = 0.218\left(10^{-6}\right)\text{ N}\cdot\text{m}$$

$$= 0.218\pi\ \mu\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

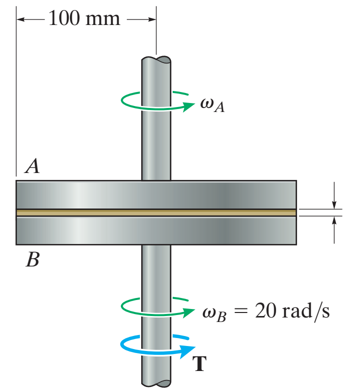


(a)



(b)

1–53. Disks A and B rotate at a constant rate of $\omega_A = 50$ rad/s and $\omega_B = 20$ rad/s respectively. Determine the torque $\mathbf{T}$ required to sustain the motion of disk B. The gap, $t = 0.1$ mm, contains SAE 10 oil for which $\mu = 0.02\ \text{N}\cdot\text{s/m}^2$. Assume the velocity profile is linear.



SOLUTION

Oil is a Newtonian fluid.

The velocities of the oil on the surfaces of disks A and B are $U_A = \omega_A r = (50r)$ m/s and $U_B = \omega_B r = (20r)$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. a,

$$\frac{du}{dy} = \frac{U_A - U_B}{t} = \frac{50r - 20r}{0.1\left(10^{-3}\right)\ \text{m}} = 300\left(10^3\right)r\ \text{s}^{-1}$$

Applying Newton's Law of viscosity,

$$\tau = \mu\frac{du}{dy} = \left(0.02\ \text{N}\cdot\text{s/m}^2\right)\left[300\left(10^3\right)r\right] = (6000r)\ \text{N/m}^2$$

The shaded differential element shown in Fig. b has an area of $dA = 2\pi r\,dr$. Thus, $dF = \tau dA = (6000r)(2\pi r\,dr) = 12\left(10^3\right)\pi r^2\,dr$. Moment equilibrium about point O in Fig. b requires

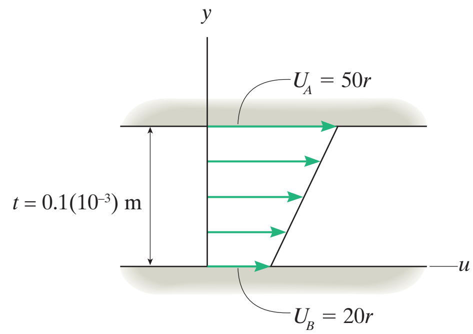
$$\circlearrowleft + \Sigma M_O = 0; \qquad T - \int r\,dF = 0$$

$$T - \int_0^{0.1\text{ m}} r\left[12\left(10^3\right)\pi r^2\,dr\right] = 0$$

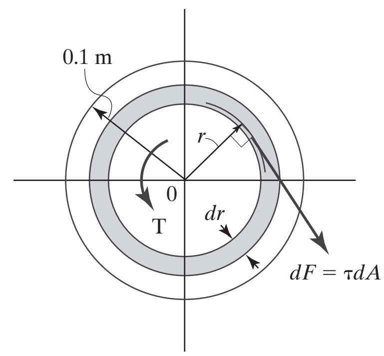
$$T = \int_0^{0.1\text{ m}} 12\left(10^3\right)\pi r^3\,dr$$

$$= 12\left(10^3\right)\pi\left(\frac{r^4}{4}\right)\Bigg|_0^{0.1\text{ m}}$$

$$= 0.942\ \text{N}\cdot\text{m}$$ **Ans.**



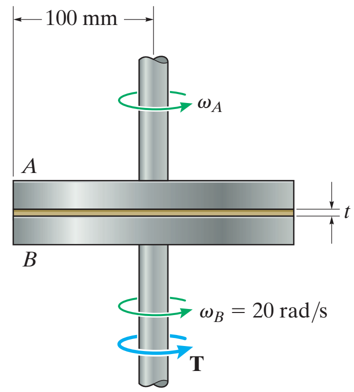
(a)



(b)

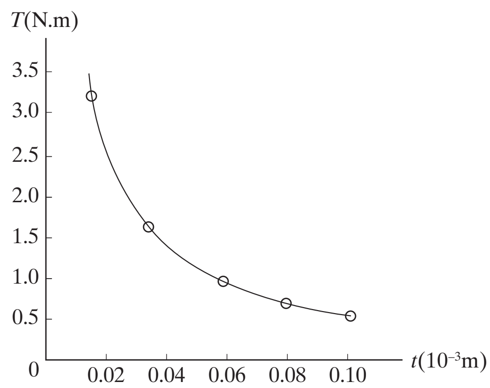
Ans:
$0.942\ \text{N}\cdot\text{m}$

1–54. If disk A is stationary, $\omega_A = 0$ and disk B rotates at $\omega_B = 20$ rad/s, determine the torque **T** required to sustain the motion. Plot your results of torque (vertical axis) versus the gap thickness for $0 \le t \le 0.1$ m. The gap contains SAE10 oil for which $\mu = 0.02\ \text{N}\cdot\text{s/m}^2$. Assume the velocity profile is linear.

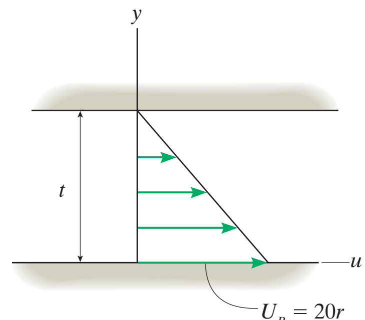


SOLUTION

$t(10^{-3})$ m	0	0.02	0.04	0.06	0.08	0.10
$T(\text{N}\cdot\text{m})$	∞	3.14	1.57	1.05	0.785	0.628



(c)



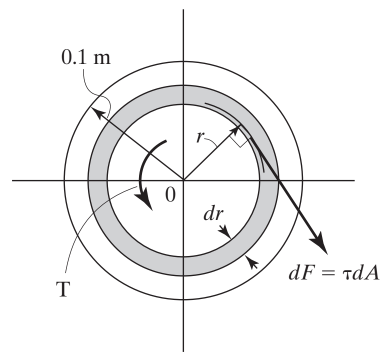
(a)

Oil is a Newtonian fluid. The velocities of the oil on the surfaces of disks A and B are $U_A = \omega_A r = 0$ and $U_B = \omega_B r = (20r)$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. *a*,

$$\frac{du}{dy} = \frac{U_A - U_B}{t} = \frac{0 - 20r}{t} = \left(-\frac{20r}{t}\right)\text{s}^{-1}$$

Applying Newton's law of viscosity,

$$\tau = \mu\left|\frac{du}{dy}\right| = \left(0.02\ \text{N}\cdot\text{s/m}^2\right)\left(\frac{20r}{t}\right) = \left(\frac{0.4r}{t}\right)\text{N/m}^2$$



(b)

1–54. (continued)

The shaded differential element shown in Fig. *b* has an area of $dA = 2\pi r\,dr$. Thus, $dF = \tau dA = \left(\dfrac{0.4r}{t}\right)(2\pi r\,dr) = \left(\dfrac{0.8\pi}{t}\right)r^2\,dr$. Moment equilibrium about point O in Fig. *b* requires

$$\circlearrowleft + \Sigma M_O = 0; \qquad T - \int r\,dF = 0$$

$$T - \int_0^{0.1\text{ m}} r\left[\left(\frac{0.8\pi}{t}\right)r^2 dr\right] = 0$$

$$T = \int_0^{0.1\text{ m}} \left(\frac{0.8\pi}{t}\right)r^3\,dr$$

$$T = \left(\frac{0.8\pi}{t}\right)\left(\frac{r^4}{4}\right)\Bigg|_0^{0.1\text{ m}}$$

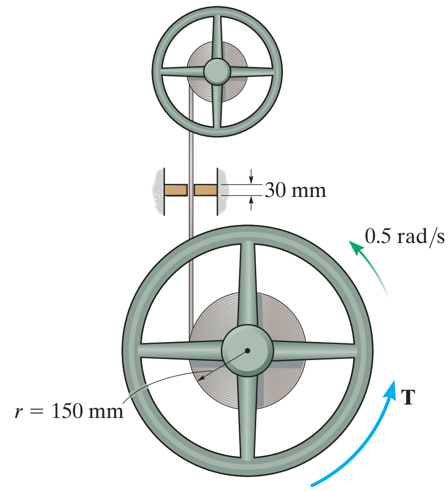
$$T = \left[\frac{20\left(10^{-6}\right)\pi}{t}\right]\text{N}\cdot\text{m} \qquad \text{where } t \text{ is in m} \qquad \textbf{Ans.}$$

The plot of T vs t is shown Fig. *c*.

Ans:

$T = \left[\dfrac{20\left(10^{-6}\right)\pi}{t}\right]\text{N}\cdot\text{m}$, where t is in m

1–55. The tape is 10 mm wide and is drawn through an applicator, which applies a liquid coating (Newtonian fluid) that has a viscosity of $\mu = 0.83\ \text{N}\cdot\text{s/m}^2$ to each side of the tape. If the gap between each side of the tape and the applicator's surface is 0.8 mm, determine the torque **T** at the instant $r = 150$ mm that is needed to rotate the wheel at 0.5 rad/s. Assume the velocity profile within the liquid is linear.



SOLUTION

Considering the moment equilibrium of the wheel, Fig. *a*,

$$\Sigma M_A = 0; \qquad T - P(0.15\ \text{m}) = 0$$

Since the velocity distribution is linear, the velocity gradient will be constant.

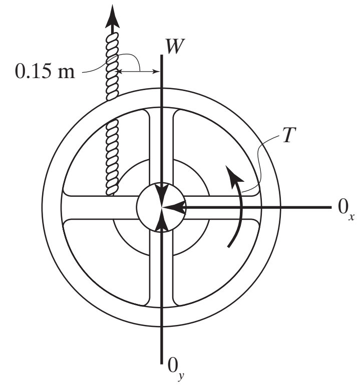
$$P = \tau(2A) = \mu(2A)\frac{du}{dy}$$

$$P = \left(0.830\ \text{N}\cdot\text{s/m}^2\right)(2)(0.03\ \text{m})(0.01\ \text{m})\left(\frac{0.5\ \text{rad/s}(0.15\ \text{m})}{0.0008\ \text{m}}\right)$$

$$P = 0.04669\ \text{N}$$

Thus

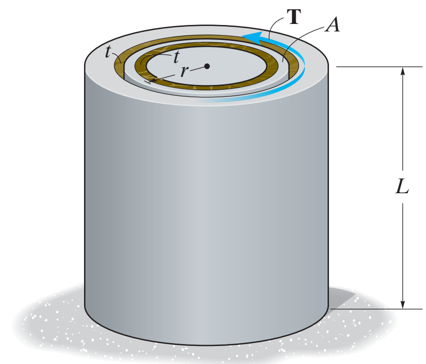
$$T = (0.04669\ \text{N})(0.15\ \text{m}) = 7.00\ \text{mN}\cdot\text{m}$$ **Ans.**



(a)

Ans:
$7.00\ \text{mN}\cdot\text{m}$

***1–56.** The very thin tube A of mean radius r and length L is placed within the fixed circular cavity as shown. If the cavity has a small gap of thickness t on each side of the tube, and is filled with a Newtonian liquid having a viscosity μ, determine the torque **T** required to overcome the fluid resistance and rotate the tube with a constant angular velocity of ω. Assume the velocity profile within the liquid is linear.

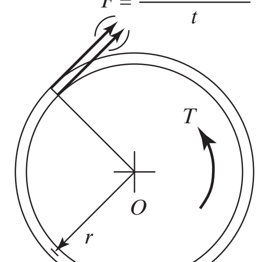


SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant.

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{(\omega r)}{t}$$



(a)

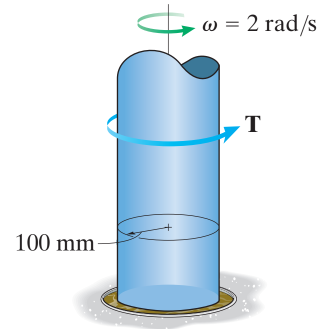
Considering the moment equilibrium of the tube, Fig. *a*,

$$\Sigma M = 0; \qquad T - 2\tau A r = 0$$

$$T = 2(\mu)\frac{(\omega r)}{t}(2\pi r L)r$$

$$T = \frac{4\pi\mu\omega r^3 L}{t} \qquad \textbf{Ans.}$$

1–57. The shaft rests on a 2-mm-thin film of oil having a viscosity of $\mu = 0.0657\ \mathrm{N \cdot s/m^2}$. If the shaft is rotating at a constant angular velocity of $\omega = 2\ \mathrm{rad/s}$, determine the shear stress in the oil at $r = 50$ mm and $r = 100$ mm. Assume the velocity profile within the oil is linear.



SOLUTION

Oil is a Newtonian fluid. Since the velocity distribution is linear, the velocity gradient will be constant.

At $r = 50$ mm,

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \left(0.0657\ \mathrm{N \cdot s/m^2}\right)\left(\frac{(2\ \mathrm{rad/s})(50\ \mathrm{mm})}{2\ \mathrm{mm}}\right)$$

$$\tau = 3.28\ \mathrm{Pa}$$ **Ans.**

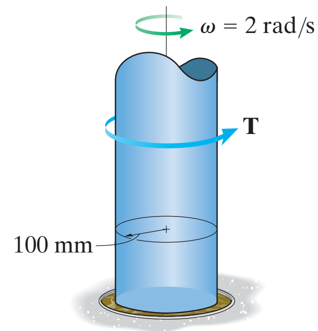
At $r = 100$ mm,

$$\tau = \left(0.0657\ \mathrm{N \cdot s/m^2}\right)\left(\frac{(2\ \mathrm{rad/s})(100\ \mathrm{mm})}{2\ \mathrm{mm}}\right)$$

$$\tau = 6.57\ \mathrm{Pa}$$ **Ans.**

Ans:
At $r = 50$ mm, $\tau = 3.28$ Pa
At $r = 100$ mm, $\tau = 6.57$ Pa

1–58. The shaft rests on a 2-mm-thin film of oil having a viscosity of $\mu = 0.0657\ \text{N}\cdot\text{s/m}^2$. If the shaft is rotating at a constant angular velocity of $\omega = 2\ \text{rad/s}$, determine the torque **T** that must be applied to the shaft to maintain the motion. Assume the velocity profile within the oil is linear.



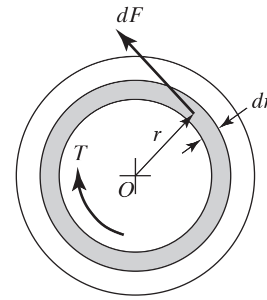
SOLUTION

Oil is a Newtonian fluid. Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus,

$$\tau = \mu\frac{du}{dy} = \frac{\mu\omega r}{t}$$

Thus, the shear force the oil exerts on the differential element of area $dA = 2\pi r\, dr$ shown shaded in Fig. *a* is

$$dF = \tau dA = \left(\frac{\mu\omega r}{t}\right)(2\pi r\, dr) = \frac{2\pi\mu\omega}{t}r^2 dr$$



(a)

Considering the moment equilibrium of the shaft, Fig. *a*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad \int_r dF - T = 0$$

$$T = \int_r dF = \frac{2\pi\mu\omega}{t}\int_0^R r^3 dr$$

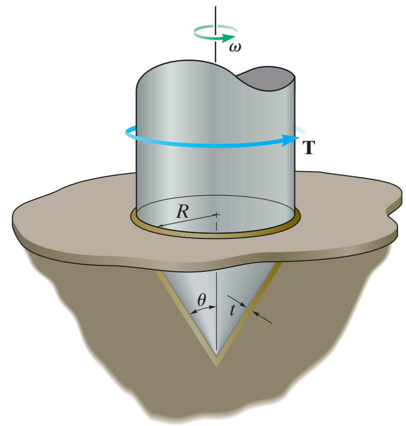
$$= \frac{2\pi\mu\omega}{t}\left(\frac{r^4}{4}\right)\Bigg|_0^R = \frac{\pi\mu\omega R^4}{2t}$$

Substituting,

$$T = \frac{\pi\left(0.0657\dfrac{\text{N}\cdot\text{s}}{\text{m}^2}\right)(2\ \text{rad/s})(0.1\ \text{m})^4}{2(0.002\ \text{m})} = 10.32\left(10^{-3}\right)\ \text{N}\cdot\text{m} = 10.3\ \text{mN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
10.3 mN · m

1–59. The conical bearing is placed in a lubricating Newtonian fluid having a viscosity μ. Determine the torque **T** required to rotate the bearing with a constant angular velocity of ω. Assume the velocity profile along the thickness t of the fluid is linear.



SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus,

$$\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{t}$$

From the geometry shown in Fig. a,

$$z = \frac{r}{\tan\theta} \qquad dz = \frac{dr}{\tan\theta} \tag{1}$$

Also, from the geometry shown in Fig. b,

$$dz = ds\cos\theta \tag{2}$$

Equating Eqs. (1) and (2),

$$\frac{dr}{\tan\theta} = ds\cos\theta \qquad ds = \frac{dr}{\sin\theta}$$

The area of the surface of the differential element shown shaded in Fig. a is $dA = 2\pi r ds = \dfrac{2\pi}{\sin\theta} r dr$. Thus, the shear force the oil exerts on this area is

$$dF = \tau dA = \left(\frac{\mu\omega r}{t}\right)\left(\frac{2\pi}{\sin\theta} r dr\right) = \frac{2\pi\mu\omega}{t\sin\theta} r^2 dr$$

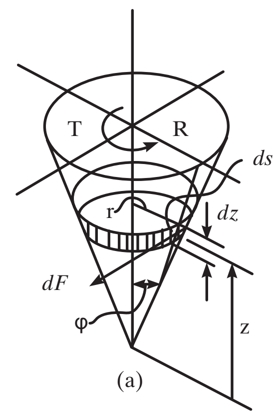
Considering the moment equilibrium of the shaft, Fig. a,

$$\Sigma M_z = 0; \qquad T - \int r dF = 0$$

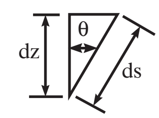
$$T = \int r dF = \frac{2\pi\mu\omega}{t\sin\theta}\int_0^R r^3 dr$$

$$= \frac{2\pi\mu\omega}{t\sin\theta}\left(\frac{r^4}{4}\right)\Big|_0^R$$

$$= \frac{\pi\mu\omega R^4}{2t\sin\theta}$$ **Ans.**



(a)



(b)

Ans:

$T = \dfrac{\pi\mu\omega R^4}{2t\sin\theta}$

***1–60.** The city of Denver, Colorado, is at an elevation of 1610 m above sea level. Determine how hot one can prepare water to make a cup of coffee.

SOLUTION

At the elevation of 1610 meters, the atmospheric pressure can be obtained by interpolating the data given in Appendix A.

$$p_{\text{atm}} = 89.88 \text{ kPa} - \left(\frac{89.88 \text{ kPa} - 79.50 \text{ kPa}}{1000 \text{ m}}\right)(610 \text{ m}) = 83.55 \text{ kPa}$$

Since water boils if the vapor pressure is equal to the atmospheric pressure, then the boiling temperature at Denver can be obtained by interpolating the data given in Appendix A.

$$T_{\text{boil}} = 90°\text{C} + \left(\frac{83.55 - 70.1}{84.6 - 70.1}\right)(5°\text{C}) = 94.6°\text{C} \qquad \textbf{Ans.}$$

Note: Compare this with $T_{\text{boil}} = 100°\text{C}$ at 1 atm.

1–61. How hot can you make a cup of tea if you climb to the top of Mt. Everest (29,000 ft) and attempt to boil water?

SOLUTION

At the elevation of 29 000 ft, the atmospheric pressure can be obtained by interpolating the data given in Appendix A

$$p_{\text{atm}} = 704.4\ \text{lb/ft}^2 - \left(\frac{704.4\ \text{lb/ft}^2 - 629.6\ \text{lb/ft}^2}{30\ 000\ \text{lb/ft}^2 - 27\ 500\ \text{lb/ft}^2}\right)(29\ 000\ \text{ft} - 27\ 500\ \text{ft})$$

$$= \left(659.52\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)^2 = 4.58\ \text{psi}$$

Since water boils if the vapor pressure equals the atmospheric pressure, the boiling temperature of the water at Mt. Everest can be obtained by interpolating the data of Appendix A

$$T_{\text{boil}} = 150°\text{F} + \left(\frac{4.58\ \text{psi} - 3.72\ \text{psi}}{4.75\ \text{psi} - 3.72\ \text{psi}}\right)(160 - 150)°\text{F} = 158°\text{F} \qquad \textbf{Ans.}$$

Note: Compare this with 212°F at 1 atm.

Ans:
$p_{\text{atm}} = 4.58\,\text{psi}$, $T_{\text{boil}} = 158\,°\text{F}$

1–62. The blades of a turbine are rotating in water that has a temperature of 30°C. What is the lowest water pressure that can be developed at the blades so that cavitation will not occur?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 30°\text{C}$ is

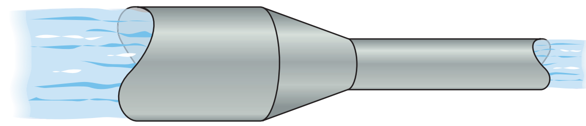
$$p_v = 4.25 \text{ kPa}$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v. Thus

$$p_{\min} = p_v = 4.25 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
4.25 kPa

1–63. As water at 40°C flows through the transition, its pressure will begin to decrease. Determine the lowest pressure it can have without causing cavitation.



SOLUTION

From Appendix A, the vapor pressure of water at $T = 40°\text{C}$ is

$$p_v = 7.38 \text{ kPa}$$

Cavitation (or boiling of water) will occur when the water pressure is equal to or less than p_v. Thus,

$$p_{\min} = 7.38 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
7.38 kPa

***1–64.** Water at 70°F is flowing through a garden hose. If the hose is bent, a hissing noise can be heard. Here cavitation has occurred in the hose because the velocity of the flow has increased at the bend, and the pressure has dropped. What would be the highest absolute pressure in the hose at this location in the hose?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 70°\text{F}$ is

$$p_v = 0.363 \text{ lb/in}^2$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v.

$$p_{\max} = p_v = 0.363 \text{ lb/in}^2 \qquad \textbf{Ans.}$$

1–65. Water at 25°C is flowing through a garden hose. If the hose is bent, a hissing noise can be heard. Here cavitation has occurred in the hose because the velocity of the flow has increased at the bend, and the pressure has dropped. What would be the highest absolute pressure in the hose at this location in the hose?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 25°\text{C}$ is

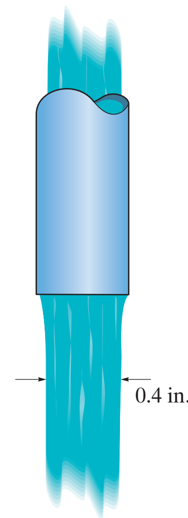
$$p_v = 3.17 \text{ kPa}$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v.

$$p_{\max} = p_v = 3.17 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
3.17 kPa

1–66. A stream of water has a diameter of 0.4 in. when it begins to fall out of the tube. Determine the difference in pressure between a point located just inside and a point just outside of the stream due to the effect of surface tension. Take $\sigma = 0.005$ lb/ft.



SOLUTION

Consider a length L of the water column. The free-body diagram of half of this column is shown in Fig. *a*.

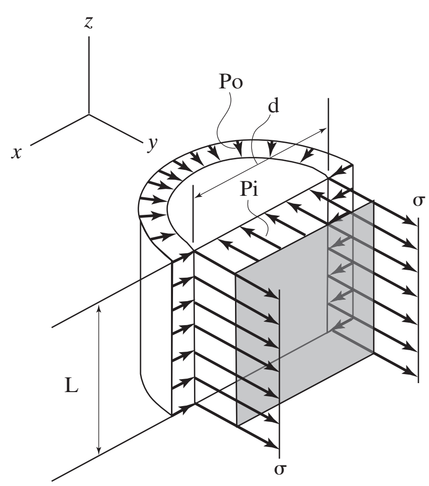
$$\Sigma F = 0$$

$$2(\sigma)(L) + p_o(d)(L) - p_i(d)(L) = 0$$

$$2\sigma = (p_i - p_o)d$$

$$p_i - p_o = \frac{2\sigma}{d}$$

$$\Delta p = \frac{2(0.005\ \text{lb/ft})}{(0.4\ \text{in.}/12)\ \text{ft}} = 0.300\ \text{lb/ft}^2 = 2.08\left(10^{-3}\right)\ \text{psi} \qquad \textbf{Ans.}$$



(a)

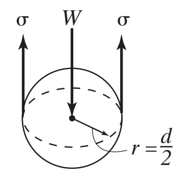
Ans:
$2.08\left(10^{-3}\right)$ psi

1–67. Steel particles are ejected from a grinder and fall gently into a tank of water. Determine the largest average diameter of a particle that will float on the water if the temperature is 80°F. Take $\gamma_{st} = 490\ \text{lb/ft}^3$ and $\sigma = 0.00492\ \text{lb/ft}^3$. Assume each particle has the shape of a sphere where $\forall = 4/3\pi r^2$.

SOLUTION

The weight of a steel particle is

$$W = \gamma_{st}\forall = \left(490\ \text{lb/ft}^3\right)\left[\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right] = \frac{245\pi}{3}d^3$$



(a)

Force equilibrium along the vertical, Fig. a, requires

$$+\uparrow \Sigma F_y = 0; \qquad (0.00492\ \text{lb/ft})\left[2\pi\left(\frac{d}{2}\right)\right] - \frac{245\pi}{3}d^3 = 0$$

$$0.00492\pi d = \frac{245\pi}{3}d^3$$

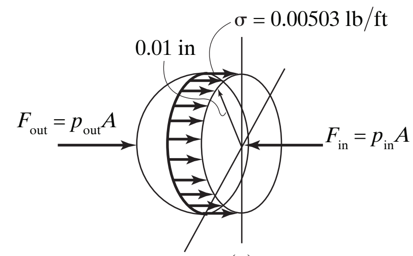
$$d = 7.762\left(10^{-3}\right)\ \text{ft}$$

$$= 0.0931\ \text{in.}$$ **Ans.**

Ans:
0.0931 in.

***1–68.** When a can of soda water is opened, small gas bubbles are produced within it. Determine the difference in pressure between the inside and outside of a bubble having a diameter of 0.02 in. The surrounding temperature is 60°F. Take $\sigma = 0.00503$ lb/ft.

SOLUTION



(a)

The FBD of a half a bubble shown in Fig. *a* will be considered. Here A is the projected area. Force equilibrium along the horizontal requires

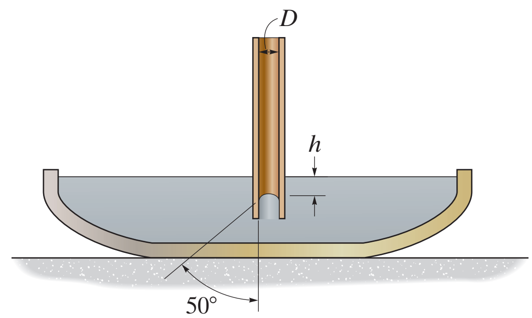
$$\xrightarrow{+} \Sigma F_x = 0; \qquad p_{\text{out}} A + (0.00503 \text{ lb/ft})\left[\pi\left(\frac{0.02}{12}\text{ ft}\right)\right] - p_{\text{in}} A = 0$$

$$(p_{\text{in}} - p_{\text{out}})\left[\frac{\pi}{4}\left(\frac{0.02}{12}\text{ft}\right)^2\right] = 8.3833(10^{-6})\pi \text{ lb}$$

$$p_{\text{in}} - p_{\text{out}} = (12.072 \text{ lb/ft}^2)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$= 0.0838 \text{ psi}$$ **Ans.**

1–69. Determine the distance h that a column of mercury in the tube will be depressed when the tube is inserted into the mercury at a room temperature of 68°F. Set $D = 0.12$ in.



SOLUTION

Using the result

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

From the table in Appendix A, for mercury $\rho = 26.3\ \text{slug/ft}^3$ and $\sigma = 31.9\left(10^{-3}\right)\dfrac{\text{lb}}{\text{ft}}$.

$$h = \frac{2\left[31.9\left(10^{-3}\right)\dfrac{\text{lb}}{\text{ft}}\right]\cos\left(180° - 50°\right)}{\left(26.3\dfrac{\text{slug}}{\text{ft}^3}\right)\left(32.2\dfrac{\text{ft}}{\text{s}^2}\right)\left[(0.06\ \text{in.})\left(\dfrac{1\ \text{ft}}{12\ \text{in.}}\right)\right]}$$

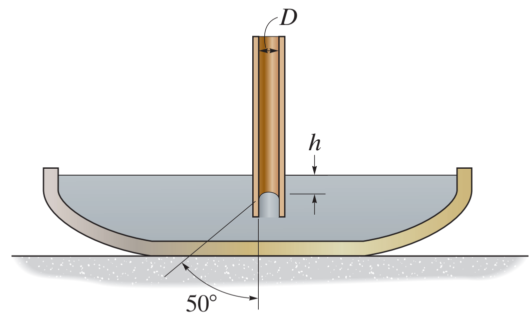
$$= \left[-9.6852\left(10^{-3}\right)\text{ft}\right]\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)$$

$$= -0.116\ \text{in.} \qquad \textbf{Ans.}$$

The negative sign indicates that a depression occurs.

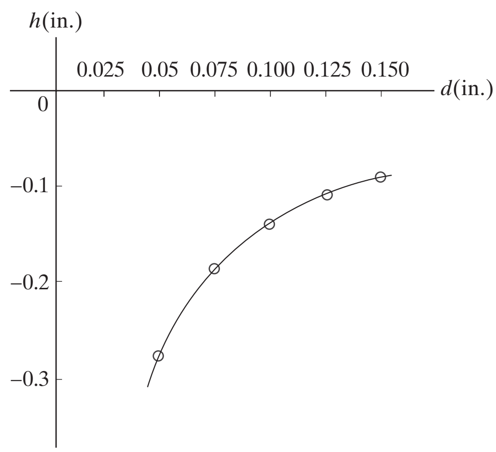
Ans:
0.116 in.

1–70. Determine the distance h that the column of mercury in the tube will be depressed when the tube is inserted into the mercury at a room temperature of 68°F. Plot this relationship of h (vertical axis) versus D for $0.05 \text{ in.} \leq D \leq 0.150$ in. Give values for increments of $\Delta D = 0.025$ in. Discuss this result.



SOLUTION

d(in.)	0.05	0.075	0.100	0.125	0.150
h(in.)	−0.279	−0.186	−0.139	−0.112	0.0930



From the table in Appendix A, for mercury at 68°F, $\rho = 26.3 \text{ slug/ft}^3$, and $\sigma = 31.9(10^{-3})$ lb/ft. Using the result

$$h = \frac{2\sigma \cos \theta}{\rho g r}$$

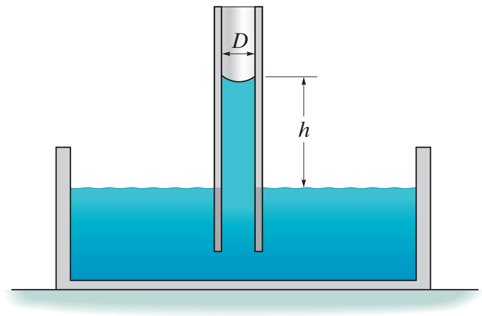
$$h = \left[\frac{2\left[31.9(10^{-3}) \text{ lb/ft}\right] \cos (180° - 50°)}{(26.3 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)\left[(d/2)(1 \text{ ft}/12 \text{ in})\right]}\right]\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)$$

$$h = \left(\frac{-0.01395}{d}\right) \text{in.} \qquad \text{where } d \text{ is in in.}$$

The negative sign indicates that a depression occurs.

Ans:
$d = 0.075$ in., $h = 0.186$ in.

1–71. Water in the glass tube is at a temperature of 40°C. Polt the height h of the water as a function of the tube's inner diameter D for $0.5 \text{ mm} \leq D \leq 3 \text{ mm}$. Use increments of 0.5 mm. Take $\sigma = 0.0696 \text{ N/m}$.



SOLUTION

When water contacts the glass wall, $\theta = 0°$. The weight of the rising column of water is

$$W = \gamma_w \forall = \rho_w g\left(\frac{\pi}{4}D^2 h\right) = \frac{1}{4}\pi\rho_w g D^2 h$$

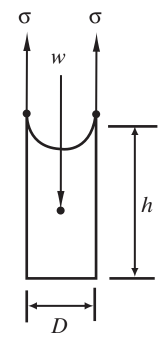
The vertical force equilibrium, Fig. *a*, requires

$$+\uparrow \Sigma F_y = 0; \qquad \sigma(\pi D) - \frac{1}{4}\pi\rho_w g D^2 h = 0$$

$$h = \frac{4\sigma}{\rho_w g D}$$

From Appendix A, $\rho_w = 992.3 \text{ kg/m}^3$ at $T = 40°\text{C}$. Then

$$h = \frac{4(0.0696 \text{ N/m})}{(992.3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)D} = \frac{28.6(10^{-6})}{D} \text{ m}$$

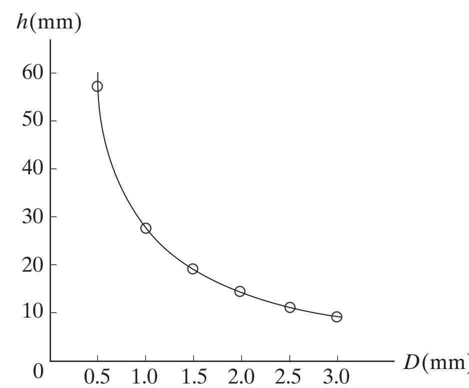


(a)

For $0.5 \text{ mm} \leq D \leq 3 \text{ mm}$

D(mm)	0.5	1.0	1.5	2.0	2.5	3.0
h(mm)	57.2	28.6	19.07	14.3	11.44	9.53

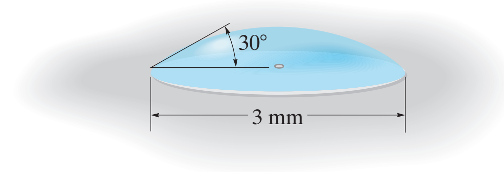
The plot of h vs D is shown in Fig. *b*.



(b)

Ans:
$D = 1.0 \text{ mm}, h = 28.6 \text{ mm}$

***1–72.** Many camera phones now use liquid lenses as a means of providing a quick auto-focus. These lenses work by electrically controlling the internal pressure within a liquid droplet, thereby affecting the angle of the meniscus of the droplet, and so creating a variable focal length. To analyze this effect, consider, for example, a segment of a spherical droplet that has a base diameter of 3 mm. The pressure in the droplet is controlled through a tiny hole under 105 Pa. If the tangent at the surface is 30°, determine the surface tension at the surface that holds it in place.

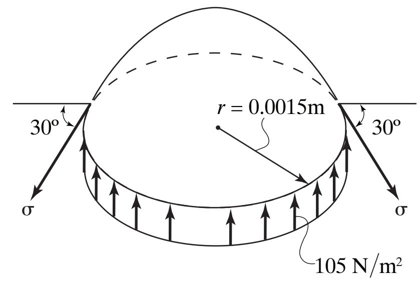


SOLUTION

Writing the force equation of equilibrium along the vertical by referring to the FBD of the droplet in Fig. *a*

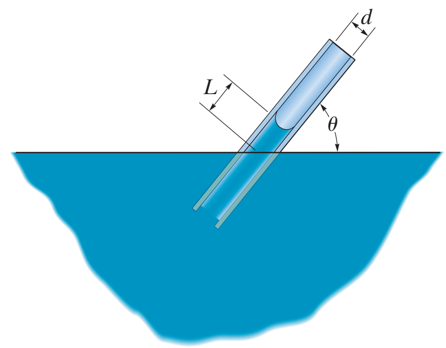
$$+\uparrow \Sigma F_z = 0; \qquad \left(105\,\frac{\text{N}}{\text{m}^2}\right)\left[\pi(0.0015\text{ m})^2\right] - (\sigma \sin 30^\circ)\left[2\pi(0.0015\text{ m})\right] = 0$$

$$\sigma = 0.158\text{ N/m} \qquad \textbf{Ans.}$$



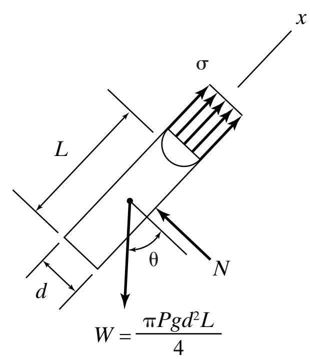
(a)

1–73. The tube has an inner diameter d and is immersed in water at an angle θ from the vertical. Determine the average length L to which water will rise along the tube due to capillary action. The surface tension of the water is σ and its density is ρ.



SOLUTION

The free-body diagram of the water column is shown in Fig. *a*. The weight of this column is $W = \rho g V = \rho g\left[\pi\left(\frac{d}{2}\right)^2 L\right] = \frac{\pi\rho g d^2 L}{4}$.



(a)

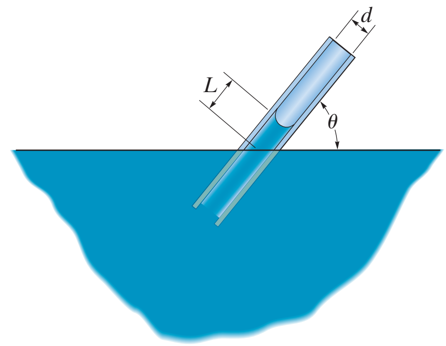
For water, its surface will be almost parallel to the surface of the tube (contact angle $\approx 0°$). Thus, σ acts along the tube. Considering equilibrium along the x axis,

$$\Sigma F_x = 0; \qquad \sigma(\pi d) - \frac{\pi\rho g d^2 L}{4}\sin\theta = 0$$

$$L = \frac{4\sigma}{\rho g d \sin\theta} \qquad \textbf{Ans.}$$

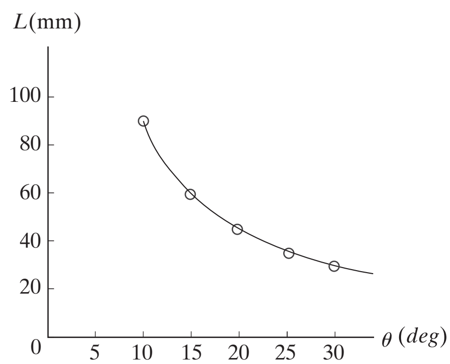
Ans:
$L = 4\sigma/(\rho g d \sin\theta)$

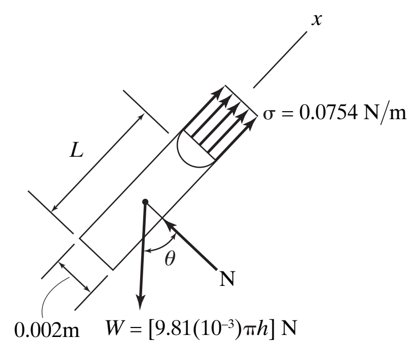
1–74. The tube has an inner diameter of $d = 2$ mm and is immersed in water. Determine the average length h to which the water will rise along the tube due to capillary action. Plot this relationship of h (vertical axis) versus the angle of tilt θ for $10° \leq \theta \leq 30°$. Give values for increments of $\Delta\theta = 5°$. The surface tension of the water is $\sigma = 0.005$ lb/ft and its density is $\rho = 1000$ kg/m^3.



SOLUTION

θ(deg.)	10	15	20	25	30
L(mm)	88.5	59.4	44.9	36.4	30.7





(a)

The FBD of the water column is shown in Fig. *a*. The weight of this column is

$$W = \rho g \forall = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[\frac{\pi}{4}(0.002\text{ m})L\right] = \left[9.81\left(10^{-3}\right)\pi L\right]\text{ N.}$$

For water, its surface will be almost parallel to the surface of the tube ($\theta \cong 0°$) at the point of contact. Thus, σ acts along the tube. Considering equilibrium along x axis,

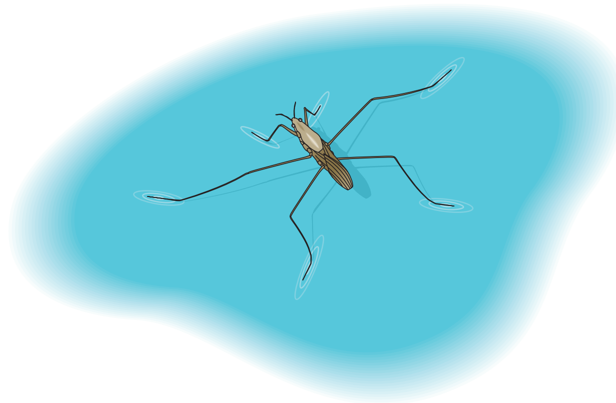
$$\Sigma F_x = 0; \qquad (0.0754\text{ N/m})\left[\pi(0.002\text{ m})\right] - \left[9.81\left(10^{-3}\right)\pi L\right]\sin\theta = 0$$

$$L = \left(\frac{0.0154}{\sin\theta}\right)\text{ m} \qquad \text{where } \theta \text{ is in deg.} \qquad \textbf{Ans.}$$

The plot of L versus θ is shown in Fig. *a*.

Ans:
$L = (0.0154/\sin\theta)$ m

1–75. The marine water strider, *Halobates*, has a mass of 0.36 g. If it has six slender legs, determine the minimum contact length of all of its legs to support itself in water having a temperature of $T = 20°\text{C}$. Take $\sigma = 0.0727\ \text{N/m}$ and assume the legs are thin cylinders.



SOLUTION

The force supported by the legs is

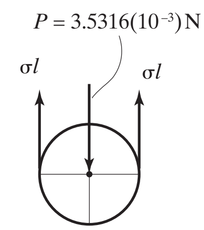
$$P = \left[0.36\left(10^{-3}\right)\text{ kg}\right]\left[9.81\text{ m/s}^2\right] = 3.5316\left(10^{-3}\right)\text{N}$$

Here, σ is most effective in supporting the weight if it acts vertically upward. This requirement is indicated on the FBD of each leg in Fig. *a*. The force equilibrium along vertical requires

$$+\uparrow \Sigma F_y = 0; \qquad 3.5316\left(10^{-3}\right)\text{N} - 2(0.0727\text{ N/m})l = 0$$

$$l = 24.3\left(10^{-3}\right)\text{m} = 24.3\text{ mm} \qquad \textbf{Ans.}$$

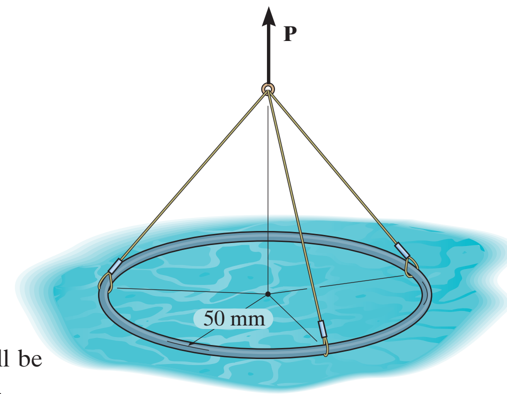
Note: Because of surface microstructure, a water strider's legs are highly hydrophobic. That is why the water surface curves *downward* with $\theta \approx 0°$, instead of upward as it does when water meets glass.



(a)

Ans:
24.3 mm

***1–76.** The ring has a weight of 0.2 N and is suspended on the surface of the water, for which $\sigma = 0.0736\ \text{N/m}$. Determine the vertical force **P** needed to pull the ring free from the surface.



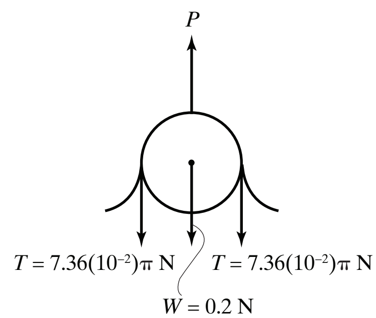
SOLUTION

The free-body diagram of the ring is shown in Fig. *a*. For water, its surface will be almost parallel to the surface of the wire ($\theta \approx 0°$) at the point of contact, Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad P - W - 2T = 0$$

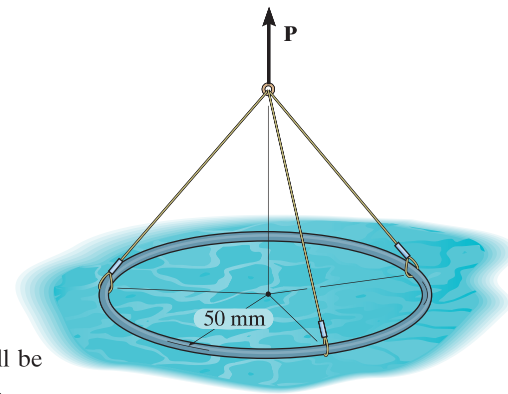
$$P - 0.2\ \text{N} - 2(0.0736\ \text{N/m})\left[2\pi(0.05\ \text{m})\right] = 0$$

$$P = 0.246\ \text{N} \qquad \textbf{Ans.}$$



(a)

1–77. The ring has a weight of 0.2 N and is suspended on the surface of the water. If it takes a force of $P = 0.245$ N to lift the ring free from the surface, determine the surface tension of the water.



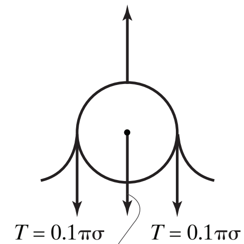
SOLUTION

The free-body diagram of the ring is shown in Fig. *a*. For water, its surface will be almost parallel to the surface of the wire ($\theta \approx 0°$) at the point of contact, Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad 0.245\text{ N} - 0.2\text{ N} - 2[\sigma(2\pi(0.05\text{ m}))] = 0$$

$$\sigma = 0.0716\text{ N/m}$$

$$= 0.0716\text{ N/m} \qquad \textbf{Ans.}$$



(a)

Ans:
0.0716 N/m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–1. Show that Pascal's law applies within a fluid that is accelerating, provided there is no shearing stresses acting within the fluid.

SOLUTION

Consider the free-body diagram of a triangular element of fluid as shown in Fig. 2–2b. If this element has acceleration components of a_x, a_y, a_z, then since $dm = \rho d\cancel{V}$ the equations of motion in the y and z directions give

$$\Sigma F_y = dma_y; \qquad p_y(\Delta x)(\Delta s \sin\theta) - [p(\Delta x \Delta s)]\sin\theta = \rho\left(\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right)a_y$$

$$\Sigma F_z = dma_z; \qquad p_z(\Delta x)(\Delta s\cos\theta) - [p(\Delta x\Delta s)]\cos\theta - \gamma\left[\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right] = \rho\left(\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right)a_z$$

Dividing by $\Delta x\Delta s$ and letting $\Delta s \rightarrow 0$, so the element reduces in size, we obtain

$$p_y = p$$
$$p_z = p$$

By a similar argument, the element can be rotated 90° about the z axis and $\Sigma F_x = dma_x$ can be applied to show $p_x = p$. Since the angle θ of the inclined face is *arbitrary,* this indeed shows that the pressure at a point is the *same in all directions* for any fluid that has no shearing stress acting within it.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–2. The water in a lake has an average temperature of 15°C. If the barometric pressure of the atmosphere is 720 mm of Hg (mercury), determine the gage pressure and the absolute pressure at a water depth of 14 m.

SOLUTION

From Appendix A, $T = 15°\mathrm{C}$.

$$\rho_w = 999.2\ \mathrm{kg/m^3}$$

$$p_g = \rho_w g h = \left(999.2\ \mathrm{kg/m^3}\right)\left(9.81\ \mathrm{m/s^2}\right)(14\ \mathrm{m})$$

$$= 137.23\left(10^3\right)\ \mathrm{Pa} = 137\ \mathrm{kPa} \qquad \textbf{Ans.}$$

$$p_{\mathrm{atm}} = \rho_{\mathrm{Hg}}\, g h = \left(13\,550\ \mathrm{kg/m^3}\right)\left(9.81\ \mathrm{m/s^2}\right)(0.720\ \mathrm{m}) = 95.71\ \mathrm{kPa}$$

$$p_{\mathrm{abs}} = p_{\mathrm{atm}} + p_g = 95.71\ \mathrm{kPa} + 137.23\ \mathrm{kPa}$$

$$= 233\ \mathrm{kPa} \qquad \textbf{Ans.}$$

Ans:
$p_g = 137\ \mathrm{kPa}$, $p_{\mathrm{abs}} = 233\ \mathrm{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–3. If the absolute pressure in a tank is 140 kPa, determine the pressure head in mm of mercury. The atmospheric pressure is 100 kPa.

SOLUTION

$$p_{\text{abs}} = p_{\text{atm}} + p_g$$

$$140\ \text{kPa} = 100\ \text{kPa} + p_g$$

$$p_g = 40\ \text{kPa}$$

From Appendix A, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$.

$$p = \gamma_{\text{Hg}} h_{\text{Hg}}$$

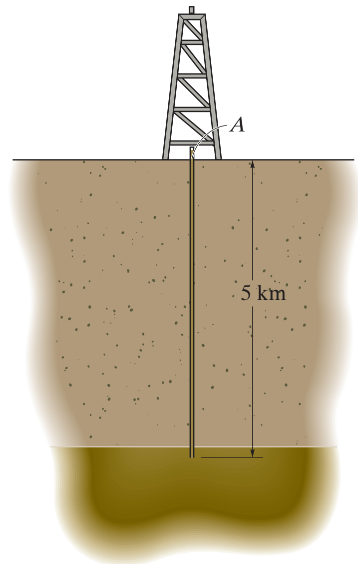
$$40(10^3)\ \text{N/m}^2 = (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)h_{\text{Hg}}$$

$$h_{\text{Hg}} = 0.3009\ \text{m} = 301\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$h_{\text{Hg}} = 301\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–4.** The oil derrick has drilled 5 km into the ground before it strikes a crude oil reservoir. When this happens, the pressure at the well head A becomes 25 MPa. Drilling "mud" is to be placed into the entire length of pipe to displace the oil and balance this pressure. What should be its density so that the pressure at A becomes zero?



SOLUTION

Consider the case when the crude oil is pushing out at A where $p_A = 25(10^6)$ Pa, Fig. *a*. Here, $\rho_o = 880\ \text{kg/m}^3$ (Appendix A) hence $p_o = \rho_o gh = (880\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5000\ \text{m}) = 43.164(10^6)$ Pa

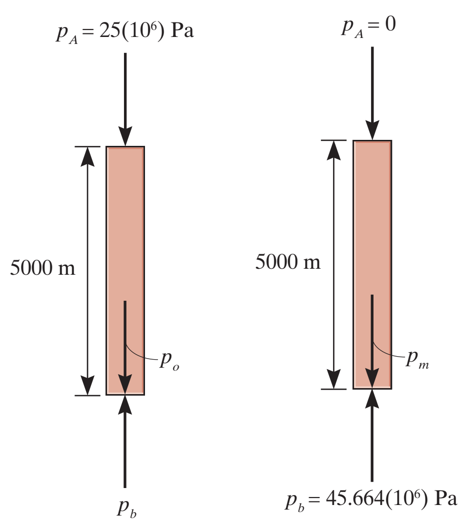
$$p_b = p_A + p_o = 25(10^6)\ \text{Pa} + 43.164(10^6)\ \text{Pa} = 68.164(10^6)\ \text{Pa}$$

It is required that $p_A = 0$, Fig. *b*. Thus

$$p_b = p_m = \rho_m gh$$

$$68.164(10^6)\frac{\text{N}}{\text{m}^2} = \rho_m(9.81\ \text{m/s}^2)(5000\ \text{m})$$

$$\rho_m = 1390\ \text{kg/m}^3$$ **Ans.**



(a) (b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–5. In 1896, S. Rova Rocci developed the prototype of the current sphygmomanometer, a device used to measure blood pressure. When it was worn as a cuff around the upper arm and inflated, the air pressure within the cuff was connected to a mercury manometer. If the reading for the high (or systolic) pressure is 120 mm and for the low (or diastolic) pressure is 80 mm, determine these pressures in psi and pascals.

SOLUTION

Mercury is considered to be incompressible. From Appendix A, the density of mercury is $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Thus, the systolic pressure is

$$p_S = \rho_{\text{Hg}} g h_s = \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.12\ \text{m}) = 15.95\ \text{kPa}$$

$$= 16.0\left(10^3\right)\ \text{Pa} \qquad \textbf{Ans.}$$

$$p_S = \left[15.95\left(10^3\right)\frac{\text{N}}{\text{m}^2}\right]\left(\frac{1\ \text{lb}}{4.4482\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2\left(\frac{1\text{ft}}{12\ \text{in.}}\right)^2 = 2.31\ \text{psi} \qquad \textbf{Ans.}$$

The diastolic pressure is

$$p_d = \rho_{\text{Hg}} g h_d = \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.08\ \text{m}) = 10.63\left(10^3\right)\ \text{Pa}$$

$$= 10.6\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_d = \left[10.63\left(10^3\right)\frac{\text{N}}{\text{m}^2}\right]\left(\frac{1\ \text{lb}}{4.4482\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.54\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$p_s = 16.0\ \text{kPa} = 2.31\ \text{psi}$
$p_d = 10.6\ \text{kPa} = 1.54\ \text{psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–6. Show why water would not be a good fluid to use for a barometer by computing the height to which standard atmospheric pressure will elevate it in a glass tube. Compare this result with that of mercury. Take $\gamma_w = 62.4\ \text{lb/ft}^3$, $\gamma_{Hg} = 846\ \text{lb/ft}^3$.

SOLUTION

For water barometer, Fig. *a*,

$$p_w = \gamma_w h_w = p_{atm}$$

$$\left(62.4 \frac{\text{lb}}{\text{ft}^3}\right) h_w = \left(14.7 \frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$h_w = 33.92\ \text{ft} = 33.9\ \text{ft} \qquad \textbf{Ans.}$$

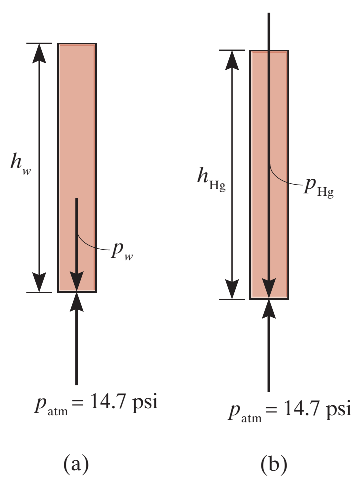
For mercury barometer, Fig. *b*,

$$p_{Hg} = \gamma_{Hg} h_{Hg} = p_{atm}$$

$$847 \frac{\text{lb}}{\text{ft}^3} h_{Hg} = \left(14.7 \frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$h_{Hg} = (2.4992\ \text{ft})\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right) = 30.0\ \text{in.} \qquad \textbf{Ans.}$$

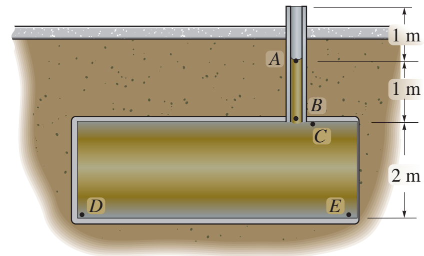
A water barometer is not suitable since it requires a very long tube.



Ans:
$h_w = 33.9$ ft
$h_{Hg} = 30.0$ in.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–7. The underground storage tank used in a service station contains gasoline filled to the level A. Determine the gage pressure at each of the five identified points. Note that point B is located in the stem, and point C is just below it in the tank. Take $\rho_g = 730\ \text{kg/m}^3$.



SOLUTION

Since the tube is open-ended, point A is subjected to atmospheric pressure, which has zero gauge pressure.

$$p_A = 0 \qquad \textbf{Ans.}$$

The pressures at points B and C are the same since they are at the same horizontal level with $h = 1$ m.

$$p_B = p_C = \left(730\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1\ \text{m}) = 7.16\ \text{kPa} \qquad \textbf{Ans.}$$

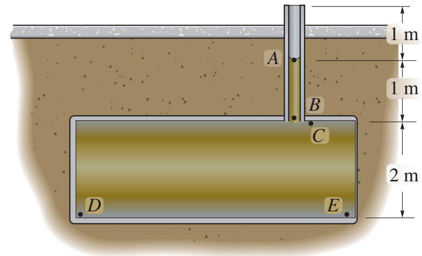
For the same reason, pressure at points D and E is the same. Here, $h = 1$ m $+ 2$ m $= 3$ m.

$$p_D = p_E = \left(730\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m}) = 21.5\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
$p_A = 0$
$p_B = p_C = 7.16$ kPa
$p_D = p_E = 21.5$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–8.** The underground storage tank contains gasoline filled to the level A. If the atmospheric pressure is 101.3 kPa, determine the absolute pressure at each of the five identified points. Note that point B is located in the stem, and point C is just below it in the tank. Take $\rho_g = 730\ \text{kg/m}^3$.



SOLUTION

Since the tube is open-ended, point A is subjected to atmospheric pressure, which has an absolute pressure of 101.3 kPa.

$$p_A = p_{\text{atm}} + p_g$$

$$p_A = 101.3(10^3)\ \text{N/m}^2 + 0 = 101.3\ \text{kPa} \qquad \textbf{Ans.}$$

The pressures at points B and C are the same since they are at the same horizontal level with $h = 1$ m.

$$p_B = p_C = 101.3(10^3)\ \text{N/m}^2 + (730\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})$$

$$= 108\ \text{kPa} \qquad \textbf{Ans.}$$

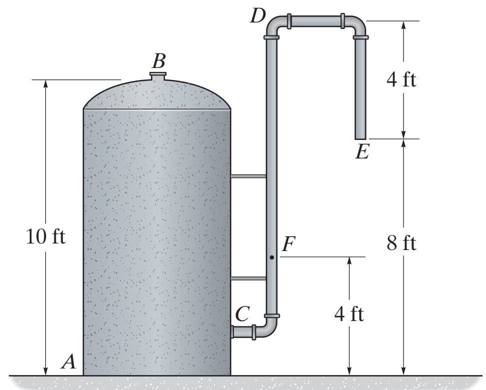
For the same reason, pressure at points D and E is the same. Here, $h = 1\ \text{m} + 2\ \text{m} = 3\ \text{m}$.

$$p_D = p_E = 101.3(10^3)\ \text{N/m}^2 + (730\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})$$

$$= 123\ \text{kPa} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–9. The field storage tank is filled with oil. This standpipe is connected to the tank at C, and the system is open to the atmosphere at B and E. Determine the maximum pressure in the tank is psi if the oil reaches a level of F in the pipe. Also, at what level should the oil be, in the tank, so that the absolute maximum pressure occurs in the tank? What is this value? Take $\rho_o = 1.78\ \text{slug/ft}^3$.



SOLUTION

Since the top of the tank is open to the atmosphere, the free surface of the oil in the tank will be the same height as that of point F. Thus, the maximum pressure which occurs at the base of the tank (level A) is

$$\begin{aligned}(p_A)_g &= \gamma h \\ &= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(4\ \text{ft}) \\ &= 229.26\ \frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.59\ \text{psi} \qquad \textbf{Ans.}\end{aligned}$$

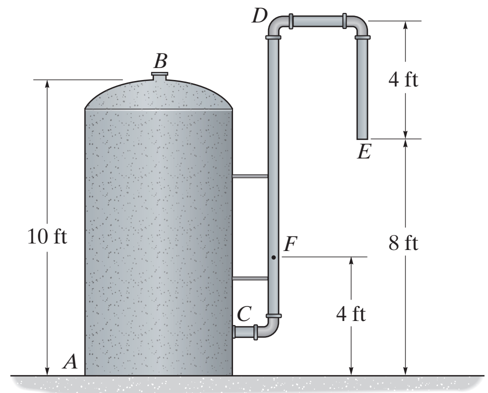
Absolute maximum pressure occurs at the base of the tank (level A) when the **oil reaches level B.**

$$\begin{aligned}(p_A)_{\substack{\text{abs}\\ \text{max}}} &= \gamma h \\ &= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(10\ \text{ft}) \\ &= 573.16\ \text{lb/ft}^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 3.98\ \text{psi} \qquad \textbf{Ans.}\end{aligned}$$

Ans:
$(p_A)_g = 1.59$ psi
Absolute maximum pressure occurs when the oil reaches level B.
$(p_A)_{\substack{\text{abs}\\ \text{max}}} = 3.98$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–10. The field storage tank is filled with oil. The standpipe is connected to the tank at C and open to the atmosphere at E. Determine the maximum pressure that can be developed in the tank if the oil has a density of $1.78\ \text{slug/ft}^3$. Where does this maximum pressure occur? Assume that there is no air trapped in the tank and that the top of the tank at B is closed.



SOLUTION

Level D is the highest the oil is allowed to rise in the tube, and the maximum gauge pressure occurs at the base of the tank (level A).

$$(p_{\max})_g = \gamma h$$

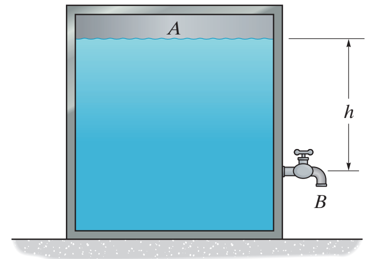
$$= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(8\ \text{ft} + 4\ \text{ft})$$

$$= \left(687.79\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 4.78\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$(p_{\max})_g = 4.78\ \text{psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–11. The closed tank was completely filled with carbon tetrachloride when the valve at B was opened, slowly letting the carbon tetrachloride level drop as shown. If the space within A is a vacuum, determine the pressure in the liquid near valve B when $h = 25$ ft. Also, determine at what level h the carbon tetrachloride will stop flowing out. The atmospheric pressure is 14.7 psi.



SOLUTION

From the Appendix, $p_{ct} = 3.09\ \text{slug/ft}^3$. Since the empty space A is a vacuum, $p_A = 0$. Thus, the absolute pressure at B when $h = 25$ ft is

$$(p_B)_{\text{abs}} = p_A + \gamma h$$
$$= 0 + \left(3.09\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(25\ \text{ft})$$
$$= \left(2487.45 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 17.274\ \text{psi}$$

The gauge pressure is given by

$$(p_B)_{\text{abs}} = p_{\text{atm}} + (p_B)_g$$
$$17.274\ \text{psi} = 14.7\ \text{psi} + (p_B)_g$$
$$(p_B)_g = 2.57\ \text{psi} \qquad \textbf{Ans.}$$

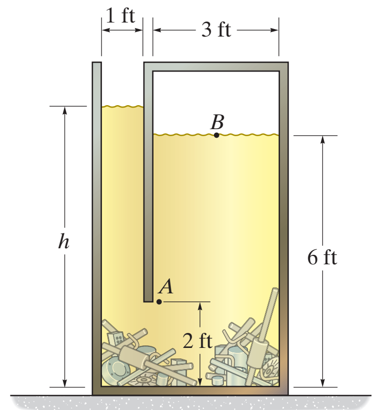
When the absolute at B equals the atmospheric pressure, the water will stop flowing. Thus,

$$(p_B)_{\text{abs}} = p_A + \gamma h$$
$$\left(14.7 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 0 + \left(3.09\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)h$$
$$h = 21.3\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$(p_B)_g = 2.57$ psi
$h = 21.3$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–12.** The soaking bin contains ethyl alcohol used for cleaning automobile parts. If $h = 7$ ft, determine the pressure developed at point A and at the air surface B within the enclosure. Take $\gamma_{ea} = 49.3\ \text{lb/ft}^3$.



SOLUTION

The gauge pressures at points A and B are

$$p_A = \gamma_{ea}\, h_A = \left(49.3\,\frac{\text{lb}}{\text{ft}^3}\right)(7\text{ft} - 2\text{ft})$$

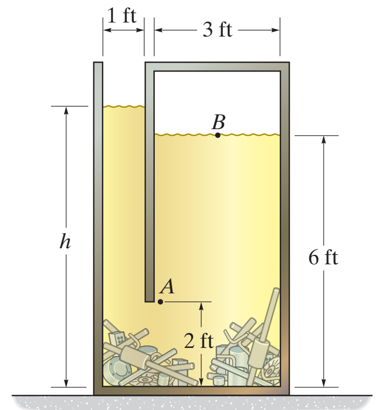
$$= \left(246.5\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.71\ \text{psi} \qquad \textbf{Ans.}$$

$$p_B = \gamma_{ea}\, h_B = (49.3\ \text{lb/ft}^3)(7\ \text{ft} - 6\ \text{ft})$$

$$= \left(49.3\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 0.342\ \text{psi} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–13. The soaking bin contains ethyl alcohol used for cleaning automobile parts. If the pressure in the enclosure is $(p_B)_g = 0.5$ psi, determine the pressure developed at point A and the height h of the ethyl alcohol level in the bin. Take $\gamma_{ea} = 49.3\ \text{lb/ft}^3$.



SOLUTION

The gauge pressure at point A is

$$(p_A)_g = (p_B)_g + \gamma_{ea}\, h_{BA}$$

$$= 0.5\ \text{psi} + \left(49.3\,\frac{\text{lb}}{\text{ft}^3}\right)(6\ \text{ft} - 2\ \text{ft})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 1.869\ \text{psi} = 1.87\ \text{psi} \qquad \textbf{Ans.}$$

The gauge pressure for the atmospheric pressure is $(p_{\text{atm}})_g = 0$. Thus,

$$(p_B)_g = (p_{\text{atm}})_g + \gamma_{ea}\, h_B$$

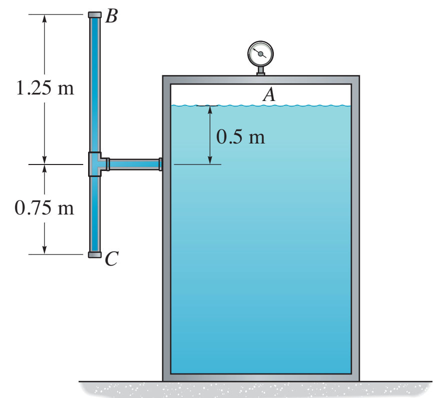
$$\left(0.5\,\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 0 + \left(49.3\,\frac{\text{lb}}{\text{ft}^3}\right)(h - 6)$$

$$h = 7.46\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$(p_A)_g = 1.87$ psi
$h = 7.46$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–14. The pipes connected to the closed tank are completely filled with water. If the absolute pressure at *A* is 300 kPa, determine the force acting on the inside of the end caps at *B* and *C* if the pipe has an inner diameter of 60 mm.



SOLUTION

Thus, the force due to pressure acting on the cap at *B* and *C* are

$$F_B = p_B A = \left[292.64(10^3)\ \mathrm{N/m^2}\right]\left[\pi(0.03\ \mathrm{m})^2\right]$$

$$= 827.43\ \mathrm{N} = 827\ \mathrm{N} \qquad \textbf{Ans.}$$

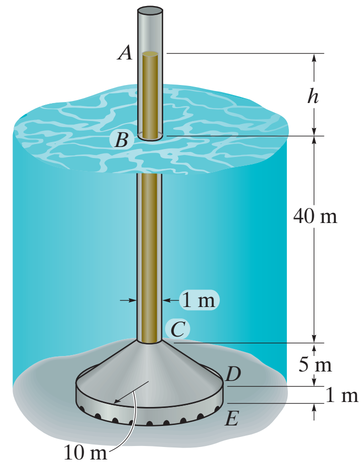
$$F_C = p_C A = \left[312.26(10^3)\ \mathrm{N/m^2}\right]\left[\pi(0.03\ \mathrm{m})^2\right]$$

$$= 882.90\ \mathrm{N} = 883\ \mathrm{N} \qquad \textbf{Ans.}$$

Ans:
$F_B = 827\ \mathrm{N}$
$F_C = 883\ \mathrm{N}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–15. The structure shown is used for the temporary storage of crude oil at sea for later loading into ships. When it is not filled with oil, the water level is a B (sea level). Why? As the oil is loaded into its stem, the water is displaced through exit ports at E. If the stem is filled with oil, that is, to depth of C, determine the height h of the oil level above sea level. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1020\ \text{kg/m}^3$.



SOLUTION

The water level remains at B when empty because the gage pressure at B must be zero. It is required that the pressure at C caused by the water and oil be the same. Then

$$(p_C)_w = (p_C)_o$$

$$\rho_w g h_w = \rho_o g h_o$$

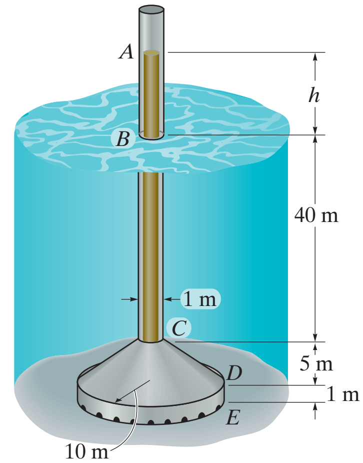
$$\left(1020\ \text{kg/m}^3\right)(g)(40\ \text{m}) = \left(900\ \text{kg/m}^3\right)g(40\ \text{m} + h)$$

$$h = 5.33\ \text{m} \qquad \textbf{Ans.}$$

Ans:
5.33 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–16.** If the water in the structure in Prob. 2–15 is displaced with crude oil to the level D at the bottom of the cone, then how high h will the oil extend above sea level? Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1020\ \text{kg/m}^3$.



SOLUTION

It is required that the pressure at D caused by the water and oil be the same.

$$(p_D)_w = (p_D)_o$$

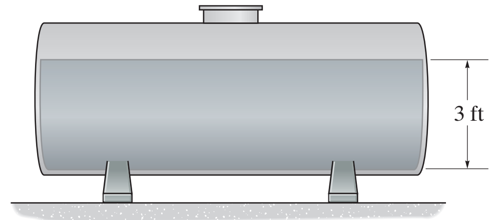
$$\rho_w g h_w = \rho_o g h_o$$

$$(1020\ \text{kg/m}^3)(g)(45\ \text{m}) = (900\ \text{kg/m}^3)(g)(45\ \text{m} + h)$$

$$h = 6.00\ \text{m} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–17. The tank is filled with aqueous ammonia (ammonium hydroxide) to a depth of 3 ft. The remaining volume of the tank contains air under absolute pressure of 20 psi. Determine the gage pressure at the bottom of the tank. Would the results be different if the tank had a square bottom rather than a curved one? Take $\rho_{am} = 1.75\ \text{slug/ft}^3$. The atmospheric pressure is $\rho_{atm} = 14.7$ psi.



SOLUTION

The gage pressure of the air in the tank is

$$(p_{air})_{abs} = p_{atm} + (p_{air})_g$$

$$20\,\frac{\text{lb}}{\text{in.}^2} = 14.7\,\frac{\text{lb}}{\text{in.}^2} + (p_{air})_g$$

$$(p_{air})_g = \left(5.3\,\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2 = 763.2\,\frac{\text{lb}}{\text{ft}^2}$$

Using this result, the gage pressure at the bottom of tank can be obtained.

$$(p_b)_g = (p_{air})_g + \gamma h$$

$$= 763.2\,\frac{\text{lb}}{\text{ft}^2} + \left(1.75\,\frac{\text{slug}}{\text{ft}^3}\right)(32.2\text{ ft/s}^2)(3\text{ ft})$$

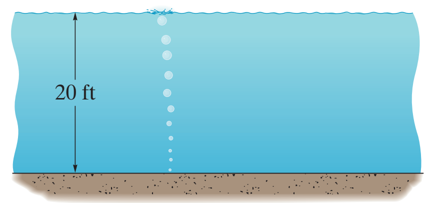
$$= \left(932.25\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = 6.47\text{ psi} \qquad \textbf{Ans.}$$

No, it does not matter what shape the bottom of the tank is.

Ans:
No, it does not matter what shape the bottom of the tank is.
$(p_b)_g = 6.47$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–18. A 0.5-in.-diameter bubble of methane gas is released from the bottom of a lake. Determine the bubble's diameter when it reaches the surface. The water temperature is 68°F and the atmospheric pressure is 14.7 lb/in².



SOLUTION

Applying the ideal gas law, $p = \rho RT$ of which T is constant in this case. Thus,

$$\frac{p}{\rho} = \text{constant}$$

Since $\rho = \dfrac{m}{V\!\!\!\!-}$, where m is also constant, then

$$\frac{p}{m/V\!\!\!\!-} = \text{constant}$$

$$pV\!\!\!\!- = \text{constant} \qquad \textbf{(1)}$$

At the bottom of the lake, the absolute pressure is

$$p_b = p_{\text{atm}} + \gamma_W h_W$$

$$= \left(14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2 + \left(62.4\text{ lb/ft}^3\right)(20\text{ ft}) = 3364.8\text{ lb/ft}^2$$

At the surface of the lake, the absolute pressure is

$$p_s = p_{\text{atm}} = \left(14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2 = 2116.8\text{ lb/ft}^2$$

Using Eq. (1), we can write

$$p_b V\!\!\!\!-_b = p_s V\!\!\!\!-_s$$

$$\left(3364.8\text{ lb/ft}^2\right)\left[\frac{4}{3}\pi\left(\frac{0.5\text{ in.}}{2}\right)^3\right] = \left(2116.8\text{ lb/ft}^2\right)\left[\frac{4}{3}\pi\left(\frac{d_s}{2}\right)^3\right]$$

$$d_s = 0.5835\text{ in.} = 0.584\text{ in.} \qquad \textbf{Ans.}$$

Ans:
$d_s = 0.584$ in.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–19. The Burj Khalifa is currently the world's tallest building. If air at 40°C is at an atmospheric pressure of 105 kPa at the ground floor (sea level), determine the absolute pressure at the top of the tower, which has an elevation of 828 m. Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible.

SOLUTION

For compressible air, with $R = 286.9\ \text{J/(kg}\cdot\text{K)}$ (Appendix A), $T_0 = 40°\text{C} + 273 = 313\ \text{K}$, $z_0 = 0$, and $z = 828\ \text{m}$,

$$p = p_0 e^{-(g/RT_0)(z - z_0)}$$

$$p = (105\ \text{kPa})e^{-[9.81/286.9(313)](828-0)}$$

$$= 95.92\ \text{kPa} \qquad \textbf{Ans.}$$

For incompressible air, with $\rho = 1.127\ \text{kg/m}^3$ at $T = 40°\text{C}$ (Appendix A),

$$p = p_0 - \rho g h$$

$$= 105(10^3)\ \text{N/m}^2 - (1.127\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(828\ \text{m})$$

$$= 95.85\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
For compressible air, $p = 95.92$ kPa
For incompressible air, $p = 95.85$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–20.** The Burj Khalifa is currently the world's tallest building. If air at 100°F is at an atmospheric pressure of 14.7 psi at the ground floor (sea level), determine the absolute pressure at the top of the building, which has an elevation of 2717 ft. Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible.

SOLUTION

For compressible air, with $R = 1716\ \text{ft}\cdot\text{lb/(slug}\cdot\text{R)}$ (Appendix A), $T_0 = (100 + 460)°\text{R} = 560°\text{R}$

$$p = p_0 e^{-(g/RT_o)(z-z_o)}$$

$$p = (14.7\ \text{psi})e^{-[32.2/1716(560)](2717-0)}$$

$$= 13.42\ \text{psi} \qquad \textbf{Ans.}$$

For incompressible air, with $\rho = 0.00220\ \text{slug/ft}^3$ at $T = 100°\text{F}$ (Appendix A),

$$p = p_0 - \gamma h$$

$$= 14.7\ \text{lb/in}^2 - \left(0.00220\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(2717\ \text{ft})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 13.36\ \text{psi} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–21. The density ρ of a fluid varies with depth h, although its bulk modulus $E_{\mathcal{V}}$ can be assumed constant. Determine how the pressure varies with depth h. The density at the surface of the fluid is ρ_0.

SOLUTION

The fluid is considered compressible.

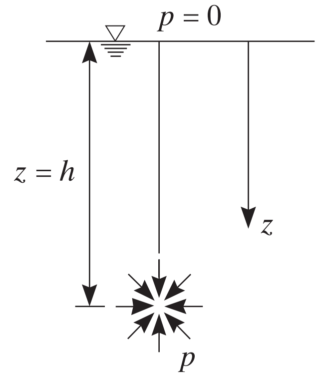
$$E_{\mathcal{V}} = -\frac{dp}{d\mathcal{V}/\mathcal{V}}$$

However, $\mathcal{V} = \dfrac{m}{\rho}$. Then,

$$\frac{d\mathcal{V}}{\mathcal{V}} = \frac{-(m/\rho^2)d\rho}{m/\rho} = -\frac{d\rho}{\rho}$$

Therefore,

$$E_{\mathcal{V}} = \frac{dp}{d\rho/\rho}$$



(a)

At the surface, where $p = 0$, $\rho = \rho_0$, Fig. a, then

$$E_{\mathcal{V}}\int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \int_0^p dp$$

$$E_{\mathcal{V}}\ln\left(\frac{\rho}{\rho_0}\right) = p$$

or

$$\rho = \rho_{0}e^{p/E_{\mathcal{V}}}$$

Also,

$$p = p_0 + \rho g z$$

$$dp = \rho g dz$$

$$\frac{dp}{\rho} = g dz$$

Since the pressure $p = 0$ at $z = 0$ and p at $z = h$, Fig. a.

$$\int_0^p \frac{dp}{\rho_{0}e^{p/E_{\mathcal{V}}}} = \int_0^h g dz$$

$$\frac{E_{\mathcal{V}}}{\rho_0}\left(1 - e^{-p/E_{\mathcal{V}}}\right) = gh$$

$$1 - e^{-p/E_{\mathcal{V}}} = \frac{\rho_0 g h}{E_{\mathcal{V}}}$$

$$p = -E_{\mathcal{V}}\ln\left(1 - \frac{\rho_0 g h}{E_{\mathcal{V}}}\right) \qquad \textbf{Ans.}$$

Ans:

$$p = -E_{\mathcal{V}}\ln\left(1 - \frac{\rho_o g h}{E_{\mathcal{V}}}\right)$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–22. Due to its slight compressibility, the density of water varies with depth, although its bulk modulus $E_{\forall} = 2.20$ GPa (absolute) can be considered constant. Accounting for this compressibility, determine the pressure in the water at a depth of 300 m, if the density at the surface of the water is $\rho = 1000\ \text{kg/m}^3$. Compare this result with assuming water to be incompressible.

SOLUTION

The water is considered compressible. Using the definition of bulk modulus,

$$E_{\forall} = \frac{dp}{d\forall/\forall}$$

However, $\forall = \dfrac{m}{\rho}$. Then

$$\frac{d\forall}{\forall} = \frac{-(m/\rho^2)d\rho}{m/\rho} = -\frac{dp}{\rho}$$

Therefore,

$$E_{\forall} = \frac{dp}{d\rho/\rho}$$

At the surface, $p = 0$ and $\rho = 1000\ kg/m^3$, Also, $E_{\forall} = 2.20$ Gpa. Then

$$\left[2.20(10^9)\ \text{N/m}^2\right]\int_{1000\ \text{kg/m}^3}^{\rho} \frac{d\rho}{\rho} = \int_0^p dp$$

$$p = 2.20(10^9)\ln\left(\frac{\rho}{1000}\right)$$

$$\rho = 1000\, e^{\frac{p}{2.20(10^9)}} \qquad \textbf{(1)}$$

Also,

$$dp = \rho g dz$$

$$\frac{dp}{\rho} = 9.81dz \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2).

$$\frac{dp}{1000e^{\frac{p}{2.20(10^9)}}} = 9.81dz$$

Since the pressure $p = 0$ at $z = 0$ and p at $z = 300$ m

$$\int_0^P \frac{dp}{1000e^{\frac{p}{2.20(10^9)}}} = \int_0^{300\ \text{m}} 9.81dz$$

$$-2.2(10^6)e^{-\frac{p}{2.20(10^9)}}\Big|_0^p = 9.81z\Big|_0^{300\ \text{m}}$$

2–22. Continued

$$-2.2\left(10^6\right)\left[e^{-\frac{p}{2.20(10^9)}} - 1\right] = 2943$$

$$e^{-\frac{p}{2.20(10^9)}} = 0.9987$$

$$\ln e^{-\frac{p}{2.20(10^9)}} = \ln 0.9987$$

$$-\frac{p}{2.20\left(10^9\right)} = -1.3386\left(10^{-3}\right)$$

Compressible:

$$p = 2.945\left(10^6\right)\text{ Pa} = 2.945\text{ MPa} \qquad \textbf{Ans.}$$

If the water is considered incompressible,

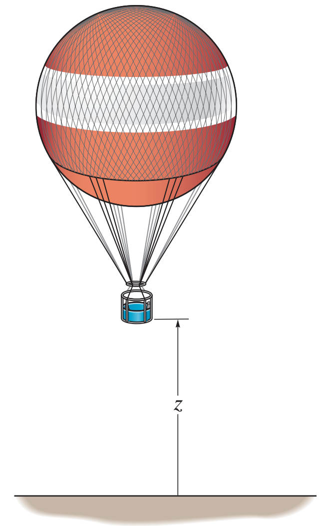
$$p = \rho_o gh = \left(1000\text{ kg/m}^3\right)\left(9.81\text{m/s}^2\right)(300\text{ m})$$

$$= 2.943\left(10^6\right)\text{ Pa} = 2.943\text{ MPa} \qquad \textbf{Ans.}$$

Ans:
Incompressible: $p = 2.943$ MPa
Compressible: $p = 2.945$ MPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–23. As the balloon ascends, measurements indicate that the temperature begins to decrease at a constant rate, from $T = 20°\text{C}$ at $z = 0$ to $T = 16°\text{C}$ at $z = 500$ m. If the absolute pressure and density of the air at $z = 0$ are $p = 101$ kPa and $\rho = 1.23\ \text{kg/m}^3$, determine these values at $z = 500$ m.



SOLUTION

We will first determine the absolute temperature as a function of z.

$$T = 293 - \left(\frac{293 - 289}{500}\right)z = (293 - 0.008z)\ \text{K}$$

Using this result to apply the ideal gas law with $R = 286.9\ \text{J/(kg}\cdot\text{K)}$

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{286.9(293 - 0.008z)}$$

$$= \frac{p}{84061.7 - 2.2952z}$$

$$dp = -\gamma dz = -\rho g dz$$

$$dp = -\frac{p(9.81)dz}{84061.7 - 2.2952z}$$

$$\frac{dp}{p} = -\frac{9.81dz}{84061.7 - 2.2952z}$$

When $z = 0$, $p = 101(10^3)$ Pa. Then

$$\int_{101(10^3)}^{p} \frac{dp}{p} = -9.81\int_0^z \frac{dz}{84061.7 - 2.2952z}$$

$$\ln p\Big|_{101(10^3)}^{p} = (-9.81)\left[-\frac{1}{2.2952}\ln(84061.7 - 2.2952z)\right]\Bigg|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 4.2741\ln\left(\frac{84061.7 - 2.2952z}{84061.7}\right)$$

$$\ln\left[\frac{p}{101(10^3)}\right] = \ln\left[\left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}\right]$$

$$\frac{p}{101(10^3)} = \left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}$$

$$p = 90.3467(10^{-18})(84061.7 - 2.2952z)^{4.2741}$$

2–23. Continued

At $z = 500$ m,

$$p = 90.3467\left(10^{-18}\right)\left[84061.7 - 2.2952(500)\right]^{4.2741}$$
$$= 95.24\left(10^{3}\right) \text{ Pa} = 95.2 \text{ kPa} \qquad \textbf{Ans.}$$

From the ideal gas law;

$$p = \rho RT; \qquad \frac{p}{\rho T} = R = \text{constant}$$

Thus,

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$$

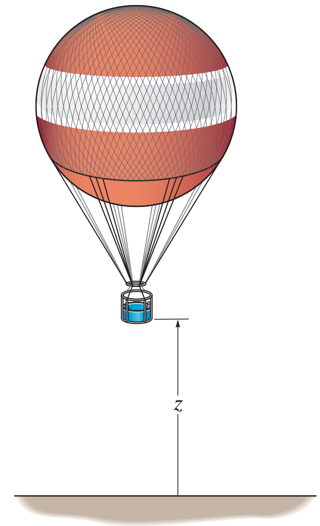
Where $p_1 = 101$ kPa, $\rho_1 = 1.23\ \text{kg/m}^3$, $T_1 = 293\ k$, $p_2 = 95.24$ kPa, $T_2 = 289$ K.
Then

$$\frac{101 \text{ kPa}}{\left(1.23 \text{ kg/m}^3\right)(293 \text{ K})} = \frac{95.24 \text{ kPa}}{\rho_2(289 \text{ K})} \qquad \rho_2 = 1.176 \text{ kg/m}^3 = 1.18 \text{ kg/m}^3 \qquad \textbf{Ans.}$$

Ans:
$p = 95.2$ kPa
$\rho = 1.18\ \text{kg/m}^3$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–24.** As the balloon ascends, measurements indicate that the temperature begins to decrease at a constant rate, from $T = 20°\text{C}$ at $z = 0$ to $T = 16°\text{C}$ at $z = 500$ m. If the absolute pressure of the air at $z = 0$ is $p = 101$ kPa, plot the variation of pressure (vertical axis) verses altitude for $0 \le z \le 3000$ m. Give values for increments of $\Delta z = 500$ m.



SOLUTION

We will first determine the absolute temperature as a function of z

$$T = 293 - \left(\frac{293 - 289}{500}\right)z = (293 - 0.008z)\text{k}$$

Using this result to apply this ideal gas law with $R = 286.9\ \text{J/kg}\cdot\text{k}$

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{286.9(293 - 0.008z)}$$

$$= \frac{p}{84061.7 - 2.2952z}$$

$$dp = -\gamma dz = -\rho g dz$$

$$dp = -\frac{p(9.81)dz}{84061.7 - 2.2952z}$$

$$\frac{dp}{p} = -\frac{9.81dz}{84061.7 - 2.2952z}$$

When $z = 0$, $p = 101(10^3)$ Pa, Then

$$\int_{101(10^3)}^{p} \frac{dp}{p} = -9.81\int_0^z \frac{dz}{84061.7 - 2.2952z}$$

$$\ln p\Big|_{101(10^3)}^{p} = (-9.81)\left[-\frac{1}{2.2952}\ln(84061.7 - 2.2952z)\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 4.2741\ln\left(\frac{84061.7 - 2.2952z}{84061.7}\right)$$

$$\ln\left[\frac{p}{101(10^3)}\right] = \ln\left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}$$

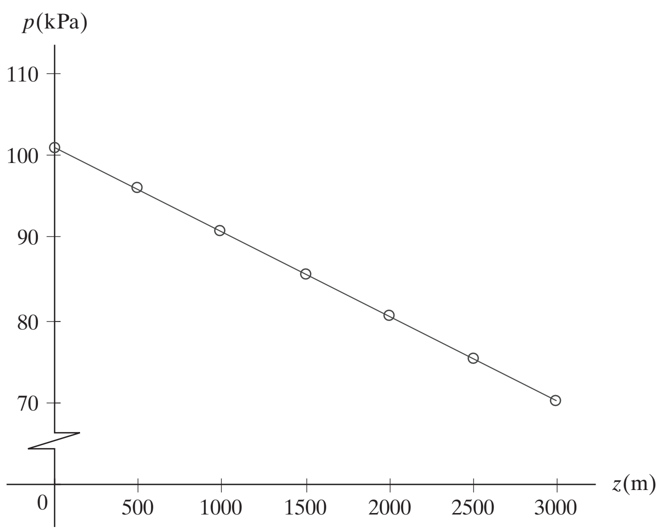
$$\frac{p}{101(10^3)} = \left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}$$

$$p = \left[90.3467(10^{-18})(84061.7 - 2.2952z)^{4.2741}\right]\ \text{Pa Where } z \text{ is in } m$$

2–24. Continued

The plot of p vs. z is shown in Fig. a

$z(m)$	0	500	1000	1500	2000	2500	3000
p (kPa)	101	95.2	89.7	84.5	79.4	74.7	70.1



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–25. In the troposphere, the absolute temperature of the air varies with elevation such that $T = T_0 - Cz$, where C is a constant. If $p = p_0$ at $z = 0$, determine the absolute pressure as a function of elevation.

SOLUTION

$$dp = -\gamma dz = -\rho g dz$$

Since the ideal gas law gives $p = \rho RT$ or $\rho = \dfrac{p}{RT}$,

$$\frac{dp}{p} = -\frac{g\,dz}{R\,T}$$

Since $p = p_0$ at $z = 0$, integrating this equation gives

$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{g}{R}\int_0^z \frac{dz}{T_0 - Cz}$$

$$\ln p\Big|_{p_0}^{p} = \frac{g}{R}\left[\frac{1}{C}\ln\left(\mathrm{T}_0 - Cz\right)\right]_0^z$$

$$\ln\frac{p}{p_0} = \frac{g}{RC}\ln\left(\frac{T_0 - Cz}{T_0}\right)$$

Therefore,

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/RC} \qquad \textbf{Ans.}$$

Ans:

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/RC}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–26. In the troposphere the absolute temperature of the air varies with elevation such that $T = T_0 - Cz$, where C is a constant. Using Fig. 2–11, determine the constants, T_0 and C. If $p_0 = 101$ kPa at $z_0 = 0$, determine the absolute pressure in the air at an elevation of 5 km.

SOLUTION

From Fig. 2–11, $T_0 = 15°\text{C}$ at $z = 0$. Then

$$15°\text{C} = T_0 - C(0)$$

$$T_0 = 15°\text{C} \qquad \textbf{Ans.}$$

Also, $T_C = -56.5°\text{C}$ at $z = 11.0(10^3)$ m. Then

$$-56.5°\text{C} = 15°\text{C} - C\left[11.0(10^3)\text{m}\right]$$

$$C = 6.50(10^{-3})°\text{C/m} \qquad \textbf{Ans.}$$

Thus,

$$T_C = \left[15 - 6.50(10^{-3})z\right]°\text{C}$$

The absolute temperature is therefore

$$T = (15 - 6.50z) + 273 = \left[288 - 6.50(10^{-3})z\right]\text{K} \qquad \textbf{(1)}$$

Substitute the ideal gas law $p = \rho RT$ or $\rho = \dfrac{p}{RT}$ into $dp = -\gamma dz = -\rho g dz$,

$$dp = -\frac{p}{RT}gdz$$

$$\frac{dp}{p} = \frac{g}{RT}dz$$

From the table in Appendix A, gas constant for air is $R = 286.9\ \text{J/(kg}\cdot\text{K)}$.

Also, $p = 101$ kPa at $z = 0$. Then

$$\int_{101(10^3)\text{ Pa}}^{p}\frac{dp}{p} = -\left(\frac{9.81\ \text{m/s}^2}{286.9\ \text{J/(kg}\cdot\text{K)}}\right)\int_0^z\frac{dz}{288 - 6.50(10^{-3})z}$$

$$\ln p\Big|_{101(10^3)\text{ Pa}}^{p} = 5.2605\ \ln\left[288 - 6.50(10^{-3})z\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 5.2605\ \ln\left[\frac{288 - 6.50(10^{-3})z}{288}\right]$$

$$p = 101(10^3)\left[\frac{288 - 6.50(10^{-3})z}{288}\right]^{5.2605}$$

At $z = 5(10^3)$ m,

$$p = 101(10^3)\left\{\frac{288 - \left[6.50(10^{-3})\right]\left[5(10^3)\right]}{288}\right\}^{5.2605}$$

$$= 53.8(10^3)\ \text{Pa} = 53.8\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
$T_0 = 15°\text{C}$
$C = 6.50(10^{-3})°\text{C/m}$
$p = 53.8$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–27. The density of a nonhomogeneous liquid varies as a function of depth h, such that $\rho = (850 + 0.2h)\ \text{kg/m}^3$, where h is in meters. Determine the pressure when $h = 20$ m.

SOLUTION

Since $p = \rho g h$, then the liquid is considered compressible.

$$dp = \rho g\, dh$$

Integrating this equation using the gage pressure $p = 0$ at $h = 0$ and p at h. Then,

$$\int_0^P dp = \int_0^h (850 + 0.2\,h)\,(9.81)\,dh$$

$$p = \left(8338.5\,h + 0.981\,h^2\right)\ \text{Pa}$$

At $h = 20$ m, this equation gives

$$p = \left[8338.5(20) + 0.981\left(20^2\right)\right]\ \text{Pa}$$

$$= 167.16\left(10^3\right)\ \text{Pa} = 167\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
167 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–28.** The density of a non-homogeneous liquid varies as a function of depth h, such that $\rho = (635 + 60h)\ \text{kg/m}^3$, where h is in meters. Plot the variation of the pressure (vertical axis) versus depth for $0 \le h < 10$ m. Give values for increments of 2 m.

SOLUTION

$h(m)$	0	2	4	6	8	10
$p(\text{kPa})$	0	13.6	29.6	48.0	68.7	91.7

The liquid is considered compressible. Use

$$dp = \rho g dh$$

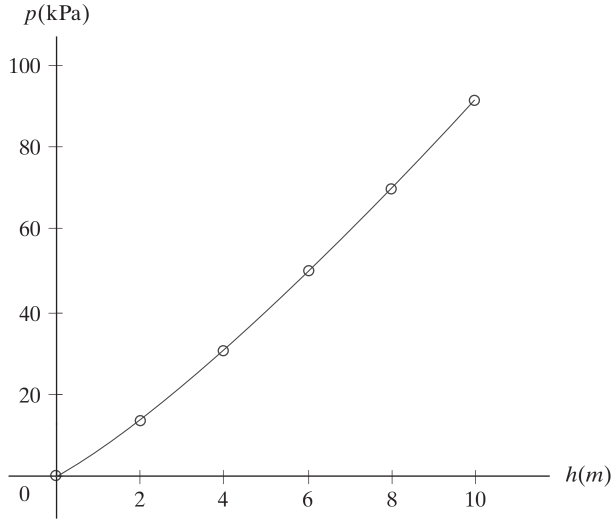
Integrate this equation using the gage pressure $p = 0$ at $h = 0$ and p at h. Then

$$\int_0^p dp = \int_0^h (635 + 60h)(9.81)\, dh$$

$$p = \left[9.81\left(635h + 30h^2\right)\right] \text{Pa}$$

$$p = \left[0.00981\left(635h + 30h^2\right)\right] \text{kPa where } h \text{ is in } m.$$

The plot of p *vs* h is shown in Fig. *a*.



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–29. In the troposphere, which extends from sea level to 11 km, it is found that the temperature decreases with altitude such that $dT/dz = -C$, where C is the constant lapse rate. If the temperature and pressure at $z = 0$ are T_0 and p_0, determine the pressure as a function of altitude.

SOLUTION

First, we must establish the relation between T, and z using $T = T_0$ at $z = 0$,

$$\int_{T_0}^{T} dT = -c\int_0^z dz$$

$$T - T_0 = -Cz$$

$$T = T_0 - Cz$$

Applying the ideal gas lan

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{R(T_0 - Cz)}$$

$$dp = -\gamma dz = -\rho g dz$$

$$dp = -\frac{gpdz}{R(T_0 - Cz)}$$

$$\frac{dp}{p} = \frac{-g}{R}\left(\frac{dz}{T_0 - Cz}\right)$$

Using $p = p_0$ at $z = 0$,

$$\int_{p_0}^{p} \frac{dp}{p} = \frac{-g}{R}\int_0^z \frac{dz}{T_0 - Cz}$$

$$\ln p\Big|_{p_0}^{p} = -\frac{g}{R}\left[\left(-\frac{1}{C}\right)\ln(T_0 - Cz)\right]\Big|_0^z$$

$$\ln\frac{p}{p_0} = \frac{g}{CR}\ln\left(\frac{T_0 - Cz}{T_0}\right)$$

$$\ln\frac{p}{p_0} = \ln\left[\left(\frac{T_0 - Cz}{T_0}\right)^{g/CR}\right]$$

$$\frac{p}{p_0} = \left(\frac{T_0 - Cz}{T_0}\right)^{g/CR}$$

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/CR} \qquad \textbf{Ans.}$$

Ans:

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/RC}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–30. At the bottom of the stratosphere the temperature is assumed to remain constant at $T = T_0$. If the pressure is $p = p_0$, where the elevation is $z = z_0$, derive an expression for the pressure as a function of elevation.

SOLUTION

$$p = \rho R T_0$$

$$dp = -\rho g dz = \frac{-pg}{RT_0} dz$$

$$\frac{dp}{p} = -\frac{g}{RT_0} dz$$

$$\ln p = -\frac{zg}{RT_0} + C$$

At $z = z_0$, $p = p_0$, so that

$$\frac{p}{p_0} = e^{-(z-z_0)g/RT_0}$$

$$p = p_0\, e^{-(z-z_0)g/RT_0}$$ **Ans.**

Ans:
$p = p_0 e^{-(z-z_0)g/RT_0}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–31. Determine the pressure at an elevation of $z = 20$ km into the stratosphere if the temperature remains constant at $T = -56.5°\text{C}$. Assume the stratosphere beings at $z = 11$ km (see Fig. 2–11).

SOLUTION

Within the Troposphere $T_C = T_0 - Cz$, and Fig. 2-7 gives $T_C = 15°\text{C}$ at $z = 0$. Then

$$15°\text{C} = T_0 - C(0)$$

$$T_0 = 15°\text{C}$$

Also, $T_C = -56.5°\text{C}$ at $z = 11.0(10^3)$ m. Then

$$-56.5°\text{C} = 15°\text{C} - C\left[11.0(10^3)\ \text{m}\right]$$

$$C = 6.50(10^{-3})\ °\text{C/m}$$

Thus

$$T_C = \left[15 - 6.50(10^{-3})z\right]\ °\text{C}$$

The absolute temperature is therefore

$$T = 15 - 6.50(10^{-3})z + 273 = \left[288 - 6.50(10^3)z\right]k \qquad \textbf{(1)}$$

Substitute the ideal gas law $p = \rho RT$ or $\rho = \dfrac{p}{RT}$ into Eq. 2–4 $dp = -\gamma dz = -\rho g dz$,

$$dp = -\frac{p}{RT} g dz$$

$$\frac{dp}{p} = -\frac{g}{RT} dz \qquad \textbf{(2)}$$

From table in Appendix A, gas constant for air is $R = 286.9\ \text{J/kg}\cdot\text{K}$. Also, $p = 101$ kPa at $z = 0$. Then

$$\int_{101(10^3)\ \text{Pa}}^{p} \frac{dp}{p} = -\left(\frac{9.81\ \text{m/s}^2}{286.9\ \text{J/kg}\cdot\text{K}}\right)\int_0^z \frac{dz}{288 - 6.50(10^{-3})z}$$

$$\ln p\Big|_{101(10^3)\ \text{Pa}}^{p} = 5.2605 \ln\left[288 - 6.50(10^{-3})z\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 5.2605 \ln\left[\frac{288 - 6.50(10^{-3})z}{288}\right]$$

$$p = 101(10^3)\left[\frac{288 - 6.50(10^{-3})z}{288}\right]^{5.2605}$$

2–31. Continued

At $z = 11.0(10^3)$ m,

$$p = 101(10^3)\left\{\frac{288 - \left[6.50(10^{-3})\right]\left[11.0(10^3)\right]}{288}\right\}^{5.2605} = 22.51(10^3)\text{ Pa}$$

Integrate Eq. (2) using this result and $T = -56.5°\text{C} + 273 = 216.5\text{ K}$

$$\int_{22.51(10^3)\text{ Pa}}^{p} \frac{dp}{p} = -\left[\frac{9.81\text{ m/s}^2}{(286.9\text{ J/kg}\cdot\text{K})(216.5\text{ K})}\right]\int_{11(10^3)\text{ m}}^{z} dz$$

$$\ln p\Big|_{22.51(10^3)\text{ Pa}}^{p} = -0.1579(10^{-3})z\Big|_{11(10^3)\text{ m}}^{z}$$

$$\ln\frac{p}{22.51(10^3)} = 0.1579(10^{-3})\left[11(10^3) - z\right]$$

$$p = \left[22.51(10^3)e^{0.1579(10^{-3})\left[11(10^3)-z\right]}\right]\text{ Pa}$$

At $z = 20(10^3)$ m,

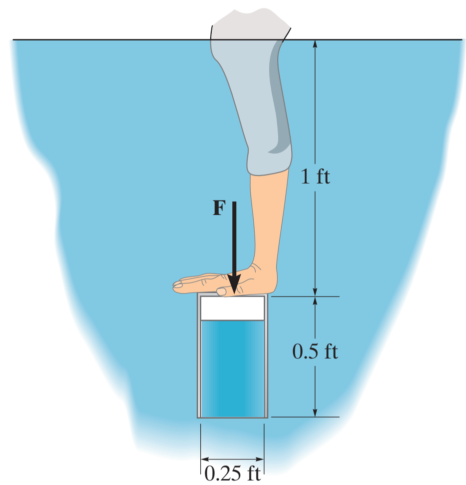
$$p = \left[22.51(10^3)e^{0.1579(10^{-3})\left[11(10^3)-20(10^3)\right]}\right]\text{ Pa}$$

$$= 5.43(10^3)\text{ Pa} = 5.43\text{ kPa}$$ **Ans.**

Ans:
5.43 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–32.** The can, which weighs 0.2 lb, has an open end. If it is inverted and pushed down into the water, determine the force **F** needed to hold it under the surface. Assume the air in the can remains at the same temperature as the atmosphere, and that is 70°F. *Hint*: Account for the change in volume of air in the can due to the pressure change. The atmospheric pressure is $p_{\text{atm}} = 14.7$ psi.



SOLUTION

When submerged, the density of the air in the can changes due to pressure changes. According to the ideal gas law,

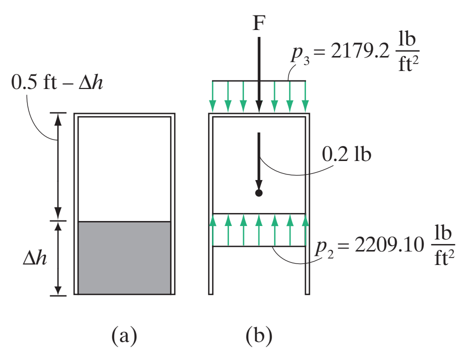
$$p\forall = mRT$$

Since the temperature T is constant, mRT is also constant. Thus,

$$p_1\forall_1 = p_2\forall_2 \qquad \textbf{(1)}$$

When $p_1 = p_{\text{atm}} = \left(14.7\dfrac{\text{lb}}{\text{in.}^2}\right)\left(\dfrac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 2116.8\dfrac{\text{lb}}{\text{ft}^2}$, $\forall_1 = \pi(0.125\ \text{ft})^2(0.5\ \text{ft})$

$$= 7.8125\left(10^{-3}\right)\pi\ \text{ft}^3.$$



When the can is submerged, the water fills the space shown shaded in Fig. *a*. Thus,

$$p_2 = p_{\text{atm}} + \gamma_w h = \left(2116.8\frac{\text{lb}}{\text{ft}^2}\right) + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1.5\ \text{ft} - \Delta h)$$

$$= (2210.4 - 62.4\Delta h)\frac{\text{lb}}{\text{ft}^2}$$

$$\forall_2 = \pi(0.125\ \text{ft})^2(0.5\ \text{ft} - \Delta h) = \left[0.015625\pi(0.5\ \text{ft} - \Delta h)\right]\text{ft}^3$$

Substituting these values into Eq. (1),

$$\left(2116.8\frac{\text{lb}}{\text{ft}^2}\right)\left[7.8125\left(10^{-3}\right)\pi\ \text{ft}^3\right] = \left[(2210.4 - 62.4\Delta h)\frac{\text{lb}}{\text{ft}^2}\right]\left[0.015625\pi(0.5 - \Delta h)\ \text{ft}^3\right]$$

$$62.4\Delta h^2 - 2241.6\Delta h + 46.8 = 0$$

Solving for the root < 0.5 ft, we obtain

$$\Delta h = 0.02089\ \text{ft}$$

Then

$$p_2 = 2210.4 - 62.4(0.02089) = 2209.10\frac{\text{lb}}{\text{ft}^2}$$

The pressure on top of the can is

$$p_3 = p_{\text{atm}} + \gamma_w h = \left(2116.8\frac{\text{lb}}{\text{ft}^2}\right) + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1\ \text{ft}) = 2179.2\frac{\text{lb}}{\text{ft}^2}$$

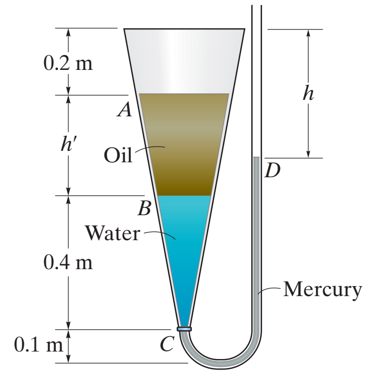
Considering the free-body diagram of the can, Fig. *b*,

$$+\uparrow\Sigma F_y = 0; \qquad \left(2209.10\frac{\text{lb}}{\text{ft}^2}\right)\left[\pi(0.125\ \text{ft})^2\right] - 0.2\ \text{lb} - \left(2179.2\frac{\text{lb}}{\text{ft}^2}\right)\left[\pi(0.125\ \text{ft})^2\right] - F = 0$$

$$F = 1.27\ \text{lb} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–33. The funnel is filled with oil and water to the levels shown. Determine the depth of oil h' that must be in the funnel so that the water remains at a depth C, and the mercury level made $h = 0.8$ m. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, $\rho_{Hg} = 13550\ \text{kg/m}^3$.



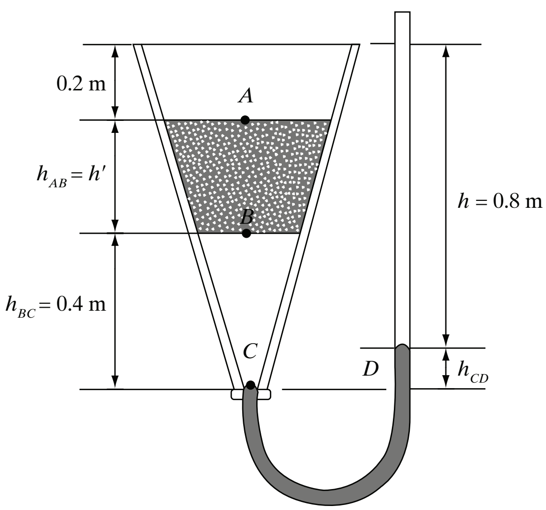
SOLUTION

Referring to Fig. a, $h_{CD} = 0.2\text{ m} + h' + 0.4\text{ m} - 0.8\text{ m} = h' - 0.2\text{ m}$. Then the manometer rule gives

$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{Hg} g h_{CD} = p_D$$

$$0 + \left(900\ \text{kg/m}^3\right)gh' + \left(1000\ \text{kg/m}^3\right)g(0.4) - \left(13\,550\ \text{kg/m}^3\right)g(h' - 0.2\text{ m}) = 0$$

$$h' = 0.2458\text{ m} = 246\text{ mm}$$ **Ans.**

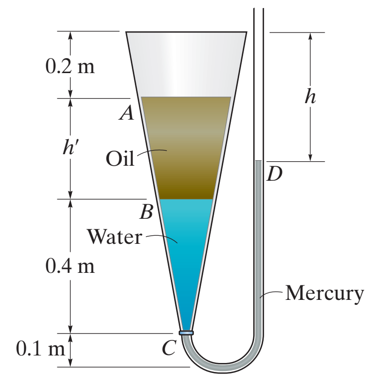


(a)

Ans:
246 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–34. The funnel is filled with oil to a depth of $h' = 0.3$ m and water to a depth of 0.4 m. Determine the distance h the mercury level is from the top of the funnel. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$.



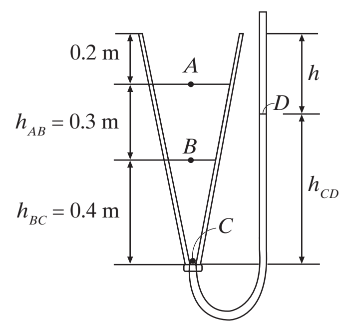
SOLUTION

Referring to Fig. a, $h_{CD} = 0.2\ \text{m} + 0.3\ \text{m} + 0.4\ \text{m} - \text{h} = 0.9\ \text{m} - h$. Then the manometer rule gives

$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{\text{Hg}} g h_{CD} = p_D$$

$$0 + \left(900\ \text{kg/m}^3\right)g(0.3\ \text{m}) + \left(1000\ \text{kg/m}^3\right)g(0.4\ \text{m}) - \left(13\,550\ \text{kg/m}^3\right)(g)(0.9\ \text{m} - h) = 0$$

$$h = 0.8506\ \text{m} = 851\ \text{mm} \qquad \textbf{Ans.}$$

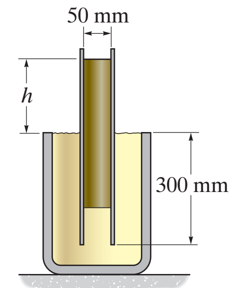


(a)

Ans:
851 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–35. The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter pipe is inserted within it to a depth of 300 mm. If 0.00075 m^3 of kerosene is then poured into the pipe, determine the height h to which the kerosene rises from the glycerin.



SOLUTION

The height of the kerosene column in the pipe, Fig. a, is

$$h_{ke} = \frac{\mathcal{V}_{ke}}{\pi r^2} = \frac{0.00075\ \text{m}^3}{\pi(0.025\ \text{m})^2} = \left(\frac{1.2}{\pi}\right)\text{m}$$

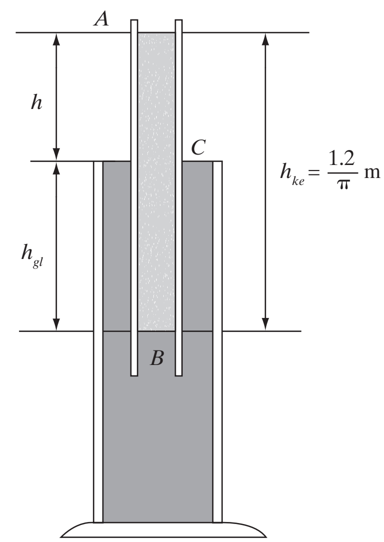
From Appendix A, $\rho_{ke} = 814\ \text{kg/m}^3$ and $\rho_{gl} = 1260\ \text{kg/m}^3$ writing the manometer equation from $A \rightarrow B \rightarrow C$ by referring to Fig. a,

$$p_{\text{atm}} + \rho_{ke} g h_{ke} - \rho_{gl} g h_{gl} = p_{\text{atm}}$$

$$h_{gl} = \left(\frac{\rho_{ke}}{\rho_{gl}}\right)h_{ke} = \left(\frac{814\ \text{kg/m}^3}{1260\ \text{kg/m}^3}\right)\left(\frac{1.2}{\pi}\ \text{m}\right) = 0.2468\ \text{m}$$

Thus,

$$h = h_{ke} - h_{gl} = \frac{1.2}{\pi}\ \text{m} - 0.2468\ \text{m} = 0.1352\ \text{m} = 135\ \text{mm} \qquad \textbf{Ans.}$$

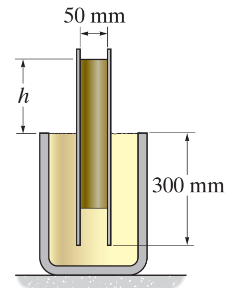


(a)

Ans:
135 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–36.** The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter thin pipe is inserted within it to a depth of 300 mm. Determine the maximum volume of kerosene that can be poured into the pipe so it does not come out from the bottom end. How high h does the kerosene rise above the glycerin?



SOLUTION

From Appendix A, $\rho_{ke} = 814\ \text{kg/m}^3$ and $\rho_{gl} = 1260\ \text{kg/m}^3$. The kerosene is required to heat the bottom of the tube as shown in Fig. *a*. Write the manometer equation from $A \rightarrow B \rightarrow C$,

$$p_{\text{atm}} + \rho_{ke} g h_{ke} - \rho_{gl} g h_{gl} = p_{\text{atm}}$$

$$h_{ke} = \frac{\rho_{gl}}{\rho_{ke}} h_{gl}$$

Here, $h_{ke} = (h + 0.3)$ m and $h_{gl} = 0.3$ m. Then

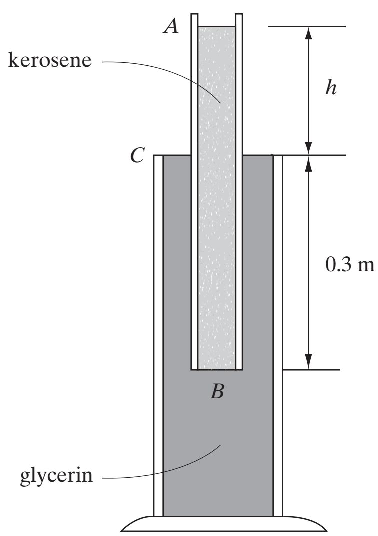
$$(h + 0.3)\ \text{m} = \left(\frac{1260\ \text{kg/m}^3}{814\ \text{kg/m}^3}\right)(0.3\ \text{m})$$

$$h_{ke} = 0.1644\ \text{m} = 164\ \text{mm} \qquad \textbf{Ans.}$$

Thus, the volume of the kerosene in the pipe is

$$\forall_{ke} = \pi r^2 h_{ke} = \pi(0.025\ \text{m})^2(0.1644\ \text{m} + 0.3\ \text{m}) = 0.9118\left(10^{-3}\right)\ \text{m}^3$$

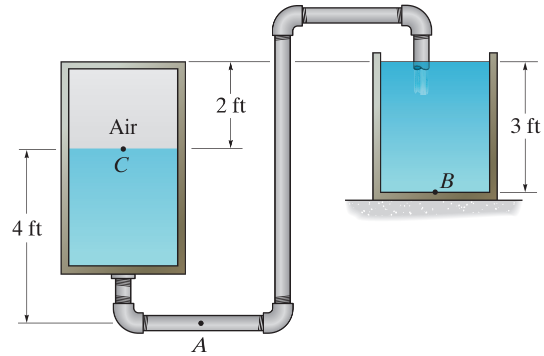
$$= 0.912\left(10^{-3}\right)\ \text{m}^3 \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–37. Determine the pressures at points A and B. The containers are filled with water.



SOLUTION

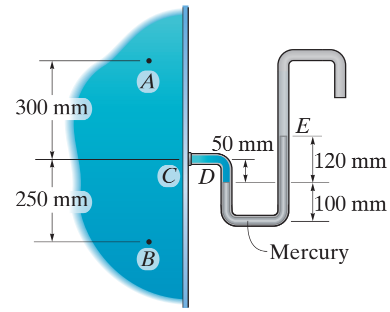
$$p_A = \gamma_A h_A = (62.4\ \text{lb/ft}^3)(2\text{ft} + 4\text{ft}) = \left(374.4\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 2.60\ \text{psi} \quad \textbf{Ans.}$$

$$p_B = \gamma_B h_B = (62.4\ \text{lb/ft}^3)(3\ \text{ft}) = \left(187.2\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.30\ \text{psi} \quad \textbf{Ans.}$$

Ans:
$p_A = 2.60$ psi, $p_B = 1.30$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–39. Butyl carbitol, used in the production of plastics, is stored in a tank having a U-tube manometer. If the U-tube is filled with mercury to level E, determine the pressure in the tank at point A. Take $S_{\text{Hg}} = 13.55$, and $S_{bc} = 0.957$.



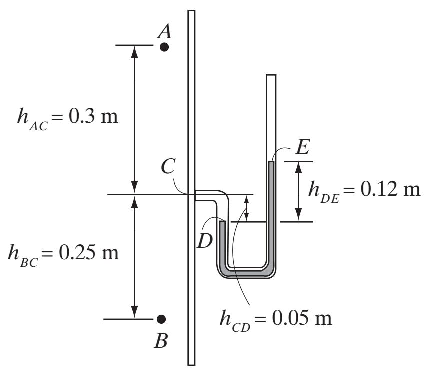
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_E + \rho_{\text{Hg}} g h_{DE} - \rho_{bc} g (h_{CD} + h_{AC}) = p_A$$

$$0 + 13.55(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.120\ \text{m}) - 0.957(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.05\ \text{m} + 0.3\ \text{m}) = p_A$$

$$p_A = 12.67(10^3)\ \text{Pa} = 12.7\ \text{kPa}$$ **Ans.**

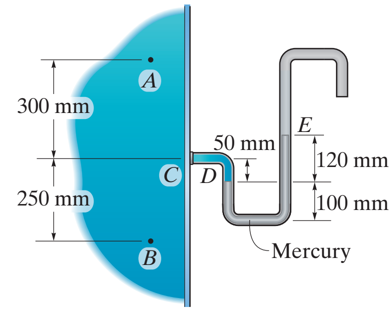


(a)

Ans:
12.7 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–40.** Butyl carbitol, used in the production of plastics, is stored in a tank having a U-tube manometer. If the U-tube is filled with mercury, determine the pressure in the tank at point B. Take $S_{\text{Hg}} = 13.55$, and $S_{bc} = 0.957$.



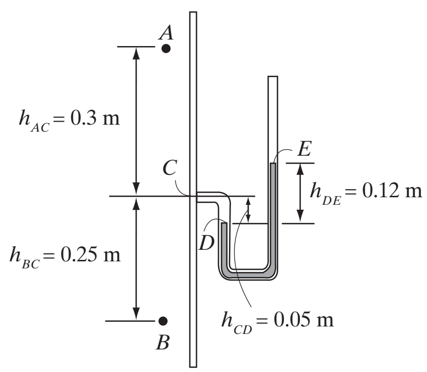
SOLUTION

Referring to Fig. *a*, the manometer rule gives

$$p_E + \rho_{\text{Hg}} g h_{DE} + \rho_{bc} g(-h_{CD} + h_{BC}) = p_B$$

$$0 + 13.55\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.120\ \text{m}) + 0.957\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(-0.05\ \text{m} + 0.25\ \text{m}) = p_B$$

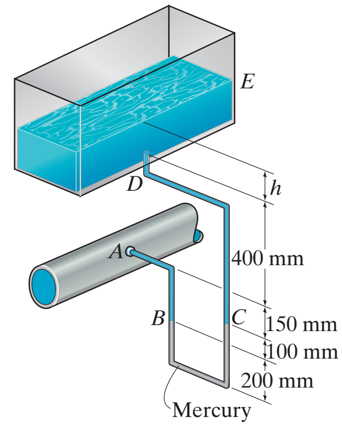
$$p_B = 17.83\left(10^3\right)\ \text{Pa} = 17.8\ \text{kPa} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–41. Water in the reservoir is used to control the water pressure in the pipe at A. If $h = 200$ mm, determine this pressure when the mercury is at the elevation shown. Take $\rho_{Hg} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipe.



SOLUTION

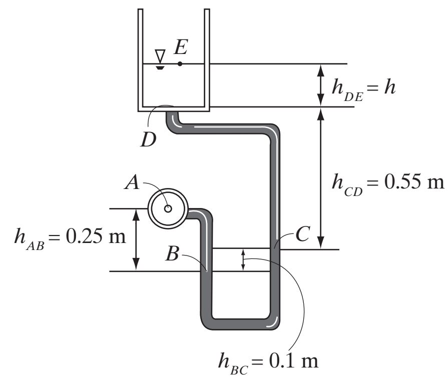
Referring to Fig. *a* with $h = 0.2$ m, the manometer rule gives

$$p_A + \rho_w g h_{AB} - \rho_{Hg} g h_{BC} - \rho_w g(h_{CD} + h_{DE}) = p_E$$

$$p_A + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25\ \text{m}) - (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m})$$

$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.55\ \text{m} + 0.2\text{m}) = 0$$

$$p_A = 18.20(10^3)\ \text{Pa} = 18.2\ \text{kPa} \qquad \textbf{Ans.}$$

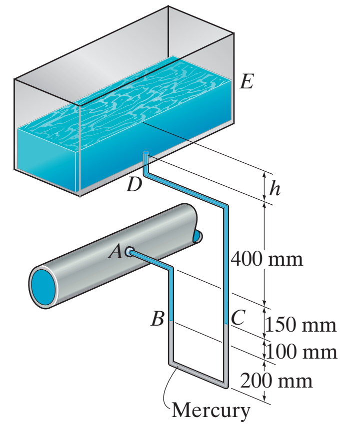


(a)

Ans:
18.2 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–42. If the water pressure in the pipe at A is to be 25 kPa, determine the required height h of water in the reservoir. Mercury in the pipe has the elevation shown. Take $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipe.



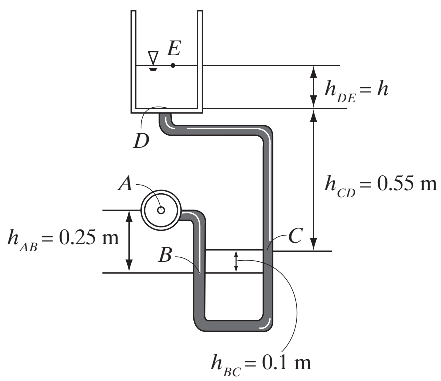
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \rho_w g h_{AB} - \rho_{\text{Hg}} g h_{BC} - \rho_w g(h_{CD} + h_{DE}) = p_E$$

$$25(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25\ \text{m}) - (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m})$$
$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.550\ \text{m} + h) = 0$$

$$h = 0.8934\ \text{m} = 893\ \text{mm}$$ **Ans.**

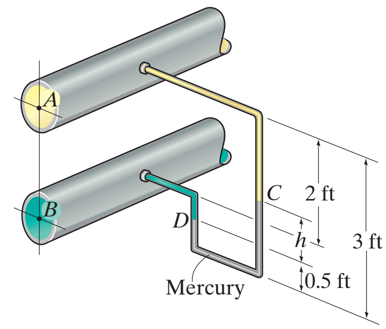


(a)

Ans:
893 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–43. A solvent used for plastics manufacturing consists of cyclohexanol in pipe A and ethyl lactate in pipe B that are being transported to a mixing tank. Determine the pressure in pipe A if the pressure in pipe B is 15 psi. The mercury in the manometer is in the position shown, where $h = 1$ ft. Neglect the diameter of the pipe. Take $S_c = 0.953$, $S_{Hg} = 13.55$, and $S_{el} = 1.03$.



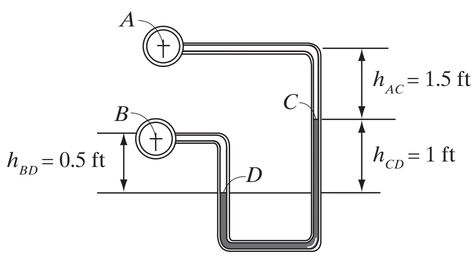
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_c g h_{AC} + \gamma_{Hg} h_{CD} - \gamma_{el} g h_{BD} = p_B$$

$$p_A + 0.953(62.4\ \text{lb/ft}^3)(1.5\ \text{ft}) + (13.55)(62.4\ \text{lb/ft}^3)(1\ \text{ft}) - (1.03)(62.4\ \text{lb/ft}^3)(0.5\ \text{ft}) = \left(15\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$p_A = 1257.42\frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 8.73\ \text{psi} \qquad \textbf{Ans.}$$

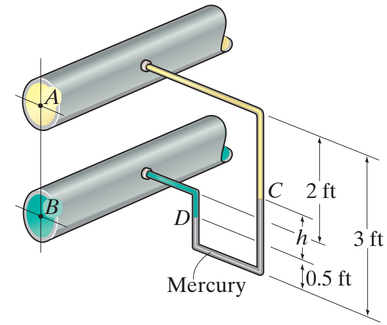


(a)

Ans:
8.73 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–44.** A solvent used for plastics manufacturing consists of cyclohexanol in pipe A and ethyl lactate in pipe B that are being transported to a mixing tank. If the pressure in pipe A is 18 psi, determine the height h of the mercury in the manometer so that a pressure of 25 psi is developed in pipe B. Neglect the diameter of the pipes. Take $S_c = 0.953$, $S_{\text{Hg}} = 13.55$, and $S_{el} = 1.03$.



SOLUTION

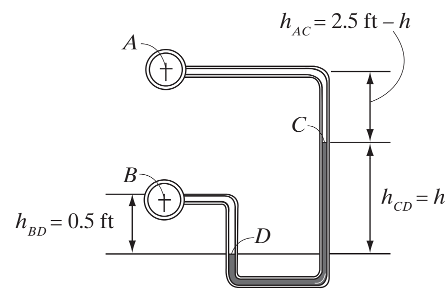
Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_c h_{AC} + \gamma_{\text{Hg}} h_{CD} - \gamma_{el} h_{BD} = p_B$$

$$\frac{18\ \text{lb}}{\text{in.}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + 0.953\left(62.4\ \text{lb/ft}^3\right)(2.5\ \text{ft} - h) + 13.55\left(62.4\ \text{lb/ft}^3\right)(h)$$

$$-(1.03)\left(62.4\ \text{lb/ft}^3\right)(0.5\ \text{ft}) = \frac{25\ \text{lb}}{\text{in.}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

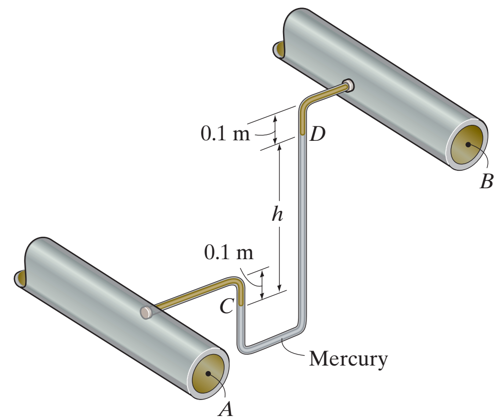
$$h = 1.134\ \text{ft} = 1.13\ \text{ft} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–45. The two pipes contain hexylene glycol, which causes the level of mercury in the manometer to be at $h = 0.3$ m. Determine the differential pressure in the pipes, $p_A - p_B$. Take $\rho_{hgl} = 923\ \text{kg/m}^3$, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipes.



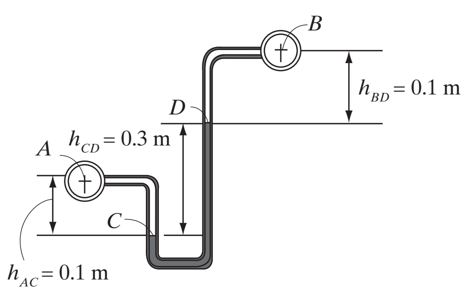
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \rho_{hgl} g h_{AC} - \rho_{\text{Hg}} g h_{CD} - \rho_{hgl} g h_{BD} = p_B$$

$$p_A + \left(923\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.1\ \text{m}) - \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.3\ \text{m})$$
$$- \left(923\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.1\ \text{m}) = p_B$$

$$p_A - p_B = 39.88\left(10^3\right)\ \text{Pa} = 39.9\ \text{kPa}$$ **Ans.**

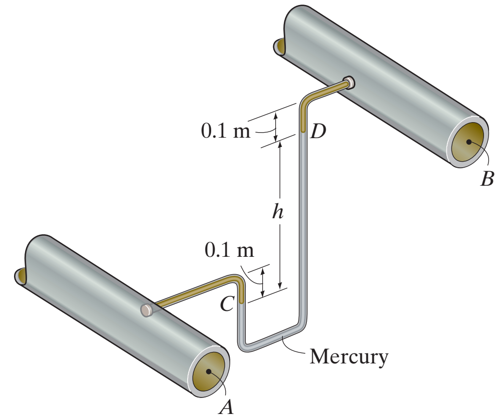


(a)

Ans:
39.9 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–46. The two pipes contain hexylene glycol, which causes the differential pressure reading of the mercury in the manometer to be at $h = 0.3$ m. If the pressure in pipe A increases by 6 kPa, and the pressure in pipe B decreases by 2 kPa, determine the new differential reading h of the manometer. Take $\rho_{hgl} = 923\ \text{kg/m}^3$, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipes.



SOLUTION

As shown in Fig. a, the mercury level is at C and D. Applying the manometer rule,

$$p_A + \rho_{hgl} g h_{AC} - \rho_{\text{Hg}} g h_{CD} - \rho_{hgl} g h_{DB} = p_B$$

$$p_A + (923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) - (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m})$$
$$-(923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) = p_B$$

$$p_A - p_B = 39\,877.65\ \text{Pa} \qquad \textbf{(1)}$$

When the pressure at A and B changes, the mercury level will be at C' and D', Fig. a. Then, the manometer rule gives

$$(p_A + \Delta p_A) + \rho_{hgl} g h_{AC'} - \rho_{\text{Hg}} g h_{C'D'} - \rho_{hgl} g h_{D'B} = (p_B - \Delta p_B)$$

$$\left[p_A + 6(10^3)\ \text{N/m}^2\right] + (923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m} + \Delta h)$$
$$-(13.550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m} + 2\Delta h)$$
$$-(923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m} - \Delta h) = \left[p_B - 2(10^3)\ \text{N/m}^2\right]$$

$$p_A - p_B = 31\,877.65 + 247741.74\ \Delta h \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2), we obtain

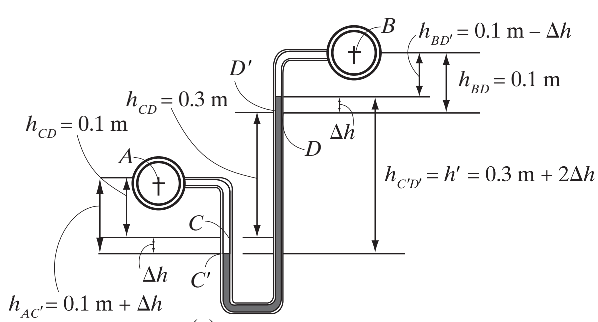
$$39\,877.65 = 31\,877.65 + 247741.74\ \Delta h$$

$$\Delta h = 0.03229$$

Thus,

$$h' = 0.3\ \text{m} + 2\Delta h$$

$$= 0.3\ \text{m} + 2(0.03229\ \text{m}) = 0.36458\ \text{m} = 365\ \text{mm} \qquad \textbf{Ans.}$$

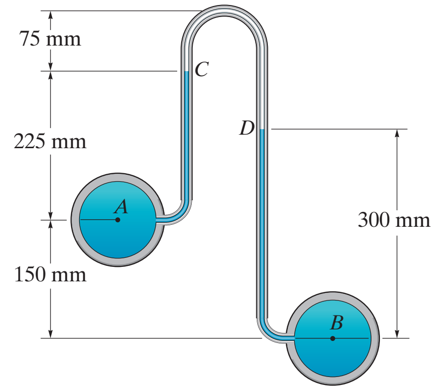


(a)

Ans:
365 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–47. The inverted U-tube manometer is used to measure the difference in pressure between water flowing in the pipes at A and B. If the top segment is filled with air, and the water levels in each segment are as indicated, determine this pressure difference between A and B. $\rho_w = 1000 \text{ kg/m}^3$.



SOLUTION

Notice that the pressure throughout the air in the tube is constant. Referring to Fig. *a*,

$$p_A = (p_w)_1 + p_a = \rho_w g (h_w)_1 + p_a$$

And

$$p_B = (p_w)_2 + p_a = \rho_w g (h_w)_2 + p_a$$

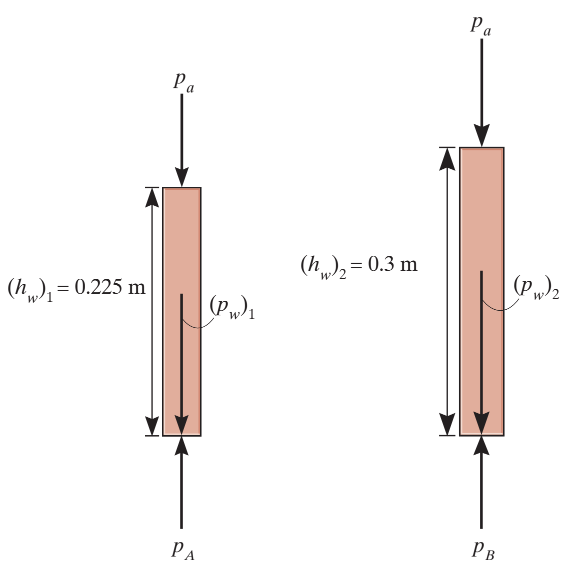
Therefore,

$$\begin{aligned} p_B - p_A &= [\rho_w g (h_w)_2 + p_a] - [\rho_w g (h_w)_1 + p_a] \\ &= \rho_w g [(h_w)_2 - (h_w)_1] \\ &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m} - 0.225 \text{ m}) \\ &= 735.75 \text{ Pa} = 736 \text{ Pa} \end{aligned}$$ **Ans.**

Also, using the manometer equation,

$$p_A - \rho_w g h_{AC} + \rho_w g h_{DB} = p_B$$

$$p_B - p_A = \rho_w g [h_{DB} - h_{AC}]$$

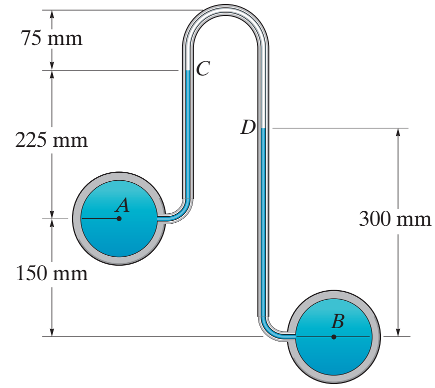


(a)

Ans:
736 Pa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–48.** Solve Prob. 2–47 if the top segment is filled with an oil for which $\rho_o = 800\ \text{kg/m}^3$.



SOLUTION

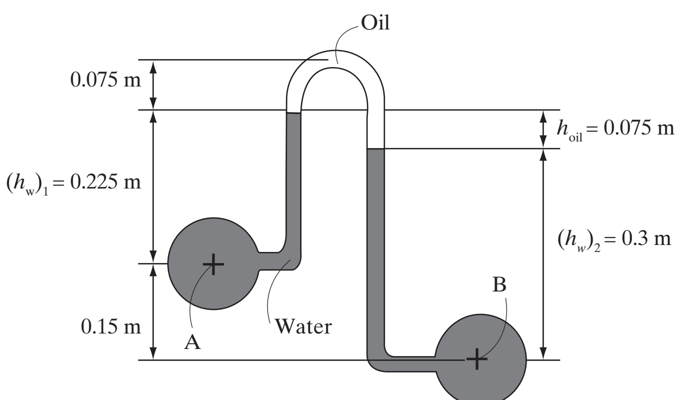
Referring to Fig. *a*, write the manometer equation starting at A and ending at B,

$$p_A - \rho_w g (h_w)_1 + \rho_{oil} g h_{oil} + \rho_w g (h_w)_2 = p_B$$

$$p_B - p_A = \rho_w g\left[(h_w)_2 - (h_w)_1\right] + \rho_{oil} g h_{oil}$$

$$= \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.3\ \text{m} - 0.225\ \text{m}) + \left(800\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.075\ \text{m})$$

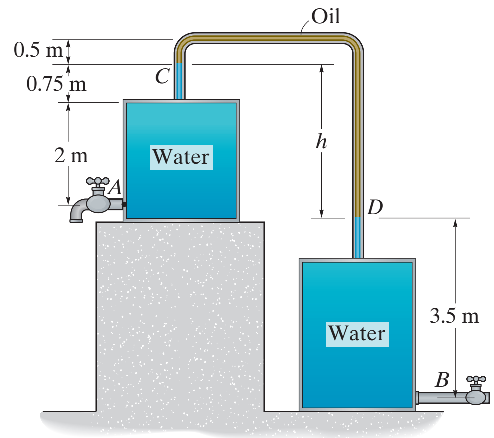
$$= 1.324\left(10^3\right)\ \text{Pa} = 1.32\ \text{kPa}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–49. The pressure in the tank at the closed valve A is 300 kPa. If the differential elevation in the oil level in $h = 2.5$ m, determine the pressure in the pipe at B. Take $\rho_o = 900\ \text{kg/m}^3$.



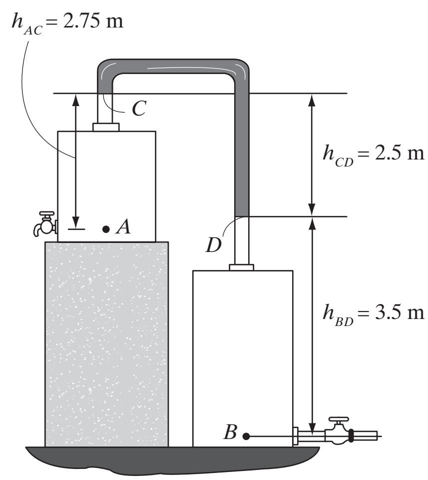
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A - \rho_w g h_{AC} + \rho_o g h_{CD} + \rho_w g h_{BD} = p_B$$

$$300(10^3)\ \text{N/m}^2 - (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.75\ \text{m}) + (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5\ \text{m})$$
$$+ (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m}) = p_B$$

$$p_B = 329.43(10^3)\ \text{Pa} = 329\ \text{kPa} \qquad \textbf{Ans.}$$

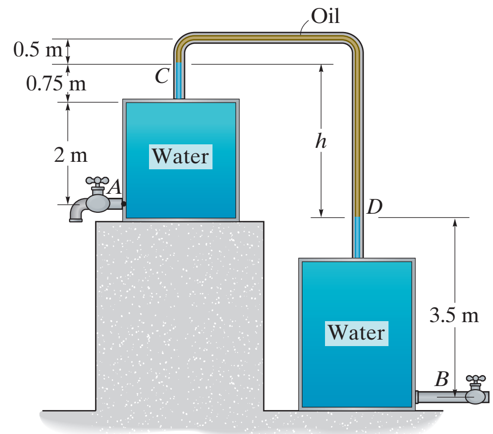


(a)

Ans:
329 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–50. The pressure in the tank at B is 600 kPa. If the differential elevation of the oil is $h = 2.25$ m, determine the pressure at the closed valve A. Take $\rho_o = 900\ \text{kg/m}^3$.



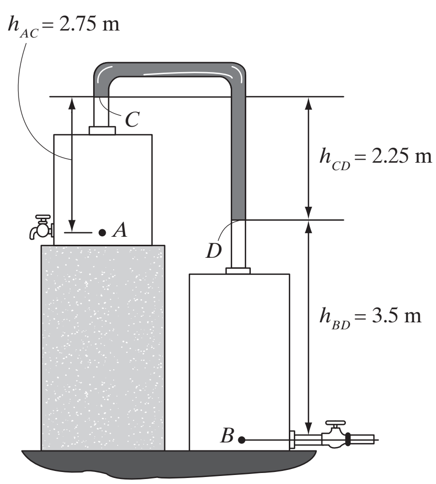
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A - \rho_w g h_{AC} + \rho_o g h_{CD} + \rho_w g h_{BD} = p_B$$

$$p_A - \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.75\ \text{m}) + \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.25\ \text{m}) + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3.5\ \text{m}) = 600\left(10^3\right)\ \text{N/m}^2$$

$$p_A = 572.78\left(10^3\right)\ \text{Pa} = 573\ \text{kPa}$$ **Ans.**

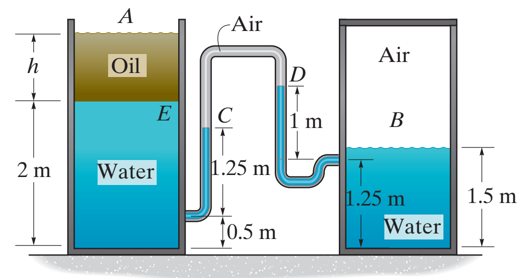


(a)

Ans:
573 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–51. The two tanks A and B are connected using a manometer. If waste oil is poured into tank A to a depth of $h = 0.6$ m, determine the pressure of the entrapped air in tank B. Air is also trapped in line CD as shown. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

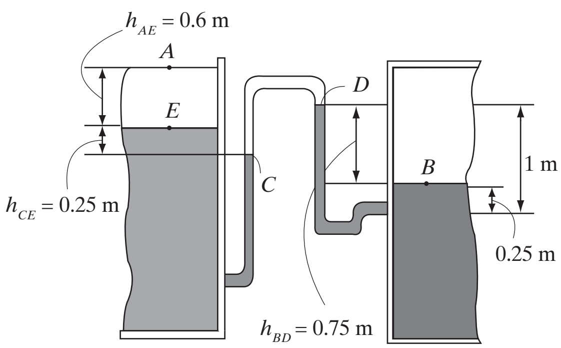
Referring to Fig. a, the manometer rule gives

$$p_A + \rho_o g h_{AE} + \rho_w g h_{CE} + \rho_w g h_{BD} = p_B$$

$$0 + (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.6\ \text{m}) + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25\ \text{m})$$

$$+ (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.75\ \text{m}) = p_B$$

$$p_B = 15.11(10^3)\ \text{Pa} = 15.1\ \text{kPa} \qquad \textbf{Ans.}$$

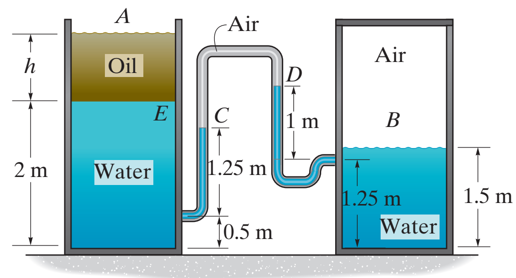


(a)

Ans:
15.1 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–52.** The two tanks A and B are connected using a manometer. If waste oil is poured into tank A to a depth of $h = 1.25$ m, determine the pressure of the trapped air in tank B. Air is also trapped in line CD as shown. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

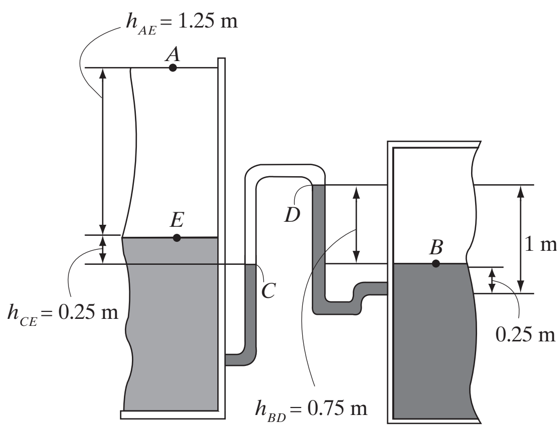
Referring to Fig. a, the manometer rule gives

$$p_A + \rho_o g h_{AE} + \rho_w g h_{CE} + \rho_w g h_{BD} = p_B$$

$$0 + \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.25\ \text{m}) + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.25\ \text{m})$$

$$+ \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m}) = p_B$$

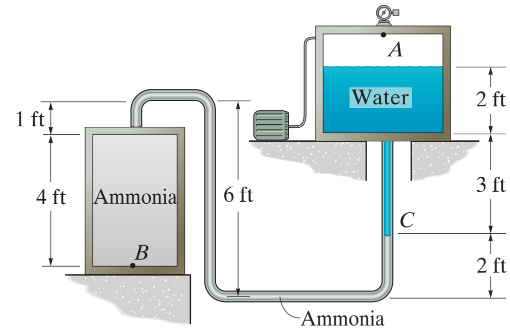
$$p_B = 20.846\left(10^3\right)\ \text{Pa} = 20.8\ \text{kPa} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–53. Air is pumped into the water tank at A such that the pressure gage reads 20 psi. Determine the pressure at point B at the bottom of the ammonia tank. Take $\rho_{am} = 1.75\ \text{slug/ft}^3$.



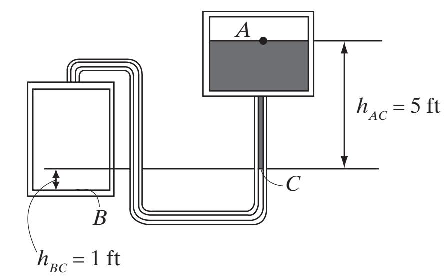
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_w h_{AC} + \gamma_{Am} h_{BC} = p_B$$

$$\frac{20\ \text{lb}}{\text{in}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + \left(62.4\ \text{lb/ft}^3\right)(5\ \text{ft}) + \left(1.75\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(1\ \text{ft}) = p_B$$

$$p_B = 3248.35\ \frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 22.6\ \text{psi}$$ **Ans.**

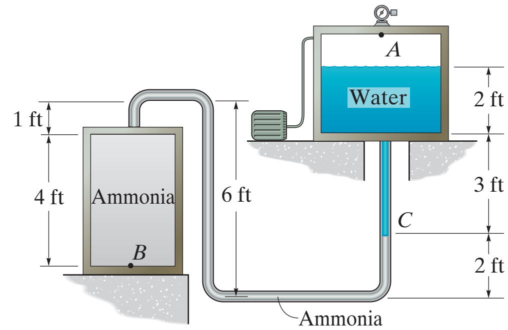


(a)

Ans:
22.6 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–54. Determine the pressure that must be supplied by the pump so that the air in the tank at A develops a pressure of 50 psi at B in the ammonia tank. Take $\rho_{am} = 1.75\ \text{slug/ft}^3$.



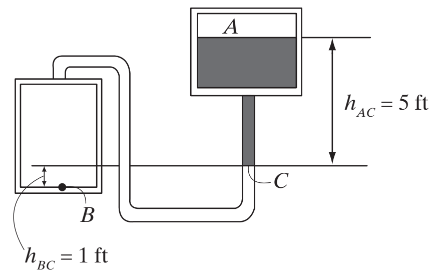
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_w h_{AC} + \gamma_{Am} h_{BC} = p_B$$

$$p_A + \left(62.4\ \text{lb/ft}^3\right)(5\ \text{ft}) + \left(1.75\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(1\ \text{ft}) = \left(50\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$p_A = \left(6831.65\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 47.4\ \text{psi}$$ **Ans.**

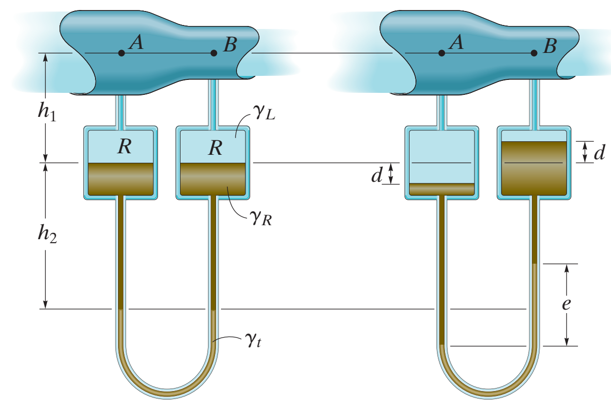


(a)

Ans:
47.4 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–55. The *micro-manometer* is used to measure small differences in pressure. The reservoirs R and upper portion of the lower tubes are filed with a liquid having a specific weight of γ_R, whereas the lower portion is filled with a liquid having a specific weight of γ_t, Fig. (*a*). When the liquid flows through the venturi meter, the levels of the liquids with respect to the original levels are shown in Fig. (*b*). If the cross-sectional area of each reservoir is A_R and the cross-sectional area of the U-tube is A_t, determine the pressure difference $p_A - p_B$. The liquid in the Venturi meter has a specific weight of γ_L.



(a) (b)

SOLUTION

Write the manometer equation starting at A and ending at B, Fig. *a*

$$p_A + \gamma_L(h_1 + d) + \gamma_R\left(h_2 - d + \frac{e}{2}\right) - \gamma_t e$$

$$-\gamma_R\left(h_2 - \frac{e}{2} + d\right) - \gamma_L(h_1 - d) = p_B$$

$$p_A - p_B = 2\gamma_R d - 2\gamma_L d + \gamma_t e - \gamma_R e \qquad \textbf{(1)}$$

Since the same amount of liquid leaving the left reservoir will enter into the left tube,

$$A_R d = A_t\left(\frac{e}{2}\right)$$

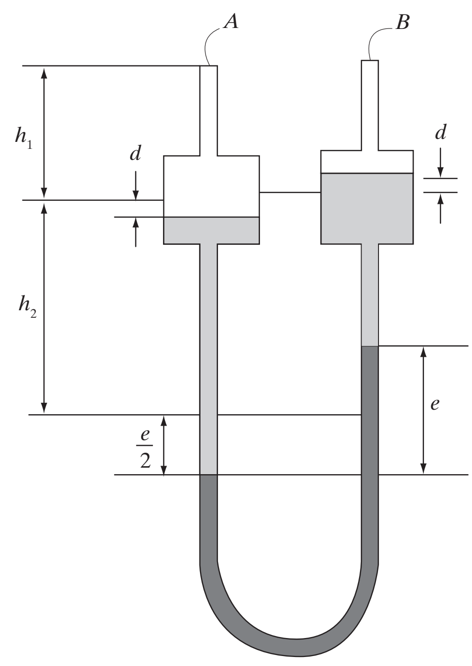
$$d = \left(\frac{A_t}{2A_R}\right)e$$

Substitute this result into Eq. (1),

$$p_A - p_B = 2\gamma_R\left(\frac{A_t}{2A_R}\right)e - 2\gamma_L\left(\frac{A_t}{2A_R}\right)e + \gamma_t e - \gamma_R e$$

$$= e\left[\left(\frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L + \gamma_t - \gamma_R\right]$$

$$= e\left[\gamma_t - \left(1 - \frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L\right] \qquad \textbf{Ans.}$$



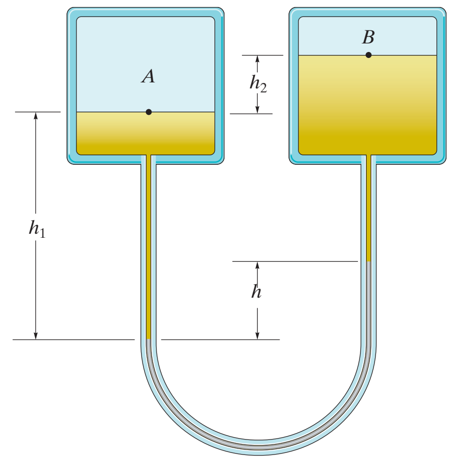
(a)

Ans:

$$p_A - p_B = e\left[\gamma_t - \left(1 - \frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L\right]$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–56.** The Morgan Company manufactures a micromanometer that works on the principles shown. Here there are two reservoirs filled with kerosene, each having a cross-sectional area of $300\ \text{mm}^2$. The connecting tube has a cross-sectional area of $15\ \text{mm}^2$ and contains mercury. Determine h if the pressure difference $p_A - p_B = 40$ Pa. What would h be if water were substituted for mercury? $\rho_{Hg} = 13550\ \text{kg/m}^3$, $\rho_{ke} = 814\ \text{kg/m}^3$.



SOLUTION

Referring to Fig. *a*, write the manometer equation starting at A and ending at B.

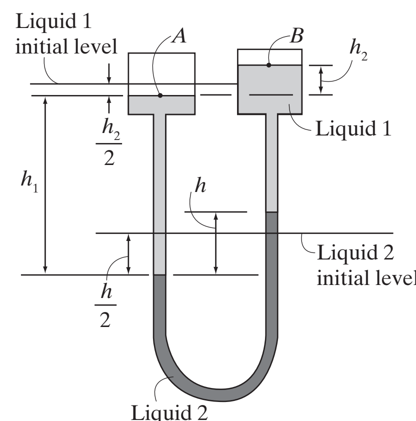
$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h + h_2) = p_B$$

$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g h_2 \qquad \textbf{(1)}$$

Since the same amount of liquid leaving the left reservoir will enter the left tube

$$A_R = \left(\frac{h_2}{2}\right) = A_t\left(\frac{h}{2}\right)$$

$$h_2 = \left(\frac{A_t}{A_R}\right)h$$



(a)

Substitute this result into Eq. (1)

$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g\left(\frac{A_t}{A_R}\right)h$$

$$p_A - p_B = h\left[\rho_2 g - \left(1 - \frac{A_t}{A_R}\right)\rho_1 g\right] \qquad \textbf{(2)}$$

When $\rho_1 = \rho_{ke} = 814\ \text{kg/m}^3$, $\rho_2 = \rho_{Hg} = 13550\ \text{kg/m}^3$ and $p_A - p_B = 40$ Pa,

$$40\ \text{N/m}^2 = h\left[(13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2) - \left(1 - \frac{15}{300}\right)(814\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\right]$$

Mercury: $h = 0.3191\ (10^{-3})\ \text{m} = 0.319\ \text{mm}$ **Ans.**

When $\rho_1 = \rho_{ke} = 814\ \text{kg/m}^3$, $\rho_2 = \rho_w = 1000\ \text{kg/m}^3$ and $P_A - P_B = 40$ Pa

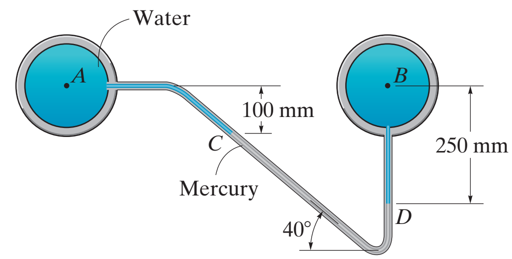
$$40\ \text{N/m}^2 = h\left[(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2) - \left(1 - \frac{15}{300}\right)(814\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\right]$$

Water: $h = 0.01799\ \text{m} = 18.0\ \text{mm}$ **Ans.**

From the results, we notice that if $\rho_2 \gg \rho_1$, h will be too small to be read. Hence, when choosing the liquid to be used, ρ_2 should be slightly larger than ρ_1 so that the sensitivity of the micromanometer is increased.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–57. Determine the difference in pressure $p_B - p_A$ between the centers A and B of the pipes, which are filled with water. The mercury in the inclined-tube manometer has the level shown $S_{Hg} = 13.55$.



SOLUTION

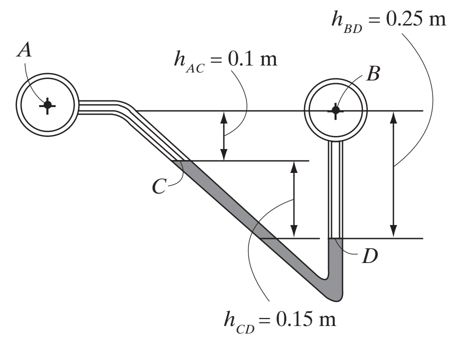
Referring to Fig. *a*, the manometer rule gives

$$p_A + \rho_w g h_{AC} + \rho_{Hg} g h_{CD} - \rho_w g h_{DB} = p_B$$

$$p_A + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) + 13.55(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.15\ \text{m})$$

$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.250\ \text{m}) = p_B$$

$$p_B - p_A = 18.47(10^3)\ \text{Pa} = 18.5\ \text{kPa} \qquad \textbf{Ans.}$$

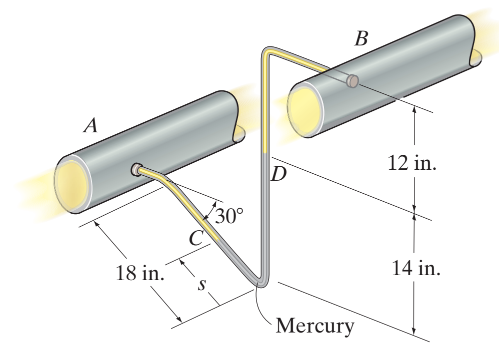


(a)

Ans:
18.5 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–58. Trichlorethylene, flowing through both pipes, is to be added to jet fuel produced in a refinery. A careful monitoring of pressure is required through the use of the inclined-tube manometer. If the pressure at A is 30 psi and the pressure at B is 25 psi, determine the position s that defines the level of mercury in the inclined-tube manometer. Take $S_{Hg} = 13.55$ and $S_t = 1.466$. Neglect the diameter of the pipes.



SOLUTION

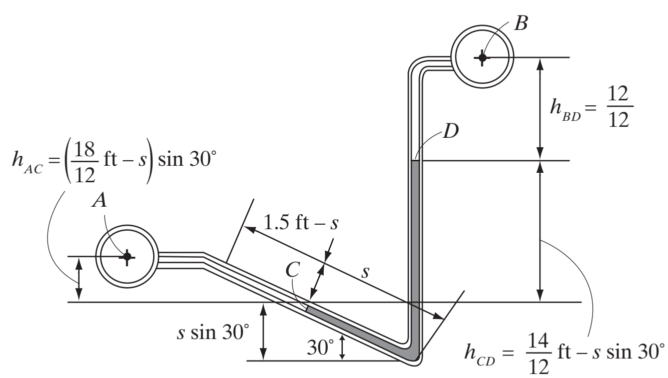
Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_t h_{AC} - \gamma_{Hg} h_{CD} - \gamma_t h_{BD} = p_B$$

$$\left(\frac{30\ \text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + 1.466\left(62.4\ \text{lb/ft}^3\right)\left(\frac{18}{12}\ \text{ft} - s\right)\sin 30^\circ - 13.55\left(62.4\ \text{lb/ft}^3\right)\left(\frac{14}{12}\ \text{ft} - s\sin 30^\circ\right)$$

$$1.466\left(62.4\ \text{lb/ft}^3\right)\left(\frac{12}{12}\ \text{ft}\right) = 25\frac{\text{lb}}{\text{in}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$s = 0.7674\ \text{ft}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right) = 9.208\ \text{in} = 9.21\ \text{in.}$$ **Ans.**

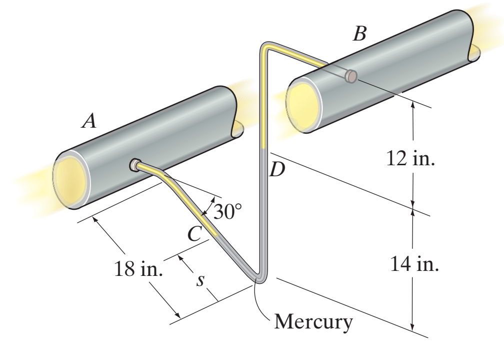


(a)

Ans:
9.21 in.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–59. Trichlorethylene, flowing through both pipes, is to be added to jet fuel produced in a refinery. A careful monitoring of pressure is required through the use of the inclined-tube manometer. If the pressure at A is 30 psi and $s = 7$ in., determine the pressure at B. Take $S_{Hg} = 13.55$ and $S_t = 1.466$. Neglect the diameter of the pipes.



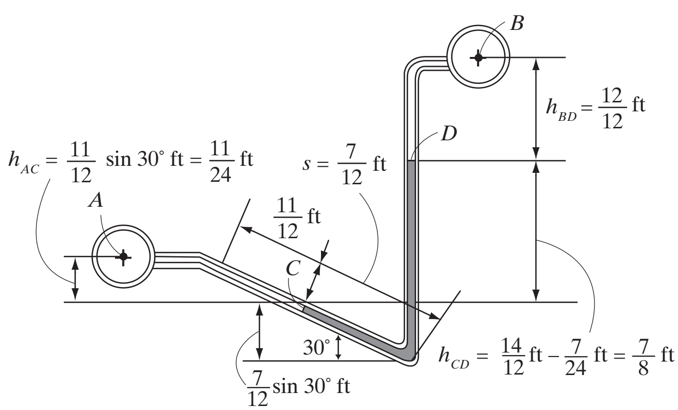
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_t h_{AC} - \gamma_{Hg} h_{CD} - \gamma_t h_{BD} = p_B$$

$$\frac{30\ \text{lb}}{\text{in.}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + 1.466(62.4\ \text{lb/ft}^3)\left(\frac{11}{24}\ \text{ft}\right) - 13.55(62.4\ \text{lb/ft}^3)\left(\frac{7}{8}\ \text{ft}\right) - 1.466(62.4\ \text{lb/ft}^3)\left(\frac{12}{12}\ \text{ft}\right) = p_B$$

$$p_B = 3530.62\ \text{lb/ft}^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 24.52\ \text{psi} = 24.5\ \text{psi} \qquad \textbf{Ans.}$$

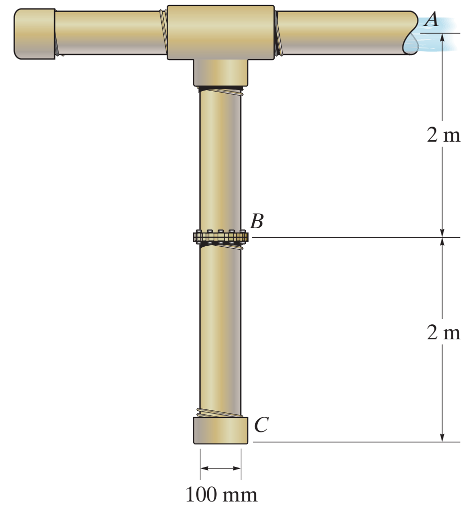


(a)

Ans:
24.5 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–60.** The vertical pipe segment has an inner diameter of 100 mm and is capped at its end and suspended from the horizontal pipe as shown. If it is filled with water and the pressure at A is 80 kPa, determine the resultant force that must be resisted by the bolts at B in order to hold the flanges together. Neglect the weight of the pipe but not the water within it.



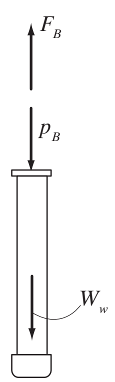
SOLUTION

The forces acting on segment BC of the pipe are indicated on its free-body diagram, Fig. *a*. Here, $\mathbf{F}_B$ is the force that must be resisted by the bolt, W_w is the weight of the water in segment BC of the pipe, and $\mathbf{P}_B$ is the resultant force of pressure acting on the cross section at B.

$$+\uparrow \Sigma F_y = 0; \qquad F_B - W_w - p_B A_B = 0$$

$$F_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m})(\pi)(0.05\ \text{m})^2 + \left[80\left(10^3\right)\text{N/m}^2 + 1000\ \text{kg/m}^3\left(9.81\ \text{m/s}^2\right)(2\ \text{m})\right]\pi(0.05\ \text{m})^2$$

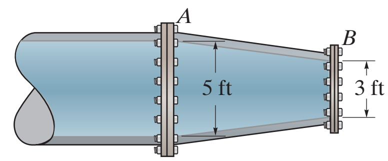
$$= 937\ \text{N} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–61. Nitrogen air in the chamber is at a pressure of 60 psi. Determine the total force the bolts at joints A and B must resist to maintain the pressure. There is a cover plate at B having a diameter of 3 ft.



SOLUTION

The force that must be resisted by the bolts at A and B can be obtained by considering the free-body diagrams in Figs. *a* and *b*, respectively. For the bolts at B, Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad p_B A_B - F_B = 0$$

$$F_B = p_B A_B = \left(60\ \text{lb/in}^2\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 \left[(\pi)(1.5\ \text{ft})^2\right]$$

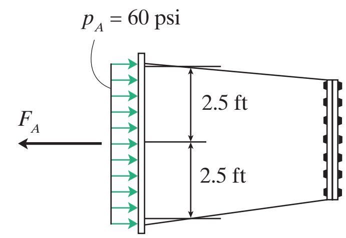
$$= 61073\ \text{lb} = 61.1\ \text{kip} \qquad \textbf{Ans.}$$

For the bolts at A, Fig. *a*,

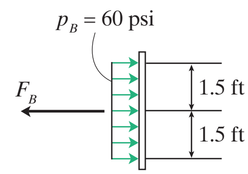
$$\xrightarrow{+} \Sigma F_x = 0; \qquad p_A A_A - F_A = 0$$

$$F_A = p_A A_A = \left(60\ \text{lb/in}^2\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 \left[(\pi)(2.5\ \text{ft})^2\right]$$

$$= 169646\ \text{lb} = 170\ \text{kip} \qquad \textbf{Ans.}$$



(a)

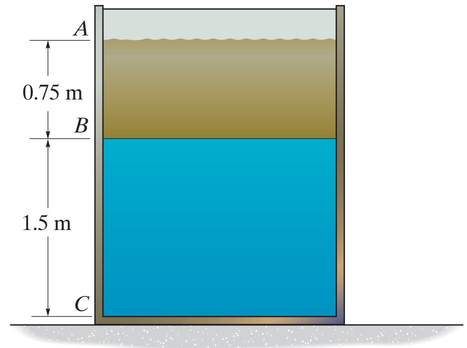


(b)

Ans:
$F_B = 61.1$ kip, $F_A = 170$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–62. The storage tank contains oil and water acting at the depths shown. Determine the resultant force that both of these liquids exert on the side ABC of the tank if the side has a width of $b = 1.25$ m. Also, determine the location of this resultant, measured from the top of the tank. Take $\rho_o = 900\ \text{kg/m}^3$.



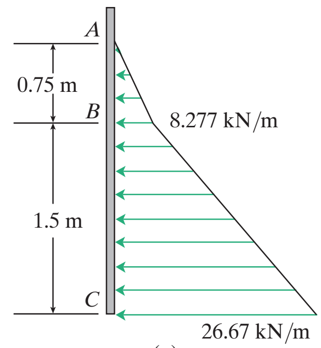
SOLUTION

Loading. Since the side of the tank has a constant width, then the intensities of the distributed loading at B and C, Fig. 2–28b, are

$$w_B = \rho_o g h_{AB} b = \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})(1.25\ \text{m}) = 8.277\ \text{kN/m}$$

$$w_C = w_B + \rho_w g h_{BC} b = 8.277\ \text{kN/m} + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.5\ \text{m})(1.25\ \text{m})$$

$$= 26.77\ \text{kN/m}$$



(a)

Resultant Force. The resultant force can be determined by adding the shaded triangular and rectangular areas in Fig. 2–28c. The resultant force is therefore

$$F_R = F_1 + F_2 + F_3$$

$$= \frac{1}{2}(0.75\ \text{m})(8.277\ \text{kN/m}) + (1.5\ \text{m})(8.277\ \text{kN/m}) + \frac{1}{2}(1.5\ \text{m})(18.39\ \text{kN/m})$$

$$= 3.104\ \text{kN} + 12.42\ \text{kN} + 13.80\ \text{kN} = 29.32\ \text{kN} = 29.3\ \text{kN} \qquad \textbf{Ans.}$$

As shown, each of these three parallel resultants acts through the centroid of its respective area.

$$y_1 = \frac{2}{3}(0.75\ \text{m}) = 0.5\ \text{m}$$

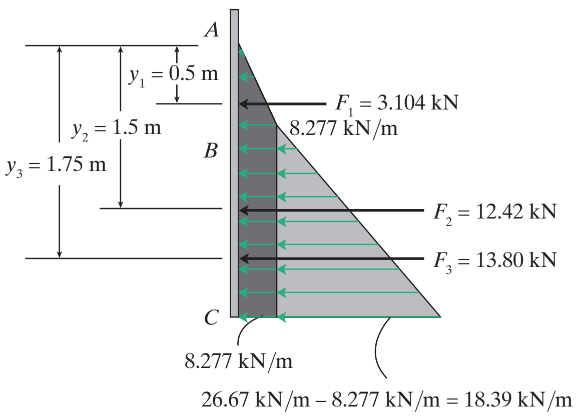
$$y_2 = 0.75\ \text{m} + \frac{1}{2}(1.5\ \text{m}) = 1.5\ \text{m}$$

$$y_3 = 0.75\ \text{m} + \frac{2}{3}(1.5\ \text{m}) = 1.75\ \text{m}$$

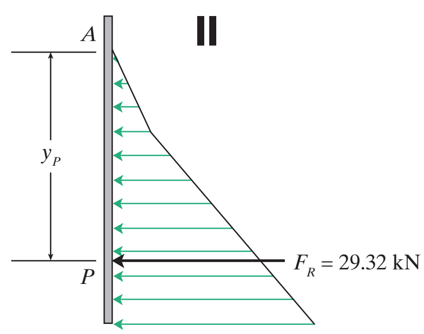
The location of the resultant force is determined by equating the moment of the resultant above A, Fig. 2–28d, to the moments of the component forces about A, Fig. 2–28c. We have,

$$\bar{y}_P F_R = \Sigma y F; \qquad \bar{y}_P(29.32\ \text{kN}) = (0.5\ \text{m})(3.104\ \text{kN}) + (1.5\ \text{m})(12.42\ \text{kN}) + (1.75\ \text{m})(13.80\ \text{kN})$$

$$\bar{y}_P = 1.51\ \text{m} \qquad \textbf{Ans.}$$



(b)

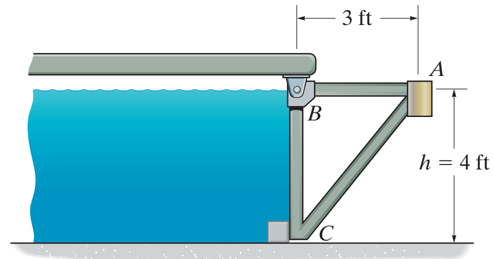


(c)

Ans:
$F_R = 29.3\ \text{kN}$, $\bar{y}_P = 1.51\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–63. Determine the weight of block A if the rectangular gate begins to open when the water level reaches the top of the channel, $h = 4$ ft. The gate has a width of 2 ft. There is a smooth stop block at C.



SOLUTION

Since the gate has a constant width of $b = 2$ ft, the intensity of the distributed load at C can be computed from

$$w_C = \gamma_w h_C b = \left(62.4\ \text{lb/ft}^3\right)(4\ \text{ft})(2\ \text{ft}) = 499.2\ \text{lb/ft}$$

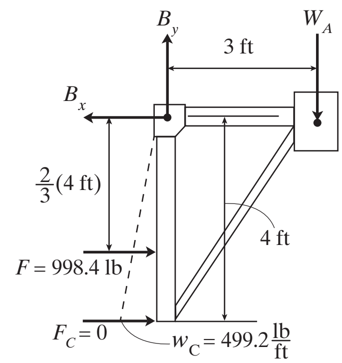
The resultant triangular distributed load is shown on the free-body diagram of the gate, Fig. *a*, and the resultant force of this load is

$$F = \frac{1}{2}w_C h_C = \frac{1}{2}(499.2\ \text{lb/ft})(4\ \text{ft}) = 998.4\ \text{lb}$$

Referring to the free-body diagram of the gate, Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 998.4\ \text{lb}\left[\frac{2}{3}(4\ \text{ft})\right] - W_A(3\ \text{ft}) = 0$$

$$W_A = 887.47\ \text{lb} = 887\ \text{lb} \qquad \textbf{Ans.}$$

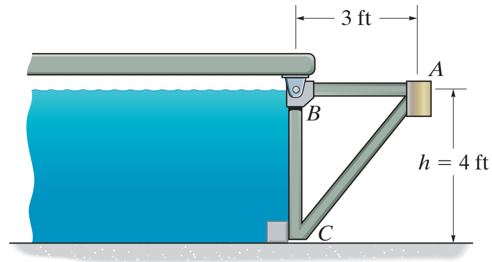


(a)

Ans:
887 lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–64.** Determine the weight of block A so that the 2 ft-radius circular gate BC begins to open when the water level reaches the top of the channel, $h = 4$ ft. There is a smooth stop block at C.



SOLUTION

Since the gate is circular in shape, it is convenient to compute the resultant force as follows.

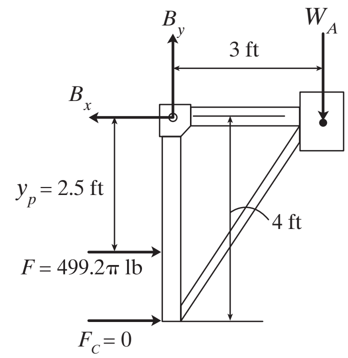
$$F_R = \gamma_w \bar{h} A$$

$$F = \left(62.4\ \text{lb/ft}^3\right)(2\ \text{ft})(\pi)(2\ \text{ft})^2 = 499.2\pi\ \text{lb}$$

The location of the center of pressure can be determined from

$$y_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

$$= \frac{\left(\dfrac{\pi(2\ \text{ft})^4}{4}\right)}{(2\ \text{ft})(\pi)(2\ \text{ft})^2} + 2\ \text{ft} = 2.50\ \text{ft}$$



(a)

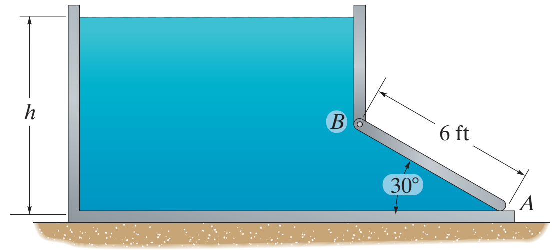
Referring to the free-body diagram of the gate, Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0;\qquad 499.2\pi\ \text{lb}(2.5\ \text{ft}) - W_A\,(3\ \text{ft}) = 0$

$W_A = 1306.90\ \text{lb} = 1.31\ \text{kip}$ **Ans.**

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–65. The uniform rectangular relief gate AB has a weight of 8000 lb and a width of 4 ft. Determine the minimum depth h of water within the canal needed to open it. The gate is pinned at B and rests on a rubber seal at A.



SOLUTION

Here $h_B = h - 6\sin 30° = (h - 3)$ ft and $h_A = h$. Thus, the intensities of the distributed load at B and A are

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(h - 3\ \text{ft})(4\ \text{ft}) = (249.6h - 748.8)\ \text{lb/ft}$$

$$w_A = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(h)(4\ \text{ft}) = (249.6h)\ \text{lb/ft}.$$

Thus,

$$\left(F_p\right)_1 = \left[(249.6h - 748.8\ \text{lb/ft})\right](6\ \text{ft}) = (1497.6h - 4492.8)\ \text{lb}$$

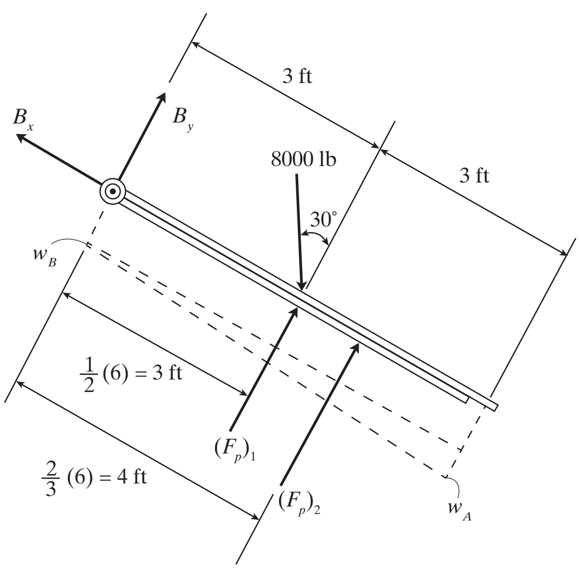
$$\left(F_p\right)_2 = \frac{1}{2}\left[(249.6h\ \text{lb/ft}) - (249.6h - 748.8\ \text{lb/ft})\right](6\ \text{ft}) = 2246.4\ \text{lb}$$

If it is required that the gate is about to open, then the normal reaction at A is equal to zero. Write the moment equation of equilibrium about B, referring to Fig. a,

$$\circlearrowleft + \Sigma M_B = 0;\quad \left[(1497.6h - 4492.8\ \text{lb})\right](3\ \text{ft}) + (2246.4\ \text{lb})(4\ \text{ft})$$

$$-(8000\ \text{lb})\cos 30°(3\ \text{ft}) = 0$$

$$h = 5.626\ \text{ft} = 5.63\ \text{ft}$$ **Ans.**

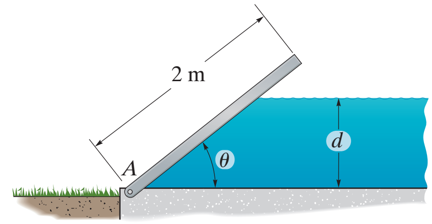


(a)

Ans:
5.63 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–66. The uniform swamp gate has a mass of 4 Mg and a width of 1.5 m. Determine the angle θ for equilibrium if the water rises to a depth of $d = 1.5$ m.



SOLUTION

Since the gate has a constant width of $b = 1.5$ m, the intensity of the distributed load at A can be computed from

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.5\ \text{m})(1.5\ \text{m})$$
$$= 22.07(10^3)\ \text{N/m}$$

The resulting triangular distributed load is shown on the free-body diagram of the gate, Fig. *a*.

$$F = \frac{1}{2}w_A L = \frac{1}{2}\left[22.07(10^3)\ \text{N/m}\right]\left(\frac{1.5\ \text{m}}{\sin\theta}\right) = \frac{16.554(10^3)}{\sin\theta}$$

Referring to the free-body diagram of the gate, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[\frac{16.554(10^3)}{\sin\theta}\right]\left[\frac{1}{3}\left(\frac{1.5\ \text{m}}{\sin\theta}\right)\right] - \left[4000(9.81)\ \text{N}\right]\cos\theta(1\ \text{m}) = 0$$

$$\sin^2\theta\cos\theta = 0.2109$$

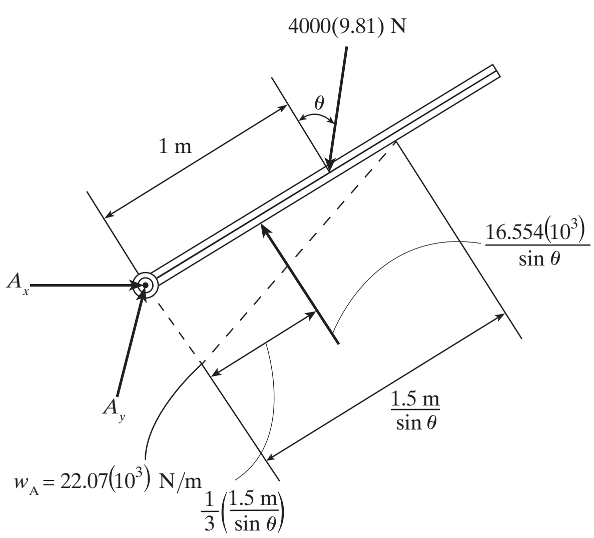
Solving numerically,

$$\theta = 29.49° \text{ or } 77.18°$$

Since $\dfrac{1.5\ \text{m}}{\sin 77.18°} = 1.54\ \text{m} < 2\ \text{m}$ and $\dfrac{1.5\ \text{m}}{\sin 29.49°} = 3.05\ \text{m} > 2\ \text{m}$ only one solution is valid.

$$\theta = 77.2° \qquad \textbf{Ans.}$$

Note: This solution represents an *unstable* equilibrium.

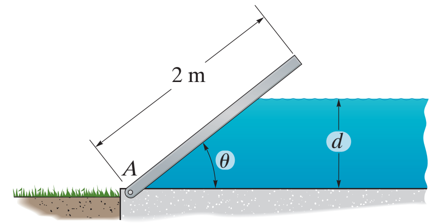


(a)

Ans:
77.2°

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–67. The uniform swamp gate has a mass of 3 Mg and a width of 1.5 m. Determine the depth of the water d if the gate is held in equilibrium at an angle of $\theta = 60°$.



SOLUTION

Since the gate has a constant width of $b = 1.5$ m, the intensity of the distributed load at A can be computed from

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(d)(1.5\ \text{m})$$

$$= 14\,715d\ \text{N/m}$$

The resulting triangular distributed load is shown on the free-body diagram of the gate, Fig. *a*.

$$F = \frac{1}{2}w_A L = \frac{1}{2}(14\,715d)\left(\frac{d}{\sin\ 60°}\right) = \frac{7357.5}{\sin\ 60°}d^2$$

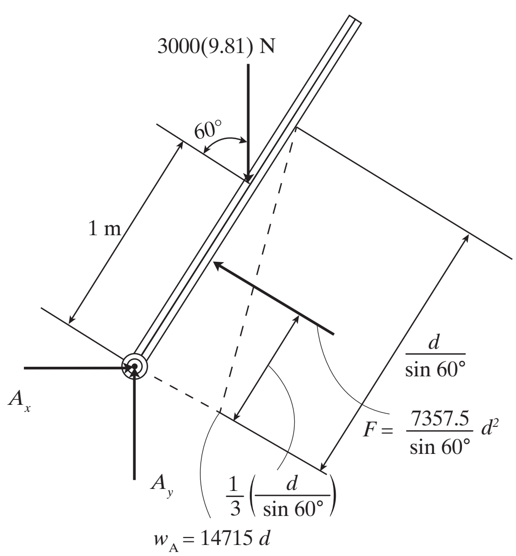
Referring to the free-body diagram of the gate, Fig. *a*,

$$\zeta + \Sigma M_A = 0; \quad \left(\frac{7357.5}{\sin 60°}d^2\right)\left[\frac{1}{3}\left(\frac{d}{\sin 60°}\right)\right] - \left[3000(9.81)\ \text{N}\right]\cos 60°(1\ \text{m}) = 0$$

$$d = 1.6510\ \text{m} = 1.65\ \text{m} \qquad \textbf{Ans.}$$

Since $\dfrac{1.6510\ \text{m}}{\sin\ 60°} = 1.906\ \text{m} < 2\ \text{m}$, this result is valid.

Note: This solution represents an unstable equilibrium. The gate is "held" in place by small external stabilizing forces.

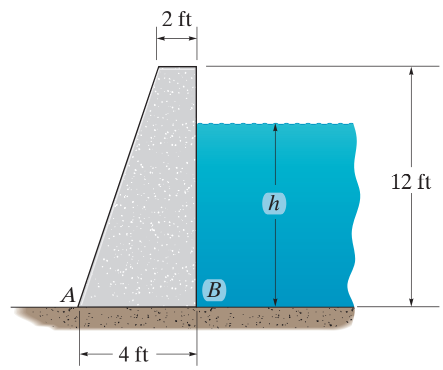


(a)

Ans:
1.65 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–68.** Determine the critical height h of the water level before the concrete gravity dam starts to tip over due to water pressure acting on its face. The specific weight of concrete is $\gamma_c = 150\ \text{lb/ft}^3$. *Hint:* Work the problem using a 1-ft width of the dam.



SOLUTION

We will consider the dam as having a width of $b = 1$ ft. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h b = (62.4\ \text{lb/ft}^3)(h)(1\ \text{ft}) = 62.4h\ \text{lb/ft}$$

The resulting triangular distributed load is shown on the free-body diagram of the dam, Fig. *a*.

$$F = \frac{1}{2}w_B h = \frac{1}{2}(62.4h)h = 31.2h^2$$

It is convenient to subdivide the dam into two parts. The weight of each part is

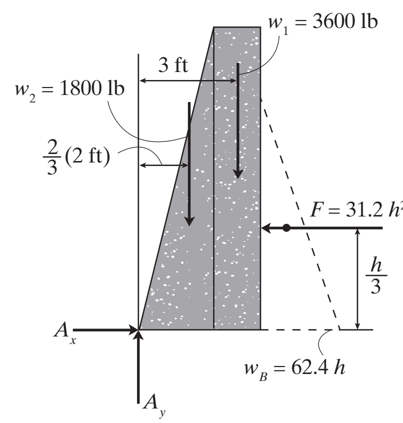
$$W_1 = \gamma_C \mathcal{V}_1 = (150\ \text{lb/ft}^3)\left[2\ \text{ft}(12\ \text{ft})(1\ \text{ft})\right] = 3600\ \text{lb}$$

$$W_2 = \gamma_C \mathcal{V}_2 = (150\ \text{lb/ft}^3)\left[\frac{1}{2}(2\ \text{ft})(12\ \text{ft})(1\ \text{ft})\right] = 1800\ \text{lb}$$

The dam will overturn about point A. Referring to the free-body diagram of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad 31.2h^2\left(\frac{h}{3}\right) - (3600\ \text{lb})(3\ \text{ft}) - (1800\ \text{lb})\left[\frac{2}{3}(2\ \text{ft})\right] = 0$$

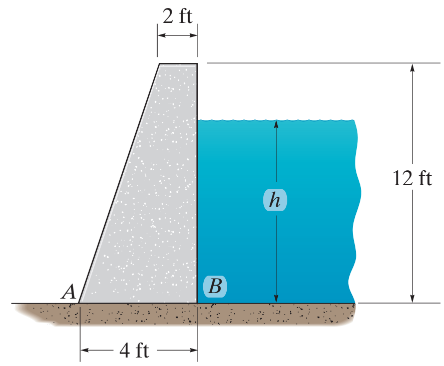
$$h = 10.83\ \text{ft} = 10.8\ \text{ft} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–69. Determine the critical height h of the water level before the concrete gravity dam starts to tip over due to water pressure acting on its face. Assume water also seeps under the base of the dam. The specific weight of concrete is $\gamma_c = 150 \text{ lb/ft}^3$. *Hint:* Work the problem using a 1-ft width of the dam.



SOLUTION

We will consider the dam having a width of $b = 1$ ft. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h_b b = (62.4 \text{ lb/ft}^3)(h)(1) = 62.4\, h \text{ lb/ft}$$

The resultant forces of the triangular distributed load and uniform distributed load due the pressure of the seepage water shown on the FBD of the dam, Fig. *a*, are

$$F_1 = \frac{1}{2} w_B h = \frac{1}{2}(62.4\, h)\, h = 31.2\, h^2$$

$$F_2 = w_B L_B = 62.4\, h(4 \text{ ft}) = 249.6\, h$$

It is convenient to subdivide the dam into two parts. The weight of each part is

$$w_1 = \gamma_C \mathcal{V}_1 = (150 \text{ lb/ft}^3)\left[(2 \text{ ft})(12 \text{ ft})(1 \text{ ft})\right] = 3600 \text{ lb}$$

$$w_2 = \gamma_C \mathcal{V}_2 = (150 \text{ lb/ft}^3)\left[\frac{1}{2}(2 \text{ ft})(12 \text{ ft})(1 \text{ ft})\right] = 1800 \text{ lb}$$

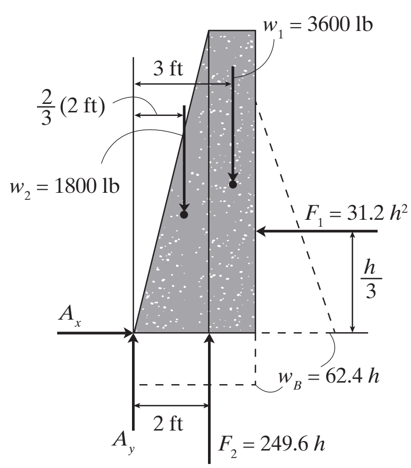
The dam will overturn about point A. Referring to the FBD of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 31.2\, h^2\left(\frac{h}{3}\right) + 249.6\, h(2 \text{ ft}) - (3600 \text{ lb})(3 \text{ ft}) - (1800 \text{ lb})\left[\frac{2}{3}(2 \text{ ft})\right] = 0$$

$$10.4\, h^3 + 499.2\, h - 13200 = 0$$

Solve numerically,

$$h = 9.3598 \text{ ft} = 9.36 \text{ ft} \qquad \textbf{Ans.}$$

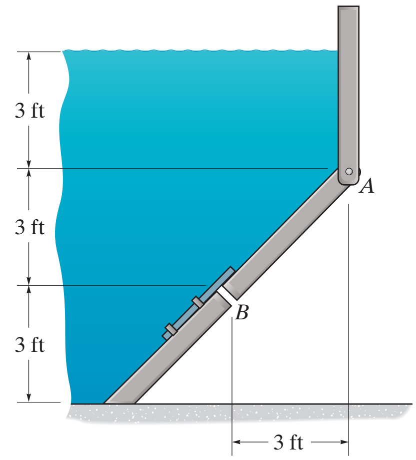


(a)

Ans:
9.36 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–70. The gate is 2 ft wide and is pinned at A and held in place by a smooth latch bolt at B that exerts a force normal to the gate. Determine this force caused by the water and the resultant force on the pin for equilibrium.



SOLUTION

Since the gate has a width of $b = 2$ ft, the intensities of the distributed loads at A and B can be computed from

$$w_A = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(3\ \text{ft})(2\ \text{ft}) = 374.4\ \text{lb/ft}$$

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(2\ \text{ft}) = 748.8\ \text{lb/ft}$$

The resulting trapezoidal distributed load is shown on the free-body diagram of the gate, Fig. a. This load can be subdivided into two parts. The resultant force of each part is

$$F_1 = w_A L_{AB} = (374.4\ \text{lb/ft})\left(3\sqrt{2}\ \text{ft}\right) = 1123.2\sqrt{2}\ \text{lb}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{AB} = \frac{1}{2}(748.8\ \text{lb/ft} - 374.4\ \text{lb/ft})\left(3\sqrt{2}\ \text{ft}\right) = 561.6\sqrt{2}\ \text{lb}$$

Considering the free-body diagram of the gate, Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 1123.2\sqrt{2}\ \text{lb}\left(\frac{1}{2}3\sqrt{2}\ \text{ft}\right) + 561.6\sqrt{2}\ \text{lb}\left(\frac{2}{3}3\sqrt{2}\ \text{ft}\right) - N_B\left(3\sqrt{2}\ \text{ft}\right) = 0$$

$$N_B = 1323.7\ \text{lb} = 1.32\ \text{kip} \qquad \textbf{Ans.}$$

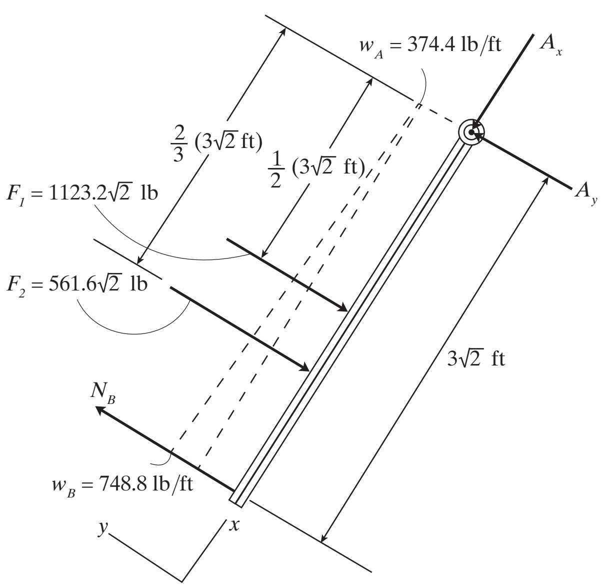
$$\Sigma F_x = 0; \qquad A_x = 0$$

$$\nwarrow + \Sigma F_y = 0; \qquad 1323.7\ \text{lb} - 1123.2\sqrt{2}\ \text{lb} - 561.6\sqrt{2}\ \text{lb} + A_y = 0$$

$$A_y = 1058.96\ \text{lb} = 1.059\ \text{kip}$$

Thus,

$$F_A = \sqrt{(0)^2 + (1.059\ \text{kip})^2} = 1.06\ \text{kip} \qquad \textbf{Ans.}$$

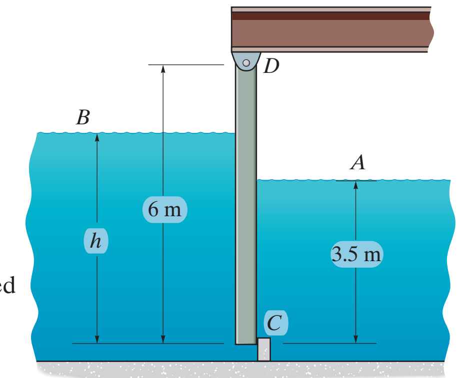


(a)

Ans:
$N_B = 1.32$ kip
$F_A = 1.06$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–71. The tide gate opens automatically when the tide water at B subsides, allowing the marsh at A to drain. For the water level $h = 4$ m, determine the horizontal reaction at the smooth stop C. The gate has a width of 2 m. At what height h will the gate be on the verge of opening?



SOLUTION

Since the gate has a constant width of $b = 2$ m, the intensities of the distributed load on the left and right sides of the gate at C are

$$(w_C)_L = \rho_w g h_{BC}(b) = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4 \text{ m})(2 \text{ m})$$
$$= 78.48(10^3) \text{ N/m}$$
$$(w_C)_R = \rho_w g h_{AC}(b) = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m})(2 \text{ m})$$
$$= 68.67(10^3) \text{ N/m}$$

The resultant triangular distributed load on the left and right sides of the gate is shown on its free-body diagram, Fig. *a*,

$$F_L = \frac{1}{2}(w_C)_L L_{BC} = \frac{1}{2}\left(78.48(10^3) \text{ N/m}\right)(4 \text{ m}) = 156.96(10^3) \text{ N}$$

$$F_R = \frac{1}{2}(w_C)_R L_{AC} = \frac{1}{2}\left(68.67(10^3) \text{ N/m}\right)(3.5 \text{ m}) = 120.17(10^3) \text{ N}$$

These results can also be obtained as follows

$$F_L = \gamma \bar{h}_L A_L = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})[(4 \text{ m})(2 \text{ m})] = 156.96(10^3) \text{ N}$$
$$F_R = \gamma \bar{h}_R A_R = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.75 \text{ m})[3.5 \text{ m}(2 \text{ m})] = 120.17(10^3) \text{ N}$$

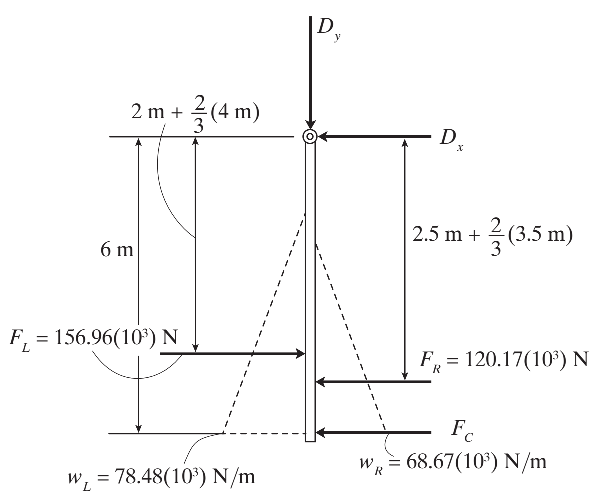
Referring to the free-body diagram of the gate in Fig. *a,*

$$\circlearrowleft + \Sigma M_D = 0; \quad \left[156.96(10^3) \text{ N}\right]\left[2 \text{ m} + \frac{2}{3}(4 \text{ m})\right] - \left[120.17(10^3) \text{ N}\right]\left[2.5 \text{ m} + \frac{2}{3}(3.5 \text{ m})\right] - F_C(6 \text{ m}) = 0$$

$$F_C = 25.27(10^3) \text{ N} = 25.3 \text{ kN} \qquad \textbf{Ans.}$$

When $h = 3.5$ m, the water levels are equal. Since $F_C = 0$, the gate will open.

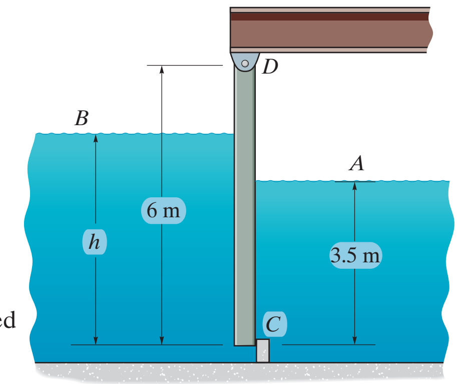
$$h = 3.5 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$F_C = 25.3$ kN
$h = 3.5$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–72.** The tide opens automatically when the tide water at B subsides, allowing the marsh at A to drain. Determine the horizontal reaction at the smooth stop C as a function of the depth h of the water level. Starting at $h = 6$ m, plot values of h for each increment of 0.5 m until the gate begins to open. The gate has a width of 2 m.



SOLUTION

Since the gate has a constant width of $b = 2$ m, the intensities of the distributed loads on the left and right sides of the gate at C are

$$(W_C)_L = \rho_w g h_{BC} b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)(2\ \text{m}) = 19.62(10^3)h$$

$$(W_C)_R = \rho_w g h_{AC} b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m})(2\ \text{m}) = 68.67(10^3)\ \text{N/m}$$

The resultant forces of the triangular distributed loads on the left and right sides of the gate shown on its FBD, Fig. a, are

$$F_L = \frac{1}{2}(w_C)_L h_{BC} = \frac{1}{2}\left[19.62(10^3)h\right]h = 9.81(10^3)h^2$$

$$F_R = \frac{1}{2}(w_C)_R h_{AC} = \frac{1}{2}\left[68.67(10^3)\ \text{N/m}\right](3.5\ \text{m}) = 120.17(10^3)\ \text{N}$$

Consider the moment equilibrium about D by referring to the FBD of the gate, Fig. a,

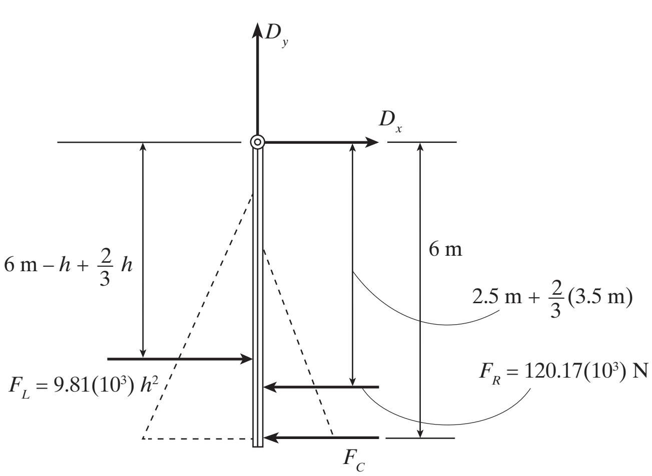
$$\circlearrowleft + \Sigma M_D = 0; \qquad \left[9.81(10^3)h^2\right]\left(6\ \text{m} - h + \frac{2}{3}h\right) - 120.17(10^3)\left[2.5\ \text{m} + \frac{2}{3}(3.5\ \text{m})\right] - F_C(6\ \text{m}) = 0$$

$$58.86(10^3)h^2 - 3.27(10^3)h^3 - 580.83(10^3) - 6F_C = 0$$

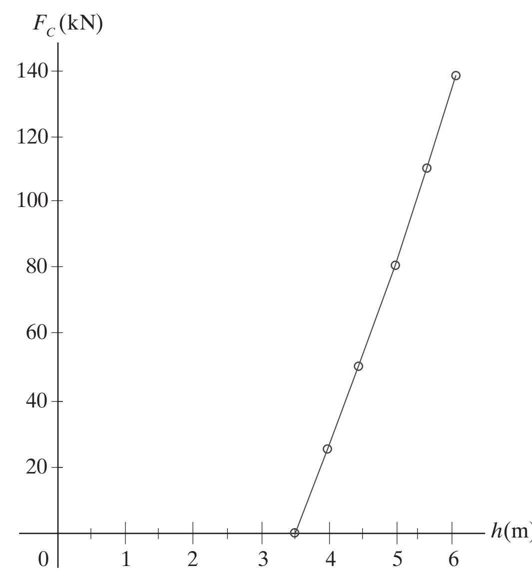
$$F_C = (9.81\,h^2 - 0.545\,h^3 - 96.806)(10^3)\ \text{N}$$

$$F_C = (9.81\,h^2 - 0.545\,h^3 - 96.8)\ \text{kN where } h \text{ is in meters}$$ **Ans.**

The gate will be on the verge of opening when the water level on both sides of the gate are equal, that is when $h = 3.5$ m. The plot of F_C vs h is shown in Fig. b.



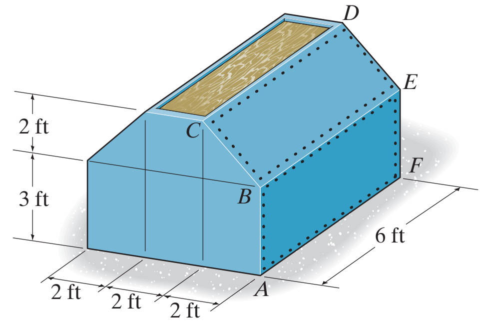
(a)



h(m)	3.5	4	4.5	5.0	5.5	6.0
F_C(kN)	0	25.3	52.2	80.3	109.3	138.6

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–73. The bin is used to store carbon tetrachloride, a cleaning agent for auto parts. If it is filled to the top, determine the magnitude of the resultant force this liquid exerts on each of the two side plates, $AFEB$ and $BEDC$, and the location of the center of pressure on each plate, measured from BE. Take $\gamma_{ct} = 99.6\ \text{lb/ft}^3$.



SOLUTION

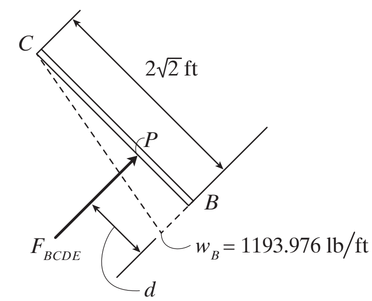
Since the side plate has a width of $b = 6$ ft, the intensities of the distributed load can be computed from

$$w_B = \rho g h_B b = (3.09\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(2\ \text{ft})(6\ \text{ft}) = 1193.976\ \text{lb/ft}$$

$$w_A = \rho g h_A b = (3.09\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(5\ \text{ft})(6\ \text{ft}) = 2984.94\ \text{lb/ft}$$

The resulting distributed load on plates $BCDE$ and $ABEF$ are shown in Figs. *a* and *b*, respectively. For plate $BCDE$,

$$F_{BCDE} = \frac{1}{2}(w_B)L_{BC} = \frac{1}{2}(1193.976\ \text{lb/ft})\left(2\sqrt{2}\ \text{ft}\right) = 1688.54\ \text{lb} = 1.69\ \text{kip} \quad \textbf{Ans.}$$



(a)

And the center of pressure of this plate from BE is

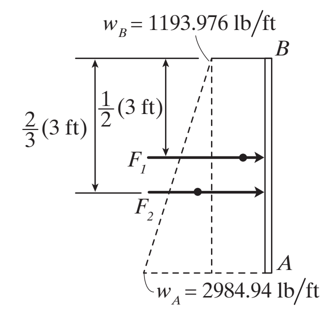
$$d = \frac{1}{3}\left(2\sqrt{2}\ \text{ft}\right) = 0.943\ \text{ft} \quad \textbf{Ans.}$$

For $ABEF$,

$$F_1 = w_B L_{AB} = (1193.976\ \text{lb/ft})(3\ \text{ft}) = 3581.93\ \text{lb}$$

$$F_2 = \frac{1}{2}(w_A - w_B)L_{AB} = \frac{1}{2}(2984.94\ \text{lb/ft} - 1193.976\ \text{lb/ft})(3\ \text{ft}) = 2686.45\ \text{lb}$$

$$F_{ABEF} = F_1 + F_2 = 3581.93\ \text{lb} + 2686.45\ \text{lb} = 6268.37\ \text{lb} = 6.27\ \text{kip} \quad \textbf{Ans.}$$

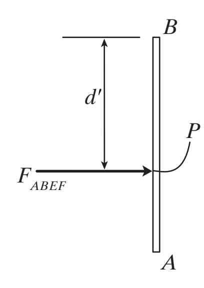


(b)

The location of the center of pressure measured from BE can be obtained by equating the sum of the moments of the forces in Figs. *b* and *c*.

$$\circlearrowleft + M_{R_B} = \Sigma M_B; \quad (6268.37\ \text{lb})d' = (3581.93\ \text{lb})\left[\frac{1}{2}(3\ \text{ft})\right] + (2686.45\ \text{lb}) + \left(\frac{2}{3}(3\ \text{ft})\right)$$

$$d' = 1.714\ \text{ft} = 1.71\ \text{ft} \quad \textbf{Ans.}$$

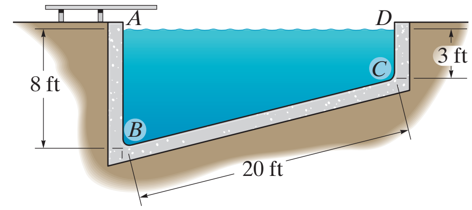


(c)

Ans:
$F_{BCDE} = 1.69$ kip, $d = 0.943$ ft
$F_{ABEF} = 6.27$ kip, $d' = 1.71$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–74. A swimming pool has a width of 12 ft and a side profile as shown. Determine the resultant force the pressure of the water exerts on walls *AB* and *DC*, and on the bottom *BC*.



SOLUTION I

Since the swimming pool has a constant width of $b = 12$ ft, the intensities of the distributed load at B and C can be computed from

$$w_B = \gamma h_{AB} b = \left(62.4 \text{ lb/ft}^3\right)(8\text{ft})(12\text{ft}) = 5990.4 \text{ lb/ft}$$

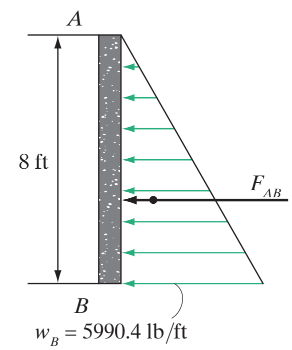
$$w_C = \gamma h_{DC} b = \left(62.4 \text{ lb/ft}^3\right)(3 \text{ ft})(12 \text{ ft}) = 2246.4 \text{ lb/ft}$$

Using these results, the distributed loads acting on walls *AB* and *CD* and bottom *BC* are shown in Figs. *a*, *b*, and *c*.

$$F_{AB} = \frac{1}{2} w_B h_{AB} = \frac{1}{2}(5990.4 \text{ lb/ft})(8 \text{ ft}) = 23\,962 \text{ lb} = 24.0 \text{ kip} \qquad \textbf{Ans.}$$

$$F_{DC} = \frac{1}{2} w_C h_{CD} = \frac{1}{2}(2246.4 \text{ lb/ft})(3 \text{ ft}) = 3369.6 \text{ lb} = 3.37 \text{ kip} \qquad \textbf{Ans.}$$

$$F_{BC} = \frac{1}{2}(w_B + w_C)L_{BC} = \frac{1}{2}\left[5990.4 \text{ lb/ft} + 2246.4\text{lb/ft}\right](20 \text{ ft})$$
$$= 82\,368 \text{ lb} = 82.4 \text{ kip} \qquad \textbf{Ans.}$$



(a)

SOLUTION II

The same result can also be obtained as follows. For wall *AB*,

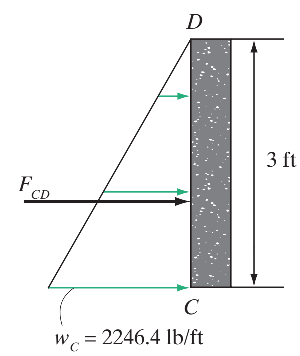
$$F_{AB} = \gamma \bar{h}_{AB} A_{AB} = \left(62.4 \text{ lb/ft}^3\right)(4 \text{ ft})\left[8 \text{ ft}(12 \text{ ft})\right] = 23\,962 \text{ lb} = 24.0 \text{ kip} \qquad \textbf{Ans.}$$

For wall *CD*,

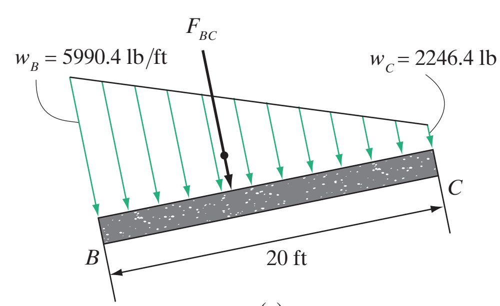
$$F_{CD} = \gamma \bar{h}_{CD} A_{CD} = \left(62.4 \text{ lb/ft}^3\right)(1.5 \text{ ft})\left[3 \text{ ft}(12 \text{ ft})\right] = 3369.6 \text{ lb} = 3.37 \text{ kip} \qquad \textbf{Ans.}$$

For floor *BC*,

$$F_{BC} = \gamma \bar{h}_{BC} A_{BC} = \left(62.4 \text{ lb/ft}^3\right)(5.5 \text{ ft})\left[20 \text{ ft}(12 \text{ ft})\right] = 82\,368 \text{ lb} = 82.4 \text{ kip} \qquad \textbf{Ans.}$$



(b)

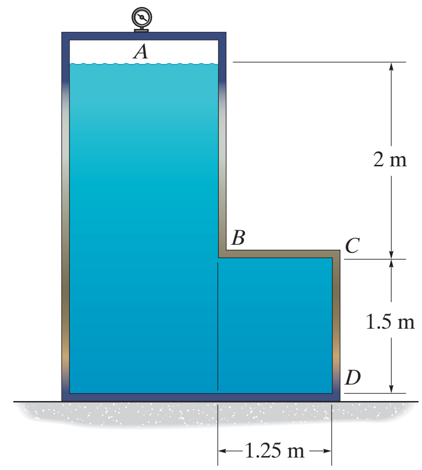


(c)

Ans:
$F_{AB} = 24.0$ kip
$F_{DC} = 3.37$ kip
$F_{BC} = 82.4$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–75. The pressure of the air at A within the closed tank is 200 kPa. Determine the resultant force acting on the plates BC and CD caused by the water. The tank has a width of 1.75 m.



SOLUTION

$$p_C = p_B = p_A + \rho g h_{AB}$$
$$= 200(10^3)\ \mathrm{N/m^2} + (1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(2\ \mathrm{m})$$
$$= 219.62(10^3)\ \mathrm{Pa}$$

$$p_D = p_A + \rho g h_{AD}$$
$$= 200(10^3)\ \mathrm{N/m^2} + (1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(3.5\ \mathrm{m})$$
$$= 234.335(10^3)\ \mathrm{Pa}$$

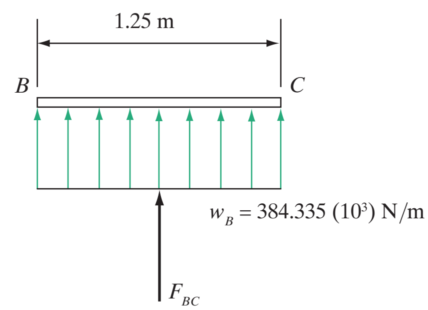
Since plates BC and CD have a constant width of $b = 1.75$ m, the intensities of the distributed load at points B (or C) and D are

$$w_C = w_B = p_B b = (219.62(10^3)\ \mathrm{N/m^2})(1.75\mathrm{m}) = 384.335(10^3)\ \mathrm{N/m}$$
$$w_D = p_D b = (234.335(10^3)\ \mathrm{N/m^2})(1.75\ \mathrm{m}) = 410.086(10^3)\ \mathrm{N/m}$$

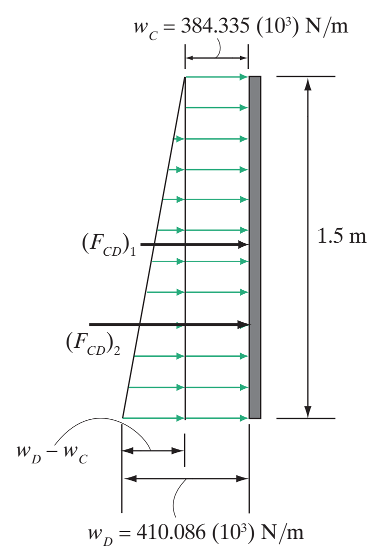
Using these results, the distributed loads acting on plates BC and CD are shown in Figs. *a* and *b*, respectively.

$$F_{BC} = w_B L_{BC} = \left[384.335(10^3)\ \frac{\mathrm{N}}{\mathrm{m}}\right](1.25\ \mathrm{m}) = 480.42(10^3)\ \mathrm{N} = 480\ \mathrm{kN} \qquad \textbf{Ans.}$$

$$F_{CD} = (F_{CD})_1 + (F_{CD})_2 = w_C L_{CD} + \frac{1}{2}(w_D - w_C)L_{CD}$$
$$= \left[384.335(10^3)\frac{\mathrm{N}}{\mathrm{m}}\right](1.5\ \mathrm{m}) + \frac{1}{2}\left[410.086(10^3)\frac{\mathrm{N}}{\mathrm{m}} - 384.335(10^3)\frac{\mathrm{N}}{\mathrm{m}}\right](1.5\ \mathrm{m})$$
$$= 595.82(10^3)\ \mathrm{N} = 596\ \mathrm{kN} \qquad \textbf{Ans.}$$



(a)

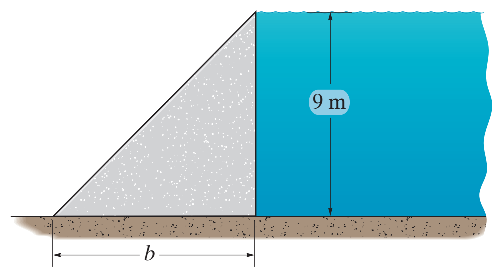


(b)

Ans:
$F_{BC} = 480$ kN, $F_{CD} = 596$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–76.** Determine the smallest base length b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. The density of concrete is $\rho_c = 2.4\ \text{Mg/m}^3$. *Hint:* Work the problem using a 1-m width of the dam.



SOLUTION

If we consider the dam as having a width of $b = 1$ m, the intensity of the distributed load at the base of the dam is

$$w_b = \rho g h(b) = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(9\ \text{m})(1\ \text{m})$$
$$= 88.29(10^3)\ \text{N/m}$$

The resultant force of the triangular distributed load shown on the free-body diagram of the dam, Fig. *a*. is

$$F = \frac{1}{2}w_b h = \frac{1}{2}\left[88.29(10^3)\ \text{N/m}\right](9\ \text{m}) = 397.305(10^3)\ \text{N}$$

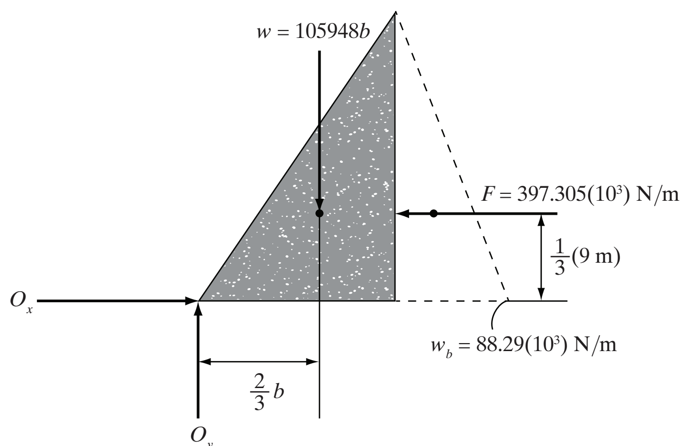
The weight of the dam is given by

$$W = \rho_C g \mathcal{V} = \left[2.4(10^3)\ \text{kg/m}^3\right](9.81\ \text{m/s}^2)\left[\frac{1}{2}(9\ \text{m})(1\ \text{m})b\right]$$
$$= 105\,948b$$

The dam will overturn about point O. Referring to the free-body diagram of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_0 = 0; \qquad \left[397.305(10^3)\ \text{N}\right]\left[\frac{1}{3}(9\ \text{m})\right] - 105\,948b\left(\frac{2}{3}b\right) = 0$$

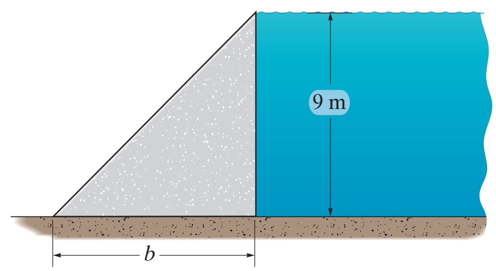
$$b = 4.108\ \text{m} = 4.11\ \text{m}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–77. Determine the smallest base length b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. Assume water also seeps under the base of the dam. The density of concrete is $\rho_c = 2.4\ \text{Mg/m}^3$. *Hint:* Work the problem using a 1-m width of the dam.



SOLUTION

If we consider the dam having a width of $b = 1$ m, the intensity of the distributed load at the base of the dam is

$$w_b = \rho g h b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(9\ \text{m})(1\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

The resultant forces of the triangular distributed load and the uniform distributed load due to the pressure of the seepage water shown on the FBD of the dam, Fig. *a* is

$$F_1 = \frac{1}{2}w_b h = \frac{1}{2}\left[88.29(10^3)\ \text{N/m}\right](9\ \text{m}) = 397.305(10^3)\ \text{N}$$

$$F_2 = w_b b = \left[88.29(10^3)\ \text{N/m}\right]b = 88.29(10^3)b$$

The weight of the dam is given by

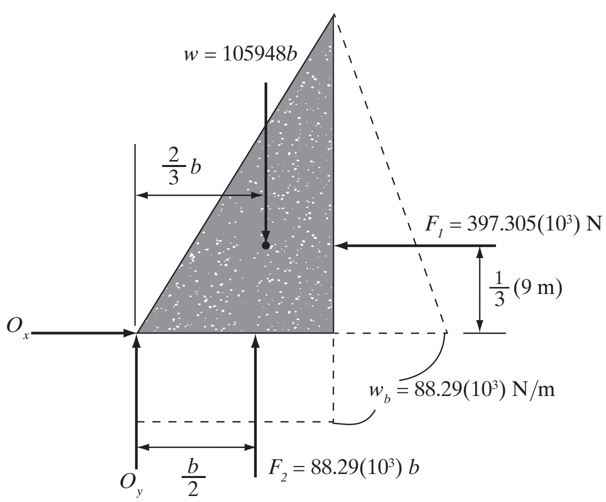
$$W = \rho_C g \forall = \left[2.4(10^3)\ \text{kg/m}^3\right](9.81\ \text{m/s}^2)\left[\frac{1}{2}(\text{b})(9\ \text{m})(1\ \text{m})\right]$$

$$= 105948b$$

The dam will overturn about point O. Referring to the FBD of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_0 = 0; \quad \left[397.305(10^3)\ \text{N}\right]\left[\frac{1}{3}(9\ \text{m})\right] + \left[88.29(10^3)b\right]\left(\frac{b}{2}\right) - 105948b\left(\frac{2}{3}b\right) = 0$$

$$b = 6.708\ \text{m} = 6.71\ \text{m} \qquad \textbf{Ans.}$$

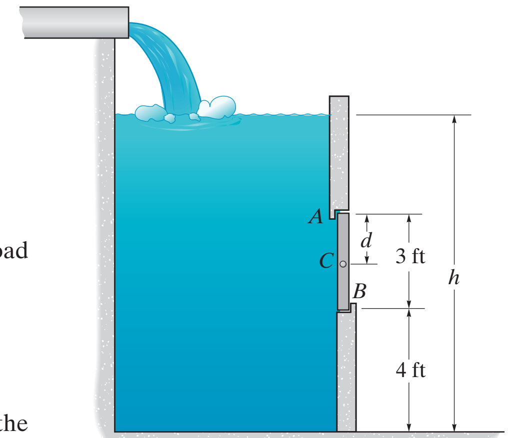


(a)

Ans:
6.71 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–78. Determine the placement d of the pin on the 2-ft-wide rectangular gate so that it begins to rotate clockwise (open) when waste water reaches a height $h = 10$ ft. What is the resultant force acting on the gate?



SOLUTION

Since the gate has a constant width of $b = 2$ ft, the intensity of the distributed load at A and B can be computed from

$$w_A = \gamma_w h_A b = \left(62.4\ \mathrm{lb/ft^3}\right)(3\ \mathrm{ft})(2\ \mathrm{ft}) = 374.4\ \mathrm{lb/ft}$$

$$w_B = \gamma_w h_B d = \left(62.4\ \mathrm{lb/ft^3}\right)(6\ \mathrm{ft})(2\ \mathrm{ft}) = 748.8\ \mathrm{lb/ft}$$

The resultant trapezoidal distributed load is shown on the free-body diagram of the gate, Fig. *a*. This load can be subdivided into two parts for which the resultant force of each part is

$$F_1 = w_A L_{AB} = 374.4\ \mathrm{lb/ft}(3\ \mathrm{ft}) = 1123.2\ \mathrm{lb}$$

$$F_2 = \frac{1}{2}\left(w_B - w_A\right)L_{AB} = \frac{1}{2}(748.8\ \mathrm{lb/ft} - 374.4\ \mathrm{lb/ft})(3\ \mathrm{ft}) = 561.6\ \mathrm{lb}$$

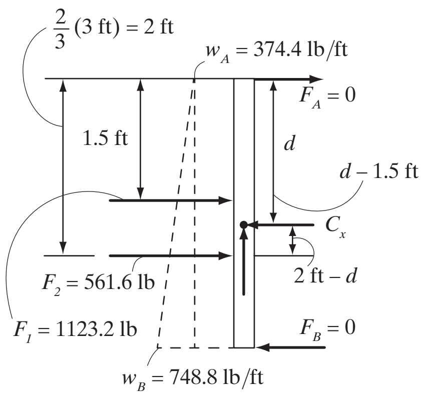
Thus, the resultant force is

$$F_R = F_1 + F_2 = 1123.2\ \mathrm{lb} + 561.6\ \mathrm{lb} = 1684.8\ \mathrm{lb} = 1.68\ \mathrm{kip} \qquad \textbf{Ans.}$$

When the gate is on the verge of opening, the normal force at A and B is zero as shown on the free-body diagram of the gate, Fig. *a*.

$$\circlearrowleft+\Sigma M_C = 0; \qquad (561.61\mathrm{b})(2\ \mathrm{ft} - d) - (1123.2\ \mathrm{lb})(d - 1.5\ \mathrm{ft}) = 0$$

$$d = 1.67\ \mathrm{ft} \qquad \textbf{Ans.}$$

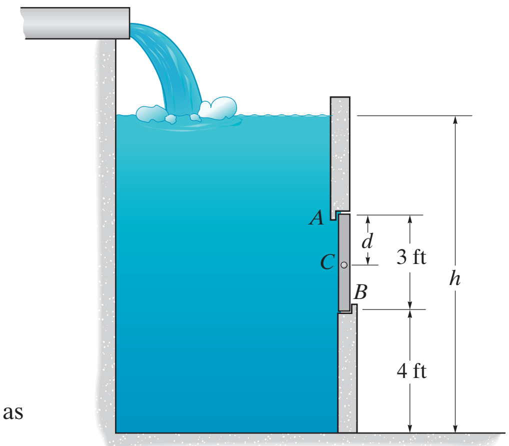


(a)

Ans:
$F_R = 1.68$ kip
$d = 1.67$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–79. Determine the placement d of the pin on the 3-ft-diameter circular gate so that it begins to rotate clockwise (open) when waste water reaches a height $h = 10$ ft. What is the resultant force acting on the gate? Use the formula method.



SOLUTION

Since the gate is circular in shape, it is convenient to compute the resultant force as follows.

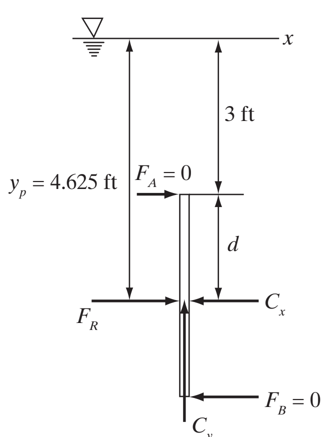
$$F_R = \gamma_w \bar{h} A = \left(62.4\ \text{lb/ft}^3\right)(10\ \text{ft} - 5.5\ \text{ft})(\pi)(1.5\ \text{ft})^2 = 1984.86\ \text{lb}$$
$$= 1.98\ \text{kip} \qquad \textbf{Ans.}$$

The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{\left(\dfrac{\pi(1.5\ \text{ft})^4}{4}\right)}{(10\ \text{ft} - 5.5\ \text{ft})(\pi)(1.5\ \text{ft})^2} + (10\ \text{ft} - 5.5\ \text{ft}) = 4.625\ \text{ft}$$

When the gate is on the verge of opening, the normal force at A and B is zero as shown on the free-body diagram of the gate, Fig. a. Summing the moments about point C requires that F_R acts through C. Thus,

$$d = y_p - 3\ \text{ft} = 4.625\ \text{ft} - 3\ \text{ft} = 1.625\ \text{ft} = 1.62\ \text{ft} \qquad \textbf{Ans.}$$

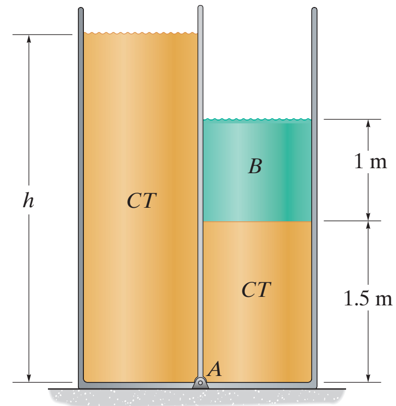


(a)

Ans:
$F_R = 1.98$ kip
$d = 1.62$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–80.** The container in a chemical plant contains carbon tetrachloride, $\rho_{ct} = 1593\ \text{kg/m}^3$, and benzene, $\rho_b = 875\ \text{kg/m}^3$, on the right side of the tank. Determine the height h of the carbon tetrachloride on the left side so that the separation plate, which is pinned at A, will remain vertical.



SOLUTION

Assume 1 m width. The intensities of the distributed load shown in Fig. *a* are,

$$w_2 = \rho_b g h_b b = (875\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})(1\ \text{m}) = 8.584(10^3)\ \text{N/m}$$

$$w_1 = w_2 + \rho_{CT} g (h_{CT})_R b = [8.584(10^3)\ \text{N/m}] + (1593\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.5\ \text{m})(1\ \text{m})$$
$$= 32.025(10^3)\ \text{N/m}$$

$$w_3 = \rho_{CT} g (h_{CT})_L b = (1593\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\ \text{h}\ (1\ \text{m})$$
$$= [15.63(10^3)h]\ \text{N/m}$$

Thus, the resultant forces of these distributed loads are

$$F_1 = \frac{1}{2}[32.025(10^3)\ \text{N/m} - 8.584(10^3)\ \text{N/m}](1.5\ \text{m}) = 17.58(10^3)\ \text{N}$$

$$F_2 = [8.584(10^3)\ \text{N/m}](1.5\ \text{m}) = 12.876(10^3)\ \text{N}$$

$$F_3 = \frac{1}{2}[8.584(10^3)\ \text{N/m}](1\ \text{m}) = 4.292(10^3)\ \text{N}$$

$$F_4 = \frac{1}{2}[15.63(10^3)h\ \text{N/m}](h) = [7.814(10^3)h^2]\ \text{N}$$

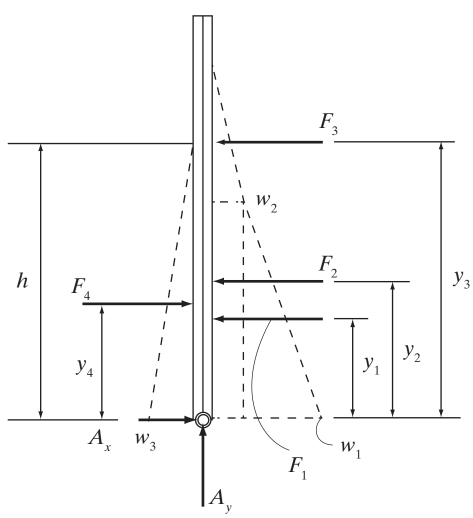
And they act at

$$y_1 = \frac{1.5\ \text{m}}{3} = 0.5\ \text{m} \quad y_2 = \frac{1.5\ \text{m}}{2} = 0.75\ \text{m} \quad y_3 = 1.5\ \text{m} + \frac{1\ \text{m}}{3} = 1.8333\ \text{m}$$
$$y_4 = \frac{h}{3}$$

For the plate to remain vertical,

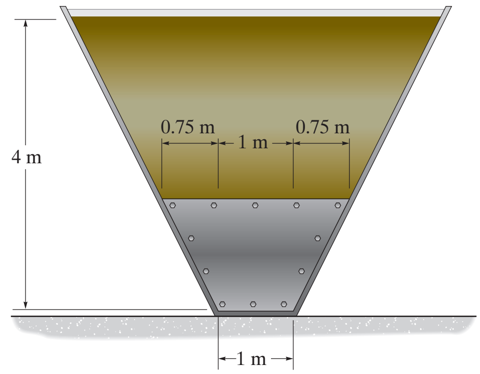
$$\circlearrowleft + \Sigma M_A = 0; \quad [17.58(10^3)\ \text{N}](0.5\ \text{m}) + [12.876(10^3)\ \text{N}](0.75\ \text{m})$$
$$+ [4.292(10^3)\ \text{N}](1.8333\ \text{m}) - [7.814(10^3)h^2\ \text{N}]\left(\frac{h}{3}\right) = 0$$

$$h = 2.167\ \text{m} = 2.16\ \text{m} \qquad \textbf{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–81. The tapered settling tank is completely filled with oil. Determine the resultant force the oil exerts on the trapezoidal clean-out plate located at its end. How far from the top does this force act on the plate? Use the formula method. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

Referring to the geometry of the plate shown in Fig. *a*

$$A = (1\ \text{m})(1.5\ \text{m}) + \frac{1}{2}(1.5\ \text{m})(1.5\ \text{m}) = 2.625\ \text{m}^2$$

$$\bar{y} = \frac{(3.25\ \text{m})[(1\ \text{m})(1.5\ \text{m})] + (3\ \text{m})\left[\frac{1}{2}(1.5\ \text{m})(1.5\ \text{m})\right]}{2.625\ \text{m}^2} = 3.1429\ \text{m}$$

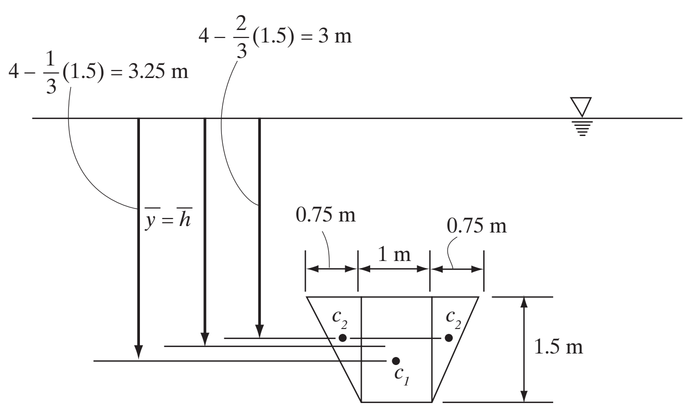
$$\bar{I}_x = \frac{1}{12}(1\ \text{m})(1.5\ \text{m})^3 + (1\ \text{m})(1.5\ \text{m})(3.25\ \text{m} - 3.1429\ \text{m})^2$$
$$+ \frac{1}{36}(1.5\ \text{m})(1.5\ \text{m})^3 + \frac{1}{2}(1.5\ \text{m})(1.5\ \text{m})(3.1429\ \text{m} - 3\ \text{m})^2$$
$$= 0.46205\ \text{m}^4$$

The resultant force is

$$F_R = \rho_o g\bar{h}A = \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3.1429\ \text{m})\left(2.625\ \text{m}^2\right)$$
$$= 72.84\left(10^3\right)\ \text{N} = 72.8\ \text{kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.46205\ \text{m}^4}{(3.1429\ \text{m})\left(2.625\ \text{m}^2\right)} + 3.1429\ \text{m} = 3.199\ \text{m} = 3.20\ \text{m} \qquad \textbf{Ans.}$$

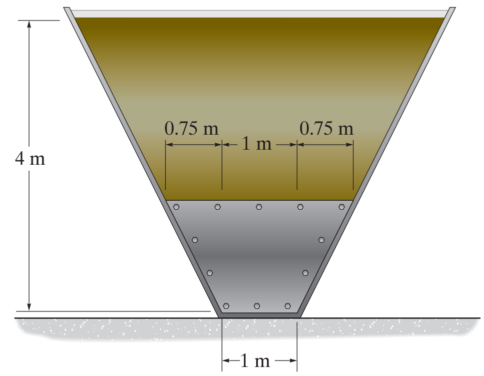


(a)

Ans:
$F_R = 72.8$ kN
$y_P = 3.20$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

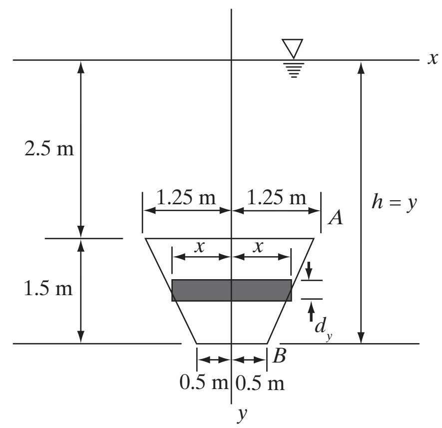
2–82. The tapered settling tank is completely filled with oil. Determine the resultant force the oil exerts on the trapezoidal clean-out plate located at its bottom. Use the integration method. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a is

$$\frac{y - 2.5}{x - 1.25} = \frac{4 - 2.5}{0.5 - 1.25}; \qquad 2x = 5 - y$$



(a)

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2x dy = 5 - y\, dy$. The pressure acting on this differential element is $p = \rho_0 g h = (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\, y = 8829y$. Thus, the resultant force acting on the entire plate is

$$F_R = \int_A p dA = \int_{2.5\ \text{m}}^{4\ \text{m}} 8829y(5 - y)dy$$

$$= 22072.5y^2 - 2943y^3 \Big|_{2.5\ \text{m}}^{4\ \text{m}}$$

$$= 72.84(10^3)\ \text{N} = 72.8\ \text{kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_P = \frac{\int_A y p dA}{F_R} = \frac{1}{72.84(10^3)\ \text{N}} \Big|_{2.5\ \text{m}}^{4\ \text{m}} y(8829y)(5 - y)dy$$

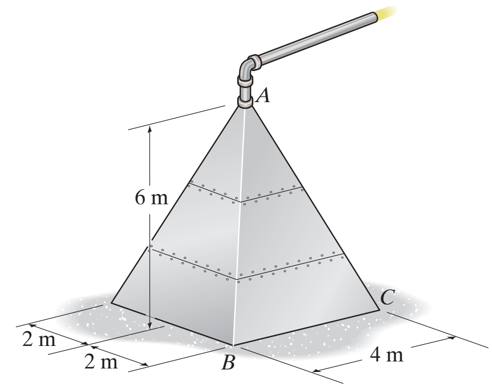
$$= \frac{1}{72.84(10^3)} \left(14715y^3 - 2207.25y^4\right) \Big|_{2.5\ \text{m}}^{4\ \text{m}}$$

$$= 3.199\ \text{m} = 3.20\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 72.8\ \text{kN}$
$y_P = 3.20\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–83. Ethyl alcohol is pumped into the tank, which has the shape of a four-sided pyramid. When the tank is completely full, determine the resultant force acting on each side, and its location measured from the top A along the side. Use the formula method. $\rho_{ea} = 789\ \text{kg/m}^3$.



SOLUTION

The geometry of the side wall of the tank is shown in Fig. *a*. In this case, it is convenient to calculate the resultant force as follows.

$$F_R = \gamma_{ea}\bar{h}A = \left(789\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{2}{3}(6\ \text{m})\right]\left(\frac{1}{2}\right)(4\ \text{m})\left(\sqrt{40}\right)$$

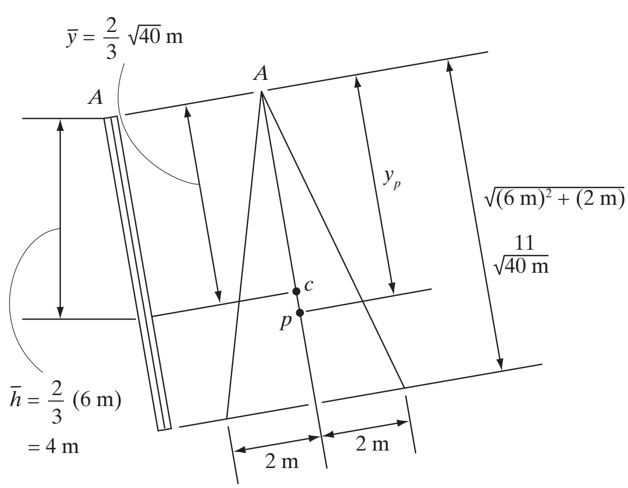
$$= 390.1\left(10^3\right)\ \text{N} = 390\ \text{kN}$$

The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

$$= \frac{\frac{1}{36}(4\ \text{m})\left(\sqrt{40}\ \text{m}\right)^3}{\frac{2}{3}\left(\sqrt{40}\ \text{m}\right)\left(\frac{1}{2}(4\ \text{m})\left(\sqrt{40}\ \text{m}\right)\right)} + \left(\frac{2}{3}\right)\left(\sqrt{40}\ \text{m}\right)$$

$$= 4.74\ \text{m}$$ **Ans.**

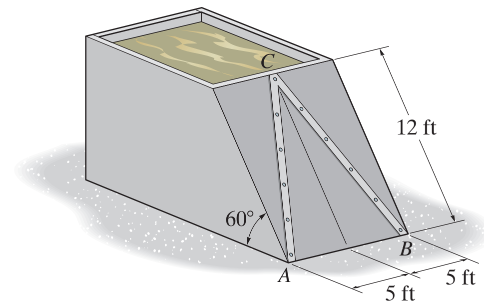


(a)

Ans:
$F_R = 390\ \text{kN}$
$y_P = 4.74\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–84.** The tank is filled to its top with an industrial solvent, ethyl ether. Determine the resultant force acting on the plate ABC, and its location on the plate measured from the base AB of the tank. Use the formula method. Take $\gamma_{ee} = 44.5\ \text{lb/ft}^3$.



SOLUTION

The resultant force is

$$F_R = \gamma_{ee}\bar{h}A = \left(44.5\ \text{lb/ft}^3\right)(8 \sin 60^\circ\ \text{ft})\left[\frac{1}{2}(10\ \text{ft})(12\ \text{ft})\right]$$

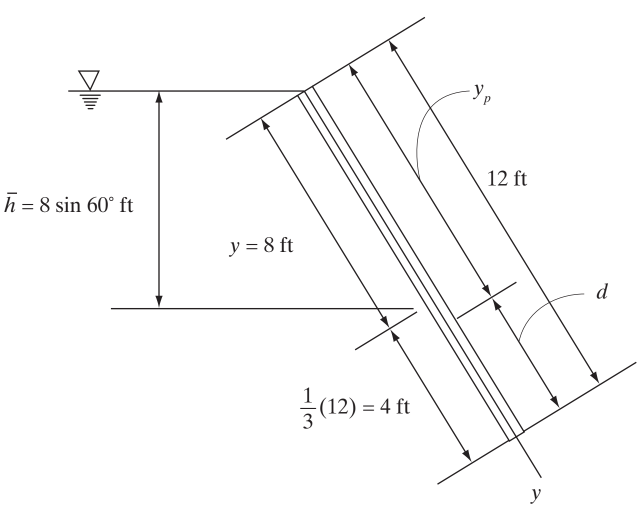
$$= 18.498\left(10^3\right)\ \text{lb} = 18.5\ \text{kip} \qquad \textbf{Ans.}$$

$$\bar{I}_x = \frac{1}{36}bh^3 = \frac{1}{36}(10\ \text{ft})(12\ \text{ft})^3 = 480\ \text{ft. Then}$$

$$y_P = \frac{\bar{I}_x}{\bar{y}_A} + \bar{y} = \frac{480\ \text{ft}}{(8\ \text{ft})\left[\frac{1}{2}(10\ \text{ft})(12\ \text{ft})\right]} + 8\ \text{ft} = 9\ \text{ft}$$

Thus,

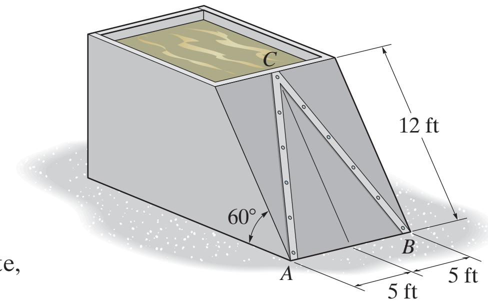
$$d = 12\ \text{ft} - y_P = 12\ \text{ft} - 9\ \text{ft} = 3\ \text{ft} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–85. Solve Prob. 2–84 using the integration method.



SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a, is

$$\frac{y-0}{x-0} = \frac{12-0}{5-0}; \qquad x = \frac{5}{12}y$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2xdy = 2\left(\frac{5}{12}y\right)dy = \frac{5}{6}ydy$. The pressure acting on this differential element is $p = \gamma h = (44.5\ \text{lb/ft}^3)(y \sin 60°) = 38.54\,y$. Thus, the resultant force acting on the entire plate is

$$F_R = \int_A pdA = \int_0^{12\text{ ft}} (38.54y)\left(\frac{5}{6}ydy\right)$$

$$= 10.71\,y^3\Big|_0^{12\text{ ft}}$$

$$= 18.50(10^3)\text{lb} = 18.5\text{ kip} \qquad \textbf{Ans.}$$

And it acts at

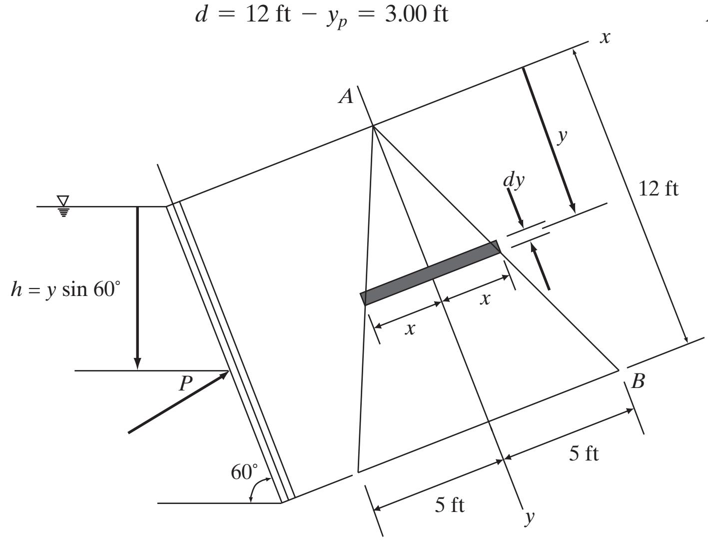
$$y_P = \frac{\int_A ypdA}{F_R} = \frac{1}{18.50(10^3)}\int_0^{12\text{ ft}} y(38.54y)\left(\frac{5}{6}ydy\right)$$

$$= \frac{1}{18.50(10^3)}\left(8.03y^4\right)\Big|_0^{12\text{ ft}}$$

$$= 9.00\text{ ft}$$

Thus,

$$d = 12\text{ ft} - y_p = 3.00\text{ ft} \qquad \textbf{Ans.}$$

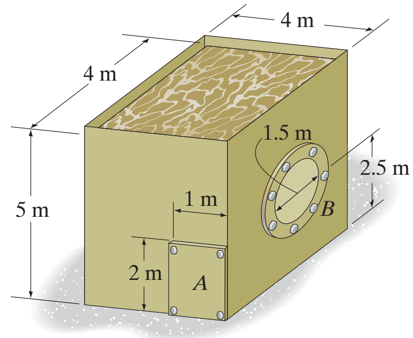


(a)

Ans:
$F_R = 18.5$ kip
$d = 3.00$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–86. Access plates on the industrial holding tank are bolted shut when the tank is filled with vegetable oil as shown. Determine the resultant force that this liquid exerts on plate A, and its location measured from the bottom of the tank. Use the formula method. $\rho_{ma} = 932\ \text{kg/m}^3$.



SOLUTION

Since the plate has a width of $b = 1$ m, the intensities of the distributed load at the top and bottom of the plate can be computed from

$$w_t = \rho_{vo}\, g h_t b = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(1\ \text{m}) = 27.429(10^3)\ \text{N/m}$$
$$w_b = \rho_{vo}\, g h_b b = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(1\ \text{m}) = 45.715(10^3)\ \text{N/m}$$

The resulting trapezoidal distributed load is shown in Fig. a, and this loading can be subdivided into two parts for which the resultant forces are

$$F_1 = w_t(L) = [27.429(10^3)\ \text{N/m}](2\ \text{m}) = 54.858(10^3)\ \text{N}$$

$$F_2 = \frac{1}{2}(w_b - w_t)(L) = \frac{1}{2}[45.715(10^3)\ \text{N/m} - 27.429(10^3)\ \text{N/m}](2\ \text{m}) = 18.286(10^3)\ \text{N}$$

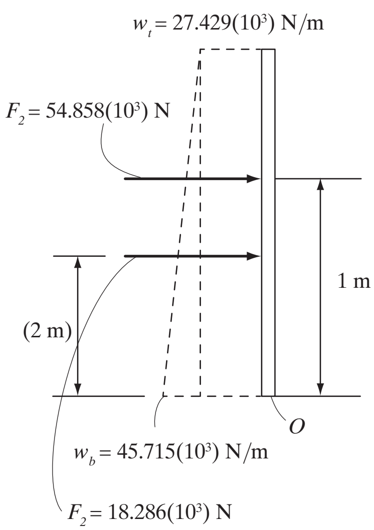
Thus, the resultant force is

$$F_R = F_1 + F_2 = 54.858(10^3)\ \text{N} + 18.286(10^3)\ \text{N} = 73.143(10^3)\ \text{N} = 73.1\ \text{kN} \quad \textbf{Ans.}$$

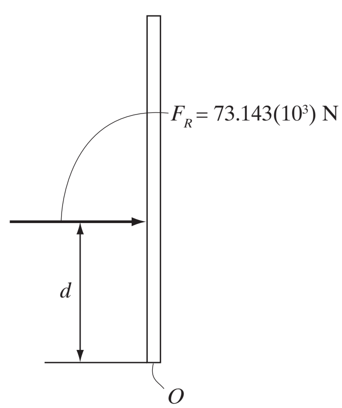
The location of the center of pressure can be determined by equating the sum of the moments of the forces in Figs. a and b about O.

$$\circlearrowleft+(M_R)_O = \Sigma M_O; \qquad [73.143(10^3)\ \text{N}]d = [54.858(10^3)\ \text{N}](1\ \text{m}) + [18.286(10^3)\ \text{N}]\left[\frac{1}{3}(2\ \text{m})\right]$$

$$d = 0.9167\ \text{m} = 917\ \text{mm} \qquad \textbf{Ans.}$$



(a)

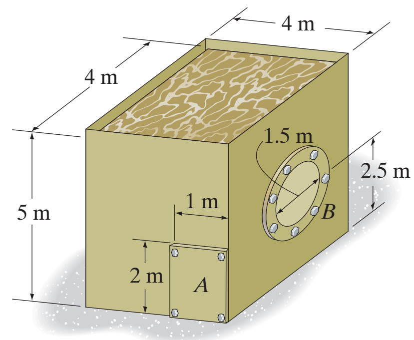


(b)

Ans:
$F_R = 73.1$ kN
$d = 917$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

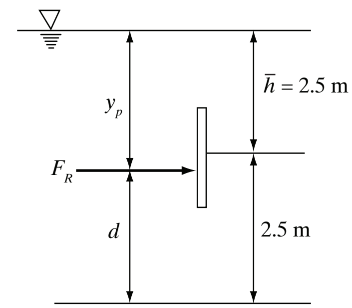
2–87. Access plates on the industrial holding tank are bolted shut when the tank is filled with vegetable oil as shown. Determine the resultant force that this liquid exerts on plate B, and its location measured from the bottom of the tank. Use the formula method. $\rho_{ma} = 932\ \text{kg/m}^3$.



SOLUTION

Since the plate is circular in shape, it is convenient to compute the resultant force as follows

$$F_R = \gamma_{vo}\,\bar{h}A = \left(932\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.5\ \text{m})\left[\pi(0.75\ \text{m})^2\right]$$
$$= 40.392\left(10^3\right)\text{N} = 40.4\ \text{kN} \qquad \textbf{Ans.}$$



(a)

The location of the center of pressure can be determined form

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{\pi\dfrac{(0.75\ \text{m})^4}{4}}{(2.5\ \text{m})(\pi)(0.75\ \text{m})^2} + 2.5\ \text{m}$$
$$= 2.556\ \text{m}$$

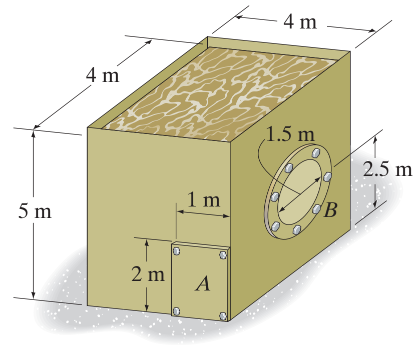
From the bottom of the tank, Fig. a,

$$d = 5\ \text{m} - y_P = 5\ \text{m} - 2.556\ \text{m} = 2.44\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 40.4$ kN
$d = 2.44$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

***2–88.** Solve Prob. 2–87 using the integration method.



SOLUTION

With respect to x and y axes established, the equation of the circumference of the circular plate is

$$x^2 + y^2 = 0.75^2; \qquad x = \sqrt{0.75^2 - y^2}$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2x\,dy = 2\sqrt{0.75^2 - y^2}\,dy$. The pressure acting on this differential element is $p = \rho_{vo} gh = \left(932\ \mathrm{kg/m^3}\right)\left(9.81\ \mathrm{m/s^2}\right)(2.5 - y) = 9142.92(2.5 - y)$. Thus, the resultant force acting on the entire plate is

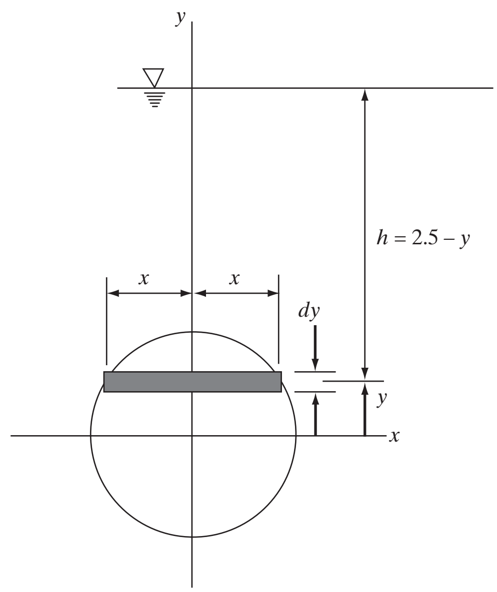
$$F_R = \int_A p\,dA = \int_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}} 9142.92(2.5 - y)\left[2\sqrt{0.75^2 - y^2}\,dy\right]$$

$$= 18285.84 \int_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}} (2.5 - y)\left(\sqrt{0.75^2 - y^2}\right)dy$$

$$= 22857.3\left[y\sqrt{0.75^2 - y^2} + 0.75^2 \sin^{-1}\frac{y}{0.75}\right]\Bigg|_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}}$$

$$+\ 6095.28\sqrt{\left(0.75^2 - y^2\right)^3}\,\Bigg|_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}}$$

$$= 40.39\left(10^3\right)\mathrm{N} = 40.4\ \mathrm{kN} \qquad \textbf{Ans.}$$



(a)

And it acts at

$$y_P = \frac{\int_A (2.5 - y)p\,dA}{F_R}$$

$$= \frac{1}{40.39\left(10^3\right)} \int_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}} (2.5 - y)\left[9142.92(2.5 - y)\right]\left(2\sqrt{0.75^2 - y^2}\,dy\right)$$

$$= 0.4527 \int_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}} \left(6.25 + y^2 - 5y\right)\left(\sqrt{0.75^2 - y^2}\right)dy$$

$$= 1.4147\left[y\sqrt{0.75^2 - y^2} + 0.75^2 \sin^{-1}\frac{y}{a}\right]\Bigg|_{-0.75\ m}^{0.75\ m} + 0.4527\left[-\frac{y}{4}\sqrt{\left(0.75^2 - y^2\right)^3}\right.$$

$$\left. + \frac{0.75^2}{8}\left(y\sqrt{0.75^2 - y^2} + a^2 \sin^{-1}\frac{y}{0.75}\right)\right]\Bigg|_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}} + 0.75450\,\Bigg|_{-0.75\ \mathrm{m}}^{0.75\ \mathrm{m}}$$

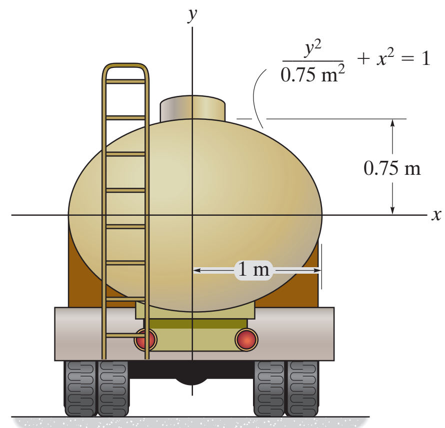
$$= 2.5562\ \mathrm{m}$$

From the bottom of tank is

$$d = 5\ \mathrm{m} - y_p = 5\ \mathrm{m} - 2.5562\ \mathrm{m} = 2.44\ \mathrm{m} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–89. The tank truck is filled to its top with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

Using Table 2-1 for the area and moment of inertia about the centroidal $\bar{x}$ axis of the elliptical plate, we get

$$F = \rho_w g \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})(\pi)(0.75\ \text{m})(1\ \text{m})$$
$$= 17.3\ \text{kN} \qquad \textbf{Ans.}$$

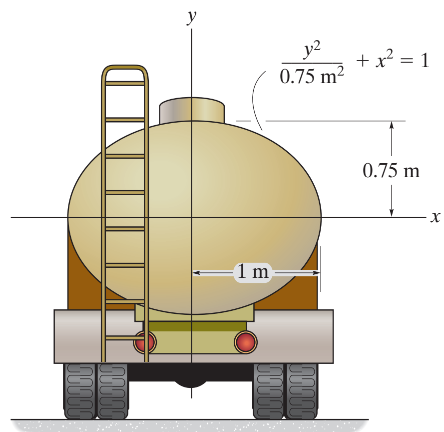
The center of pressure is at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{\left[\frac{1}{4}\pi(1\ \text{m})(0.75\ \text{m})^3\right]}{(0.75\ \text{m})\pi(1\ \text{m})(0.75\ \text{m})} + 0.75\ \text{m}$$
$$= 0.9375\ \text{m} = 0.938\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F = 17.3\ \text{kN}$
$y_P = 0.938\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–90. Solve Prob. 2–89 using the integration method.



SOLUTION

By integration of a horizontal strip of area

$$dF = p\,dA = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.75\ \text{m} - y)(2x\,dy)$$

$$F = 19\,620\int_{-0.75\ \text{m}}^{0.75\ \text{m}}(0.75 - y)\left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}}dy$$

$$= 19\,620\left[\int_{-0.75\ \text{m}}^{0.75\ \text{m}}\sqrt{(0.75)^2 - y^2}\,dy - \frac{1}{0.75}\int_{-0.75\ \text{m}}^{0.75\ \text{m}}y\sqrt{(0.75)^2 - y^2}\,dy\right]$$

$$= \frac{19\,620}{2}\left[y\sqrt{(0.75)^2 - y^2} + (0.75)^2\sin^{-1}\frac{y}{0.75}\right]_{-0.75}^{0.75} - \frac{19\,620}{0.75}\left[-\frac{1}{3}\sqrt{((0.75)^2 - y^2)^3}\right]_{-0.75}^{0.75}$$

$$= \frac{19\,620\pi(0.75)^2}{2} - 0 = 17\,336\ \text{N} = 17.3\ \text{kN} \qquad \textbf{Ans.}$$

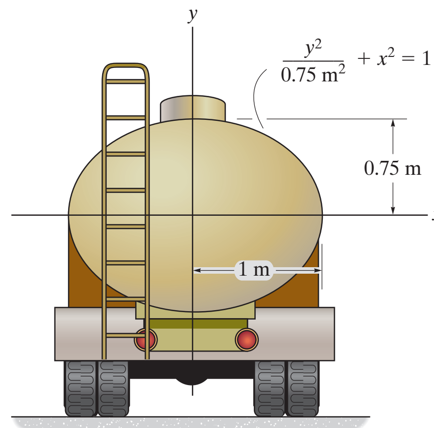
$$y_P = \frac{19\,620\int_{-0.75\ \text{m}}^{0.75\ \text{m}}y(0.75 - y)\left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}}dy}{17\,336\ \text{N}} = -0.1875\ \text{m}$$

$$y_P = 0.75\ \text{m} + 0.1875\ \text{m} = 0.9375\ \text{m} = 0.938\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F = 17.3\ \text{kN}$
$y_P = 0.938\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–91. The tank truck is half-filled with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the x axis. Solve the problem using formula method. *Hint*: The centroid of a semi-ellipse measured from the x axis is $\bar{y} = \dfrac{4b}{3\pi}$.



SOLUTION

From Table 2-1, the area and moment of inertia about the x axis of the half-ellipse plate are

$$A = \frac{\pi}{2}ab = \frac{\pi}{2}(1\ \text{m})(0.75\ \text{m}) = 0.375\pi\ \text{m}^2$$

$$I_x = \frac{1}{2}\left(\frac{\pi}{4}ab^3\right) = \frac{1}{2}\left[\frac{\pi}{4}(1\ \text{m})(0.75\ \text{m})^3\right] = 0.05273\pi\ \text{m}^4$$

Thus, the moment of inertia of the half of ellipse about its centroidal $\bar{x}$ axis can be determined by using the parallel-axis theorem.

$$I_x = \bar{I}_x + Ad_y^2$$

$$0.05273\pi\ \text{m}^4 = \bar{I}_x + (0.375\ \pi)\left[\frac{4(0.75\ \text{m})}{3\pi}\right]^2$$

$$\bar{I}_x = 0.046304\ \text{m}^4$$

Since $\bar{h} = \dfrac{4(0.75\ \text{m})}{3\pi} = 0.3183\ \text{m}$, then

$$F_R = \gamma\bar{h}A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.3183\ \text{m})\left(0.375\pi\ \text{m}^2\right)$$
$$= 3.679\left(10^3\right)\ \text{N} = 3.68\ \text{kN} \qquad \textbf{Ans.}$$

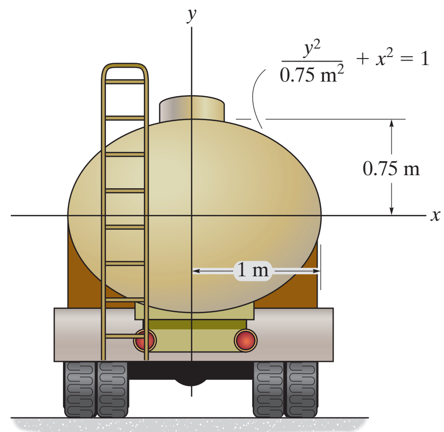
Since $\bar{y} = \bar{h} = 0.3183\ \text{m}$,

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{0.046304\ \text{m}^4}{(0.3183\ \text{m})\left(0.375\pi\ \text{m}^2\right)} + 0.3183\ \text{m}$$
$$= 0.4418\ \text{m} = 442\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$F_R = 3.68\ \text{kN}$
$y_P = 442\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–92.** Solve Prob. 2–91 using the integration method.



SOLUTION

Using a horizontal strip of area dA,

$$dF = p\,dA$$

$$dF = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-y)(2x\,dy)$$

$$F = -19\,620\int_{-0.75\ \text{m}}^{0}(y)\left(1-\frac{y^2}{0.75^2}\right)^{\frac{1}{2}}dy$$

$$= -\frac{19\,620}{0.75}\int_{-0.75\ \text{m}}^{0} y\sqrt{0.75^2-y^2}\,dy$$

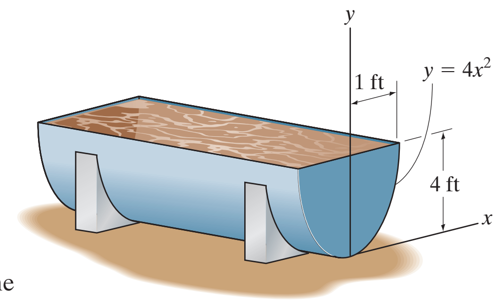
$$= \frac{26\,160}{3}\left[\sqrt{(0.75^2-y^2)^3}\right]_{-0.75\ \text{m}}^{0}$$

$$= 3.679(10^3)\ \text{N} = 3.68\ \text{kN} \qquad \textbf{Ans.}$$

$$y_P = -\frac{-19\,620\displaystyle\int_{-0.75\ \text{m}}^{0} y(y)\left(1-\frac{y^2}{0.75^2}\right)^{\frac{1}{2}}dy}{3678.75\ \text{N}} = 0.4418\ \text{m} = 442\ \text{mm} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–93. The trough is filled to its top with carbon disulphide. Determine the magnitude of the resultant force acting on the parabolic end plate, and the location of the center of pressure measured from the top. $\rho_{cd} = 2.46\ \text{slug/ft}^3$. Solve the problem using the formula method.



SOLUTION

From Table 2-1, the area and moment of inertia about the centroidal $\bar{x}$ axis of the parabolic plate are

$$A = \frac{2}{3}bh = \frac{2}{3}(2\ \text{ft})(4\ \text{ft}) = 5.3333\ \text{ft}^2$$

$$\bar{I}_x = \frac{8}{175}bh^3 = \frac{8}{175}(2\ \text{ft})(4\ \text{ft})^3 = 5.8514\ \text{ft}^4$$

With $\bar{h} = \frac{2}{5}h = \frac{2}{5}(4\ \text{ft}) = 1.6\ \text{ft}$,

$$F_R = \gamma\bar{h}A = \left(2.46\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(1.6\ \text{ft})\left(5.3333\ \text{ft}^2\right)$$
$$= 675.94\ \text{lb} = 676\ \text{lb}$$ **Ans.**

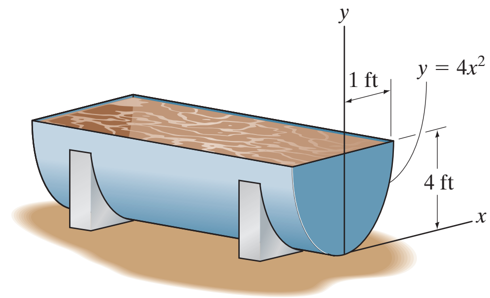
Since $\bar{y} = \bar{h} = 1.6\ \text{ft}$,

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{5.8514\ \text{ft}^4}{(1.6\ \text{ft})\left(5.3333\ \text{ft}^2\right)} + 1.6\ \text{ft}$$
$$= 2.2857\ \text{ft} = 2.29\ \text{ft}$$ **Ans.**

Ans:
$F_R = 676\ \text{lb}$
$y_P = 2.29\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–94. Solve Prob. 2–93 using the integration method.



SOLUTION

Using a horizontal strip of area,

$$F_R = \int_A p\,dA = \int_0^{4\text{ ft}} (2.46\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(4 - y)2x\,dy$$

$$= 158.424 \int_0^{4\text{ ft}} (4 - y)\left(\frac{1}{2}\right)\left(y^{\frac{1}{2}}\right)dy$$

$$= 79.212\left(\int_0^{4\text{ ft}} \left(4y^{\frac{1}{2}} - y^{\frac{3}{2}}\right)dy\right)$$

$$= 79.212\left(\frac{8}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right)\Bigg|_0^{4\text{ ft}}$$

$$= 675.94\ \text{lb} = 676\ \text{lb} \qquad \textbf{Ans.}$$

$$F_R(d) = \int_A y(p\,dA) = 158.424 \int_0^{4\text{ ft}} y(4 - y)\left(\frac{1}{2}\right)\left(y^{\frac{1}{2}}\right)dy$$

$$(675.94\ \text{lb})(d) = 158.424 \int_0^{4\text{ ft}} y(4 - y)\left(\frac{1}{2}\right)y^{\frac{1}{2}}\,dy$$

$$= 79.212 \int_0^{4\text{ ft}} \left(4y^{\frac{3}{2}} - y^{\frac{5}{2}}\right)dy$$

$$= 79.212\left(\frac{8}{3}y^{\frac{5}{2}} - \frac{2}{5}y^{\frac{7}{2}}\right)\Bigg|_0^{4\text{ ft}}$$

$$= 1158.76\ \text{lb}\cdot\text{ft}$$

$$d = \frac{1158.76\ \text{lb}\cdot\text{ft}}{675.94\ \text{lb}} = 1.7143\ \text{ft}$$

$$y_P = 4\ \text{ft} - d$$

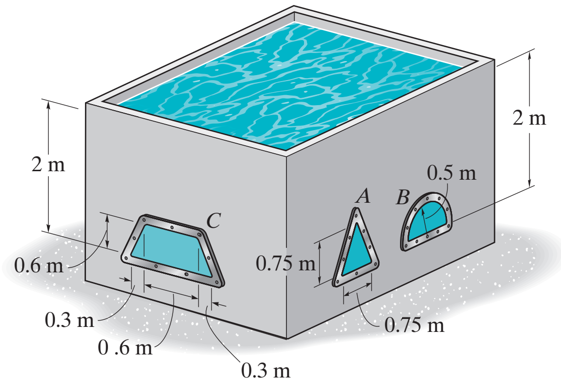
$$= 4\ \text{ft} - 1.7143\ \text{ft}$$

$$= 2.2857\ \text{ft} = 2.29\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 676$ lb
$y_P = 2.29$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–95. The tank is filled with water. Determine the resultant force acting on the triangular plate A and the location of the center of pressure, measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

The resultant force is

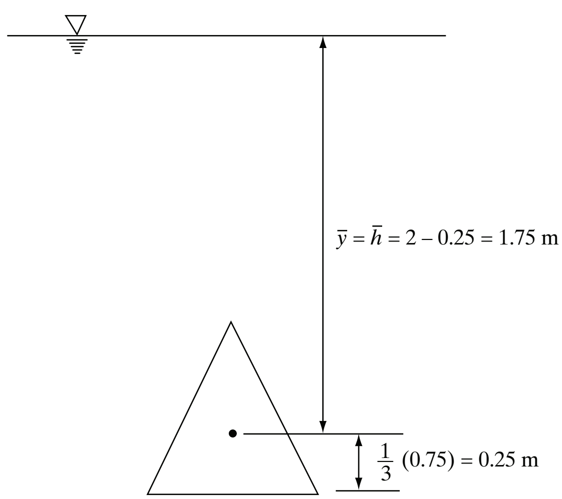
$$F_R = \rho_w g\bar{h}A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.75\ \text{m})\left[\frac{1}{2}(0.75\ \text{m})(0.75\ \text{m})\right]$$

$$= 4828.36\ \text{N} = 4.83\ \text{kN} \qquad \textbf{Ans.}$$

$$\bar{I}_x = \frac{1}{36}(bh^3) = \frac{1}{36}(0.75\ \text{m})(0.75\ \text{m})^3 = 8.7891(10^{-3})\ \text{m}^4$$

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{8.7891(10^{-3})\ \text{m}^4}{(1.75\ \text{m})\left[\frac{1}{2}(0.75\ \text{m})(0.75\ \text{m})\right]} + 1.75\ \text{m}$$

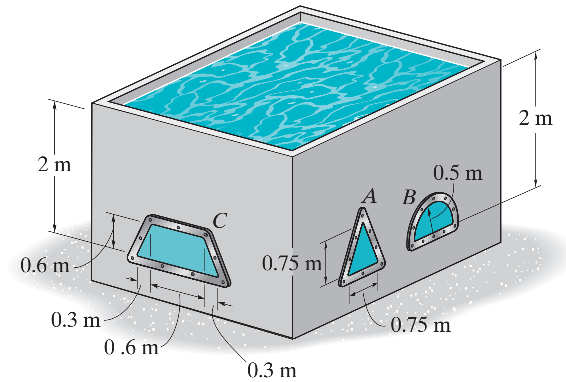
$$= 1.768\ \text{m} = 1.77\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$F_R = 4.83$ kN
$y_P = 1.77$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

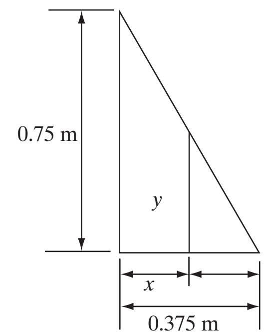
***2–96.** Solve Prob. 2–95 using the integration method.



SOLUTION

From the geometry shown in Fig. *a*

$$\frac{y}{0.75\ \text{m}} = \frac{0.375\ \text{m} - x}{0.375\ \text{m}} \qquad x = (0.375 - 0.5y)\ \text{m}$$



(a)

Referring to Fig. *b*, the pressure as a function of *y* can be written as

$$p = \rho_w g h = \left(1000\ \text{kg/m}^3\right)\left(9.81\text{m/s}^2\right)(2\text{m} - y) = [9810(2 - y)]\ \text{N/m}^2$$

This pressure acts on the element of area $dA = 2x dy = 2(0.375 - 0.5y)dy$. Thus,

$$dF = p dA = \left[9810(2 - y)\ \text{N/m}^2\right][2(0.375 - 0.5y)dy]$$

$$= 19\,620\left(0.5y^2 - 1.375y + 0.75\right)dy$$

Then

$$F_R = \int dF = 19\,620\int_0^{0.75\ \text{m}}\left(0.5y^2 - 1.375y + 0.75\right)dy$$

$$= 19\,620\left[\frac{0.5y^3}{3} - \frac{1.375y^2}{2} + 0.75y\right]\Bigg|_0^{0.75\ \text{m}}$$

$$= 4828.36\ \text{N} = 4.83\ \text{kN}$$ **Ans.**

And

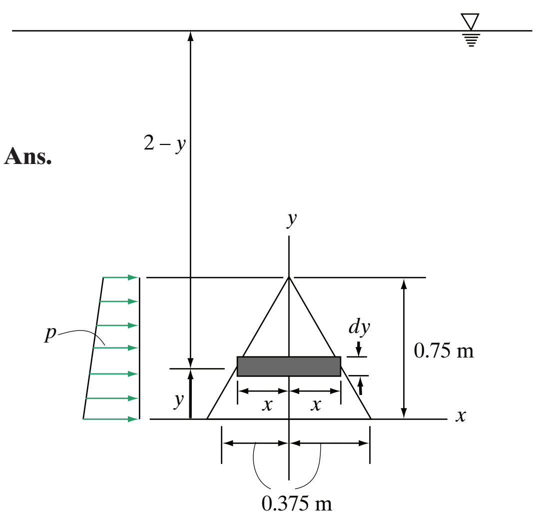
$$y_p = \frac{\int (2 - y)dF}{F_R}$$

$$= \frac{\int_0^{0.75\ \text{m}} (2 - y)\left[19\,620\left(0.5y^2 - 1.375y + 0.75\right)dy\right]}{4828.36}$$

$$= \frac{19\,620\int_0^{0.75\ \text{m}}\left(-0.5y^3 + 2.375y^2 - 3.5y + 1.5\right)dy}{4828.36}$$

$$= \frac{19\,620\left(-0.125y^4 + 0.79167y^3 - 1.75y^2 + 1.5y\right)\Big|_0^{0.75\ \text{m}}}{4828.36}$$

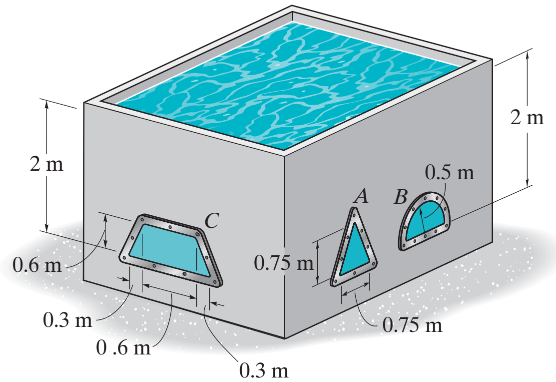
$$= 1.768 = 1.77\ \text{m}$$ **Ans.**



(b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–97. The tank is filled with water. Determine the resultant force acting on the semicircular plate B and the location of the center of pressure, measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

The resultant force is

$$F_R = \rho_w g \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\left(2 - \frac{2}{3\pi}\right)\text{m}\right]\left[\frac{1}{2}\pi(0.5\ \text{m})^2\right]$$

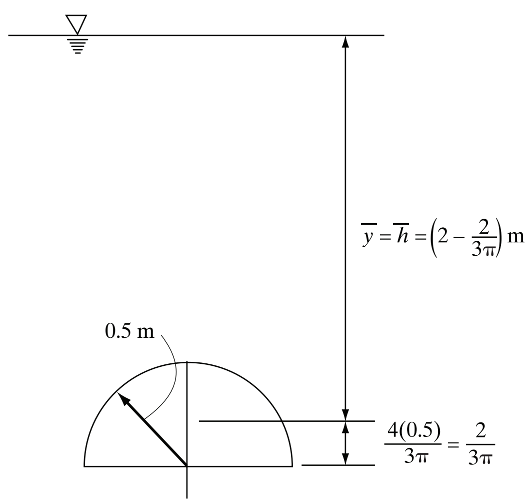
$$= 6887.26\ \text{N} = 6.89\ \text{kN} \qquad \textbf{Ans.}$$

$$\bar{I}_x = 0.1098\, r^4 = 0.1098(0.5\ \text{m})^4 = 6.8625\left(10^{-3}\right)\text{m}^4$$

Then

$$y_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{6.8625\left(10^{-3}\right)\text{m}^4}{\left[\left(2 - \frac{2}{3\pi}\right)\text{m}\right]\left[\frac{1}{2}\pi(0.5\ \text{m})^2\right]} + \left(2 - \frac{2}{3\pi}\right)\text{m} = 1.798\ \text{m}$$

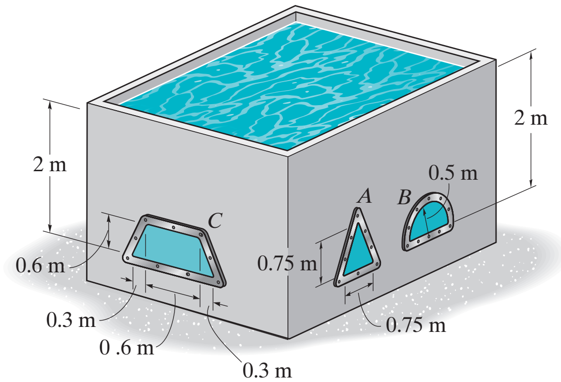
$$= 1.80\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$F_R = 6.89$ kN
$y_P = 1.80$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–98. Solve Prob. 2–97 using the integration method.



SOLUTION

Referring to Fig. *a*, the pressure as a function of *y* can be written as

$$p = \rho_w g h = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(2\text{m} - y) = \left[9810(2 - y)\right]\ \text{N/m}^2$$

This pressure acts on the strip element of area $dA = 2x\,dy$. Here, $x = (0.25 - y^2)^{\frac{1}{2}}$. Thus, $dA = 2(0.25 - y^2)^{\frac{1}{2}}dy$. Then

$$dF = p\,dA = 9810(2 - y)\left[2(0.25 - y^2)^{\frac{1}{2}}dy\right]$$

$$= 19\,620\left[2(0.25 - y^2)^{\frac{1}{2}} - y(0.25 - y^2)^{\frac{1}{2}}\right]dy$$

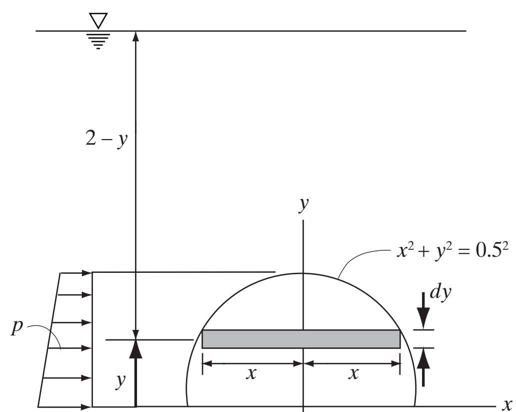
Then

$$F_R = \int dF = 19\,620\int_0^{0.5\text{ m}}\left[2(0.25 - y^2)^{\frac{1}{2}} - y\,(0.25 - y^2)^{\frac{1}{2}}\right]dy$$

$$= 19\,620\left[y(0.25 - y^2)^{\frac{1}{2}} + 0.25\sin^{-1}\frac{y}{0.5} + \frac{1}{3}(0.25 - y^2)^{\frac{3}{2}}\right]_0^{0.5\text{ m}}$$

$$= 6887.26\ \text{N}$$

$$= 6.89\ \text{kN} \qquad \textbf{Ans.}$$



(a)

And

$$y_P = \frac{\int (2 - y)\,dF}{F_R}$$

$$= \frac{\int_0^{0.5\text{ m}}(2 - y)\left\{19\,620\left[2(0.25 - y^2)^{\frac{1}{2}} - y(0.25 - y^2)^{\frac{1}{2}}\right]dy\right\}}{6887.26}$$

$$= \frac{19\,620\int_0^{0.5\text{ m}}\left[4(0.25 - y^2)^{\frac{1}{2}} - 4y(0.25 - y^2)^{\frac{1}{2}} + y^2(0.25 - y^2)^{\frac{1}{2}}\right]dy}{6887.26}$$

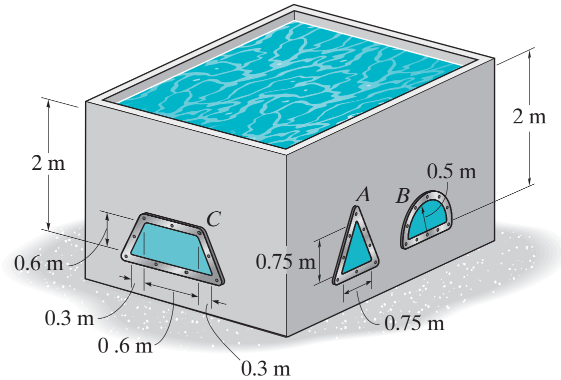
$$= \frac{19\,620\left[2y(0.25 - y^2)^{\frac{1}{2}} + 0.5\sin^{-1}\frac{y}{0.5} + \frac{4}{3}(0.25 - y^2)^{\frac{3}{2}} + \frac{y}{4}(0.25 - y^2)^{\frac{3}{2}} + \frac{y}{32}(0.25 - y^2)^{\frac{1}{2}} + \frac{1}{128}\sin^{-1}\frac{y}{0.5}\right]\Bigg|_0^{0.5\text{m}}}{6887.26}$$

$$= \frac{12380.29}{6887.26} = 1.798\ \text{m} = 1.80\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 6.89\ \text{kN}$
$y_P = 1.80\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–99. The tank is filled with water. Determine the resultant force acting on the trapezoidal plate C and the location of the center of pressure, measured from the top of the tank. Solve the problem using formula method.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$A = (0.6\text{ m})(0.6\text{ m}) + \frac{1}{2}(0.6\text{ m})(0.6\text{ m}) = 0.54\text{ m}^2$$

$$\bar{y} = \frac{(1.7\text{ m})(0.6\text{ m})(0.6\text{ m}) + (1.8\text{m})\left[\frac{1}{2}(0.6\text{ m})(0.6\text{ m})\right]}{0.54\text{ m}^2} = 1.7333\text{ m}$$

$$\bar{I}_x = \frac{1}{12}(0.6\text{ m})(0.6\text{ m})^3 + (0.6\text{ m})(0.6\text{ m})(1.7333\text{ m} - 1.7\text{ m})^2$$

$$+ \frac{1}{36}(0.6\text{ m})(0.6\text{ m})^3 + \frac{1}{2}(0.6\text{ m})(0.6\text{ m})(1.8\text{ m} - 1.7333\text{ m})^2$$

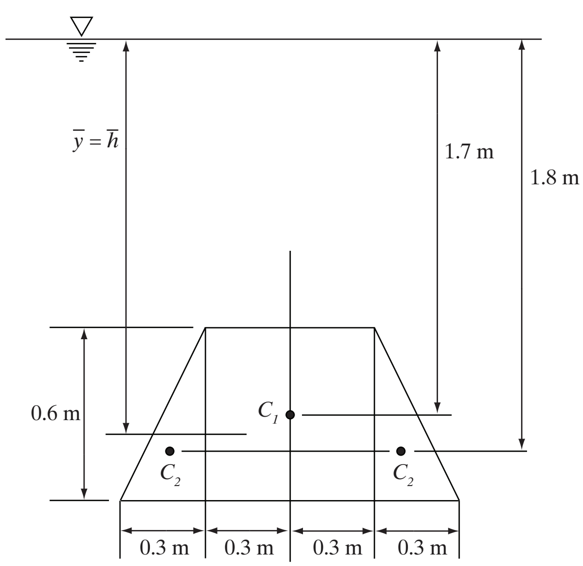
$$= 0.0156\text{ m}^4$$

The resultant force is

$$F_R = \rho_w g\bar{h}A = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1.7333\text{ m})\left(0.54\text{ m}^2\right) = 9182.16\text{ N}$$
$$= 9.18\text{ kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.0156\text{ m}^4}{(1.7333\text{ m})(0.54\text{ m})} + 1.7333\text{ m} = 1.75\text{ m} \qquad \textbf{Ans.}$$

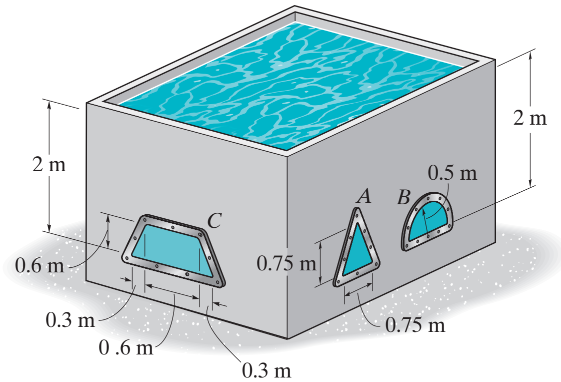


(a)

Ans:
$F_R = 9.18$ kN
$y_P = 1.75$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

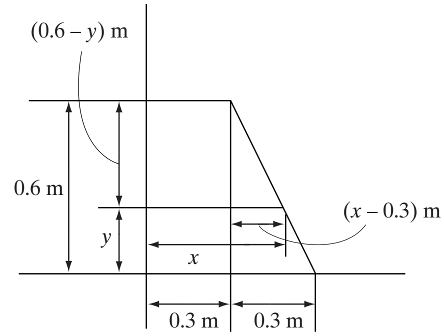
***2–100.** Solve Prob. 2–99 using the integration method.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$\frac{0.6\ \mathrm{m} - y}{0.6\ \mathrm{m}} = \frac{x - 0.3\ \mathrm{m}}{0.3\ \mathrm{m}}; \qquad x = (0.6 - 0.5y)\ \mathrm{m}$$



(a)

Referring to Fig. *b*, the pressure as a function of y can be written as

$$p = \rho_w g h = \left(1000\ \mathrm{kg/m^3}\right)\left(9.81\mathrm{m/s^2}\right)(2 - y)\ \mathrm{m} = \left[9810(2 - y)\right]\ \mathrm{N/m^2}$$

This pressure acts on the element of area $dA = 2x dy = 2(0.6 - 0.5y)dy = (1.2 - y)dy$. Thus

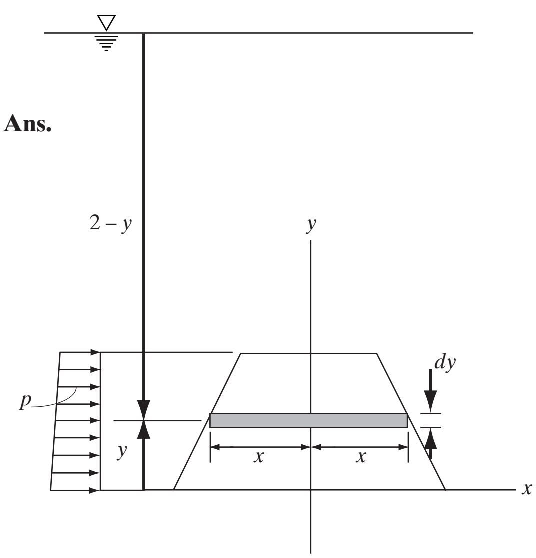
$$dF = p dA = 9810(2 - y)(1.2 - y)dy$$
$$= 9810\left(y^2 - 3.2y + 2.4\right)dy$$

Then

$$F_R = \int dF = 9810\int_0^{0.6\ \mathrm{m}}\left(y^2 - 3.2y + 2.4\right)dy$$
$$= 9810\left(\frac{y^3}{3} - 1.6y^2 + 2.4y\right)\Bigg|_0^{0.6\ \mathrm{m}}$$
$$= 9182.16\ \mathrm{N} = 9.18\ \mathrm{kN} \qquad \textbf{Ans.}$$

And it acts at

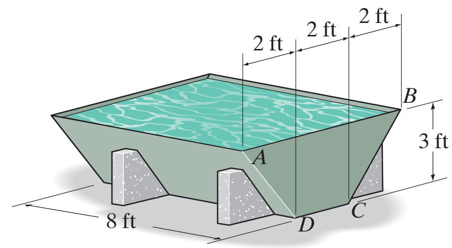
$$y_P = \frac{\int (2 - y)\, dF}{F_R}$$
$$= \frac{\int_0^{0.6\ \mathrm{m}} (2 - y)\left[9810\left(y^2 - 3.2y + 2.4\right)dy\right]}{9182.16}$$
$$= \frac{9810\int_0^{0.6\ \mathrm{m}}\left(-y^3 + 5.2y^2 - 8.8y + 4.8\right)dy}{9182.16}$$
$$= \frac{9810\left(-\frac{y^4}{4} + 1.7333y^3 - 4.4y^2 + 4.8y\right)\Big|_0^{0.6\ \mathrm{m}}}{9182.16}$$
$$= 1.75\ \mathrm{m} \qquad \textbf{Ans.}$$



(b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–101. The open wash tank is filled to its top with butyl alcohol, an industrial solvent. Determine the magnitude of the resultant force on the end plate *ABCD* and the location of the center of pressure, measured from *AB*. Solve the problem using the formula method. Take $\gamma_{ba} = 50.1\ \text{lb/ft}^3$.



SOLUTION

First, the location of the centroid of plate *ABCD*, Fig. *a*, measured from edge *AB* must be determined.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(1.5\ \text{ft})\left[3\ \text{ft}(2\ \text{ft})\right] + (1\ \text{ft})\left[\frac{1}{2}(4\ \text{ft})(3\ \text{ft})\right]}{3\ \text{ft}(2\ \text{ft}) + \frac{1}{2}(4\ \text{ft})(3\ \text{ft})} = 1.25\ \text{ft}$$

Then, the moment of inertia of plate *ABCD* about its centroid $\bar{x}$ axis is

$$\bar{I}_x = \left[\frac{1}{12}(2\ \text{ft})(3\ \text{ft})^3 + 2\ \text{ft}(3\ \text{ft})(1.5\ \text{ft} - 1.25\ \text{ft})^2\right] + \left[\frac{1}{36}(4\ \text{ft})(3\ \text{ft})^3 + \frac{1}{2}(4\ \text{ft})(3\ \text{ft})(1.25\ \text{ft} - 1\ \text{ft})^2\right] = 8.25\ \text{ft}^4$$

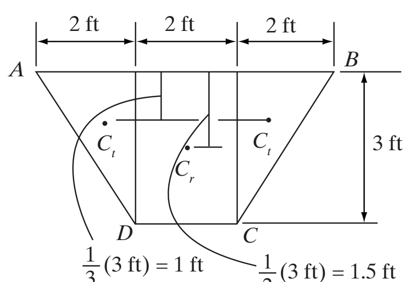
The area of plate *ABCD* is

$$A = 3\ \text{ft}(2\ \text{ft}) + \frac{1}{2}(4\ \text{ft})(3\ \text{ft}) = 12\ \text{ft}^2$$

Thus,

$$F_R = \gamma \bar{h} A = \left(50.1\ \text{lb/ft}^3\right)(1.25\ \text{ft})\left(12\ \text{ft}^2\right) = 751.5\ \text{lb} = 752\ \text{lb} \qquad \textbf{Ans.}$$

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{8.25}{1.25(12)} + 1.25 = 1.80\ \text{ft} \qquad \textbf{Ans.}$$

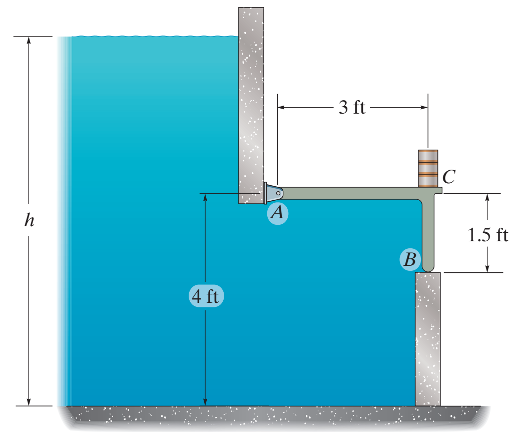


(a)

Ans:
$F_R = 752$ lb
$y_P = 1.80$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–102. The control gate ABC is pinned at A and rest on the smooth surface at B. Determine the amount of weight that should be placed at C in order to maintain a reservoir depth of $h = 10$ ft. The gate has a width of 3 ft. Neglect its weight.



SOLUTION

The intensities of the distributed load at C and B shown in Fig. a are

$$w_C = \gamma_w h_C b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(3\ \text{ft}) = 1123.2\ \text{lb/ft}$$

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(7.5\ \text{ft})(3\ \text{ft}) = 1404\ \text{lb/ft}$$

Thus,

$$F_1 = (1123.2\ \text{lb/ft})(3\ \text{ft}) = 3369.6\ \text{lb}$$

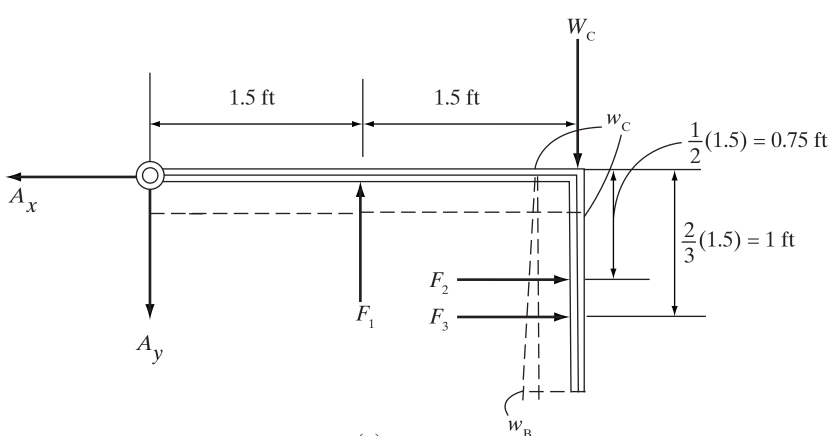
$$F_2 = (1123.2\ \text{lb/ft})(1.5\ \text{ft}) = 1684.8\ \text{lb}$$

$$F_3 = \frac{1}{2}\left[(1404 - 1123.2\ \text{lb/ft})\right](1.5\ \text{ft}) = 210.6\ \text{lb}$$

Since the gate is about to be opened, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad (3369.6\ \text{lb})(1.5\ \text{ft}) + (1684.8\ \text{lb})(0.75\ \text{ft}) + (210.6\ \text{lb})(1\ \text{ft}) - w_C(3\ \text{ft}) = 0$$

$$W_C = 2176.2\ \text{lb} = 2.18\ \text{kip} \qquad \textbf{Ans.}$$

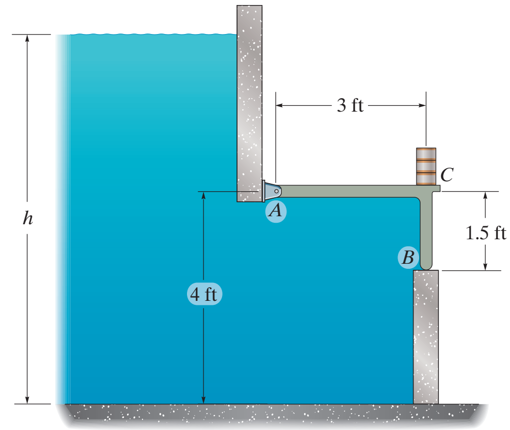


(a)

Ans:
2.18 kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–103. The control gate ABC is pinned at A and rest on the smooth surface at B. If the counterweight C is 2000 lb, determine the maximum depth of water h in the reservoir before the gate begins to open. The gate has a width of 3 ft. Neglect its weight.



SOLUTION

The intensities of the distributed loads at C and B are show in Fig. *a*

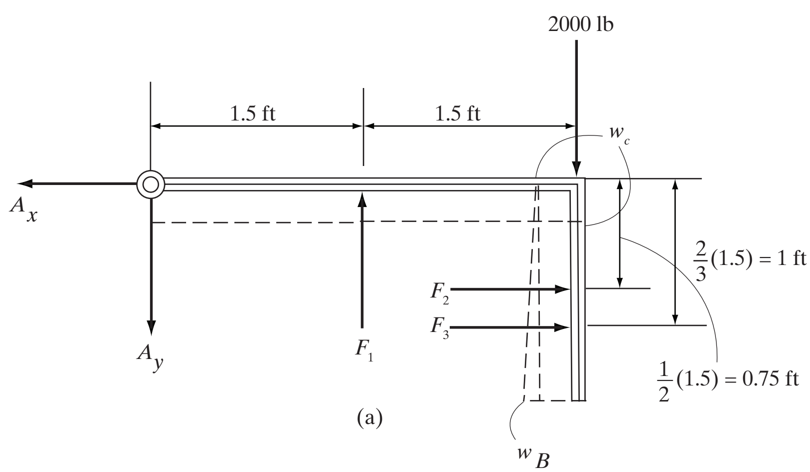
$$w_C = \gamma_w h_C b = (62.4\ \text{lb/ft}^3)(h - 4\ \text{ft})(3\ \text{ft}) = [187.2(h - 4)]\ \text{lb/ft}$$
$$w_B = \gamma_w h_B b = (62.4\ \text{lb/ft}^3)(h - 2.5\ \text{ft})(3\ \text{ft}) = [187.2(h - 2.5)]\ \text{lb/ft}$$

Thus,

$$F_1 = (187.2(h - 4)\ \text{lb/ft})(3\ \text{ft}) = 561.6(h - 4)\ \text{lb}$$
$$F_2 = (187.2(h - 4)\ \text{lb/ft})(1.5\ \text{ft}) = 280.8(h - 4)\ \text{lb}$$
$$F_3 = \frac{1}{2}[187.2(h - 2.5)\ \text{lb/ft} - (187.2(h - 4)\ \text{lb/ft}](1.5\ \text{ft}) = 210.6\ \text{lb}$$

Since the gate is required to be opened $N_B = 0$. Write the moment equation of equilibrium about point A by referring to Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \quad [561.6(h - 4)\ \text{lb}](1.5\ \text{ft}) + [280.8(h - 4)\ \text{lb}](0.75\ \text{ft})$$
$$+ (210.6\ \text{lb})(1\ \text{ft}) - (2000\ \text{lb})(3\ \text{ft}) = 0$$
$$1053(h - 4) = 5789.4$$
$$h = 9.498\ \text{ft} = 9.50\ \text{ft}$$ **Ans.**

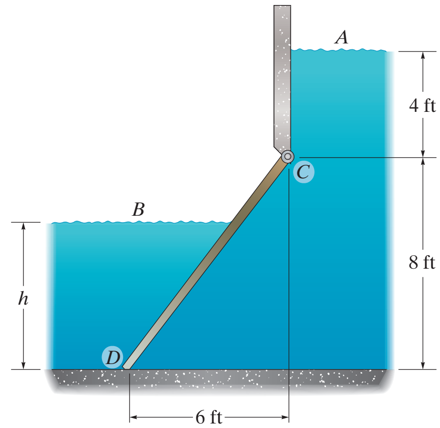


(a)

Ans:
9.50 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

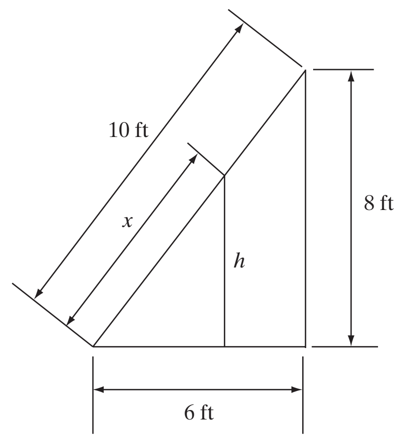
***2–104.** The uniform plate, which is hinged at C, is used to control the level of the water at A to maintain its constant depth of 12 ft. If the plate has a width of 8 ft and a weight of $50(10^2)$ lb, determine the minimum height h of the water at B so that seepage will not occur at D.



SOLUTION

Referring to the geometry in Fig. a

$$\frac{x}{10} = \frac{h}{8}; \qquad x = \frac{5}{4}h$$



(a)

The intensities of the distributed load shown in Fig. b are

$$w_1 = \gamma_w h_1 b = \left(62.4\ \text{lb/ft}^3\right)(4\ \text{ft})(8\ \text{ft}) = 1996.8\ \text{lb/ft}$$
$$w_2 = \gamma_w h_2 b = \left(62.4\ \text{lb/ft}^3\right)(12\ \text{ft})(8\ \text{ft}) = 5990.4\ \text{lb/ft}$$
$$w_3 = \gamma_w h_3 b = \left(62.4\ \text{lb/ft}^3\right)(h)(8\ \text{ft}) = (499.2h)\ \text{lb/ft}$$

Thus, the resultant forces of these distributed loads are

$$F_1 = (1996.8\ \text{lb/ft})(10\ \text{ft}) = 19968\ \text{lb}$$
$$F_2 = \frac{1}{2}(5990.4\ \text{lb/ft} - 1996.8\ \text{lb/ft})(10\ \text{ft}) = 19968\ \text{lb}$$
$$F_3 = \frac{1}{2}(499.2h\ \text{lb/ft})\left(\frac{5}{4}h\right) = \left(312h^2\right)\ \text{lb}$$

and act at

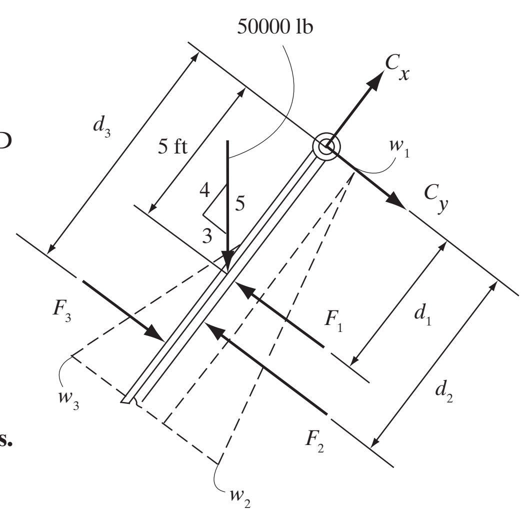
$$d_1 = \frac{10\ \text{ft}}{2} = 5\ \text{ft}$$
$$d_2 = \frac{2}{3}(10\ \text{ft}) = 6.667\ \text{ft}$$
$$d_3 = 10\ \text{ft} - \frac{1}{3}\left(\frac{5}{4}h\right) = (10 - 0.4167h)\ \text{ft}$$

For seepage to occur, the reaction at D, must be equal to zero. Referring to the FBD of the gate, Fig. b,

$$\circlearrowleft + \Sigma M_C = 0; \qquad (50000\ \text{lb})\left(\frac{3}{5}\right)(5\ \text{ft}) + \left(312h^2\ \text{lb}\right)(10 - 0.4167h)\ \text{ft}$$
$$- (19968\ \text{lb})(5\ \text{ft}) - (19968\ \text{lb})(6.667\ \text{ft}) = 0$$
$$-130h^3 + 3120h^2 - 82960 = 0$$

Solving numerically,

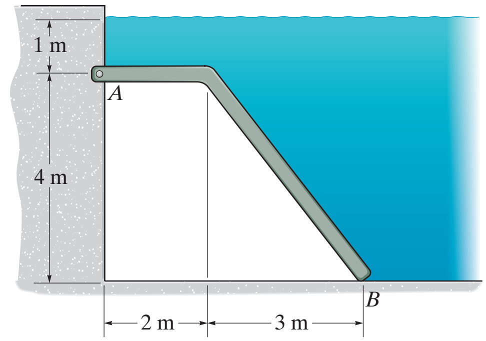
$$h = 5.945\ \text{ft} = 5.95\ \text{ft} < 8\ \text{ft}$$ **Ans.**



(b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–105. The bent plate is 1.5 m wide and is pinned at A and rests on a smooth support at B. Determine the horizontal and vertical components of reaction at A and the vertical reaction at the smooth support B for equilibrium. The fluid is water.

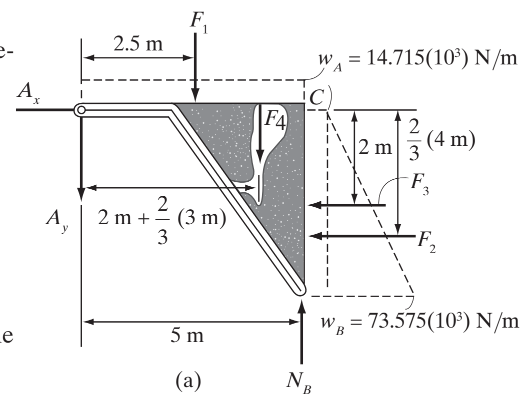


SOLUTION

Since the gate has a width of $b = 1.5$ m, the intensities of the distributed loads at A and B can be computed from

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1\ \text{m})(1.5\ \text{m}) = 14.715\left(10^3\right)\ \text{N/m}$$
$$w_B = \rho_w g h_B b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(5\ \text{m})(1.5\ \text{m}) = 73.575\left(10^3\right)\ \text{N/m}$$

Using these results, the distributed load acting on the plate is shown on the free-body diagram of the gate, Fig. a.



(a)

$$F_1 = w_A L_{AB} = \left(14.715\left(10^3\right)\ \text{N/m}\right)(5\ \text{m}) = 73.575\left(10^3\right)\ \text{N}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{BC} = \frac{1}{2}\left(73.575\left(10^3\right)\ \text{N/m} - 14.715\left(10^3\right)\ \text{N/m}\right)(4\ \text{m})$$

$$= 117.72\left(10^3\right)\ \text{N}$$

$$F_3 = w_A L_{BC} = \left(14.715\left(10^3\right)\ \text{N/m}\right)(4\ \text{m}) = 58.86\left(10^3\right)\ \text{N}$$

$\mathbf{F}_4$ on the free-body diagram is equal to the weight of the water contained in the shaded triangular block, Fig. a.

$$F_4 = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{1}{2}(3\ \text{m})(4\ \text{m})(1.5\ \text{m})\right] = 88.29\left(10^3\right)\ \text{N}$$

Considering the free-body diagram of the gate, Fig. a.

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(5\ \text{m}) - 73.575\left(10^3\right)\text{N}(2.5\ \text{m}) - 58.86\left(10^3\right)\text{N}(2\ \text{m}) - 117.72\left(10^3\right)\text{N}\left(\frac{2}{3}(4\ \text{m})\right)$$
$$- 88.29\left(10^3\right)\text{N}\left(2\ \text{m} + \frac{2}{3}(3\ \text{m})\right) = 0$$

$$N_B = 193.748\left(10^3\right)\ \text{N} = 194\ \text{kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 58.86\left(10^3\right)\ \text{N} - 117.72\left(10^3\right)\ \text{N} = 0$$

$$A_x = 176.58\left(10^3\right)\ \text{N} = 177\ \text{kN} \qquad \textbf{Ans.}$$

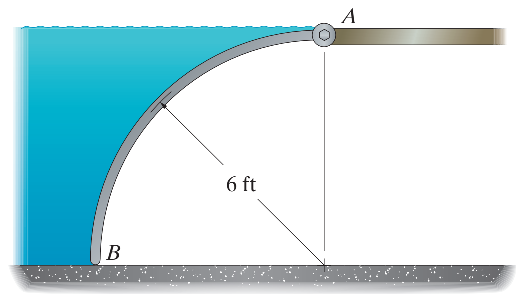
$$+\uparrow \Sigma F_y = 0; \quad -A_y - 73.575\left(10^3\right)\ \text{N} - 88.29\left(10^3\right)\ \text{N} + 193.748\left(10^3\right)\ \text{N} = 0$$

$$A_y = 31.88\left(10^3\right)\ \text{N} = 31.9\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 194$ kN
$A_x = 177$ kN
$A_y = 31.9$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–106. The quarter-circular arched gate is 3 ft wide, is pinned at A, and rests on the smooth support at B. Determine the reactions at these supports due to the water pressure.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$A_{ADB} = (6\ \text{ft})(6\ \text{ft}) - \frac{\pi}{4}(6\ \text{ft})^2 = (36 - 9\pi)\ \text{ft}^2$$

$$\bar{x} = \frac{(3\ \text{ft})[(6\ \text{ft})(6\ \text{ft})] - \left(\dfrac{8}{\pi}\ \text{ft}\right)\left[\dfrac{\pi}{4}(6\ \text{ft})^2\right]}{(36 - 9\pi)\ \text{ft}^2} = 4.6598\ \text{ft}$$

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *b*

$$N_B = \gamma_w h_B b = (62.4\ \text{lb/ft}^3)(6\ \text{ft})(3\ \text{ft}) = 1123.2\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(1123.2\ \text{lb/ft})(6\ \text{ft}) = 3369.6\ \text{lb}$$

The vertical component of the resultant force acting on the gate is equal to the weight of the column of water above the gate (shown shaded in Fig. *b*).

$$F_v = \gamma_w \forall = \gamma_w A_{ADB} b = (62.4\ \text{lb/ft}^3)\left[(36 - 9\pi)\ \text{ft}^2\right](3\ \text{ft}) = 1446.24\ \text{lb}$$

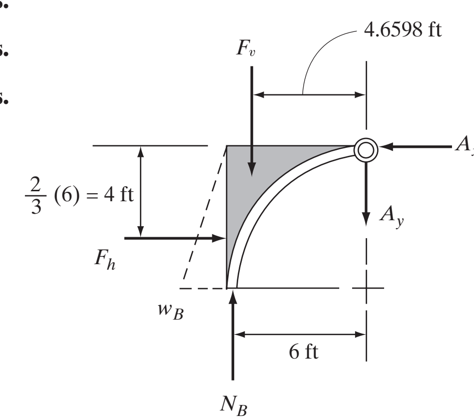
Considering the equilibrium of the FBD of the gate in Fig. *b*,

$\zeta+\Sigma M_A = 0;\quad (1446.24\ \text{lb})(4.6598\ \text{ft}) + (3369.6\ \text{lb})(4\ \text{ft}) - N_B(6\ \text{ft}) = 0$

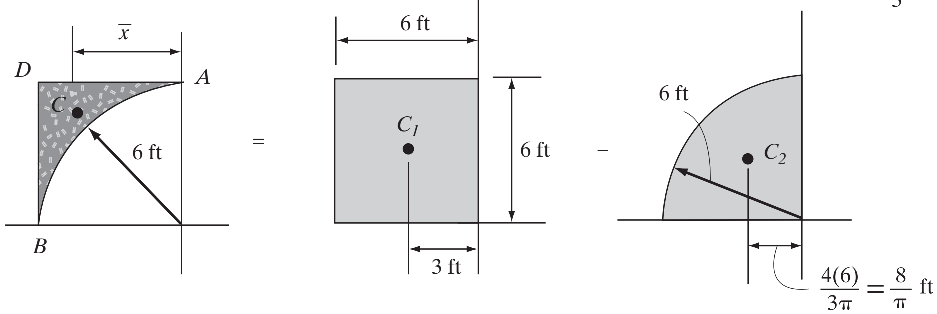
$N_B = 3369.6\ \text{lb} = 3.37\ \text{kip}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 3369.6\ \text{lb} - A_x = 0\quad A_x = 3369.6\ \text{lb} = 3.37\ \text{kip}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_y = 0;\quad 3369.6\ \text{lb} - 1446.24\ \text{lb} - A_y = 0\quad A_y = 1923.36\ \text{lb} = 1.92\ \text{kip}$ **Ans.**



(b)

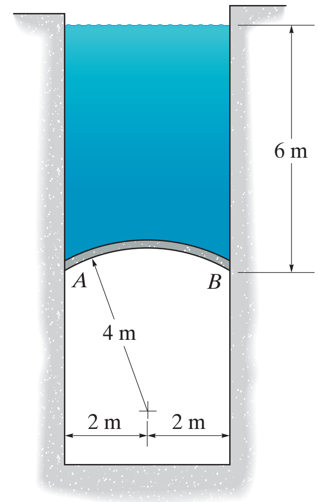


(a)

Ans:
$N_B = 3.37\ \text{kip}$
$A_x = 3.37\ \text{kip}$
$A_y = 1.92\ \text{kip}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–107. Water is confined in the vertical chamber, which is 2 m wide. Determine the resultant force it exerts on the arched roof AB.



SOLUTION

Due to symmetry, the resultant force that the water exerts on arch AB will be vertically downward, and its magnitude is equal to the weight of water of the shaded block in Fig. a. This shaded block can be subdivided into two parts as shown in Figs. b and c. The block in Fig. c should be considered a negative part since it is a hole. From the geometry in Fig. a,

$$\theta = \sin^{-1}\left(\frac{2\text{ m}}{4\text{ m}}\right) = 30^\circ$$

$$h = 4\cos 30^\circ\text{ m}$$

Then, the area of the parts in Figs. b and c are

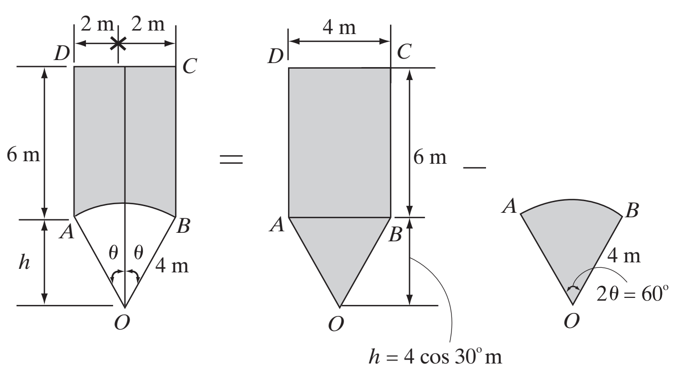
$$A_{OBCDAO} = 6\text{ m}(4\text{ m}) + \frac{1}{2}(4\text{ m})(4\cos 30^\circ\text{ m}) = 30.928\text{ m}^2$$

$$A_{OBAO} = \frac{60^\circ}{360^\circ}(\pi r^2) = \frac{60^\circ}{360^\circ}\left[\pi(4\text{ m})^2\right] = 2.6667\pi\text{ m}^2$$

Therefore,

$$F_R = W = \rho_w g \forall = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[\left(30.928\text{ m}^2 - 2.6667\pi\text{ m}^2\right)(2\text{ m})\right]$$

$$= 442.44\left(10^3\right)\text{ N} = 442\text{ kN} \qquad \textbf{Ans.}$$

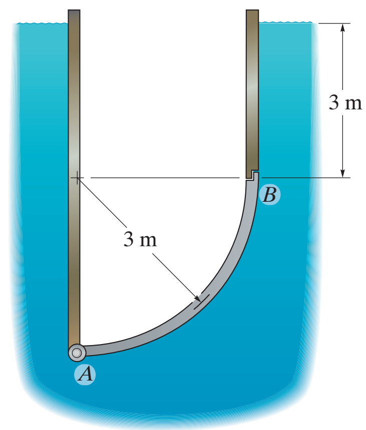


(a) (b) (c)

Ans:
442 kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–108.** Determine the horizontal and vertical components of reaction at the hinge A and the normal reaction at B caused by the water pressure. The gate has a width of 3 m.



SOLUTION

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(3\ \text{m}) = 176.58(10^3)\ \text{N/m}$$

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(3\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

Thus,

$$(F_h)_1 = \left[88.29(10^3)\ \text{N/m}\right](3\ \text{m}) = 264.87(10^3)\ \text{N} = 264.87\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}\left[176.58(10^3)\ \text{N/m} - 88.29(10^3)\ \text{N/m}\right](3\ \text{m}) = 132.435(10^3)\ \text{N} = 132.435\ \text{kN}$$

They act at

$$\tilde{y}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{y}_2 = \frac{1}{3}(3\ \text{m}) = 1\ \text{m}$$

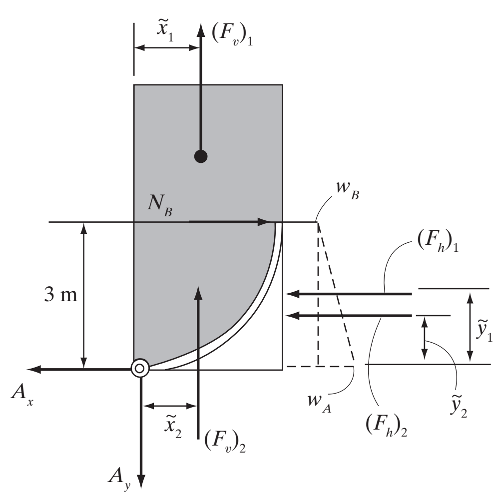
The vertical component of the resultant force acting on the gate is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*) but acts upward

$$(F_v)_1 = \rho_w g V_1 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(3\ \text{m})(3\ \text{m})(3\ \text{m})\right] = 264.87(10^3)\ \text{N} = 264.87\ \text{kN}$$

$$(F_v)_2 = \rho_w g V_2 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{4}(3\ \text{m})^2(3\ \text{m})\right] = 66.2175\pi(10^3)\ \text{N} = 66.2175\pi\ \text{kN}$$

They act at

$$\tilde{x}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{x}_2 = \frac{4(3\ \text{m})}{3\pi} = \left(\frac{4}{\pi}\right)\text{m}$$



(a)

2–108. Continued

Considering the equilibrium of the FBD of the gate in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \quad (264.87\text{ kN})(1.5\text{ m}) + (132.435\text{ kN})(1\text{ m}) + (264.87\text{ kN})(1.5\text{ m}) + (66.2175\pi\text{ kN})\left(\frac{4}{\pi}\text{ m}\right) - N_B(3\text{ m}) = 0$$

$$N_B = 397.305\text{ kN} = 397\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 397.305\text{ kN} - 264.87\text{ kN} - 132.435\text{ kN} - A_x = 0$$

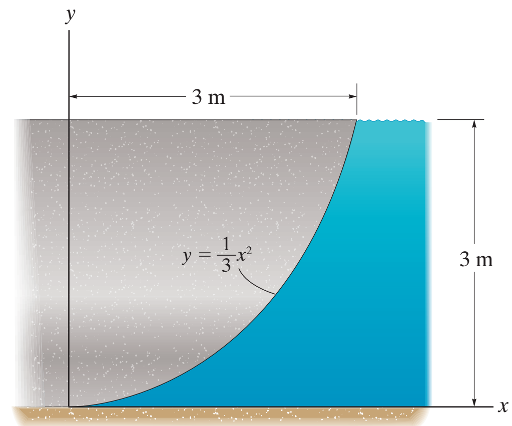
$$A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 264.87\text{ kN} + 66.2175\pi\text{ kN} - A_y = 0$$

$$A_y = 472.90\text{ kN} = 473\text{ kN} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–109. The 5-m-wide overhang is in the form of a parabola, as shown. Determine the magnitude and direction of the resultant force on the overhang.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(5\ \text{m}) = 147.15(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2}w_A h_A = \frac{1}{2}\left[147.15(10^3)\ \text{N/m}\right](3\ \text{m}) = 220.725(10^3)\ \text{N} = 220.725\ \text{kN}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above surface *AB* of the wall (shown shaded in Fig. *a*) but acts upward. The volume of this column of water is

$$\forall = \frac{2}{3}ahb = \frac{2}{3}(3\ \text{m})(3\ \text{m})(5\ \text{m}) = 30\ \text{m}^3$$

Thus,

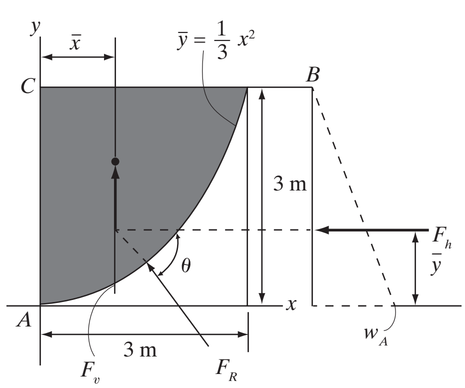
$$F_v = \rho_w g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(30\ \text{m}^3) = 294.3(10^3)\ \text{N} = 294.3\ \text{kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(220.725\ \text{kN})^2 + (294.3\ \text{kN})^2} = 367.875\ \text{kN} = 368\ \text{kN} \quad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{294.3\ \text{kN}}{220.725\ \text{kN}}\right) = 53.13° = 53.1° \quad \textbf{Ans.}$$

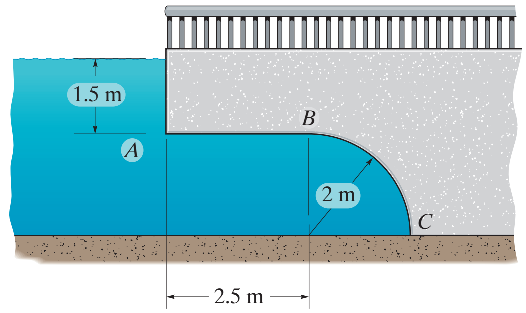


(a)

Ans:
$F_R = 368\ \text{kN}$
$\theta = 53.1°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

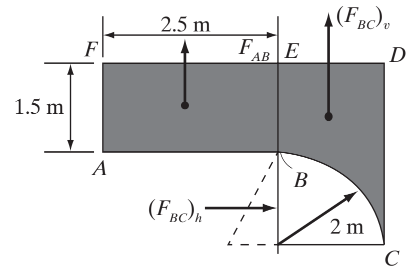
2–110. Determine the resultant force that water exerts on the overhang sea wall along *ABC*. The wall is 2 m wide.



SOLUTION

Horizontal Component. Since *AB* is along the horizontal, no horizontal component exists. The horizontal component of the force on *BC* is

$$(F_{BC})_h = \gamma_w \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(1.5\ \text{m} + \frac{1}{2}(2\ \text{m})\right)(2\ \text{m}(2\ \text{m})) = 98.1\left(10^3\right)\text{N}$$



(a)

Vertical Component. The force on *AB* and the vertical component of the force on *BC* is equal to the weight of the water contained in blocks *ABEFA* and *BCDEB* (shown shaded in Fig. *a*), but it acts upwards. Here, $A_{ABEFA} = 1.5\ \text{m}(2.5\ \text{m}) = 3.75\ \text{m}^2$ and $A_{BCDEB} = (3.5\ \text{m})(2\ \text{m}) - \frac{\pi}{4}(2\ \text{m})^2 = (7 - \pi)\ \text{m}^2$. Then,

$$F_{AB} = \gamma_w \forall_{ABEFA} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\left(3.75\ \text{m}^2\right)(2\ \text{m})\right]$$
$$= 73.575\left(10^3\right)\text{N} = 73.6\ \text{kN}$$

$$(F_{BC})_v = \gamma_w \forall_{BCDEB} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(7 - \pi)\ \text{m}^2(2\ \text{m})\right]$$
$$= 75.702\left(10^3\right)\text{N}$$

Therefore,

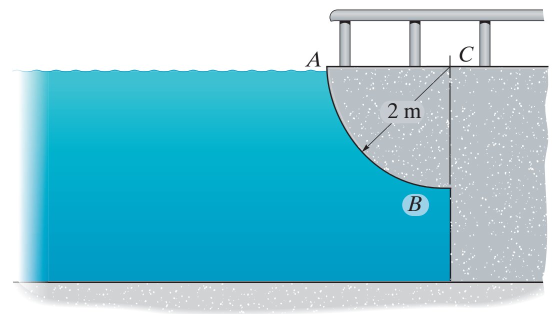
$$F_{BC} = \sqrt{(F_{BC})_h^2 + (F_{BC})_v^2} = \sqrt{\left[98.1\left(10^3\right)\text{N}\right]^2 + \left[75.702\left(10^3\right)\text{N}\right]^2}$$
$$= 123.91\left(10^3\right)\text{N} = 124\ \text{kN}$$

$$F_R = \sqrt{(F_{BC})_h^2 + \left[F_{AB} + (F_{BC})_v\right]^2}$$
$$= \sqrt{\left[98.1\left(10^3\right)\text{N}\right]^2 + \left[73.6\left(10^3\right)\text{N} + 75.702\left(10^3\right)\text{N}\right]^2}$$
$$= 178.6\left(10^3\right)\text{N} = 179\ \text{kN}$$ **Ans.**

Ans:
179 kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–111. Determine the magnitude and direction of the resultant hydrostatic force the water exerts on the face AB of the overhang if it is 2 m wide.



SOLUTION

Horizontal Component. The intensity of the distributed load at B is

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m})(2\ \text{m}) = 39.24(10^3)\ \text{N/m}$$

Then,

$$(F_R)_h = \frac{1}{2}(w_B)h_B = \frac{1}{2}(39.24(10^3)\ \text{N/m})(2\ \text{m}) = 39.24(10^3)\ \text{N}$$

Vertical Component. This component is equal to the weight of the water contained in the block shown shaded in Fig. *a*, but it acts upwards. Then

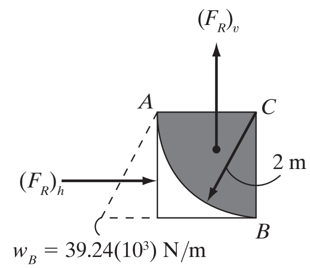
$$(F_R)_v = \rho_w g \forall_{ABCA} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{4}(2\ \text{m})^2(2\ \text{m})\right]$$
$$= 61.638(10^3)\ \text{N}\uparrow$$

Thus, the magnitude of the resultant force is

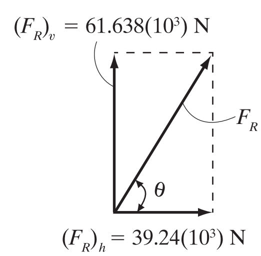
$$F_R = \sqrt{(F_R)_h^{\,2} + (F_R)_v^{\,2}} = \sqrt{[39.24(10^3)\ \text{N}]^2 + [61.638(10^3)\ \text{N}]^2}$$
$$= 73.07(10^3)\ \text{N} = 73.1\ \text{kN} \qquad \textbf{Ans.}$$

And its direction, Fig. *b*, is defined by

$$\theta = \tan^{-1}\left(\frac{(F_R)_v}{(F_R)_h}\right) = \tan^{-1}\left[\frac{61.638(10^3)\ \text{N}}{39.24(10^3)\ \text{N}}\right] = 57.5° \;\measuredangle \qquad \textbf{Ans.}$$



(a)

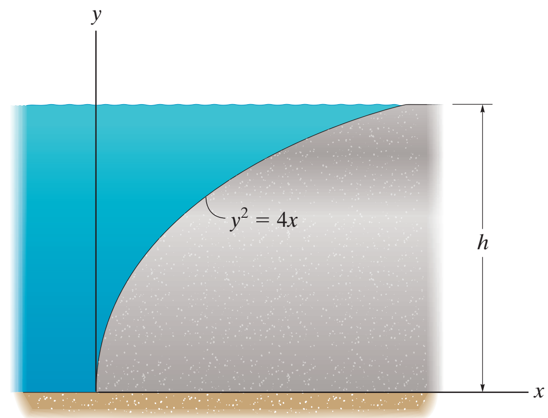


(b)

Ans:
$F_R = 73.1$ kN
$\theta = 57.5° \;\measuredangle$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–112.** The 5-m-wide wall is in the form of a parabola. If the depth of the water is $h = 4$ m, determine the magnitude and direction of the resultant force on the wall.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*.

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(4\ \text{m})(5\ \text{m}) = 196.2\left(10^3\right)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[196.2\left(10^3\right)\ \text{N/m}\right](4\ \text{m}) = 392.4\left(10^3\right)\ \text{N} = 392.4\ \text{kN}$$

It acts at

$$\tilde{y} = \frac{1}{3} h_A = \frac{1}{3}(4\ \text{m}) = \frac{4}{3}\text{m}$$

The vertical component of the resultant force is equal to the weight of the column of water above surface AB of the wall (shown shaded in Fig. *a*). The volume of this column of water is

$$\forall = \frac{1}{3} ahb = \frac{1}{3}(4\ \text{m})(4\ \text{m})(5\ \text{m}) = 26.67\ \text{m}^3$$

Thus,

$$F_r = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(26.67\ \text{m/s}) = 261.6\left(10^3\right)\ \text{N} = 261.6\ \text{kN}$$

It acts at

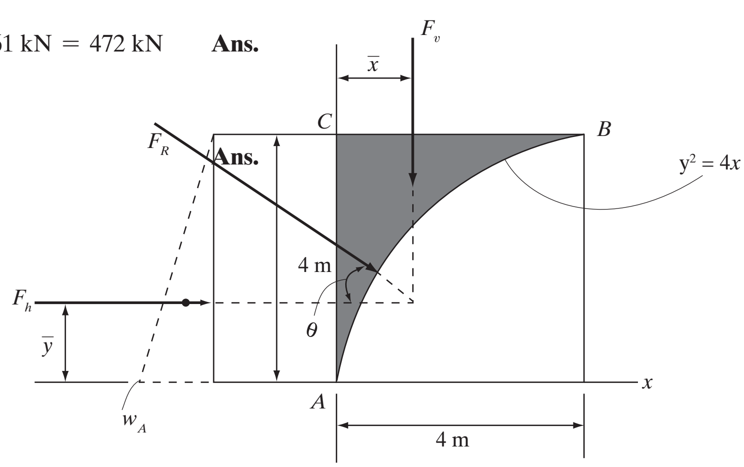
$$\tilde{x} = \frac{3}{10} a = \frac{3}{10}(4\ \text{m}) = \frac{6}{5}\text{m}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(392.4\ \text{kN})^2 + (261.6\ \text{kN})^2} = 471.61\ \text{kN} = 472\ \text{kN}$$ **Ans.**

And its direction is

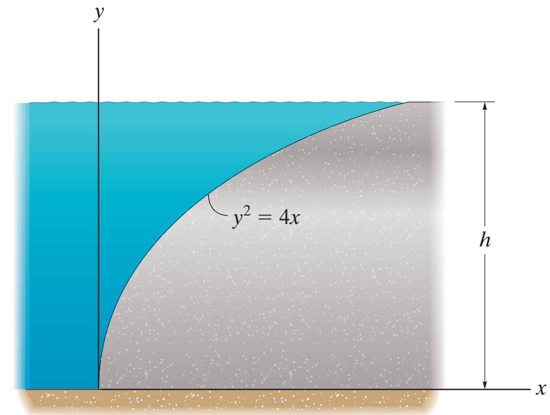
$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{261.6\ \text{kN}}{392.4\ \text{kN}}\right) = 33.69°$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–113. The 5-m wide wall is in the form of a parabola. Determine the magnitude of the resultant force on the wall as a function of depth h of the water. Plot the results of force (vertical axis) versus depth h for $0 \le h \le 4$ m. Give values for increments of $\Delta h = 0.5$ m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. a,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(h)(5\ \text{m}) = 49.05(10^3)h$$

Thus,

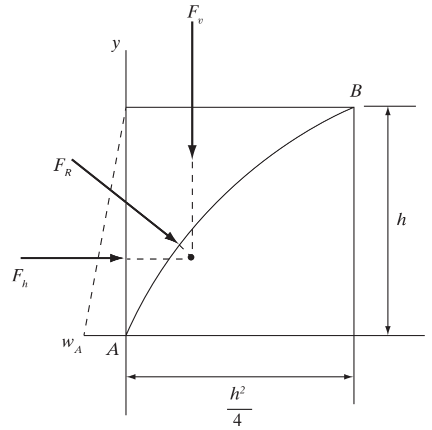
$$F_h = \frac{1}{2}w_A h_A = \frac{1}{2}\left[49.05(10^3)h\right]h = 24.525(10^3)h^2$$

The vertical component of the resultant force is equal to the weight of the column of water above surface AB of the wall (shown shaded in Fig. a) The volume of this column of water is

$$\forall = \frac{1}{3}ahb = \frac{1}{3}\left(\frac{h^2}{4}\right)(h)(5\ \text{m}) = \frac{5}{12}h^3$$

Thus,

$$F_v = \rho_w g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{5}{12}\text{h}^3\right) = 4087.5\,h^3$$



(a)

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}}$$

$$F_R = \sqrt{\left[24.525(10^3)h^2\right]^2 + \left[4087.5\,h^3\right]^2}$$

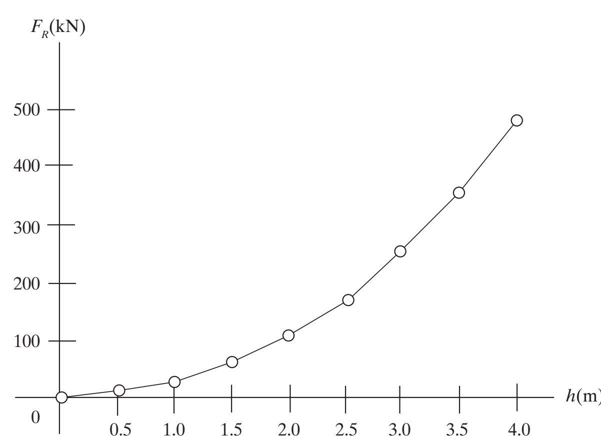
$$F_R = \sqrt{601.48(10^6)h^4 + 16.71(10^6)h^6}$$

$h(m)$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
F_R(kN)	0	6.15	24.9	56.9	103.4	166.1	246.8	347.8	471.6

The plot of F_R vs h is shown in Fig. b

$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6}\right]\text{N}$$

where h is in m.



(b)

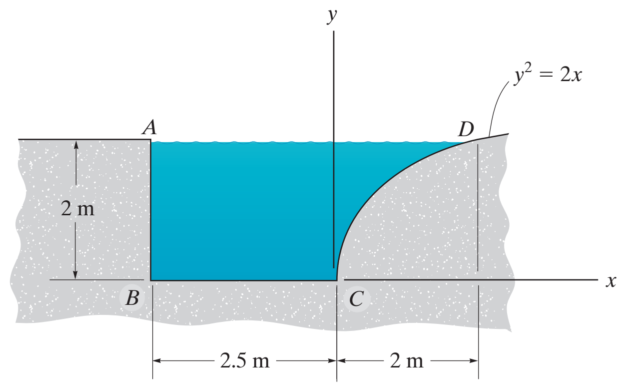
Ans:

$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6}\right]\text{N}$$

where h is in m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–114. Determine the resultant force the water exerts on AB, BC, and CD of the enclosure, which is 3 m wide.



SOLUTION

Horizontal Component. The horizontal component of the force CD is the same as the force on AB. Its magnitude can be determined from

$$F_{AB} = (F_{CD})_h = \gamma_w \bar{h} A = (100\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})(2\ \text{m}(3\ \text{m}))$$

$$= 58.86(10^3)\ \text{N} = 58.9\ \text{kN} \qquad \textbf{Ans.}$$

Vertical Component. The force on BC and the vertical component of the force on CD is equal to the weight of the water contained in blocks $ABCEA$ and $CDEC$ (shown shaded in Fig. a). Here, $A_{ABCEA} = 2\ \text{m}(2.5\ \text{m}) = 5\ \text{m}^2$ and $A_{CDEC} = \frac{1}{3}bh = \frac{1}{3}(2\ \text{m})(2\ \text{m}) = 1.3333\ \text{m}^2$ (Table 2-1). Then,

$$F_{BC} = \gamma_w \forall_{ABCEA} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[5\ \text{m}^2(3\ \text{m})\right]$$

$$= 147.15(10^3)\ \text{N} = 147\ \text{kN} \qquad \textbf{Ans.}$$

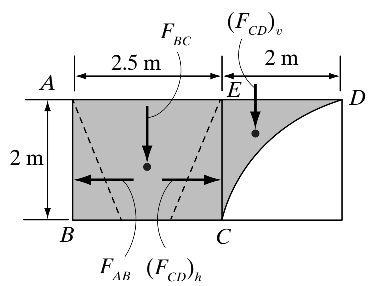
$$(F_{CD})_V = \gamma_w \forall_{CDEC} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[1.3333\ \text{m}^2(3\ \text{m})\right]$$

$$= 39.24(10^3)\ \text{N}$$

Therefore,

$$F_{CD} = \sqrt{(F_{CD})_h^2 + (F_{CD})_v^2} = \sqrt{\left[58.86(10^3)\ \text{N}\right]^2 + \left[39.24(10^3)\ \text{N}\right]^2}$$

$$= 70.74(10^3)\ \text{N} = 70.7\ \text{kN} \qquad \textbf{Ans.}$$

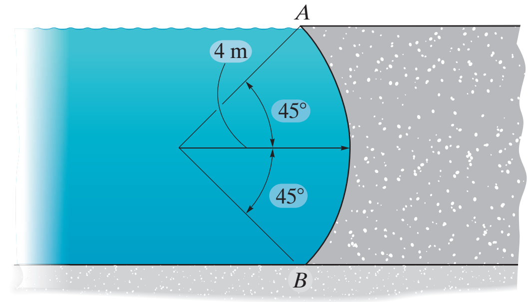


(a)

Ans:
$F_{AB} = 58.9$ kN
$F_{BC} = 147$ kN
$F_{CD} = 70.7$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–115. Determine the magnitude of the resultant force the water exerts on the curved vertical wall. The wall is 2 m wide.



SOLUTION

Horizontal Component. This component can be determined by applying

$$(F_{AB})_h = \gamma_w \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\text{m/s}^2\right)(4 \sin 45°)\left[2(4 \sin 45°\ \text{m})(2\ \text{m})\right]$$
$$= 313.92\left(10^3\right)\ \text{N}$$

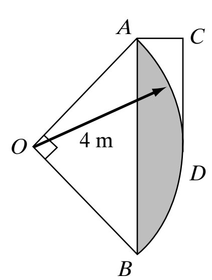
Vertical Component. The downward force on BD and the upward force on AD is equal to the weight of the water contained in blocks $ACDBA$ and $ACDA$, respectively. Thus, the net downward force on ADB is equal to the weight of water contained in block $ADBA$ shown shaded in Fig. *a*. Here, $A_{ADBA} = \frac{\pi}{4}(4\ \text{m})^2 - 2\left[\frac{1}{2}(4 \sin 45°)(4 \cos 45°)\right] = (4\pi - 8)\ \text{m}^2$.

Then,

$$(F_{AB})v = \gamma_w \forall_{ADBA} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(4\pi - 8)\text{m}^2(2\ \text{m})\right]$$
$$= 89.59\left(10^3\right)\ \text{N}$$

Then,

$$F_{AB} = \sqrt{(F_{AB})_h^2 + (F_{AB})_v^2} = \sqrt{\left[313.92\left(10^3\right)\text{N}\right]^2 + \left[89.59\left(10^3\right)\text{N}\right]^2}$$
$$= 326.45\left(10^3\right)\ \text{N} = 326\ \text{kN} \qquad \textbf{Ans.}$$

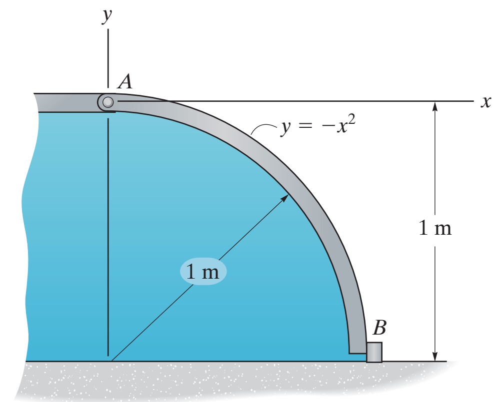


(a)

Ans:
326 kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–116.** Gate AB has a width of 0.5 m and a radius of 1 m. Determine the horizontal and vertical components of reaction at the pin A and the horizontal reaction at the smooth stop B due to the water pressure.



SOLUTION

Vertical Component. This component is equal to the weight of the water contained in the block shown shaded in Fig. *a*, but it acts upward. This block can be subdivided into parts (1) and (2) as shown in Figs. *b* and *c*. Part (2) is a hole and should be considered as a negative part. Thus, the area of the block, Fig. *a*, is $\Sigma A = (1\text{ m})(1\text{ m}) - \frac{\pi}{4}(1\text{ m})^2 = 0.2146\text{ m}^2$ and the horizontal distance measured form its centroid to point A is

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{0.5\text{ m}(1\text{ m})(1\text{ m}) - \frac{4(1\text{ m})}{3\pi}\left[\frac{\pi}{4}(1\text{ m})^2\right]}{0.2146\text{ m}^2} = 0.7766\text{ m}$$

The magnitude of the vertical component is

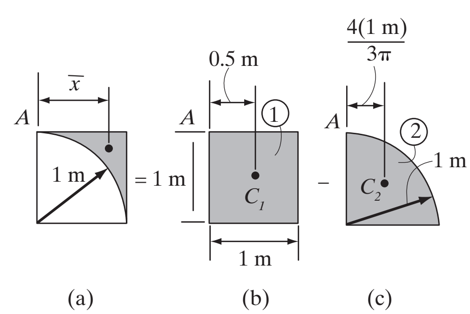
$$(F_R)_v = \rho_w g V = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[0.2146\text{ m}^2(0.5\text{ m})\right]$$
$$= 1052.62\text{ N}$$

Horizontal Component. The intensity of the distributed load at B is

$$w_B = \rho_w g h_B b = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1\text{ m})(0.5\text{ m}) = 4.905\left(10^3\right)\text{ N/m}$$

Then,

$$(F_R)_h = \frac{1}{2}w_B h_B = \frac{1}{2}\left[4.905\left(10^3\right)\text{ N/m}\right](1\text{ m}) = 2452.5\text{ N}$$



Considering the free-body diagram of the gate in Fig. *d*,

$$\circlearrowleft +\Sigma M_A = 0;\quad (2452.5\text{ N})\left[\frac{2}{3}(1\text{ m})\right] + (1052.62\text{ N})(0.7766\text{ N}) - F_B(1\text{ m}) = 0$$

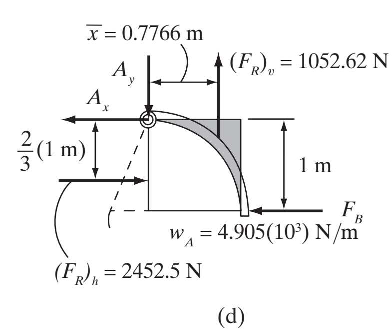
$$F_B = 2452.5\text{ N} = 2.45\text{ kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0;\quad 2452.5\text{ N} - 2452.5\text{ N} - A_x = 0$$

$$A_x = 0 \qquad \textbf{Ans.}$$

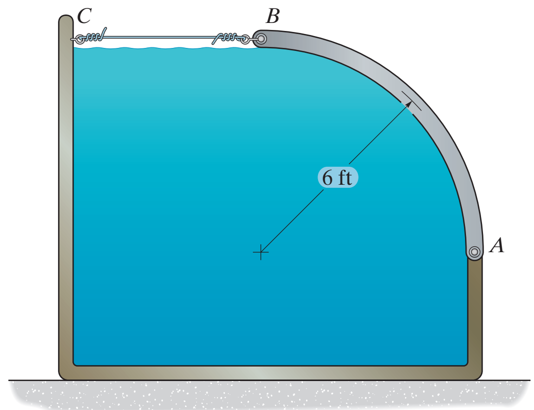
$$+\uparrow \Sigma F_y = 0;\quad 1052.62\text{ N} - A_y = 0$$

$$A_y = 1052.62\text{ N} = 1.05\text{ kN} \qquad \textbf{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–117. A quarter-circular shell is pinned at A and tied to the tank's wall using the cable BC. If the tank and shell are each 4 ft wide, determine the horizontal and vertical components of reaction at A, and the tension in the cable due to the water pressure.



SOLUTION

Referring to the geometry shown in Fig. *a*

$$A_{ADB} = (6\ \text{ft})(6\ \text{ft}) - \frac{\pi}{4}(6\ \text{ft})^2 = (36 - 9\pi)\ \text{ft}^2$$

$$\bar{x} = \frac{(3\ \text{ft})\left[(6\ \text{ft})(6\ \text{ft})\right] - \left[\left(6 - \frac{8}{\pi}\right)\text{ft}\right]\left[\frac{\pi}{4}(6\ \text{ft})^2\right]}{(36 - 9\pi)\ \text{ft}^2} = 1.3402\ \text{ft}$$

The horizontal component of the resultant force acting on the shell is equal to the pressure force on the vertically projected area of the shell. Referring to Fig. *b*

$$w_{\bar{A}} = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(4\ \text{ft}) = 1497.6\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(1497.6\ \text{lb/ft})(6\ \text{ft}) = 4492.8\ \text{lb}$$

The vertical component of the resultant force acting on the shell is equal to the weight of the imaginary column of water above the shell (shown shaded in Fig. *b*) but acts upwards.

$$F_v = \gamma_w \forall = \gamma_w A_{ADB} b = \left(62.4\ \text{lb/ft}^3\right)\left[(36 - 9\pi)\ \text{ft}^2\right](4\ \text{ft}) = 1928.33\ \text{lb}$$

Write the moment equation of equilibrium about A by referring to Fig. *b*,

$$\circlearrowleft + \Sigma M_A = 0;\ T_{BC}(6\ \text{ft}) - (1928.33\ \text{lb})(1.3402\ \text{ft}) - (4492.8\ \text{lb})(2\ \text{ft}) = 0$$

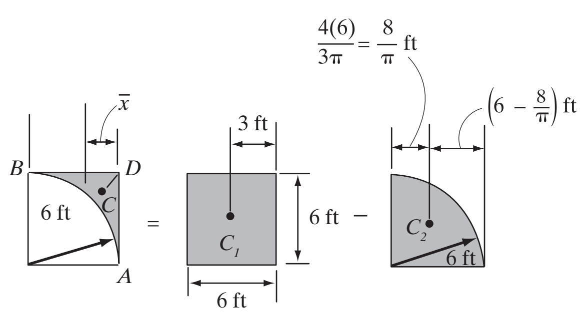
$$T_{BC} = 1928.33\ \text{lb} = 1.93\ \text{kip}$$ **Ans.**

$$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad -A_x + 4492.8\ \text{lb} - 1928.32\ \text{lb} = 0$$

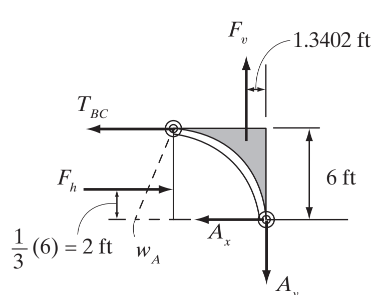
$$A_x = 2564.5\ \text{lb} = 2.56\ \text{kip}$$ **Ans.**

$$\uparrow + \Sigma F_y = 0 \quad 1928.33\ \text{lb} - A_y = 0$$

$$A_y = 1928.33\ \text{lb} = 1.93\ \text{kip}$$ **Ans.**



(a)

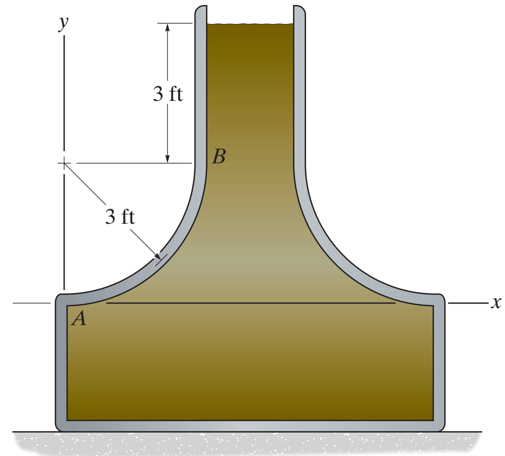


(b)

Ans:
$T_{BC} = 1.93$ kip
$A_x = 2.56$ kip
$A_y = 1.93$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–118. The bin is 4 ft wide and filled with linseed oil. Determine the horizontal and vertical components of the force the oil exerts on the curved segment AB. Also, find the location of the points of application of these components acting on the segment, measured from point A. $\gamma_{lo} = 58.7\ \text{lb/ft}^3$.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of curve AB of the bin. Referring to Fig. a,

$$w_B = \gamma_o h_B b = (58.7\ \text{lb/ft}^3)(3\ \text{ft})(4\ \text{ft}) = 704.4\ \text{lb/ft}$$

$$w_A = \gamma_o h_A b = (58.7\ \text{lb/ft}^3)(6\ \text{ft})(4\ \text{ft}) = 1408.8\ \text{lb/ft}$$

Then,

$$(F_h)_1 = (704.4\ \text{lb/ft})(3\ \text{ft}) = 2113.2\ \text{lb}$$

$$(F_h)_2 = \frac{1}{2}(1408.8\ \text{lb/ft} - 704.4\ \text{lb/ft})(3\ \text{ft}) = 1056.6\ \text{lb}$$

Thus,

$$F_h = (F_h)_1 + (F_h)_2 = 2113.2\ \text{lb} + 1056.6\ \text{lb} = 3169.8\ \text{lb} = 3.17\ \text{kip} \qquad \textbf{Ans.}$$

Here, $\tilde{y}_1 = \frac{1}{2}(3\ \text{ft}) = 1.5\ \text{ft}$ and $\tilde{y}_2 = \frac{1}{3}(3\ \text{ft}) = 1\ \text{ft}$. The location of the point of application of F_h can be determined from

$$\bar{y} = \frac{\Sigma \bar{y} F}{\Sigma F} = \frac{(1.5\ \text{ft})(2113.2\ \text{lb}) + (1\ \text{ft})(1056.6\ \text{lb})}{3169.8\ \text{lb}} = 1.3333\ \text{ft}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above curve AB of the bin (shown shaded in Fig. a) but acts upwards

$$(F_v)_1 = \gamma_w \mathcal{V}_1 = (58.7\ \text{lb/ft}^3)\left[(3\ \text{ft})(3\ \text{ft})(4\ \text{ft})\right] = 2113.2\ \text{lb}$$

$$(F_v)_2 = \gamma_w \mathcal{V}_2 = (58.7\ \text{lb/ft}^3)\left[\frac{\pi}{4}(3\ \text{ft})^2(4\ \text{ft})\right] = 1659.70\ \text{lb}$$

Thus,

$$F_v = (F_v)_1 + (F_v)_2 = 2113.2\ \text{lb} + 1659.70\ \text{lb} = 3772.90\ \text{lb} = 3.77\ \text{kip} \qquad \textbf{Ans.}$$

Here, $\tilde{x}_1 = \frac{1}{2}(3\ \text{ft}) = 1.5\ \text{ft}$ and $\tilde{x}_2 = \frac{4(3\ \text{ft})}{3\pi} = \frac{4}{\pi}\ \text{ft}$. The location of the point of application of F_v can be determined from

$$\bar{x} = \frac{\Sigma \bar{x} F}{\Sigma F} = \frac{(1.5\ \text{ft})(2113.2\ \text{lb}) + \left(\frac{4}{\pi}\ \text{ft}\right)(1659.70\ \text{lb})}{3772.90\ \text{lb}} = 1.4002\ \text{ft}$$

The equation of the line of action of F_R is given by

$$y - \bar{y} = -\frac{F_v}{F_h}(x - \bar{x})$$

$$y - 1.3333 = -\frac{3772.9}{3169.8}(x - 1.4002)$$

$$y = -1.1903x + 3$$

2–118. Continued

Use substitution to find the intersection of this line and the circle $x^2 + (y - 3)^2 = 9$:

$$x^2 + \left[(-1.1903x + 3) - 3\right]^2 = 9$$

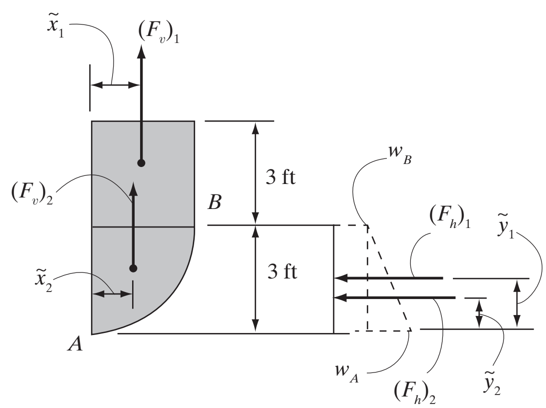
$$2.4167x^2 = 9$$

$$x = 1.9298 \text{ m} = 1.93 \text{ m} \qquad \textbf{Ans.}$$

Back-substituting,

$$y = -1.1903(1.9298) + 3$$

$$= 0.7035 \text{ m} = 0.704 \text{ m} \qquad \textbf{Ans.}$$

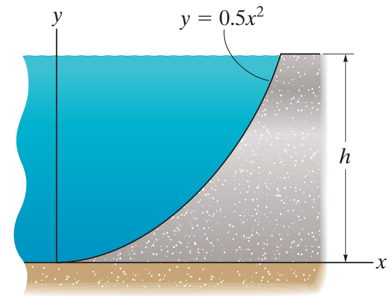


(a)

Ans:
$F_h = 3.17$ kip
$F_v = 3.77$ kip
$x = 1.93$ m
$y = 0.704$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–119. If the water depth is $h = 2$ m, determine the magnitude and direction of the resultant force, due to water pressure acting on the parabolic surface of the dam, which has width of 5 m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the dam. Referring to Fig. *a*

$$w_A = \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})(5 \text{ m}) = 98.1(10^3) \text{ N/m}$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[98.1(10^3) \text{ N/m}\right](2 \text{ m}) = 98.1(10^3) \text{ N} = 98.1 \text{ kN}$$

The vertical component of the resultant force is equal to the weight of the column of water above the dam surface (shown shaded in Fig. *a*). The volume of this column of water is

$$\forall = \frac{2}{3} ahb = \frac{2}{3}(2 \text{ m})(2 \text{ m})(5 \text{ m}) = 13.33 \text{ m}^3$$

Thus,

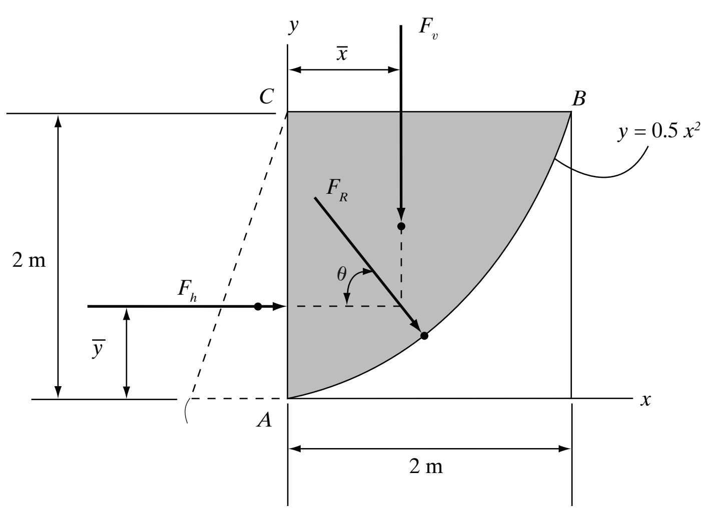
$$F_v = \rho_w g \forall = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(13.33 \text{ m}^3) = 130.80(10^3) \text{ N} = 130.80 \text{ kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(98.1 \text{kN})^2 + (130.80 \text{ kN})^2} = 163.5 \text{ kN} \qquad \textbf{Ans.}$$

and its direction is

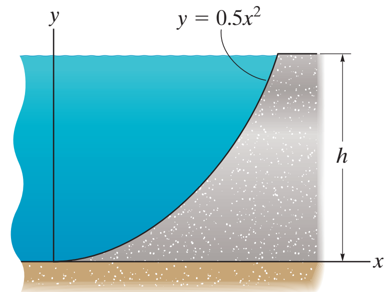
$$\theta = \tan^{-1}\frac{F_v}{F_h} = \tan^{-1}\left(\frac{130.80 \text{ kN}}{98.1 \text{ kN}}\right) = 53.1° \qquad \textbf{Ans.}$$



Ans:
$F_R = 163.5$ kN
$\theta = 53.1°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–120.** Determine the magnitude of the resultant force due to water pressure acting on the parabolic surface of the dam as a function of the depth h of the water. Plot the results of force (vertical axis) versus depth h for $0 \le h \le 2$ m. Give values for increments of $\Delta h = 0.5$ m. The dam has a width of 5 m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the dam. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(h)(5\ \text{m}) = 49.05\left(10^3\right)h$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[49.05\left(10^3\right)h\right]h = 24.525\left(10^3\right)h^2$$

The vertical component of the resultant force is equal to the weight of the column of water above the dam surface (shown shaded in Fig. *a*). The volume of this column of water is

$$\mathcal{V} = \frac{2}{3}ahb = \frac{2}{3}\left(\sqrt{2h}\right)(h)(5) = 4.7140\,h^{3/2}$$

Thus,

$$F_v = \rho_w g \mathcal{V} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(4.7140\,h^{3/2}\right) = 46.2447\left(10^3\right)h^{3/2}$$

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}}$$

$$F_R = \sqrt{\left[24.525\left(10^3\right)h^2\right]^2 + \left[46.2447\left(10^3\right)h^{3/2}\right]^2}$$

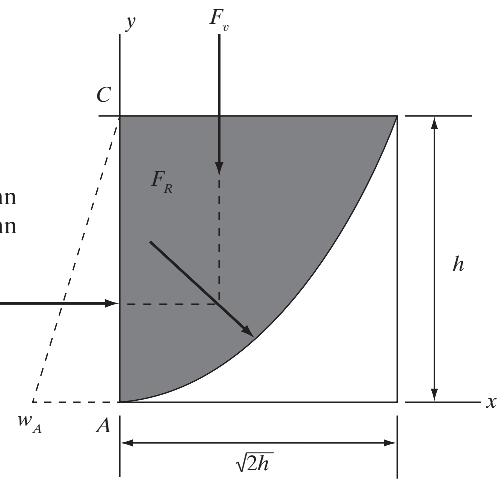
$$F_R = \sqrt{601.476\left(10^6\right)h^4 + 2.13858\left(10^9\right)h^3}$$

The plot of F_R vs h is shown in Fig. *b*.

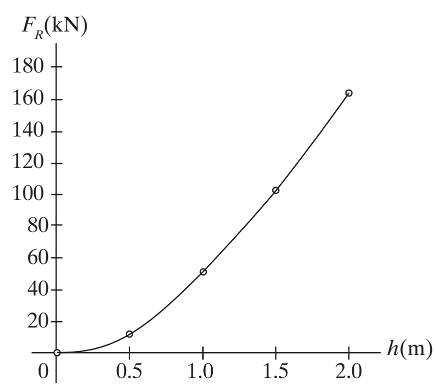
$$F_R = \left[\sqrt{601\left(10^6\right)h^4 + 2.14\left(10^9\right)h^3}\right]\text{N}$$

where h is in m. **Ans.**

$h(m)$	0	0.5	1.0	1.5	2.0
F_R(kN)	0	17.5	52.3	101.3	163.5



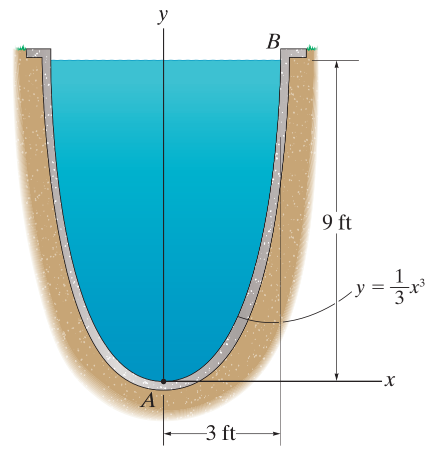
(a)



(b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–121. The canal transports water and has the cross section shown. Determine the magnitude and direction of the resultant force per unit length acting on wall AB, and the location of the center of pressure on the wall, measured with respect to the x and y axes.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of curve AB of the canal. Referring to Fig. a,

$$w_A = \gamma_w h_A b = (62.4\ \text{lb/ft}^3)(9\ \text{ft})(1\ \text{ft}) = 561.6\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(561.6\ \text{lb/ft})(9\ \text{ft}) = 2527.2\ \text{lb}$$

And it acts at

$$\bar{y} = \frac{1}{3}(9\ \text{ft}) = 3\ \text{ft}$$

The vertical component of the resultant force is equal to the weight of the column of water above curve AB of the canal

$$F_v = \int dF = \int \gamma_w\, d\cancel{V} = \gamma_w \int b\,dA = \gamma_w \int (1)(x\,dy)$$

However, $y = \frac{1}{3}x^3$ or $x = 3^{\frac{1}{3}} y^{\frac{1}{3}}$. Then

$$F_v = 62.4 \int_0^{9\ \text{ft}} 3^{\frac{1}{3}} y^{\frac{1}{3}}\, dy = 62.4\left(3^{\frac{1}{3}}\right)\left[\frac{3}{4} y^{\frac{4}{3}}\right]\Bigg|_0^{9\ \text{ft}} = 1263.6\ \text{lb}$$

The location of its point of application can be determined from

$$\bar{x} = \frac{\int \tilde{x}\,dF}{F_v} \quad \text{where } \tilde{x} = \frac{x}{2} \text{ and } dF = \gamma_w x\,dy = 62.4x\,dy$$

Thus,

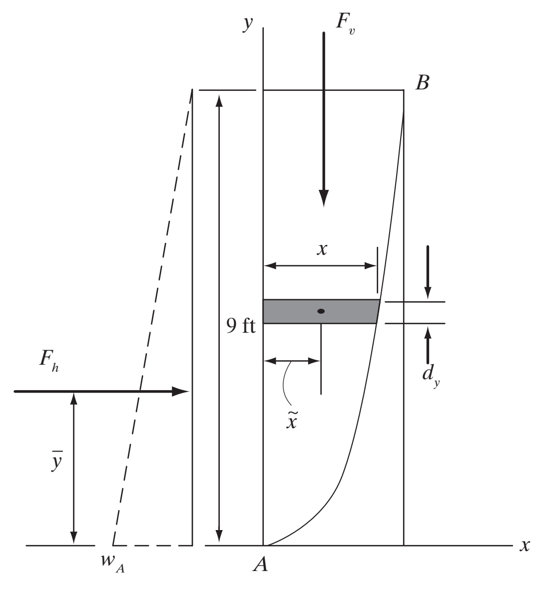
$$\bar{x} = \frac{\int_0^{9\ \text{ft}} \left(\frac{x}{2}\right)(62.4x\,dy)}{1263.6}$$

$$= \frac{31.2 \int_0^{9\ \text{ft}} x^2\,dy}{1263.6}$$

$$= \frac{31.2 \int_0^{9\ \text{ft}} 3^{\frac{2}{3}} y^{\frac{2}{3}}\,dy}{1263.6}$$

$$= \frac{31.2\left(3^{\frac{2}{3}}\right)\left(\frac{3}{5} y^{\frac{5}{3}}\right)\Big|_0^{9\ \text{ft}}}{1263.6}$$

$$= 1.20\ \text{ft}$$



(a)

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(2527.2\ \text{lb})^2 + (1263.6\ \text{lb})^2} = 2825.50\ \text{lb} = 2.83\ \text{kip} \quad \textbf{Ans.}$$

2–121. Continued

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{1263.6 \text{ lb}}{2527.2 \text{ lb}}\right) = 26.67° \quad \textbf{Ans.}$$

The equation of the line of action of F_R is

$$y - \bar{y} = m(x - \bar{x});\ y - 3 = -\tan\ 26.57°(x - 1.20)$$

$$y = -0.5x + 3.6$$

The intersection point of the line of action of F_R and surface AB can be obtained by solving simultaneously this equation and that of AB.

$$\frac{1}{3}x^3 = -0.5x + 3.6$$

$$\frac{1}{3}x^3 + 0.5x - 3.6 = 0$$

Solving numerically

$$x = 1.9851 \text{ ft} = 1.99 \text{ ft} \quad \textbf{Ans.}$$

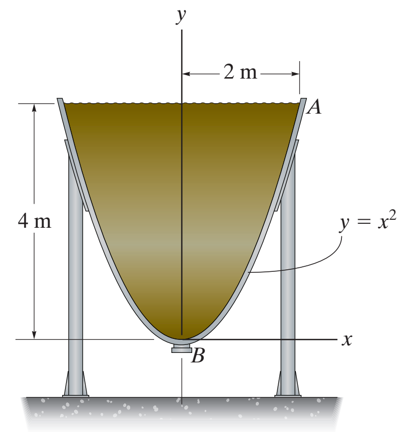
when $x = 1.9851$ ft,

$$y = \frac{1}{3}\left(1.9851^3\right) = 2.61 \text{ ft} \quad \textbf{Ans.}$$

Ans:
$F_R = 2.83$ kip
$\theta = 26.6°$
$x = 1.99$ ft
$y = 2.61$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–122. The settling tank is 3 m wide and contains turpentine having a density of 860 kg/m^3. If the parabolic shape is defined by $y = (x^2)$ m, determine the magnitude and direction of the resultant force the turpentine exerts on the side AB of the tank.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of surface AB. Referring to Fig. a,

$$w_B = \rho_t g h_B b = (860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4\ \text{m})(3\ \text{m}) = 101.24(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2}\left[101.24(10^3)\ \text{N/m}\right](4\ \text{m}) = 202.48(10^3)\ \text{N} = 202.48\ \text{kN}$$

The vertical component of the resultant forced is equal to the weight of the column of turpentine above surface AB of the wall (shown shaded in Fig. a). The volume of this column of water is

$$\cancel{V} = \frac{2}{3}ahb = \frac{2}{3}(2\ \text{m})(4\ \text{m})(3\ \text{m}) = 16\ \text{m}^3$$

Thus,

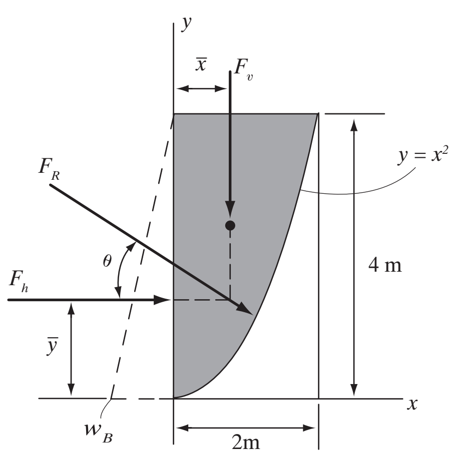
$$F_v = \rho_t g \cancel{V} = (860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(16\ \text{m}^3) = 134.99(10^3)\ \text{N} = 134.99\ \text{kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(202.48\ \text{kN})^2 + (134.99\ \text{kN})^2} = 243.35\ \text{kN} = 243\ \text{kN} \quad \textbf{Ans.}$$

And its direction as defined by

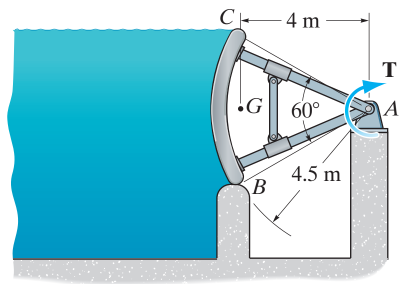
$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{134.99\ \text{kN}}{202.48\ \text{kN}}\right) = 33.79° \quad \textbf{Ans.}$$



Ans:
$F_R = 243\ \text{kN}$
$\theta = 33.7°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–123. The radial gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the torque **T** that must be applied at the pin A in order to open the gate. The gate has a mass of 5 Mg and a center of mass at G. It is 3 m wide.



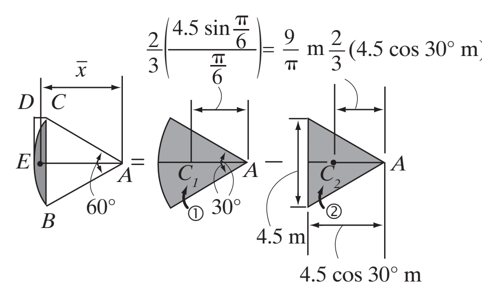
SOLUTION

Horizontal Component. This component can be determined by applying

$$(F_{BC})_h = \gamma_w \bar{h} A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.5 \sin 30°\ \text{m})\left[2(4.5 \sin 30°\ \text{m})(3\ \text{m})\right]$$
$$= 297.98(10^3)\ \text{N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$ shown shaded in Fig. *a*. This block can be subdivided into parts (1) and (2), Figs. *a* and *b*, respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{1}{2}(4.5\ \text{m})(4.5 \cos 30°\ \text{m}) = 1.8344\ \text{m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi}\ \text{m}\right)\left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{2}{3}(4.5 \cos 30°\ \text{m})\left[\frac{1}{2}(4.5\ \text{m})(4.5 \cos 30°\ \text{m})\right]}{1.8344\ \text{m}^2}$$
$$= 4.1397\ \text{m}$$



(a) (b) (c)

The magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w \forall_{BCEB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[1.8344\ \text{m}^2(3\ \text{m})\right]$$
$$= 53.985(10^3)\ \text{N}$$

When the gate is on the verge of opening, $N_B = 0$. Referring to the free-body diagram of the gate in Fig. *d*,

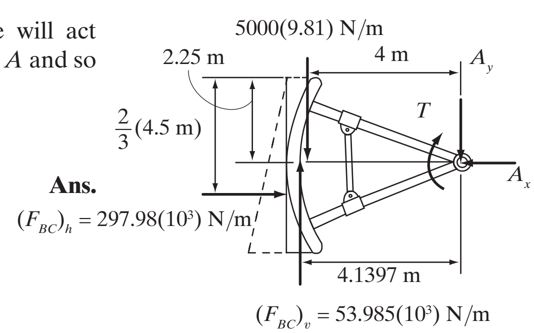
$$\circlearrowleft + \Sigma M_A = 0; \quad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) + \left[297.98(10^3)\ \text{N}\right]\left[\frac{2}{3}(4.5\ \text{m}) - 2.25\ \text{m}\right]$$
$$- \left[53.985(10^3)\ \text{N}\right](4.1397\ \text{m}) - T = 0$$

$$T = 196.2(10^3)\ \text{N}\cdot\text{m} = 196\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

This solution can be simplified if one realizes that the resultant force will act perpendicular to the circular surface. Therefore, F_{BC} will act through point A and so produces no moment about this point. Hence,

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) - T = 0$$

$$T = 196.2(10^3)\ \text{N}\cdot\text{m} = 196\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

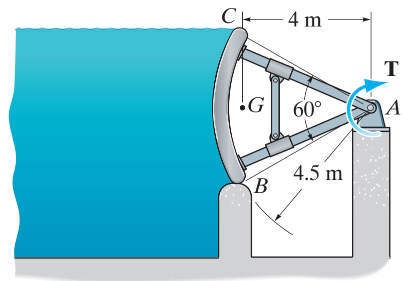


(d)

Ans:
196 kN · m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–124.** The radial gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the horizontal and vertical components of reaction at pin A and the vertical reaction at the spillway crest B. The gate has a weight of 5 Mg and a center of gravity at G. It is 3 m wide. Take $T = 0$.



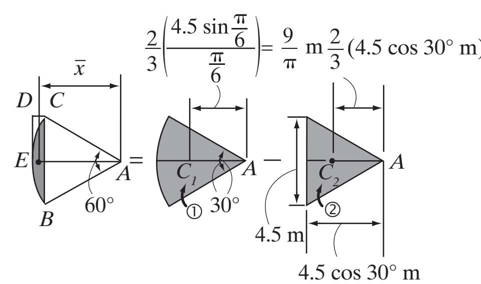
SOLUTION

Horizontal Component. This component can be determined from

$$(F_{BC})_h = \gamma_w \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(4.5\ \sin 30°\ \text{m})\left[2(4.5\ \sin 30°\ \text{m})(3\ \text{m})\right]$$
$$= 297.98\left(10^3\right)\ \text{N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$ shown shaded in Fig. a. This block can be subdivided into parts (1) and (2), Fig. a and b, respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{1}{2}(4.5\ \text{m})(4.5\cos 30°\ \text{m}) = 1.8344\ \text{m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi}\ \text{m}\right)\left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{2}{3}(4.5\cos 30°\ \text{m})\left[\frac{1}{2}(4.5\ \text{m})(4.5\cos 30°\ \text{m})\right]}{1.8344\ \text{m}^2}$$
$$= 4.1397\ \text{m}$$



(a) (b) (c)

Thus, the magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w \mathcal{V}_{BCEB} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[1.8344\ \text{m}^2(3\ \text{m})\right]$$
$$= 53.985\left(10^3\right)\ \text{N}$$

Considering the free-body diagram of the gate in Fig. d,

$\zeta + \Sigma M_A = 0;$ $\quad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) + \left[297.98\left(10^3\right)\ \text{N}\right]\left[\frac{2}{3}(4.5\ \text{m}) - 2.25\ \text{m}\right] - \left[53.985\left(10^3\right)\ \text{N}\right](4.1397\ \text{m})$

$-N_B(4.5\ \cos\ 30°\ \text{m}) = 0$

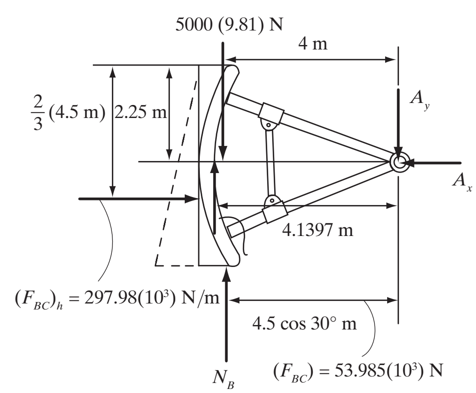
$N_B = 50.345\left(10^3\right)\ \text{N} = 50.3\ \text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 50.345\left(10^3\right)\ \text{N} + 53.985\left(10^3\right)\ \text{N} - 5000(9.81)\ \text{N} - A_y = 0$

$A_y = 55.28\left(10^3\right)\ \text{N} = 55.3\ \text{kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 297.98\left(10^3\right)\ \text{N} - A_x$

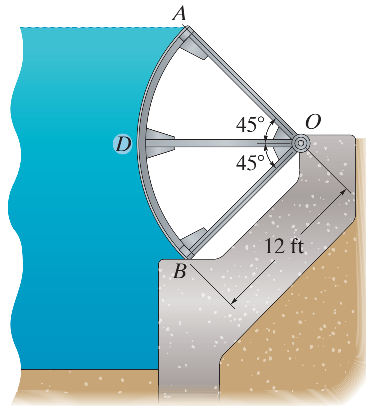
$A_x = 297.98\left(10^3\right)\ \text{N} = 298\ \text{kN}$ **Ans.**



(d)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–125. The 6-ft-wide plate in the form of a quarter-circular arc is used as a sluice gate. Determine the magnitude and direction of the resultant force of the water on the bearing O of the gate. What is the moment of this force about the bearing at O?



SOLUTION

Referring to the geometry in Fig. *a*,

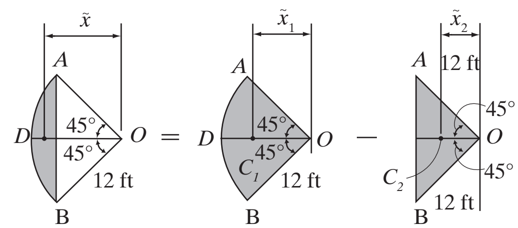
$$A_{ADB} = \frac{\pi}{4}(12\ \text{ft}^2) - \frac{1}{2}\left[(2)(12 \sin 45°\ \text{ft})(12 \cos 45°\ \text{ft})\right] = 41.097\ \text{ft}^2$$

$$\tilde{x}_1 = \frac{2}{3}\left(\frac{12\ \sin 45°\ \text{ft}}{\pi/4}\right) = 7.2025\ \text{ft}$$

$$\tilde{x}_2 = \frac{2}{3}(12 \cos 45°\ \text{ft}) = 5.6569\ \text{ft}$$

$$\bar{x} = \frac{(7.2025\ \text{ft})\left[\frac{\pi}{4}(12\ \text{ft})^2\right] - (5.6569\ \text{ft})\left[\frac{1}{2}(2)(12 \sin 45°\ \text{ft})(12 \cos 45°\ \text{ft})\right]}{41.097\ \text{ft}^2}$$

$$= 9.9105\ \text{ft}$$



(a)

The horizontal component of the resultant force is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *b*

$$w_B = \gamma_w h_B b = (62.4\ \text{lb/ft}^3)(16.971\ \text{ft})(6\ \text{ft}) = 6353.78\ \text{lb/ft}$$

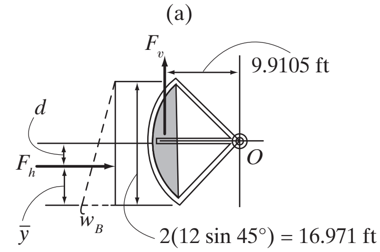
Thus,

$$F_h = \frac{1}{2}(6353.78\ \text{lb/ft})(16.971\ \text{ft}) = 53.9136(10^3)\ \text{lb} = 53.9136\ \text{kip}$$

It acts at

$$\bar{y} = \frac{1}{3}(16.971\ \text{ft}) = 5.657\ \text{ft}$$

$$d = 12\ \sin 45°\ \text{ft} - 5.657\ \text{ft} = 2.8284\ \text{ft}$$



(b)

The vertical component of the resultant force is equal to the weight of the block of water contained in sector ADB shown in Fig. *a* but acts upward.

$$F_v = \gamma_w \cancel{V}_{ADB} = \gamma_w A_{ADB} b = (62.4\ \text{lb/ft}^3)(41.097\ \text{ft}^2)(6\ \text{ft}) = 15.3868(10^3)\ \text{lb} = 15.3868\ \text{kip}$$

Thus, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(53.9136\ \text{kip})^2 + (15.3868\ \text{kip})^2} = 56.07\ \text{kip} = 56.1\ \text{kip} \qquad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{15.3868\ \text{kip}}{53.9136\ \text{kip}}\right) = 15.93° = 15.9° \ \measuredangle \qquad \textbf{Ans.}$$

By referring to Fig. *b*, the moment of F_R about O is

$$\circlearrowleft + (M_R)_O = \Sigma M_O;\ (M_R)_O = (53.9136\ \text{kip})(2.8284\ \text{ft}) - (15.3868\ \text{kip})(9.9105\ \text{ft})$$

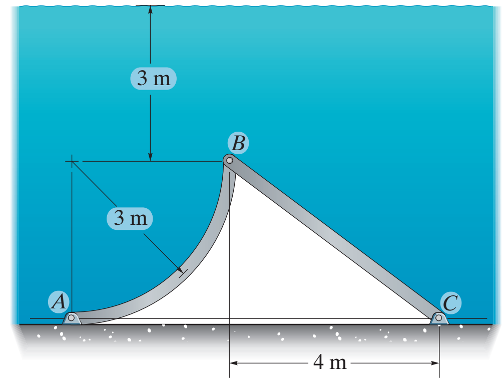
$$= 0 \qquad \textbf{Ans.}$$

This result is expected since the gate is circular in shape. Thus, F_R is always directed toward center O of the circular gate.

Ans:
$F_R = 56.1$ kip
$\theta = 15.9° \ \measuredangle$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–126. The curved and flat plates are pin connected at A, B, and C. They are submerged in water at the depth shown. Determine the horizontal and vertical components of reaction at pin B. The plates have a width of 4 m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force on the vertically projected area of the plate. Referring to Fig. *a*

$$w_B = \rho_w g h_B b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m})(4\ \text{m}) = 117.72\left(10^3\right)\ \text{N/m}$$

$$w_A = w_C = \rho_w g h_C b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(6\ \text{m})(4\ \text{m}) = 235.44\left(10^3\right)\ \text{N/m}$$

Thus,

$$(F_h)_{AB1} = (F_h)_{BC1} = \left[117.72\left(10^3\right)\ \text{N/m}\right](3\ \text{m}) = 353.16\left(10^3\right)\ \text{N} = 353.16\ \text{kN}$$

$$(F_h)_{AB2} = (F_h)_{BC2} = \frac{1}{2}\left[235.44\left(10^3\right)\ \text{N/m} - 117.72\left(10^3\right)\ \text{N/m}\right](3\ \text{m}) = 176.58\left(10^3\right)\ \text{N} = 176.58\ \text{kN}$$

They act at

$$\tilde{y}_2 = \tilde{y}_4 = \frac{1}{2}(3) = 1.5\ \text{m} \quad \tilde{y}_1 = \tilde{y}_3 = \frac{1}{3}(3\ \text{m}) = 1\ \text{m}$$

The vertical component of the resultant force is equal to the weight of the column of water above the plates shown shaded in Fig. a

$$(F_v)_{AB_1} = \rho_w g V = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(3\ \text{m})(3\ \text{m})(4\ \text{m})\right] = 353.16\left(10^3\right)\ \text{N} = 353.16\ \text{kN}$$

$$(F_v)_{AB_2} = \rho_w g V = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{1}{4}\pi(3\ \text{m})^2(4\ \text{m})\right] = 88.29\pi\left(10^3\right)\ \text{N} = 88.29\pi\ \text{kN}$$

$$(F_v)_{BC_1} = \rho_w g V = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(3\ \text{m})(4\ \text{m})(4\ \text{m})\right] = 470.88\left(10^3\right)\ \text{N} = 470.88\ \text{kN}$$

$$(F_v)_{BC_2} = \rho_w g V = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{1}{2}(3\ \text{m})(4\ \text{m})(4\ \text{m})\right] = 235.44\left(10^3\right)\ \text{N} = 235.44\ \text{kN}$$

They act at

$$\tilde{x}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{x}_2 = \frac{4(3\ \text{m})}{3\pi} = \frac{4}{\pi}\ \text{m} \qquad \tilde{x}_3 = \frac{1}{2}(4\ \text{m}) = 2\ \text{m} \qquad \tilde{x}_4 = \frac{1}{3}(4\ \text{m}) = \frac{4}{3}\ \text{m}$$

Referring to Fig. *a* and writing the moment equations of equilibrium about A and C

$$\circlearrowleft + \Sigma M_A = 0; \quad B_x(3\ \text{m}) - B_y(3\ \text{m}) - (353.16\ \text{kN})(1.5\ \text{m}) - (88.29\pi\ \text{kN})\left(\frac{4}{\pi}\ \text{m}\right)$$

$$- (353.16\ \text{kN})(1.5\ \text{m}) - (176.58\ \text{kN})(1\ \text{m}) = 0$$

$$B_x - B_y = 529.74 \qquad \textbf{(1)}$$

2–126. Continued

$$\circlearrowleft +\Sigma M_C = 0; \qquad (470.88\text{ kN})(2\text{ m}) + (235.44\text{ kN})\left(\frac{4}{3}\text{m}\right) + (353.16\text{ kN})(1.5\text{ m})$$

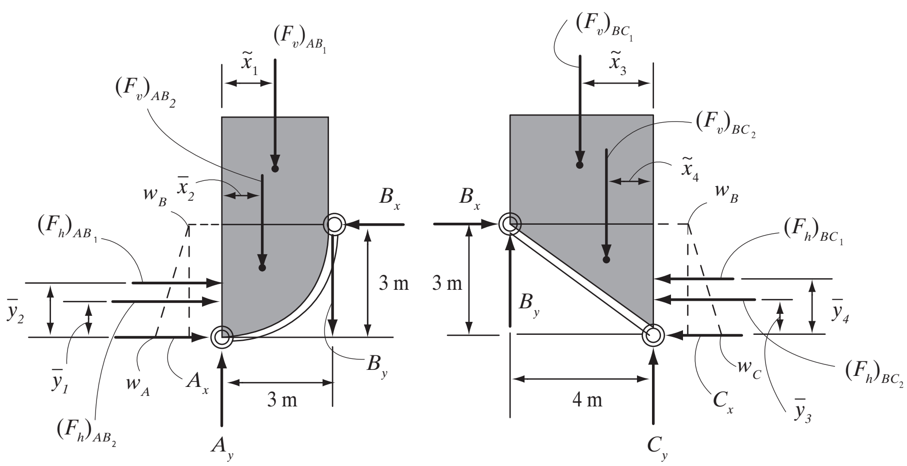
$$+ (176.58\text{ kN})(1\text{ m}) - B_x(3\text{ m}) - B_y(4\text{ m}) = 0$$

$$3B_x - 4B_y = 1962 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2)

$B_x = 582.99\text{ kN} = 583\text{ kN}$ **Ans.**

$B_y = 53.2\text{ kN}$ **Ans.**

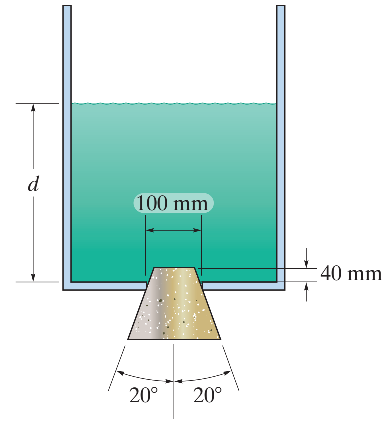


(a)

Ans:
$B_x = 583\text{ kN}$
$B_y = 53.2\text{ kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–127. The stopper in the shape of a frustum is used to plug the 100-mm-diameter hole in the tank that contains amyl acetate. If the greatest vertical force the stopper can resist is 100 N, determine the depth d before it becomes unplugged. Take $\rho_{aa} = 863\ \text{kg/m}^3$. *Hint:* The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



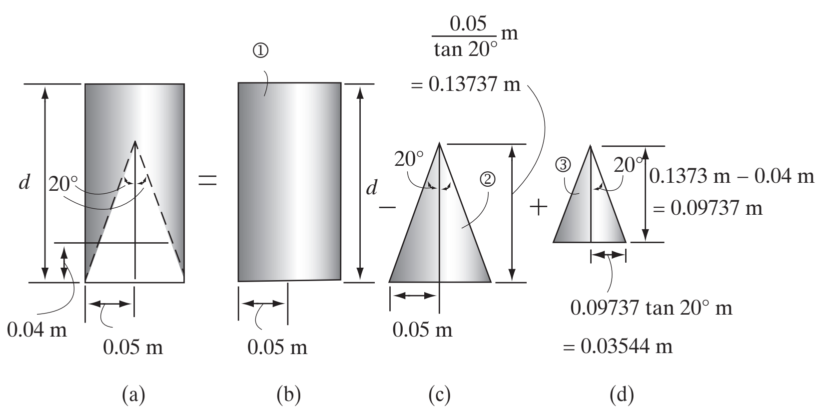
SOLUTION

The vertical downward force on the conical stopper is due to the weight of the liquid contained in the block shown shaded in Fig *a*. This block can be subdivided into parts (1), (2), and (3), shown in Figs. *b*, *c*, and *d*, respectively. Part (2) is a hole and should be considered as a negative part. Thus, the volume of the shaded block in Fig. *a* is

$$\begin{aligned} V &= V_1 - V_2 + V_3 \\ &= \pi(0.05\ \text{m})^2 d - \frac{1}{3}\pi(0.05\ \text{m})^2(0.13737\ \text{m}) + \frac{1}{3}\pi(0.03544\ \text{m})^2(0.09737\ \text{m}) \\ &= \left[2.5\left(10^{-3}\right)\pi d - 0.2316\left(10^{-3}\right)\right]\ \text{m}^3 \end{aligned}$$

The vertical force on the stopper is required to be equal to 100 N. Then,

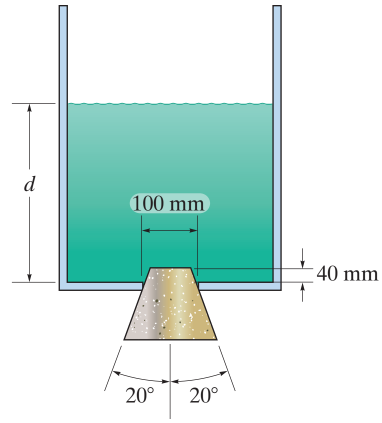
$$\begin{aligned} F &= \rho w g V \\ 100\ \text{N} &= \left(863\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[2.5\left(10^{-3}\right)\pi d - 0.2316\left(10^{-3}\right)\right] \\ d &= 1.5334\ \text{m} = 1.53\ \text{m} \end{aligned}$$ **Ans.**



Ans:
1.53 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–128.** The stopper in the shape of a frustum is used to plug the 100-mm-diameter hole in the tank that contains amyl acetate. Determine the vertical force this liquid exerts on the stopper. Take $d = 0.6$ m and $\rho_{aa} = 863\ \text{kg/m}^3$. *Hint:* The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



SOLUTION

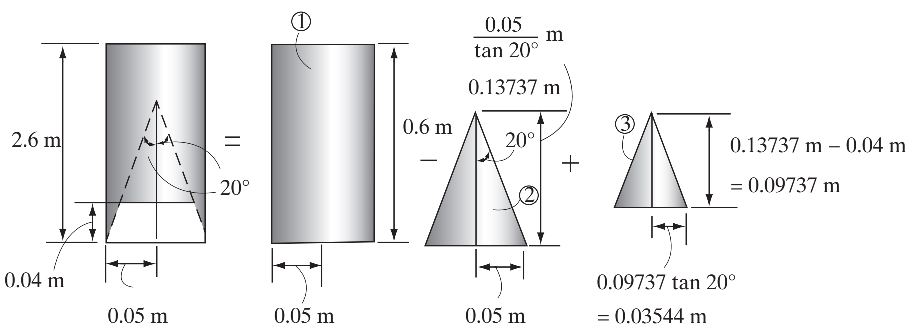
The vertical downward force on the conical stopper is due to the weight of the liquid contained in the block shown shaded in Fig. *a*. This block can be subdivided into parts (1), (2), and (3), shown in Figs. *b*, *c*, and *d*, respectively. Part (2) is a hole and should be considered as a negative part. Thus, the volume of the shaded block in Fig. *a* is

$$\begin{aligned} V &= V_1 - V_2 + V_3 \\ &= \pi(0.05\ \text{m})^2(0.6\ \text{m}) - \frac{1}{3}\pi(0.05\ \text{m})^2(0.13737\ \text{m}) + \frac{1}{3}\pi(0.03544\ \text{m})^2\,(0.09737\ \text{m}) \\ &= 4.4808\left(10^{-3}\right)\ \text{m}^3 \end{aligned}$$

Then,

$$\begin{aligned} F &= \rho_w g V \\ &= \left(863\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[4.4808\left(10^{-3}\right)\ \text{m}^3\right] \\ &= 37.93\ \text{N} = 37.9\ \text{N} \end{aligned}$$

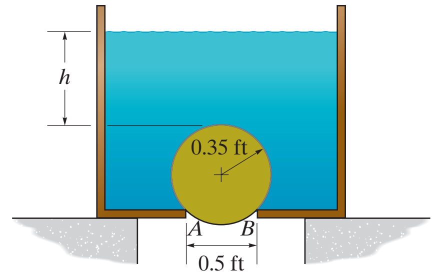
Ans.



(a) (b) (c) (d)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–129. The steel cylinder has a specific weight of 490 lb/ft³ and acts as a plug for the 1-ft-long slot in the tank. Determine the resultant force the bottom of the tank exerts on the cylinder when the water in the tank is at a depth of $h = 2$ ft.



SOLUTION

The vertical downward force and the vertical upward force are equal to the weight of the water contained in the blocks shown shaded in Figs. *a* and *b*, respectively. The volume of the shaded block in Fig. *a* is

$$\forall_1 = \left[2.35\ \text{ft}(0.7\ \text{ft}) - \frac{\pi}{2}(0.35\ \text{ft})^2\right](1\ \text{ft}) = 1.4526\ \text{ft}^3$$

The volume of the shaded block in Fig. *b* is

$$\forall_2 = 2\left\{0.1\ \text{ft}(2.35\ \text{ft}) + \left[\frac{44.42°}{360°}(\pi)(0.35\ \text{ft})^2 - \frac{1}{2}(0.25\ \text{ft})(0.2449\ \text{ft})\right]\right\}(1\ \text{ft})$$

$$= 0.5037\ \text{ft}^3$$

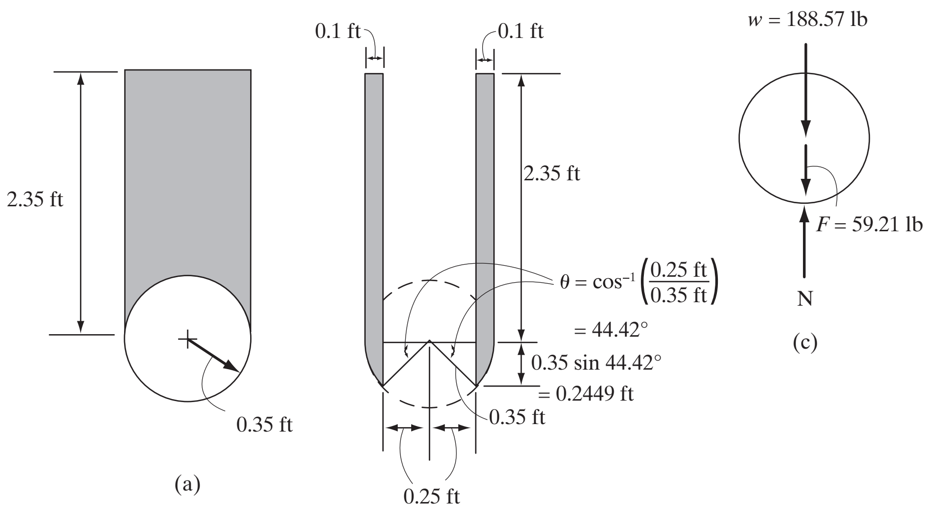
Then,

$$F = \gamma_w(\forall_1 - \forall_2)$$
$$= (62.4\ \text{lb/ft}^3)(1.4526\ \text{ft}^3 - 0.5037\ \text{ft}^3)$$
$$= 59.21\ \text{lb}\ \downarrow$$

The weight of the cylinder is $W = \gamma_{st}\forall_C = (490\ \text{lb/ft}^2)\left[\pi(0.35\ \text{ft})^2(1\ \text{ft})\right] = 188.57\ \text{lb}$. Considering the free-body diagram of the cylinder, Fig. *c*, we have

$+\uparrow\Sigma F_y = 0;\qquad N - 59.21\ \text{lb} - 188.57\ \text{lb} = 0$

$N = 247.78\ \text{lb} = 248\ \text{lb}$ **Ans.**

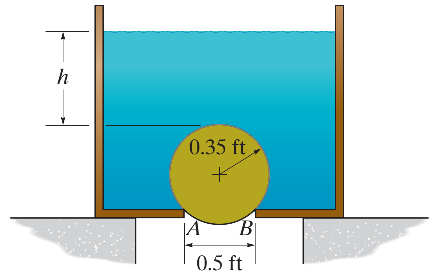


(a) (b) (c)

Ans:
248 lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–130. The steel cylinder has a specific weight of 490 lb/ft^3 and acts as a plug for the 1-ft-long slot in the tank. Determine the resultant force the bottom of the tank exerts on the cylinder when the water in the tank just covers the top of the cylinder, $h = 0$.



SOLUTION

The vertical downward force and the vertical upward force are equal to the weight of the water contained in the blocks shown shaded in Figs. *a* and *b*, respectively. The volume of the shaded block in Fig. *a* is

$$V_1 = \left[0.35 \text{ ft}(0.7 \text{ ft}) - \frac{\pi}{2}(0.35 \text{ ft})^2\right](1 \text{ ft}) = 0.05258 \text{ ft}^3$$

The volume of the shaded block in Fig. *b* is

$$V_2 = 2\left\{0.35 \text{ ft}(0.1 \text{ ft}) + \left[\frac{44.42^\circ}{360^\circ}(\pi)(0.35 \text{ ft})^2 - \frac{1}{2}(0.25 \text{ ft})(0.2449 \text{ ft})\right]\right\}(1 \text{ ft})$$

$$= 0.10372 \text{ ft}^3$$

Then,

$$F = \gamma_w(V_1 - V_2)$$

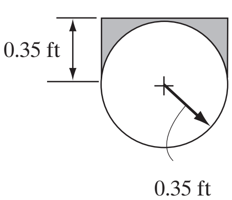
$$= \left(62.4 \text{ lb/ft}^3\right)\left(0.10372 \text{ ft}^3 - 0.05258 \text{ ft}^3\right)$$

$$= 3.192 \text{ lb} \uparrow$$

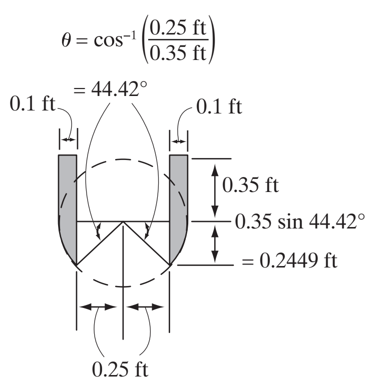
The weight of the cylinder is $W = \gamma_{st} V_C = \left(490 \text{ lb/ft}^2\right)\left[\pi(0.35 \text{ ft})^2(1 \text{ ft})\right] = 188.57 \text{ lb}$. Considering the force equilibrium vertically by free-body diagram of the cylinder, Fig. *c*, we have

$$+\uparrow \Sigma F_y = 0; \qquad N + 3.192 \text{ lb} - 188.57 \text{ lb} = 0$$

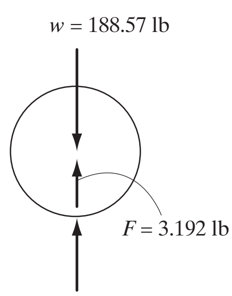
$$N = 185.38 \text{ lb} = 185 \text{ lb}$$ **Ans.**



(a)



(b)

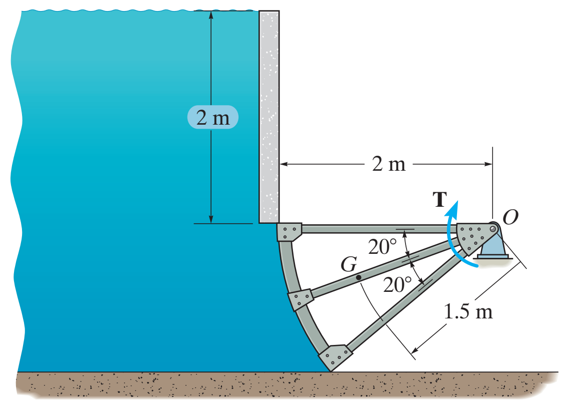


(c)

Ans:
185 lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–131. The sluice gate for a water channel is 1.5 m wide and in the closed position, as shown. Determine the magnitude of the resultant force of the water acting on the gate. Solve the problem by considering the fluid acting on the horizontal and vertical projections of the gate. Determine the smallest torque **T** that must be applied to open the gate if its weight is 30 kN and its center of gravity is at G?



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the gate. Referring to Fig. *a*

$$w_1 = \rho_w g h_1 b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m})(1.5\ \text{m}) = 29.43\left(10^3\right)\ \text{N/m}$$

$$w_2 = \rho_w g h_2 b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m} + 2\text{m}\sin 40°)(1.5\ \text{m}) = 48.347\left(10^3\right)\ \text{N/m}$$

Then

$$(F_h)_1 = \left[29.43\left(10^3\right)\ \text{N/m}\right](2\sin 40°\text{m}) = 37.834\left(10^3\right)\ \text{N} = 37.834\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}\left[(48.347 - 29.43)\left(10^3\right)\ \text{N/m}\right](2\ \sin\ 40°\text{m}) = 12.160\ \left(10^3\right)\ \text{N} = 12.160\ \text{kN}$$

$$F_h = (F_h)_1 + (F_h)_2 = 37.834\left(10^3\right)\ \text{N} + 12.160\left(10^3\right)\ \text{N} = 49.994\left(10^3\right)\ \text{N} = 49.994\ \text{kN}$$

Also

$$\tilde{y}_1 = \frac{1}{2}(2\ \text{m}\sin 40°) = 0.6428\ \text{m} \quad \text{and} \quad \tilde{y}_2 = \frac{2}{3}(2\ \text{m}\sin 40°) = 0.8571\ \text{m}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*) but acts upward. The volume of this column of water is

$$\forall = \left[(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + \frac{1}{2}(2\ \text{m})^2\left(\frac{40°}{180°}\pi\ \text{rad}\right) - \frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)\right](1.5\ \text{m})$$

$$= 2.0209\ \text{m}^3$$

$$F_v = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(2.0209\ \text{m}^3\right) = 19.825\left(10^3\right)\ \text{N} = 19.825\ \text{kN}$$

Referring to Fig *b* and *c*

$$\bar{r} = \frac{2}{3}\left(\frac{2\ \text{m}\sin 20°}{\frac{20}{180}\pi}\right) = 1.3064\ \text{m} \qquad \tilde{x}_2 = 1.3064\ \text{m}\cos 20° = 1.2276\ \text{m}$$

$$\tilde{x}_1 = 2\ \text{m}\cos 40° + \left(\frac{2\ \text{m} - 2\ \text{m}\cos 40°}{2}\right) = 1.7660\ \text{m}$$

$$\tilde{x}_3 = \frac{2}{3}(2\ \text{m}\cos 40°) = 1.0214\ \text{m}$$

Thus, F_v acts at

$$\bar{x} = \frac{(1.7660\ \text{m})(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + (1.2276\ \text{m})\left[\frac{1}{2}(2\ \text{m})^2\left(\frac{40}{180}\pi\ \text{rad}\right)\right] - (1.0214\ \text{m})\left[\frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)\right]}{(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + \frac{1}{2}(2\ \text{m})^2\left(\frac{40}{180}\pi\ \text{rad}\right) - \frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)}$$

$$= 1.7523\ \text{m}$$

2–131. Continued

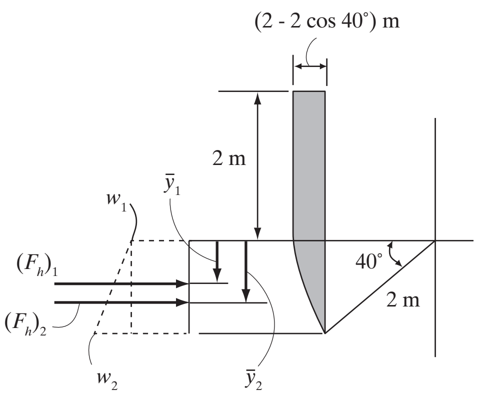
The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(49.994\text{ kN})^2 + (19.825\text{ kN})^2} = 53.78\text{ kN} = 53.8\text{ kN} \quad \textbf{Ans.}$$

Referring to the FBD of the gate shown in Fig *d*,

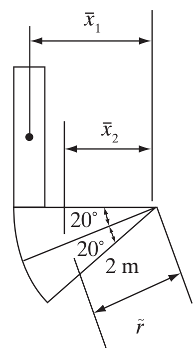
$$\circlearrowleft +\Sigma M_0 = 0; \quad (30\text{ kN})(1.5\ \cos\ 20°\text{m}) + (37.834\text{ kN})(0.6428\text{ m}) + (12.160\text{ kN})(0.8571\text{ m})$$
$$-(19.825\text{ kN})(1.7524\text{ m}) - T = 0$$
$$T = 42.29\text{ kN}\cdot\text{m} = 42.3\text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$

Note that the resultant force of the write acting on the give must act normal to its surface, and therefore it will pass through the pin at O. Therefore it produces moment about the pin.



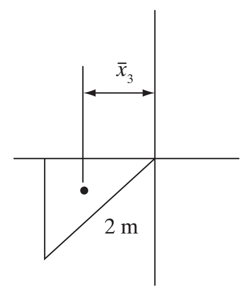
(a)

=

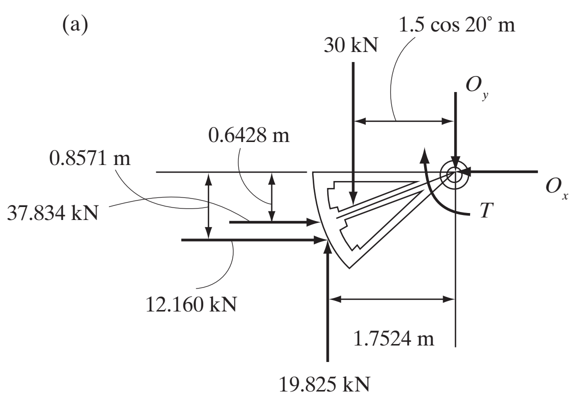


(b)

−



(c)

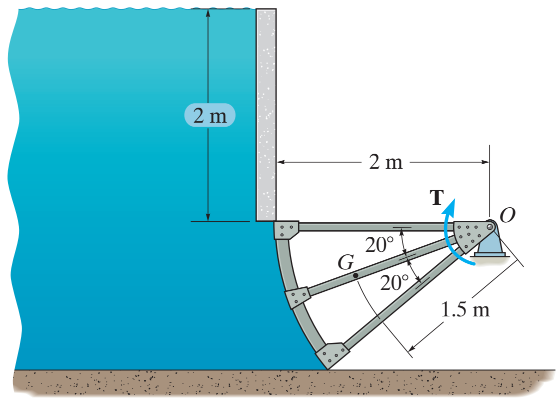


(d)

Ans:
$F_R = 53.8$ kN
$T = 42.3$ kN·m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–132.** Solve the first part of Prob. 2-131 by the integration method using polar coordinates.

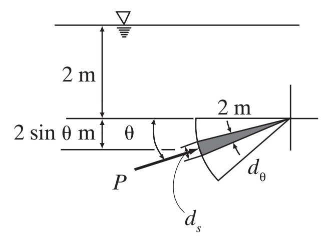


SOLUTION

Referring to Fig *a*, $h = (2 + 2\sin\theta)$ m. Thus, the pressure acting on the gate as a function of θ is

$$p = \rho_w g h = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2 + 2\sin\theta)\ \text{m} = [19620(1 + \sin\theta)]\ \text{N/m}^2$$

This pressure is acting on the element of area $dA = b\,ds = 1.5\,ds = 1.5\,(2\,d\theta) = 3\,d\theta$.



(a)

Thus,

$$dF = p\,d_A = 19620(1 + \sin\theta)(3\,d\theta). = 58.86\left(10^3\right)(1 + \sin\theta)\,d\theta$$

The horizontal and vertical components of $d\mathbf{F}$ are

$$(dF)_h = 58.86\left(10^3\right)(1 + \sin\theta)\cos\theta\,d\theta$$
$$= 58.86\left(10^3\right)(\cos\theta + \sin\theta\cos\theta)\,d\theta$$
$$(dF)_v = 58.86\left(10^3\right)(1 + \sin\theta)\sin\theta\,d\theta$$
$$= 58.86\left(10^3\right)\left(\sin\theta + \sin^2\theta\right)d\theta$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$, then

$$(dF)_h = 58.86\left(10^3\right)\left(\cos\theta + \frac{1}{2}\sin 2\theta\right)d\theta$$

$$(dF)_v = 58.86\left(10^3\right)\left(\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

The horizontal and vertical components of the resultant force are

$$F_h = \int (dF)_h = 58.86\left(10^3\right)\int_0^{\frac{2\pi}{9}}\left(\cos\theta + \frac{1}{2}\sin 2\theta\right)d\theta$$

$$= 58.86\left(10^3\right)\left[\sin\theta - \frac{1}{4}\cos 2\theta\right]\Bigg|_0^{\frac{2\pi}{9}}$$

$$= 49.994\left(10^3\right)\ \text{N} = 49.994\ \text{kN}$$

$$F_v = \int (dF)_v = 58.86\left(10^3\right)\int_0^{\frac{2\pi}{9}}\left(\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= 58.86\left(10^3\right)\left(-\cos\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Bigg]\Bigg|_0^{\frac{2\pi}{9}}$$

$$= 19.825\left(10^3\right)\ \text{N} = 19.825\ \text{kN}$$

Thus, the magnitude of the resultant force is

$$F_R = \sqrt{F_h{}^2 + F_v{}^2} = \sqrt{(49.994\ \text{kN})^2 + (19.825\ \text{kN})^2} = 53.78\ \text{kN} = 53.8\ \text{kN} \quad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–133. A flat-bottomed boat has vertical sides and a bottom surface area of $0.75\ \text{m}^2$. It floats in water such that its draft (depth below the surface) is 0.3 m. Determine the mass of the boat. What is the draft when a 50-kg man stands in the center of the boat?

SOLUTION

Equilibrium requires that the weight of the empty boat is equal to the buoyant force.

$$W_b = F_b = \rho_w g \forall Disp = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(0.75\ \text{m}^2)(0.3\ \text{m})$$

$$= 2207.25\ \text{N}$$

Thus, the mass of the boat is given by

$$m_b = \frac{W_b}{g} = \frac{2207.25\ \text{N}}{9.81\ \text{m/s}^2} = 225\ \text{kg} \qquad \textbf{Ans.}$$

When the man steps into the boat, the total mass is $m_{b+m} = 225\ \text{kg} + 50\ \text{kg} = 275\ \text{kg}$. Then $W_{b+m} = m_{b+m}g = 275\ \text{kg}(9.81\ \text{m/s}^2) = 2697.75\ \text{N}$. Under this condition, the boat will sink further to create a greater buoyancy force to balance the additional weight. Thus,

$$W_{b+m} = \rho_w g \forall' Disp$$

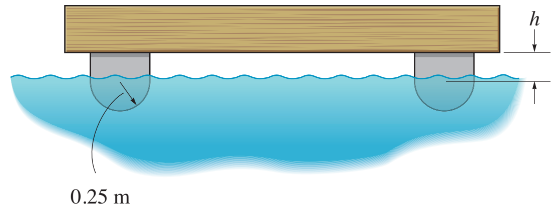
$$2697.75\ \text{N} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.75\ \text{m}^2)(h)$$

$$h = 0.367\ \text{m} \qquad \textbf{Ans.}$$

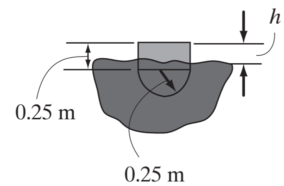
Ans:
$m_b = 225\ \text{kg}$
$h = 0.367\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–134. The raft consists of a uniform platform having a mass of 2 Mg and four floats, each having a mass of 120 kg and a length of 4 m. Determine the height h at which the platform floats from the water surface. Take $\rho_w = 1\text{Mg/m}^3$.



SOLUTION



(a)

Each float must support a weight of

$$W = \left[\frac{1}{4}(2000\ \text{kg}) + 120\ \text{kg}\right]9.81\ \text{m/s}^2 = 6082.2\ \text{N}$$

For equilibrium, the buoyant force on each float is required to be

$$+\uparrow\Sigma F_y = 0; \qquad F_b - 6082.2\ \text{N} = 0 \qquad F_b = 6082.2\ \text{N}$$

Therefore, the volume of water that must be displaced to generate this force is

$$F_b = \gamma V; \qquad 6082.2\ \text{N} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)V$$

$$V = 0.620\ \text{m}^3$$

Since the semicircular segment of a float has a volume of $\frac{1}{2}(\pi)(0.25\ \text{m})^2(4\ \text{m}) = 0.3927\ \text{m}^3 < 0.620\ \text{m}^3$, then it must be fully submerged to develop F_b. As shown in Fig. *a*, we require

$$0.620\ \text{m}^3 = \frac{1}{2}(\pi)(0.25\ \text{m})^2(4\ \text{m}) + (0.25\ \text{m} - h)(0.5\ \text{m})(4\ \text{m})$$

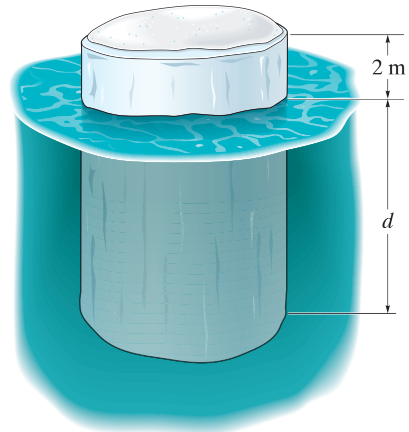
Thus,

$$h = 0.136\ \text{m} = 136\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
136 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–135. Consider an iceberg to be in the form of a cylinder of arbitrary diameter and floating in the ocean as shown. If the cylinder extends 2 m above the ocean's surface, determine the depth of the cylinder below the surface. The density of ocean water is $\rho_{sw} = 1024\ \mathrm{kg/m^3}$, and the density of the ice is $\rho_i = 935\ \mathrm{kg/m^3}$.



SOLUTION

The weight of the iceberg is

$$W = \rho_i g \mathcal{V}_i = (935\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})\left[\pi r^2(2 + d)\right]$$

The buoyant force is

$$F_b = \rho_{sw} g \mathcal{V}_{\mathrm{sub}} = (1024\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(\pi r^2 d)$$

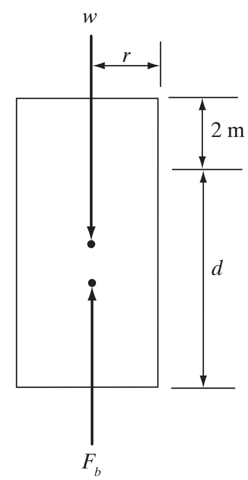
Referring to the FBD of the iceberg, Fig. *a*, equilibrium requires,

$$+\uparrow \Sigma F_y = 0; \qquad F_b - w = 0$$

$$(1024\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(\pi r^2 d) - (935\ \mathrm{kg/m^3})(9.81\mathrm{m/s^2})\left[\pi r^2(2 + d)\right] = 0$$

$$1024\, d = 935(2 + d)$$

$$d = 21.01\ \mathrm{m} = 21.0\ \mathrm{m} \qquad \textbf{Ans.}$$

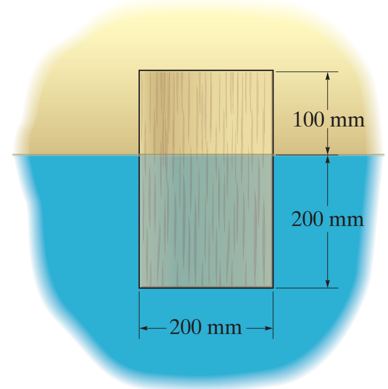


(a)

Ans:
21.0 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–136.** The cylinder floats in the water and oil to the level shown. Determine the weight of the cylinder. $\rho_o = 910\ \text{kg/m}^3$



SOLUTION

The buoyant force fuel to the submerging in oil and water are

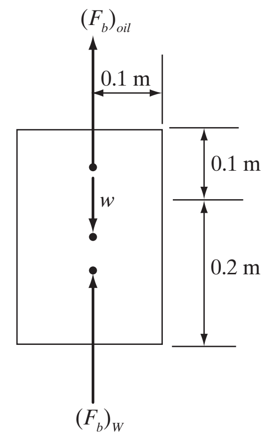
$$(F_b)_{oil} = \rho_{oil} g (V_{\text{sub}})_{oil} = (910\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.1\text{m})^2(0.1\ \text{m})\right] = 8.9271\ \pi\ \text{N}$$

$$(F_b)_N = \rho_w g (V_{\text{sub}})_w = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.1\ \text{m})^2(0.2\ \text{m})\right] = 19.62\ \pi\ \text{N}$$

Referring to the FBD of the cylinder, Fig. *a*, equilibrium requires,

$$+\uparrow \Sigma F_y = 0; \qquad 8.9271\ \pi\ \text{N} + 19.62\ \pi\ \text{N} - W = 0$$

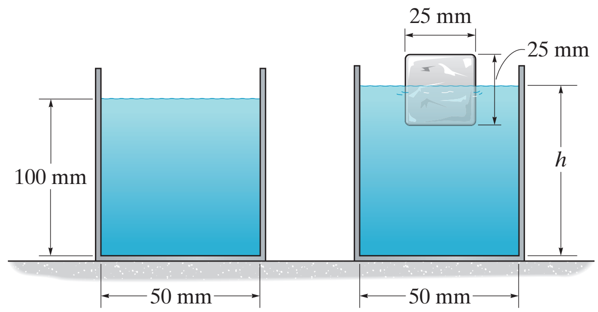
$$W = 89.68\ \text{N} = 89.7\ \text{N} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–137. A glass having a diameter of 50 mm is filled with water to the level shown. If an ice cube with 25 mm sides is placed into the glass, determine the new height h of the water surface. Take $\rho_w = 1000\ \text{kg/m}^3$ and, $\rho_z = 920\ \text{kg/m}^3$. What will the water level h be when the ice cube completely melts?



SOLUTION

Since the ice floats, the buoyant force is equal to the weight of the ice cube which is

$$F_b = W_i = \rho_i \mathcal{V}_i g = (920\ \text{kg/m}^3)(0.025\ \text{m})^3(9.81\ \text{m/s}^2) = 0.1410\ \text{N}$$

This buoyant force is also equal to the weight of the water displaced by the submerged ice cube with at a depth h_s.

$$F_b = \rho_w g \mathcal{V}_s; \qquad 0.1410\ \text{N} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(0.025\ \text{m})^2 h_s\right]$$

$$h_s = 0.023\ \text{m}$$

Referring to Fig. a,

$$\mathcal{V}_1 = \mathcal{V}_2 - \mathcal{V}_3$$

$$\left[\pi(0.025\ \text{m})^2\right](0.1\ \text{m}) = \left[\pi(0.025\text{m})^2\right]h - (0.025\text{m})^2(0.023\ \text{m})$$

$$h = 0.1073\ \text{m} = 107\ \text{mm} \qquad \textbf{Ans.}$$

The mass of ice cube is

$$M_i = \rho_i \mathcal{V}_i = (920\ \text{kg/m}^3)(0.025\ \text{m})^3 = 0.014375\ \text{kg}.$$

Thus, the nice in water level due to the additional water of the melting ice cubs: can be determined from

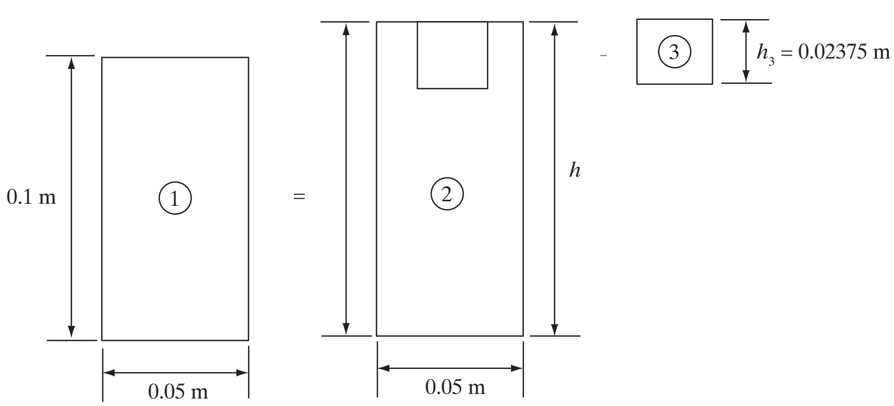
$$M_i = \rho_w \mathcal{V}_w; \qquad 0.014375\ \text{kg} = (1000\ \text{kg/m}^3)\left[\pi(0.025\ \text{m})^2 \Delta h\right]$$

$$\Delta h = 0.007321$$

Thus,

$$h' = 0.1\ \text{m} + 0.007321\ \text{m} = 107\ \text{mm} \qquad \textbf{Ans.}$$

Note The water level h remains unchanged as the cube melts.

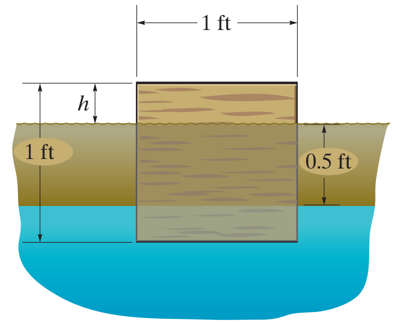


(a)

Ans:
107 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–138. The wood block has a specific weight of 45 lb/ft^3. Determine the depth h at which it floats in the oil–water system. The block is 1 ft wide. Take $\rho_o = 1.75 \text{ slug/ft}^3$.



SOLUTION

The weight of the block is

$$W = \gamma_b \forall_b = \left(45 \text{ lb/ft}^3\right)\left[(1\text{ft})^3\right] = 45 \text{ lb}$$

Assume that block floats in both oil and water, Fig, *a*. Then, the volume of the water and oil being displaced is

$$(\forall_{oil})_{Disp} = 0.5 \text{ ft}(1 \text{ ft})(1 \text{ ft}) = 0.5 \text{ ft}^3$$

$$(\forall_w)_{Disp} = (0.5 \text{ ft} - h)(1 \text{ ft})(1 \text{ ft}) = (0.5 - h) \text{ ft}^3$$

Thus, the buoyancy forces on the block due to the oil and water are

$$(F_b)_{oil} = \gamma_{oil}(\forall_{oil})_{Disp} = \left(1.75 \text{ slug/ft}^3\right)\left(32.2 \text{ ft/s}^2\right)\left(0.5 \text{ ft}^2\right) = 28.175 \text{ lb}$$

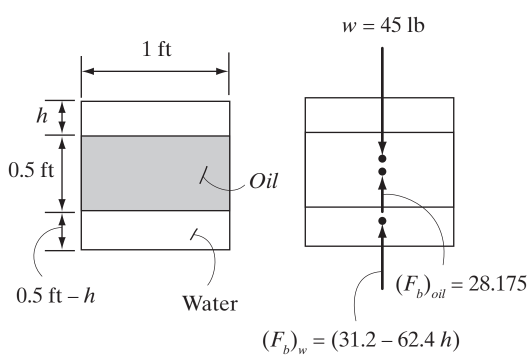
$$(F_b)_w = \gamma_w(\forall_w)_{Disp} = \left(62.4 \text{ slug/ft}^3\right)(0.5 - h) \text{ ft}^3 = (31.2 - 62.4h) \text{ lb}$$

Considering the free-body diagram in Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad (31.2 - 62.4h) \text{ lb} + 28.175 \text{ lb} - 45 \text{ lb} = 0$

$h = 0.2304 \text{ ft} = 0.230 \text{ ft}$ **Ans.**

Since $h < 0.5$ ft, the assumption was correct and the result is valid.

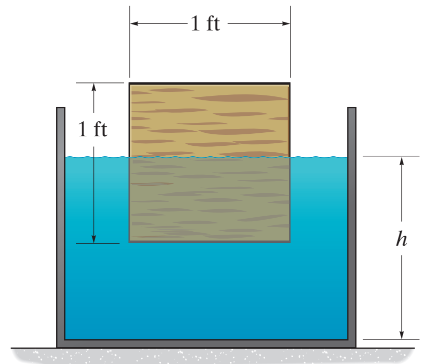


(a) (b)

Ans:
0.230 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–139. Water in the container is originally at a height of $h = 3$ ft. If a block having a specifit weight of $50\ \text{lb/ft}^3$ is placed in the water, determine the new level h of the water. The base of the block is 1 ft square, and the base of the container is 2 ft square.



SOLUTION

The weight of the block is

$$W_b = \gamma_b \mathcal{V}_b = (50\ \text{lb/ft}^3)\left[(1\text{ft})^3\right] = 50\ \text{lb}$$

Equilibrium requires that the buoyancy force equal the weight of the block, so that $F_b = 50$ lb. Thus, the displaced volume is

$$F_b = \gamma_w \mathcal{V}_{Disp} \qquad 50\ \text{lb} = (62.4\ \text{lb/ft}^3)\mathcal{V}_{Disp}$$

$$\mathcal{V}_{Disp} = 0.8013\ \text{ft}^3$$

The volume of the water is

$$\mathcal{V}_w = 2\ \text{ft}(2\ \text{ft})(3\ \text{ft}) = 12\ \text{ft}^3$$

When the level of the water in the container has a height of h,

$$\mathcal{V}_w = \mathcal{V}' - \mathcal{V}_{Disp}$$

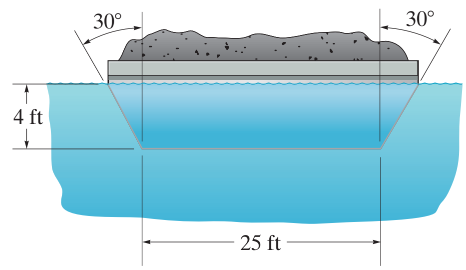
$$12\ \text{ft}^3 = 4\,h\ \text{ft}^3 - 0.8013\ \text{ft}^3$$

$$h = 3.20\ \text{ft}$$ **Ans.**

Ans:
3.20 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–140.** The cross section of the front of a barge is shown. Determine the buoyant force acting per foot length of the hull when the water level is at the indicated depth.



SOLUTION

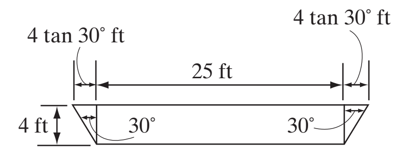
Referring to the geometry shown in Fig. *a*, the volume of the water displaced per foot length of the hull is

$$\forall_{Disp} = 25\ \text{ft}(4\ \text{ft}) + 2\left[\frac{1}{2}(4\ \tan\ 30°\ \text{ft})(4\ \text{ft})\right] = 109.24\ \text{ft}^3/\text{ft}$$

Thus, the buoyancy force acting per foot length of the hull is

$$F_b = \gamma_w \forall_b = \left(62.4\ \text{lb/ft}^3\right)\left(109.24\ \text{ft}^3/\text{ft}\right)$$

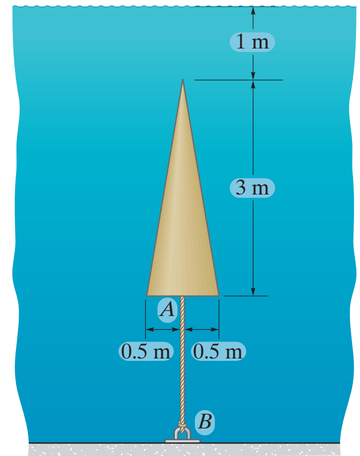
$$= 6816\ \text{lb/ft} = 6.82\ \text{kip/ft} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–141. The cone is made of wood having a density of $\rho_{wood} = 650\ \text{kg/m}^3$. Determine the tension in rope AB if the cone is submerged in the water at the depth shown. Will this force increase, decrease, or remain the same if the cord is shortened? Why? *Hint*: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



SOLUTION

The weight of the wooden cone is

$$W = \rho_{wood}\, g V_c = \left(650\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{1}{3}\pi(0.5\ \text{m})^2(3\ \text{m})\right] = 1594.125\pi\ \text{N}$$

The volume of water that is displaced is the same as the volume of the cone. Thus, the buoyancy force is

$$F_b = \rho_w g V_c = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{1}{3}\pi(0.5\ \text{m})^2(3\ \text{m})\right] = 2452.5\pi\ \text{N}$$

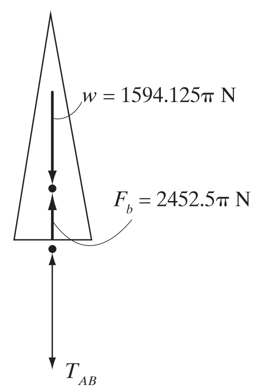
Considering the free-body diagram of the cone in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2452.5\pi\ \text{N} - 1594.125\pi\ \text{N} - T_{AB} = 0$$

$$T_{AB} = 2696.66\ \text{N} = 2.70\ \text{kN} \qquad \textbf{Ans.}$$

The tension in rope AB *remains the same* since the buoyancy force does not change. For a fully submerged body, the buoyancy force is independent of the depth to which the body is submerged.

Remains the same **Ans.**

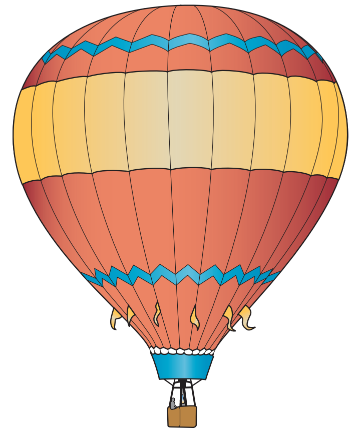


(a)

Ans:
$T_{AB} = 2.70\ \text{kN}$
Remains the same

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–142. The hot-air balloon contains air having a temperature of 180°F, while the surrounding air having a temperature of 60°F. Determine the maximum weight of the load the balloon can lift if the volume of air it contains is $120(10^3)\ \text{ft}^3$. The empty weight of the balloon is 200 lb.



SOLUTION

From the Appendix, the densities of the air inside the balloon where $T = 180°\text{ F}$ and outside the balloon where $T = 60°\text{ F}$, are

$$\rho_a\big|_{T=60°\text{ F}} = 0.00237\ \text{slug/ft}^3$$

$$\rho_a\big|_{T=180°\text{ F}} = 0.00193\ \text{slug/ft}^3$$

Thus, the weight of the air inside the balloon is

$$W_a\big|_{T=180°\text{ F}} = \rho_a\big|_{T=180°\text{F}} g\mathcal{V} = \left(0.00193\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left[120\left(10^3\right)\text{ft}^3\right]$$

$$= 7457.52\ \text{lb}$$

The buoyancy force is equal to the weight of the displaced air outside of the balloon. This gives

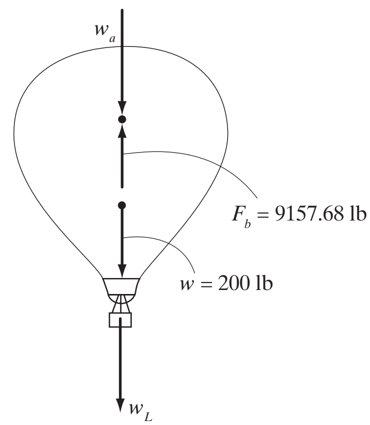
$$F_b = \rho_a\big|_{T=60°\text{F}} g\mathcal{V} = \left(0.00237\ \text{slug/ft}^3\right)\left(32.2\text{ft/s}^2\right)\left[120\left(10^3\right)\text{ft}^3\right]$$

$$= 9157.68\ \text{lb}$$

Considering the free-body diagram of the balloon in Fig. *a*,

$+\uparrow \Sigma F_y = 0;\qquad 9157.68\ \text{lb} - 7457.52\ \text{lb} - 200\ \text{lb} - W_L = 0$

$W_L = 1500.16\ \text{lb} = 1.50\ \text{kip}$ **Ans.**

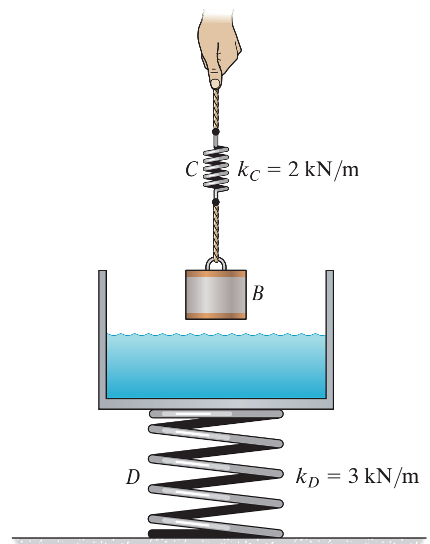


(a)

Ans:
1.50 kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–143. The container with water in it has a mass of 30 kg. Block B has a density of $8500\ \text{kg/m}^3$ and a mass of 15 kg. If springs C and D have an unstretched length of 200 mm and 300 mm, respectively, determine the length of each spring when the block is submerged in the water.



SOLUTION

The volume of block B is,

$$\forall_B = \frac{m_B}{\rho} = \frac{15\ \text{kg}}{8500\ \text{kg/m}^3} = 1.7647\left(10^{-3}\right)\ \text{m}^3$$

Thus, the bouyant force is

$$F_b = \rho_w g \forall_{\text{sub}} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(1.7647\left(10^{-3}\right)\ \text{m}^3\right) = 17.31\ \text{N}$$

Referring to the *FBD* of block B, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad (F_{sp})_c + 17.31\ \text{N} - \left[15(9.81)\ \text{N}\right] = 0 \qquad (F_{sp})_c = 129.84\ \text{N}$$

Referring to the FBD of the container, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad (F_{sp})_D - 17.31\ \text{N} - \left[30(9.81)\ \text{N}\right] = 0 \qquad (F_{sp})_D = 311.61\ \text{N}$$

Thus, the deformations of springs C dand D are

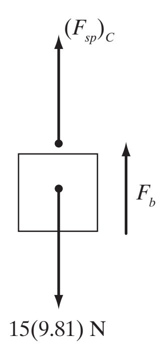
$$\delta_C = \frac{(F_{sp})_C}{k_C} = \frac{129.84\ \text{N}}{2000\ \text{N/m}} = 0.06492\ \text{m} = 64.92\ \text{mm}$$

$$\delta_D = \frac{(F_{sp})_D}{k_D} = \frac{311.61\ \text{N}}{3000\ \text{N/m}} = 0.1039\ \text{m} = 103.87\ \text{mm}$$

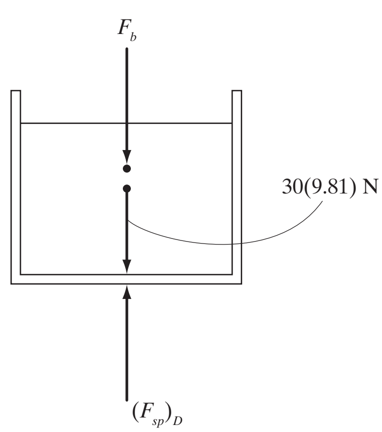
Thus

$l_C = (l_o)_C + \delta_c = 200\ \text{mm} + 64.92\ \text{mm} = 264.92\ \text{mm} = 265\ \text{mm}$ **Ans.**

$l_D = (l_o)_D + \delta_D = 300\ \text{mm} - 103.87\ \text{mm} = 196.13\ \text{mm} = 196\ \text{mm}$ **Ans.**



(a)

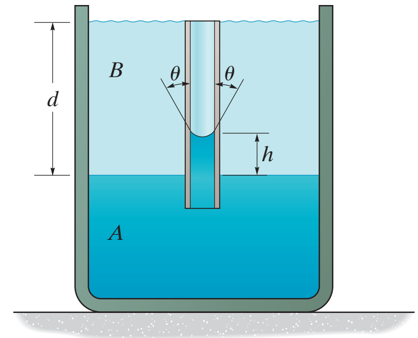


(b)

Ans:
265 mm
196 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–144.** An open-ended tube having an inner radius r is placed in a wetting liquid A having a density ρ_A. The top of the tube is just below the surface of a surrounding liquid B, which has a density ρ_B, where $\rho_A > \rho_B$. If the surface tension σ causes liquid A to make a wetting angle θ with the tube wall as shown, determine the rise h of liquid A within the tube. $\rho_A > \rho_B$ show that the result is independent of the depth d of liquid B.



SOLUTION

The volume of the column of liquid A in the rise is $\mathcal{V} = \pi r^2 h$. Thus, its weight is

$$W = \gamma_A \mathcal{V} = \rho_A g\left(\pi r^2 h\right) = \pi g \rho_A r^2 h$$

The force on the top and bottom of the column is $p_t A = \rho_B(\bar{d} - h)\left(\pi r^2\right)$ and $\rho_B(d)\left(\pi r^2\right)$. The difference in these forces is the bouyont force.

$$F_B = \gamma_B \mathcal{V} = \rho_B g\left(\pi r^2 h\right) = \pi g \rho_B r^2 h$$

The force equilibrium along the vertical, Fig. a, requires.

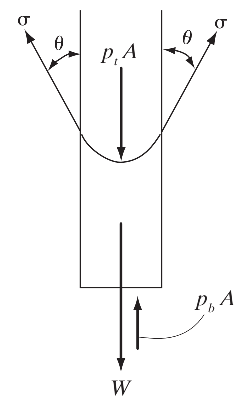
$+\uparrow \Sigma F_y = 0;$ $\qquad \sigma(2\pi r)\cos\theta + \pi g \rho_B r^2 h - \pi g \rho_A r^2 h = 0$

$$2\pi r \sigma \cos\theta + \pi g r^2 h(\rho_B - \rho_A) = 0$$

$$2\pi r \sigma \cos\theta = (\rho_A - \rho_B)\pi g r^2 h$$

$$h = \frac{2\sigma \cos\theta}{g r(\rho_A - \rho_B)} \qquad \textbf{Ans.}$$

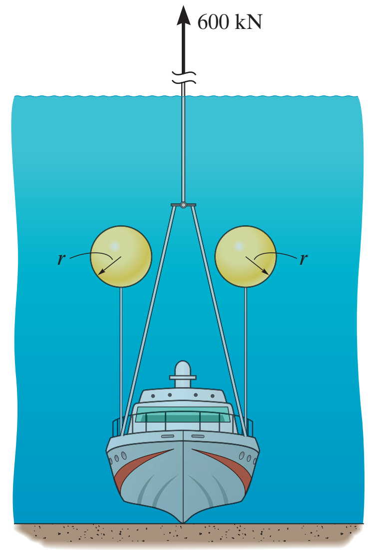
Note that the result is independent of d.



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–145. A boat having a mass of 80 Mg rests on the bottom or the lake and displaces $10.25\ \text{m}^3$ of water. Since the lifting capacity of the crane is only 60 kN, two balloons are attached to the sides of the boat and filled with air. Determine the smallest radius of each spherical balloon that is needed to lift the boat. What is the mass of air in each balloon if the water temperature is 12°C? The balloons are at an average depth of 20 m. Neglect the mass of air and of the balloon for the calculation required for the lift. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.



SOLUTION

The bouyant force acting on the boat and a balloon are

$$(F_b)_B = \rho_w g (V_B)_{\text{sub}} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(10.25\ \text{m}^3\right) = 100.55\left(10^3\right)\ \text{N}$$
$$= 100.55\ \text{kN}$$

$$(F_b)_b = \rho_w g (V_b)_{\text{sub}} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{4}{3}\pi r^3\right] = 13.08\pi r^3\left(10^3\right)\ \text{N}$$
$$= 13.08\pi r^3\ \text{kN}$$

Referring to the FBD of the boat, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2T + 100.55\ \text{kN} + 600\ \text{kN} - \left[80(9.81)\ \text{kN}\right] = 0$$
$$T = 42.124\ \text{kN}$$

Referring to the FBD of the balloon Fig. *b*

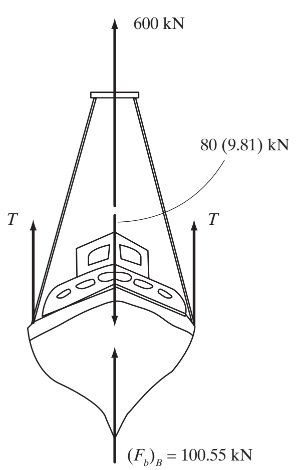
$$+\uparrow \Sigma F_y = 0; \qquad 13.08\pi r^3 - 42.125\ \text{kN} = 0$$
$$r = 1.008\ \text{m} = 1.01\ \text{m} \qquad \textbf{Ans.}$$

Here, $p = p_{\text{atm}} + \rho_w g h = 101\left(10^3\right)\ \text{Pa} + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(20\ \text{m}) = 297.2\left(10^3\right)\ \text{Pa}$ and $T = 12°\ \text{C} + 273 = 285\ \text{K}$. From Appendix A, $R = 286.9\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law,

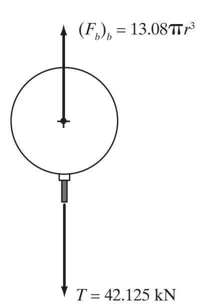
$$p = \rho R T; \qquad \rho = \frac{p}{RT} = \frac{297.2\left(10^3\right)\ \text{N/m}^2}{(286.9\ \text{J/kg}\cdot\ \text{K})(285\ \text{K})} = 3.6347\ \text{kg/m}^3$$

Thus,

$$m = \rho V = \left(3.6347\ \text{kg/m}^3\right)\left[\frac{4}{3}\pi(1.008\ \text{m})^3\right] = 15.61\ \text{kg} = 15.6\ \text{kg} \qquad \textbf{Ans.}$$



(a)

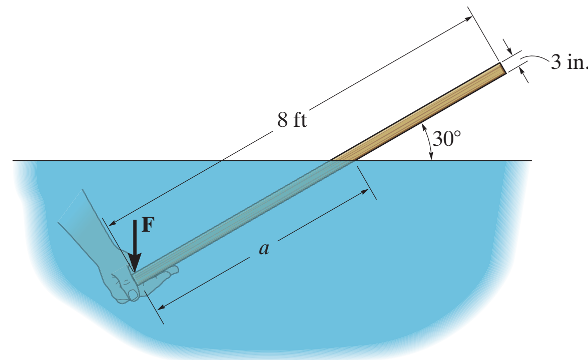


(b)

Ans:
$r = 1.01$ m
$m = 15.6$ kg

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–146. The uniform 8-ft board is pushed down into the water so it makes an angle of $\theta = 30°$ with the water surface. If the cross section of the board measures 3 in. by 9 in., and its specific weight is $\gamma_b = 30\ \text{lb/ft}^3$, determine the length a that will be submerged and the vertical force **F** needed to hold its end in this position.



SOLUTION

The weight of the board is

$$W = \gamma_b \forall_b = \left(30\ \text{lb/ft}^3\right)\left[\left(\frac{3}{12}\ \text{ft}\right)\left(\frac{9}{12}\ \text{ft}\right)(8\ \text{ft})\right] = 45\ \text{lb}$$

$$F_b = \gamma_w \forall_{\text{sub}} = \left(62.4\ \text{lb/ft}^3\right)\left[\left(\frac{3}{12}\ \text{ft}\right)\left(\frac{9}{12}\ \text{ft}\right)a\right] = 11.7a\ \text{lb}$$

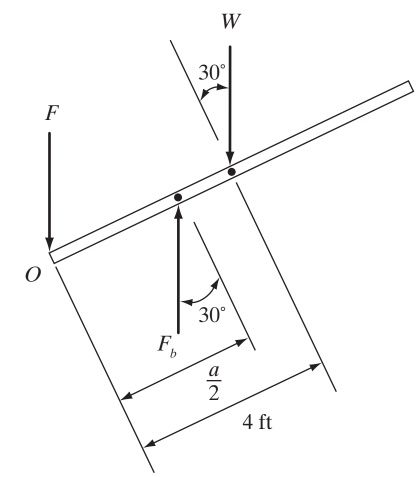
Referring to the *FBD* of the board, Fig. *a*, equilibrium requires,

$\circlearrowleft + \Sigma M_o = 0;\qquad 11.7\,a\,(\cos 30°)\left(\frac{a}{2}\right) - (45\cos 30°\ \text{lb})(4\ \text{ft}) = 0$

$$a = 5.547\ \text{ft} = 5.55\ \text{ft} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0;\qquad 11.7(5.547) - 45\ \text{lb} - F = 0$

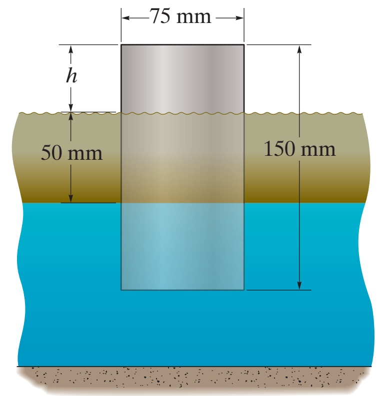
$$F = 19.90\ \text{lb} = 19.9\ \text{lb} \qquad \textbf{Ans.}$$



Ans:
$a = 5.55$ ft
$F = 19.9$ lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–147. The cylinder has a diameter of 75 mm and a mass of 600 g. If it is placed in the tank, which contains oil and water, determine the height h above the surface of the oil at which it will float if maintained in the vertical position. Take $\rho_0 = 980\ \text{kg/m}^3$



SOLUTION

Since the cylinder floats, the buoyant force is equal to the weight of the cylinder.

$$F_b = (0.6\ \text{kg})(9.81\ \text{m/s}^2) = 5.886\ \text{N}$$

Assuming that the cylinder is submerged below the oil layer, then, the buoyant force produced by the oil layer is

$$(F_b)_{oil} = \rho_{oil}\, g(\forall_s)_{oil} = (940\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.0375\ \text{m})^2(0.05\ \text{m})\right]$$

$$= 2.037\ \text{N} < F_b \qquad \textbf{(O.K!)}$$

The buoyant force produced by the water layer is

$$(F_b)_w = \rho_w g(\forall_s)_w = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.0375\ \text{m})^2(0.15\ \text{m} - 0.05\ \text{m} - h)\right]$$

$$= 43.339\,(0.1 - h)$$

We require

$$F_b = (F_b)_{oil} + (F_b)_w$$

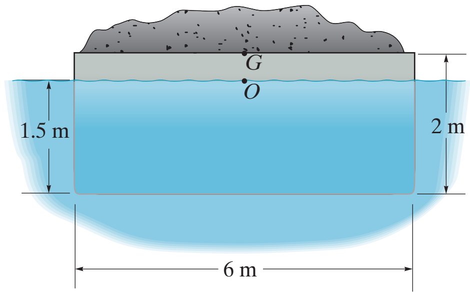
$$5.886\ \text{N} = 2.037\ \text{N} + 43.339(0.1 - h)$$

$$h = 0.01119\ \text{m} = 11.2\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$h = 11.2\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–148.** When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is located at G determine whether the barge will restore itself when a wave causes it to roll slightly at 9°.



SOLUTION

When the barge tips 9°, the submerged portion is trapezoidal in shape, as shown in Fig. *a*. The new center of buoyancy, C_b', is located at the centroid of this area. Then

$$x = \frac{(0)(6)(1.0248) + (1)\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.3168\ \text{m}$$

$$y = \frac{\frac{1}{2}(1.0248)(6)(1.0248) + \left[1.0248 + \frac{1}{3}(0.9503)\right]\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.7751\ \text{m}$$

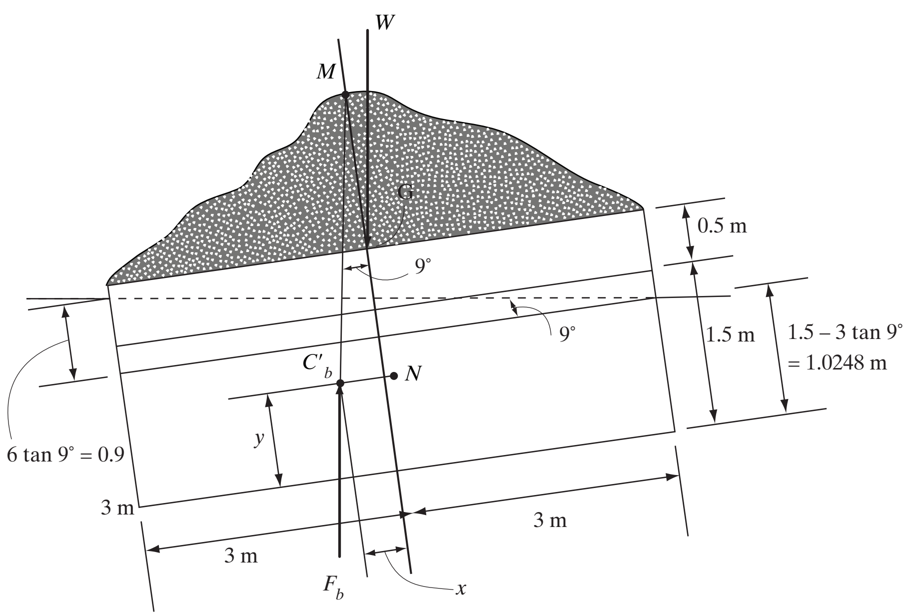
The intersection point M of the line of action of $\mathbf{F}_b$ and the centerline of the barge is the metacenter, Fig. *a*. From the geometry of triangle MNC_b' we have

$$MN = \frac{x}{\tan 9°} = \frac{0.3168}{\tan 9°} = 2\ \text{m}$$

Also,

$$GN = 2 - y = 2 - 0.7751 = 1.2249\ \text{m}$$

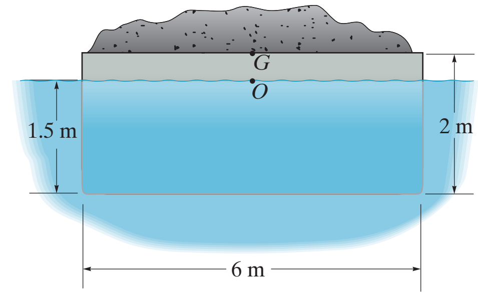
Since $MN > GN$, point M is above G. Therefore, **the barge will restore itself.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–149. When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is located at G determine whether the barge will restore itself when a wave causes it to tip slightly.



SOLUTION

The barge is tilted counterclockwise slightly and the new center of buoyancy $C_b{}'$ is located to the left of the old one. The metacenter M is at the intersection point of the center line of the barge and the line of action of $\mathbf{F}_b$, Fig. *a*. The location of $C_b{}'$ can be obtained by referring to Fig. *b*.

$$\bar{x} = \frac{(1\,\text{m})\left[\frac{1}{2}(6\,\text{m})(6\tan\phi\,\text{m})\right]}{(1.5\,\text{m})(6\,\text{m})} = 2\tan\phi\ \text{m}$$

Then

$$\delta = \bar{x}\cos\phi = 2\,\text{m}\tan\phi\ \cos\phi = (2\,\text{m})\left(\frac{\sin\phi}{\cos\phi}\right)(\cos\phi) = (2\sin\phi)\ \text{m}$$

Since ϕ is very small $\sin\phi = \phi$, hence

$$\delta = 2\phi\ \text{m} \qquad \textbf{(1)}$$

From the geometry shown in Fig. *a*

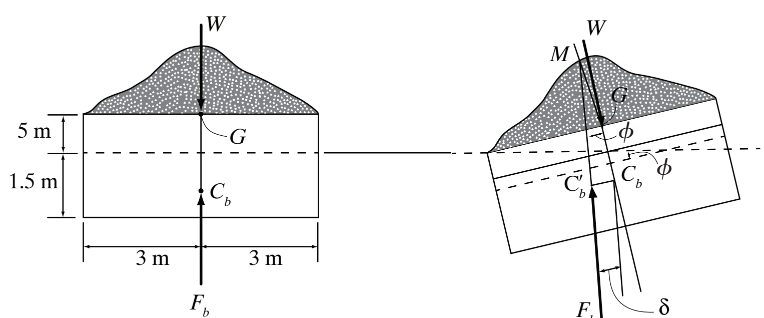
$$\delta = MC_b\ \sin\phi = MC_b\phi \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

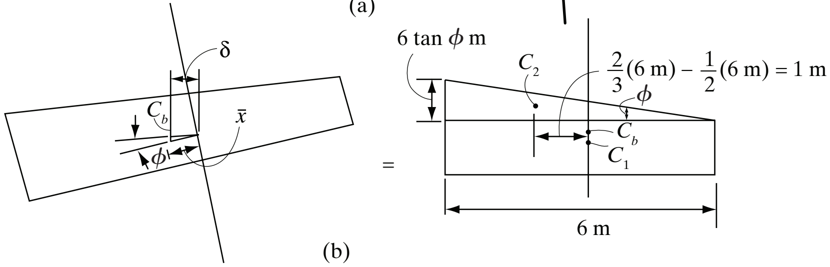
$$2\phi = MC_b\phi$$

$$MC_b = 2\,\text{m}$$

Here, $GC_b = 2\,\text{m} - 0.75\,\text{m} = 1.25\,\text{m}$. Since $MC_b > GC_b$, the barge is in stable equilibrium. Thus, it will restore itself if tilted slightly.



(a)

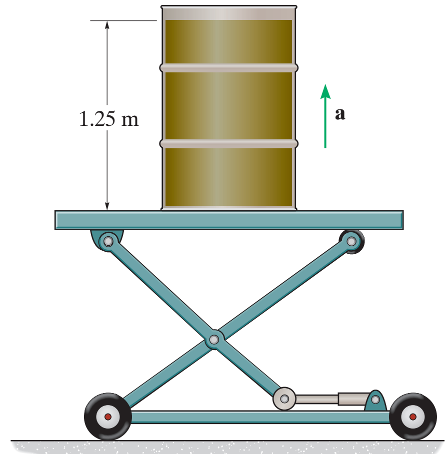


(b)

Ans:
It will restore itself.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–150. The barrel of oil rests on the surface of the scissors lift. Determine the maximum pressure developed in the oil if the lift is moving upward with (a) a constant velocity of 4 m/s, and (b) a constant acceleration of 2 m/s^2. Take $\rho_o = 900\ \text{kg/m}^3$. The top of the barrel is open to the atmosphere.



SOLUTION

a) Equilibrium

$$p = \rho_o g h = 900\ \text{kg/m}^3(9.81\ \text{m/s}^2)(1.25\ \text{m})$$

$$= 11.0\ \text{kPa} \qquad \textbf{Ans.}$$

b) $p = \rho_o g h\left(1 + \dfrac{a_C}{g}\right)$

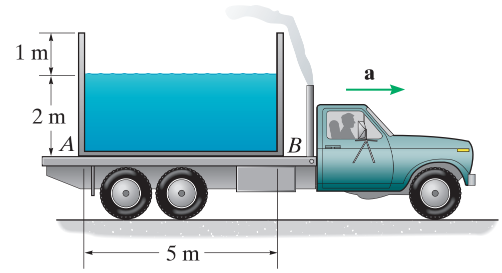
$$p = 900\ \text{kg/m}^3(9.81\ \text{m/s}^2)(1.25\ \text{m})\left(1 + \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2}\right)$$

$$p = 13.3\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
a) 11.0 kPa
b) 13.3 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–151. The truck carries an open container of water as shown. If it has a constant acceleration $2\ \text{m/s}^2$, determine the angle of inclination of the surface of the water and the pressure at the bottom corners A and B.



SOLUTION

The free surface of the water in the accelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_c}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2}$$

$$\theta = 11.52° = 11.5° \qquad \textbf{Ans.}$$

From the geometry in Fig. *a*.

$$\Delta h_A = \Delta h_B = (2.5\ \text{m})\ \tan\ 11.52°$$

$$= 0.5097\ \text{m}$$

Thus,

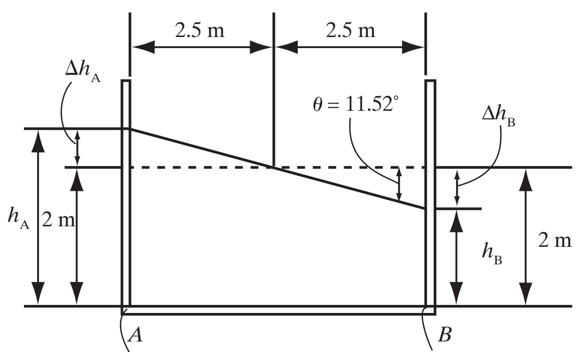
$$p_A = \rho_w g h_A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m} + 0.5095\ \text{m})$$

$$= 24.6\ \text{kPa}$$

Ans.

$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m} - 0.5095\ \text{m})$$

$$= 14.6\ \text{kPa} \qquad \textbf{Ans.}$$

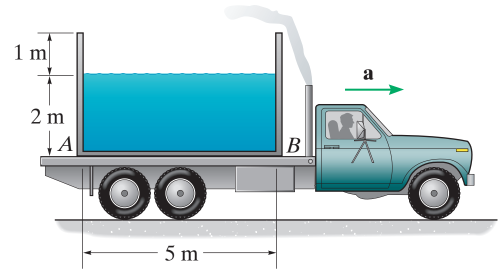


(a)

Ans:
$\theta = 11.5°$
$p_A = 24.6\ \text{kPa}$
$p_B = 14.6\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–152.** The truck carries an open container of water as shown. Determine the maximum constant acceleration it can have without causing the water to spill out of the container.

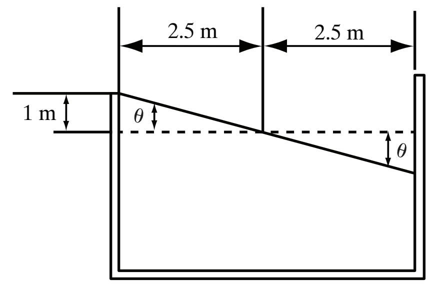


SOLUTION

When the tank accelerates, the water spill from the left side wall. The surface of the water under this condition is shown in Fig. *a*.

$$\tan\theta = \frac{1\ \text{m}}{2.5\ \text{m}} = \frac{a_c}{9.81\ \text{m/s}^2}$$

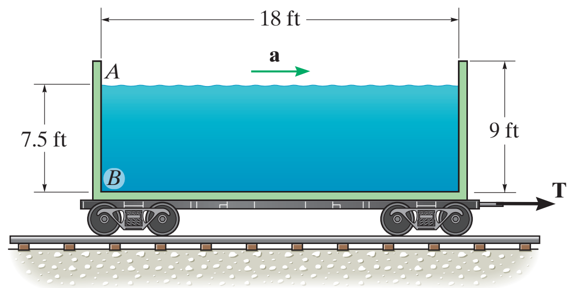
$$a_c = 3.92\ \text{m/s}^2 \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–153. The open rail car is 6 ft wide and filled with water to the level shown. Determine the pressure that acts at point B both when the car is at rest and when the car is moving with a constant acceleration of 10 ft/s^2. How much water spills out of the car?



SOLUTION

When the car is at rest, the water is at the level shown by the dashed line shown in Fig. *a*

At rest: $p_B = \gamma_w h_B = \left(62.4\ \text{lb/ft}^3\right)(7.5\ \text{ft}) = 468\ \text{lb/ft}^2$ **Ans.**

When the car accelerates, the angle θ the water level makes with the horizontal can be determined.

$$\tan\theta = \frac{a_c}{g} = \frac{10\ \text{ft/s}^2}{32.2\ \text{ft/s}^2}; \qquad \theta = 17.25°$$

Assuming that the water will spill out. Then the water level when the car accelerates is indicated by the solid line shown in Fig. *a*. Thus,

$$h = 9\ \text{ft} - 18\ \text{ft}\tan 17.25° = 3.4099\ \text{ft}$$

The original volume of water is

$$V\!\!\!/ = (7.5\ \text{ft})(18\ \text{ft})(6\ \text{ft}) = 810\ \text{ft}^3$$

The volume of water after the car accelerate is

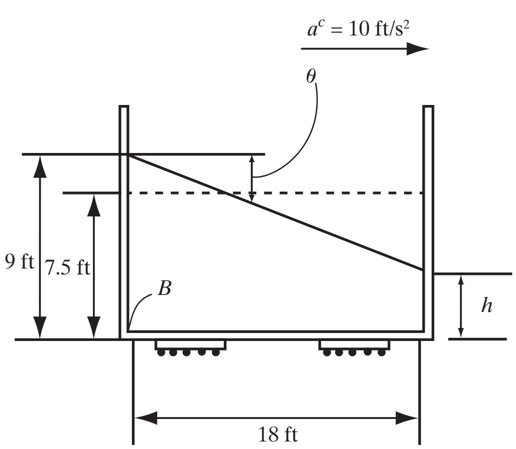
$$V\!\!\!/' = \frac{1}{2}(9\ \text{ft} + 3.4099\ \text{ft})(18\ \text{ft})(6\ \text{ft}) = 670.14\ \text{ft}^3 < 810\ \text{ft}^3 \qquad \textbf{(OK!)}$$

Thus, the amount of water spilled is

$$\Delta V\!\!\!/ = V\!\!\!/ - V\!\!\!/' = 810\ \text{ft}^3 - 670.14\ \text{ft}^3 = 139.86\ \text{ft}^3 = 140\ \text{ft}^3 \qquad \textbf{Ans.}$$

The pressure at B when the car accelerates is

With acceleration: $p_B = \gamma_w h_B = \left(62.4\ \text{lb/ft}^3\right)(9\ \text{ft}) = 561.6\ \text{lb/ft}^2 = 562\ \text{lb/ft}^2$ **Ans.**

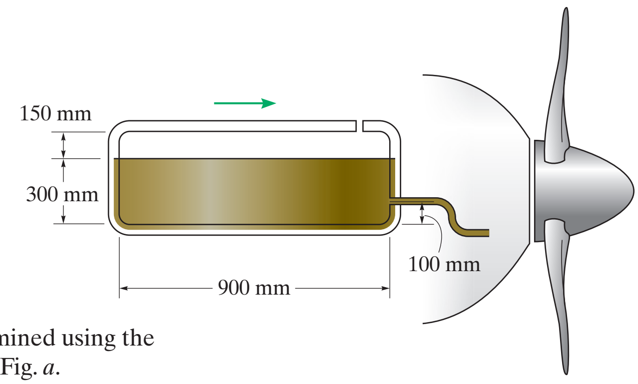


(a)

Ans:
At rest: $p_B = 468\ \text{lb/ft}^2$
With acceleration: $\Delta V\!\!\!/ = 140\ \text{ft}^3$
$p_B = 562\ \text{lb/ft}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–154. The fuel tank, supply line, and engine for an airplane are shown. If the gas tank is filled to the level shown, determine the largest constant acceleration a that the plane can have without causing the engine to be starved of fuel. The plane is accelerating to the right for this to happen. Suggest a safer location for attaching the fuel line.



SOLUTION

If the fuel width of the tank is w, the volume of the fuel can be determined using the fuel level when the airplane is at rest indicated by the dashed line in Fig. *a*.

$$\forall_f = (0.9\text{ m})(0.3\text{ m})w = 0.27w$$

It is required that the fuel level is about to drop lower than the supply line. In this case, the fuel level is indicated by the solid line in Fig. *a*.

$$\forall_f = (0.9\text{ m})(0.45\text{ m})w - \frac{1}{2}(0.45\text{ m} - 0.1\text{ m})(0.9\text{ m} - b)w = 0.27w$$

$$b = 0.1286\text{ m}$$

Thus,

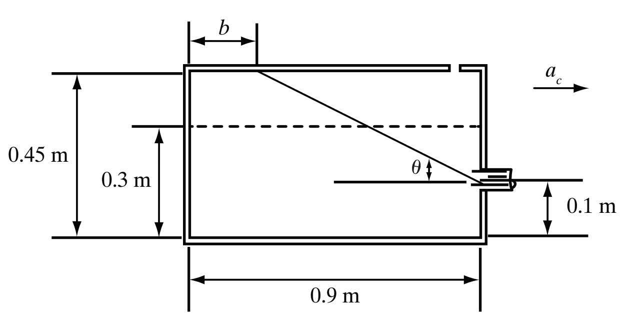
$$\tan\theta = \frac{0.45\text{ m} - 0.1\text{ m}}{0.9\text{ m} - 0.1286\text{ m}} = 0.4537$$

And so

$$\tan\theta = \frac{a_c}{g}; \qquad 0.4537 = \frac{a_c}{9.81\text{ m/s}^2}$$

$$a_c = 4.45\text{ m/s}^2 \qquad \textbf{Ans.}$$

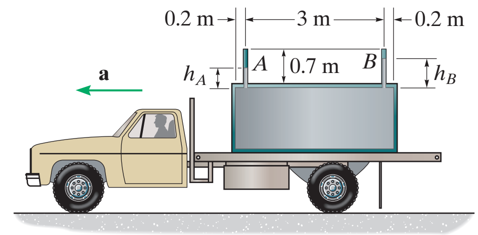
The safer location for attaching the fuel line is at the bottom of the tank.



Ans:
$a_c = 4.45\text{ m/s}^2$
The safer location is at the bottom of the tank.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–155. A large container of benzene is transported on the truck. Determine the level in each of the vent tubes A and B if the truck accelerates at $a = 1.5\ \text{m/s}^2$. When the truck is at rest, $h_A = h_B = 0.4$ m.



SOLUTION

The imaginary surface of the benzene in the accelerated tank is shown in Fig. a.

$$\tan\theta = \frac{a_c}{g} = \frac{1.5\ \text{m/s}^2}{9.81\ \text{m/s}^2}$$

$$\theta = 8.6935^\circ$$

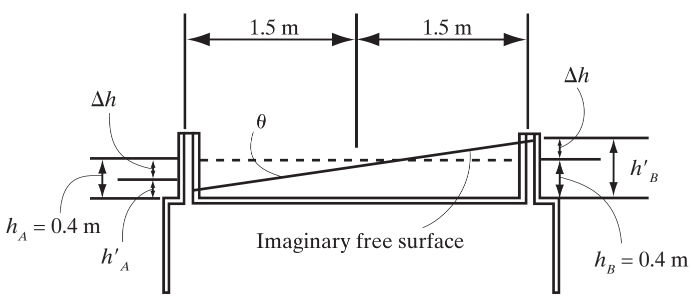
Then,

$$\Delta h = (1.5\ \text{m})\ \tan\ 8.6935^\circ = 0.2294\ \text{m}$$

Thus,

$$h'_A = h_A - \Delta h = 0.4\ \text{m} - 0.2294\ \text{m} = 0.171\ \text{m} \qquad \textbf{Ans.}$$

$$h'_B = h_B + \Delta h = 0.4\ \text{m} + 0.2294\ \text{m} = 0.629\ \text{m} \qquad \textbf{Ans.}$$

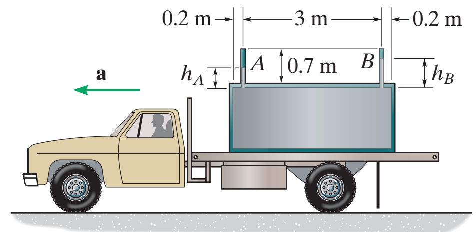


(a)

Ans:
$h'_A = 0.171$ m
$h'_B = 0.629$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–156.** A large container of benzene is being transported by the truck. Determine its maximum constant acceleration so that no benzenes will spill from the vent tubes A or B. When the truck is at rest, $h_A = h_B = 0.4$ m.

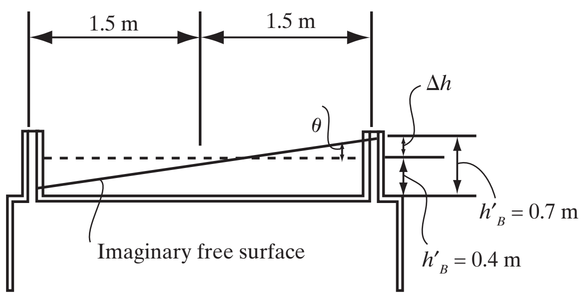


SOLUTION

The imaginary surface of the benzene in the accelerated tank is shown in Fig. a. Under this condition, the water will spill from vent B. Thus, $\Delta h = h'_B - h_B = 0.7 \text{ m} - 0.4 \text{ m} = 0.3 \text{ m}$.

$$\tan\theta = \frac{0.3 \text{ m}}{1.5 \text{ m}} = 0.2 = \frac{a_c}{g}$$

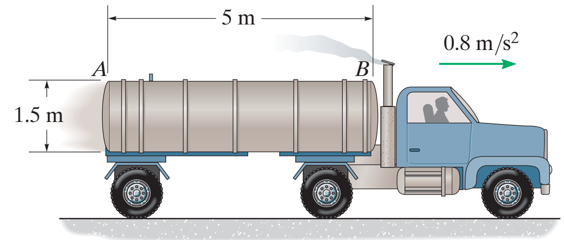
$$a_c = 0.2\left(9.81 \text{ m/s}^2\right) = 1.96 \text{ m/s}^2 \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–157. The closed cylindrical tank is filled with milk, for which $\rho_m = 1030\ \text{kg/m}^3$. If the inner diameter of the tank is 1.5 m, determine the difference in pressure within the tank between corners A and B when the truck accelerates at $0.8\ \text{m/s}^2$.



SOLUTION

The imaginary surface of the milk in the accelerated tank is shown in Fig. *a*.

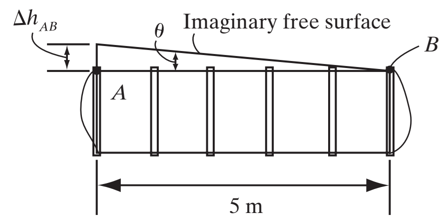
$$\tan\theta = \frac{a_c}{g} = \frac{0.8\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.08155$$

Then,

$$\Delta h_{AB} = L_{AB}\tan\theta = (5\ \text{m})(0.08155) = 0.4077\ \text{m}$$

Finally,

$$\Delta p_{AB} = \rho_m g \Delta h_{AB} = \left(1030\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.4077\ \text{m})$$
$$= 4.12\left(10^3\right)\ \text{Pa} = 4.12\ \text{kPa} \qquad \textbf{Ans.}$$

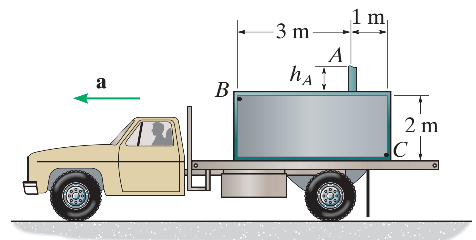


(a)

Ans:
4.12 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–158. Determine the water pressure at points B and C in the tank if the truck has a constant acceleration $a_c = 2\ \text{m/s}^2$. When the truck is at rest, the water level in the vent tube A is at $h_A = 0.3$ m.



SOLUTION

The water level at vent tube A will not change when the tank is accelerated since the water in the tank is confined (no other vent tube). Thus, the imaginary free surface must pass through the free surface at vent tube A.

$$\tan\theta = \frac{a_C}{g}; \qquad \tan\theta = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2} \qquad \theta = 11.52°$$

From the geometry in Fig. a,

$$\Delta h_B = (3\ \text{m})\ \tan\ 11.52° = 0.6116\ \text{m}$$

$$\Delta h_C = (1\ \text{m})\ \tan\ 11.52° + 0.3\ \text{m} = 0.5039\ \text{m}$$

Then,

$$h_B = -(\Delta h_B - 0.3\ \text{m}) = -(0.6116\ \text{m} - 0.3\ \text{m}) = -0.3116\ \text{m}$$

$$h_C = \Delta h_C + 2\ \text{m} = 0.5039\ \text{m} + 2\ \text{m} = 2.5039\ \text{m}$$

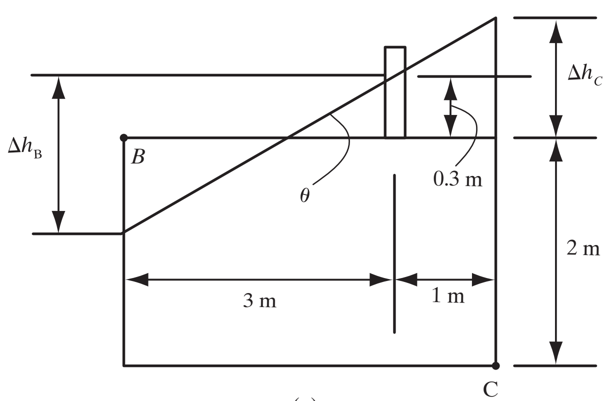
Thus,

$$p_B = \rho_w g h_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-0.3116\ \text{m})$$

$$= -3.057(10^3)\ \text{Pa} = -3.06\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_C = \rho_w g h_C = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5039\ \text{m})$$

$$= 24.563(10^3)\ \text{Pa} = 24.6\ \text{kPa} \qquad \textbf{Ans.}$$

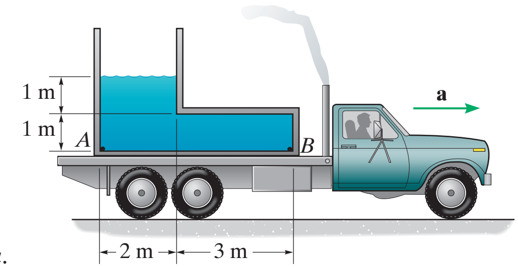


(a)

Ans:
$p_B = -3.06$ kPa
$p_C = 24.6$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–159. If the truck has a constant acceleration of $2\ \text{m/s}^2$, determine the water pressure at the bottom corners A and B of the water rank.



SOLUTION

The imaginary free surface of the water in the accelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_C}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.2039$$

From the geometry in Fig. *a*,

$$\Delta h_A = (1\ \text{m})\tan\theta = (1\ \text{m})(0.2039) = 0.2039\ \text{m}$$

$$\Delta h_B = (1\ \text{m} + 3\ \text{m})\tan\theta = (4\ \text{m})(0.2039) = 0.8155\ \text{m}$$

Then

$$h_A = 2\ \text{m} + \Delta h_A = 2\ \text{m} + 0.2039\ \text{m} = 2.2039\ \text{m}$$

$$h_B = 2\ \text{m} - \Delta h_B = 2\ \text{m} - 0.8155\ \text{m} = 1.1845\ \text{m}$$

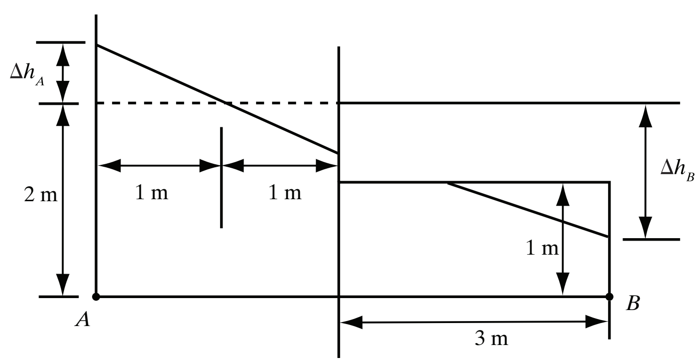
Finally,

$$p_A = \rho_w g h_A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.2039\ \text{m})$$

$$= 21.62\left(10^3\right)\ \text{Pa} = 21.6\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.1845\ \text{m})$$

$$= 11.62\left(10^3\right)\ \text{Pa} = 11.6\ \text{kPa} \qquad \textbf{Ans.}$$

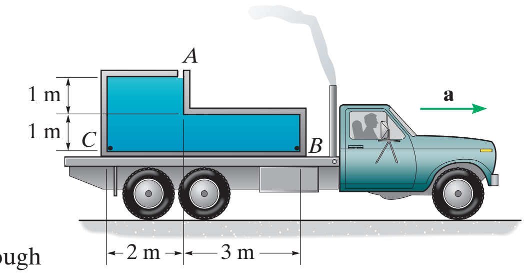


(a)

Ans:
$p_A = 21.6\ \text{kPa}$
$p_B = 11.6\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–160.** If the truck has a constant acceleration of $2\ \text{m/s}^2$, determine the water pressure at the bottom corners B and C of the water tank. There is a small opening at A.



SOLUTION

Since the water in the tank is confined, the imaginary free surface must pass through A as shown in Fig. a. We have

$$\tan\theta = \frac{a_C}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.2039$$

From the geometry in Fig. a,

$$\Delta h_C = (2\ \text{m})\tan\theta = (2\ \text{m})(0.2039) = 0.4077\ \text{m}$$

$$\Delta h_B = (3\ \text{m})\tan\theta = (3\ \text{m})(0.2039) = 0.6116\ \text{m}$$

Then

$$h_C = 2\ \text{m} + \Delta h_A = 2\ \text{m} + 0.4077\ \text{m} = 2.4077\ \text{m}$$

$$h_B = 2\ \text{m} - \Delta h_B = 2\ \text{m} - 0.6116\ \text{m} = 1.3884\ \text{m}$$

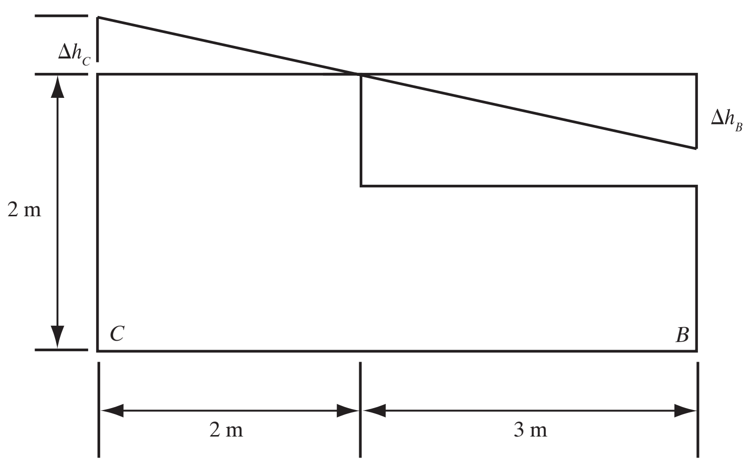
Finally,

$$p_C = \rho_w g h_C = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.4077\ \text{m})$$

$$= 23.62\left(10^3\right)\ \text{Pa} = 23.6\ \text{kPa} \qquad \textbf{Ans.}$$

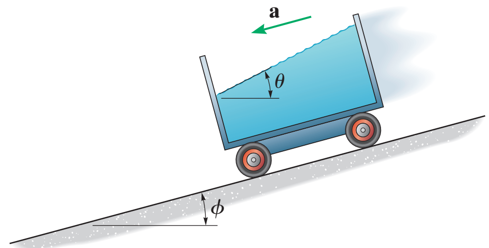
$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.3884\ \text{m})$$

$$= 13.62\left(10^3\right)\ \text{Pa} = 13.6\ \text{kPa} \qquad \textbf{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–161. The cart is allowed to roll freely down the inclined plane due to its weight. Show that the slope of the surface of the liquid θ, during the motion is $\theta = \phi$.



SOLUTION

Referring to the free-body diagram of the container in Fig. *a*,

$\swarrow + \Sigma F_{x'} = m a_{x'}$

$$w \sin\phi = \frac{w}{g} a$$

$$a = g \sin\phi$$

Referring to Fig. *b*,

$$a_x = -(g \sin\phi)\cos\phi$$

$$a_y = -(g \sin\phi)\sin\phi$$

We will now apply Newton's equations of notation, Fig. *c*.

$$\xrightarrow{+} \Sigma F_x = m a_x; \qquad -\left(p_x + \frac{\partial p_x}{\partial x}dx\right)dydz + p_x\, dydz = \frac{\gamma(dxdydz)}{g}a_x$$

$$dp_x = -\frac{\gamma dx}{g}a_x$$

In y direction,

$$+\uparrow \Sigma F_y = m a_y; \qquad p_y dxdz - \left(p_y + \frac{\partial p_y}{\partial y}dy\right)dxdz - \gamma dxdydz = \frac{\gamma dxdydz}{g}a_y$$

$$dp_y = -\gamma dy\left(1 + \frac{a_y}{g}\right)$$

At the surface, p is constant, so that $dp_x + dp_y = 0$, or $dp_x = -dp_y$.

$$\frac{\gamma dx}{g}a_x = -\gamma dy\left(1 + \frac{a_y}{g}\right)$$

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} = \frac{g \sin\phi\cos\phi}{g - g\sin\phi\sin\phi} = \frac{\sin\phi\cos\phi}{\cos^2\phi} = \frac{\sin\phi}{\cos\phi} = \tan\phi$$

Since at the surface,

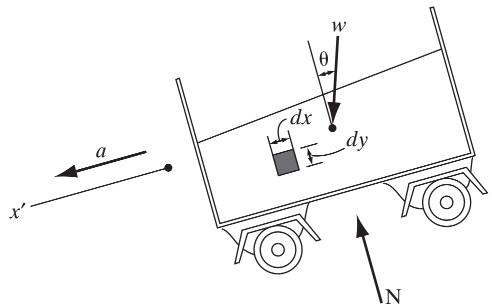
$$\frac{dy}{dx} = -\tan\theta$$

then

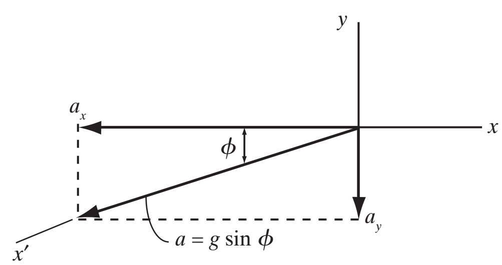
$$\tan\theta = \tan\phi$$

or

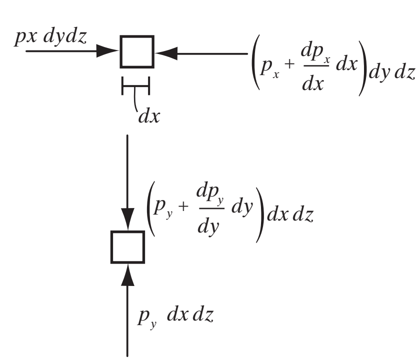
$$\theta = \phi \qquad \textbf{Q.E.D.}$$



(a)



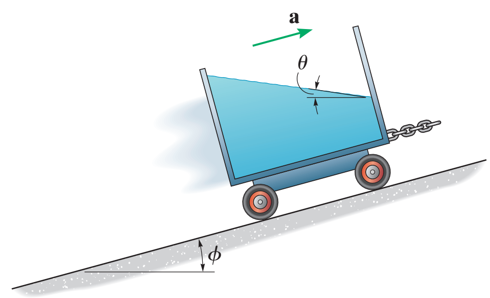
(b)



(c)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–162. The cart is given a constant acceleration **a** up the plane, as shown. Show that the lines of constant pressure *within* the liquid have a slope of $\tan\theta = -(a\cos\phi)/(a\sin\phi + g)$.



SOLUTION

As in the preceding solution, we determine that

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} \quad \textbf{(1)}$$

Here, the slope of the surface of the liquid, Fig. *a*, is

$$\frac{dy}{dx} = -\tan\theta \quad \textbf{(2)}$$

Equating Eqs. (1) and (2), we obtain

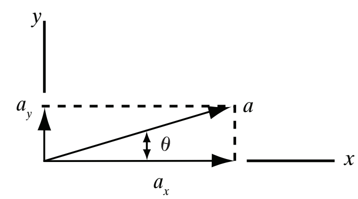
$$\tan\theta = \frac{a_x}{g + a_y} \quad \textbf{(3)}$$

By establishing the x and y axes shown in Fig. *a*,

$$a_x = a\ \cos\phi \qquad\qquad a_y = a\ \sin\phi$$

Substituting these values into Eq. (3),

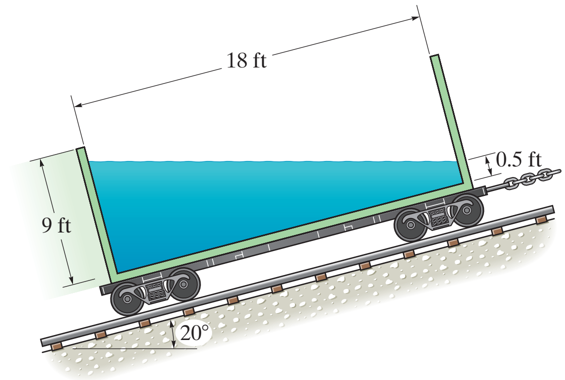
$$\tan\theta = \frac{a\ \cos\phi}{a\ \sin\phi + g} \quad \textbf{Q.E.D}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–163. The open railcar is used to transport water up the 20° incline. When the car is at rest, the water level is as shown. Determine the maximum acceleration the car can have when it is pulled up the incline so that no water will spill out.



SOLUTION

The volume of the water can be determined by using the water level when the car is at rest, indicated by dashed line in Fig. *a*. Here

$$a = 0.5\ \text{ft} + 18 \tan 20°\ \text{ft} = 7.0515\ \text{ft}$$

If the width of the car is w

$$\cancel{V}_w = \frac{1}{2}(0.5\ \text{ft} + 7.0515\ \text{ft})(18\ \text{ft})w = 67.9632\ w$$

It is required that the water is about to spill out. In this case, the water is indicated by the solid line in Fig. *a*,

$$\cancel{V}_w = \frac{1}{2}(9\ \text{ft})(\text{b})\ w = 67.9632\ w$$

$$b = 15.1029\ \text{ft} < 18\ \text{ft} \qquad \textbf{(O.K.)}$$

Then,

$$\theta = \tan^{-1}\left(\frac{9\ \text{ft}}{15.1029\ \text{ft}}\right) - 20° = 10.7912°$$

Consider the vertical block of water of weight $\delta_w = \gamma_w h \delta A$ shown shaded in Fig. *a*

$$+\uparrow \Sigma F_y = ma_y; \qquad p\delta_A - \gamma_w h \delta_A = \frac{\gamma_w h \delta_A}{g} a \sin 20°$$

$$p = \frac{\gamma_w h}{g} a \sin 20° + \gamma_w h$$

$$p = \frac{\gamma_w h}{g}(a \sin 20° + g) \qquad \textbf{(1)}$$

Consider the horizontal block of water of weight $\delta_w = \gamma_w x \delta A$ shown shaded in Fig. *a*

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad p_2 \delta_A - p_1 \delta_A = \frac{\gamma_w x \delta_A}{g} a \cos 20°$$

$$p_2 - p_1 = \frac{\gamma_w x}{g} a \cos 20° \qquad \textbf{(2)}$$

However, from Eq. (1), p_2 is at h_2 and p_1 is at h_1, so that

$$p_2 - p_1 = \frac{\gamma_w}{g}(h_2 - h_1)(a \sin 20° + g)$$

Substituting this result into Eq. (2), we have

$$\frac{\gamma_w}{g}(h_2 - h_1)(a \sin 20° + g) = \frac{\gamma_w x}{g} a \cos 20°$$

$$\frac{h_2 - h_1}{x} = \frac{a \cos 20°}{a \sin 20° + g}$$

2–163. Continued

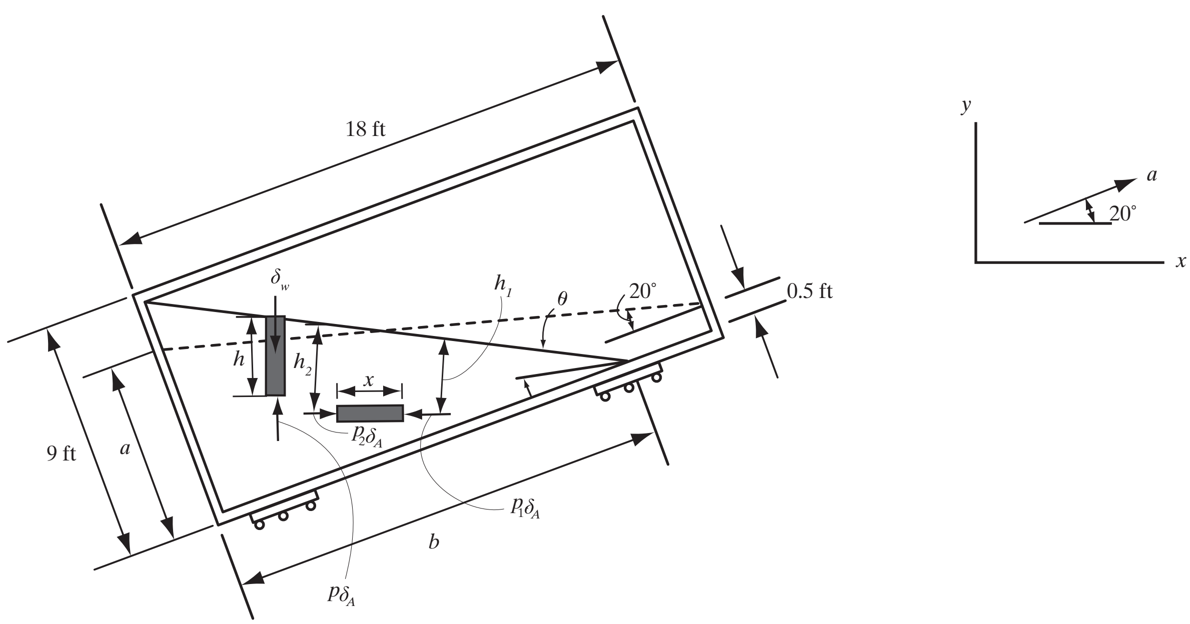
However, $\tan\theta = \dfrac{h_2 - h_1}{x}$. Thus

$$\tan\theta = \frac{a\cos 20^\circ}{a\sin 20^\circ + g}$$

Here, $\theta = 10.7912^\circ$. Then

$$\tan 10.7912^\circ = \frac{a\cos 20^\circ}{a\sin 20^\circ + 32.2\ \text{ft/s}^2}$$

$$a = 7.02\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

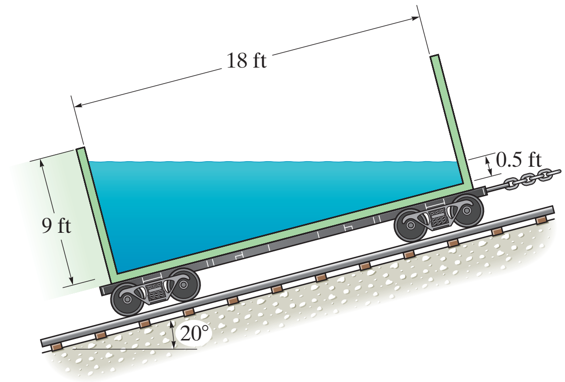


(a)

Ans:
$7.02\ \text{ft/s}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–164.** The open railcar is used to transport water up the 20° incline. When the car is at rest, the water level is as shown. Determine the maximum deceleration the car can have when it is pulled up the incline so that no water will spill out.



SOLUTION

The volume of water can be determined using the water level when the car is at rest indicated by the dashed line in Fig. *a*. Here,

$$a = 0.5\ \text{ft} + 18\ \text{ft}\ \tan 20° = 7.0515\ \text{ft}$$

If the width of the car is w,

$$V_w = \frac{1}{2}(0.5\ \text{ft} + 7.0515\ \text{ft})(18\ \text{ft})w = 67.9632\ w$$

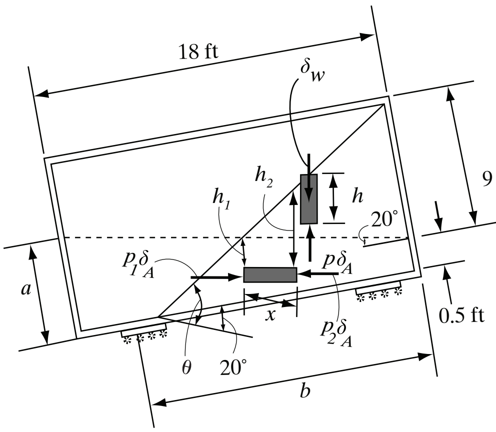
It is required that the water is about to spill out. In this case, the water level is indicated by the solid line in Fig. *a*

$$V_w = \frac{1}{2}(9\ \text{ft})(b)(w) = 67.9632\ w$$

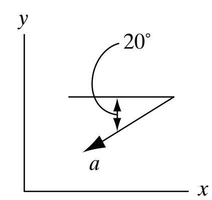
$$b = 15.1029\ \text{ft} < 18\ \text{ft} \qquad \textbf{(O.K)}$$

Then

$$\theta = \tan^{-1}\left(\frac{9\ \text{ft}}{15.1029\ \text{ft}}\right) + 20° = 50.7912°$$



(a)



Consider the vertical block of water of weight $\delta_w = \gamma_w h \delta_A$ shown shaded in Fig. *a*.

$$+\uparrow \Sigma F_y = ma_y; \qquad p\,\delta_A - \gamma_w h \delta_A = \frac{\gamma_w h \delta_A}{g}(-a \sin 20°)$$

$$p = \frac{\gamma_w h}{g}(g - a \sin 20°) \qquad \textbf{(1)}$$

Consider the horizontal block of water of weight $\delta_w = \gamma_w x \delta_A$ shown shaded in Fig. *a*

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad p_1\delta_A - p_2\delta_A = \frac{\gamma_w x \delta_A}{g}(-a \cos 20°)$$

$$p_2 - p_1 = \frac{\gamma_w x}{g} a \cos 20° \qquad \textbf{(2)}$$

However, from Eq. (1), since p_1 is at h_1 and p_2 is at h_2.

$$p_2 - p_1 = \frac{\gamma_w}{g}(h_2 - h_1)(g - a \sin 20°)$$

Substituting this result into Eq. (2)

$$\frac{\gamma_w}{g}(h_2 - h_1)(g - a \sin 20°) = \frac{\gamma_w x}{g} a \cos 20°$$

$$\frac{h_2 - h_1}{x} = \frac{a \cos 20°}{g - a \sin 20°}$$

However, $\tan\theta = \dfrac{h_2 - h_1}{x}$. Thus

$$\tan\theta = \frac{a\ \cos 20°}{g - a\ \sin 20°}$$

Here, $\theta = 50.7912°$. Then

$$\tan 50.7912° = \frac{a \cos 20°}{32.2\ \text{ft/s}^2 - a \sin 20°}$$

$$a = 29.04\ \text{ft/s}^2 = 29.0\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–165. A woman stands on a horizontal platform that is rotating at 1.5 rad/s. If she is holding a cup of coffee, and the center of the cup is 4 m from the axis of rotation, determine the slope angle of the coffee's surface. Neglect the size of the cup.

SOLUTION

Since the coffee cup is rotating at a constant velocity about the vertical axis of rotation, then its acceleration is always directed horizontally toward the axis of rotation and its magnitude is given by

$$a_r = \omega^2 r = (1.5\ \text{rad/s})^2(4\ \text{m}) = 9\ \text{m/s}^2$$

Thus, the slope of coffee surface is

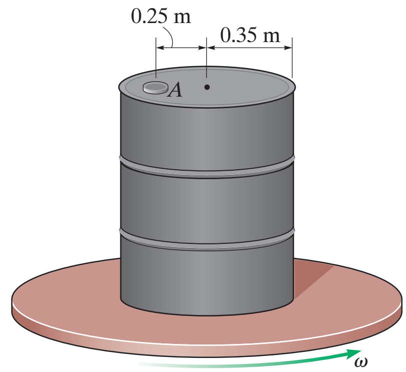
$$m = \tan\theta = \frac{a_r}{g} = \frac{9\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.917$$

$$\theta = 42.5^\circ \qquad \textbf{Ans.}$$

Ans:
42.5°

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–166. The drum is filled to the top with oil and placed on the platform. If the platform is given a rotation of $\omega = 12\ \text{rad/s}$, determine the pressure the oil will exert on the cap at A. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

We observe from Fig. a that $h = h_A$ at $r = 0.25$ m.

$$h_A = \frac{\omega^2}{2g} r^2$$

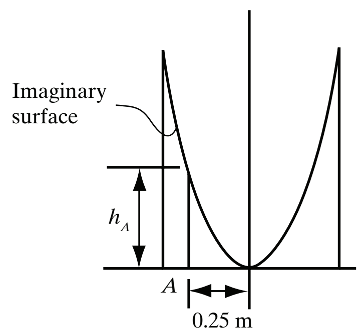
$$h_A = \left[\frac{(12\ \text{rad/s})^2}{2(9.81\ \text{m/s}^2)}\right](0.25\ \text{m})^2$$

$$= 0.4587\ \text{m}$$

$$p_A = \rho_o g h_A = (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.4587\ \text{m})$$

$$= 4.05(10^3)\ \text{Pa}$$

$$= 4.05\ \text{kPa}$$ **Ans.**

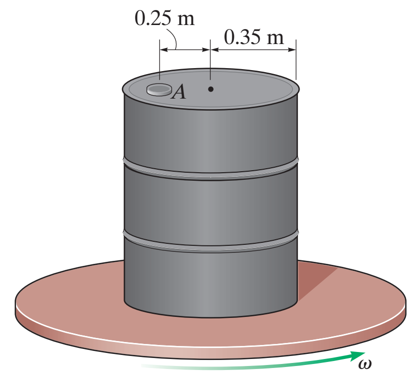


(a)

Ans:
4.05 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–167. The drum is filled to the top with oil and placed on the platform. Determine the maximum rotation of the platform if the maximum pressure the cap at A can sustain before it opens is 40 kPa. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

It is required that $p_A = 40$ kPa. Thus, the pressure head for the oil is

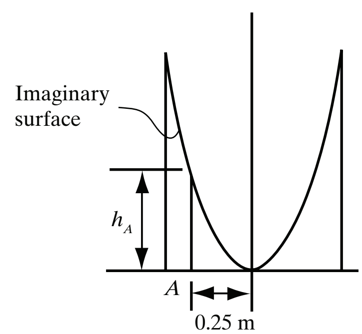
$$h_A = \frac{p_A}{\gamma_O} = \frac{40(10^3)\ \text{N/m}^2}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 4.531\ \text{m}$$

We observe from Fig. a that $h = h_A$ at $r = 0.25$ m.

$$h_A = \frac{\omega^2}{2g}r^2$$

$$4.531\ \text{m} = \left[\frac{\omega^2}{2(9.81\ \text{m/s}^2)}\right](0.25\ \text{m})^2$$

$$\omega = 37.7\ \text{rad/s}$$ **Ans.**

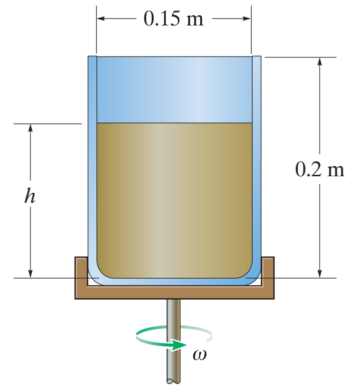


(a)

Ans:
37.7 rad/s

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–168.** The beaker is filled to a height of $h = 0.1$ m with kerosene and place on the platform. What is the maximum angular velocity ω it can have so that no kerosene spills out of the beaker?



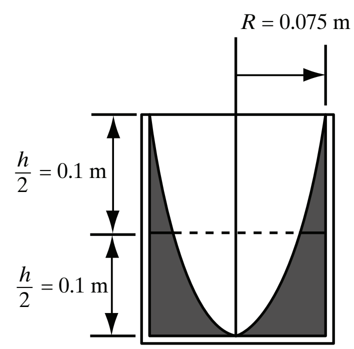
SOLUTION

When the kerosene is about to spill out of the beaker, Fig. *a*, $h/2 = 0.1$ m or $h = 0.2$ m.

$$h = \frac{\omega^2}{2g}r^2$$

$$0.2\text{ m} = \frac{\omega^2}{2(9.81\text{ m/s}^2)}\left(\frac{0.15\text{ m}}{2}\right)^2$$

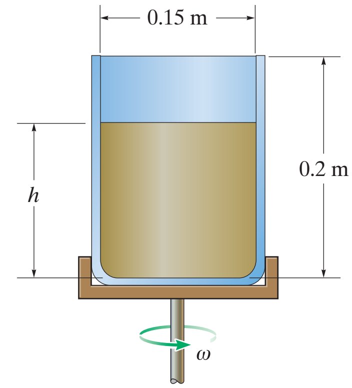
$$\omega = 26.4\text{ rad/s}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–169. The beaker is filled to a height of $h = 0.1$ m with kerosene and placed on the platform. To what height $h = h'$ does the kerosene rise against the wall of the beaker when the platform has an angular velocity of $\omega = 15$ rad/s?



SOLUTION

$$H = \frac{\omega^2}{2g} r^2$$

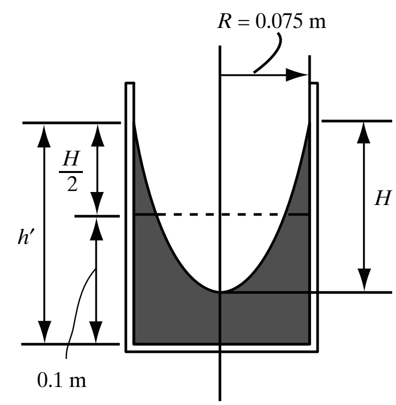
$$H = \frac{(15\ \text{rad/s})^2}{2(9.81\ \text{m/s}^2)}\left(\frac{0.15\ \text{m}}{2}\right)^2$$

$$= 0.0645\ \text{m}$$

From Fig. *a*, we observe that

$$h' = 0.1\ \text{m} + \frac{0.0645}{2}\ \text{m}$$

$$h' = 0.132\ \text{m} = 132\ \text{mm} \qquad \textbf{Ans.}$$

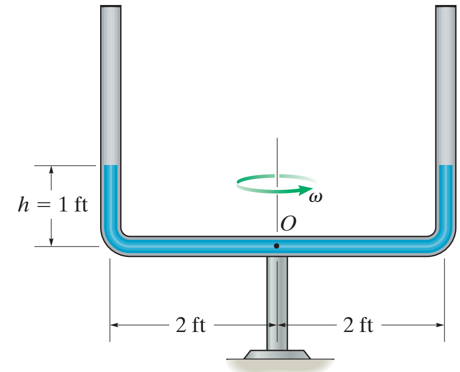


(a)

Ans:
132 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–170. The tube is filled with water to the level $h = 1$ ft. Determine the pressure at point O when the tube has an angular velocity of $\omega = 8$ rad/s.



SOLUTION

The level of the water in the tube will not change. Therefore, the imaginary surface will be as shown in Fig. *a*.

$$H = \frac{\omega^2 R^2}{2g} = \frac{(8\ \text{rad/s})^2(2\ \text{ft})^2}{2(32.2\ \text{ft/s}^2)} = 3.9752\ \text{ft}$$

We observe from Fig. *a* that

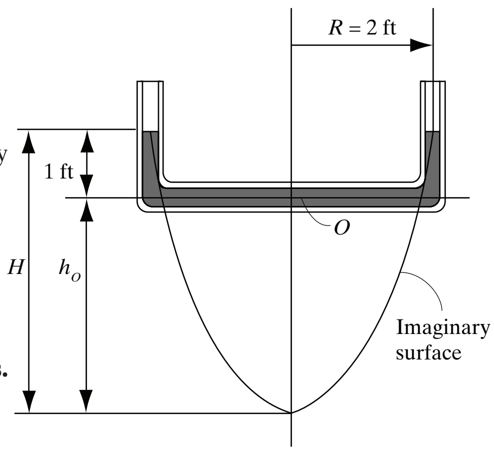
$$h_O = H - 1\ \text{ft} = 3.9752\ \text{ft} - 1\ \text{ft} = 2.9752\ \text{ft}$$

Finally, the pressure at O must be negative since it is 2.9752 ft *above* the imaginary surface of the liquid.

$$p_O = \gamma h_O = (62.4\ \text{lb/ft}^3)(-2.9752\ \text{ft})$$

$$= \left(-185.65\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= -1.29\ \text{psi} \qquad \textbf{Ans.}$$

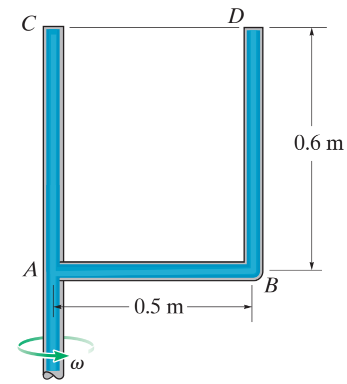


(a)

Ans:
-1.29 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–171. The sealed assembly is completely filled with water such that the pressures at C and D are zero. If the assembly is given an angular velocity of $\omega = 15\ \text{rad/s}$, determine the difference in pressure between points C and D.



SOLUTION

$$H = \frac{\omega^2 R^2}{2g}$$

$$= \frac{(15\ \text{rad/s})^2(0.5\ \text{m})^2}{2(9.81\ \text{m/s}^2)}$$

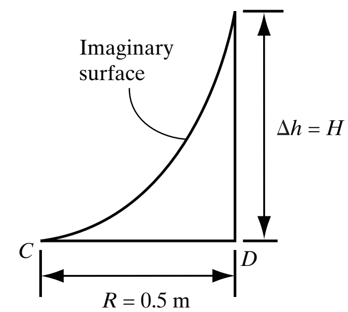
$$= 2.867\ \text{m}$$

From Fig. a, $\Delta h = H = 2.867\ \text{m}$. Then,

$$\Delta p = p_D - p_C = \rho_w g \Delta h$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.867\ \text{m})$$

$$= 28.13(10^3)\ \text{Pa} = 28.1\ \text{kPa}$$ **Ans.**

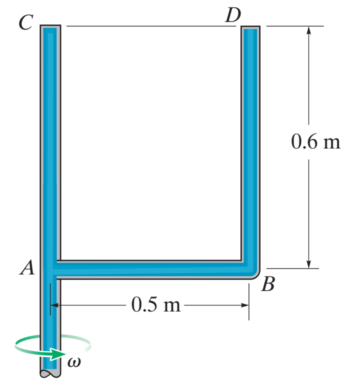


(a)

Ans:
28.1 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–172.** The sealed assembly is completely filled with water such that the pressures at C and D are zero. If the assembly is given an angular velocity of $\omega = 15\ \text{rad/s}$, determine the difference in pressure between points A and B.



SOLUTION

$$H = \frac{\omega^2 R^2}{2g}$$

$$= \frac{(15\ \text{rad/s})^2(0.5\ \text{m})^2}{2(9.81\ \text{m/s}^2)}$$

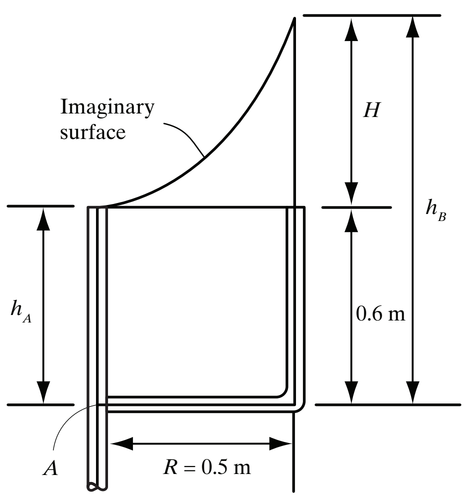
$$= 2.867\ \text{m}$$

From Fig. a, $\Delta h = h_B - h_A = H = 2.867\ \text{m}$. Then,

$$\Delta p = p_B - p_A = \rho_w g \Delta h$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.867\ \text{m})$$

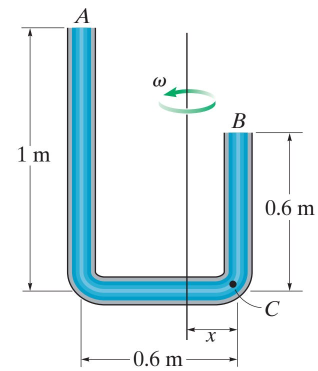
$$= 28.13(10^3)\ \text{Pa} = 28.1\ \text{kPa}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–173. The U-tube is filled with water and A is open while B is closed. If the axis of rotation is at $x = 0.2$ m, determine the constant rate of rotation so that the pressure at B is zero.



SOLUTION

Since points A and B have zero gauge pressure, the imaginary free surface must pass through them as shown in Fig. *a*.

$$h'_A = \frac{\omega^2 r_A^2}{2g} = \frac{\omega^2(0.4\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.008155\omega^2$$

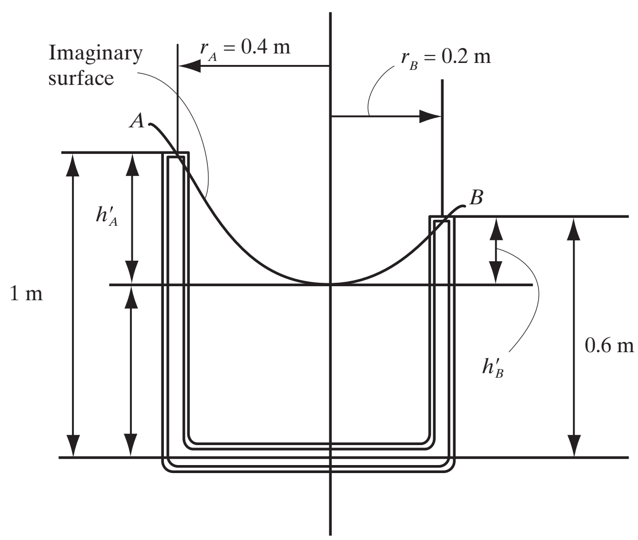
$$h'_B = \frac{\omega^2 r_B^2}{2g} = \frac{\omega^2(0.2\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.002039\omega^2$$

From Fig. *a*,

$$1\ \text{m} - h'_A = 0.6\ \text{m} - h'_B$$

$$1\ \text{m} - 0.008155\omega^2 = 0.6\ \text{m} - 0.002039\omega^2$$

$$\omega = 8.09\ \text{rad/s}$$ **Ans.**

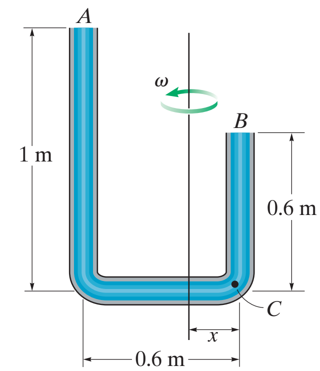


(a)

Ans:
8.09 rad/s

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–174. The U-tube is filled with water and A is open while B is closed. If the axis of rotation is at $x = 0.2$ m and the tube is rotating at a constant rate of $\omega = 10\ \text{rad/s}$, determine the pressure at points B and C.



SOLUTION

Since point A has zero gauge pressure, the imaginary free surface must pass through this point as shown in Fig. a.

$$H = \frac{\omega^2 R^2}{2g} = \frac{(10\ \text{rad/s})^2(0.4\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.8155\ \text{m}$$

and

$$h' = \frac{\omega^2 r^2}{2g} = \frac{(10\ \text{rad/s})^2(0.2\ \text{m})^2}{2(9.18\ \text{m/s}^2)} = 0.2039\ \text{m}$$

From Fig. a,

$$a = 1\ \text{m} - H = 1\ \text{m} - 0.8155\ \text{m} = 0.1845\ \text{m}$$

Then,

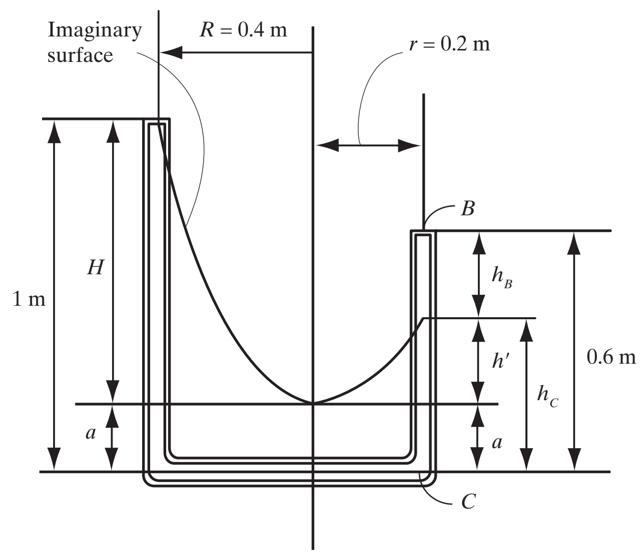
$$h_B = -(0.6\ \text{m} - h' - a) = -(0.6\ \text{m} - 0.2039\ \text{m} - 0.1845\ \text{m}) = -0.2116\ \text{m}$$

$$h_C = h' + a = 0.2039\ \text{m} + 0.1845\ \text{m} = 0.3884\ \text{m}$$

Finally,

$$p_B = \rho_w g h_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-0.2116\ \text{m})$$
$$= -2.076(10^3)\ \text{Pa}$$
$$= -2.08\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_C = \rho_w g h_C = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3884\ \text{m})$$
$$= 3.81(10^3)\ \text{Pa}$$
$$= 3.81\ \text{kPa} \qquad \textbf{Ans.}$$

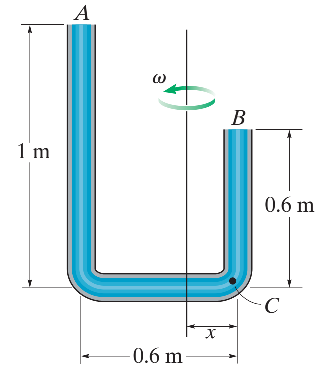


(a)

Ans:
$p_B = -2.08\ \text{kPa}$
$p_C = 3.81\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–175. The U-tube is filled with water and A is open while B is closed. If the axis of rotation is at $x = 0.4$ m and the tube is rotating at a constant rate of $\omega = 10$ rad/s, determine the pressure at points B and C.



SOLUTION

Since point A has zero gauge pressure, the imaginary free surface must pass through this point as shown in Fig. a.

$$H = \frac{\omega^2 R^2}{2g} = \frac{(10\ \text{rad/s})^2(0.4\ \text{m})^2}{2(9.18\ \text{m/s}^2)} = 0.8155\ \text{m}$$

and

$$h'_A = \frac{\omega^2 {r_A}^2}{2g} = \frac{(10\ \text{rad/s})^2(0.2\ \text{m})^2}{2(9.18\ \text{m/s}^2)} = 0.2039\ \text{m}$$

From Fig. a,

$$a = 1\ \text{m} - h'_A = 0.7961\ \text{m}$$

Then,

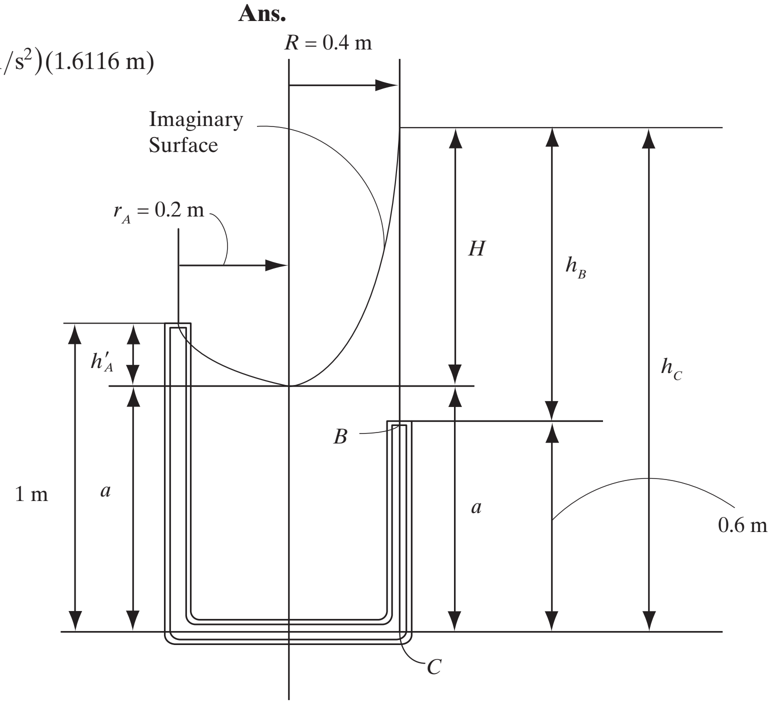
$$h_C = H + a = 0.8155\ \text{m} + 0.7961\ \text{m} = 1.6116\ \text{m}$$

$$h_B = h_C - 0.6\ \text{m} = 1.6116\ \text{m} - 0.6\ \text{m} = 1.0116\ \text{m}$$

Finally,

$$p_B = \rho_w g h_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.0116\ \text{m})$$
$$= 9.924(10^3)\ \text{Pa}$$
$$= 9.92\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_C = \rho_w g h_C = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.6116\ \text{m})$$
$$= 15.81(10^3)\ \text{Pa}$$
$$= 15.8\ \text{kPa} \qquad \textbf{Ans.}$$

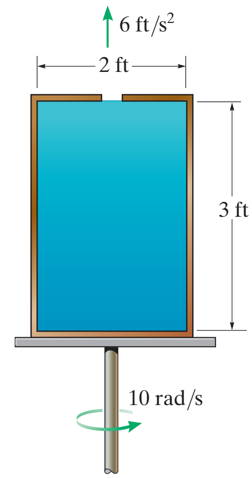


(a)

Ans:
$p_B = 9.92$ kPa
$p_C = 15.8$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–176.** The cylindrical container has a height of 3 ft and a diameter of 2 ft. If it is filled with water through the hole in its center, determine the maximum pressure the water exerts on the container when it undergoes the motion shown.



SOLUTION

The container undergoes an upward acceleration of $a_C = 6\ \text{ft/s}^2$, the maximum pressure occurs at the bottom of the container.

$$(Pa_c)_{max} = \gamma h\left(1 + \frac{a_C}{g}\right) = \left(62.4\ \text{lb/ft}^3\right)(3\ \text{ft}) + \left(1 + \frac{6\ \text{ft/s}^2}{32.2\ \text{ft/s}^2}\right) = 222.08\ \text{lb/ft}^2$$

Since the container is fully filled and the pressure at the center O, of the lid is atmospheric pressure, the imaginary parabolic surface above the lid will be formed as if there were no lid. Fig. *a*

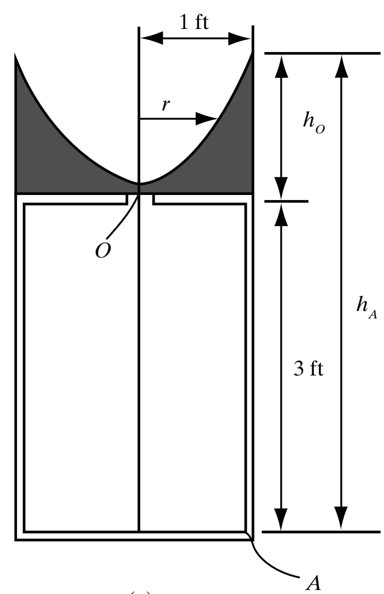
$$h = \frac{\omega^2}{2g}r^2; \qquad h_o = \left[\frac{(10\ \text{rad/}s)^2}{2\left(32.2\ \text{ft/s}^2\right)}\right](1\ \text{ft})^2 = 1.5528\ \text{ft}$$

Here, the maximum pressure due to the rotation occurs at point A where $h_A = 3\ \text{ft} + 1.5528\ \text{ft} = 4.5528\ \text{ft}$. Thus

$$(P_w)_{max} = \gamma h_A = \left(62.4\ \text{lb/ft}^3\right)(4.5528\ \text{ft}) = 284.09\ \text{lb/ft}^2$$

The maximum pressure is

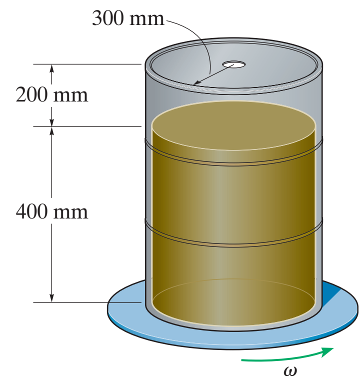
$$P_{max} = (Pa_c)_{max} + (P_w)_{max}$$
$$= 222.08\ \text{lb/ft}^2 + 284.09\ \text{lb/ft}^2$$
$$= \left(506.18\ \text{lb/ft}^2\right)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) = 3.515\ \text{psi} = 3.52\ \text{psi} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–177. The drum has a hole in the center of its lid and contains kerosene to a level of 400 mm when $\omega = 0$. If the drum is placed on the platform and it attains an angular velocity of 12 rad/s, determine the resultant force the kerosene exerts on the lid.



SOLUTION

The volume of the air contained in the paraboloid must be the same as the volume of air in the drum when it is not rotating. Since the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height, then

$$\forall_{Air} = \forall_{pb}$$

$$\pi(0.3 \text{ m})^2(0.2) = \frac{1}{2}(\pi r_i^2 h)$$

$$r_i^2 h = 0.036 \qquad \textbf{(1)}$$

Then

$$h = \frac{w^2}{2g} r^2; \qquad h = \left[\frac{12^2}{2(9.81)}\right] r_i^2 = 7.3394 r_i^2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$h = 0.5140 \text{ m} \qquad r_i = 0.2646 \text{ m}$$

From Section 2–14, beneath the lid,

$$p = \left(\frac{\gamma\omega^2}{2g}\right) r^2 + C$$

Since $\gamma = \rho g$, this equation becomes

$$p = \left(\frac{\rho\omega^2}{2}\right) r^2 + C$$

At $r = r_i$, $p = 0$. Then

$$0 = \frac{\rho\omega^2}{2} r_i^2 + C$$

$$C = -\frac{\rho\omega^2}{2} r_i^2$$

Thus,

$$p = \frac{\rho\omega^2}{2}(r^2 - r_i^2)$$

2–177. Continued

Then the differential force dF acting on the differential annular element of area $dA = 2\pi r dr$ shown shaded in Fig. *a* is

$$dF = p dA = \frac{\rho\omega^2}{2}\left(r^2 - r_i^2\right)(2\pi r dr)$$

$$= \pi\rho\omega^2\left(r^3 - r_i^2 r\right)dr$$

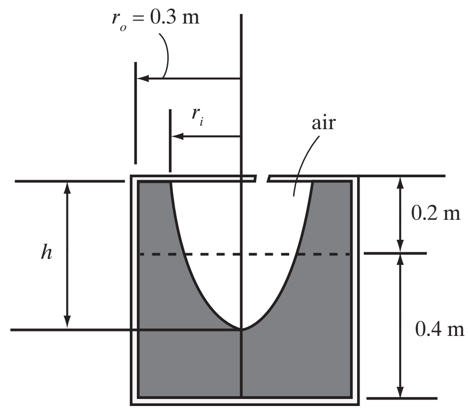
$$F = \int dF = \pi\rho\omega^2\int_{r_i}^{r_o}\left(r^3 - r_i^2 r\right)dr$$

$$= \pi\rho\omega^2\left(\frac{r^4}{4} - \frac{r_i^2}{2}r^2\right)\Bigg|_{r_i}^{r_o}$$

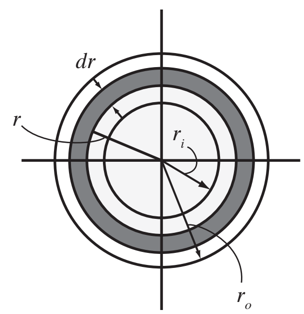
$$= \pi\rho\omega^2\left(\frac{r_o^4}{4} - \frac{r_i^2 r_o^2}{2} + \frac{r_i^4}{4}\right)$$

$$= \frac{\pi}{4}\rho\omega^2\left(r_o^4 - 2r_i^2 r_o^2 + r_i^4\right)$$

$$= \frac{\pi}{4}\rho\omega^2\left(r_o^2 - r_i^2\right)^2$$



(a)



(b)

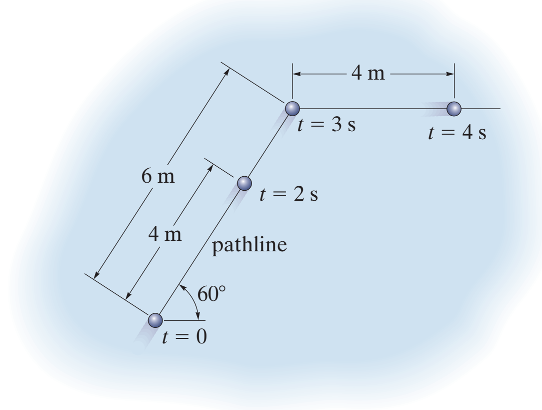
Here $\rho = \rho_{ke} = 814\ \text{kg/m}^3$, $\omega = 12\ \text{rad/s}$, $r_o = 0.3$ m and $r_i = 0.2646$ m

$$F = \frac{\pi}{4}\left(814\ \text{kg/m}^3\right)(12\ \text{rad/s})^2\left[(0.3\ \text{m})^2 - (0.2646\ \text{m})^2\right]^2$$

$$= 36.69\ \text{N} = 36.7\ \text{N}$$ **Ans.**

Ans:
36.7 N

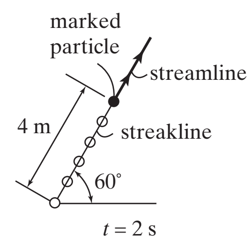
3–1. A marked particle is released into a flow when $t = 0$, and the pathline for a particle is shown. Draw the streakline, and the streamline for the particle when $t = 2$ s and $t = 4$ s.



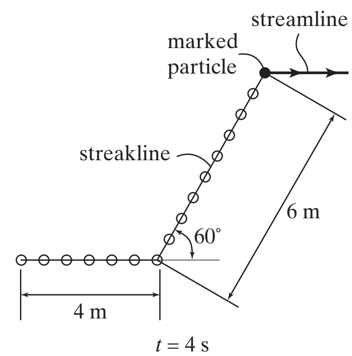
SOLUTION

Since the streamlines have a constant direction for the time interval $0 \leq t < 3$ s, the pathline and streakline coincide with the streamline when $t = 2$ s as shown in Fig. *a*.

The pathline and streakline will coincide with the streamline until $t = 3$ s, after which the streamline makes a sudden change in direction. Thus, the streamline of the marked particle and the streakline when $t = 4$ s will be as shown in Fig. *b*.



(a)



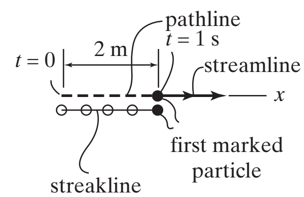
(b)

3–2. The flow of a liquid is originally along the positive x axis at 2 m/s for 3 s. If it then suddenly changes to 4 m/s along the positive y axis for $t > 3$ s, draw the pathline and streamline for the first marked particle when $t = 1$ s and $t = 4$ s. Also, draw the streaklines at these two times.

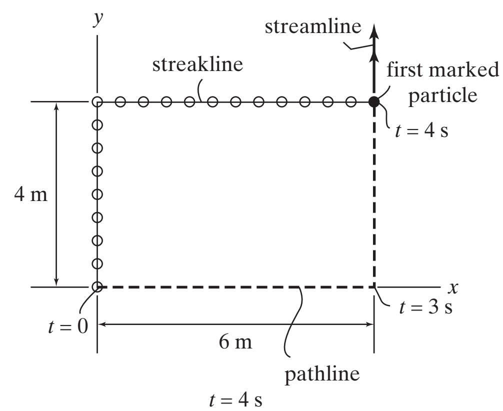
SOLUTION

Since the streamlines have a constant direction along the positive x axis for the time interval $0 \le t < 3$ s, the pathline and streakline coincide with the streamline when $t = 1$ s as shown in Fig. a.

The pathline and streakline will coincide with the streamline until $t = 3$ s, after which the streamline makes a sudden change in direction. Thus, the streamline and pathline of the first marked particle and the streakline when $t = 4$ s will be as shown in Fig. b.



(a)



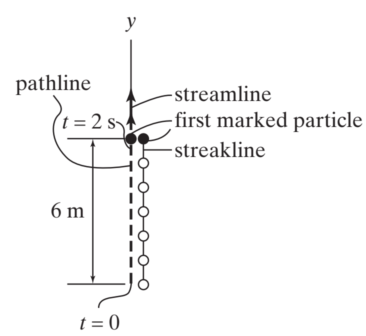
(b)

3–3. The flow of a liquid is originally along the positive y axis at 3 m/s for 4 s. If it then suddenly changes to 2 m/s along the positive x axis for $t > 4$ s, draw the pathline and streamline for the first marked particle when $t = 2$ s and $t = 6$ s. Also, draw the streakline at these two times.

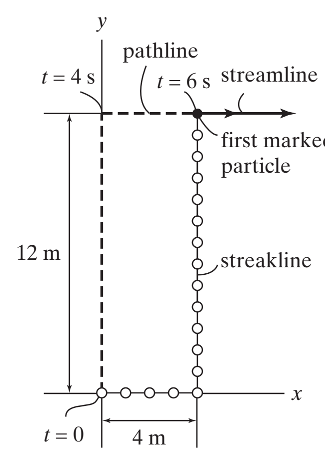
SOLUTION

Since the streamlines have a constant direction along the positive y axis, $0 \leq t < 4$ s, the pathline and streakline coincide with the streamline when $t = 2$ s as shown in Fig. *a*.

The pathline and streakline will coincide with the streamline until $t = 4$ s, when the streamline makes a sudden change in direction. The pathline, streamline, and streakline are shown in Fig. *b*.

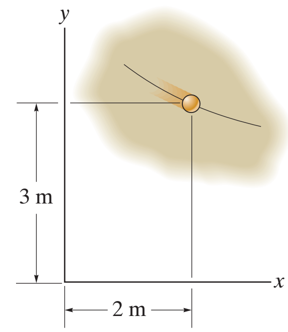


(a)



(b)

***3–4.** A two-dimensional flow field for a fluid be described by $\mathbf{V} = [(2x + 1)\mathbf{i} - (y + 3x)\mathbf{j}]$ m/s, where x and y are in meters. Determine the magnitude of the velocity of a particle located at (2 m, 3 m), and its direction measured counterclockwise from the x axis.



SOLUTION

The velocity vector for a particle at $x = 2$ m and $y = 3$ m is

$$\begin{aligned}\mathbf{V} &= \{(2x + 1)\mathbf{i} - (y + 3x)\mathbf{j}\} \text{ m/s} \\ &= [2(2) + 1]\mathbf{i} - [3 + 3(2)]\mathbf{j} \\ &= \{5\mathbf{i} - 9\mathbf{j}\} \text{ m/s}\end{aligned}$$

The magnitude of $\mathbf{V}$ is

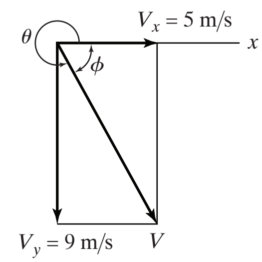
$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(5 \text{ m/s})^2 + (-9 \text{ m/s})^2} = 10.3 \text{ m/s} \qquad \textbf{Ans.}$$

As indicated in Fig. *a*, the direction of $\mathbf{V}$ is defined by $\theta = 360° - \phi$, where

$$\phi = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{9 \text{ m/s}}{5 \text{ m/s}}\right) = 60.95°$$

Thus,

$$\theta = 360° - 60.95° = 299° \qquad \textbf{Ans.}$$



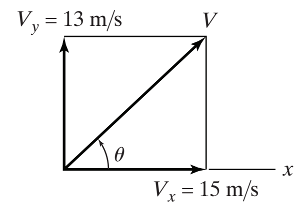
(a)

3–5. A two-dimensional flow field for a liquid can be described by $\mathbf{V} = \left[(5y^2 - x)\mathbf{i} + (3x + y)\mathbf{j} \right]$ m/s, where x and y are in meters. Determine the magnitude of the velocity of a particle located at $(5\text{ m}, -2\text{ m})$, and its direction measured counterclockwise from the x axis.

SOLUTION

The velocity vector of a particle at $x = 5$ m and $y = -2$ m is

$$\begin{aligned}\mathbf{V} &= \left\{ (5y^2 - x)\mathbf{i} + (3x + y)\mathbf{j} \right\} \text{ m/s} \\ &= \left[5(-2)^2 - 5 \right]\mathbf{i} + \left[3(5) + (-2) \right]\mathbf{j} \\ &= \{ 15\mathbf{i} + 13\mathbf{j} \} \text{ m/s}\end{aligned}$$



(a)

The magnitude of **V** is

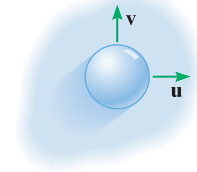
$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(15\text{ m/s})^2 + (13\text{ m/s})^2} = 19.8\text{ m/s} \qquad \textbf{Ans.}$$

As indicated in Fig. *a*, the direction of **V** is defined by

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{13\text{ m/s}}{15\text{ m/s}}\right) = 40.9^\circ \qquad \textbf{Ans.}$$

Ans:
$V = 19.8$ m/s
$\theta = 40.9^\circ$

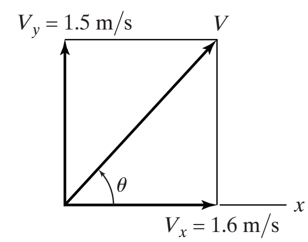
3–6. The soap bubble is released in the air and rises with a velocity of $\mathbf{V} = \left[(0.8x)\mathbf{i} + \left(0.06t^2\right)\mathbf{j}\right]$ m/s where x is meters and t is in seconds. Determine the magnitude of the bubble's velocity, and its directions measured counterclockwise from the x axis, when $t = 5$ s, at which time $x = 2$ m and $y = 3$ m. Draw its streamline at this instant.



SOLUTION

The velocity vector of a particle at $x = 2$ m and the corresponding time $t = 5$ s is

$$\mathbf{V} = \left\{(0.8x)\mathbf{i} + \left(0.06t^2\right)\mathbf{j}\right\} \text{ m/s}$$
$$= \left[0.8(2)\mathbf{i} + 0.06(5)^2\mathbf{j}\right]$$
$$= \{1.6\mathbf{i} + 1.5\mathbf{j}\} \text{ m/s}$$



(a)

The magnitude of **V** is

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(1.6 \text{ m/s})^2 + (1.5 \text{ m/s})^2} = 2.19 \text{ m/s} \qquad \textbf{Ans.}$$

As indicated in Fig. *a*, the direction of **V** is defined by

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{1.5 \text{ m/s}}{1.6 \text{ m/s}}\right) = 43.2°$$

Using the definition of the slope of the streamline and initial condition at $x = 2$ m, $y = 3$ m.

$$\frac{dy}{dx} = \frac{v}{u}; \quad \frac{dy}{dx} = \frac{0.06t^2}{0.8x}$$

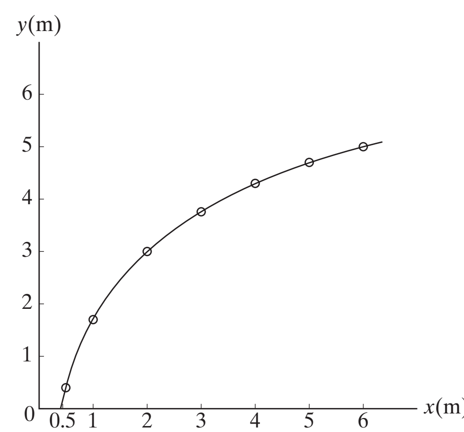
Note that since we are finding the streamline, which represents a single instant in time, $t = 5$ s, t is a constant.

$$\int_{3\text{ m}}^{y} \frac{dy}{t^2} = \int_{2\text{ m}}^{x} \frac{0.075dx}{x}$$
$$\frac{1}{t^2}(y - 3) = 0.075 \ln\frac{x}{2}$$
$$y = \left(0.075t^2 \ln\frac{x}{2} + 3\right) \text{ m}$$

When $t = 5$ s,

$$y = 0.075\left(5^2\right) \ln\left(\frac{x}{2}\right) + 3$$
$$y = \left[1.875 \ln\left(\frac{x}{2}\right) + 3\right] \text{ m}$$

x(m)	0.5	1	2	3	4	5	6
y(m)	0.401	1.700	3	3.760	4.300	4.718	5.060



(a)

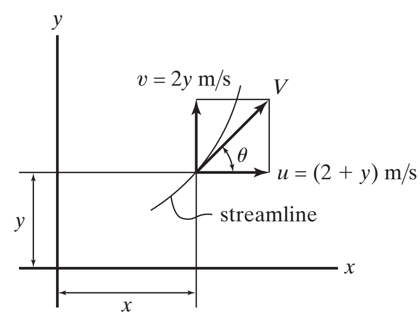
The plot of the streamline is shown in Fig. *a*

Ans:
$V = 2.19$ m/s
$\theta = 43.2°$

3–7. A flow field for a fluid described by $u = (2 + y)$ m/s and $v = (2y)$ m/s, where y is in meters. Determine the equation of the streamline that passes through point (3 m, 2 m), and find the velocity of a particle located at this point. Draw this streamline.

SOLUTION

As indicated in Fig *a*, the velocity **V** of a particle on the streamline is always directed along the tangent of the streamline. Therefore,



(a)

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$$\frac{dy}{dx} = \frac{2y}{2 + y}$$

$$\int \frac{2 + y}{2y}\,dy = \int dx$$

$$\ln y + \frac{1}{2}y = x + C$$

At point (3 m, 2 m), we obtain

$$\ln(2) + \frac{1}{2}(2) = 3 + C$$

$$C = -1.31$$

Thus,

$$\ln y + \frac{1}{2}y = x - 1.31$$

$$\ln y^2 + y = 2x - 2.61 \qquad \textbf{Ans.}$$

At point (3 m, 2 m)

$$u = (2 + 2)\text{ m/s} = 4\text{ m/s} \rightarrow$$

$$v = 2(2) = 4\text{ m/s}\uparrow$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(4\text{ m/s})^2 + (4\text{ m/s})^2} = 5.66\text{ m/s} \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{4\text{ m/s}}{4\text{ m/s}}\right) = 45^\circ \qquad \textbf{Ans.}$$

Ans:
$\ln y^2 + y = 2x - 2.61$
$V = 5.66$ m/s
$\theta = 45^\circ$

***3–8.** A two-dimensional flow field is described by $u = (x^2 + 5)$ m/s and $v = (-6xy)$ m/s. Determine the equation of the streamline that passes through point (5 m, 1 m), and find the velocity of a particle located at this point. Draw this streamline.

SOLUTION

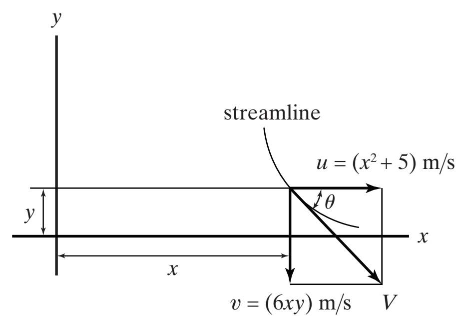
As indicated in Fig a, the velocity **V** of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-6xy}{x^2 + 5}$$

$$\int \frac{dy}{y} = -6\int \frac{x}{x^2 + 5}dx$$

$$\ln y = -3\ln(x^2 + 5) + C$$



(a)

At $x = 5$ m, $y = 1$ m. Then,

$$\ln 1 = -3\ln\left[(5)^2 + 5\right] + C$$

$$C = 3\ln 30$$

Thus

$$\ln y = -3\ln(x^2 + 5) + 3\ln 30$$

$$\ln y + \ln(x^2 + 5)^3 = 3\ln 30$$

$$\ln\left[y(x^2 + 5)^3\right] = \ln 30^3$$

$$y(x^2 + 5)^3 = 30^3$$

$$y = \frac{27(10^3)}{(x^2 + 5)^3} \qquad \textbf{Ans.}$$

At point (5 m, 1m),

$$u = (5^2 + 5)\text{ m/s} = 30\text{ m/s}\rightarrow$$

$$v = -6(5)(1) = -30\text{ m/s} = 30\text{ m/s}\downarrow$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(30\text{ m/s})^2 + (30\text{ m/s})^2} = 42.4\text{ m/s} \qquad \textbf{Ans.}$$

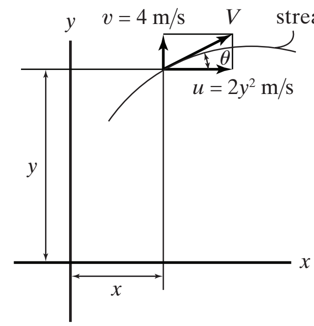
And its direction is

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{30\text{ m/s}}{30\text{ m/s}}\right) = 45^\circ \qquad \textbf{Ans.}$$

3–9. Particles travel within a flow field defined by $\mathbf{V} = \left[2y^2\mathbf{i} + 4\mathbf{j}\right]$ m/s, where x and y are in meters. Determine the equation of the streamline passing through point (1 m, 2 m), and find the velocity of a particle located at this point. Draw this streamline.

SOLUTION

As indicated in Fig. *a*, the velocity **V** of a particle on the streamline is always directed along the tangent of the streamline. Therefore,



(a)

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{4}{2y^2}$$

$$\int y^2\,dy = \int 2dx$$

$$\frac{1}{3}y^3 = 2x + C$$

At $x = 1$ m, $y = 2$ m. Then

$$\frac{1}{3}(2)^3 = 2(1) + C$$

$$C = \frac{2}{3}$$

Thus,

$$\frac{1}{3}y^3 = 2x + \frac{2}{3}$$

$$y^3 = 6x + 2 \qquad \textbf{Ans.}$$

At point (1 m, 2 m)

$$u = 2\left(2^2\right) = 8\text{ m/s}\rightarrow$$

$$v = 4\text{ m/s}\uparrow$$

The magnitude of the velocity is

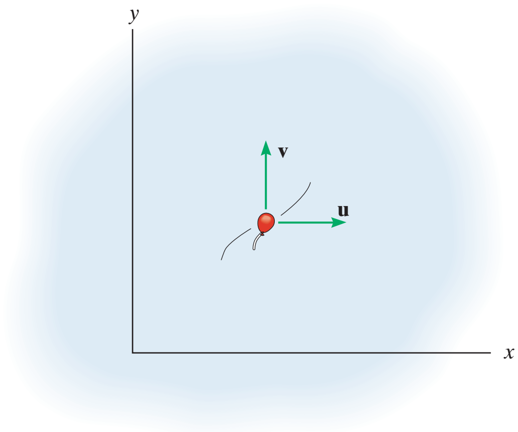
$$V = \sqrt{u^2 + v^2} = \sqrt{(8\text{ m/s})^2 + (4\text{ m/s})^2} = 8.94\text{ m/s} \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{4}{8}\right) = 26.6° \qquad \textbf{Ans.}$$

Ans:
$y^3 = 6x + 2$
$V = 8.94$ m/s
$\theta = 26.6°$

3–10. A ballon is released into the air from the origin and carried along by the wind, which blows at a constant rate of $u = 0.5$ m/s. Also, buoyancy and thermal winds cause the balloon to rise at a rate of $v = (0.8 + 0.6y)$ m/s. Determine the equation of the streamline for the balloon, and draw this streamline.



SOLUTION

As indicated in Fig. *a*, the velocity **V** of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{0.8 + 0.6y}{0.5} = 1.6 + 1.2y$$

Since the balloon starts at $y = 0, x = 0$, using these values,

$$\int_0^y \frac{dy}{1.6 + 1.2y} = \int_0^x dx$$

$$\frac{1}{1.2}\ln(1.6 + 1.2y)\Big|_0^y = x$$

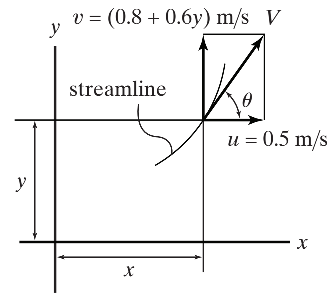
$$\ln\left(\frac{1.6 + 1.2y}{1.6}\right) = 1.2x$$

$$\ln\left(1 + \frac{3}{4}y\right) = 1.2x$$

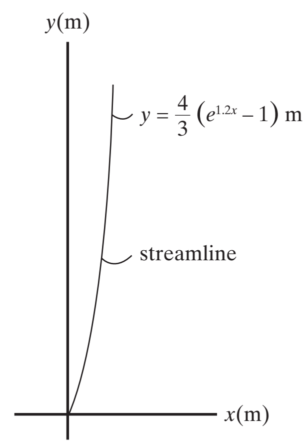
$$1 + \frac{3}{4}y = e^{1.2x}$$

$$y = \frac{4}{3}\left(e^{1.2x} - 1\right)\text{ m} \qquad \textbf{Ans.}$$

Using this result, the streamline is shown in Fig. *b*.



(a)

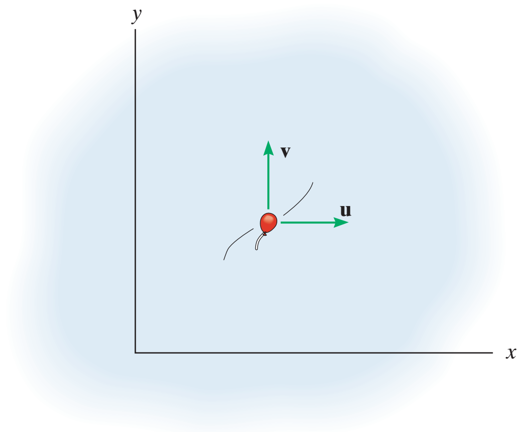


(b)

Ans:

$y = \frac{4}{3}\left(e^{1.2x} - 1\right)$

3–11. A ballon is released into the air from point (1 m, 0) and carried along by the wind, which blows at a rate of $u = (0.8x)$ m/s. Also, buoyancy and thermal winds cause the balloon to rise at a rate of $v = (1.6 + 0.4y)$ m/s. Determine the equation of the streamline for the balloon, and draw this streamline.



SOLUTION

As indicated in Fig. *a*, the velocity **V** of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{1.6 + 0.4y}{0.8x}$$

The balloon starts at point (1 m, 0).

$$\int_0^y \frac{dy}{1.6 + 0.4y} = \int_1^x \frac{dx}{0.8x}$$

$$\frac{1}{0.4}\ln(1.6 + 0.4y)\Big|_0^y = \frac{1}{0.8}\ln x\Big|_1^x$$

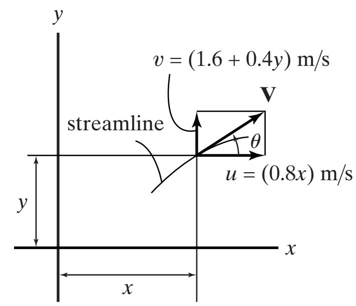
$$\frac{1}{0.4}\ln\left(\frac{1.6 + 0.4y}{1.6}\right) = \frac{1}{0.8}\ln x$$

$$\ln\left(1 + \frac{1}{4}y\right)^2 = \ln x$$

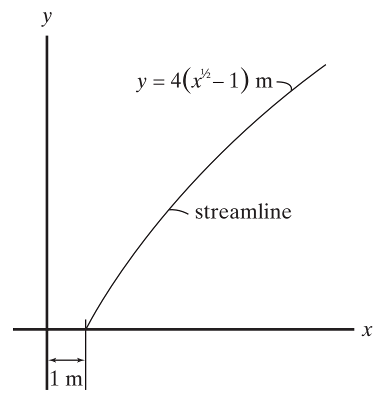
$$\left(1 + \frac{1}{4}y\right)^2 = x$$

$$y = 4\left(x^{1/2} - 1\right)\text{ m} \qquad \textbf{Ans.}$$

Using this result, the streamline is shown in Fig. *b*.



(a)



(b)

Ans:
$y = 4\left(x^{1/2} - 1\right)$

***3–12.** A flow field is defined by $u = (8y)$ m/s, $v = (6x)$ m/s where x and y are in meters. Determine the equation of the streamline that passes through point (1 m, 2 m). Draw this streamline.

SOLUTION

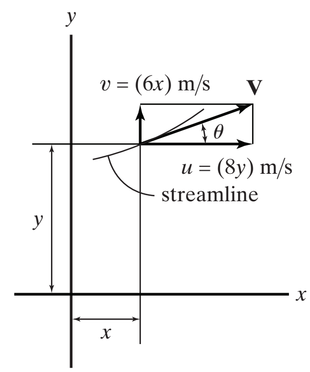
As indicated in Fig. *a*, the velocity **V** of a particle on the streamline is always directed along the tangent of the streamline. Therefore,

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{6x}{8y}$$

$$\int 8y\,dy = \int 6x\,dx$$

$$4y^2 = 3x^2 + C$$



(a)

At $x = 1$ m, $y = 2$ m. Then

$$4(2)^2 = 3(1)^2 + C$$

$$C = 13$$

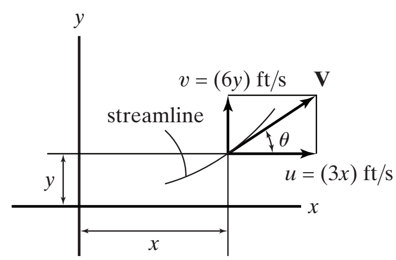
Thus

$$4y^2 = 3x^2 + 13 \qquad \textbf{Ans.}$$

3–13. A flow field is defined by $u = (3x)$ ft/s and $v = (6y)$ ft/s, where x and y are in feet. Determine the equation of the streamline passing through point (3 ft, 1 ft). Draw this streamline.

SOLUTION

As indicated in Fig. a, the velocity $\mathbf{V}$ of a particle on the streamline is always directed along the tangent of the streamline. Therefore,



(a)

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{6y}{3x}$$

$$\int \frac{dy}{2y} = \int \frac{dx}{x}$$

$$\frac{1}{2}\ln y = \ln x + C$$

At $x = 3$ ft, $y = 1$ ft. Then

$$\frac{1}{2}\ln y = \ln x - \ln 3$$

$$\frac{1}{2}\ln y = \ln\frac{x}{3}$$

$$\ln y = \ln\left(\frac{x}{3}\right)^2$$

$$y = \frac{x^2}{9} \qquad \textbf{Ans.}$$

Ans:
$y = x^2/9$

3–14. A flow of water is defined by $u = 5\text{ m/s}$ and $v = 8\text{ m/s}$. If metal flakes are released into the flow at the origin $(0, 0)$, draw the streamlines and pathlines for these particles.

SOLUTION

Since the velocity $\mathbf{V}$ is constant, Fig. *a*, the streamline will be a straight line with a slope.

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{8}{5}$$

$$y = 1.6x + C$$

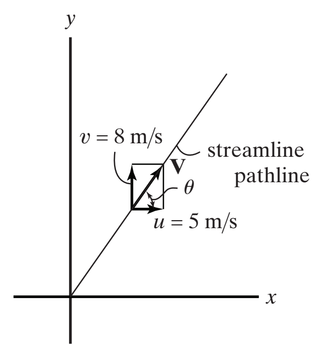
At $x = 0$, $y = 0$. Then

$$C = 0$$

Thus

$$y = 1.6x \qquad \textbf{Ans.}$$

Since the direction of velocity $\mathbf{V}$ remains constant so does the streamline, and the flow is steady. Therefore, the pathline coincides with the streamline and shares the same equation.



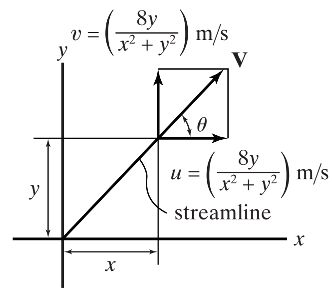
(a)

Ans:
$y = 1.6x$

3–15. A flow field is defined by $u = \left[8x/\left(x^2 + y^2\right)\right]$ m/s and $v = \left[8y/\left(x^2 + y^2\right)\right]$ m/s, where x and y are in meters. Determine the equation of the streamline passing through point (1 m, 1 m). Draw this streamline.

SOLUTION

As indicated in Fig. *a*, the velocity **V** of a particle on the streamline is always directed along the tangent of the streamline. Therefore,



(a)

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{8y/\left(x^2 + y^2\right)}{8x/\left(x^2 + y^2\right)} = \frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln\frac{y}{x} = C$$

$$\frac{y}{x} = C'$$

At $x = 1$ m, $y = 1$ m. Then

$$C' = 1$$

Thus,

$$\frac{y}{x} = 1$$

$$y = x \qquad \textbf{Ans.}$$

Ans:
$y = x$

***3–16.** A fluid has velocity components of $u = [30/(2x + 1)]$ m/s and $v = 2ty$ m/s where x and y are in meters and t is in seconds. Determine the pathline that passes through the point (2 m, 6 m) at time $t = 2$ s. Plot this pathline for $0 \le x \le 4$ m.

SOLUTION

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform flow. Because we are finding a pathline, t is not a constant but a variable. We must first find equations relating x to t and y to t, and then eliminate t. Using the definition of velocity

$$\frac{dx}{dt} = u = \frac{30}{2x+1}; \qquad \int_{2\text{ m}}^{x} (2x+1)dx = 30\int_{2\text{ s}}^{t} dt$$

$$\left(x^2 + x\right)\Big|_{2\text{ m}}^{x} = 30\,t\Big|_{2\text{ s}}^{t}$$

$$x^2 + x - 6 = 30(t - 2)$$

$$t = \frac{1}{30}\left(x^2 + x + 54\right) \qquad \textbf{(1)}$$

$$\frac{dy}{dt} = v = 2ty; \qquad \int_{6\text{ m}}^{y} \frac{dy}{y} = 2\int_{2\text{ s}}^{t} t\,dt$$

$$\ln y\Big|_{6\text{ m}}^{y} = t^2\Big|_{2\text{ s}}^{t}$$

$$\ln\frac{y}{6} = t^2 - 4$$

$$\frac{y}{6} = e^{t^2-4}$$

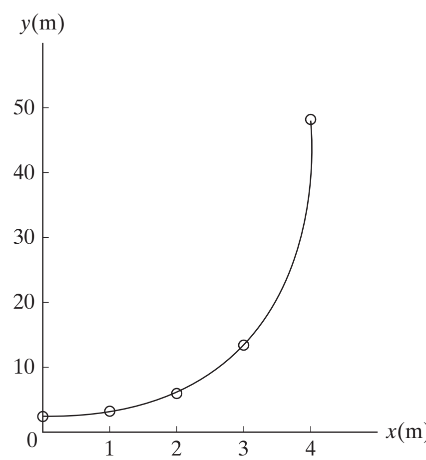
$$y = 6e^{t^2-4} \qquad \textbf{(2)}$$

Substitute Eq. (1) into Eq. (2),

$$y = 6e^{\frac{1}{900}(x^2+x+54)^2-4} \qquad \textbf{Ans.}$$

The plot of the pathline is shown in Fig. *a*.

x(m)	0	1	2	3	4
y(m)	2.81	3.58	6.00	13.90	48.24



(a)

3–17. A fluid has velocity components of $u = [30/(2x + 1)]$ m/s and $v = (2ty)$ m/s where x and y are in meters and t is in seconds. Determine the streamlines that passes through point (1 m, 4 m) at times $t = 1$s, $t = 2$s, and $t = 3$s. Plot each of these streamlines for $0 \le x \le 4$ m.

SOLUTION

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform. The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{2ty}{30/(2x+1)} = \frac{1}{15}ty(2x+1)$$

$$\int_{4\text{ m}}^{y} \frac{dy}{y} = \frac{1}{15}t\int_{1\text{ m}}^{x}(2x+1)dx$$

$$\ln y\Big|_{4\text{ m}}^{y} = \frac{1}{15}t(x^2 + x)\Big|_{1\text{ m}}^{x}$$

$$\ln\frac{y}{4} = \frac{1}{15}t(x^2 + x - 2)$$

$$y = 4e^{t(x^2+x-2)/15}$$

For $t = 1$ s,

$$y = 4e^{(x^2+x-2)/15} \qquad \textbf{Ans.}$$

For $t = 2$ s,

$$y = 4e^{2(x^2+x-2)/15} \qquad \textbf{Ans.}$$

For $t = 3$ s,

$$y = 4e^{(x^2+x-2)/5} \qquad \textbf{Ans.}$$

The plot of these streamlines are shown in Fig. *a*

For $t = 1$ s

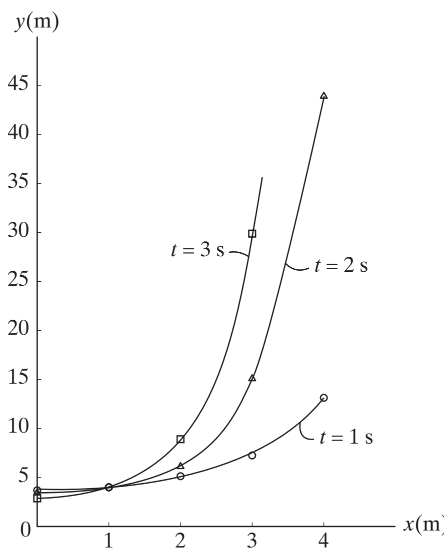
x(m)	0	1	2	3	4
y(m)	3.50	4	5.22	7.79	13.3

For $t = 2$ s

x(m)	0	1	2	3	4
y(m)	3.06	4	6.82	15.2	44.1

For $t = 3$ s

x(m)	0	1	2	3	4
y(m)	2.68	4	8.90	29.6	146



(a)

Ans:
For $t = 1$ s, $y = 4e^{(x^2 + x - 2)/15}$
For $t = 2$ s, $y = 4e^{2(x^2 + x - 2)/15}$
For $t = 3$ s, $y = 4e^{(x^2 + x - 2)/5}$

3–18. A fluid has velocity components of $u = [30/(2x + 1)]$ m/s and $v = (2ty)$ m/s where x and y are in meters and t is in seconds. Determine the streamlines that pass through point (2 m, 6 m) at times $t = 2$ s and $t = 5$ s. Plot these streamlines for $0 \le x \le 4$ m.

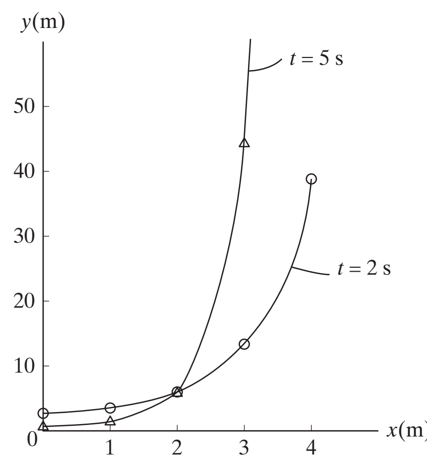
SOLUTION

For $t = 2$ s

x(m)	0	1	2	3	4
y(m)	2.70	3.52	6.00	13.35	38.80

For $t = 5$ s

x(m)	0	1	2	3	4
y(m)	0.812	1.58	6.00	44.33	638.06



(a)

Since the velocity components are a function of time and position the flow can be classified as unsteady nonuniform. The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{2ty}{30/(2x + 1)} = \frac{1}{15}ty(2x + 1)$$

Note that since we are finding the streamline, which represents a single instant in time, either $t = 2$ s or $t = 5$ s, t is a constant.

$$\int_{6\text{ m}}^{y} \frac{dy}{y} = \frac{1}{15}t\int_{2\text{ m}}^{x} (2x + 1)dx$$

$$\ln y\Big|_{6\text{ m}}^{y} = \frac{1}{15}t\left(x^2 + x\right)\Big|_{2\text{ m}}^{x}$$

$$\ln \frac{y}{6} = \frac{1}{15}t\left(x^2 + x - 6\right)$$

$$y = 6e^{\frac{1}{15}t(x^2+x-6)}$$

For $t = 2$ s,

$$y = 6e^{\frac{2}{15}(x^2+x-6)} \qquad \textbf{Ans.}$$

For $t = 5$ s,

$$y = 6e^{\frac{1}{3}(x^2+x-6)} \qquad \textbf{Ans.}$$

The plots of these two streamlines are show in Fig. *a*.

Ans:
For $t = 2$ s, $y = 6e^{2(x^2 + x - 6)/15}$
For $t = 5$ s, $y = 6e^{(x^2 + x - 6)/3}$

3–19. A particle travels along the streamline defined by $y^3 = 8x - 12$. If its speed is 5 m/s when it is at $x = 1$ m, determined the two components of its velocity at this point. Sketch the velocity on the streamline.

SOLUTION

x(m)	0	1	1.5	2	3	4	5
y(m)	−2.29	−1.59	0	1.59	2.29	2.71	3.04

The plot of the streamline is shown in Fig. *a*. Taking the derivative of the streamline equation,

$$3y^2\frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \tan\theta = \frac{8}{3y^2}$$

When $x = 1$ m,

$$y^3 = 8(1) - 12; \qquad y = -1.5874$$

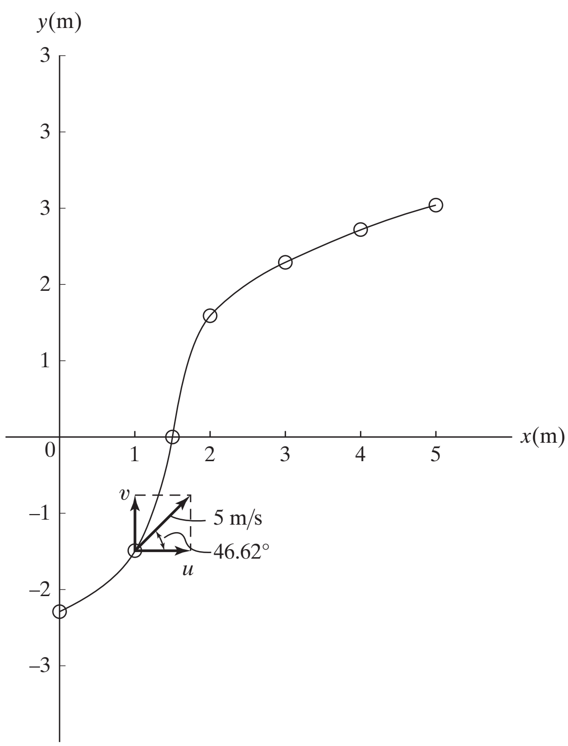
Then

$$\left.\frac{dy}{dx}\right|_{x=1\text{ m}} = \tan\theta\Big|_{x=1\text{ m}} = \frac{8}{3(-1.5874)^2}; \qquad \theta\big|_{x=1\text{ m}} = 46.62°$$

Therefore, the horizontal and vertical components of the velocity are

$$u = (5\text{ m/s})\cos 46.62° = 3.43\text{ m/s} \qquad \textbf{Ans.}$$

$$v = (5\text{ m/s})\sin 46.62° = 3.63\text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$u = 3.43$ m/s
$v = 3.63$ m/s

***3–20.** A flow field is defined by $u = (0.8t)$ m/s and $v = 0.4$ m/s, where t is in seconds. Plot the pathline for a particle that passes through the origin when $t = 0$. Also, draw the streamline for the particle when $t = 4s$.

SOLUTION

Here, $u = \dfrac{dx}{dt}$. Then,

$$dx = udt$$

Using $x = 0$ when $t = 0$ as the integration limit,

$$\int_0^x dx = \int_0^t \left[(0.8t)\ \text{m/s}\right] dt$$

$$x = 0.4t^2 \qquad \textbf{(1)}$$

Also, $v = \dfrac{dy}{dt}$. Then

$$dy = vdt$$

Using $y = 0$ when $t = 0$ as the integration limit,

$$\int_0^y dy = \int_0^t (0.4\ \text{m/s})dt$$

$$y = 0.4t \qquad \textbf{(2)}$$

Eliminating t from Eqs. (1) and (2)

$$y^2 = 0.4x$$

This equation represents the pathline of the particle. The x and y values of the pathline for the first five seconds are tabulated below.

t	x	y
1	0.4	0.4
2	1.6	0.8
3	3.6	1.2
4	6.4	1.6
5	10	2

A plot of the pathline is shown in Fig. a.

From Eqs. (1) and (2), when $t = 4$ s,

$$x = 0.4(4^2) = 6.4\ \text{m} \qquad y = 0.4(4) = 1.6\ \text{m}$$

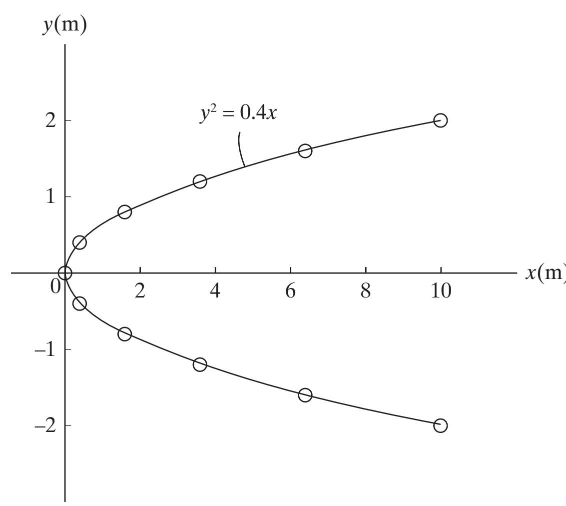
Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{0.4}{0.8t}$$

$$t\int_{1.6\ \text{m}}^{y} dy = \frac{1}{2}\int_{6.4\ \text{m}}^{x} dx$$

$$t(y - 1.6) = \frac{1}{2}(x - 6.4)$$

$$y = \left[\frac{1}{2t}(x - 6.4) + 1.6\right]\ \text{m}$$



(a)

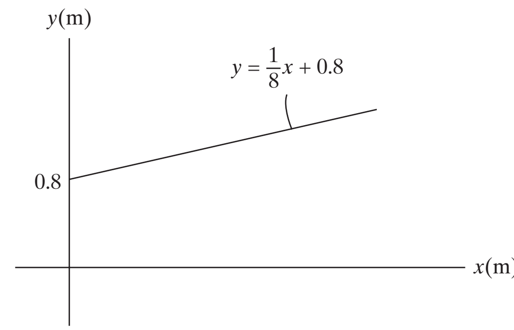
***3–20. (continued)**

When $t = 4$ s,

$$y = \frac{1}{2(4)}(x - 6.4) + 1.6$$

$$y = \frac{1}{8}x + 0.8$$

The plot of the streamline is shown is Fig. *b*.



(b)

3–21. The velocity for an oil flow is defined by $\mathbf{V} = \left(3y^2\,\mathbf{i} + 8\,\mathbf{j}\right)$ m/s, where y is in meters. What is the equation of the streamline that passes through point (2 m, 1 m)? If a particle is at this point when $t = 0$, at what point is it located when $t = 1$ s?

SOLUTION

Since the velocity components are a function of position only, the flow can be classified as steady nonuniform. Here, $u = \left(3y^2\right)$ m/s and $v = 8$ m/s. The slope of the streamline is defined by

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{8}{3y^2}$$

$$\int_{1\text{ m}}^{y} 3y^2 dy = 8\int_{2\text{ m}}^{x} dx$$

$$y^3\Big|_{1\text{ m}}^{y} = 8x\Big|_{2\text{ m}}^{x}$$

$$y^3 - 1 = 8x - 16$$

$$y^3 = 8x - 15 \qquad \textbf{(1)}$$ **Ans.**

From the definition of velocity

$$\frac{dy}{dt} = 8$$

$$\int_{1\text{ m}}^{y} dy = \int_{0}^{1\text{ s}} 8\,dt$$

$$y\Big|_{1\text{ m}}^{y} = 8t\Big|_{0}^{1\text{ s}}$$

$$y - 1 = 8$$

$$y = 9\text{ m}$$ **Ans.**

Substituting this result into Eq. (1)

$$9^3 = 8x - 15$$

$$x = 93\text{ m}$$ **Ans.**

Ans:
$y^3 = 8x - 15$, $y = 9$ m
$x = 93$ m

3–22. The circulation of a fluid is defined by the velocity field $u = (6 - 3x)$ m/s and $v = 2$ m/s where x is in meters. Plot the streamline that passes through the origin for $0 \leq x < 2$ m.

SOLUTION

x(m)	0	0.25	0.5	0.75	1
y(m)	0	0.089	0.192	0.313	0.462
x(m)	1.25	1.5	1.75	2	
y(m)	0.654	0.924	1.386	∞	

Since the velocity component is a function of position only, the flow can be classified as steady nonuniform. Using the definition of the slope of a streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{2}{6 - 3x}$$

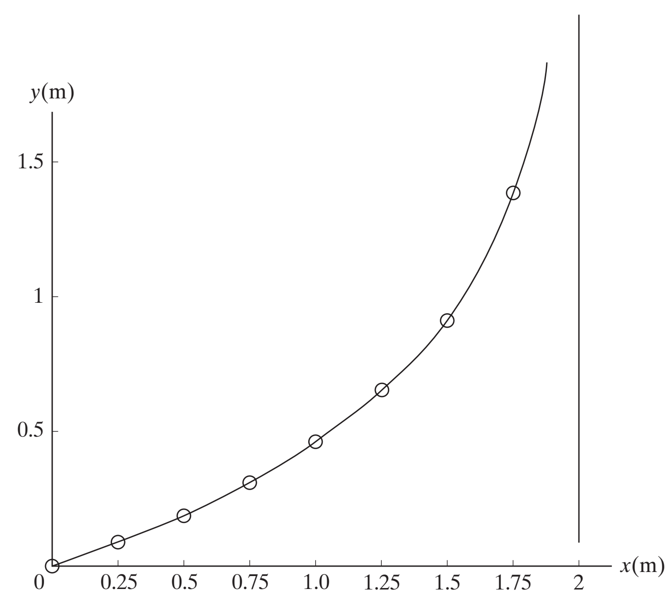
$$\int_0^y dy = 2\int_0^x \frac{dx}{6 - 3x}$$

$$y = -\frac{2}{3}\ln(6 - 3x)\Big|_0^x$$

$$y = -\frac{2}{3}\ln\left(\frac{6 - 3x}{6}\right)$$

$$y = \frac{2}{3}\ln\left(\frac{2}{2 - x}\right) \qquad \textbf{Ans.}$$

The plot of this streamline is show in Fig. *a*



(a)

Ans:

$$y = \frac{2}{3}\ln\left(\frac{2}{2 - x}\right)$$

3–23. A stream of water has velocity components of $u = -2\text{ m/s}$, $v = 3\text{ m/s}$ for $0 \le t < 10\text{ s}$; and $u = 5\text{ m/s}$, $v = -2\text{ m/s}$ for $10\text{ s} < t \le 15\text{ s}$. Plot the pathline and streamline for a particle released at point $(0, 0)$ when $t = 0\text{ s}$.

SOLUTION

Using the definition of velocity, for $0 \le t < 10\text{ s}$

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = -2$$

$$\int_0^x dx = -2\int_0^t dt$$

$$x = (-2t)\text{ m} \qquad \textbf{(1)}$$

When $t = 10\text{ s}$, $x = -2(10) = -20\text{ m}$

$$\frac{dy}{dt} = v; \qquad \frac{dy}{dt} = 3$$

$$\int_0^y dy = 3\int_0^t dt$$

$$y = (3t)\text{ m} \qquad \textbf{(2)}$$

When $t = 10\text{ s}$, $y = 3(10) = 30\text{ m}$

The equation of the streamline can be determined by eliminating t from Eq. (1) and (2).

$$y = -\frac{3}{2}x \qquad \textbf{Ans.}$$

For $10 < t \le 15\text{ s}$.

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = 5$$

$$\int_{-20\text{ m}}^x dx = 5\int_{10\text{ s}}^t dt$$

$$x - (-20) = 5(t - 10)$$

$$x = (5t - 70)\text{ m} \qquad \textbf{(3)}$$

At $t = 15\text{ s}$, $x = 5(15) - 70 = 5\text{ m}$

$$\frac{dy}{dt} = v; \qquad \frac{dy}{dt} = -2$$

$$\int_{30\text{ m}}^y dy = -2\int_{10\text{ s}}^t dt$$

$$y - 30 = -2(t - 10)$$

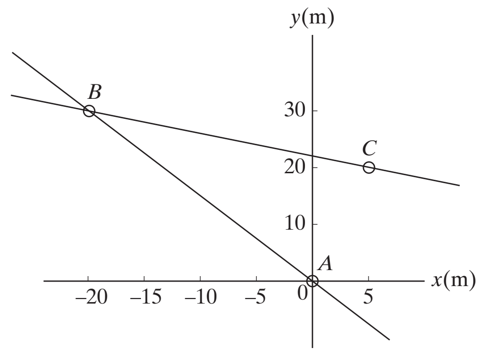
$$y = (-2t + 50)\text{ m} \qquad \textbf{(4)}$$

When $t = 15\text{ s}$, $y = -2(15) + 50 = 20\text{ m}$

Eliminate t from Eqs. (3) and (4),

$$y = \left(-\frac{2}{5}x + 22\right) \qquad \textbf{Ans.}$$

The two streamlines intersect at $(-20, 30)$, point B in Fig. (a). The pathline is the path ABC.



(a)

Ans:

For $0 \le t < 10\text{ s}$, $y = -\frac{3}{2}x$

For $10\text{ s} < t \le 15\text{ s}$, $y = -\frac{2}{5}x + 22$

***3–24.** The velocity field is defined by $u = (4x)$ m/s and $v = (2t)$ m/s, where t is in seconds and x is in meters. Determine the equation of the streamline that passes through point (2 m, 6 m) for $t = 1$ s. Plot this streamline for $0.25 \text{ m} \le x \le 4$ m.

SOLUTION

x(m)	0.25	0.5	0.75	1	2	3	4
y(m)	4.96	5.31	5.51	5.65	6	6.20	6.35

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform. Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{2t}{4x} = \frac{t}{2x}$$

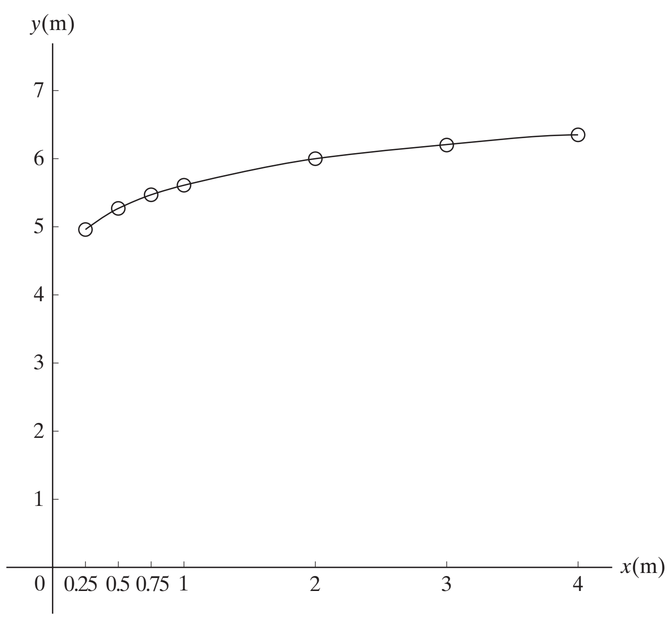
$$\int_{6\text{ m}}^{y} dy = \frac{t}{2}\int_{2\text{ m}}^{x} \frac{dx}{x}$$

$$y - 6 = \frac{t}{2}\ln\frac{x}{2}$$

$$y = \frac{t}{2}\ln\frac{x}{2} + 6$$

For $t = 1$ s, $$y = \left(\frac{1}{2}\ln\frac{x}{2} + 6\right)\text{m}$$ **Ans.**

The plot of this streamline is shown in Fig. *a*.



(a)

3–25. The velocity field is defined by $u = (4x)$ m/s and $v = (2t)$ m/s, where t is in seconds and x is in meters. Determine the pathline that passes through point (2 m, 6 m) when $t = 1$ s. Plot this pathline for $0.25 \text{ m} \le x \le 4 \text{ m}$.

SOLUTION

x(m)	0.25	0.50	0.75	1	2	3	4
y(m)	5.23	5.43	5.57	5.68	6	6.21	6.38

Since the velocity components are a function of time and position, the flow can be classified as unsteady nonuniform. Using the definition of velocity,

$$\frac{dx}{dt} = u = 4x; \qquad \int_{2\text{ m}}^{x} \frac{dx}{4x} = \int_{1\text{ s}}^{t} dt$$

$$\frac{1}{4}\ln x\Big|_{2\text{ m}}^{x} = t\Big|_{1\text{ s}}^{t}$$

$$\frac{1}{4}\ln\frac{x}{2} = t - 1$$

$$t = \frac{1}{4}\ln\frac{x}{2} + 1 \qquad \textbf{(1)}$$

$$\frac{dy}{dt} = v = 2t; \qquad \int_{6\text{ m}}^{y} dy = \int_{1\text{ s}}^{t} 2t\,dt$$

$$y - 6 = t^2\Big|_{1\text{ s}}^{t}$$

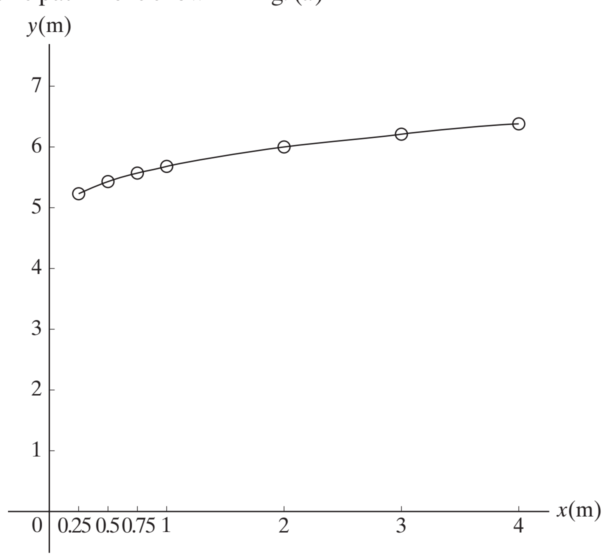
$$y = t^2 + 5 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2),

$$y = \left(\frac{1}{4}\ln\frac{x}{2} + 1\right)^2 + 5$$

$$y = \left(\frac{1}{16}\ln^2\frac{x}{2} + \frac{1}{2}\ln\frac{x}{2} + 6\right) \qquad \textbf{Ans.}$$

The plot of this pathline is shown in Fig. (*a*)



(a)

Ans:
$y = \frac{1}{16}\ln^2\frac{x}{2} + \frac{1}{2}\ln\frac{x}{2} + 6$

3–26. The velocity field of a fluid is defined by $u = \left(\frac{1}{2}x\right)$ m/s, $v = \left(\frac{1}{8}y^2\right)$ m/s for $0 \le t < 5$ s and by $u = \left(-\frac{1}{4}x^2\right)$ m/s, $v = \left(\frac{1}{4}y\right)$ m/s for $5 < t \le 10$ s, where x and y are in meters. Plot the streamline and pathline for a particle released at point (1 m, 1 m) when $t = 0$ s.

SOLUTION

Using the definition of velocity, for $0 \le t < 5$ s,

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = \frac{1}{2}x$$

$$\int_{1\text{ m}}^{x} \frac{dx}{x} = \int_0^t \frac{1}{2}\,dt$$

$$\ln x = \frac{1}{2}t$$

$$x = \left(e^{\frac{1}{2}t}\right)\text{ m} \qquad \textbf{(1)}$$

When $t = 5$ s, $\qquad x = e^{\frac{1}{2}(5)} = 12.18$ m

$$\frac{dy}{dt} = v; \qquad \frac{dy}{dt} = \frac{1}{8}y^2$$

$$\int_{1\text{ m}}^{y} \frac{dy}{y^2} = \int_0^t \frac{1}{8}\,dt$$

$$-\left(\frac{1}{y}\right)\Bigg|_{1\text{ m}}^{y} = \frac{1}{8}t$$

$$1 - \frac{1}{y} = \frac{1}{8}t$$

$$\frac{y-1}{y} = \frac{1}{8}t$$

$$y\left(1 - \frac{1}{8}t\right) = 1$$

$$y = \left(\frac{8}{8-t}\right)\text{ m} \qquad t \neq 8\text{ s} \qquad \textbf{(2)}$$

When $t = 5$ s, $y = \dfrac{8}{8-5} = 2.667$ m

The equation of the streamline and pathline can be determined by eliminating t from Eqs. (1) and (2)

$$y = \left(\frac{8}{8 - 2\ln x}\right)\text{m}$$

x(m)	1	3	5	7	9	11	12.18
y(m)	1	1.38	1.67	1.95	2.22	2.50	2.67

For $5\text{ s} < t \le 10$ s,

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = -\frac{1}{4}x^2$$

$$\int_{12.18\text{ m}}^{x} \frac{dx}{x^2} = -\frac{1}{4}\int_{5\text{ s}}^{t} dt$$

3–26. (continued)

$$-\left(\frac{1}{x} - \frac{1}{12.18}\right) = -\frac{1}{4}(t-5)$$

$$\frac{1}{x} = \frac{t}{4} - 1.1679$$

$$x = \left(\frac{4}{t - 4.6717}\right)\text{ m} \qquad t \neq 4.6717\text{ s} \qquad \textbf{(3)}$$

When $t = 10$ s, $\quad x = \dfrac{4}{10 - 4.6717} = 0.751$ m

$$\frac{dy}{dt} = v; \qquad \frac{dy}{dt} = \frac{1}{4}y$$

$$\int_{2.667\text{ m}}^{y} \frac{dy}{y} = \frac{1}{4}\int_{5\text{ s}}^{t} dt$$

$$\ln\frac{y}{2.667} = \frac{1}{4}(t-5)$$

$$\frac{y}{2.667} = e^{\frac{1}{4}(t-5)}$$

$$y = \left[2.667e^{\frac{1}{4}(t-5)}\right]\text{ m} \qquad \textbf{(4)}$$

When $t = 10$ s, $\quad y = 2.667e^{\frac{1}{4}(10-5)} = 9.31$ m

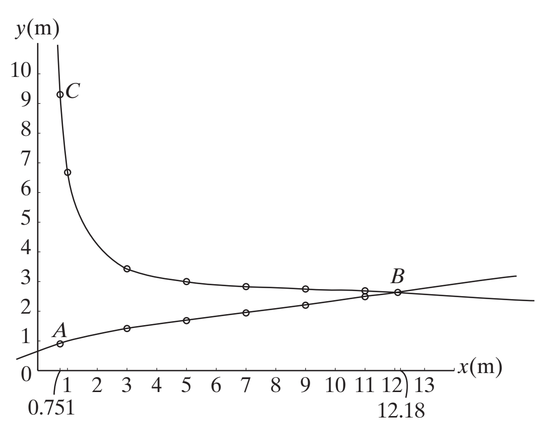
Eliminate t from Eqs. (3) and (4),

$$y = 2.667e^{\frac{1}{4}\left[4\left(\frac{1}{x}+1.1679\right)-5\right]}$$

$$= \left[2.667e^{\left(\frac{1}{x}-0.08208\right)}\right]\text{m}$$

x(m)	0.751	1	3	5	7	9	11	12.18
y(m)	9.31	6.68	3.43	3.00	2.83	2.75	2.69	2.67

The two streamlines intersect at (12.18, 2.67), point B in Fig. (a). The pathline is the path ABC.



Ans:

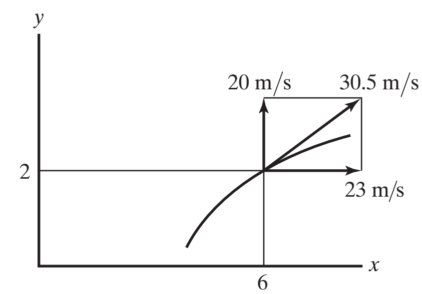
For $0 \leq t < 5$ s, $y = \dfrac{8}{8 - 2\ln x}$

For $5\text{ s} < t \leq 10$ s, $y = 2.67e^{(1/x\, -\, 0.0821)}$

3–27. A two-dimensional flow field for a liquid can be described by $\mathbf{V} = [(6y^2 - 1)\mathbf{i} + (3x + 2)\mathbf{j}]$ m/s, where x and y are in meters. Determine a streamline that passes through points (6 m, 2 m) and determine the velocity at this point. Sketch the velocity on the streamline.

SOLUTION

We have steady flow since the velocity does not depend upon time.



$$u = 6y^2 - 1$$

$$v = 3x + 2$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{3x + 2}{6y^2 - 1}$$

$$\int_2^y (6y^2 - 1)dy = \int_6^x (3x + 2)dx$$

$$2y^3 - y\Big|_2^y = 1.5\,x^2 + 2x\Big|_6^x$$

$$2y^3 - y - [2(2)^3 - 2] = 1.5x^2 + 2x - [1.5(6)^2 + 2(6)]$$

$$2y^3 - 1.5x^2 - y - 2x + 52 = 0 \qquad \textbf{Ans.}$$

At (6 m, 2 m)

$$u = 6(2)^2 - 1 = 23 \text{ m/s} \rightarrow$$

$$v = 3(6) + 2 = 20 \text{ m/s}\uparrow$$

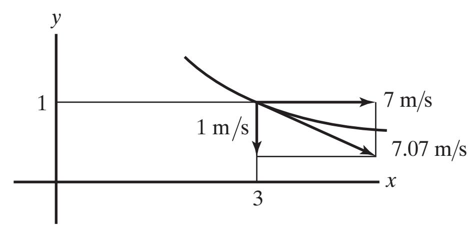
$$V = \sqrt{(23 \text{ m/s})^2 + (20 \text{ m/s})^2} = 30.5 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$2y^3 - 1.5x^2 - y - 2x + 52 = 0$
$V = 30.5$ m/s

***3–28.** A flow field for a liquid can be described by $\mathbf{V} = \{(2x + 1)\mathbf{i} - y\mathbf{j}\}$ m/s, where x and y are in meters. Determine the magnitude of the velocity of a particle located at points $(3\text{ m}, 1\text{ m})$. Sketch the velocity on the streamline.

SOLUTION

We have steady flow since the velocity does not depend upon time.



$$u = 2x + 1$$

$$v = -y$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-y}{2x + 1}$$

$$-\int \frac{dy}{y} = \int \frac{dx}{(2x + 1)}$$

$$-\ln y = \frac{1}{2}\ln(2x + 1) + C$$

$$-y = (2x + 1)^{\frac{1}{2}} + C'$$

$$-1 = (2(3) + 1)^{\frac{1}{2}} + C'$$

$$C' = -3.65$$

$$-y\Big|_1^y = (2x + 1)^{\frac{1}{2}}\Big|_3^x$$

$$-y + 1 = (2x + 1)^{\frac{1}{2}} - \left[2(3) + 1\right]^{\frac{1}{2}}$$

$$y = 3.65 - (2x + 1)^{\frac{1}{2}} \qquad \textbf{Ans.}$$

$$u = 2(3) + 1 = 7\text{ m/s}$$

$$v = -1\text{ m/s}$$

$$V = \sqrt{(7\text{ m/s})^2 + (-1\text{ m/s})^2} = 7.07\text{ m/s} \qquad \textbf{Ans.}$$

3–29. Air flows uniformly through the center of a horizontal duct with a velocity of $V = \left(6t^2 + 5\right)$ m/s, where t is in seconds. Determine the acceleration of the flow when $t = 2$ s.

SOLUTION

Since the flow is along the horizontal ($\bar{x}$ axis) $v = w = 0$. Also, the velocity is a function of time t only. Therefore, the convective acceleration is zero, so that $u\dfrac{\partial V}{\partial x} = 0$.

$$\begin{aligned} a &= \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} \\ &= 12t + 0 \\ &= (12t)\ \text{m/s}^2 \end{aligned}$$

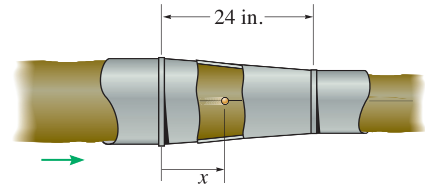
When $t = 2$ s,

$$a = 12(2) = 24\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Note: The flow is unsteady since its velocity is a function of time.

Ans:
$a = 24\ \text{m/s}^2$

3–30. Oil flows through the reducer such that particles along its centerline have a velocity of $V = (4xt)$ in./s, where x is in inches and t is in seconds. Determine the acceleration of the particles at $x = 16$ in. when $t = 2$ s.



SOLUTION

Since the flow is along the x axis, $v = w = 0$

$$\begin{aligned} a &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \\ &= 4x + (4xt)(4t) \\ &= 4x + 16xt^2 \\ &= \left[4x\left(1 + 4t^2\right)\right] \text{ in./s}^2 \end{aligned}$$

When $t = 2$ s, $x = 16$ in. Then

$$a = \left[4(16)\left[1 + 4\left(2^2\right)\right]\right] \text{ in./s}^2 = 1088 \text{ in./s}^2$$

Note: The flow is unsteady since its velocity is a function of time.

Ans:
1088 in./s^2

3–31. A fluid has velocity components of $u = (6y + t)$ ft/s and $v = (2tx)$ ft/s where x and y are in feet and t is in seconds. Determine the magnitude of acceleration of a particle passing through the point (1 ft, 2 ft), when $t = 1$ s.

SOLUTION

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar component of this equation along the x and y axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \\ &= 1 + (6y + t)(0) + (2tx)(6) \\ &= (1 + 12tx)\ \text{ft/s}^2 \\ a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \\ &= 2x + (6y + t)(2t) + (2tx)(0) \\ &= \left(2x + 12ty + 2t^2\right)\ \text{ft/s}^2 \end{aligned}$$

When $t = 1$ s, $x = 1$ ft and $y = 2$ ft, then

$$a_x = [1 + 12(1)(1)] = 13\ \text{ft/s}^2$$

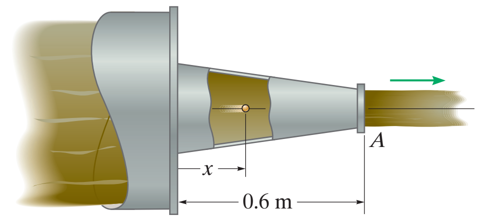
$$a_y = \left[2(1) + 12(1)(2) + 2\left(1^2\right)\right] = 28\ \text{ft/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(13\ \text{ft/s}^2\right)^2 + \left(28\ \text{ft/s}^2\right)^2} = 30.9\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$30.9\ \text{ft/s}^2$

***3–32.** The velocity for the flow of a gas along the center streamline of the pipe is defined by $u = (10x^2 + 200t + 6)$ m/s, where x is in meters and t is in seconds. Determine the acceleration of a particle when $t = 0.01$ s and it is at A, just before leaving the nozzle.



SOLUTION

$$a = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial t} = 200 \qquad \frac{\partial u}{\partial x} = 20\,x$$

$$a = \left[200 + (10x^2 + 200t + 6)(20x)\right] \text{ m/s}^2$$

When $t = 0.01$ s, $x = 0.6$ m.

$$a = \left\{200 + \left[10(0.6^2) + 200(0.01) + 6\right]\left[20(0.6)\right]\right\} \text{ m/s}^2$$

$$= 339 \text{ m/s}^2$$ **Ans.**

3–33. A fluid has velocity components of $u = (2x^2 - 2y^2 + y)$ m/s and $v = (y + xy)$ m/s, where x and y are in meters. Determine the magntiude of the velocity and acceleration of a particle at point (2 m, 4 m).

SOLUTION

Velocity.

At $x = 2$ m, $y = 4$ m,

$$u = 2(2^2) - 2(4^2) + 4 = -20 \text{ m/s}$$

$$v = 4 + 2(4) = 12 \text{ m/s}$$

The magnitude of the particle's velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(-20 \text{ m/s})^2 + (12 \text{ m/s})^2} = 23.3 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration. The x and y components of the particle's acceleration, with $w = 0$ are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (2x^2 - 2y^2 + y)(4x) + (y + xy)(-4y + 1)$$

At $x = 2$ m, $y = 4$ m,

$$a_x = -340 \text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (2x^2 - 2y^2 + y)(y) + (y + xy)(1 + x)$$

At $x = 2$ m, $y = 4$ m,

$$a_y = -44 \text{ m/s}^2$$

The magnitude of the particle's acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-340 \text{ m/s}^2)^2 + (-44 \text{ m/s}^2)^2} = 343 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$V = 23.3$ m/s
$a = 343$ m/s^2

3–34. A fluid velocity components of $u = (5y^2 - x)$ m/s and $v = (4x^2)$ m/s, where x and y are in meters. Determine the velocity and acceleration of particles passing through point (2 m, 1 m).

SOLUTION

Since the velocity components are a function of position only the flow can be classified as steady nonuniform. At point $x = 2$ m and $y = 1$ m,

$$u = 5(1^2) - 2 = 3 \text{ m/s}$$

$$v = 4(2^2) = 16 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(3 \text{ m/s})^2 + (16 \text{ m/s})^2} = 16.3 \text{ m/s} \qquad \textbf{Ans.}$$

Its direction is

$$\theta_v = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{16 \text{ m/s}}{3 \text{ m/s}}\right) = 79.4° \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description is

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axis

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (5y^2 - x)(-1) + 4x^2(10y)$$

$$= (x - 5y^2) + 40x^2y$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (5y^2 - x)(8x) + 4x^2(0)$$

$$= 8x(5y^2 - x)$$

At point $x = 2$ m and $y = 1$ m,

$$a_x = \left[2 - 5(1^2)\right] + 40(2^2)(1) = 157 \text{ m/s}^2$$

$$a_y = 8(2)\left[5(1^2) - 2\right] = 48 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(157 \text{ m/s}^2)^2 + (48 \text{ m/s}^2)^2} = 164 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Its direction is

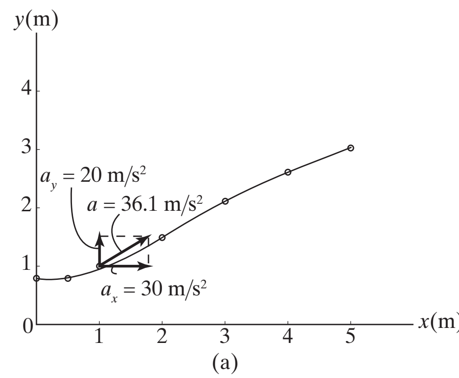
$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{48 \text{ m/s}^2}{157 \text{ m/s}^2}\right) = 17.0° \qquad \textbf{Ans.}$$

Ans:
$V = 16.3$ m/s
$\theta_v = 79.4°$
$a = 164$ m/s^2
$\theta_a = 17.0°$

3–35. A fluid has velocity components of $u = (5y^2)$ m/s and $v = (4x - 1)$ m/s, where x and y are in meters. Determine the equation of the streamline passing through point $(1\text{ m}, 1\text{ m})$. Find the components of the acceleration of a particle located at this point and sketch the acceleration on the streamline.

SOLUTION

Since the velocity components are independent of time but are a function of position, the flow can be classified as steady nonuniform. The slope of the streamline is



(a)

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{4x - 1}{5y^2}$$

$$\int_{1\text{ m}}^{y} 5y^2 dy = \int_{1\text{ m}}^{x} (4x - 1)dx$$

$$y^3 = \frac{1}{5}(6x^2 - 3x + 2) \text{ where } x \text{ is in } m$$

For two dimensional flow, the Eulerian description is

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along x and y axes,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + 5y^2(0) + (4x - 1)(10y)$$

$$= 40xy - 10y$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 5y^2(4) + (4x - 1)(0)$$

$$= 20y^2$$

At point $x = 1$ m and $y = 1$ m,

$$a_x = 40(1)(1) - 10(1) = 30\text{ m/s}^2$$

$$a_y = 20(1^2) = 20\text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(30\text{ m/s}^2)^2 + (20\text{ m/s}^2)^2} = 36.1\text{ m/s}^2$$ **Ans.**

Its direction is

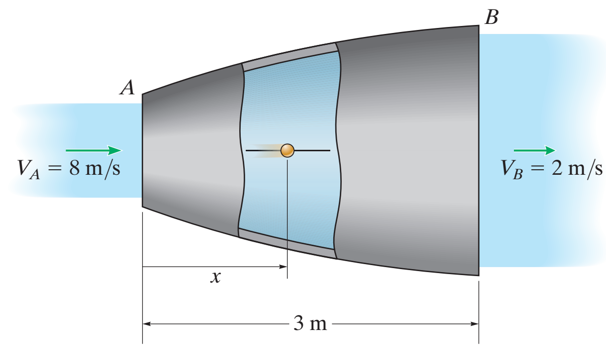
$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{20\text{ m/s}^2}{30\text{ m/s}^2}\right) = 33.7°$$ **Ans.**

The plot of the streamline and the acceleration on point (1 m, 1 m) is shown in Fig. *a*.

x(m)	0	0.5	1	2	3	4	5
y(m)	0.737	0.737	1	1.59	2.11	2.58	3.01

Ans:
$a = 36.1\text{ m/s}^2$
$\theta = 33.7°$

***3–36.** Air flowing through the center of the duct has been found to decrease in speed from $V_A = 8$ m/s to $V_B = 2$ m/s in a linear manner. Determine the velocity and acceleration of a particle moving horizontally through the duct as a function of its position x. Also, find the position of the particle as a function of time if $x = 0$ when $t = 0$.



SOLUTION

Since the velocity is a function of position only, the flow can be classified as steady nonuniform. Since the velocity varies linearly with x,

$$V = V_A + \left(\frac{V_B - V_A}{L_{AB}}\right)x = 8 + \left(\frac{2 - 8}{3}\right)x = (8 - 2x)\text{ m/s} \qquad \textbf{Ans.}$$

For one dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{dt} + V\frac{\partial \mathbf{V}}{dx}$$

$$= 0 + (8 - 2x)(-2)$$

$$= 4(x - 4)\text{ m/s}^2 \qquad \textbf{Ans.}$$

using the definition of velocity,

$$\frac{dx}{dt} = V = 8 - 2x; \qquad \int_0^x \frac{dx}{8 - 2x} = \int_0^t dt$$

$$-\frac{1}{2}\ln(8 - 2x)\Big|_0^x = t$$

$$\frac{1}{2}\ln\left(\frac{8}{8 - 2x}\right) = t$$

$$\ln\left(\frac{8}{8 - 2x}\right) = 2t$$

$$\frac{8}{8 - 2x} = e^{2t}$$

$$x = 4\left(1 - e^{-2t}\right)\text{ m} \qquad \textbf{Ans.}$$

3–37. A fluid has velocity components of $u = (8t^2)$ m/s and $v = (7y + 3x)$ m/s, where x and y are in meters and t is in seconds. Determine the velocity and acceleration of a particle passing through point $x = 1$ m, $y = 1$ m when $t = 2$ s.

SOLUTION

Since the velocity components are functions of time and position the flow can be classified as unsteady nonuniform. When $t = 2$ s, $x = 1$ m and $y = 1$ m.

$$u = 8(2^2) = 32 \text{ m/s}$$

$$v = 7(1) + 3(1) = 10 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(32 \text{ m/s})^2 + (10 \text{ m/s})^2} = 33.5 \text{ m/s} \qquad \textbf{Ans.}$$

Its direction is

$$\theta_v = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{10 \text{ m/s}}{32 \text{ m/s}}\right) = 17.4° \;\measuredangle\theta_v \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \\ &= 16t + 8t^2(0) + (7y + 3x)(0) \\ &= (16t) \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \\ &= 0 + (8t^2)(3) + (7y + 3x)(7) \\ &= \left[24t^2 + 7(7y + 3x)\right] \text{ m/s}^2 \end{aligned}$$

When $t = 2$ s, $x = 1$ m and $y = 1$ m.

$$a_x = 16(2) = 32 \text{ m/s}^2$$

$$a_y = 24(2^2) + 7[7(1) + 3(1)] = 166 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(32 \text{ m/s}^2)^2 + (166 \text{ m/s}^2)^2} = 169 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Its direction is

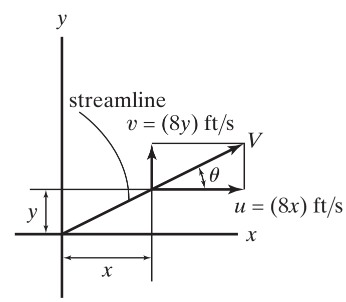
$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{166 \text{ m/s}^2}{32 \text{ m/s}^2}\right) = 79.1° \;\measuredangle\theta_a \qquad \textbf{Ans.}$$

Ans:
$V = 33.5$ m/s
$\theta_V = 17.4°$
$a = 169$ m/s^2
$\theta_a = 79.1°$ $\measuredangle$

3–38. A fluid has velocity components of $u = (8x)$ ft/s and $v = (8y)$ ft/s, where x and y are in feet. Determine the equation of the streamline and the acceleration of particles passing through point (2 ft, 1 ft). Also find the acceleration of a particle located at this point. Is the flow steady or unsteady?

SOLUTION

Since the velocity components are the function of position but not the time, **the flow is steady** (**Ans.**) but nonuniform. Using the definition of the slope of the streamline,



(a)

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{8y}{8x} = \frac{y}{x}$$

$$\int_{1\text{ ft}}^{y} \frac{dy}{y} = \int_{2\text{ ft}}^{x} \frac{dx}{x}$$

$$\ln y \Big|_{1\text{ ft}}^{y} = \ln x \Big|_{2\text{ ft}}^{x}$$

$$\ln y = \ln \frac{x}{2}$$

$$y = \frac{1}{2}x \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + 8x(8) + 8y(0)$$

$$= (64x)\text{ ft/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (8x)(0) + 8y(8)$$

$$= (64y)\text{ ft/s}^2$$

At $x = 2$ ft, $y = 1$ ft. Then

$$a_x = 64(2) = 128\text{ ft/s}^2 \qquad a_y = 64(1) = 64\text{ ft/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(128\text{ ft/s}^2)^2 + (64\text{ ft/s}^2)^2} = 143\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Its direction is

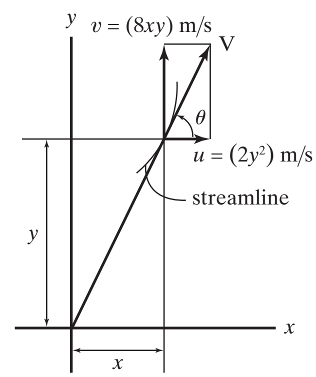
$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{64\text{ ft/s}^2}{128\text{ ft/s}^2}\right) = 26.6° \measuredangle\theta \qquad \textbf{Ans.}$$

Ans:
$y = x/2, a = 143\text{ ft/s}^2$
$\theta = 26.6° \measuredangle$

3–39. A fluid velocity components of $u = (2y^2)$ m/s and $v = (8xy)$ m/s, where x and y are in meters. Determine the equation of the streamline passing through point (1 m, 2 m). Also, what is the acceleration of a particle at this point? Is the flow steady or unsteady?

SOLUTION

Since the velocity components are the function of position, not of time, the flow can be classified as **steady** (**Ans.**) but nonuniform. Using the definition of the slope of the streamline,



(a)

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{8xy}{xy^2} = \frac{4x}{y}$$

$$\int_{2\text{ m}}^{y} y\,dy = \int_{1\text{ m}}^{x} 4x\,dx$$

$$\left.\frac{y^2}{2}\right|_{2\text{ m}}^{y} = \left.2x^2\right|_{1\text{m}}^{x}$$

$$\frac{y^2}{2} - 2 = 2x^2 - 2$$

$$y^2 = 4x^2$$

$$y = 2x \qquad \textbf{Ans.}$$

(Note that $x = 1$, $y = 2$ is not a solution to $y = -2x$.)
For two dimensional flow, the Eulerian description gives.

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + 2y^2(0) + 8xy(4y)$$

$$= \left(32xy^2\right)\text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 2y^2(8y) + (8xy)(8x)$$

$$= \left(16y^3 + 64x^2y\right)\text{ m/s}^2$$

At point $x = 1$ m and $y = 2$ m,

$$a_x = 32(1)\left(2^2\right) = 128\text{ m/s}^2$$

$$a_y = \left[16\left(2^3\right) + 64\left(1^2\right)(2)\right] = 256\text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(128\text{ m/s}^2\right)^2 + \left(256\text{ m/s}^2\right)^2} = 286\text{ m/s}^2 \qquad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{256\text{ m/s}^2}{128\text{ m/s}^2}\right) = 63.4^\circ\ \measuredangle\theta \qquad \textbf{Ans.}$$

Ans:
$y = 2x$
$a = 286\text{ m/s}^2$
$\theta = 63.4^\circ\ \measuredangle$

***3–40.** The velocity of a flow field is defined by $\mathbf{V} = \{4y\mathbf{i} + 2x\mathbf{j}\}$ m/s, where x and y are in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point (2 m, 1 m). Find the equation of the streamline passing through this point, and sketch the velocity and acceleration at the point on this streamline.

SOLUTION

The flow is steady but nonuniform since the velocity components are a function of position, but not time. At point (2 m, 1 m)

$$u = 4y = 4(1) = 4\text{ m/s}$$

$$v = 2x = 2(2) = 4\text{ m/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(4\text{ m/s})^2 + (4\text{ m/s})^2} = 5.66\text{ m/s} \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + 4y(0) + (2x)(4)$$

$$= (8x)\text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 4y(2) + 2x(0)$$

$$= (8y)\text{ m/s}^2$$

At point (2 m, 1 m),

$$a_x = 8(2) = 16\text{ m/s}^2$$

$$a_y = 8(1) = 8\text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{(16\text{ m/s})^2 + (8\text{ m/s})^2}$$

$$= 17.9\text{ m/s}^2 \qquad \textbf{Ans.}$$

Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{2x}{4y} = \frac{x}{2y}$$

$$\int_{1\text{ m}}^{y} 2y\,dy = \int_{2\text{ m}}^{x} x\,dx$$

$$y^2\Big|_{1\text{ m}}^{y} = \frac{x^2}{2}\Big|_{2\text{ m}}^{x}$$

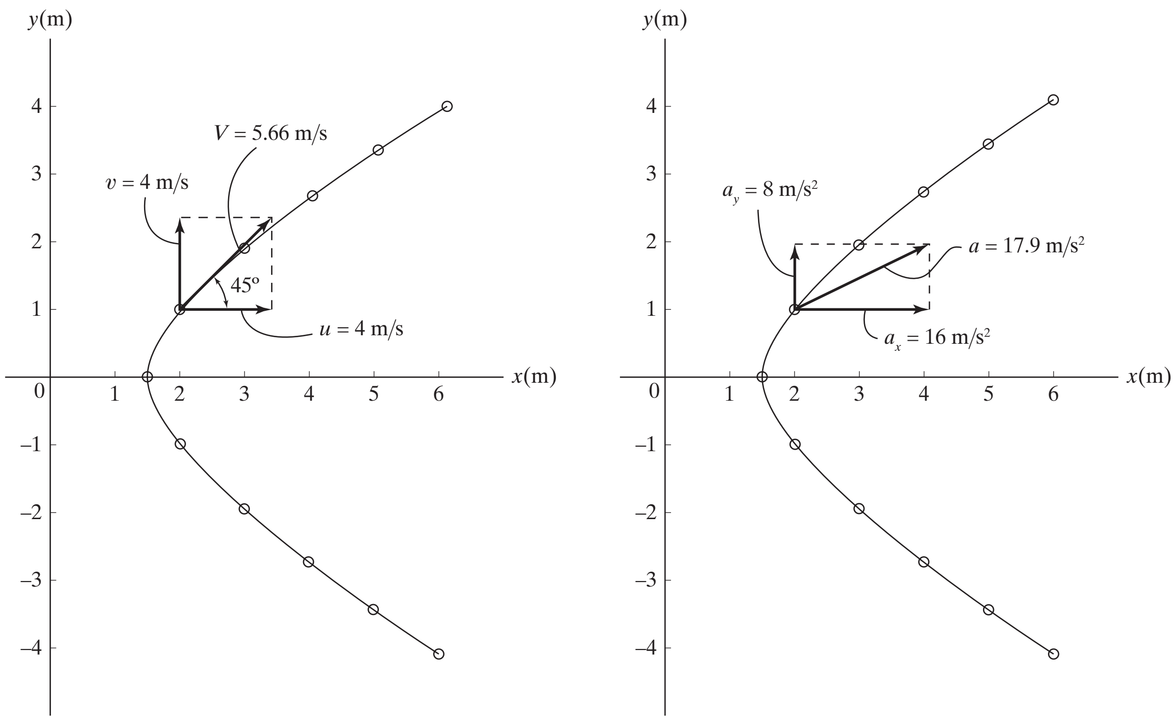
$$y^2 - 1 = \frac{x^2}{2} - 2$$

$$y^2 = \frac{1}{2}x^2 - 1$$

***3–40. (continued)**

The plot of this streamline is shown is Fig. *a*

x(m)	$\sqrt{2}$	2	3	4	5	6
y(m)	0	±1	±1.87	±2.65	±3.39	±4.12



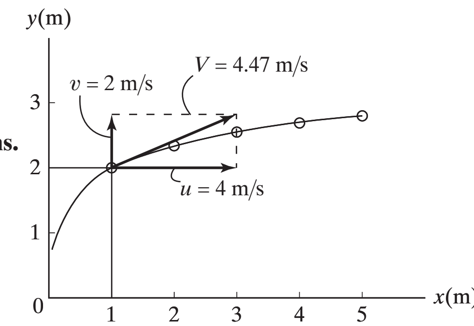
(a)

3–41. The velocity of a flow field is defined by $\mathbf{V} = \{4x\mathbf{i} + 2\mathbf{j}\}$ m/s, where x is in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point (1 m, 2 m). Find the equation of the streamline passing through this point, and sketch these vectors on this streamline.

SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady nonuniform. At point (1 m, 2 m),

$$u = 4x = 4(1) = 4\text{ m/s}$$
$$v = 2\text{ m/s}$$



The magnitude of velocity is

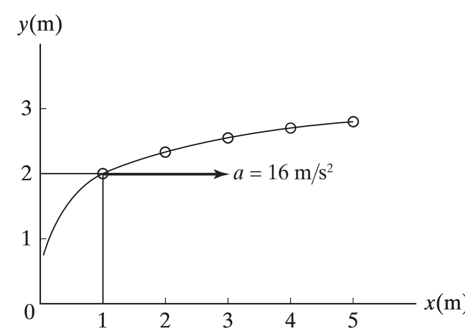
$$V = \sqrt{u^2 + v^2} = \sqrt{(4\text{ m/s})^2 + (2\text{ m/s})^2} = 4.47\text{ m/s}$$ **Ans.**

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$
$$= 0 + 4x(4) + 2(0) = 16x$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$
$$= 0 + 4x(0) + 2(0) = 0$$



At point (1 m, 2 m),

$$a_x = 16(1) = 16\text{ m/s}^2 \qquad a_y = 0$$

Thus, the magnitude of the acceleration is

$$a = a_x = 16\text{ m/s}^2$$ **Ans.**

Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{2}{4x} = \frac{1}{2x}$$

$$\int_{2\text{ m}}^{y} dy = \frac{1}{2}\int_{1\text{ m}}^{x} \frac{dx}{x}$$

$$y - 2 = \frac{1}{2}\ln x$$

$$y = \left(\frac{1}{2}\ln x + 2\right)$$ **Ans.**

The plot of this streamline is shown in Fig. *a*

x(m)	e^{-4}	1	2	3	4	5
y(m)	0	2	2.35	2.55	2.69	2.80

Ans:
$V = 4.47\text{ m/s}, a = 16\text{ m/s}^2$
$y = \frac{1}{2}\ln x + 2$

3–42. The velocity of a flow field is defined by $u = (2x^2 - y^2)$ m/s and $v = (-4xy)$ m/s, where x and y are in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point (1 m, 1 m). Find the equation of the streamline passing through this point, and sketch the velocity and acceleration at the point on this streamline.

SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady but nonuniform. At point (1 m, 1 m),

$$u = 2x^2 - y^2 = 2(1^2) - 1^2 = 1 \text{ m/s}$$
$$v = -4xy = -4(1)(1) = -4 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(1 \text{ m/s})^2 + (-4 \text{ m/s})^2} = 4.12 \text{ m/s} \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \\ &= 0 + (2x^2 - y^2)(4x) + (-4xy)(-2y) \\ &= 4x(2x^2 - y^2) + 8xy^2 \end{aligned}$$

$$\begin{aligned} a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \\ &= 0 + (2x^2 - y^2)(-4y) + (-4xy)(-4x) \\ &= -4y(2x^2 - y^2) + 16x^2y \end{aligned}$$

At point (1 m, 1 m),

$$a_x = 4(1)\left[2(1^2) - 1^2\right] + 8(1)(1^2) = 12 \text{ m/s}^2$$
$$a_y = -4(1)\left[2(1^2) - 1^2\right] + 16(1^2)(1) = 12 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(12 \text{ m/s}^2)^2 + (12 \text{ m/s}^2)^2} = 17.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = -\frac{4xy}{2x^2 - y^2}$$

$$(2x^2 - y^2)dy = -4xydx$$
$$2x^2dy + 4xydx - y^2dy = 0$$

However, $d(2x^2y) = 2(2xydx + x^2dy) = 2x^2dy + 4xydx$. Then

$$d(2x^2y) - y^2dy = 0$$

3–42. (continued)

Integrating this equation,

$$2x^2y - \frac{y^3}{3} = C$$

with the condition $y = 1$ m when $x = 1$ m,

$$2(1^2)(1) - \frac{1^3}{3} = C$$

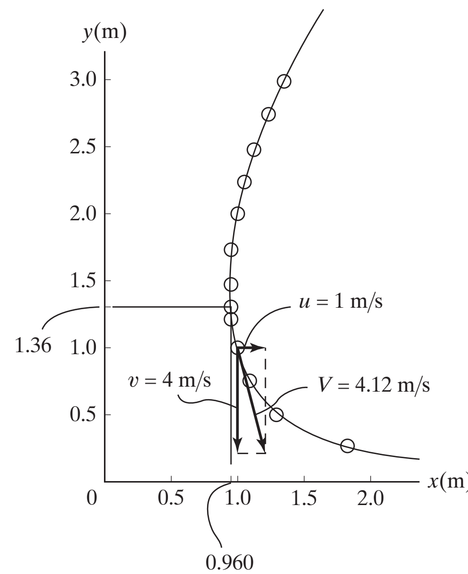
$$C = \frac{5}{3}$$

Thus,

$$2x^2y - \frac{y^3}{3} = \frac{5}{3}$$

$$6x^2y - y^3 = 5$$

$$x^2 = \frac{y^3 + 5}{6y} \qquad \textbf{Ans.}$$



Taking the derivative of this equation with respect to y

$$2x\frac{dx}{dy} = \frac{6y(3y^2) - (y^3 + 5)(6)}{(6y)^2} = \frac{2y^3 - 5}{6y^2}$$

$$\frac{dx}{dy} = \frac{2y^3 - 5}{12xy^2}$$

Set $\frac{dx}{dy} = 0$;

$$2y^3 - 5 = 0$$

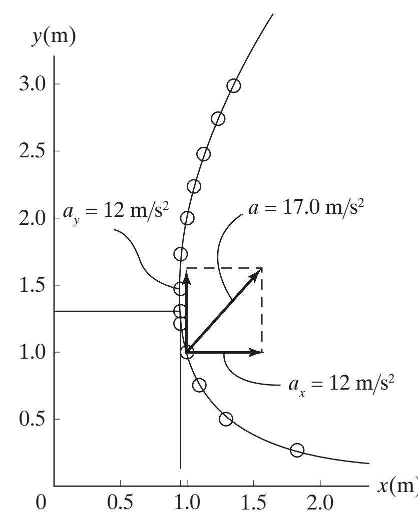
$$y = 1.357 \text{ m}$$

The corresponding x is

$$x^2 = 1.357^3 + 5$$

$$x = 0.960 \text{ m}$$

y(m)	0.25	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
x(m)	1.83	1.31	1.10	1.00	0.963	0.965	0.993	1.04	1.10	1.17	1.25	1.33



Ans:
$V = 4.12$ m/s
$a = 17.0$ m/s^2
$x^2 = \frac{y^3 + 5}{6y}$

3–43. The velocity of a flow field is defined by $u = (-y/4)$ m/s and $v = (x/9)$ m/s, where x and y are in meters. Determine the magnitude of the velocity and acceleration of a particle that passes through point (3 m, 2 m). Find the equation of the streamline passing through this point, and sketch the velocity and acceleration at the point on this streamline.

SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady nonuniform. At point (3 m, 2m)

$$u = \frac{-y}{4} = -\frac{2}{4} = -0.5 \text{ m/s}$$

$$v = \frac{x}{9} = \frac{3}{9} = 0.3333 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(-0.5 \text{ m/s})^2 + (0.333\text{m/s})^2} = 0.601 \text{ m/s} \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y are

$$(\overset{+}{\rightarrow})\ a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + \left(\frac{-y}{4}\right)(0) + \left(\frac{x}{9}\right)\left(-\frac{1}{4}\right)$$

$$= \left(-\frac{1}{36}x\right) \text{ m/s}^2$$

$$(+\uparrow)\ a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial y} + v\frac{\partial v}{\partial y}$$

$$= 0 + \left(\frac{-y}{4}\right)\left(\frac{1}{9}\right) + \left(\frac{x}{9}\right)(0)$$

$$= \left[-\frac{1}{36}y\right] \text{ m/s}^2$$

At point (3 m, 2 m),

$$a_x = -\frac{1}{36}(3) = -0.08333 \text{ m/s}^2$$

$$a_y = -\frac{1}{36}(2) = -0.05556 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{ax^2 + ay^2}$$

$$= \sqrt{(-0.08333 \text{ m/s}^2)^2 + (-0.05556 \text{ m/s}^2)}$$

$$= 0.100 \text{ m/s}^2 \qquad \textbf{Ans.}$$

3–43. (continued)

Using the definition of slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{x/9}{-y/4} = -\frac{4x}{9y}$$

$$9\int_{2\text{ m}}^{y} y\,dy = -4\int_{3\text{ m}}^{x} x\,dx$$

$$\left.\frac{9y^2}{2}\right|_{2\text{ m}}^{y} = -\left.\left(2x^2\right)\right|_{3\text{ m}}^{x}$$

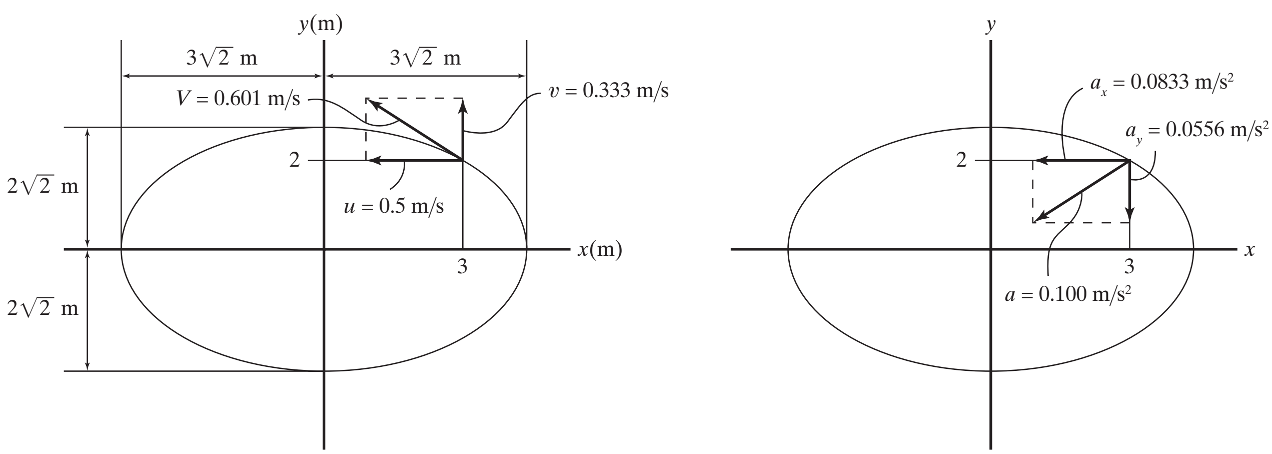
$$\frac{9y^2}{2} - 18 = -2x^2 + 18$$

$$9y^2 + 4x^2 = 72$$

$$\frac{x^2}{72/4} + \frac{y^2}{72/9} = 1$$

$$\frac{x^2}{(4.24)^2} + \frac{y^2}{(2.83)^2} = 1 \qquad \textbf{Ans.}$$

This is an equation of an ellipse with center at (0, 0). The plot of this streamline is shown in Fig. *a*



(a)

Ans:
$V = 0.601\text{ m/s}$
$a = 0.100\text{ m/s}^2$
$x^2/(4.24)^2 + y^2/(2.83)^2 = 1$

***3–44.** The velocity of gasoline, along the centerline of a tapered pipe, is given by $u = (4tx)$ m/s, where t is in seconds and x is in meters. Determine the acceleration of a particle when $t = 0.8$ s if $u = 0.8$ m/s when $t = 0.1$ s.

SOLUTION

The flow is unsteady nonuniform. For one dimensional flow,

$$a = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

Here, $u = (4\,tx)$ m/s. Then $\dfrac{\partial u}{\partial t} = 4x$ and $\dfrac{\partial u}{\partial x} = 4t$. Thus,

$$a = 4x + (4tx)(4t) = \left(4x + 16t^2x\right)\text{ m/s}^2$$

Since $u = 0.8$ m/s when $t = 0.1$ s,

$$0.8 = 4(0.1)\,x \qquad x = 2\text{ m}$$

The position of the particle can be determined from

$$\frac{dx}{dt} = u = 4\,tx; \qquad \int_{2\text{ m}}^{x}\frac{dx}{x} = 4\int_{0.15}^{t} t\,dt$$

$$\ln x\Big|_{2\text{m}}^{x} = 2t^2\Big|_{0.15}^{t}$$

$$\ln\frac{x}{2} = 2t^2 - 0.02$$

$$e^{2t^2-0.02} = \frac{x}{2}$$

$$x = 2e^{2t^2-0.02}$$

$$x = 2e^{2(0.8^2)-0.02} = 7.051\text{ m}$$

Thus, $t = 0.8$ s,

$$\begin{aligned} a &= 4(7.051) + 16\left(0.8^2\right)(7.051) \\ &= 100.40\text{ m/s}^2 \\ &= 100\text{ m/s}^2 \end{aligned}$$

Ans.

3–45. The velocity field for a flow of water is defined by $u = (2x)$ m/s, $v = (6tx)$ m/s, and $w = (3y)$ m/s, where t is in seconds and x, y, z are in meters. Determine the acceleration and the position of a particle when $t = 0.5$ s if this particle is at (1 m, 0, 0) when $t = 0$.

SOLUTION

The flow is unsteady nonuniform. For three dimensional flow,

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial t} + v\frac{\partial \mathbf{V}}{\partial t} + w\frac{\partial \mathbf{V}}{\partial t}$$

Thus,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial t}$$

$$= 0 + 2x(2) + (6tx)(0) + 3y(0)$$

$$= (4x)\text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

$$= 6x + 2x(6t) + 6tx(0) + 3y(0)$$

$$= (6x + 12tx)\text{ m/s}^2$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$= 0 + 2x(0) + 6tx(3) + 3y(0)$$

$$= (18tx)\text{ m/s}^2$$

The position of the particle can be determined from

$$\frac{dx}{dt} = u = 2x; \qquad \int_{1\text{ m}}^{x} \frac{dx}{x} = 2\int_0^t dt$$

$$\ln x = 2t$$

$$x = \left(e^{2t}\right)\text{ m}$$

$$\frac{dy}{dt} = v = 6tx = 6te^{2t}; \qquad \int_0^y dy = 6\int_0^t te^{2t}dt$$

$$y = \frac{3}{2}\left(2te^{2t} - e^{2t}\right)\Big|_0^t$$

$$y = \frac{3}{2}\left(2te^{2t} - e^{2t} + 1\right)$$

$$\frac{dz}{dt} = w = 3y = \frac{9}{2}\left[2te^{2t} - e^{2t} + 1\right];$$

$$\int_0^z dz = \frac{9}{2}\int_0^t \left(2te^{2t} - e^{2t} + 1\right)dt$$

$$z = \frac{9}{2}\left[te^{2t} - \frac{1}{2}e^{2t} - \frac{1}{2}e^{2t} + t\right]\Bigg|_0^t$$

$$z = \frac{9}{2}\left(te^{2t} - e^{2t} + t + 1\right)\text{ m}$$

3–45. (continued)

When, $t = 0.5$ s,

$$x = e^{2(0.5)} = 2.7183 \text{ m} = 2.72 \text{ m} \qquad \textbf{Ans.}$$

$$y = \frac{3}{2}\left[2(0.5)e^{2(0.5)} - e^{2(0.5)} + 1\right] = 1.5 \text{ m} \qquad \textbf{Ans.}$$

Thus, $z = \frac{9}{2}\left[0.5e^{2(0.5)} - e^{2(0.5)} + 0.5 + 1\right] = 0.6339 \text{ m} = 0.634 \text{ m}$ **Ans.**

$$a_x = 4(2.7183) = 10.87 \text{ m/s}^2$$

$$a_y = 6(2.7183) + 12(0.5)(2.7183) = 32.62 \text{ m/s}^2$$

$$a_z = 18(0.5)(2.7183) = 24.46 \text{ m/s}^2$$

Then

$$\mathbf{a} = \{10.9\mathbf{i} + 32.6\mathbf{j} + 24.5\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$x = 2.72$ m
$y = 1.5$ m
$z = 0.634$ m
$\mathbf{a} = \{10.9\mathbf{i} + 32.6\mathbf{j} + 24.5\mathbf{k}\}$ m/s^2

3–46. A flow field has velocity components of $u = -(4x + 6)$ m/s and $v = (10y + 3)$ m/s where x and y are in meters. Determine the equation for the streamline that passes through point (1 m, 1 m), and find the acceleration of a particle at this point.

SOLUTION

Since the velocity components are the function of position but not of time, the flow can be classified as steady but nonuniform. Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{10y + 3}{-(4x + 6)}$$

$$\int_{1\,\text{m}}^{y} \frac{dy}{10y + 3} = -\int_{1\,\text{m}}^{x} \frac{dx}{4x + 6}$$

$$\frac{1}{10}\ln(10y + 3)\Big|_{1\text{m}}^{y} = -\frac{1}{4}\ln(4x + 6)\Big|_{1\text{m}}^{x}$$

$$\frac{1}{10}\ln\left(\frac{10y + 3}{13}\right) = \frac{1}{4}\ln\left(\frac{10}{4x + 6}\right)$$

$$\ln\left(\frac{10y + 3}{13}\right)^{\frac{1}{10}} = \ln\left(\frac{10}{4x + 6}\right)^{\frac{1}{4}}$$

$$\left(\frac{10y + 3}{13}\right)^{\frac{1}{10}} = \left(\frac{10}{4x + 6}\right)^{\frac{1}{4}}$$

$$\frac{10y + 3}{13} = \left(\frac{10}{4x + 6}\right)^{\frac{5}{2}}$$

$$y = \left[\frac{411}{(4x + 6)^{5/2}} - 0.3\right]\text{m} \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\delta \mathbf{V}}{\delta t} + u\frac{\delta \mathbf{V}}{\delta x} + v\frac{\delta \mathbf{V}}{\delta y}$$

Writing the scalar components of this equation along the x and y axes,

$$a_x = \frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y}$$

$$= 0 + [-(4x + 6)(-4)] + (10y + 3)(0)$$

$$= [4(4x + 6)]\ \text{m/s}^2$$

$$a_y = \frac{\delta v}{\delta t} + u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y}$$

$$= 0 + [-(4x + 6)(0)] + (10y + 3)(10)$$

$$= [10(10y + 3)]\ \text{m/s}^2$$

At point (1m, 1m),

$$a_x = 4[4(1) + 6] = 40\ \text{m/s}^2 \rightarrow$$

$$a_y = 10[10(1) + 3] = 130\ \text{m/s}^2 \uparrow$$

3–46. (continued)

The magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(40\text{m/s}^2)^2 + (130\text{m/s}^2)^2} = 136 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{130 \text{ m/s}^2}{40 \text{ m/s}^2}\right) = 72.9° \measuredangle \theta \qquad \textbf{Ans.}$$

Ans:

$$y = \frac{411}{(4x+6)^{5/2}} - 0.3$$

$$a = 136 \text{ m/s}^2$$

$$\theta = 72.9° \measuredangle$$

3–47. A velocity field for oil is defined by $u = (100y)$ m/s, $v = (0.03\,t^2)$ m/s, where t is in seconds and y is in meters. Determine the acceleration and the position of a particle when $t = 0.5$ s. The particle is at the origin when $t = 0$.

SOLUTION

Since the velocity components are a function of both position and time, the flow can be classified as unsteady nonuniform. Using the defination of velocity,

$$\frac{dy}{dt} = v = 0.03t^2; \qquad \int_0^y dy = 0.03\int_0^t t^2\,dt$$

$$y = (0.01t^3)\text{ m}$$

When $t = 0.5$ s,

$$y = 0.01(0.5^3) = 0.00125\text{ m} = 1.25\text{ mm} \qquad \textbf{Ans.}$$

$$\frac{dx}{dt} = u = 100y = 100(0.01t^3) = t^3; \qquad \int_0^y dx = \int_0^t t^3\,dt$$

$$x = \left(\frac{1}{4}t^4\right)\text{ m}$$

When $t = 0.5$ s,

$$x = \frac{1}{4}(0.5^4) = 0.015625\text{ m} = 15.6\text{ mm} \qquad \textbf{Ans.}$$

For a two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Write the scalar components of this equation along the x and y axes,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (100y)(0) + (0.03t^2)(100)$$

$$= (3t^2)\text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0.06t + (100y)(0) + 0.03t^2(0)$$

$$= (0.06t)\text{ m/s}^2$$

When $t = 0.5$ s,

$$a_x = 3(0.5^2) = 0.75\text{ m/s}^2 \rightarrow$$

$$a_y = 0.06(0.5) = 0.03\text{ m/s}^2 \uparrow$$

The magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.75\text{ m/s}^2)^2 + (0.03\text{ m/s}^2)^2} = 0.751\text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0.03\text{ m/s}^2}{0.75\text{ m/s}^2}\right) = 2.29^\circ \measuredangle\theta \qquad \textbf{Ans.}$$

Ans:
$y = 1.25$ mm
$x = 15.6$ mm
$a = 0.751\text{ m/s}^2$
$\theta = 2.29^\circ$ $\measuredangle$

***3–48.** If $u = (2x^2)$ m/s and $v = (-y)$ m/s where x and y are in meters, determine the equation of the streamline that passes through point (2 m, 6 m), and find the acceleration of a particle at this point. Sketch the streamline for $x > 0$, and find the equations that define the x and y components of acceleration of the particle as a function of time if $x = 2$ m and $y = 6$ m when $t = 0$.

SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady but nonuniform. Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{-y}{2x^2}$$

$$\int_{6\text{ m}}^{y} \frac{dy}{y} = -\frac{1}{2}\int_{2\text{ m}}^{x} \frac{dx}{x^2}$$

$$\ln y\Big|_{6\text{ m}}^{y} = \frac{1}{2}\left(\frac{1}{x}\right)\Big|_{2\text{ m}}^{x}$$

$$\ln\frac{y}{6} = \frac{1}{2}\left(\frac{1}{x} - \frac{1}{2}\right)$$

$$\ln\frac{y}{6} = \frac{2 - x}{4x}$$

$$\frac{y}{6} = e^{\left(\frac{2-x}{4x}\right)}$$

$$y = \left[6e^{\left(\frac{2-x}{4x}\right)}\right]\text{ m} \qquad \textbf{Ans.}$$

The plot of this streamline is shown in Fig. a.

For two dimensional flow, the Eulerian description gives.

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (2x^2)(4x) + (-y)(0)$$

$$= (8x^3)\text{ m/s}^2$$

$$a_y = \frac{\delta v}{\delta t} + u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y}$$

$$= 0 + (2x^2)(0) + (-y)(-1)$$

$$= (y)\text{m/s}^2$$

At point (2 m, 6 m),

$$a_x = 8(2^3) = 64\text{ m/s}^2 \rightarrow$$

$$a_y = 6\text{m/s}^2 \uparrow$$

The magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(64\text{ m/s}^2)^2 + (6\text{m/s}^2)^2} = 64.3\text{m/s}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{6\text{ m/s}^2}{64\text{ m/s}^2}\right) = 5.36° \angle\theta \qquad \textbf{Ans.}$$

3–48. (continued)

Using the definition of the velocity,

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = 2x^2$$

$$\int_{2\text{ m}}^{x} \frac{dx}{2x^2} = \int_0^t dt$$

$$-\frac{1}{2}\left(\frac{1}{x}\right)\Bigg|_{2\text{ m}}^{x} = t$$

$$-\frac{1}{2}\left(\frac{1}{x} - \frac{1}{2}\right) = t$$

$$\frac{x-2}{4x} = t$$

$$x = \left(\frac{2}{1-4t}\right) \text{m}$$

$$\frac{dy}{dt} = v; \qquad \frac{dy}{dt} = -y$$

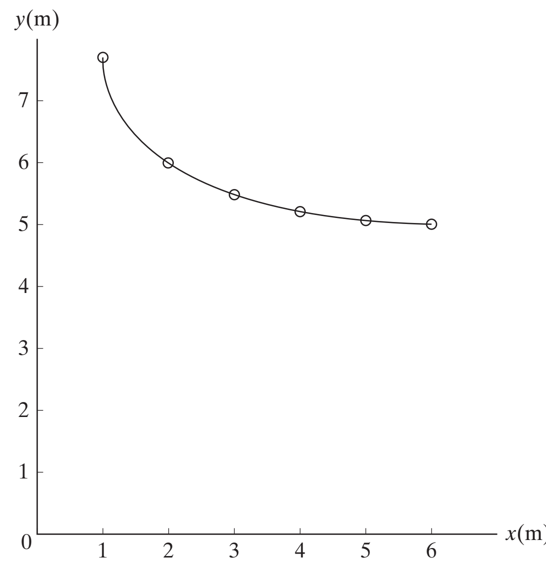
$$-\int_{6\text{ m}}^{y} \frac{dy}{y} = \int_0^t dt$$

$$-\ln y\Bigg|_{6\text{ m}}^{y} = t$$

$$\ln \frac{6}{y} = t$$

$$\frac{6}{y} = e^t$$

$$y = \left(6e^{-t}\right) \text{m}$$



(a)

Thus,

$$u = 2x^2 = 2\left(\frac{2}{1-4t}\right)^2 = \left[\frac{8}{(1-4t)^2}\right] \text{m/s and } v = -y = \left(-6e^{-t}\right) \text{m/s}$$

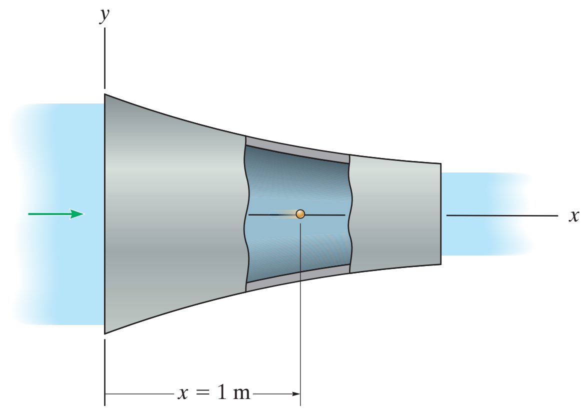
Then,

$$a_x = \frac{du}{dt} = -16(1-4t)^{-3}(-4) = \left[\frac{64}{(1-4t)^3}\right] \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_y = \frac{dv}{dt} = \left(6e^{-t}\right) \text{m/s}^2 \qquad \textbf{Ans.}$$

x(m)	0.5	1	2	3	4	5	6
y(m)	12.70	7.70	6.00	5.52	5.29	5.16	5.08

3–49. Air flow through the duct is defined by the velocity field $u = (2x^2 + 8)$ m/s $v = (-8x)$ m/s where x is in meters. Determine the acceleration of a fluid particle at the origin (0, 0) and at point (1 m, 0). Also, sketch the streamlines that pass through these points.



SOLUTION

Since the velocity component are a function of position but not time, the flow can be classified as steady but nonuniform. For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= \left[0 + (2x^2 + 8)(4x) + (-8x)(0)\right]$$

$$= \left[4x(2x^2 + 8)\right] \text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (2x^2 + 8)(-8) + (-8x)(0)$$

$$= \left[-8(2x^2 + 8)\right] \text{ m/s}^2$$

At point (0, 0),

$$a_x = 4(0)\left[2(0^2) + 8\right] = 0$$

$$a_y = -8\left[2(0^2) + 8\right] = -64 \text{ m/s}^2 = 64 \text{ m/s}^2\downarrow$$

Thus,

$$a = a_y = 64 \text{ m/s}^2\downarrow \qquad \textbf{Ans.}$$

At point (1 m, 0),

$$a_x = 4(1)\left[2(1^2) + 8\right] = 40 \text{ m/s}^2 \rightarrow$$

$$a_y = -8\left[2(1^2) + 8\right] = -80 \text{ m/s}^2 = 80 \text{ m/s}^2\downarrow$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(40 \text{ m/s}^2)^2 + (80 \text{ m/s}^2)^2} = 89.4 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{80 \text{ m/s}^2}{40 \text{ m/s}^2}\right) = 63.4° \ \searrow\theta \qquad \textbf{Ans.}$$

Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-8x}{2x^2 + 8}; \qquad \int dy = -8\int \frac{x\,dx}{2x^2 + 8}$$

$$y = -2\ln\left(2x^2 + 8\right) + C$$

3–49. (continued)

For the streamline passing through point (0, 0),

$$0 = -2\ln\left[2\left(0^2\right) + 8\right] + C \qquad C = 2\ln 8$$

Then $y = \left[2\ln\left(\dfrac{8}{2x^2+8}\right)\right]$ m **Ans.**

For the streamline passing through point (1 m, 0),

$$0 = -2\ln\left[2\left(1^2\right) + 8\right] + C \qquad C = 2\ln\ 10$$

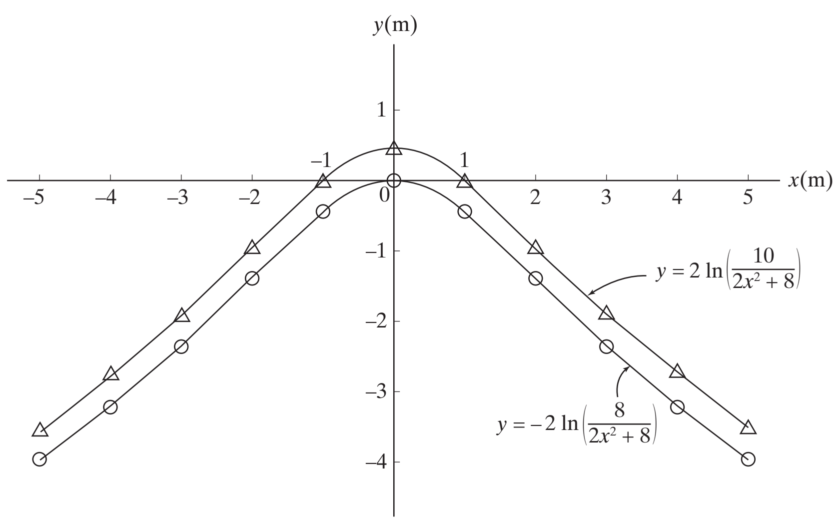
$$y = \left[2\ln\left(\frac{10}{2x^2+8}\right)\right] \text{ m}$$ **Ans.**

For point (0, 0)

x(m)	0	±1	±2	±3	±4	±5
y(m)	0	−0.446	−1.39	−2.36	−3.22	−3.96

For point (1 m, 0)

x(m)	0	±1	±2	±3	±4	±5
y(m)	0.446	0	−0.940	−1.91	−2.77	−3.52



Ans:
At point (0, 0),
$a = 64\ \text{m/s}^2 \downarrow$
At point (1 m, 0),
$a = 89.4\ \text{m/s}^2, \theta = 63.4°$
For the streamline passing through point (0, 0),
$y = \left[2\ln\left(\dfrac{8}{2x^2+8}\right)\right]$ m
For the streamline passing through point (1 m, 0),
$y = \left[2\ln\left(\dfrac{10}{2x^2+8}\right)\right]$ m

3–50. The velocity field for a fluid is defined by $u = y/(x^2+y^2)$ and $v = \left[4x/(x^2+y^2)\right]$ m/s, where x and y are in meters. Determine the acceleration of particles located at point (2 m, 0) and that of a particle located at point (4 m, 0). Sketch the equations that define the streamlines that pass through these points.

SOLUTION

Since the velocity components are a function of position but not time, the flow can be classified as steady but nonuniform. For two dimensional flow, the Eulerian description gives

$$a = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Write the scalar components of this equation along x and y axes,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + \left(\frac{y}{x^2+y^2}\right)\left[\frac{(x^2+y^2)(0) - y(2x)}{(x^2+y^2)^2}\right] + \left(\frac{4x}{x^2+y^2}\right)\left[\frac{(x^2+y^2)(1) - y(2y)}{(x^2+y^2)^2}\right]$$

$$= \left[\frac{4x^3 - 6xy^2}{(x^2+y^2)^3}\right]\text{m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + \left(\frac{y}{x^2+y^2}\right)\left[\frac{(x^2+y^2)(4) - 4x(2x)}{(x^2+y^2)^2}\right] + \left(\frac{4x}{x^2+y^2}\right)\left[\frac{(x^2+y^2)(0) - 4x(2y)}{(x^2+y^2)^2}\right]$$

$$= \left[\frac{4y^3 - 36x^2y}{(x^2+y^2)^3}\right]\text{m/s}^2$$

At point (2 m, 0)

$$a_x = \frac{4(2^3) - 6(2)(0^2)}{(2^2+0^2)^3} = 0.5\text{ m/s}^2 \rightarrow$$

$$a_y = \frac{4(0^3) - 36(2^2)(0)}{(2^2+0^2)^3} = 0$$

Thus $a = a_x = 0.5\text{ m/s}^2 \rightarrow$ **Ans.**

3–50. (continued)

At point (4 m, 0)

$$a_x = \frac{4(4^3) - 6(4)(0)}{(4^2 + 0^2)^3} = 0.0625 \text{ m/s}^2 \rightarrow$$

$$a_y = \frac{4(0^3) - 36(4^2)(0)}{(4^2 + 0^2)^3} = 0$$

Thus

$$a = a_x = 0.0625 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans.}$$

Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u} = \frac{4x/(x^2 + y^2)}{y/(x^2 + y^2)} = \frac{4x}{y}; \qquad \int y\,dy = 4\int x\,dx$$

$$\frac{y^2}{2} = 2x^2 + C'$$

$$y^2 = 4x^2 + C$$

For the streamline passing through point (2 m, 0),

$$0^2 = 4(2^2) + C \qquad C = -16$$

Then

$$y^2 = 4x^2 - 16$$

$$y = \pm\sqrt{4x^2 - 16} \qquad x \geq 2\text{m} \qquad \textbf{Ans.}$$

For the streamline passes through point (4 m, 0)

$$0^2 = 4(4^2) + C \qquad C = -64$$

Then

$$y^2 = 4x^2 - 64$$

$$y = \pm\sqrt{4x^2 - 64} \qquad x \geq 4\text{m}$$

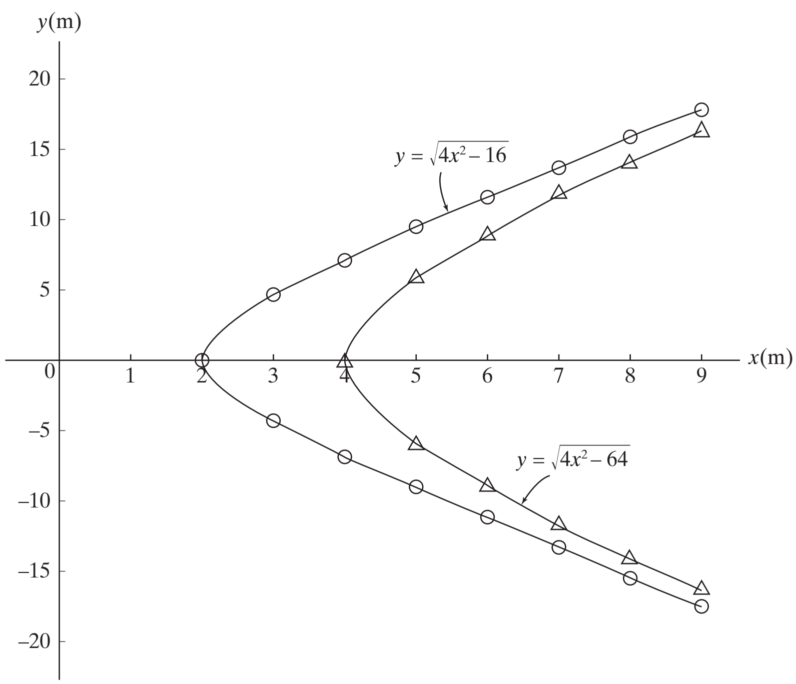
3–50. (continued)

For the streamline passing through point (2 m, 0)

x(m)	2	3	4	5	6	7	8	9
y(m)	0	±4.47	±6.93	±9.17	±11.31	±13.42	±15.49	±17.55

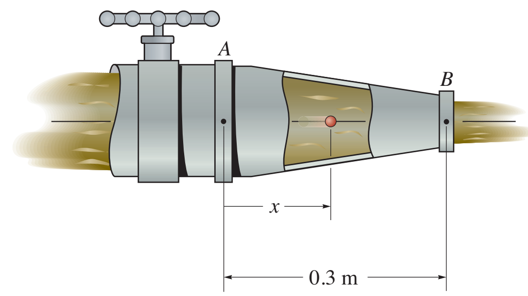
For the streamline passing through point (4 m, 0)

x(m)	4	5	6	7	8	9
y(m)	0	±6.00	±8.94	±11.49	±13.86	±16.12



Ans:
For point (2 m, 0),
$a = 0.5 \text{ m/s}^2$
$y = \pm\sqrt{4x^2 - 16}$
For point (4 m, 0),
$a = 0.0625 \text{ m/s}^2$
$y = \pm\sqrt{4x^2 - 64}$

3–51. As the value is closed, oil flows through the nozzle such that along the center streamline it has a velocity of $V = 6(1 + 0.4x^2)(1 - 0.5t)$ m/s where x is in meters and t is on seconds. Determine the acceleration of an oil particle at $x = 0.25$ m when $t = 1$ s.



SOLUTION

Here V only has an x component, so that $V = u$. Since V is a function of time at each x, the flow is unsteady. Since $v = w = 0$, we have

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

$$= \frac{\partial}{\partial x}\left[6(1 + 0.4x^2)(1 - 0.5t)\right] + \left[6(1 + 0.4x^2)(1 - 0.5t)\right]\frac{\partial}{\partial x}\left[6(1 + 0.4x^2)(1 - 0.5t)\right]$$

$$= \left[6(1 + 0.4x^2)(0 - 0.5)\right] + \left[6(1 + 0.4x^2)(1 - 0.5t)\right]\left[6(0 + 0.4(2x))(1 - 0.5t)\right]$$

Evaluating this expression at $x = 0.25$ m, $t = 1$ s, we get

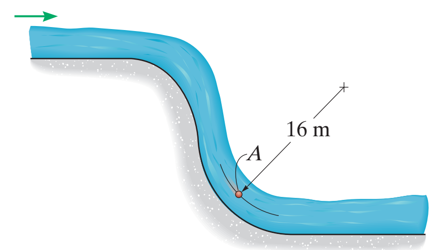
$$a_s = -3.075 \text{ m/s}^2 + 1.845 \text{ m/s}^2$$

$$= -1.23 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Note that the local acceleration component (-3.075 m/s^2) indicates a deceleration since the valve is being closed to decrease the flow. The convective acceleration (1.845 m/s^2) is positive since the nozzle constricts as x increases. The net result causes the particle to decelerate at 1.23 m/s^2.

Ans:
-1.23 m/s^2

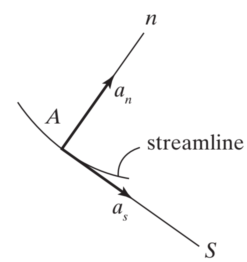
***3–52.** As water flows *steadily* over the spillway, one of its particles follows a streamline that has a radius of curvature of 16 m. If its speed at point A is 5 m/s which is increasing at 3 m/s^2, determine the magnitude of acceleration of the particle.



SOLUTION

The $n - s$ coordinate system is established with origin at point A as shown in Fig. *a*. Here, the component of the particle's acceleration along the s axis is

$$a_s = 3\text{ m/s}^2$$



(a)

Since the streamline does not rotate, the local acceleration along the n axis is zero, so that $\left(\dfrac{\partial V}{\partial t}\right)_n = 0$. Therefore, the component of the particle's acceleration along the n axes is

$$a_n = \left(\frac{\partial V}{\partial t}\right)_n + \frac{V^2}{R}$$

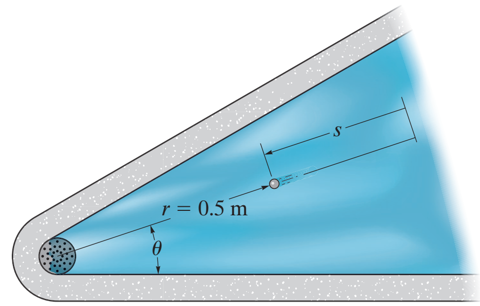
$$= 0 + \frac{(5\text{ m/s})^2}{16\text{ m}} = 1.5625\text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_s^2 + a_n^2} = \sqrt{(3\text{ m/s}^2)^2 + (1.5625\text{ m/s}^2)^2}$$

$$= 3.38\text{ m/s}^2 \qquad \textbf{Ans.}$$

3–53. Water flows into the drainpipe such that it only has a radial velocity component $V = (-3/r)$ m/s, where r is in meters. Determine the acceleration of a particle located at point $r = 0.5$ m, $\theta = 20°$. At $s = 0$, $r = 1$ m.



SOLUTION

Fig. a is based on the initial condition when $s = 0$, $r = r_D$. Thus, $r = 1 - s$. Then the radial component of velocity is

$$V = -\frac{3}{r} = \left(-\frac{3}{1-s}\right) \text{ m/s}$$

This is one dimensional steady flow since the velocity is along the straight radial line. The Eulerian description gives

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

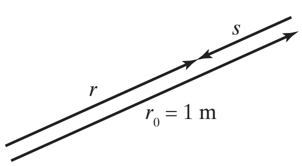
$$= 0 + \left(-\frac{3}{1-s}\right)\left[-\frac{3}{(1-s)^2}\right]$$

$$= \left[\frac{9}{(1-s)^3}\right] \text{ m/s}^2$$

When $1 - s = r = 0.5$ m, this equation gives

$$a = \left(\frac{9}{0.5^3}\right) \text{ m/s}^2 = 72 \text{ m/s}^2 \qquad \textbf{Ans.}$$

The positive sign indicates that a is directed towards positive s.
Note there is no normal component for motion along a straightline.

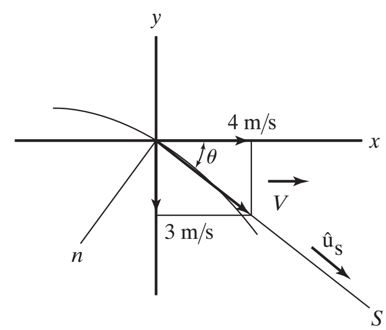


(a)

Ans:
72 m/s^2

3–54. A particle located at a point within a fluid flow has velocity components of $u = 4\text{ m/s}$ and $v = -3\text{ m/s}$, and acceleration components of $a_x = 2\text{ m/s}^2$ and $a_y = 8\text{ m/s}^2$. Determine the magnitude of the streamline and normal components of acceleration of the particle.

SOLUTION



$$V = \sqrt{(4\text{ m/s})^2 + (-3\text{ m/s})^2} = 5\text{ m/s}$$

$$a = \sqrt{(2\text{ m/s}^2)^2 + (8\text{ m/s}^2)^2} = 8.246\text{ m/s}^2$$

$$\mathbf{a} = 2\mathbf{i} + 8\mathbf{j}$$

$$\theta = \tan^{-1}\frac{3}{4} = 36.870°$$

$$u_s = \cos 36.870°\mathbf{i} - \sin 36.870°\mathbf{j}$$

$$= 0.8\mathbf{i} - 0.6\mathbf{j}$$

$$a_s = \mathbf{a}\cdot\mathbf{u}_s = (2\mathbf{i} + 8\mathbf{j})\cdot(0.8\mathbf{i} - 0.6\mathbf{j})$$

$$a_s = -3.20\text{ m/s}^2$$

$$a_s = 3.20\text{ m/s}^2 \qquad \textbf{Ans.}$$

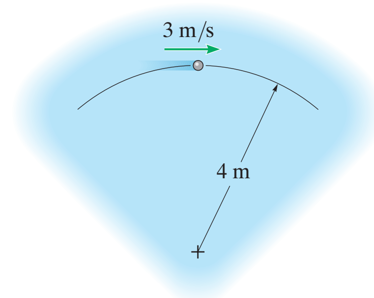
$$a = \sqrt{a_s^2 + a_n^2}$$

$$(8.246)^2 = (3.20)^2 + a_n^2$$

$$a_n = 7.60\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_s = 3.20\text{ m/s}^2$
$a_n = 7.60\text{ m/s}^2$

3–55. A particle moves along the circular streamline, such that it has a velocity of 2 m/s, which is increasing at 3 m/s^2. Determine the acceleration of the particle, and show the acceleration on the streamline.



SOLUTION

The normal component of the acceleration is

$$a_n = \frac{V^2}{\rho} = \frac{(3\ \text{m/s})^2}{4\ \text{m}} = 2.25\ \text{m/s}^2$$

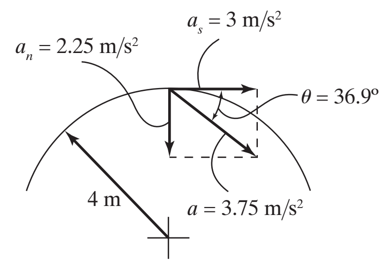
Thus, the magnitude of the acceleration is

$$a = \sqrt{a_s^2 + a_n^2} = \sqrt{(3\ \text{m/s}^2)^2 + (2.25\ \text{m/s}^2)^2} = 3.75\ \text{m/s}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{a_n}{a_s}\right) = \tan^{-1}\left(\frac{2.25\ \text{m/s}^2}{3\ \text{m/s}^2}\right) = 36.9^\circ \qquad \textbf{Ans.}$$

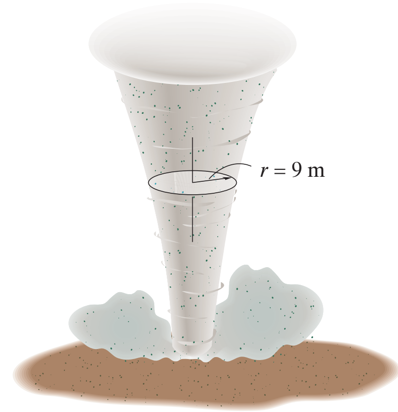
The plot of the acceleration on the streamline is shown in Fig. *a*.



(a)

Ans:
$a = 3.75\ \text{m/s}^2$
$\theta = 36.9^\circ$

***3–56.** The motion of a tornado can, in part, be described by a free vortex, $V = k/r$ where k is a constant. Consider the steady motion at the radial distance $r = 3$ m, where $V = 18$ m/s. Determine the magnitude of the acceleration of a particle traveling on the streamline having a radius of $r = 9$ m.



SOLUTION

Using the condition at $r = 3$ m, $V = 18$ m/s.

$$V = \frac{k}{r}; \qquad 18\text{ m/s} = \frac{k}{3\text{ m}} \qquad k = 54\text{ m}^2/\text{s}$$

Then

$$V = \left(\frac{54}{r}\right)\text{ m/s}$$

At $r = 9$ m, $V = \left(\frac{54}{9}\right)$ m/s $= 6$ m/s. Since the velocity is constant, the streamline component of acceleration is

$$a_s = 0$$

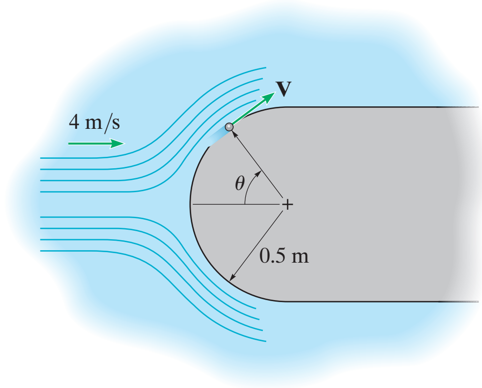
The normal component of acceleration is

$$a_n = \left(\frac{\partial V}{\partial t}\right)_n + \frac{V^2}{r} = 0 + \frac{(6\text{ m/s})^2}{9\text{ m}} = 4\text{ m/s}^2$$

Thus, the acceleration is

$$a = a_n = 4\text{ m/s}^2$$ **Ans.**

3–57. Air flows around the front circular surface. If the steady-steam velocity is 4 m/s upstream from the surface, and the velocity along the surface is defined by $V = (16 \sin \theta)$ m/s, determine the magnitude of the streamline and normal components of acceleration of a particle located at $\theta = 30°$.



SOLUTION

The streamline component of acceleration can be determined from

$$a_s = \left(\frac{\partial V}{\partial t}\right)_s + V\frac{\partial V}{\partial s}$$

However, $s = r\theta$. Thus, $\partial s = r\,\partial\theta = 0.5\,\partial\theta$. Also, the flow is steady, $\left(\frac{\partial V}{\partial t}\right)_s = 0$ and $\frac{\partial V}{\partial s} = \frac{\partial V}{0.5\partial\theta} = 2\frac{\partial V}{\partial\theta} = 2(16\cos\theta) = 32\cos\theta$. Then

$$a_s = 0 + 16\sin\theta(32\cos\theta) = 512\sin\theta\cos\theta = 256\sin 2\theta$$

When $\theta = 30°$,

$$a_s = 256\sin 2(30°) = 221.70\ \text{m/s}^2 = 222\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$V = (16\sin 30°)\ \text{m/s} = 8\ \text{m/s}$$

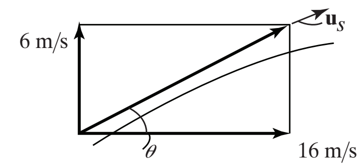
The normal component of acceleration can be determined from

$$a_n = \left(\frac{\partial V}{\partial t}\right)_n + \frac{V^2}{R} = 0 + \frac{(8\ \text{m/s})^2}{0.5\ \text{m}} = 128\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_s = 222\ \text{m/s}^2$
$a_n = 128\ \text{m/s}^2$

3–58. Fluid particles have velocity components of $u = (8y)$, $v = (6x)$ m/s, where x and y are in meters. Determine the streamline and the normal component of acceleration of a particle located at point (1 m, 2 m).

SOLUTION



At $x = 1$ m, $y = 2$ m

$$u = 8(2) = 16 \text{ m/s}$$

$$v = 6(1) = 6 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{6}{16} = 20.55°$$

$$u_s = \cos 20.55°\mathbf{i} + \sin 20.55°\mathbf{j}$$
$$= 0.9363\mathbf{i} + 0.3511\mathbf{j}$$

Acceleration. With $w = 0$, we have

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$
$$= 0 + 8y(0) + 6x(8) = 48x$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$
$$= 0 + 8y(6) + 6x(0) = 48y$$

At $x = 1$ m, and $y = 2$ m,

$$a_x = 48(1) = 48 \text{ m/s}^2$$
$$a_y = 48(2) = 96 \text{ m/s}^2$$

Therefore, the acceleration is

$$\mathbf{a} = \{48\mathbf{i} + 96\mathbf{j}\} \text{ m/s}^2$$

And its magnitude is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(48 \text{ m/s}^2)^2 + (96 \text{ m/s}^2)^2} = 107.33 \text{ m/s}^2$$

Since the direction of the s axis is defined by $\mathbf{u}_s$, the component of the particle's acceleration along the s axis can be determined from

$$a_s = \mathbf{a}\cdot\mathbf{u}_s = [48\mathbf{i} + 96\mathbf{j}]\cdot[0.9363\mathbf{i} + 0.3511\mathbf{j}]$$
$$= 78.65 \text{ m/s}^2 = 78.7 \text{ m/s}^2 \qquad \textbf{Ans.}$$

The normal component of the particle's acceleration is

$$a_n = \sqrt{a^2 - a_s^2} = \sqrt{(107.33 \text{ m/s}^2)^2 - (78.65 \text{ m/s}^2)^2} = 73.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_s = 78.7 \text{ m/s}^2$
$a_n = 73.0 \text{ m/s}^2$

3–59. Fluid particles have velocity components of $u = (8y)$ m/s and $v = (6x)$ m/s, where x and y are in meters. Determine the acceleration of a particle located at point (1 m, 1 m). Determine the equation of the streamline, passing through this point.

SOLUTION

Since the velocity components are independent of time, but a function of position, the flow can be classified as steady nonuniform. For two dimensional flow, $(w = 0)$, the Eulerian description is

$$a = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \\ &= 0 + 8y(0) + 6x(8) \\ &= 48x \\ a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \\ &= 0 + 8y(6) + 6x(0) \\ &= 48y \end{aligned}$$

At point $x = 1$ m and $y = 1$ m

$$a_x = 48(1) = 48\ \text{m/s}^2$$
$$a_y = 48(1) = 48\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(48\ \text{m/s}^2)^2 + (48\ \text{m/s}^2)^2} = 67.9\ \text{m/s}^2$$ **Ans.**

Its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{48\ \text{m/s}^2}{48\ \text{m/s}^2}\right) = 45^\circ \quad ⦣$$ **Ans.**

The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{6x}{8y} \tag{1}$$

$$\int_{1\,\text{m}}^{y} 8y\,dy = \int_{1\,\text{m}}^{x} 6x\,dx$$

$$4y^2\Big|_{1\,\text{m}}^{y} = 3x^2\Big|_{1\,\text{m}}^{x}$$

$$4y^2 - 3x^2 = 1$$ **Ans.**

Ans:
$a = 67.9\ \text{m/s}^2$
$\theta = 45^\circ$ ⦣
$4y^2 - 3x^2 = 1$

***3–60.** A fluid has velocity components of $u = (2y^2)$ m/s and $v = (8\,x\,y)$ m/s, where x and y are in meters. Determine the magnitude of the streamline and normal components of acceleration of a particle located at point (1 m, 2 m).

SOLUTION

$$\frac{dy}{dx} = \frac{r}{u}; \quad \frac{dy}{dx} = \frac{8xy}{2y^2} = \frac{4x}{y}$$

$$\int_{2\text{ m}}^{y} y\,dy = \int_{1\text{ m}}^{x} 4x\,dx$$

$$\left.\frac{y^2}{2}\right|_{2\text{ m}}^{y} = 2x^2\Big|_{1\text{ m}}^{x}$$

$$\frac{y^2}{2} - 2 = 2x^2 - 2$$

$$y^2 = 4x^2$$

$$y = 2x$$

(Note that $x = 1$, $y = 2$ is not a solution $y = -2x$.) Two equation from streamline is $y = 2x$ (A straight line). Thus, $R \rightarrow \infty$ and since the flow is steady,

$$a_n = \frac{V^2}{R} = 0 \qquad \textbf{Ans.}$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + 2y^2(0) + (8xy)(4y) = (32xy^2)\text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 2y^2(8y) + (8xy)(8x)$$

$$= (16y^3 + 64x^2y)\text{ m/s}^2$$

At (1 m, 2 m),

$$a_x = 32(1)(2^2) = 128\text{ m/s}^2$$

$$a_y = \left[16(2^3) + 64(1^2)(2)\right] = 256\text{ m/s}^2$$

$$a = a_s = \sqrt{a_x^2 + a_y^2} = \sqrt{(128\text{ m/s}^2)^2 + (256\text{ m/s}^2)}$$

$$a_s = 286\text{ m/s}^2 \qquad \textbf{Ans.}$$

3–61. A fluid has velocity components of $u = (2y^2)$ m/s and $v = (8x\,y)$ m/s, where x and y are in meters. Determine the magnitude of the streamline and normal components of the acceleration of a particle located at point (1 m, 1 m). Find the equation of the streamline passing through this point, and sketch the streamline and normal components of the velocity and acceleration at point.

SOLUTION

x(m)	$\sqrt{3}/2$	1.0	2.0	3.0	4.0	5.0
y(m)	0	1.0	3.61	5.74	7.81	9.85

Since the velocity component is independent of time and is a function of position, the flow can be classified as steady nonuniform. The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{8xy}{2y^2} = \frac{4x}{y}$$

$$\int_{1\,\text{m}}^{y} y\,dy = \int_{1\,\text{m}}^{x} 4x\,dx$$

$$\left.\frac{y^2}{2}\right|_{1\,\text{m}}^{y} = \left.2x^2\right|_{1\,\text{m}}^{x}$$

$$4x^2 - y^2 = 3 \text{ where } x \text{ and } y \text{ are in } m \qquad \textbf{Ans.}$$

For two dimensional flow ($w = 0$), the Eulerian description is

$$a = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (2y^2)(0) + (8xy)(4y)$$

$$= (32xy^2)\ \text{m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (2y^2)(8y) + (8xy)(8x)$$

$$= (16y^3 + 64x^2y)\ \text{m/s}^2$$

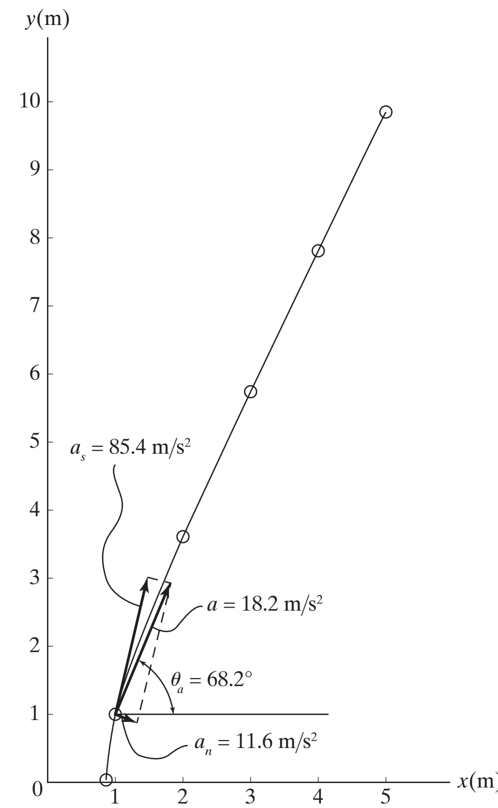
At point $x = 1$ m and $y = 1$ m

$$a_x = 32(1)(1^2) = 32\ \text{m/s}^2$$

$$a_y = 16(1^3) + 64(1^2)(1) = 80\ \text{m/s}^2$$

Thus,

$$a = \{32\mathbf{i} + 80\mathbf{j}\}\ \text{m/s}^2$$



(a)

3–61. (continued)

The magnitude of the acceleration is

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}} = \sqrt{\left(32\text{ m/s}^2\right)^2 + \left(80\text{ m/s}^2\right)} = 86.16\text{ m/s}^2 = 18.2\text{ m/s}^2$$

and its direction is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{80\text{ m/s}^2}{32\text{ m/s}^2}\right) = 68.2°$$

At point (1 m, 1 m),

$$\tan\theta = \left.\frac{dy}{dx}\right|_{\substack{x=1\text{ m}\\ y=1\text{ m}}} = \frac{4(1)}{1} = 4;\ \theta = 75.96°$$

Thus, the unit vector along the streamline is

$$u = \cos 75.96°\mathbf{i} + \sin 75.96°\mathbf{j} = 0.2425\mathbf{i} + 0.9701\mathbf{j}$$

Thus, the streamline component of the acceleration is

$$a_s = \mathbf{a}\cdot\mathbf{u}_s = (32\mathbf{i} + 80\mathbf{j})\cdot(0.2425\mathbf{i} + 0.9701\mathbf{j})$$

$$= 85.37\text{ m/s}^2 = 85.4\text{ m/s}^2 \qquad \textbf{Ans.}$$

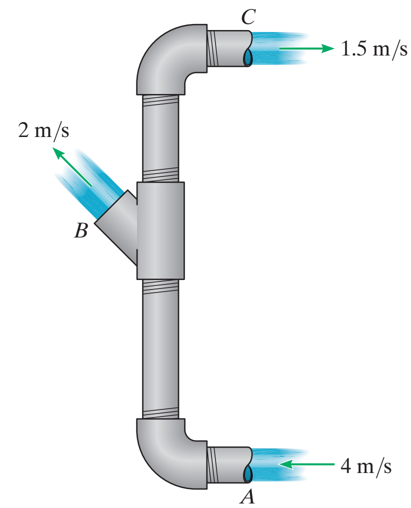
Then

$$a_n = \sqrt{a^2 - a_s^{\,2}} = \sqrt{\left(86.16\text{ m/s}^2\right)^2 - \left(85.37\text{ m/s}^2\right)^2}$$

$$= 11.64\text{ m/s}^2 = 11.6\text{ m/s}^2 \qquad \textbf{Ans.}$$

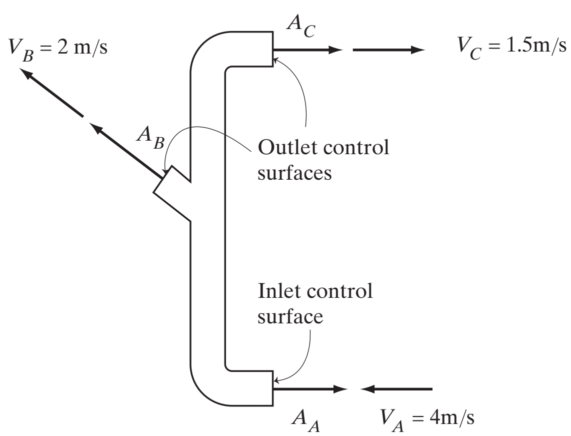
Ans:
$4x^2 - y^2 = 3$
$a_s = 85.4\text{ m/s}^2$
$a_n = 11.6\text{ m/s}^2$

4–1. Water flows steadily through the pipes with the average velocities shown. Outline the control volume that contains the water in the pipe system. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

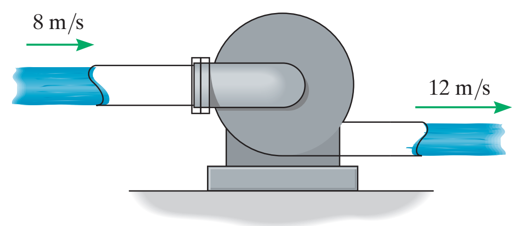


SOLUTION

Since the flow is steady, no local change occurs. However, the water flows in and out of the control volume through the open (inlet and outlet) control surfaces. Thus, convective changes take place.

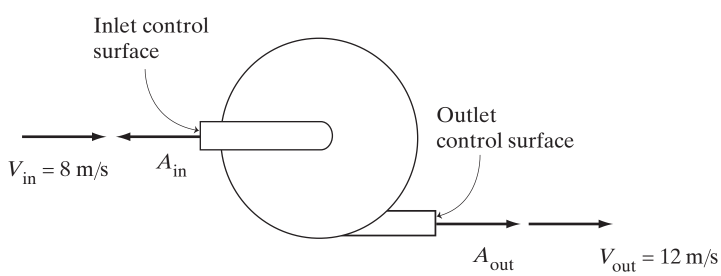


4–2. Water is drawn steadily through the pump. The average velocities are indicated. Select a control volume that contains the water in the pump and extends slightly past it. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.



SOLUTION

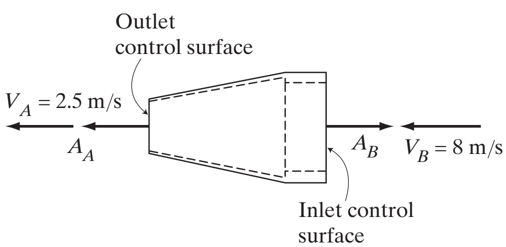
Since the flow is steady, there is no local change. However, the water flows in and out of the control volume through the open (inlet and outlet) control surfaces. Thus, convective changes take place.

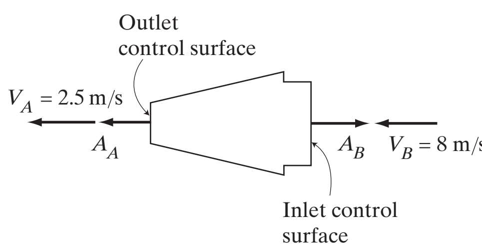


4–3. The average velocities of water flowing steadily through the nozzle are indicated. If the nozzle is glued onto the end of the hose, outline the control volume to be the entire nozzle and the water inside it. Also, select another control volume to be just the water inside the nozzle. In each case, indicate the open control surfaces, and show the positive direction of their areas. Specify the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

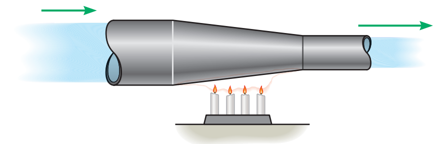
SOLUTION

Since the flow is steady, no local change occurs. However, the water flows in and out of the control volume through the opened (inlet and outlet) control surfaces. Thus, convective changes take place.



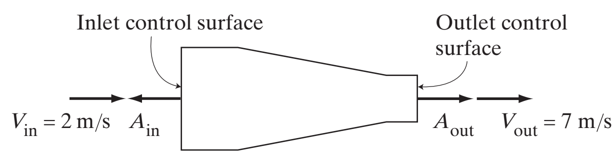


***4–4.** Air flows through the tapered duct, and during this time heat is being added that changes the density of the air within the duct. The average velocities are indicated. Select a control volume that contains the air in the duct. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume the air is incompressible.

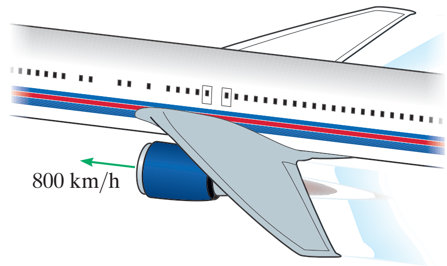


SOLUTION

Since the density of the air within the control volume changes with time, local changes occur. Also, air flows/in and out of the control volume through the opened (inlet and outlet) control surfaces. This causes a convective change to take place.

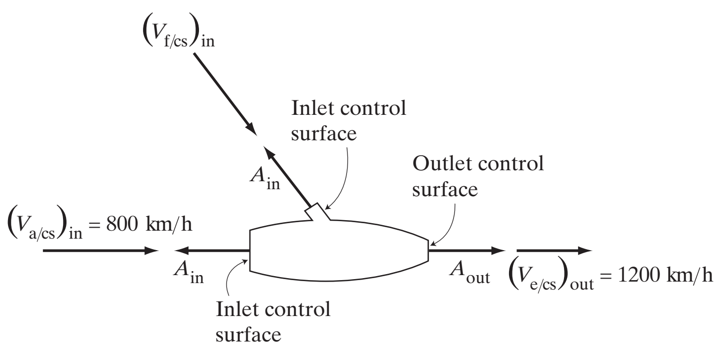


4–9. The jet engine is moving forward with a constant speed of 800 km/h. Fuel from a tank enters the engine and is mixed with the intake air, burned, and exhausted with an average relative velocity of 1200 km/h. Outline the control volume as the jet engine and the air and fuel within it. For an analysis, why is it best to consider this control volume to be moving? Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the magnitudes of the relative velocities and their directions through these surfaces. Identify the local and convective changes that occur. Assume the fuel is incompressible and the air is compressible.

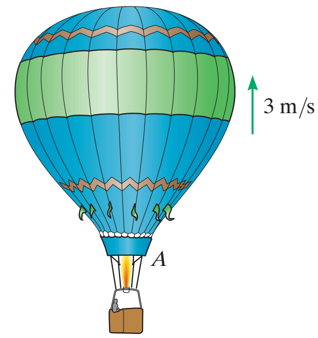


SOLUTION

The control volume is considered moving, so that the Reynolds transport theorem can be applied using relative velocities since the masses are conserved within the control volume the flow is steady, no local changes occur. Also, mass flows in and out from the control volume through the open (inlet and outlet) control surfaces. This causes convective changes to take place.

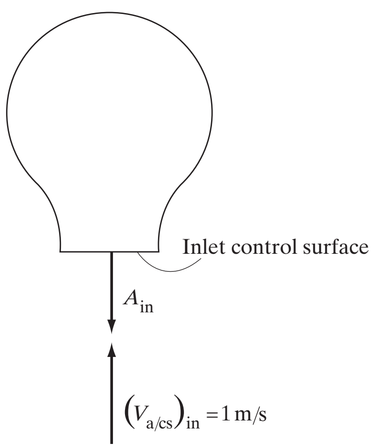


4–10. The balloon is rising at a constant velocity of 3 m/s. Hot air enters from a burner and flows into the balloon at A at an average velocity of 1 m/s, measured relative to the balloon. For an analysis, why is it best to consider the control volume as moving? Outline this moving control volume that contains the air in the balloon. Indicate the open control surface, and show the positive direction of its area. Also, indicate the magnitude of the velocity and its direction through this surface. Identify the local and convective changes that occur. Assume the air to be incompressible.

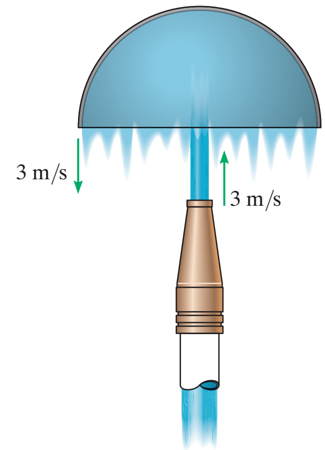


SOLUTION

The control volume is considered moving so that the Reynolds transport theorem can be applied using relative velocities. Since the volume of the control volume (ballon) changes with time, local changes occur. Also, the air flows in the control volume through the opened (inlet) control surface. This causes convective changes to take place.

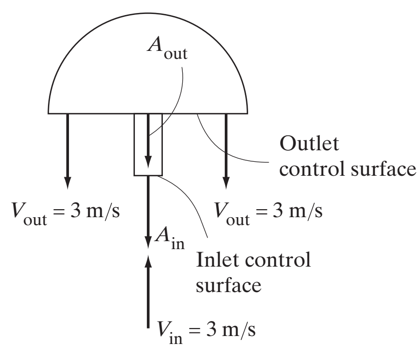


4–11. The hemispherical bowl is suspended in the air by the water stream that flows into and then out of the bowl at the average velocities indicated. Outline a control volume that contains the bowl and the water entering and leaving it. Indicate the open control surfaces, and show the positive direction of their areas. Also, indicate the direction of the velocities through these surfaces. Identify the local and convective changes that occur. Assume water to be incompressible.

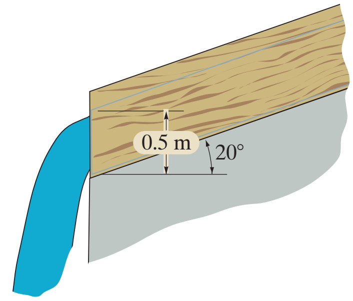


SOLUTION

Since the flow is steady, there is no local change. However, the water flows in and out of the control volume through the open (inlet and outlet) control surface, thus convective changes take place.



***4–12.** Water flows along a rectangular channel having a width of 0.75 m. If the average velocity is 2 m/s, determine the volumetric discharge.



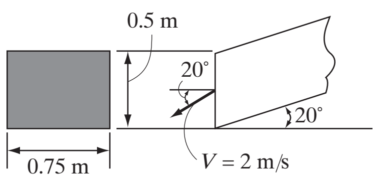
SOLUTION

The vertical cross section of the flow of area $A = (0.75\text{ m})(0.5\text{ m}) = 0.375\text{ m}^2$ is shown shaded in Fig. *a*. The component of velocity perpendicular to this cross section is $V_\perp = (2\text{ m/s})\cos 20° = 1.8794\text{ m/s}$. Thus,

$$Q = \mathbf{V}\cdot\mathbf{A}$$

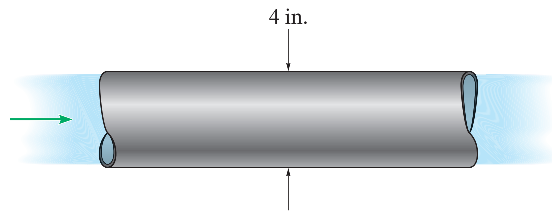
$$= (1.8794\text{ m/s})(0.375\text{ m}^2)$$

$$= 0.705\text{ m}^3/\text{s}$$ **Ans.**



(a)

***4–16.** Carbon dioxide gas flows through the 4-in.-diameter duct. If it has an average velocity of 10 ft/s and the gage pressure is maintained at 8 psi, plot the variation of mass flow (vertical axis) versus temperature for the temperature range $0°\text{F} \le T \le 100°\text{F}$. Give values for increments of $\Delta T = 20°$.



SOLUTION

From Appendix A, $R = 1130\ \text{ft}\cdot\text{lb}/(\text{slug}\cdot\text{R})$ for CO_2. Here the absolute pressure is $p = p_{\text{atm}} + p_g = 14.7\text{ psi} + 8\text{ psi} = \left(22.7\dfrac{\text{lb}}{\text{in}^2}\right)\left(\dfrac{12\text{ in.}}{1\text{ ft}}\right)^2 = 3268.8\ \text{lb}/\text{ft}^2$ and $T = T_F + 460$.

$$p = \rho R T$$

$$3268.8\ \text{lb}/\text{ft}^2 = \rho\left(1130\ \text{ft}\cdot\text{lb}/(\text{slug}\cdot\text{R})\right)(T_F + 460)$$

$$\rho = \left(\frac{2.8927}{T_F + 460}\right)\text{slug}/\text{ft}^3$$

The mass flow is

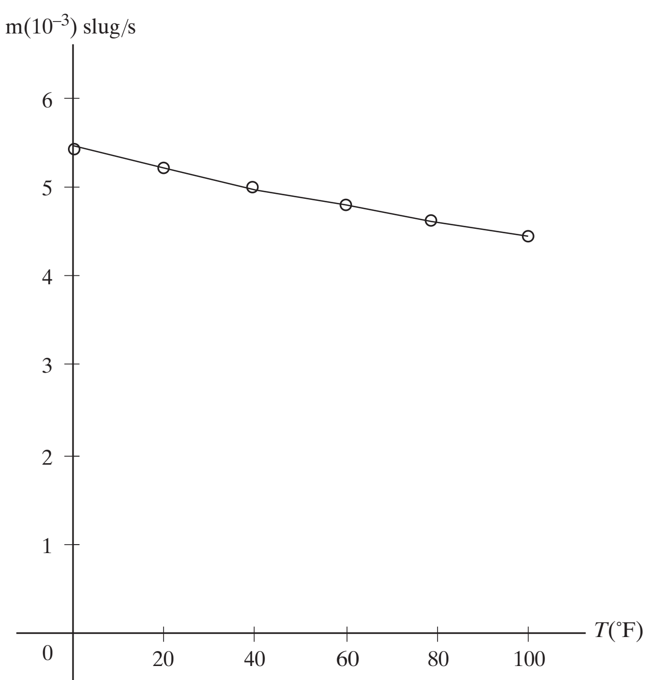
$$\dot{m} = \rho V A$$

$$\dot{m} = \left[\left(\frac{2.8927}{T_F + 460}\right)\text{slug}/\text{ft}^3\right](10\ \text{ft/s})\left[\pi\left(\frac{2}{12}\text{ft}\right)^2\right]$$

$$\dot{m} = \left(\frac{2.5244}{T_F + 460}\right)\text{slug/s where } T_F \text{ is in °F.}$$

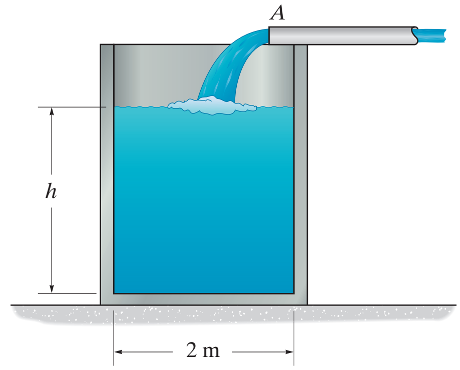
The plot of $\dot{m}$ vs T_F is shown in Fig. *a*.

T(°F)	0	20	40	60	80	100
$\dot{m}\,(10^{-3})$ slug/s	5.49	5.26	5.05	4.85	4.67	4.51



(a)

4–17. Water flowing at a constant rate fills the tank to a height of $h = 3$ m in 5 minutes. If the tank has a width of 1.5 m, determine the average velocity of the flow from the 0.2-m-diameter pipe at A.



SOLUTION

The volume of water in the tank when $t = 5(60)\text{ s} = 300\text{ s}$ is

$$\forall = (3\text{ m})(2\text{ m})(1.5\text{ m}) = 9\text{ m}^3$$

Thus, the discharge through the pipe at A is

$$Q = \frac{\forall}{t} = \frac{9\text{ m}^3}{300\text{ s}} = 0.03\text{ m}^3/\text{s}$$

Then, the average velocity of the water flow through A is

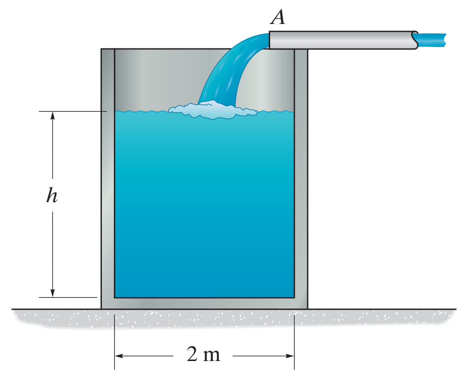
$$Q = VA$$

$$0.03\text{ m}^3/\text{s} = V(\pi)(0.1\text{ m})^2$$

$$V = 0.955\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
0.955 m/s

4–18. Water flows through the pipe at a constant average velocity of 0.5 m/s. Determine the relation between the time needed to fill the tank to a depth of $h = 3$ m and the diameter D of the pipe at A. The tank has a width of 1.5 m. Plot the time in minutes (vertical axis) versus the diameter $0.05 \text{ m} \le D \le 0.25 \text{ m}$. Give values for increments of $\Delta D = 0.05$ m.



SOLUTION

The volume of water filled to $h = 3$ m in time t is

$$\forall = (2 \text{ m})(1.5 \text{ m})(3 \text{ m}) = 9 \text{ m}^3$$

Thus, the discharge through the pipe is

$$Q = \frac{\forall}{t} = \left(\frac{9}{t}\right) \text{m}^3/\text{s}$$

Applying

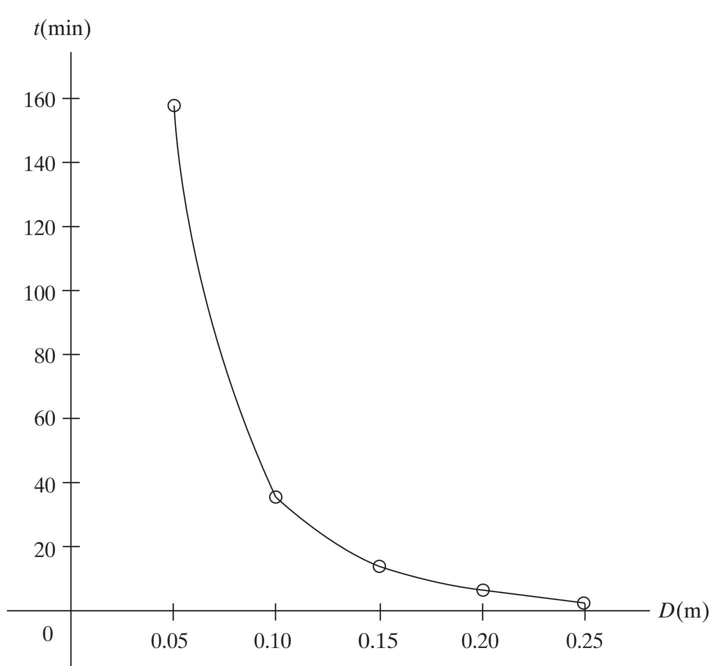
$$Q = VA; \qquad \frac{9}{t} = (0.5 \text{ m/s})\left[\frac{\pi}{4}(D^2)\right]$$

$$t = \left[\left(\frac{22.9183}{D^2}\right)\text{s}\right]\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$t = \left(\frac{0.382}{D^2}\right) \text{min where } d \text{ is in } m$$ **Ans.**

The plot of t vs D is shown in Fig. a

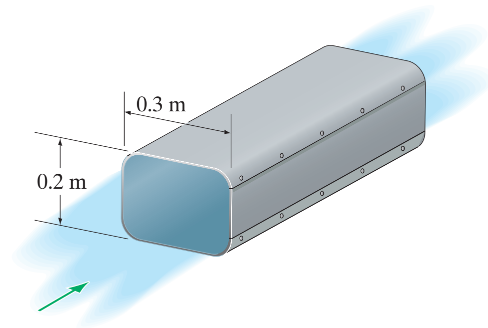
D(m)	0.05	0.10	0.15	0.20	0.25
t(min)	153	38.2	17.0	9.55	6.11



(a)

Ans:
$t = \left(\frac{0.382}{D^2}\right)$ min, where d is in m

4–19. Determine the mass flow of air in the duct if it has an average velocity of 15 m/s. The air has a temperature of 30°C, and the (gage) pressure is 50 kPa.



SOLUTION

From Appendix A, $R = 286.9\ \mathrm{J/kg \cdot K}$ for air.

$$p = \rho R T$$

$$(50 + 101)\left(10^3\right)\mathrm{N/m^2} = \rho(286.9\ \mathrm{J/(kg \cdot K)}(30^\circ + 273)$$

$$\rho = 1.737\ \mathrm{kg/m^3}$$

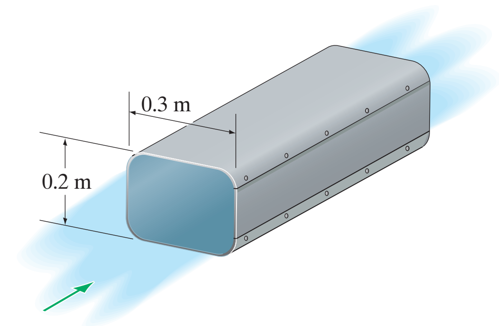
$$\dot{m} = \rho V A$$

$$= \left(1.737\ \mathrm{kg/m^3}\right)(15\ \mathrm{m/s})(0.3\ \mathrm{m})(0.2\ \mathrm{m})$$

$$= 1.56\ \mathrm{kg/s}$$ **Ans.**

Ans:
1.56 kg/s

***4–20.** Air flows through the duct at an average velocity of 20 m/s. If the temperature is maintained at 20°C, plot the variation of the mass flow (vertical axis) versus the (gage) pressure for the range of $0 \le p \le 100$ kPa. Give values for increments of $\Delta p = 20$ kPa. The atmospheric pressure is 101.3 kPa.



SOLUTION

From Appendix A, $R = 286.9\ \mathrm{J/(kg \cdot K)}$ for air. Here, the absolute pressure is $p = p_g + p_{\text{atm}} = (p_g + 101.3)$ kPa

$$p = \rho R T$$

$$(p_g + 101.3)(10^3)\ \mathrm{N/m^2} = \rho(286.9\ \mathrm{J/(kg \cdot K)})(20 + 273)\ \mathrm{K}$$

$$\rho = \left[0.01190(p_g + 101.3)\right]\ \mathrm{kg/m^3}$$

The mass flow is

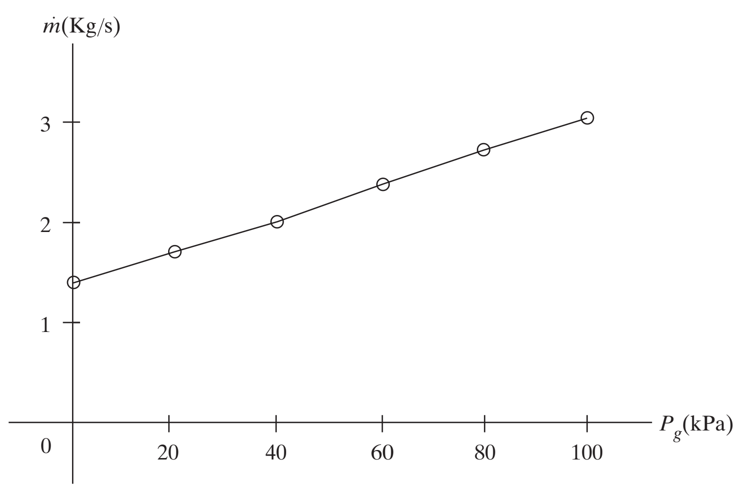
$$\dot{m} = \rho V A$$

$$\dot{m} = \left[0.01190(p_g + 101.3)\mathrm{kg/m^3}\right](20\ \mathrm{m/s})\left[0.3\ \mathrm{m}(0.2\ \mathrm{m})\right]$$

$$\dot{m} = [0.01428(p_g + 101.3)]\mathrm{kg/s} \text{ where } p_g \text{ is in kPa}$$

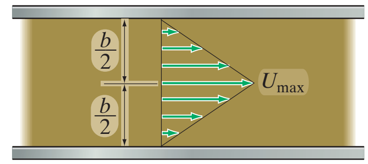
The plot of $\dot{m}$ vs p_g is shown in Fig. *a*.

P_g(kPa)	0	20	40	60	80	100
$\dot{m}$(kg/s)	1.45	1.73	2.02	2.30	2.59	2.87



(a)

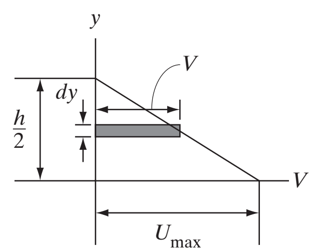
4–21. A fluid flowing between two plates has a velocity profile that is assumed to be linear as shown. Determine the average velocity and volumetric discharge in terms of U_{max}. The plates have a width of w.



SOLUTION

The velocity profile in Fig. a can be expressed as

$$\frac{-\ 0}{y - \frac{h}{2}} = \frac{U_{max} - 0}{0 - \frac{h}{2}}; \qquad = U_{max}\left(1 - \frac{2}{h}y\right)$$



(a)

The differential rectangular element of the thickness dy on the cross section will be considered. Thus, $dA = wdy$.

$$Q = \int_A = dA$$

$$= 2\int_0^{\frac{h}{2}}\left[U_{max}\left(1 - \frac{2}{h}y\right)\right](wdy)$$

$$= 2wU_{max}\int_0^{\frac{h}{2}}\left(1 - \frac{2}{h}y\right)dy$$

$$= 2wU_{max}\left(y - \frac{y^2}{h}\right)\Bigg|_0^{\frac{h}{2}}$$

$$= \frac{wU_{max}h}{2} \qquad \textbf{Ans.}$$

Also,

$$Q = \int_A \cdot d\mathbf{A} = \text{volume under velocity diagram}$$

$$= \frac{1}{2}(h)(w)(U_{max}) = \frac{wU_{max}h}{2} \qquad \textbf{Ans.}$$

Therefore,

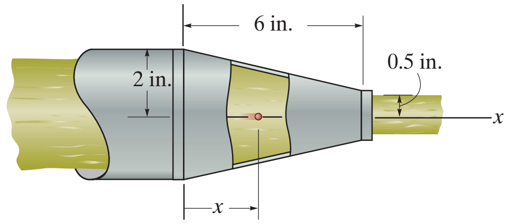
$$V = \frac{Q}{A} = \frac{wU_{max}h}{2(w)(h)} = \frac{U_{max}}{2} \qquad \textbf{Ans.}$$

Ans:

$$Q = \frac{wU_{max}h}{2}$$

$$V = \frac{U_{max}}{2}$$

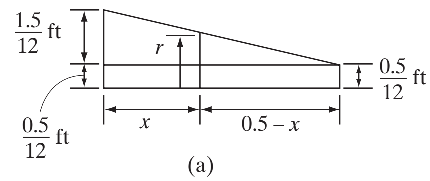
4–41. Acetate flows through the nozzle at $2\ \text{ft}^3/\text{s}$. Determine the time it takes for a particle on the x axis to pass through the nozzle, from $x = 0$ to $x = 6$ in. if $x = 0$ at $t = 0$. Plot the distance-versus-time graph for the particle.



SOLUTION

Since the flow is assumed to be one dimensional and incompressible, its velocity can be determined using

$$u = \frac{Q}{A}$$



(a)

From the geometry shown in Fig. *a*, the radius r of the nozzle's cross-section as a function of x is

$$r = \frac{0.5}{12}\ \text{ft} + \left(\frac{0.5 - x}{0.5}\right)\left(\frac{1.5}{12}\ \text{ft}\right) = \frac{1}{24}(4 - 6x)\ \text{ft}$$

Thus, the cross-sectional area of the nozzle is

$$A = \pi r^2 = \pi\left[\frac{1}{24}(4 - 6x)\right]^2 = \frac{\pi}{576}(4 - 6x)^2\ \text{ft}^2$$

Then

$$u = \frac{Q}{A} = \frac{2\ \text{ft}^3/\text{s}}{\frac{\pi}{576}(4 - 6x)^2\ \text{ft}^2} = \left[\frac{1152}{\pi(4 - 6x)^2}\right]\text{ft/s}$$

Using the definition of velocity and the initial condition of $x = 0$ at $t = 0$,

$$\frac{dx}{dt} = u = \frac{1152}{\pi(4 - 6x)^2}$$

$$\int_0^t dt = \frac{\pi}{1152}\int_0^x (4 - 6x)^2 dx$$

$$t = \frac{\pi}{1152}\int_0^x \left(36x^2 - 48x + 16\right)dx$$

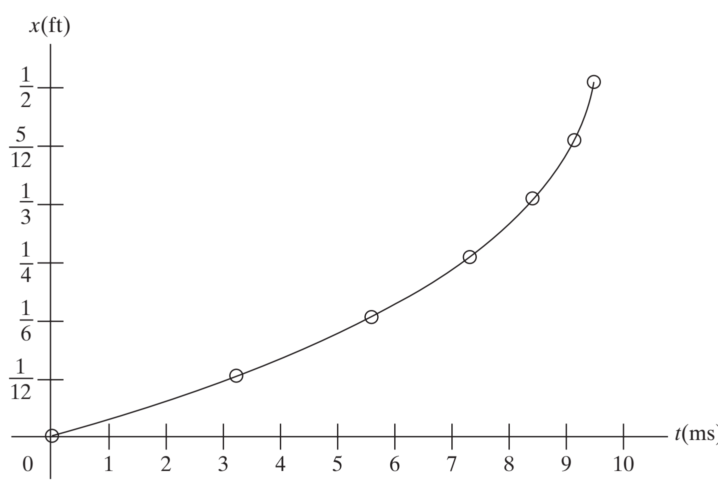
$$t = \frac{\pi}{1152}\left(12x^3 - 24x^2 + 16x\right)$$

$$t = \frac{\pi x}{288}\left(3x^2 - 6x + 4\right) \qquad \textbf{(1)}$$

When $x = \frac{6}{12}\ \text{ft} = 0.5\ \text{ft}$,

$$t = \frac{\pi(0.5)}{288}\left[3\left(0.5^2\right) - 6(0.5) + 4\right] = 9.5448\left(10^{-3}\right)$$

$$= 9.54\ \text{ms} \qquad \textbf{Ans.}$$



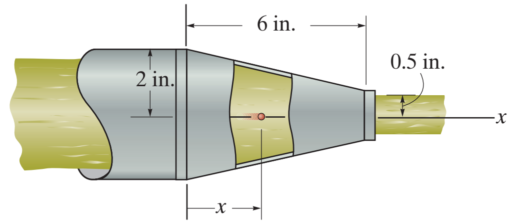
(b)

Using Eq (1), the following tabulation can be computed and the plot of x vs t is shown in Fig. *b*.

x(ft)	0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{12}$	$\frac{1}{2}$
t(ms)	0	3.20	5.61	7.33	8.48	9.18	9.54

Ans:
9.54 ms

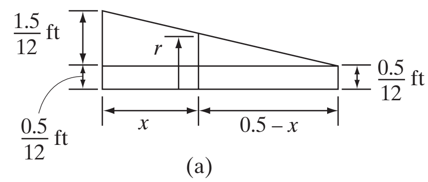
4–42. Acetate flows through the nozzle at $2\ \text{ft}^3/\text{s}$. Determine the velocity and acceleration of a particle on the x axis at $x = 3$ in. When $t = 0, x = 0$.



SOLUTION

Since the flow is assumed to be one dimensional and incompressible, its velocity can be determined using

$$u = \frac{Q}{A}$$



(a)

From the geometry shown in Fig. a, the radius r of the nozzle's cross-section as a function of x is

$$r = \frac{0.5}{12}\ \text{ft} + \left(\frac{0.5 - x}{0.5}\right)\left(\frac{1.5}{12}\ \text{ft}\right) = \frac{1}{24}(4 - 6x)\ \text{ft}$$

Thus,

$$A = \pi r^2 = \pi\left[\frac{1}{24}(4 - 6x)\right]^2 = \frac{\pi}{576}(4 - 6x)^2\ \text{ft}^2$$

Then,

$$u = \frac{Q}{A} = \frac{2\ \text{ft}^3/\text{s}}{\dfrac{\pi}{576}(4 - 6x)^2\ \text{ft}^2} = \left[\frac{1152}{\pi(4 - 6x)^2}\right]\ \text{ft/s}$$

Thus, when $x = \left(\frac{3}{12}\right)\ \text{ft} = 0.25\ \text{ft}$

$$u = \frac{1152}{\pi\left[4 - 6(0.25)\right]^2} = 58.67\ \text{ft/s} = 58.7\ \text{ft/s} \qquad \textbf{Ans.}$$

The acceleration can be determined using

$$a = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

Since Q is constant $\frac{\partial u}{\partial t} = 0$, that is, there is no local change at the print. Since,

$$\frac{\partial u}{\partial x} = \frac{1152}{\pi}\left[(-2)\left[(4 - 6x)^{-3}\right](-6)\right] = \frac{13824}{\pi}\left[\frac{1}{(4 - 6x)^3}\right]$$

Then

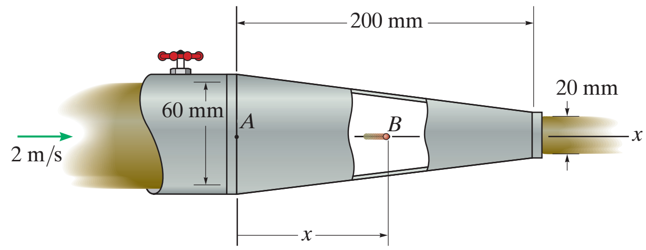
$$a = 0 + u\frac{\partial u}{\partial x}$$
$$= \frac{13824}{\pi}\left[\frac{u}{(4 - 6x)^3}\right]\ \text{ft/s}^2$$

When $x = \frac{2}{12}\ \text{ft} = 0.25\ \text{ft}, \quad u = 58.67\ \text{ft/s}$. Then

$$a = \frac{13824}{\pi}\left\{\frac{58.67}{\left[4 - 6(0.25)\right]^3}\right\} = 16523\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$u = 58.7\ \text{ft/s}, a = 16\,523\ \text{ft/s}^2$

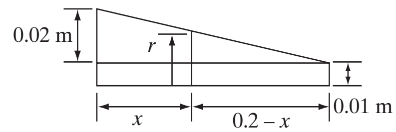
4–43. The tapered pipe transfers ethyl alcohol to a mixing tank such that a particle at A has a velocity of 2 m/s. Determine the velocity and acceleration of a particle at B, where $x = 75\text{ mm}$.



SOLUTION

From the geometry shown in Fig. a, the radius r of the pipe as a function of x is

$$r = 0.01\text{ m} + \left(\frac{0.2 - x}{0.2}\right)(0.02\text{ m}) = (0.03 - 0.1x)\text{ m}$$



(a)

Thus, the cross-sectional area of the pipe as a function of x is

$$A = \pi r^2 = \left[\pi(0.03 - 0.1x)^2\right]\text{ m}^2$$

The flow rate is constant which can be determined from

$$Q = u_A A_A = (2\text{ m/s})\left[\pi(0.03\text{ m})^2\right] = 1.8\pi\left(10^{-3}\right)\text{ m}^3/\text{s}$$

Thus, the velocity of the flow as a function of x is

$$u = \frac{Q}{A} = \frac{1.8\pi\left(10^{-3}\right)\text{ m}^3/\text{s}}{\pi(0.03 - 0.1x)^2\text{ m}^2} = \left[\frac{1.8\left(10^{-3}\right)}{(0.03 - 0.1x)^2}\right]\text{ m/s}$$

At $x = 0.075\text{ m}$,

$$u = \frac{1.8\left(10^{-3}\right)}{\left[0.03 - 0.1(0.075)\right]^2} = 3.556\text{ m/s} = 3.56\text{ m/s} \qquad \textbf{Ans.}$$

The acceleration of the flow can be determined using

$$a = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

Since the flow rate is constant there is no local changes, so that at the point, $\frac{\partial u}{\partial t} = 0$. Here

$$\frac{\partial u}{\partial x} = 1.8\left(10^{-3}\right)(-2)(0.03 - 0.1x)^{-3}(-0.1)$$

$$= \frac{0.36\left(10^{-3}\right)}{(0.03 - 0.1x)^3}$$

Thus

$$a = 0 + u\frac{\partial u}{\partial x}$$

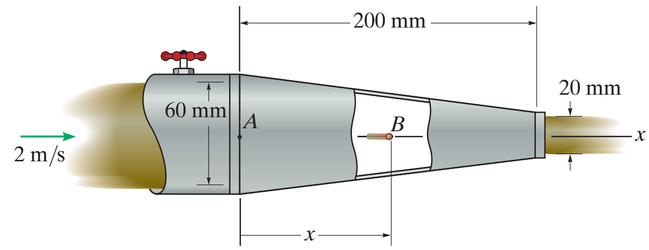
$$= \left[\frac{0.36\left(10^{-3}\right)u}{(0.03 - 0.1x)^3}\right]\text{ m/s}^2$$

When $x = 0.075\text{ m}$, $u = 3.556\text{ m/s}$. Then

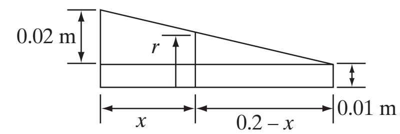
$$a = \frac{0.36\left(10^{-3}\right)(3.556)}{\left[0.03 - 0.1(0.075)\right]^3} = 112.37\text{ m/s}^2 = 112\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$u = 3.56\text{ m/s}$, $a = 112\text{ m/s}^2$

***4–44.** The tapered pipe transfers ethyl alcohol to a mixing tank such that when a valve is opened, a particle at A has a velocity at A of 2 m/s, which is increasing at 4 m/s^2. Determine the velocity of the same particle when it arrives at B, where $x = 75\text{ mm}$.



SOLUTION



(a)

From the geometry shown in Fig. a, the radius r of the pipe as a function of x is

$$r = 0.01\text{ m} + \left(\frac{0.2 - x}{0.2}\right)(0.02\text{ m}) = (0.03 - 0.1x)\text{ m}$$

Thus, the cross-sectional area of the pipe as a function of x is

$$A = \pi r^2 = \left[\pi(0.03 - 0.1x)^2\right]\text{ m}^2$$

The velocity of a particle passing through the cross-section at A can be determined from

$$\frac{du}{dt} = a; \quad \int_{2\text{ m/s}}^{u_A} du = \int_0^t 4\,dt$$

$$u_A - 2 = 4t$$

$$u_A = (4t + 2)\text{ m/s}$$

Thus, the flow is

$$Q = u_A A_A = \left[(4t + 2)\text{ m/s}\right]\left[\pi(0.03\text{ m})^2\right]$$

$$= \pi(0.0036t + 0.0018)\text{ m}^3/\text{s}$$

Then, the velocity of the flow as a function of t and x is

$$u = \frac{Q}{A} = \frac{\pi(0.0036t + 0.0018)\text{ m}^3/\text{s}}{\pi(0.03 - 0.1x)^2\text{ m}^2}$$

$$= \left[\frac{0.0036t + 0.0018}{(0.03 - 0.1x)^2}\right]\text{ m/s}$$

Using the definition of velocity,

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = \frac{0.0036t + 0.0018}{(0.03 - 0.1x)^2}$$

$$\int_0^x (0.03 - 0.1x)^2 dx = \int_0^t (0.0036t + 0.0018)dt$$

$$\int_0^x \left(0.01x^2 - 0.006x + 0.0009\right)dx = \int_0^t (0.0036t + 0.0018)dt$$

$$0.003333x^3 - 0.003x^2 + 0.0009x = 0.0018t^2 + 0.0018t$$

***4–44. Continued**

When $x = 0.075$ m,

$$0.003333\left(0.075^3\right) - 0.003\left(0.075^2\right) + 0.0009(0.075) = 0.0018t^2 + 0.0018t$$

$$t^2 + t - 0.02890625 = 0$$

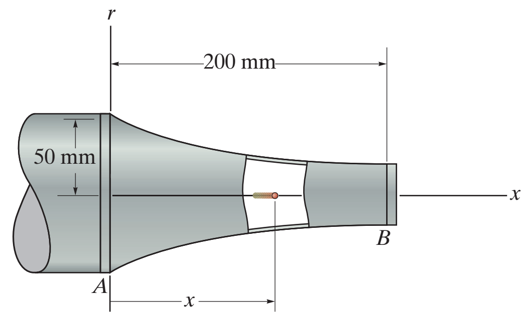
$$t = 0.02812 \text{ s}$$

When $x = 0.075$ m, $t = 0.02812$ s,

$$u = \frac{0.0036(0.02812) + 0.0018}{\left[0.03 - 0.1(0.075)\right]^2}$$

$$= 3.7555 \text{ m/s} = 3.76 \text{ m/s} \qquad \textbf{Ans.}$$

4–45. The radius of the circular duct varies as $r = \left(0.05e^{-3x}\right)$ m, where x is in meters. The flow of a fluid at A is $Q = 0.004\ \text{m}^3/\text{s}$ at $t = 0$, and it is increasing at $dQ/dt = 0.002\ \text{m}^3/\text{s}^2$. If a fluid particle is originally located at $x = 0$ when $t = 0$, determine the time for this particle to arrive at $x = 100$ mm.



SOLUTION

The discharge as a function of time t is

$$Q = 0.004\ \text{m}^3/\text{s} + \left(0.002\ \text{m}^3/\text{s}^2\right)t$$
$$= (0.004 + 0.002\,t)\ \text{m}^3/\text{s}$$

The cross-sectional Area of the duct as a function of x is

$$A = \pi r^2 = \pi\left(0.05e^{-3x}\right)^2 = \left(0.0025\pi e^{-6x}\right)\ \text{m}^2$$

Thus, the velocity of the flow is

$$u = \frac{Q}{A} = \frac{(0.004 + 0.002\,t)\ \text{m}^3/\text{s}}{\left(0.0025\pi e^{-6x}\right)\ \text{m}^2}$$
$$= \frac{1.6}{\pi}e^{6x} + \frac{0.8\,t}{\pi}e^{6x}$$
$$= \frac{4}{5\pi}e^{6x}(t + 2)$$

Using the definition of velocity,

$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = \frac{4}{5\pi}e^{6x}(t + 2)$$
$$\int_0^x \frac{dx}{e^{6x}} = \frac{4}{5\pi}\int_0^t (t + 2)dt$$
$$-\left(\frac{1}{6}e^{-6x}\right)\Bigg|_0^x = \frac{4}{5\pi}\left(\frac{t^2}{2} + 2t\right)\Bigg|_0^t$$
$$\frac{1}{6}\left(1 - e^{-6x}\right) = \frac{2}{5\pi}\left(t^2 + 4t\right)$$

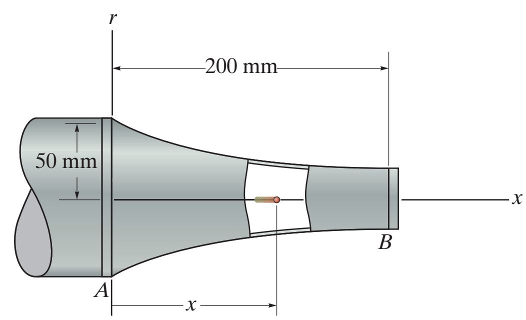
When $x = 0.1$ m,

$$\frac{1}{6}\left[1 - e^{-6(0.1)}\right] = \frac{2}{5\pi}\left(t^2 + 4t\right)$$
$$t^2 + 4t - 0.5906 = 0$$
$$t = \frac{-4 \pm \sqrt{4^2 - 4(1)(-0.5906)}}{2(1)} > 0$$

$$t = 0.143\ \text{s}$$ **Ans.**

Ans:
0.413 s

4–46. The radius of the circular duct varies as $r = \left(0.05e^{-3x}\right)$ m, where x is in meters. If the flow of the fluid at A is $Q = 0.004\ \text{m}^3/\text{s}$ at $t = 0$, and it is increasing at $dQ/dt = 0.002\ \text{m}^3/\text{s}^2$, determine the time for this particle to arrive at $x = 200$ mm.



SOLUTION

The discharge as a function of time t is

$$Q = 0.004\ \text{m}^3/\text{s} + \left(0.002\ \text{m}^3/\text{s}^2\right)t$$
$$= (0.004 + 0.002t)\ \text{m}^3/\text{s}$$

The cross-sectional area of the duct as a function of x is

$$A = \pi r^2 = \pi\left(0.05e^{-3x}\right)^2 = \left(0.0025\pi e^{-6x}\right)\ \text{m}^2$$

Thus, the velocity of the flow is

$$u = \frac{Q}{A} = \frac{(0.004 + 0.002t)\ \text{m}^3/\text{s}}{\left(0.0025\pi e^{-6x}\right)\ \text{m}^2}$$
$$= \frac{1.6}{\pi}e^{6x} + \frac{0.8t}{\pi}e^{6x}$$
$$= \frac{4}{5\pi}e^{6x}(t + 2)$$

Using the definition of velocity,

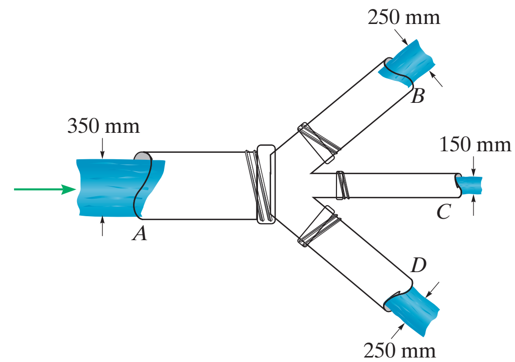
$$\frac{dx}{dt} = u; \qquad \frac{dx}{dt} = \frac{4}{5\pi}e^{6x}(t + 2)$$
$$\int_0^x \frac{dx}{e^{6x}} = \frac{4}{5\pi}\int_0^t (t + 2)dt$$
$$-\frac{1}{6}\left(e^{-6x}\right)\Big|_0^x = \frac{4}{5\pi}\left(\frac{t^2}{2} + 2t\right)\Big|_0^t$$
$$\frac{1}{6}\left(1 - e^{-6x}\right) = \frac{2}{5\pi}\left(t^2 + 4t\right)$$

At point E, $x = 0.2$ m.

$$\frac{1}{6}\left[1 - e^{-6(0.2)}\right] = \frac{2}{5\pi}\left(t^2 + 4t\right)$$
$$t^2 + 4t - 0.9147 = 0$$
$$t = \frac{-4 \pm \sqrt{4^2 - 4(1)(-0.9147)}}{2(1)} > 0$$
$$t = 0.217\ \text{s}$$ **Ans.**

Ans:
0.217 s

4–47. Water flows through the pipe at A at 300 kg/s, and then out the double wye with an average velocity of 3 m/s through B and an average velocity of 2 m/s through C. Determine the average velocity at which it flows through D.



SOLUTION

$$\dot{m} = \rho V_A A_A$$

$$300 \text{ kg/s} = \left(1000 \text{ kg/m}^3\right)(V_A)\left[\pi(0.175 \text{ m})^2\right]$$

$$V_A = 3.118 \text{ m/s}$$

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation. Since the fluid is water which has a constant density, then

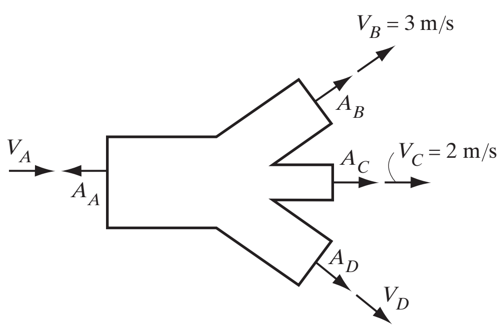
$$\frac{\partial}{\partial t}\int_{cv} \rho d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$-V_A A_A + V_B A_B + V_C A_C + V_D A_D = 0$$

$$-(3.118 \text{ m/s})\left[\pi(0.175 \text{ m})^2\right] + (3 \text{ m/s})\left[\pi(0.125 \text{ m})^2\right] + (2 \text{ m/s})\left[\pi(0.075 \text{ m})^2\right]$$

$$+ V_D\left[\pi(0.125 \text{ m})^2\right] = 0$$

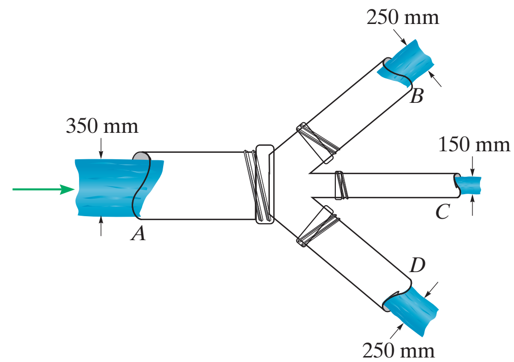
$$V_D = 2.39 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
2.39 m/s

*4–48. If water flows at 150 kg/s through the double wye at B, at 50 kg/s through C, and at 150 kg/s through D, determine the average velocity of flow through the pipe at A.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation. Since $\dot{m} = \rho V \cdot A$, then

$$\frac{\partial}{dt}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \dot{m}_A + \dot{m}_B + \dot{m}_C + \dot{m}_D = 0$$

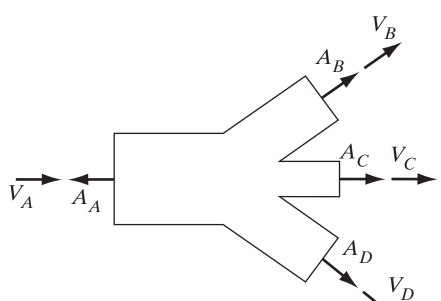
$$-\dot{m}_A + 150\text{ kg/s} + 50\text{ kg/s} + 150\text{ kg/s} = 0$$

$$\dot{m}_A = 350\text{ kg/s}$$

$$\dot{m}_A = \rho V_A A_A$$

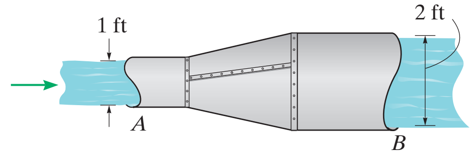
$$350\text{ kg/s} = \left(1000\text{ kg/m}^3\right)(V_A)\left[\pi(0.175\text{ m})^2\right]$$

$$V_A = 3.64\text{ m/s}$$ **Ans.**



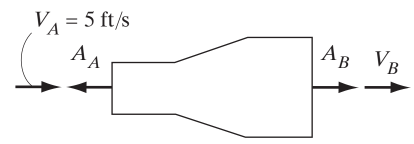
(a)

4–49. Air having a specific weight of 0.0795 lb/ft^3 flows into the duct at A with an average velocity of 5 ft/s. If its density at B is $0.00206 \text{ slug/ft}^3$, determine its average velocity at B.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.



(a)

Continuity Equation. The density of the air at A and B is different. The density at A is $\rho_A = \dfrac{\gamma_A}{g} = \dfrac{0.0795 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} = 0.002469 \text{ slug/ft}^3$. Then,

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

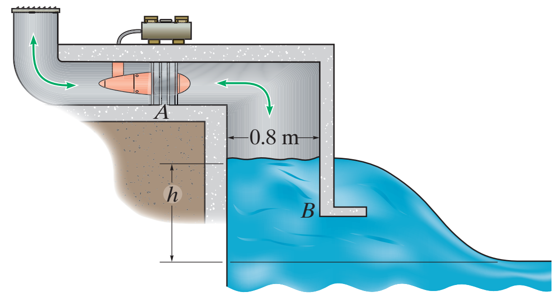
$$0 - \rho_A V_A A_A + \rho_B V_B A_B = 0$$

$$-\left(0.002469 \text{ slug/ft}^3\right)\left(5 \text{ ft/s}\right)\left[\pi(0.5 \text{ ft})^2\right] + \left(0.00206 \text{ slug/ft}^3\right)(V_B)\left[\pi(1 \text{ ft})^2\right] = 0$$

$$V_B = 1.50 \text{ ft/s}$$ **Ans.**

Ans:
1.50 ft/s

4–50. An oscillating water column (OWC), or gully generator, is a device for producing energy created by ocean waves. As noted, a wave will push water up into the air chamber, forcing the air to pass through a turbine, producing energy. As the wave falls back, the air is drawn into the chamber, reversing the rotational direction of the turbine, but still creating more energy. Assuming a wave will reach an average height of $h = 0.5$ m in the 0.8-m-diameter chamber at B, and it falls back at an average speed of 1.5 m/s determine the speed of the air as it moves through the turbine at A, which has a net area of 0.26 m^2. The air temperature at A is $T_A = 20°\text{C}$, and at B it is $T_B = 10°\text{C}$.



SOLUTION

Here, the control volume contains the air in the air chamber from A to B. Thus, it is a fixed control Volume,

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

Since the flow is steady and the volume of the control volume does not change with time, there are no local changes. Thus,

$$0 - \rho_A V_A A_A + \rho_B V_B A_B = 0$$

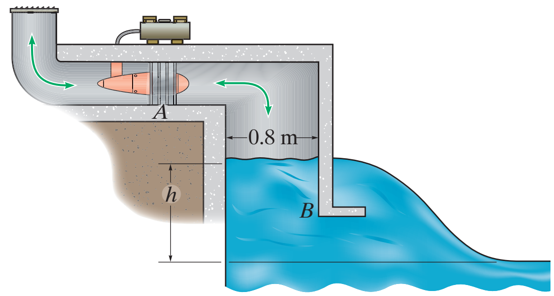
From Appendix A, at $T_A = 20°\text{C}$, $\rho_A = 1.202\ \text{kg/m}^3$ and at $T_B = 10°\text{C}$, $\rho_B = 1.247\ \text{kg/m}^3$. Thus,

$$-(1.202\ \text{kg/m}^3)(V_A)(0.26\ \text{m}^2) + (1.247\ \text{kg/m}^3)(1.5\ \text{m/s})\left[\pi(0.4\ \text{m})^2\right] = 0$$

$$V_A = 3.01\ \text{m/s}$$ **Ans.**

Ans:
3.01 m/s

4–51. An oscillating water column (OWC), or gully generator, is a device for producing energy created by ocean waves. As noted, a wave will push water up into the air chamber, forcing the air to pass through a turbine, producing energy. As the wave falls back, the air is drawn into the chamber, reversing the rotational direction of the turbine, but still creating more energy. Determine the speed of the air as it moves through the turbine at A, which has a net open area of $0.26\ \text{m}^2$, if the speed of the water in the 0.8-m diameter chamber is 5 m/s. The air temperature at A is $T_A = 20°\text{C}$, and at B it is $T_B = 10°\text{C}$.



SOLUTION

Here, the control volume contains the air in the air chamber from A to B. Thus, it is a fixed control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

Since the flow is steady and the volume of the control volume does not change with time, there are no local changes. Thus

$$0 + \rho_A V_A A_A + \rho_B(-V_B A_B) = 0$$

From Appendix A, at $T_A = 20°\text{C}$, $\rho_A = 1.202\ \text{kg/m}^3$ and at $T_B = 10°\text{C}$, $\rho_B = 1.247\ \text{kg/m}^3$. Thus,

$$(1.202\ \text{kg/m}^3)(V_A)(0.26\ \text{m}^2) + (1.247\ \text{kg/m}^3)\left\{-V_B\left[\pi(0.4\ \text{m})^2\right]\right\} = 0$$

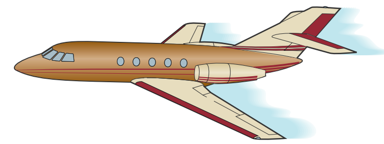
$$V_A = 2.0057\, V_B$$

Here, $V_B = 5\ \text{m/s}$, so that

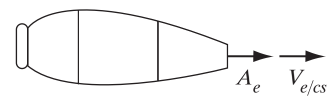
$$V_A = 2.0057(5\ \text{m/s}) = 10.0\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
10.0 m/s

***4–52.** A jet engine draws in air at 25 kg/s and jet fuel at 0.2 kg/s. If the density of the expelled air-fuel mixture is 1.356 kg/m^3, determine the average velocity of the exhaust relative to the plane. The exhaust nozzle has a diameter of 0.4 m.



SOLUTION



(a)

Control Volume. If the control volume moves with the plane, the flow is steady if viewed from the plane. No local changes occur within this control volume.

Continuity Equation. Since $\dot{m} = \rho \mathbf{V}_{f/cs} \cdot \mathbf{A}$, then

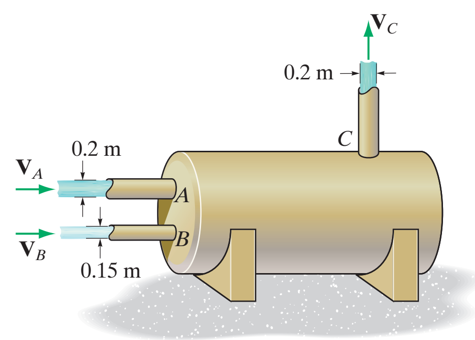
$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}_{f/cs} \cdot d\mathbf{A} = 0$$

$$0 - \dot{m}_a - \dot{m}_f + \rho V_{e/cs} A_e = 0$$

$$-25\text{ kg/s} - 0.2\text{ kg/s} + \left(1.356\text{ kg/m}^3\right)\left(V_{e/cs}\right)\left[\pi(0.2\text{ m})^2\right] = 0$$

$$V_{e/cs} = 148\text{ m/s} \qquad \textbf{Ans.}$$

4–53. Carbon dioxide flows into the tank at A at $V_A = 4\text{ m/s}$, and nitrogen flows in at B at $V_B = 3\text{ m/s}$. Both enter at a gage pressure of 300 kPa and a temperature of 250°C. Determine the steady mass flow of the mixed gas at C.



SOLUTION

From Appendix A, the values of the gas constants for CO_2 and nitrogen are $R_{CO_2} = 188.9\text{ J/kg}\cdot\text{K}$ and $R_N = 296.8\text{ J/kg}\cdot\text{K}$.

$$p = \rho RT$$

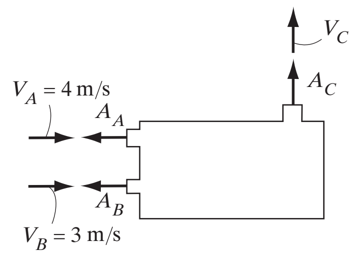
$$(300 + 101.3)(10^3) = \rho_{CO_2}(188.9\text{ J/kg}\cdot\text{K})(250° + 273)\text{ K}$$

$$\rho_{CO_2} = 4.062\text{ kg/m}^3$$

and

$$(300 + 101.3)(10^3) = \rho_N(296.8\text{ J/kg}\cdot\text{K})(250° + 273)\text{ K}$$

$$\rho_N = 2.585\text{ kg/m}^3$$



(a)

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation. Since the densities of the fluids through the open control surfaces are different but of constant value, then

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

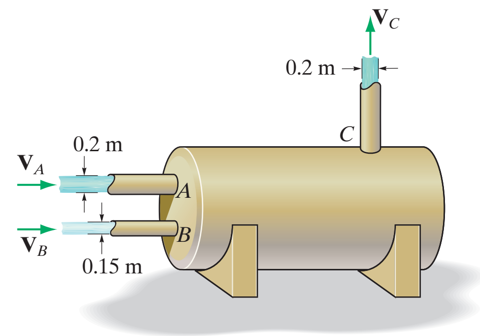
$$0 - \rho_{CO_2}V_A A_A - \rho_N V_B A_B + \dot{m}_m = 0$$

$$-(4.062\text{ kg/m}^3)(4\text{ m/s})\left[\pi(0.1\text{ m})^2\right] - (2.585\text{ kg/m}^3)(3\text{ m/s})\left[\pi(0.075\text{ m})^2\right] + \dot{m}_m = 0$$

$$\dot{m}_m = 0.647\text{ kg/s} \qquad \textbf{Ans.}$$

Ans:
0.647 kg/s

4–54. Carbon dioxide flows into the tank at A at $V_A = 10$ m/s, and nitrogen flows in at B with a velocity of $V_B = 6$ m/s. Both enter at a pressure of 300 kPa and a temperature of 250°C. Determine the average velocity of the mixed gas leaving the tank at a steady rate at C. The mixture has a density of $\rho = 1.546\ \text{kg/m}^3$.



SOLUTION

From Appendix A, the values of the gas constants for CO_2 and nitrogen are $R_{CO_2} = 188.9\ \text{J/kg}\cdot\text{K}$ and $R_N = 296.8\ \text{J/kg}\cdot\text{K}$.

$$p = \rho RT$$

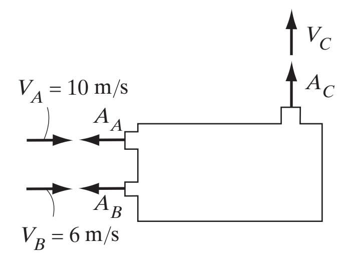
$$(300 + 101.3)(10^3) = \rho_{CO_2}(188.9\ \text{J/kg}\cdot\text{K})(250 + 273)\ \text{K}$$

$$\rho_{CO_2} = 4.062\ \text{kg/m}^3$$

and

$$(300 + 101.3)(10^3) = \rho_N(296.8\ \text{J/kg}\cdot\text{K})(250 + 273)\ \text{K}$$

$$\rho_N = 2.585\ \text{kg/m}^3$$



(a)

Control Volume. The fixed control volume is shown in Fig. a. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - \rho_{CO_2}V_A A_A - \rho_N V_B A_B + \rho_m V_C A_C = 0$$

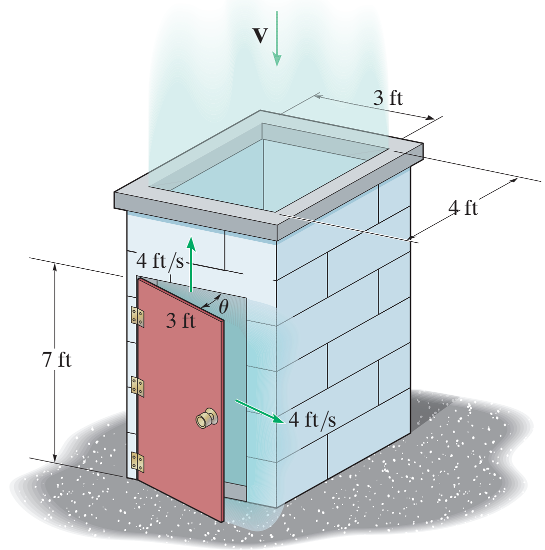
$$-(4.062\ \text{kg/m}^3)(10\ \text{m/s})\left[\pi(0.1\ \text{m})^2\right] - (2.585\ \text{kg/m}^3)(6\ \text{m/s})\left[\pi(0.075\ \text{m})^2\right]$$

$$+ (1.546\ \text{kg/m}^3)(V_C)\left[\pi(0.1\ \text{m})^2\right] = 0$$

$$V_C = 31.9\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
31.9 m/s

4–57. Pressurized air in a building well flows out through the partially opened door with an average velocity of 4 ft/s. Determine the average velocity of the air as it flows down from the top of the building well. Assume the door is 3 ft wide and $\theta = 30°$.



SOLUTION

The control volume contains the air in the building well and in the circular sector of the door opening. It can be considered fixed. Also, since in this case the air is assumed to be incompressible, the flow is steady, thus there is no local change. Here, the density of the air remains constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + (V_{out})_1(A_{out})_1 + (V_{out})_2(A_{out})_2 = 0 \qquad \textbf{(1)}$$

Here, the entrance open control surface is the cross-section of the building well above the door.

$$A_{in} = (4\text{ ft})(3\text{ ft}) = 12\text{ ft}^2$$

The exit open control surfaces are the top and side of the door opening.

$$(A_{out})_1 = \frac{1}{2}r^2\theta = \frac{1}{2}(3\text{ ft})^2\left(\frac{30°}{180°}\pi\text{ rad}\right) = 0.75\pi\text{ ft}^2$$

$$(A_{out})_2 = r\,\theta h = (3\text{ ft})\left(\frac{30°}{180°}\pi\text{ rad}\right)(7\text{ ft}) = 3.5\pi\text{ ft}^2$$

Substituting these values into Eq. (1),

$$-V_{in}(12\text{ ft}^2) + (4\text{ ft/s})(0.75\pi\text{ ft}^2) + (4\text{ ft/s})(3.5\pi\text{ ft}^2) = 0$$

$$V_{in} = 4.45\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
4.45 ft/s

4–58. Pressurized air in a building well flows out through the partially opened door with an average velocity of 4 ft/s. Determine the average velocity of the air as it flows down from the top of the building well as a function of the door opening θ. Plot this function of velocity (vertical axis) versus θ for $0° \le \theta \le 50°$. Give values for increments of $\Delta\theta = 10°$.

SOLUTION

The control volume contains the air in the building well and in the circular sector of the door opening. It can be considered fixed. Also, since in this case the air is assumed to be incompressible, the flow is steady, thus there is no local change. Here, the density of the air remains constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + (V_{out})_1(A_{out})_1 + (V_{out})_2(A_{out})_2 = 0 \qquad \textbf{(1)}$$

Here, the entrance open control surface is the cross-section of the building well above the door.

$$A_{in} = (4\text{ ft})(3\text{ ft}) = 12\text{ ft}^2$$

The exit open control surfaces are the top and side of the door opening.

$$(A_{out})_1 = \frac{1}{2}r^2\theta = \frac{1}{2}(3\text{ ft})^2\left(\frac{\theta}{180°}\pi\text{ rad}\right) = (0.025\pi\theta)\text{ ft}^2$$

$$(A_{out})_2 = r\theta h = (3\text{ ft})\left(\frac{\theta}{180°}\pi\text{ rad}\right)(7\text{ ft}) = (0.1167\pi\theta)\text{ ft}^2$$

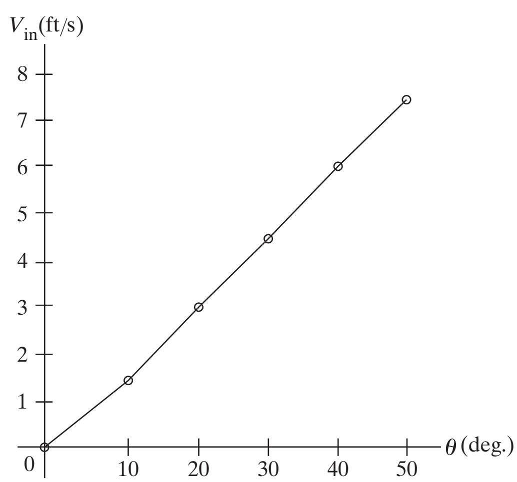
Substituting these values into Eq. (1),

$$-V_{in}(12\text{ ft}^2) + (4\text{ ft/s})(0.025\pi\theta\text{ ft}^2) + (4\text{ ft/s})(0.1167\pi\theta\text{ ft}^2) = 0$$

$$V_{in} = (0.0472\pi\theta)\text{ ft/s where } \theta \text{ is in degrees.} \qquad \textbf{Ans.}$$

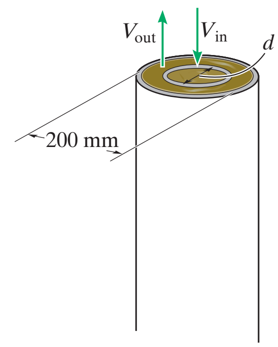
The plot of V_{in} vs θ is shown in Fig. *a*.

θ(deg.)	0	10	20	30	40	50
V_{in}(ft/s)	0	1.48	2.97	4.45	5.93	7.42



Ans:
$V_{in} = (0.0472\pi\theta)$ ft/s, where θ is in degrees

4–59. Drilling fluid is pumped down through the center pipe of a well and then rises up within the annulus. Determine the diameter d of the inner pipe so that the average velocity of the fluid remains the same in both regions. Also, what is this average velocity if the discharge is 0.02 m^3/s? Neglect the thickness of the pipes.



SOLUTION

The control volume considered is the volume of drilling fluid in pipe which is fixed. Here, the flow is steady, thus there are no local changes. Also, the density of the fluid is constant and the average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + V_{out}A_{out} = 0 \qquad \textbf{(1)}$$

Here, it is required that $V_{in} = V_{out}$. Also, $A_{in} = \frac{\pi}{4}d^2$ and $A_{out} = \frac{\pi}{4}(0.2\text{ m})^2 - \frac{\pi}{4}d^2$.

Then

$$-V\left(\frac{\pi}{4}d^2\right) + V\left[\frac{\pi}{4}(0.2\text{ m})^2 - \frac{\pi}{4}d^2\right] = 0$$

$$2d^2 = 0.04$$

$$d = 0.1414\text{ m} = 141\text{ mm} \qquad \textbf{Ans.}$$

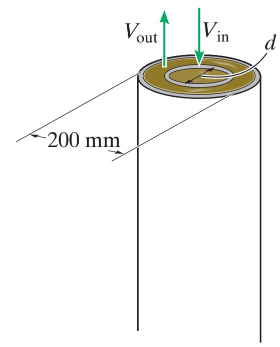
Considering the flow in the center pipe,

$$Q = VA; \quad 0.02\text{ m}^3/\text{s} = V\left[\frac{\pi}{4}(0.1414\text{ m})^2\right]$$

$$V = 1.27\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$d = 141\text{ mm}$
$V = 1.27\text{ m/s}$

***4–60.** Drilling fluid is pumped down through the center pipe of a well and then rises up within the annulus. Determine the velocity of the fluid forced out of the well as a function of the diameter d of the inner pipe, if the velocity of the fluid forced into the well is maintained at $V_{in} = 2$ m/s. Neglect the thickness of the pipes. Plot this velocity (vertical axis) versus the diameter for 50 mm $\leq d \leq$ 150 mm. Give values for increments of $\Delta d = 25$ mm.



SOLUTION

The control volume is the volume of the drilling fluid in the pipe which is fixed. Here, the flow is steady thus there is no local change. Also the density of the fluid is constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + V_{out}A_{out} = 0$$

Here, $$A_{in} = \frac{\pi}{4}\left(\frac{d}{1000}\right)^2 = 0.25(10^{-6})\pi\, d^2 \text{ and}$$

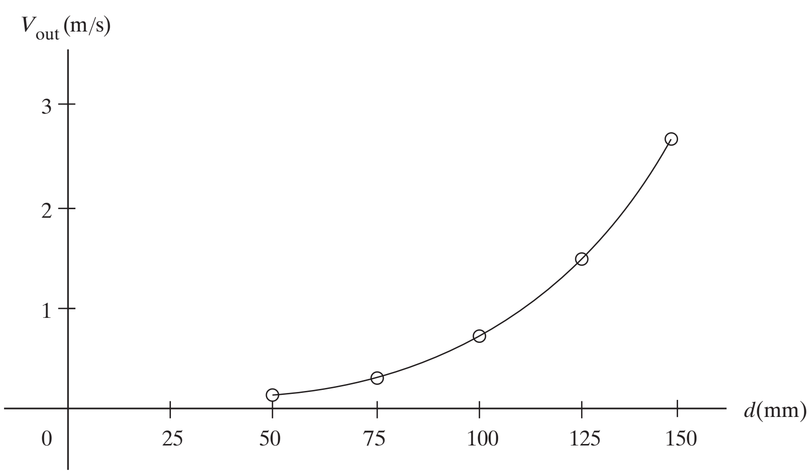
$$A_{out} = \frac{\pi}{4}\left[(0.2\text{ m})^2 - \left(\frac{d}{1000}\right)^2\right] = \frac{\pi}{4}\left[0.04 - (10^{-6})d^2\right]$$

$$-(2\text{ m/s})\left[0.25(10^{-6})\pi\, d^2\right] + V_{out}\left\{\frac{\pi}{4}\left[0.04 - (10^{-6})d^2\right]\right\} = 0$$

$$V_{out} = \left[\frac{2(10^{-6})d^2}{0.04 - (10^{-6})d^2}\right]\text{ m/s where } d \text{ is in mm} \qquad \textbf{Ans.}$$

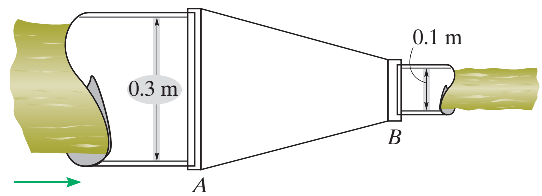
The plot of V_{out} vs d is shown in Fig. *a*.

d(mm)	50	75	100	125	150
V_{out}(m/s)	0.133	0.327	0.667	1.28	2.57



(a)

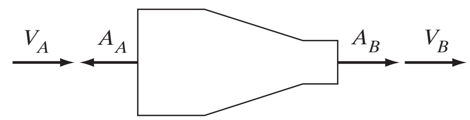
4–61. The unsteady flow of glycerin through the reducer is such that at A its velocity is $V_A = (0.8t^2)$ m/s, where t is in seconds. Determine its average velocity at B, and its average acceleration at A, when $t = 2$ s. The pipes have the diameters shown.



SOLUTION

When $t = 2$ s, the velocity of the flow at A is

$$V_A = 0.8(2)^2 = 3.20 \text{ m/s}$$



(a)

Control Volume. The fixed control volume is shown in Fig. *a*. Since the volume of the control volume does not change over time, no local changes occur within this control volume.

Continuity Equation. Since the water has a constant density, then

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(3.20 \text{ m/s})\left[\pi\left(\frac{0.3 \text{ m}}{2}\right)^2\right] + V_B\left[\pi\left(\frac{0.1 \text{ m}}{2}\right)^2\right] = 0$$

$$V_B = 28.8 \text{ m/s} \qquad \textbf{Ans.}$$

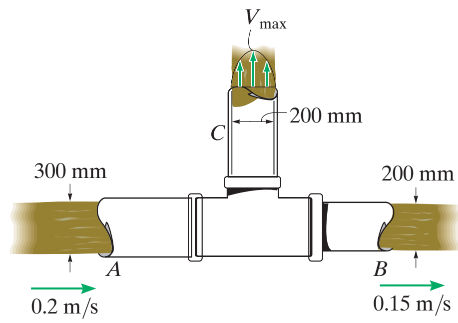
With $u = V_A$ and $v = w = 0$, we have

$$a_A = \frac{\partial V_A}{\partial t}$$

$$= 1.6t\big|_{t = 2\text{ s}} = 3.20 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$V_B = 28.8$ m/s
$a_A = 3.20$ m/s^2

4–62. Oil flows into the pipe at A with an average velocity of 0.2 m/s and through B with an average velocity of 0.15 m/s. Determine the maximum velocity V_{max} of the oil as it emerges from C if the velocity distribution is parabolic, defined by $v_C = V_{max}(1 - 100r^2)$, where r is in meters measured from the centerline of the pipe.



SOLUTION

The control volume considered is fixed as it contain the oil in the pipe. Also, the flow is steady and so no local changes occur. Here, the density of the oil is constant. Then

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

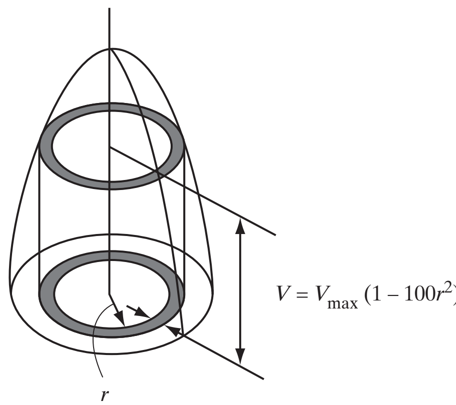
$$0 - V_A A_A + V_B A_B + \int_A V_c dA = 0$$

$$-(0.2 \text{ m/s})\left[\pi(0.15 \text{ m})^2\right] + (0.15 \text{ m/s})\left[\pi(0.1 \text{ m})^2\right]$$

$$+ \int_0^{0.1 \text{ m}} V_{max}(1 - 100r^2)(2\pi r dr) = 0$$

$$-3(10^{-3})\pi \text{m}^3 + 5(10^{-3})\pi V_{max} = 0$$

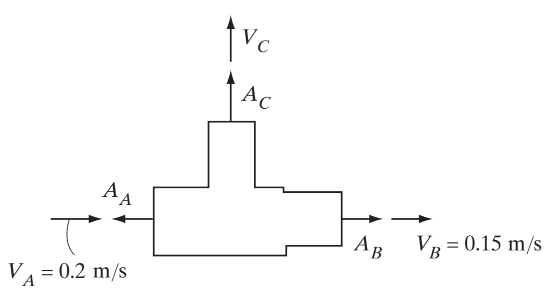
$$V_{max} = 0.6 \text{ m/s}$$ **Ans.**



(a)

Note: The integral in the above equation is equal to the volume under the velocity profile, while in this case is a paraboloid.

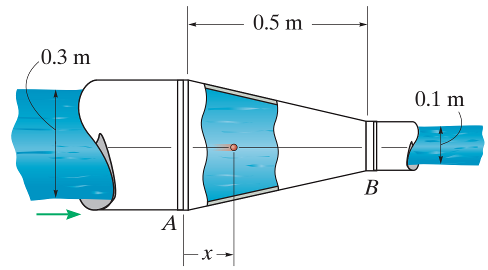
$$\int_A V_c dA = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(0.1 \text{ m})^2(V_{max}) = 5(10^{-3})\pi V_{max}.$$



(b)

Ans:
0.6 m/s

4–63. The unsteady flow of linseed oil is such that at A it has a velocity of $V_A = (0.7t + 4)$ m/s, where t is in seconds. Determine the acceleration of a fluid particle located at $x = 0.2$ m when $t = 1$ s. *Hint*: Determine $V = V(x, t)$, then use Eq. 3–4.

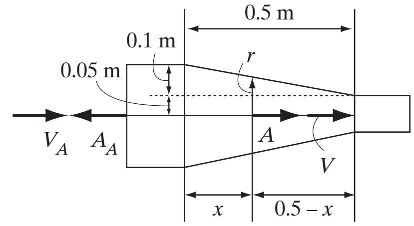


SOLUTION

Control Volume. The fixed control volume is shown in Fig. a. Since the volume does not change over time, no local changes occur within this control volume.

Continuity Equation. Referring to the geometry shown in Fig. a, the radius of the pipe at an arbitrary distance x is

$$\frac{r - 0.05}{0.1} = \frac{0.5 - x}{0.5}; \quad r = (0.15 - 0.2x)\text{ m}$$



(a)

Then,

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho d\forall + \int_{\text{cs}} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + VA = 0$$

$$-(0.7t + 4)\left[\pi(0.15\text{ m})^2\right] + V\left[\pi(0.15 - 0.2x)^2\right] = 0$$

$$V = \frac{0.0225(0.7t + 4)}{(0.15 - 0.2x)^2}$$

For A differential control volume at $x = 0.2$ m, with $u = V$ and $v = w = 0$,

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

$$= \frac{0.0225(0.7)}{(0.15 - 0.2x)^2} + \left(\frac{0.0225(0.7t + 4)}{(0.15 - 0.2x)^2}\right)\left(\frac{0.0225\left[(0.15 - 0.2x)^2\right](0) - (0.7t + 4)(2)(0.15 - 0.2x)(-0.2)\right]}{(0.15 - 0.2x)^4}\right)$$

$$= \frac{0.01575}{(0.15 - 0.2x)^2} + \left[\frac{0.0225(0.7t + 4)}{(0.15 - 0.2x)^2}\right]\left[\frac{0.009(0.7t + 4)}{(0.15 - 0.2x)^3}\right]$$

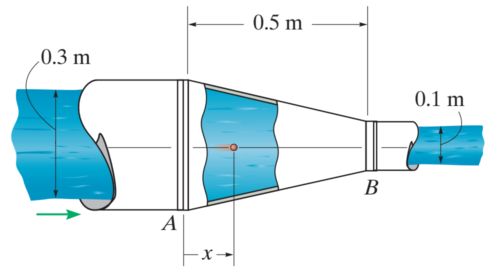
For $t = 1$ s, $x = 0.2$ m,

$$a = 1.3017 + 8.7397(31.781)$$

$$= 279\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
279 m/s^2

***4–64.** The unsteady flow of linseed oil is such that at A it has a velocity of $V_A = (0.4t^2)$ m/s, where t is in seconds. Determine the acceleration of a fluid particle located at $x = 0.25$ m when $t = 2$ s *Hint*: Determine $V = V(x, t)$, then use Eq. 3–4.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the volumes does not change over time, no local changes occur within this control volume.

Continuity Equation. Referring to the geometry shown in Fig. *a*, the radius of the pipe at an arbitrary distance x is

$$\frac{r - 0.05}{0.1} = \frac{0.5 - x}{0.5}; \quad r = (0.15 - 0.2x)\text{ m}$$

Then,

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + VA = 0$$

$$-(0.4t^2)\left[\pi(0.15\text{ m})^2\right] + V\left[\pi(0.15 - 0.2x)^2\right] = 0$$

$$V = \frac{0.009t^2}{(0.15 - 0.2x)^2}$$

For A differential control volume at $x = 0.25$ m, with $u = V$ and $v = w = 0$,

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

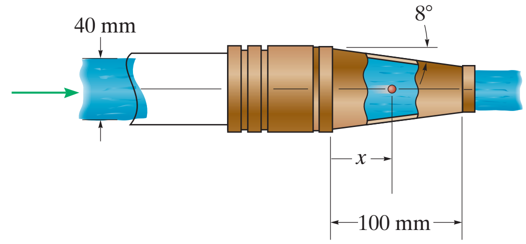
$$= \frac{0.018t}{(0.15 - 0.2x)^2} + \left[\frac{0.009t}{(0.15 - 0.2x)^2}\right]\left[\frac{(0.15 - 0.2x)^2(0) - (0.009t^2)(2)(0.15 - 0.2x)(-0.2)}{(0.15 - 0.2x)^4}\right]$$

$$= \frac{0.018t}{(0.15 - 0.2x)^2} + \left[\frac{0.009t^2}{(0.15 - 0.2x)^2}\right]\left[\frac{0.0036t^2}{(0.15 - 0.2x)^3}\right]$$

For $t = 2$ s, $x = 0.25$ m,

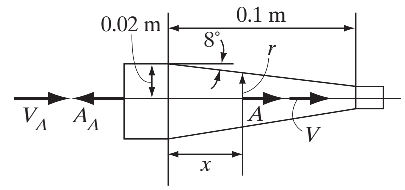
$$a = 3.6 + 3.6(14.4)$$
$$= 55.4\text{ m/s}^2$$ **Ans.**

4–65. Water flows through the nozzle at a rate of $0.2\ \text{m}^3/\text{s}$. Determine the velocity V of a particle as it moves along the centerline as a function of x.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.



(a)

Continuity Equation. From the geometry shown in Fig. *a*,

$$r = 0.02\ \text{m} - x\tan 8° = (0.02 - 0.1405x)\ \text{m}$$

Realizing that $Q_A = V_A A_A = 0.2\ \text{m}^3/\text{s}$,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - Q_A + VA = 0$$

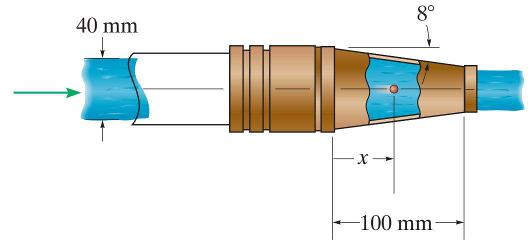
$$-0.2\ \text{m}^3/\text{s} + V\left[\pi(0.02 - 0.1405x)^2\right] = 0$$

$$V = \frac{0.0637}{(0.02 - 0.141x)^2} \qquad \textbf{Ans.}$$

Ans:

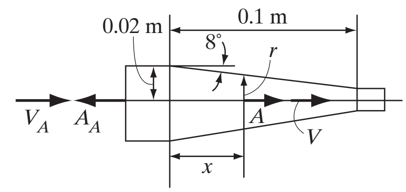
$$V = \frac{0.0637}{(0.02 - 0.141x)^2}$$

4–66. Water flows through the nozzle at a rate of $0.2\ \text{m}^3/\text{s}$. Determine the acceleration of a particle as it moves along the centerline as a function of x.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume.



(a)

Continuity Equation. From the geometry shown in Fig. *a*,

$$r = 0.02\ \text{m} - x\tan 8° = (0.02 - 0.1405x)\ \text{m}$$

Realizing that $Q_A = V_A A_A = 0.2\ \text{m}^3/\text{s}$,

$$\frac{\partial}{\partial t}\int_{cv}\rho\, d\mathcal{V} + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - Q_A + VA = 0$$

$$-0.2\ \text{m}^3/\text{s} + V\left[\pi(0.02 - 0.1405x)^2\right] = 0$$

$$V = \frac{0.06366}{(0.02 - 0.1405x)^2}$$

Since the flow is one dimensional, the acceleration can be determined using

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

Here, $\dfrac{\partial V}{\partial t} = 0$ and

$$\frac{\partial V}{\partial x} = \frac{(0.02 - 0.1405x)^2(0) - 0.06366(2)(0.02 - 0.1405x)(-0.1405)}{(0.02 - 0.1405x)^4}$$

$$= \frac{0.01789}{(0.02 - 0.1405x)^3}$$

Thus,

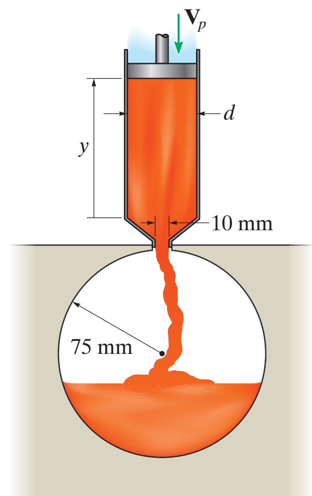
$$a = 0 + \left[\frac{0.06366}{(0.02 - 0.1405x)^2}\right]\left[\frac{0.01789}{(0.02 - 0.1405x)^3}\right]$$

$$= \left[\frac{1.14(10^{-3})}{(0.02 - 0.141x)^5}\right]\ \text{m/s}^2$$ **Ans.**

Ans:

$$\left[\frac{1.14\left(10^{-3}\right)}{(0.02 - 0.141x)^5}\right]\ \text{m/s}^2$$

4–67. The cylindrical plunger traveling at $V_p = (0.004t^{1/2})$ m/s, where t is in seconds, injects a liquid plastic into the mold to make a solid ball. If $d = 50$ mm, determine the amount of time needed to do this if the volume of the ball is $\forall = \frac{4}{3}\pi r^3$.



SOLUTION

The control volume considered is the volume of the liquid plastic contained in the plunger. Its volume changes with time, Fig. *a*. The volume $\forall_0$ of the lower portion of the control volume is constant.

$$\frac{\partial}{\partial t}\int_{cv} \rho_p d\forall + \int_{cs} \rho_p \mathbf{V}_{p/cs} \cdot d\mathbf{A} = 0$$

Since ρ_p is constant, it can be factored out of the integrals. Also, the average velocity will be used. Thus, the above equation becomes

$$\rho_p \frac{d\forall}{dt} + \rho_p V_A A_A = 0$$

Since, $Q_A = V_A A_A$,

$$\frac{d\forall}{dt} + Q_A = 0 \qquad \textbf{(1)}$$

The volume of the control volume is

$$\forall = \pi(0.025 \text{ m})^2 y + \forall_0 = 0.625(10^{-3})\pi y + \forall_0$$

$$\frac{d\forall}{dt} = 0.625(10^{-3})\pi \frac{dy}{dt}$$

However, $\frac{dy}{dt} = -V_p = (-0.004t^{\frac{1}{2}})$m/s. The negative sign indicates that $\mathbf{V}_p$ is directed in the opposite sense to positive y.

$$\frac{d\forall}{dt} = 0.625(10^{-3})\pi(-0.004t^{\frac{1}{2}}) = \left[-2.5(10^{-6})\pi t^{\frac{1}{2}}\right] \text{m}^3/\text{s}$$

The negative sign indicates that the volume is decreasing.

$$-2.5(10^{-6})\pi t^{\frac{1}{2}} + Q_A = 0$$

$$Q_A = \left(2.5(10^{-6})\pi t^{\frac{1}{2}}\right) \text{m}^3/\text{s}$$

The volume of the ball is

$$\forall_s = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.075 \text{ m})^3 = 0.5625(10^{-3})\pi \text{ m}^3$$

The time required to fill up the mold is given by

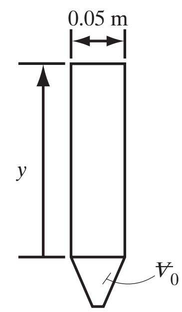
$$\int Q_A dt = \forall_s$$

$$\int_0^T 2.5(10^{-6})\pi t^{\frac{1}{2}} dt = 0.5625(10^{-3})\pi$$

$$\int_0^T t^{\frac{1}{2}}\, dt = 225$$

$$\frac{2}{3}T^{\frac{3}{2}} = 225$$

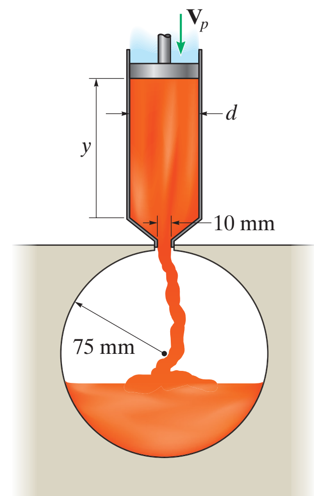
$$T = (337.5)^{\frac{2}{3}} = 48.5 \text{ s} \qquad \textbf{Ans.}$$



(a)

Ans:
48.5 s

***4–68.** The cylindrical plunger traveling at $V_p = \left(0.004\,t^{\frac{1}{2}}\right)$ m/s, where t is in seconds, injects a liquid plastic into the mold to make a solid ball. Determine the time needed to fill the mold as a function of the plunger diameter d. Plot the time needed to fill the mold (vertical axis) versus the diameter of the plunger for 10 mm $\le d \le$ 50 mm. Give values for increments of $\Delta d = 10$ mm. The volume of the ball is $\forall = \frac{4}{3}\pi r^3$.



SOLUTION

The control volume is the volume of the liquid plastic contained in the plunger for which its volume changes with time, Fig. *a*. The volume $\forall_0$ of the lower portion of the control volume is constant.

$$\frac{\partial}{\partial t}\int_{cv} \rho_p d\forall + \int_{cs} \rho_p \mathbf{V}_{p/cs} \cdot d\mathbf{A} = 0$$

Since ρ_p is constant, it can be factored out of the integral. Also, the average velocity will be used. Thus, the above equation becomes

$$\rho_p \frac{\partial \forall}{\partial t} + \rho_p V_A A_A = 0$$

Since $Q_A = V_A A_A$,

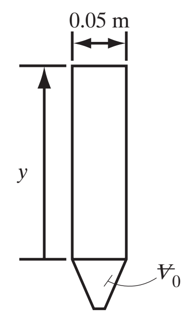
The volume of the control volume is

$$\forall = \left(\frac{\pi}{4}d^2\right)y + V_0$$

$$\frac{\partial \forall}{\partial t} = \frac{\pi}{4}d^2\frac{\partial y}{\partial t}$$

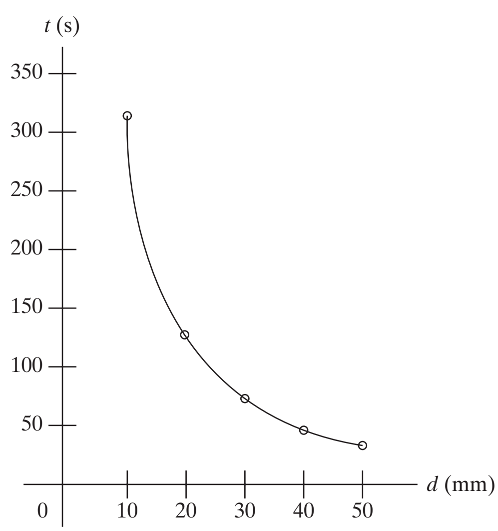
However, $\frac{\partial y}{\partial t} = -V_p = \left(-0.004\,t^{\frac{1}{2}}\right)$ m/s. The negative sign indicates that V_p is directed in the opposite sense to that of positive y.

$$\frac{\partial \forall}{\partial t} = \frac{\pi}{4}d^2\left(-0.004t^{\frac{1}{2}}\right) = \left(-0.001\pi d^2 t^{\frac{1}{2}}\right)\text{ m}^3/\text{s}$$



(a)

d(mm)	10	20	30	40	50
t(s)	316	126	73.1	49.8	37.0



(b)

***4–68. Continued**

The negative sign indicates that the volume is decreasing. Substituting into Eq (1),

$$= -0.001\pi d^2 t^{\frac{1}{2}} + Q_A = 0$$

$$Q_A = \left(0.001\pi d^2 t^{\frac{1}{2}}\right)\text{m}^3/\text{s}$$

The volume of the sphere (mold) is

$$V_s = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.075\text{ m})^3 = 0.5625\left(10^{-3}\right)\pi\text{ m}^3$$

The time to fill up the sphere is

$$t = \frac{V_s}{Q_A}; \qquad t = \frac{0.5625\left(10^{-3}\right)\pi\text{ m}^3}{0.001\pi d^2 t^{\frac{1}{2}}}$$

$$t^{\frac{3}{2}} = \frac{0.5625}{d^2}$$

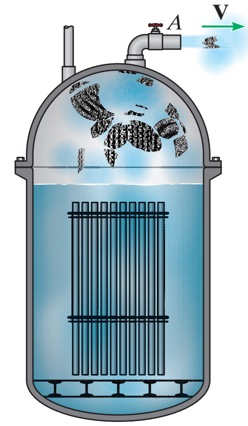
$$t = \left(\frac{0.6814}{d^{\frac{4}{3}}}\right)\text{s}$$

Or, converting d to mm,

$$t = \frac{0.6814}{\left(\frac{d}{1000}\right)^{\frac{4}{3}}}$$

$$t = \left(\frac{6814}{d^{\frac{4}{3}}}\right)\text{s where d is in mm} \qquad \textbf{Ans.}$$

4–69. The pressure vessel of a nuclear reactor is filled with boiling water having a density of $\rho_w = 850\ \text{kg/m}^3$. Its volume is $185\ \text{m}^3$. Due to failure of a pump, needed for cooling, the pressure release valve A is opened and emits steam having a density of $\rho_s = 35\ \text{kg/m}^3$ and an average speed of $V = 400\ \text{m/s}$. If it passes through the 40-mm-diameter pipe, determine the time needed for all the water to escape. Assume that the temperature of the water and the velocity at A remain constant.



SOLUTION

The steam has a steady flow and the density of the water in the pressure vessel is constant since the temperature is assumed to be constant. Here, the control volume is changing since it contains the water in the vessel.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho_w d\forall + \int_{\text{cs}} \rho_s \mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_w and ρ_s are constant, they can be factored out from the integrals. Also, the average velocity of the steam will be used. Then $\int_{\text{cs}} \mathbf{V}\cdot d\mathbf{A} = V_s A$.

$$\rho_w \frac{\partial}{\partial t}\int_{\text{cv}} d\forall + \rho_s V_s A = 0$$

$$\rho_w \frac{\partial \forall}{\partial t} + \rho_s V_s A = 0$$

$$\frac{\partial \forall}{\partial t} = -\frac{\rho_s V_s A}{\rho_w} = -\frac{(35\ \text{kg/m}^3)(400\ \text{m/s})\left[\pi(0.02\ \text{m})^2\right]}{850\ \text{kg/m}^3}$$

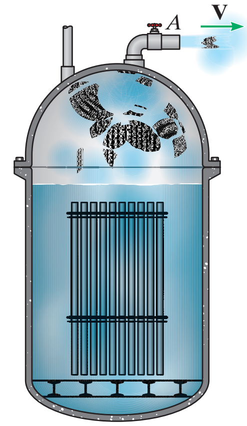
$$= -0.02070\ \text{m}^3/\text{s}$$

The negative sign indicates that the volume of water is decreasing. Thus, the time needed for all the water to escape is

$$t = \frac{\forall}{\partial \forall/\partial t} = \frac{185\ \text{m}^3}{0.02070\ \text{m}^3/\text{s}} = (8938\ \text{s})\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) = 2.48\ \text{hr} \qquad \textbf{Ans.}$$

Ans:
2.48 hr

4–70. The pressure vessel of a nuclear reactor is filled with boiling water having a density of $\rho_w = 850\ \text{kg/m}^3$. Its volume is 185 m^3. Due to failure of a pump, needed for cooling, the pressure release valve is opened and emits steam having a density of $\rho_s = 35\ \text{kg/m}^3$. If the steam passes through the 40-mm-diameter pipe, determine the average speed through the pipe as a function of the time needed for all the water to escape. Plot the speed (vertical axis) versus the time for $0 \le t \le 3\ \text{h}$. Give values for increments of $\Delta t = 0.5\ \text{h}$. Assume that the temperature of the water remains constant.



SOLUTION

The steam has a steady flow and the densities of the water in the pressure vessel and the steam are constant since the temperature is assumed to be constant. Here the control volume is changing since it contains the water in the vessel.

$$\frac{\partial}{\partial t}\int_{cv} \rho_w d\forall + \int_{cs} \rho_s \mathbf{V}_s \cdot d\mathbf{A} = 0$$

Since ρ_w and ρ_s are constants, they can be factored out from the integrals. Also the average velocity of the steam will be used. Then

$$\int_{cs} \mathbf{V}_s \cdot d\mathbf{A} = V_s A.$$

$$\rho_w \frac{\partial}{\partial t}\int_{cv} d\forall + \rho_s V_s A = 0$$

$$\frac{\partial \forall}{\partial t} = -\frac{\rho_s V_s A}{\rho_w} = -\frac{(35\ \text{kg/m}^3)(V_s)\left[\pi(0.02\ \text{m})^2\right]}{850\ \text{kg/m}^3}$$

$$\frac{\partial \forall}{\partial t} = \left[-51.74(10^{-6})\ V_s\right]\text{m}^3/\text{s}$$

The negative sign indicates that the volume of water is decreasing. Thus, the time needed for all the water to escape is

$$t = -\frac{\forall}{\partial \forall/\partial t} = \frac{185\ \text{m}^3}{\left[51.74(10^{-6})\ V_s\right]\text{m}^3/\text{s}}$$

$$t = \left\{\left[\frac{3.5753(10^6)}{V_s}\right]\text{s}\right\}\left(\frac{1\ \text{hr}}{3600}\right)$$

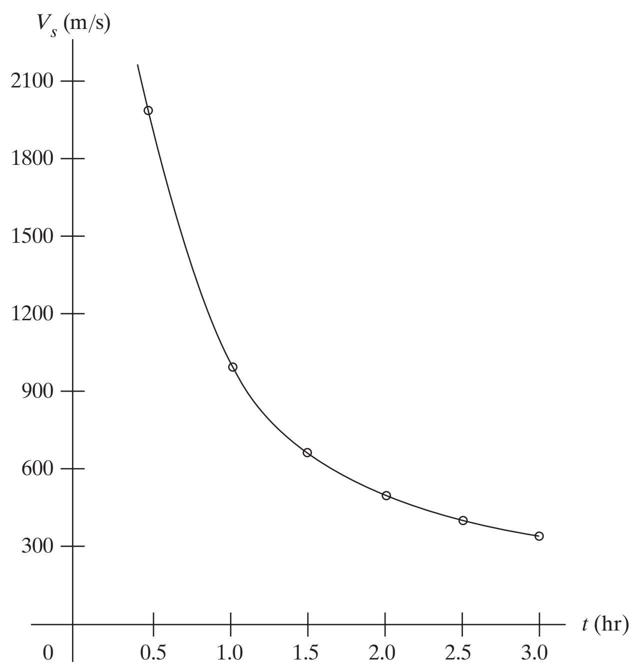
4–70. Continued

$$t = \frac{993.14}{V_s}$$

$$V_s = \left(\frac{993}{t}\right) \text{ m/s where } t \text{ is in hrs}$$ **Ans.**

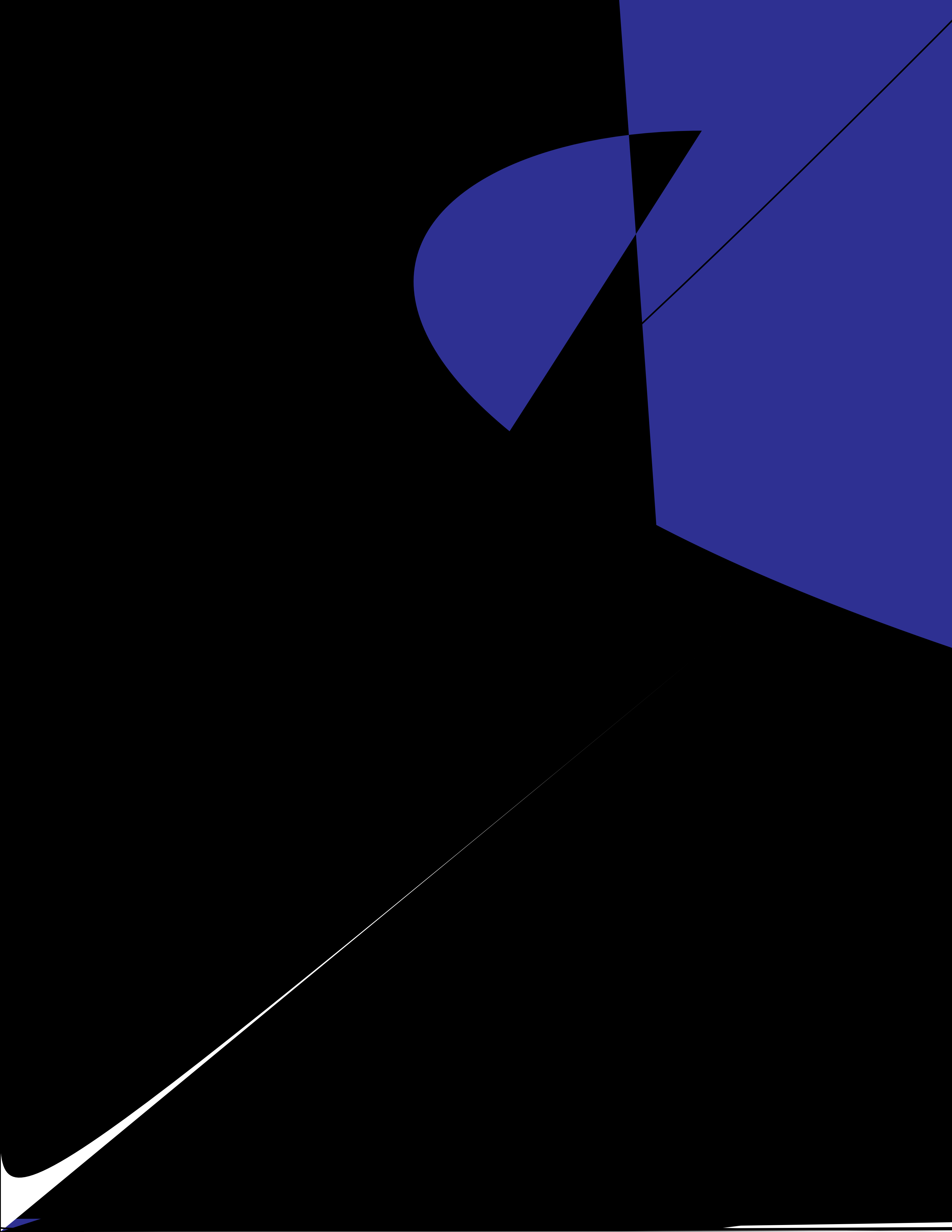
The plot of V_s vs t is shown in Fig. *a*

t(hr)	0	0.5	1.0	1.5	2.0	2.5	3.0
V_s(m/s)	∞	1986	993	662	497	397	331

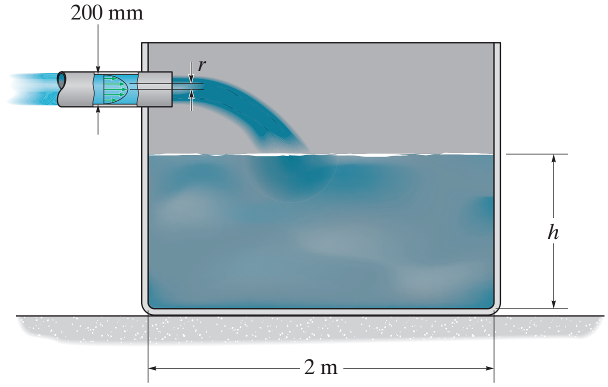


(a)

Ans:
$V_s = \left(\frac{993}{t}\right)$ m/s, where t is in hrs



***4–72.** Water flows through the pipe such that it has a parabolic velocity profile $V = 3(1 - 100r^2)$ m/s, where r is in meters. Determine the time needed to fill the tank to a depth of $h = 1.5$ m if $h = 0$ when $t = 0$. The width of the tank is 3 m.



SOLUTION

The control volume is the volume of the water in the tank. Thus, its volume changes with time

$$\frac{\partial}{\partial t}\int_{cv} \rho_w d\forall + \int_{cs} \rho_w \mathbf{V} \cdot d\mathbf{A} = 0$$

Since ρ_w is constant (incompressible), it can be factor out of the integrals.

$$\rho_w \frac{\partial \forall}{\partial t} + \rho_w \int_{cs} \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\frac{\partial \forall}{\partial t} + Q_{out} - Q_{in} = 0 \qquad \textbf{(1)}$$

Here, $Q_{out} = 0$ and

$$Q_{in} = \int_A v dA = \int_0^{0.1\text{m}} 3(1 - 100r^2)(2\pi r dr) = (0.015\pi)\ \text{m}^3/\text{s}$$

The integral $\int_A v dA$ can also be determined by computing the volume under the velocity profile, which in this case is a paraboloid.

$$\int_A v dA = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(0.1\ \text{m})^2\ (3\ \text{m/s}) = (0.015\pi)\ \text{m}^3/\text{s}$$

Also, the volume of the control volume at a particular instant is

$$\forall = (2\text{m})(3\text{m})(h) = 6h$$

Thus,

$$\frac{d\forall}{dt} = 6\frac{dh}{dt}$$

Substituting these results into Eq (1)

$$6\frac{dh}{dt} - 0.015\pi = 0$$

$$\frac{dh}{dt} = 0.0025\pi$$

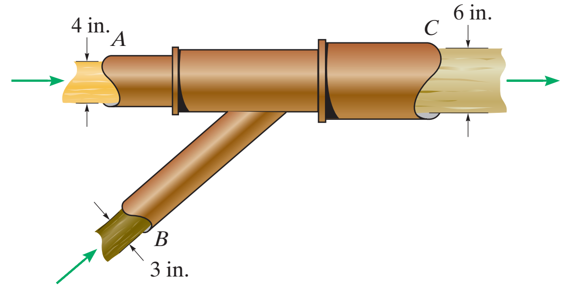
$$\int_0^{1.5\ \text{m}} dh = 0.0025\pi \int_0^t dt$$

$$1.5 = 0.0025\pi t$$

$$t = (190.99\ \text{s})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= 3.18\ \text{min} \qquad \textbf{Ans.}$$

4–73. Ethyl alcohol flows through pipe A with an average velocity of 4 ft/s, and oil flows through pipe B at 2 ft/s. Determine the average density at which the mixture flows through the pipe at C. Assume uniform mixing of the fluids occurs within a 200 in^3 volume of the pipe assembly. Take $\rho_{ea} = 1.53\ \text{slug/ft}^3$ and $\rho_o = 1.70\ \text{slug/ft}^3$.



SOLUTION

The fluids are assumed to be incompressible, and so their volumes remain constant. Also the volume within the pipe is constant. Therefore

$$-V_A A_A - V_B A_B + V_C A_C = 0$$

$$-(4\ \text{ft/s})\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right] - (2\ \text{ft/s})\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right] + V_C\left[\pi\left(\frac{3}{12}\ \text{ft}\right)^2\right] = 0$$

$$V_C = 2.278\ \text{ft/s}$$

Applying the conservation of mass for steady flow.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \rho_{ea} V_A A_A - \rho_o V_B A_B + \rho_C V_C A_C = 0$$

Thus

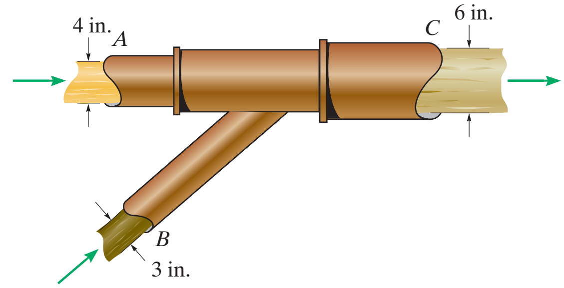
$$\rho_C = \frac{\rho_{ea} V_A A_A + \rho_o V_B A_B}{V_c A_c}$$

$$\rho_C = \frac{(1.53\ \text{slug/ft}^3)(4\ \text{ft/s})\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right] + (1.70\ \text{slug/ft}^3)(2\ \text{ft/s})\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right]}{(2.278\ \text{ft/s})\left(\pi\left(\frac{3}{12}\ \text{ft}\right)^2\right)}$$

$\rho_C = 1.57\ \text{slug/ft}^3$ **Ans.**

Ans:
$1.57\ \text{slug/ft}^3$

4–74. Ethyl alcohol flows through pipe A at $0.05\ \text{ft}^3/\text{s}$, and oil flows through pipe B at $0.03\ \text{ft}^3/\text{s}$. Determine the average density of the two fluids as the mixture flows through the pipe at C. Assume uniform mixing of the fluids occurs within a $200\ \text{in}^3$ volume of the pipe assembly. Take $\rho_{ea} = 1.53\ \text{slug/ft}^3$ and $\rho_o = 1.70\ \text{slug/ft}^3$.



SOLUTION

The fluids are assumed to be incompressible, so their volumes remain constant. Also, the volume within the pipe is constant. Therefore

$$-Q_A - Q_B + Q_C = 0$$

$$-0.05\ \text{ft}^3/\text{s} - 0.03\ \text{ft}^3/\text{s} + Q_C = 0$$

$$Q_C = 0.08\ \text{ft}^3/\text{s}$$

Applying the conservation of mass for steady flow

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\Psi + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

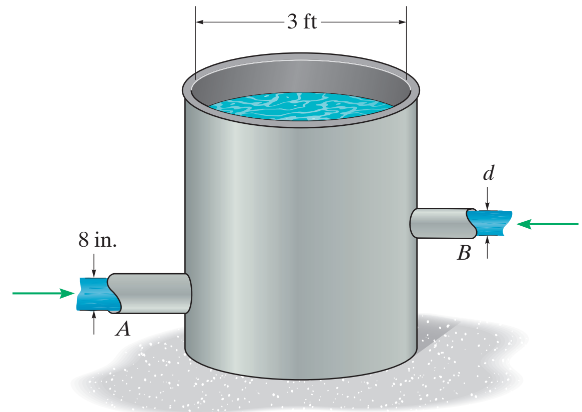
$$0 - \rho_{ea} Q_A - \rho_o Q_B + \rho_m Q_C = 0$$

$$0 - \left(1.53\ \text{slug/ft}^3\right)\left(0.05\ \text{ft}^3/\text{s}\right) - \left(1.70\ \text{slug/ft}^3\right)\left(0.03\ \text{ft}^3/\text{s}\right) + \rho_C\left(0.08\ \text{ft}^3/\text{s}\right) = 0$$

$$\rho_C = 1.59\ \text{slug/ft}^3 \qquad \textbf{Ans.}$$

Ans:
$1.59\ \text{slug/ft}^3$

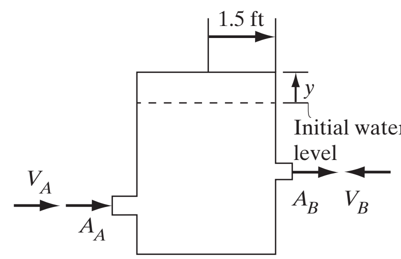
4–75. Water flows into the tank through two pipes. At A the flow is 400 gal/h, and at B it is 200 gal/h when $d = 6$ in. Determine the rate at which the level of water is rising in the tank. There are 7.48 gal/ft^3.



SOLUTION

Control volume. The deformable control volume shown in Fig. *a* will be considered. If the initial control volume is $\forall_0$, then its volume at any given instant is

$$\forall = \forall_0 + \pi(1.5\text{ ft})^2 y = \left[\forall_0 + 2.25\pi y\right]\text{ ft}^3$$



(a)

Continuity Equation. Realizing that $Q = \int_{cs} \mathbf{V}\cdot d\mathbf{A}$ and

$$Q_A = \left(400\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.01485\text{ ft}^3/\text{s}$$

$$Q_B = \left(200\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.007427\text{ ft}^3/\text{s}$$

Since the density of water is constant,

$$\rho_w\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs}\mathbf{V}\cdot d\mathbf{A}\right] = 0$$

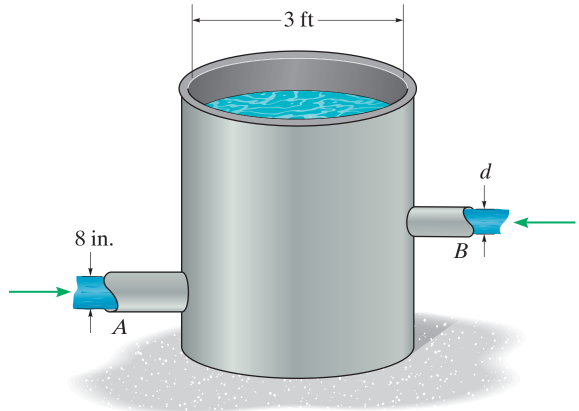
$$\frac{\partial}{\partial t}\left(\forall_0 + 2.25\pi y\right) - 0.01485\text{ ft}^3/\text{s} - 0.007427\text{ ft}^3/\text{s} = 0$$

$$2.25\pi\frac{\partial y}{\partial t} = 0.02228$$

$$\frac{\partial y}{\partial t} = 3.15\left(10^{-3}\right)\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$3.15\left(10^{-3}\right)$ ft/s

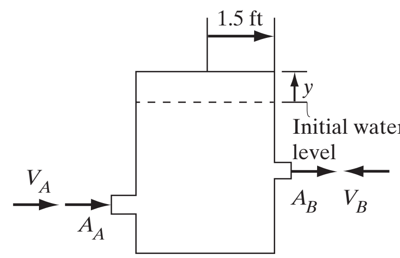
***4–76.** Water flows into the tank through two pipes. At A the flow is 400 gal/h. Determine the rate at which the level of water is rising in the tank as a function of the discharge of the inlet pipe B. Plot this rate (vertical axis) versus the discharge for $0 \le Q_B \le 300$ gal/h. Give values for increments of $\Delta Q_B = 50$ gal/h. There are 7.48 gal/ft^3.



SOLUTION

The deformable control volume shown in Fig. *a* will be considered. If the initial volume of this control volume is $\forall_0$, then its volume at any given instant is

$$\forall = \forall_0 + \pi(1.5\text{ ft})^2 y = (\forall_0 + 2.25\pi y)\text{ ft}^3$$



(a)

The discharges at A and B are

$$Q_A = \left(400\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.01485\text{ ft}^3/\text{s}$$

$$\left(Q_B\frac{\text{gal}}{\text{h}}\right)\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \left[37.14(10^{-6})Q_B\right]\text{ ft}^3/\text{s}$$

Since the density of water is constant and $Q = \int_{cs}\mathbf{V}\cdot d\mathbf{A}$,

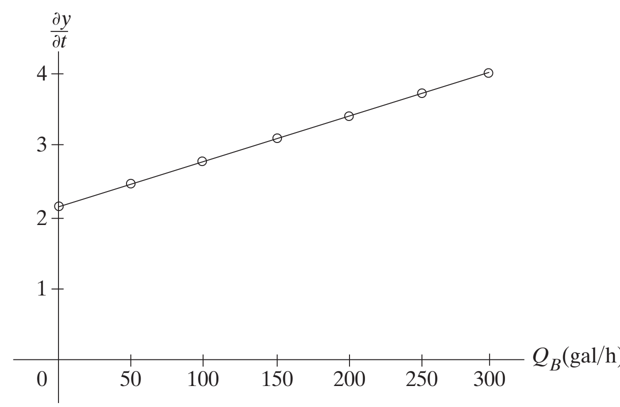
$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$\rho\left(\frac{\partial\forall}{\partial t} - Q_A - Q_B\right) = 0$$

$$\frac{\partial}{\partial t}\left(\forall_0 + 2.25\,\pi y\right) = Q_A + Q_B$$

$$2.25\pi\frac{\partial y}{\partial t} = 0.01485 + 37.14(10^{-6})Q_B$$

$$\frac{\partial y}{\partial t} = \left[2.10(10^{-3}) + 5.25(10^{-6})Q_B\right]\text{ ft/s where } Q_B \text{ is in gal/h}$$ **Ans.**

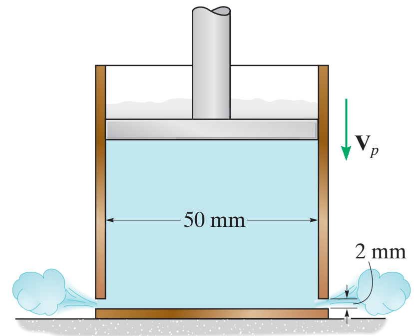


(b)

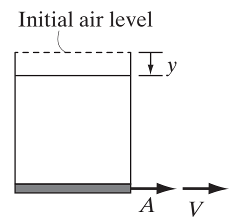
The plot of $\frac{\partial y}{\partial t}$ vs Q_B is shown in Fig. *b*.

Q_B(gal/h)	0	50	100	150	200	250	300
$\frac{\partial y}{\partial t}(10^{-3}\text{ft/s})$	2.10	2.36	2.63	2.89	3.15	3.41	3.68

4–77. The piston is traveling downwards at $V_p = 3\ \text{m/s}$, and as it does, air escapes radially outward through the entire bottom of the cylinder. Determine the average speed of the escaping air. Assume the air is incompressible.



SOLUTION



(a)

Control Volume. The deformable control volume shown in Fig. *a* will be considered. If the initial control volume is $\mathcal{V}_0$, then its volume at any given instant is

$$\mathcal{V} = \mathcal{V}_0 - \pi(0.025\ \text{m})^2 y = \left[\mathcal{V}_0 - 0.625\left(10^{-3}\right)\pi y\right]\text{m}^3$$

Continuity Equation. Since the air is assumed to be incompressible, its density is constant.

$$\rho\left[\frac{\partial}{\partial t}\int_{\text{cv}} d\mathcal{V} + \int_{\text{cs}} \mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\left[\mathcal{V}_0 - 0.625\left(10^{-3}\right)\pi y\right] + V\left[2\pi(0.025\ \text{m})(0.002\ \text{m})\right] = 0$$

$$-0.625\left(10^{-3}\right)\pi\frac{\partial y}{\partial t} + 0.1\left(10^{-3}\right)\pi V = 0$$

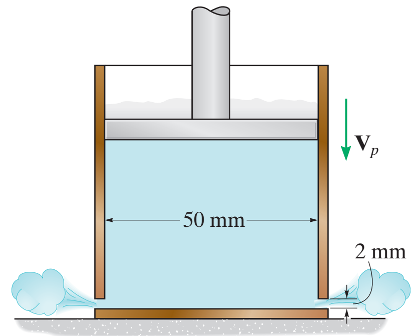
$$V = 6.25\frac{dy}{dt}$$

However, $\dfrac{dy}{dt} = 3\ \text{m/s}$. Then

$$V = 6.25\left(3\ \text{m/s}\right) = 18.8\ \text{m/s}$$ **Ans.**

Ans:
18.8 m/s

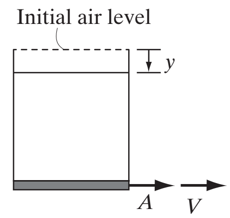
4–78. The piston is travelling downwards with a velocity V_p, and as it does, air escapes radially outward through the entire bottom of the cylinder. Determine the average velocity of the air at the bottom as a function of V_p. Plot this average velocity of the escaping air (vertical axis) versus the velocity of the piston for $0 \le V_p \le 5$ m/s. Give values for increments of $\Delta V_p = 1$ m/s. Assume the air is incompressible.



SOLUTION

The deformable control volume shown in Fig. *a* will be considered. If the initial control volume is $\forall_0$, then its volume at any given instant is

$$\forall = \forall_0 - \pi(0.025\text{ m})^2 y = \left[\forall_0 - 0.625\left(10^{-3}\right)\pi y\right]\text{ m}^3$$



(a)

Since the air is assumed to be incompressible, its density is constant.

$$\rho\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs}\mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\left[\forall_0 - 0.625\left(10^{-3}\right)\pi y\right] + V\left[2\pi(0.025\text{ m})(0.002\text{ m})\right] = 0$$

$$-0.625\left(10^{-3}\right)\pi\frac{\partial y}{\partial t} + 0.1\left(10^{-3}\right)\pi V = 0$$

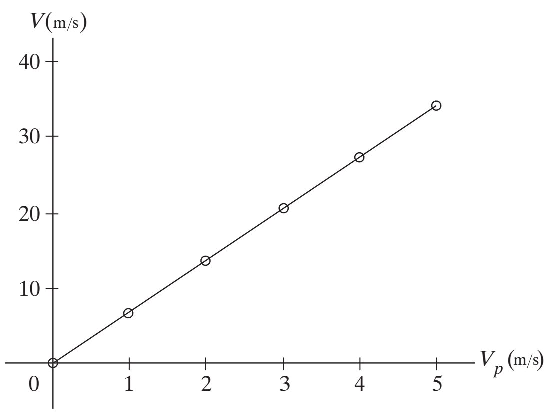
$$V = 6.25\frac{\partial y}{\partial t}$$

However, $\dfrac{\partial y}{\partial t} = V_p$. Then

$$V = (6.25\,V_p)\text{ m/s} \qquad \textbf{Ans.}$$

The plot of V vs V_p is shown in Fig. *b*

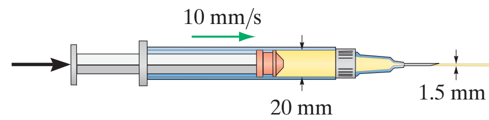
V_p(m/s)	0	1	2	3	4	5
V(m/s)	0	6.25	12.5	18.75	25.0	31.25



(b)

Ans:
$V = (6.25V_p)$ m/s

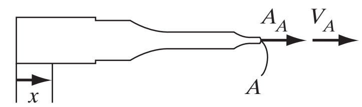
4–79. The cylindrical syringe is actuated by applying a force on the plunger. If this causes the plunger to move forward at 10 mm/s, determine the average velocity of the fluid passing out of the needle.



SOLUTION

Control Volume. The deformable control volume is shown in Fig. *a*. If the volume of the control volume is initially $\mathcal{V}_0$ then at any instant its volume is

$$\mathcal{V} = \mathcal{V}_0 - \frac{\pi}{4}(0.02\text{ m})^2 x = \left[\mathcal{V}_0 - 0.1\left(10^{-3}\right)\pi x\right]\text{ m}^3$$



(a)

Continuity Equation. With the fluid assumed to be incompressible, ρ is constant. since $\mathbf{V}_A$ and $\mathbf{A}_A$ are in the same sense, $Q_A = V_A A_A = V_A\left[\frac{\pi}{4}(0.0015\text{ m})^2\right] = 0.5625\left(10^{-6}\right)\pi V_A$.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho d\mathcal{V} + \int_{\text{cs}} p\mathbf{V}\cdot d\mathbf{A} = 0$$

$$\rho_w\left[\frac{\partial}{\partial t}(\mathcal{V}) + V_A A_A\right] = 0$$

$$\frac{\partial}{\partial t}\left[\mathcal{V}_0 - 0.1\left(10^{-3}\right)\pi x\right] + 0.5625\left(10^{-6}\right)\pi V_A = 0$$

$$-0.1\left(10^{-3}\right)\pi\frac{\partial x}{\partial t} + 0.5625\left(10^{-6}\right)\pi V_A = 0$$

However,

$$\frac{\partial x}{\partial t} = 10\text{ mm/s} = 0.01\text{ m/s}$$

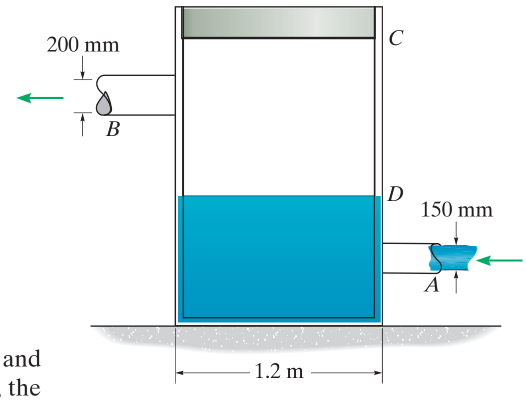
Then

$$\left[0.1\left(10^{-3}\right)\pi\right](0.01) + 0.5625\left(10^{-6}\right)\pi V_A = 0$$

$$V_A = 1.78\text{ m/s}$$ **Ans.**

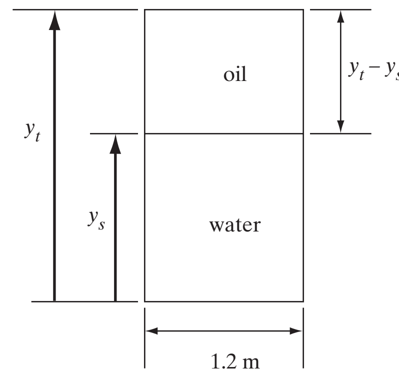
Ans:
1.78 m/s

***4–80.** Water enters the cylindrical tank at A with an average velocity of 2 m/s, and oil exits the tank at B with an average velocity of 1.5 m/s. Determine the rates at which the top level C and interface level D are moving. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

We will consider two control volumes separately namely one contains water and the other contains oil in the tank their volume changes with time, Fig. *a*. Here, the densities of water and oil are constant and the average velocities will be used.



(a)

$$\frac{\partial}{\partial t}\int_{cv} \rho dV + \int \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

For water,

$$\rho_w \frac{\partial \mathcal{V}_w}{\partial t} - \rho_w V_A A_A = 0$$

$$\frac{\partial \mathcal{V}_w}{\partial t} - V_A A_A = 0 \quad \textbf{(1)}$$

Here,

$$\mathcal{V}_w = \left[\pi(0.6\text{ m})^2\right]y_s = 0.36\,\pi y_s$$

$$\frac{\partial \mathcal{V}_w}{\partial t} = 0.36\pi\frac{\partial y_s}{\partial t}$$

Substitute this result into Eq (1)

$$0.36\pi\frac{\partial y_s}{\partial t} - (2\text{ m/s})\left[\pi(0.075\text{ m})^2\right] = 0$$

$$V_p = \frac{\partial y_s}{\partial t} = 0.0312\text{ m/s} \quad \textbf{Ans.}$$

Positive sign indicates that the separation level is rising.

For the oil,

$$\rho_o\frac{\partial \mathcal{V}_0}{\partial t} + \rho_o V_B A_B = 0$$

$$\frac{\partial \mathcal{V}_0}{\partial t} + V_B A_B = 0 \quad \textbf{(2)}$$

Here

$$\mathcal{V}_0 = \left[\pi(0.6\text{ m})^2\right](y_t - y_s)$$

$$= 0.36\pi(y_t - y_s)$$

$$\frac{\partial \mathcal{V}_0}{\partial t} = 0.36\pi\left(\frac{\partial y_t}{\partial t} - \frac{\partial y_s}{\partial t}\right)$$

$$= 0.36\,\pi\left(\frac{\partial y_t}{\partial t} - 0.03125\text{ m/s}\right)$$

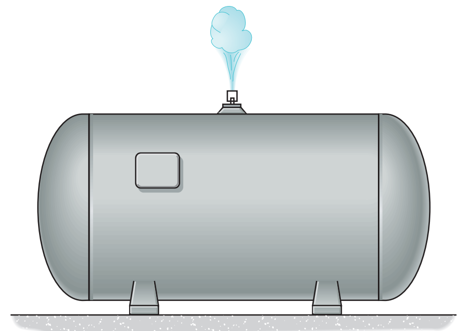
Substituting this result into Eq. (2),

$$0.36\,\pi\left(\frac{\partial y_t}{\partial t} - 0.03125\right) + (1.5\text{ m/s})\left[\pi(0.1\text{ m})^2\right] = 0$$

$$V_C = \frac{\partial y_t}{\partial t} = -0.0104\text{ m/s} \quad \textbf{Ans.}$$

The negative sign indicates that the top level descends.

4–81. The tank contains air at a temperature of 20°C and absolute pressure of 500 kPa. Using a valve, the air escapes with an average speed of 120 m/s through a 15-mm-diameter nozzle. If the volume of the tank is 1.25 m^3, determine the rate of change in the density of the air within the tank at this instant. Is the flow steady or unsteady?



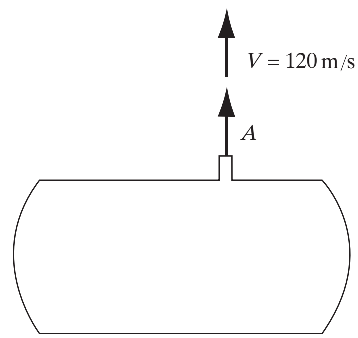
SOLUTION

From Appendix A, the gas constant for air is $R = 286.9\ \text{J}/(\text{kg}\cdot\text{K})$

$$p = \rho RT$$

$$500(10^3)\ \text{N/m}^2 = \rho\big(286.9\ \text{J}/(\text{kg}\cdot\text{K})\big)(20^\circ\text{C} + 273)$$

$$\rho = 5.948\ \text{kg/m}^3$$



(a)

Control Volume. The control volume is show in Fig. *a*. The control volume does not change, but the density of the air changes and therefore results in local changes.

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cv} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$\frac{\partial \rho}{\partial t}(\forall) + \rho VA = 0$$

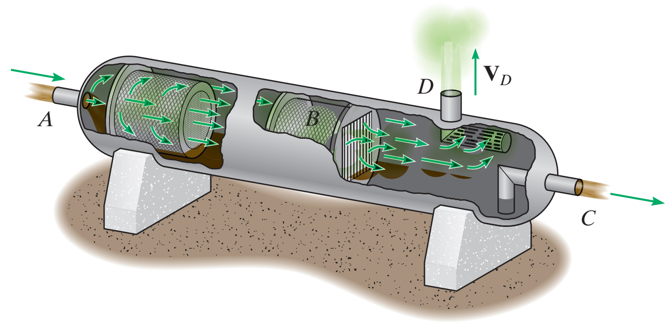
$$\frac{\partial \rho}{\partial t}(1.25\ \text{m}^3) + (5.948\ \text{kg/m}^3)(120\text{m/s})\left[\pi(0.0075\ \text{m})^2\right] = 0$$

$$\frac{\partial \rho}{\partial t} = -0.101\ \text{kg}/(\text{m}^3\cdot\text{s}) \qquad \textbf{Ans.}$$

The negative sign indicates that the density of the air is decreasing. Flow is unsteady, since the pressure within the tank is decreasing and this affects the flow.

Ans:
$-0.101\ \text{kg}/(\text{m}^3\cdot\text{s})$, unsteady

4–82. The natural gas (methane) and crude oil mixture enters the separator at A at $6\ \text{ft}^3/\text{s}$ and passes through the mist extractor at B. Crude oil flows out at $800\ \text{gal/min}$ through the pipe at C, and natural gas leaves the 2-in-diameter pipe at D at $V_D = 300\ \text{ft/s}$. Determine the specific weight of the mixture that enters the separator at A. The process takes place at a constant temperature of 68°F. Take $\rho_o = 1.71\ \text{slug/ft}^3$, $\rho_{me} = 1.29\left(10^{-3}\right)\ \text{slug/ft}^3$. Note $1\ \text{ft}^3 = 7.48\ \text{gal}$.



SOLUTION

The control volume is fixed which is the volume of the crude oil and natural gas contained in the tank. Here, the flow is steady. Thus, no local changes take place. Also, the densities of the gas oil mixture, gas and oil separation are constant, and the average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - \rho_{\text{mix}} V_A A_A + \rho_{CD} V_C A_C + \rho_{me} V_D A_D = 0 \qquad \textbf{(1)}$$

From Appendix A, $\rho_{CD} = 1.71\ \text{slug/ft}^3$ and $\rho_m = 1.29\left(10^{-3}\right)\ \text{slug/ft}^3$ when $T = 68°\text{F}$. Also, $Q_C = V_C A_C = \left(800\dfrac{\text{gal}}{\text{min}}\right)\left(\dfrac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 1.783\ \text{ft}^3/\text{s}$ and

$Q_A = V_A A_A = 6\ \text{ft}^3/\text{s}$. Substituting these results into Eq. (1),

$$-\rho_{\text{mix}}(6\ \text{ft}^3/\text{s}) + \left(1.71\ \text{slug/ft}^3\right)\left(1.783\ \text{ft}^3/\text{s}\right)$$

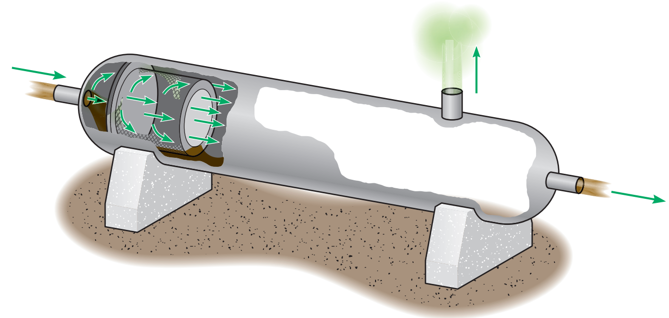
$$+ \left[1.29\left(10^{-3}\right)\text{slug/ft}^3\right]\left(300\ \text{ft/s}\right)\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] = 0$$

$$\rho_{\text{mix}} = 0.5094\ \text{slug/ft}^3$$

$$\gamma_{\text{mix}} = \rho_{\text{mix}}\, g = \left(0.5094\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right) = 16.4\ \text{lb/ft}^3 \qquad \textbf{Ans.}$$

Ans:
$16.4\ \text{lb/ft}^3$

4–83. The natural gas (methane) and crude oil mixture having a density of 0.51 slug/ft^3 enters the separator at A at 6 ft^3/s, and crude oil flows out through the pipe at C at 800 gal/min. Determine the average velocity of the natural gas that leaves the 2-in.-diameter pipe at D. The process takes place at a constant temperature of 68°F. Take $\rho_o = 1.71\ \text{slug/ft}^3$, $\rho_{\text{me}} = 1.29(10^{-3})\ \text{slug/ft}^3$. Note $1\ \text{ft}^3 = 7.48$ gal.



SOLUTION

The control volume is fixed which is the volume of the mixture of crude oil and natural gas contained in the tank. Here, the flow is steady. Thus, no local changes take place. Also the densities of the oil gas mixture, gas and oil separation, are constant and average velocities will be used.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \rho_{\text{mix}} V_A A_A + \rho_{CO} V_C A_C + \rho_{\text{me}} V_D A_D = 0 \qquad \textbf{(1)}$$

From Appendix A, $\rho_{CO} = 1.71\ \text{slug/ft}^3$ and $\rho_{\text{me}} = 1.29(10^{-3})\ \text{slug/ft}^3$ at $T = 68°\text{F}$

Also, $Q_C = \left(800 \dfrac{\text{gal}}{\text{min}}\right)\left(\dfrac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 1.783\ \text{ft}^3/\text{s}$ and $Q_A = V_A A_A = 6\ \text{ft}^3/\text{s}$.

Substituting these results into Eq 1,

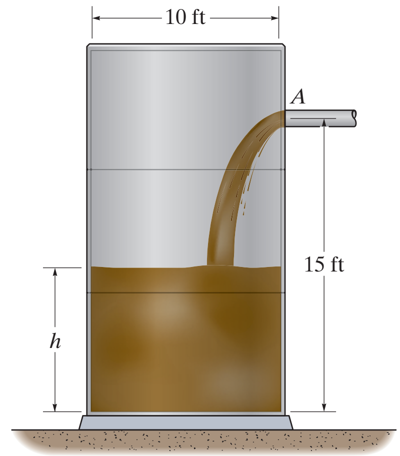
$$-(0.51\ \text{slug/ft}^3)(6\ \text{ft}^3/\text{s}) + (1.71\ \text{slug/ft}^3)(1.783\ \text{ft}^3/\text{s})$$

$$+ \left[1.29(10^{-3})\ \text{slug/ft}^3\right](V_D)\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] = 0$$

$$V_D = 422\ \text{ft/s} \qquad \textbf{Ans.}$$

Ans:
422 ft/s

***4–84.** The cylindrical storage tank is being filled using a pipe having a diameter of 3 in. Determine the rate at which the level in the tank is rising if the flow into the tank at A is 40 gal/min . Note 1 ft^3 = 7.48 gal.



SOLUTION

The control volume is the volume of oil contained in the tank, which changes with time. Here, the density of the oil is constant and the average velocity will be used

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V} + \int_{cs} \rho \, \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\rho \frac{\partial \mathcal{V}}{\partial t} - \rho V_A A_A = 0$$

Since $Q_A = V_A A_A = \left(40 \dfrac{\text{gal}}{\text{min}}\right)\left(\dfrac{1 \text{ ft}^3}{7.48 \text{ gal}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = 0.08913 \text{ ft}^3/\text{s}$. Then

$$\frac{\partial \mathcal{V}}{\partial t} = 0.08913 \qquad \textbf{(1)}$$

Here, the volume of the control volume at a particular instant is

$$\mathcal{V} = \pi r^2 h = \pi (5 \text{ ft})^2 h = 25\pi h$$

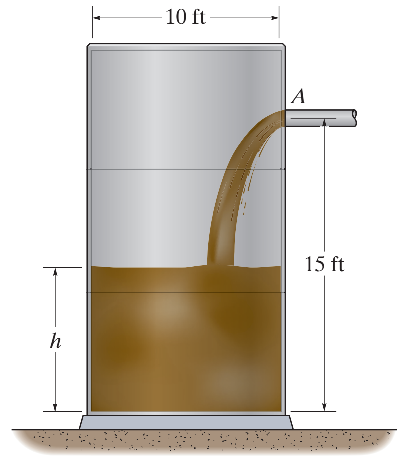
$$\frac{\partial \mathcal{V}}{\partial t} = 25\pi \frac{\partial h}{\partial t}$$

Substituting this result into Eq (1)

$$25\pi \frac{\partial h}{\partial t} = 0.08913$$

$$\frac{\partial h}{\partial t} = 1.13\left(10^{-3}\right) \text{ ft/s} \qquad \textbf{Ans.}$$

4–85. The cylindrical storage tank is being filled using a pipe having a diameter of D. Determine the rate at which the level is rising as a function of D if the velocity of the flow into the tank is 6 ft/s. Plot this rate (vertical axis) versus the diameter for $0 \le D \le 6$ in. Give values for increments of $\Delta D = 1$ in.



SOLUTION

The control volume is the volume of oil contained in the tank of which its volume changes with time. Here, the density of the oil is constant and the average velocities will be used.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\rho\left(\frac{\partial \forall}{\partial t} - V_A A_A\right) = 0$$

$$\frac{\partial \forall}{\partial t} = V_A A_A = (6\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{D}{12}\right)^2\right]$$

$$\frac{\partial \forall}{\partial t} = 0.03272\, D^2 \qquad \textbf{(1)}$$

Here, the volume of the control volume at a particular instant is

$$\forall = \pi r^2 h = \pi(5\text{ ft})^2 h = 25\pi h$$

$$\frac{\partial \forall}{\partial t} = 25\pi \frac{\partial h}{\partial t}$$

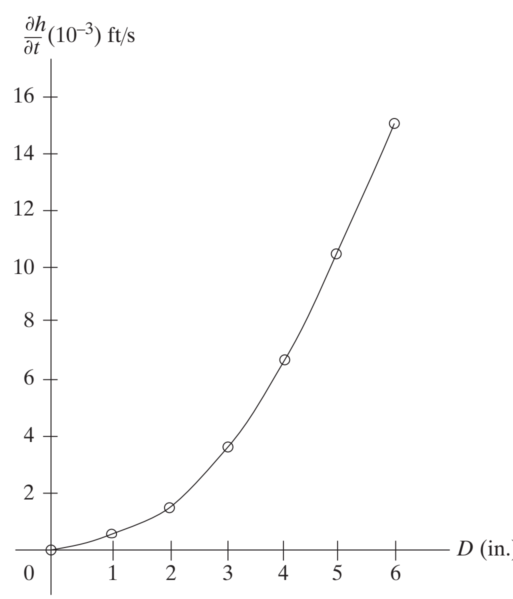
Substituting this result into Eq (1),

$$25\pi \frac{\partial h}{\partial t} = 0.03272\, D^2$$

$$\frac{\partial h}{\partial t} = \left[0.417\left(10^{-3}\right)D^2\right]\text{ ft/s where } D \text{ is in inches.} \qquad \textbf{Ans.}$$

The plot of $\frac{\partial h}{\partial t}$ vs D is shown in Fig. a.

D(in.)	0	1	2	3	4	5	6
$\frac{\partial h}{\partial t}(10^{-3})$ft/s	0	0.417	1.67	3.75	6.67	10.4	15.0

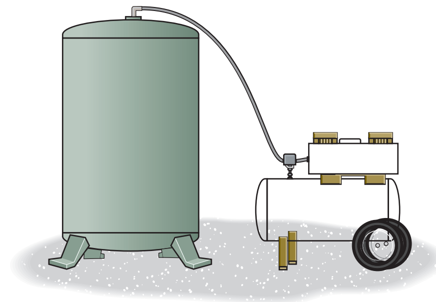


(a)

Ans:

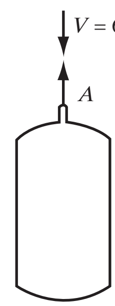
$$\frac{\partial h}{\partial t} = \left[0.417\left(10^{-3}\right)D^2\right]\text{ ft/s, where } D \text{ is in in.}$$

4–86. Air is pumped into the tank using a hose having an inside diameter of 6 mm. If the air enters the tank with an average speed of 6 m/s and has a density of 1.25 kg/m^3, determine the initial rate of change in the density of the air within the tank. The tank has a volume of 0.04 m^3.



SOLUTION

Control Volume. The control volume is shown in Fig. *a*. The control volume does not change but the density of the air changes and therefore results in local changes.



(a)

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

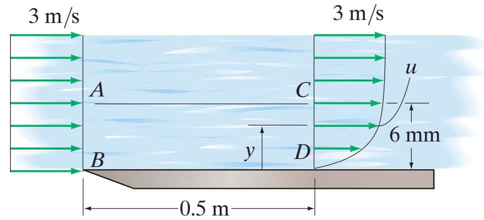
$$\frac{\partial \rho}{\partial t}(\forall) - \rho V A = 0$$

$$\frac{\partial \rho}{\partial t}\left(0.04 \text{ m}^3\right) - \left(1.25 \text{ kg/m}^3\right)\left(6 \text{ m/s}\right)\left[\pi (0.003 \text{ m})^2\right] = 0$$

$$\frac{\partial \rho}{\partial t} = 0.00530 \text{ kg/}\left(\text{m}^3 \cdot \text{s}\right)$$ **Ans.**

Ans:
$0.00530 \text{ kg/}\left(\text{m}^3 \cdot \text{s}\right)$

4–87. As air flows over the plate, frictional effects on its surface tend to form a boundary layer in which the velocity profile changes from that of being uniform to one that is parabolic, defined by $u = \left[1000y - 83.33\left(10^3\right)y^2\right]$ m/s, where y is in meters, $0 \le y < 6$ mm. If the plate is 0.2 m wide and this change in velocity occurs within the distance of 0.5 m, determine the mass flow through the sections AB and CD. Since these results will not be the same, how do you account for the mass flow difference? Take $\rho = 1.226\ \text{kg/m}^3$.



SOLUTION

Mass Flow Rate. For section AB, since ρ is constant and the velocity has a constant magnitude,

$$\dot{m}_{AB} = \rho V_{AB} A_{AB}$$

$$= \left(1.226\ \text{kg/m}^3\right)(3\ \text{m/s})\left[0.006\ \text{m}(0.2\ \text{m})\right] = 0.00441\ \text{kg/s} = 4.41\ \text{g/s} \qquad \textbf{Ans.}$$

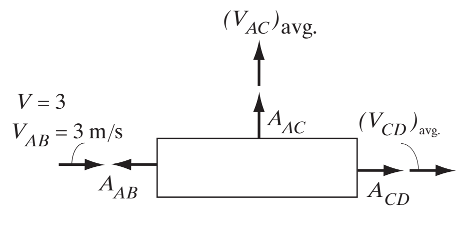
For section CD, since the velocity is a function of y, a differential element of thickness dy, which has an area $dA = b\,dy = (0.2\ \text{m})dy$, is chosen. Thus,

$$\dot{m}_{CD} = \rho \int u\,dA$$

$$= \left(1.226\ \text{kg/m}^3\right)\left(\int_0^{0.006\ \text{m}} \left[1000y - 83.33\left(10^3\right)y^2\right]\ \text{m/s}\right)(0.2\ \text{m})dy$$

$$= 0.2452\left[500y^2 - 27.78\left(10^3\right)y^3\right]\Big|_0^{0.006\ \text{m}}$$

$$= 0.00294\ \text{kg/s} = 2.94\ \text{g/s} \qquad \textbf{Ans.}$$

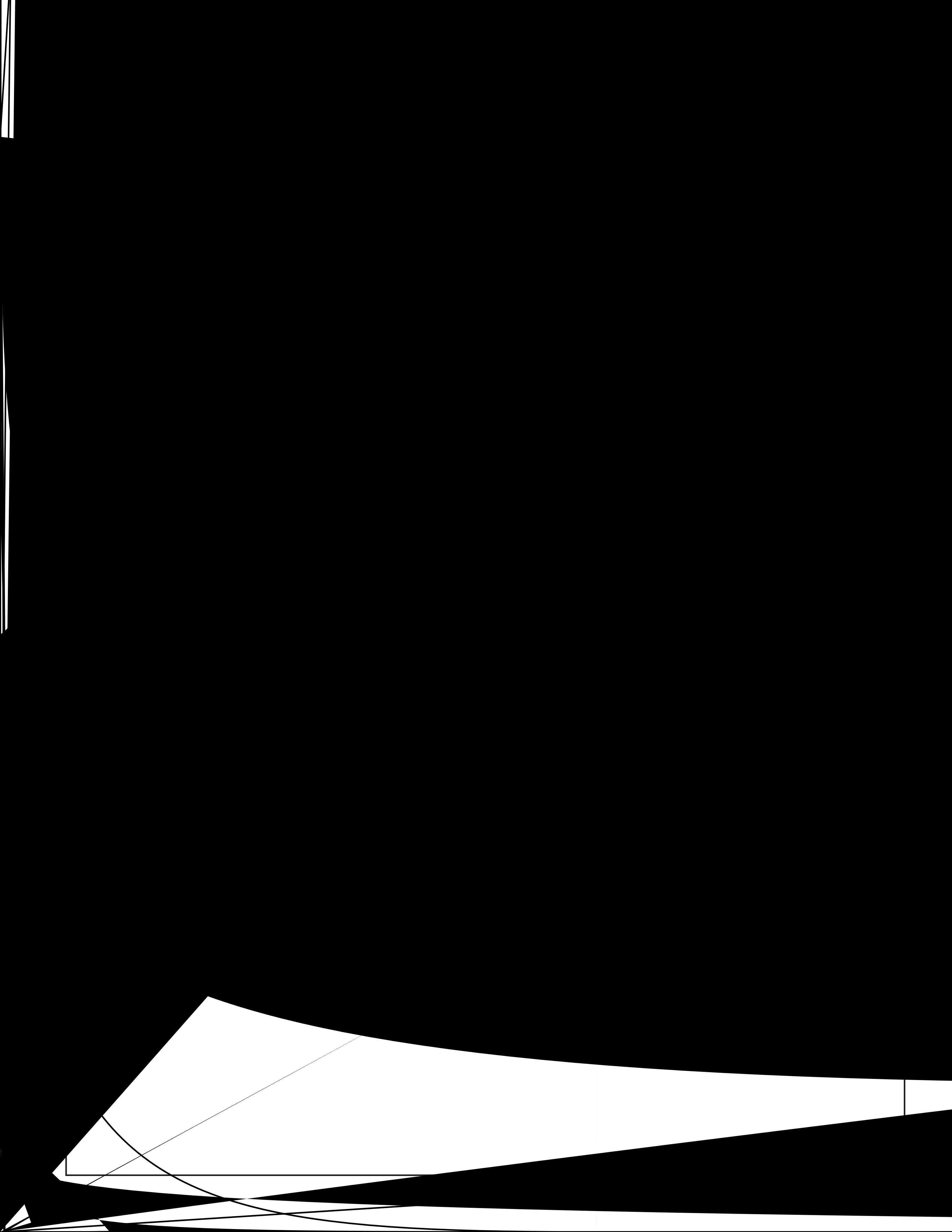


(a)

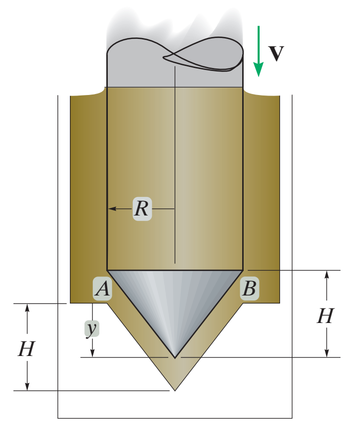
To satisfy continuity the difference between $\dot{m}_{AB}$ and $\dot{m}_{CD}$ requires that mass flows through the control surface AC as indicated on the control volume in Fig. *a*.

What this means is that the streamlines in fact cannot be horizontal, as the figure implies. The fluid velocity must have a vertical component in addition to the horizontal one.

Ans:
2.94 g/s



4–90. The conical shaft is forced into the conical seat at a constant speed of V_0. Determine the average velocity of the liquid as it is ejected from the horizontal section AB as a function of y. *Hint:* The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a* will be considered.

$$\frac{r}{R} = \frac{y}{H};\ r = \frac{R}{H}y$$

Then, the volume of the control volume at any instant is

$$V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\left(\frac{R}{H}y\right)^2 y = \frac{\pi R^2}{3H^2}(H^3 - y^3)$$

Continuity Equation. Since the density of the liquid is constant,

$$\rho\left[\frac{\partial}{\partial t}\int_{cv} dV + \int_{cs} \mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}V + V(A\cos\theta) = 0$$

$$\frac{\partial}{\partial t}\left[\frac{\pi R^2}{3H^2}(H^3 - y^3)\right] + V\left[\pi\left[R^2 - \left(\frac{R}{H}y\right)^2\right]\cos\theta\right] = 0$$

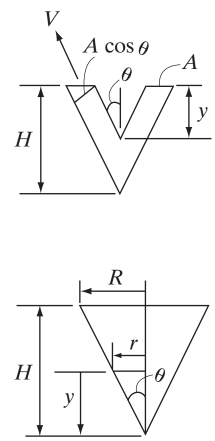
$$\frac{\pi R^2}{3H^2}(-3y^2)\frac{dy}{dt} + V\left[\frac{\pi R^2}{H^2}(H^2 - y^2)\right]\cos\theta = 0$$

$$V = \left(\frac{y^2}{H^2 - y^2}\right)\frac{dy}{dt}(\sec\theta)$$

However, $\dfrac{dy}{dt} = V_0$ and $\sec\theta = \dfrac{\sqrt{H^2 + R^2}}{H}$ Then,

$$V = \left(\frac{y^2}{H^2 - y^2}\right)V_0\frac{\sqrt{H^2 + R^2}}{H}$$

$$V = V_0\frac{y^2\sqrt{H^2 + R^2}}{H(H^2 - y^2)} \qquad \textbf{Ans.}$$

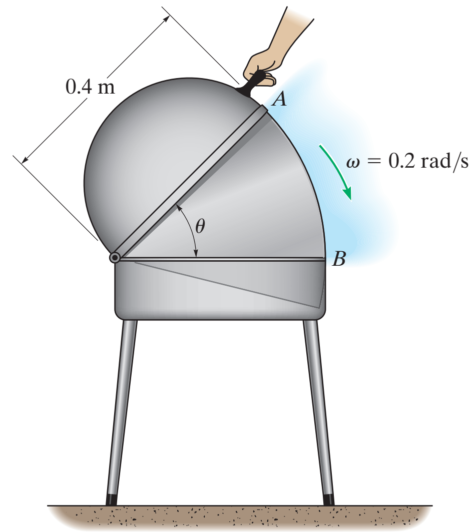


(a)

Ans:

$$V = V_0\frac{y^2\sqrt{H^2 + R^2}}{H\left(H^2 - y^2\right)}$$

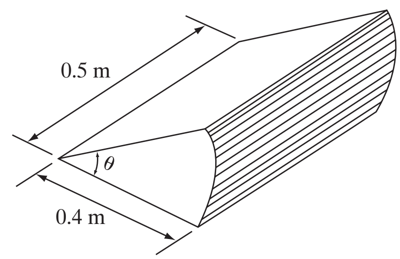
4–91. The 0.5-m-wide lid on the barbecue grill is being closed at a constant angular velocity of $\omega = 0.2$ rad/s, starting at $\theta = 90°$. In the process, the air between A and B will be pushed out in the *radial direction* since the sides of the grill are covered. Determine the average velocity of the air that emerges from the front of the grill at the instant $\theta = 45°$ rad. Assume that the air is incompressible.



SOLUTION

The flow is considered one dimensional since its velocity is directed in the radial direction only. The control volume is shown in Fig. *a*, and its volume changes with time. At a particular instant it is

$$\mathcal{V} = \frac{1}{2}r^2\theta b = \frac{1}{2}(0.4\text{ m})^2\theta(0.5\text{ m}) = 0.04\theta\text{ m}^3$$



(a)

Thus,

$$\frac{d\mathcal{V}}{dt} = 0.04\frac{d\theta}{dt}$$

However,

$$\frac{d\theta}{dt} = \omega = -0.2\text{ rad/s}.$$

Then

$$\frac{d\mathcal{V}}{dt} = -0.008\text{ m}^3/\text{s}$$

Notice that negative sign indicates that the volume is decreasing with time. The opened control surface is shown shaded in Fig. *a*. Its area is

$$A = r\theta b = (0.4\text{ m})\theta(0.5\text{ m}) = 0.2\theta$$

Thus,

$$\frac{\partial}{\partial t}\int_{cv}\rho_a\, d\mathcal{V} + \int_{cs}\rho_a\mathbf{V}\cdot d\mathbf{A} = 0$$

Since here air is assumed to be incompressible, ρ_a is constant. Also, the average velocity of the air is directed radially outward. Thus, it always acts perpendicular to the opened control surface. Hence, the above equation becomes

$$\rho_a\frac{\partial\mathcal{V}}{\partial t} + \rho_a V_a A = 0$$

$$\frac{\partial\mathcal{V}}{\partial t} = -V_a A$$

$$V_a = -\frac{\partial\mathcal{V}/\partial t}{A} = -\frac{-0.008}{0.2\theta} = \frac{0.04}{\theta}\text{ m/s}$$

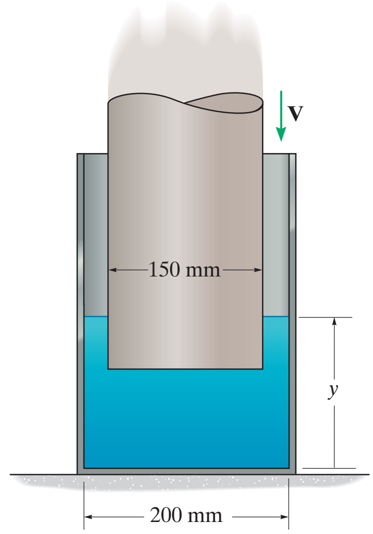
When

$$\theta = 45° = \frac{\pi}{4}\text{ rad},$$

$$V_A = \frac{0.04}{\pi/4} = 0.0509\text{ m/s}$$ **Ans.**

Ans:
0.0509 m/s

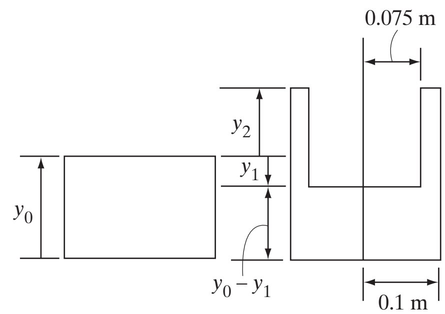
***4–92.** The cylinder is pushed down into the tube at a rate of $V = 5\text{ m/s}$. Determine the average velocity of the liquid as it rises in the tube.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a* will be considered. If the initial water level in the tube is y_0, Fig. *b*, then the control volume at any instant is

$$\cancel{V} = \pi(0.1\text{ m})^2(y_0 - y_1) + \pi(0.1\text{ m})^2(y_1 + y_2) - \pi(0.075\text{ m})^2(y_1 + y_2)$$
$$= \pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)$$



(b) (a)

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho_w d\cancel{V} + \int_{cs} \rho_w \mathbf{V}_{f/cs} \cdot d\mathbf{A} = 0$$

Since no water enters or leaves the control volume at any instant, $\int_{cs} \rho_w \mathbf{V}_{f/cs} \cdot d\mathbf{A} = 0$. Then,

$$\frac{\partial}{\partial t}\int_{cv} \rho_w d\cancel{V} = 0$$

$$\rho_w \frac{\partial}{\partial t}\cancel{V} = 0$$

$$\frac{\partial}{\partial t}\left[\pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)\right] = 0$$

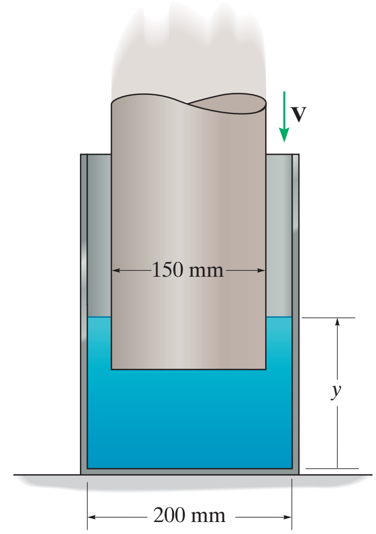
$$-0.005625\frac{\partial y_1}{\partial t} + 0.004375\frac{\partial y_2}{\partial t} = 0$$

$$\frac{\partial y_2}{\partial t} = 1.2857\frac{\partial y_1}{\partial t}$$

However, $\frac{\partial y_1}{\partial t} = V_r = 5\text{ m/s}$. Then

$$\frac{\partial y_2}{\partial t} = 1.2857(5\text{ m/s}) = 6.43\text{ m/s}$$ **Ans.**

4–93. Determine the speed V at which the cylinder must be pushed down into the tube so that the liquid in the tube rises with an average velocity of 4 m/s.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a* will be considered. If the initial water level in the tube is y_0, Fig. *b*, then the control volume at any instant is

$$\forall = \pi(0.1\text{ m})^2(y_0 - y_1) + \pi(0.1\text{ m})^2(y_1 + y_2) - \pi(0.075\text{ m})^2(y_1 + y_2)$$
$$= \pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)$$

Continuity Equation. $$\frac{\partial}{\partial t}\int_{cv} \rho_w d\forall + \int_{cs} \rho_w \mathbf{V}_{f/cs} \cdot d\mathbf{A} = 0$$

Since no water enters or leaves the control volume at any instant, $\int_{cs} \rho_w \mathbf{V}_{f/cs} \cdot d\mathbf{A} = 0$.

Then,

$$\frac{\partial}{\partial t}\int_{cv} \rho_w d\forall = 0$$

$$\rho_w \frac{\partial}{\partial t}\forall = 0$$

$$\frac{\partial}{\partial t}\left[\pi(0.01y_0 - 0.005625y_1 + 0.004375y_2)\right] = 0$$

$$-0.005625\frac{\partial y_1}{\partial t} + 0.004375\frac{\partial y_2}{\partial t} = 0$$

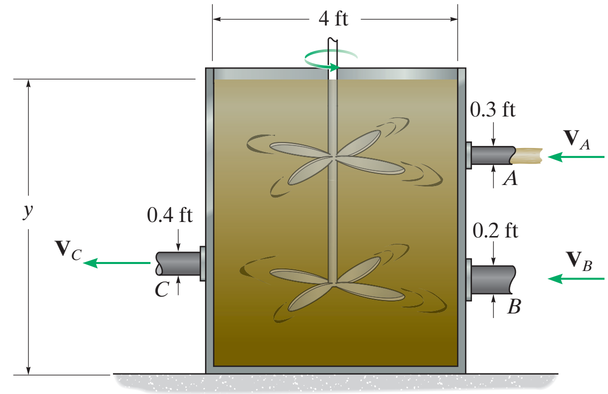
$$\frac{\partial y_1}{\partial t} = 0.7778\frac{\partial y_2}{\partial t}$$

However, $\frac{\partial y_1}{\partial t} = V_r$ and $\frac{\partial y_2}{\partial t} = 4\text{ m/s}$. Then

$$V_r = 0.7778(4\text{ m/s}) = 3.11\text{ m/s}$$ **Ans.**

Ans:
3.11 m/s

***4–96.** Benzene flows through the pipe at A with an average velocity of 4 ft/s, and kerosene flows through the pipe at B with an average velocity of 6 ft/s. If the average velocity of the mixture leaving the tank at C is $V_C = 5$ ft/s, determine the rate at which the level in the tank is changing. The tank has a width of 3 ft. Is the level rising or falling? What is the density of the mixture leaving the tank at C? Take $\rho_b = 1.70$ slug/ft^3 and $\rho_{ke} = 1.59$ slug/ft^3.



SOLUTION

The density of the mixture can be determined from

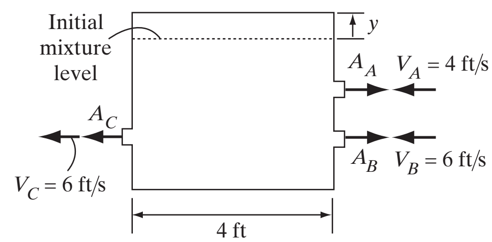
$$\rho_m = \frac{\rho_b Q_A + \rho_{ke} Q_B}{Q_A + Q_B}$$

Here,

$$Q_A = V_A A_A = (4\text{ ft/s})\left[\pi(0.15\text{ ft})^2\right] = 0.09\pi\text{ ft}^3/\text{s}$$

$$Q_B = V_B A_B = (6\text{ ft/s})\left[\pi(0.1\text{ ft})^2\right] = 0.06\pi\text{ ft}^3/\text{s}$$

$$Q_C = V_C A_C = (5\text{ ft/s})\left[\pi(0.2\text{ ft})^2\right] = 0.2\pi\text{ ft}^3/\text{s}$$



(a)

Then

$$\rho_m = \frac{\left(1.70\text{ slug/ft}^3\right)\left(0.09\pi\text{ ft}^3/\text{s}\right) + \left(1.59\text{ slug/ft}^3\right)\left(0.06\pi\text{ ft}^3/\text{s}\right)}{0.09\pi\text{ ft}^3/\text{s} + 0.06\pi\text{ ft}^3/\text{s}}$$

$$= 1.656\text{ slug/ft}^3 \qquad \textbf{Ans.}$$

Here, the volume of the control volume changes with time since it contains the mixture in the tank. Its volume is

$$\forall = (4\text{ ft})(3\text{ ft})y = 12y$$

$$\frac{\partial \forall}{\partial t} = 12\frac{\partial y}{\partial t}$$

Here, the densities of the liquids are constant and the average velocity will be used.

$$\rho_m \frac{\partial \forall}{\partial t} - \rho_b V_A A_A - \rho_a V_B A_B + \rho_m V_C A_C = 0$$

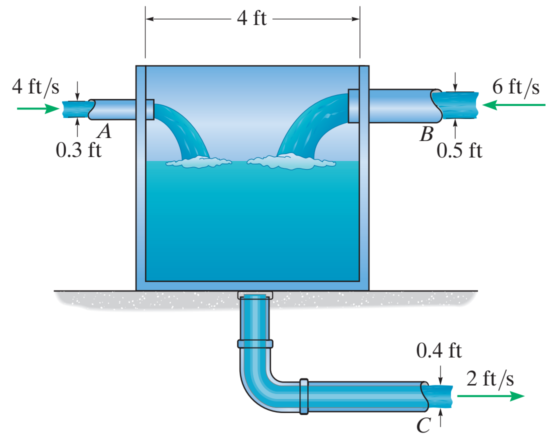
$$\left(1.656\text{ slug/ft}^3\right)\left(12\frac{\partial y}{\partial t}\right) - \left(1.70\text{ slug/ft}^3\right)\left(0.09\pi\text{ ft}^3/\text{s}\right)$$

$$- \left(1.59\text{ slug/ft}^3\right)\left(0.06\pi\text{ ft}^3/\text{s}\right) + \left(1.656\text{ slug/ft}^3\right)\left(0.2\pi\text{ ft}^3/\text{s}\right) = 0$$

$$\frac{\partial y}{\partial t} = -0.0131\text{ ft/s} \qquad \textbf{Ans.}$$

The negative sign indicates the level of the mixture is falling.

4–97. The three pipes are connected to the water tank. If the average velocities of water flowing through the pipes are $V_A = 4$ ft/s, $V_B = 6$ ft/s, and $V_C = 2$ ft/s, determine the rate at which the water level in the tank changes. The tank has a width of 3 ft.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a* will be considered.

$$\forall = (4\text{ ft})(3\text{ ft})y = (12y)\text{ ft}^3$$

Continuity Equation. Since water has a constant density,

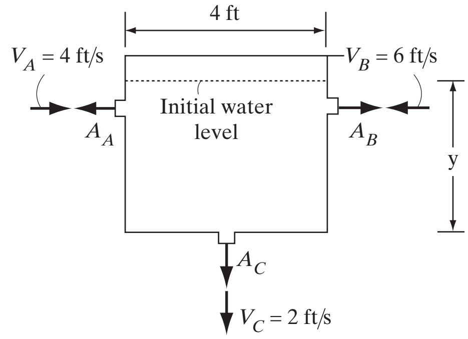
$$\rho_w\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs}\mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\forall - V_A A_A - V_B A_B + V_C A_C = 0$$

$$\frac{\partial}{\partial t}(12y) - (4\text{ ft/s})\left[\pi(0.15\text{ ft})^2\right] - (6\text{ft/s})\left[\pi(0.25\text{ ft})^2\right] + (2\text{ ft/s})\left[\pi(0.2\text{ ft})^2\right] = 0$$

$$12\frac{\partial y}{\partial t} = 1.2095$$

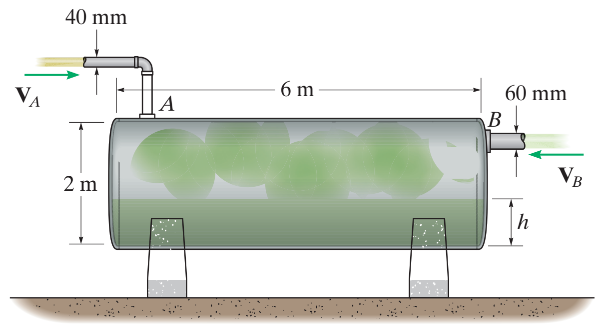
$$\frac{\partial y}{\partial t} = 0.101\text{ ft/s} \qquad \textbf{Ans.}$$



(a)

Ans:
0.101 ft/s

4–98. The 2-m-diameter cylindrical emulsion tank is being filled at A with cyclohexanol at an average rate of $V_A = 4\text{ m/s}$ and at B with thiophene at an average rate of $V_B = 2\text{ m/s}$. Determine the rate at which the depth increases as a function of length h.

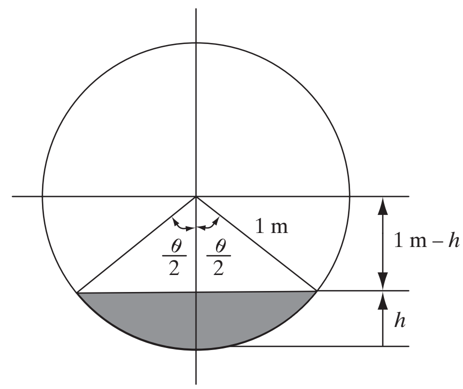


SOLUTION

The control volume considered is the volume of liquid mixture contained in the sector of the tank (shown shaded in Fig. a) which changes with time. The volume of this control volume at a particular instant is

$$\forall = \left\{\frac{1}{2}(1\text{ m})^2\theta - \frac{1}{2}\left[2(1\text{ m})\sin\frac{\theta}{2}(1\text{ m})\cos\frac{\theta}{2}\right]\right\}(6\text{ m}) = 3(\theta - \sin\theta)$$

$$\frac{\partial\forall}{\partial t} = 3\left(\frac{\partial\theta}{\partial t} - \cos\theta\frac{\partial\theta}{\partial t}\right) = 3(1 - \cos\theta)\frac{\partial\theta}{\partial t}$$



(a)

However, $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$ and $\cos\frac{\theta}{2} = \frac{1-h}{1} = 1 - h$. Thus $\cos\theta = 2(1-h)^2 - 1$. Then

$$\frac{\partial\forall}{\partial t} = 3\left\{1 - \left[2(1-h)^2 - 1\right]\right\}\frac{d\theta}{dt} = 6(2h - h^2)\frac{d\theta}{dt} \qquad \textbf{(1)}$$

Here,

$$\cos\frac{\theta}{2} = 1 - h$$

$$\left(-\frac{1}{2}\sin\frac{\theta}{2}\right)\frac{d\theta}{dt} = -\frac{dh}{dt}$$

$$\frac{d\theta}{dt} = \frac{2}{\sin\frac{\theta}{2}}\frac{dh}{dt}$$

However, $\sin\frac{\theta}{2} = \frac{\sqrt{1^2 - (1-h)^2}}{1} = \sqrt{2h - h^2}$. Thus,

$$\frac{d\theta}{dt} = \frac{2}{\sqrt{2h - h^2}}\frac{dh}{dt}$$

Substituting this result into Eq. (1),

$$\frac{d\forall}{dt} = 6(2h - h^2)\left(\frac{2}{\sqrt{2h - h^2}}\frac{dh}{dt}\right) = 12\sqrt{2h - h^2}\frac{dh}{dt} \qquad \textbf{(2)}$$

Since the liquids are assumed incompressible, there volume remain the same. Thus,

$$\left(\frac{d\forall}{dt}\right) = V_A A_A + V_B A_B = (4\text{ m/s})(\pi)(0.02\text{ m})^2 + (2\text{ m/s})(\pi)(0.03\text{ m})^2$$
$$= 0.010681\text{ m}^3/\text{s}$$

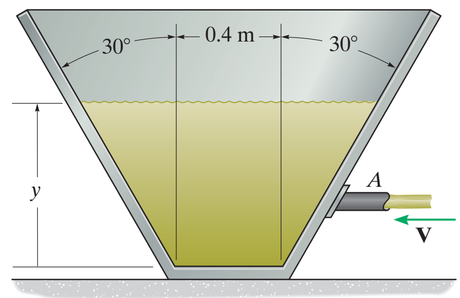
From Eq. (2),

$$0.010681 = 12\sqrt{2h - h^2}\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{0.890(10^{-3})}{\sqrt{2h - h^2}}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\frac{0.890(10^{-3})}{\sqrt{2h - h^2}}\text{ m/s}$

4–101. Hexylene glycol is flowing into the container at a constant rate of 600 kg/min. Determine the rate at which the level is rising when $y = 0.5$ m. The container is in the form of a conical frustum. *Hint*: the volume of a cone is $\forall = \frac{1}{3}\pi r^2 h$. $\rho_{hg} = 924\ \text{kg/m}^3$.



SOLUTION

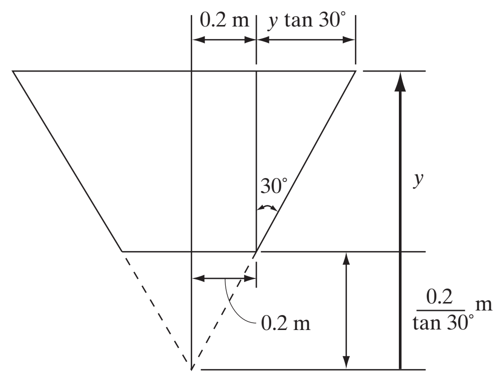
Since the control volume contains hexylene glycol in the tank, Fig. *a*, its volume is

$$\forall = \frac{1}{3}\pi(0.2 + y\tan 30°)^2 + \left(y + \frac{0.2}{\tan 30°}\text{ m}\right) - \frac{1}{3}\pi(0.2\text{ m})^2\left(\frac{0.2}{\tan 30°}\text{ m}\right)$$

$$= \frac{1}{3}\pi\left(\frac{1}{3}y^3 + 0.2\sqrt{3}y^2 + 0.12y\right)$$

$$\frac{\partial\forall}{\partial t} = \frac{1}{3}\pi\left(y^2\frac{\partial y}{\partial t} + 0.4\sqrt{3}y\frac{\partial y}{\partial t} + 0.12\frac{\partial y}{\partial t}\right)$$

$$= \frac{1}{3}\pi\left(y^2 + 0.4\sqrt{3}y + 0.12\right)\frac{\partial y}{\partial t}$$



(a)

Thus,

$$\frac{\partial}{\partial t}\int_{cv}\rho_{hg}\,d\forall + \int_{cs}\rho_{hg}\mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_{hg} is constant, it can be factored out from the integrals. Also, the average velocity will be used. Thus, the equation reduces to

$$\rho_{hg}\frac{\partial\forall}{\partial t} - \rho_{hg}V_A A_A = 0$$

Here, $\rho_{hg}V_A A_A = \dot{m}_{hg} = \left(600\frac{\text{kg}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 10\ \text{kg/s}$ .Then

$$\left(924\ \text{kg/m}^3\right)\left[\frac{1}{3}\pi\left(y^2 + 0.4\sqrt{3}y + 0.12\right)\frac{\partial y}{\partial t}\right] - 10\ \text{kg/s} = 0$$

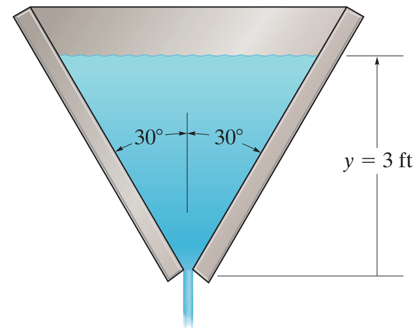
$$\frac{\partial y}{\partial t} = \left[\frac{15}{462\pi\left(y^2 + 0.4\sqrt{3}y + 0.12\right)}\right]\text{m/s}$$

When $y = 0.5$ m

$$\frac{\partial y}{\partial t} = \frac{15}{462\pi\left[0.5^2 + 0.4\sqrt{3}(0.5) + 0.12\right]} = 0.0144\ \text{m/s}$$ **Ans.**

Ans:
0.0144 m/s

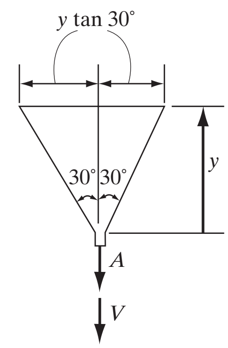
4–102. Water in the triangular trough is at a depth of $y = 3$ ft. If the drain is opened at the bottom, and water flows out at a rate of $V = (8.02y^{1/2})$ ft/s, where y is in feet, determine the time needed to fully drain the trough. The trough has a width of 2 ft. The slit at the bottom has a cross-sectional area of 24 in^2.



SOLUTION

Control Volume. The deformable control volume shown in Fig. *a*. will be considered. Its volume at any instant is

$$\forall = 2\left[\frac{1}{2}(y\tan 30°)y\right](2\text{ ft}) = (1.1547y^2)\text{ ft}^3$$



(a)

Continuity Equation. Since water has a constant density.

$$\rho_w\left[\frac{\partial}{\partial t}\int_{cv} d\forall + \int_{cs}\mathbf{V}\cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\forall + VA = 0$$

$$\frac{\partial}{\partial t}(1.1547y^2) + (8.02y^{\frac{1}{2}})\left(\frac{24}{144}\text{ ft}^2\right) = 0$$

$$2.3094y\frac{\partial y}{\partial t} = -1.3367y^{\frac{1}{2}}$$

$$\frac{\partial y}{\partial t} = -0.5788y^{-\frac{1}{2}}$$

Integrating,

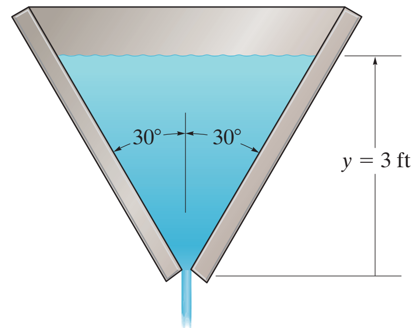
$$\int_0^t dt = \int_{3\text{ ft}}^0 -1.7277y^{\frac{1}{2}}dy$$

$$t = -1.7277\left(\frac{2}{3}y^{\frac{3}{2}}\right)\Bigg|_{3\text{ ft}}^0$$

$$t = 5.99\text{ s}$$ **Ans.**

Ans:
5.99 s

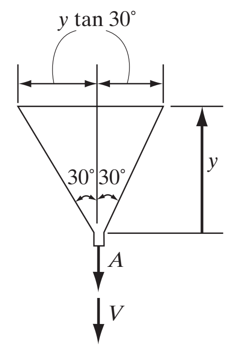
4–103. Water in the triangular trough is at a depth of $y = 3$ ft. If the drain is opened at the bottom, and water flows out at a rate of $V = \left(8.02y^{1/2}\right)$ ft/s, where y is in feet, determine the time needed for the water to reach a depth of $y = 2$ ft. The trough has a width of 2 ft. The slit at the bottom has a cross-sectional area of 24 in^2.



SOLUTION

Control volume. The deformable control volume shown in Fig. *a* will be considered. Its volume at any instant is

$$\forall = 2\left[\frac{1}{2}(y \tan 30°)y\right](2\text{ ft}) = \left(1.1547y^2\right)\text{ ft}^3$$



(a)

Continuity Equation. Since water has a constant density.

$$\rho_w\left[\frac{\partial}{\partial t}\int_{\text{cv}} d\forall + \int_{\text{cs}} \mathbf{V}_{f/\text{cs}} \cdot d\mathbf{A}\right] = 0$$

$$\frac{\partial}{\partial t}\forall + VA = 0$$

$$\frac{\partial}{\partial t}\left(1.1547y^2\right) + \left(8.02y^{\frac{1}{2}}\right)\left(\frac{24}{144}\text{ ft}^2\right) = 0$$

$$2.3094y\frac{\partial y}{\partial t} = -1.3367y^{\frac{1}{2}}$$

$$\frac{\partial y}{\partial t} = -0.5788y^{-\frac{1}{2}}$$

Integrating,

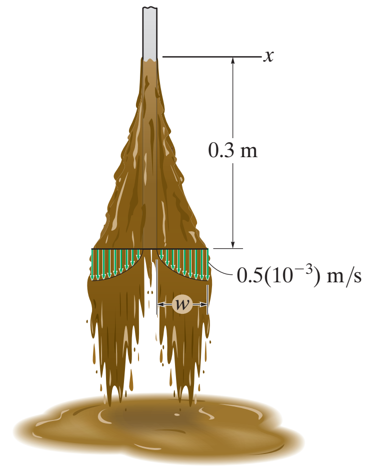
$$\int_0^t dt = \int_{3\text{ ft}}^{2\text{ ft}} -1.7277y^{\frac{1}{2}}dy$$

$$t = -1.7277\left(\frac{2}{3}y^{\frac{3}{2}}\right)\Bigg|_{3\text{ ft}}^{2\text{ ft}}$$

$$t = 2.73\text{ s}$$ **Ans.**

Ans:
2.73 s

***4–104.** As part of a manufacturing process, a 0.1-m-wide plate is dipped into hot tar and then lifted out, causing the tar to run down and then off the sides of the plate as shown. The thickness w of the tar at the bottom of the plate decreases with time t, but it still is assumed to maintain a linear variation along the plate as shown. If the velocity profile at the bottom of the plate is approximately parabolic, such that $u = \left[0.5\left(10^{-3}\right)(x/w)^{1/2}\right]$ m/s, where x and w are in meters, determine w as a function of time. Initially, when $t = 0$, $w = 0.02$ m.



SOLUTION

The flow is considered one dimensional since its velocity is directed downward. The control volume is shown in Fig. *a* and its volume changes with time. At a particular instant it is

$$\forall = \frac{1}{2}w(0.3\text{ m})(0.1\text{ m}) = (0.015w)\text{ m}^3$$

Thus,

$$\frac{\partial \forall}{\partial t} = 0.015\frac{\partial w}{\partial t}$$

The opened control surface is shown shaded in Fig. *a*. The differential area element is $dA = bdx$.

$$\frac{\partial}{\partial t}\int_{cv}\rho_t d\forall + \int_{cs}\rho_t \mathbf{V}\cdot d\mathbf{A} = 0$$

Since the tar is assumed to be incompressible, ρ_t is constant. Also, the velocity of the tar is always directed perpendicular to the opened surface. Hence the above equation reduces to

$$\rho_t\frac{\partial \forall}{\partial t} + \rho_t\int_{cs} udA = 0$$

$$\frac{\partial \forall}{\partial t} = -\int_{cs} udA$$

The negative sign indicates that $\forall$ is decreasing with time. Then

$$0.015\frac{dw}{dt} = -\int_0^w 0.5\left(10^{-3}\right)\left(\frac{x}{w}\right)^{\frac{1}{2}}(0.1dx)$$

$$0.015\frac{dw}{dt} = -\frac{0.5\left(10^{-3}\right)(0.1)}{w^{\frac{1}{2}}}\left(\frac{2}{3}x^{\frac{3}{2}}\right)\Bigg|_0^w$$

$$0.015\frac{dw}{dt} = -3.333\left(10^{-5}\right)w$$

Or, the integral $\int_{cs} udA$ is equal to the volume of the parabolic block under the velocity profile, ie, $\int_{cs} udA = \frac{2}{3}\left[0.5\left(10^{-3}\right)\right]w(0.1) = 0.0333\left(10^{-3}\right)w$

$$\frac{dw}{dt} = -2.222\left(10^{-3}\right)w$$

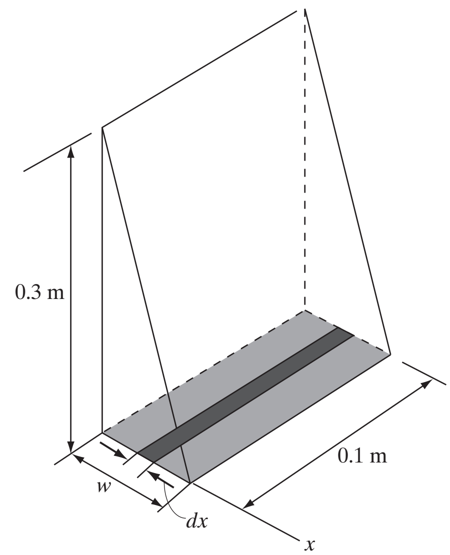
$$-\int_{0.02\text{ m}}^{w}\frac{dw}{2.222\left(10^{-3}\right)w} = \int_0^t dt$$

$$-450\ln\frac{w}{0.02} = t$$

$$\ln\frac{w}{0.02} = -\frac{t}{450}$$

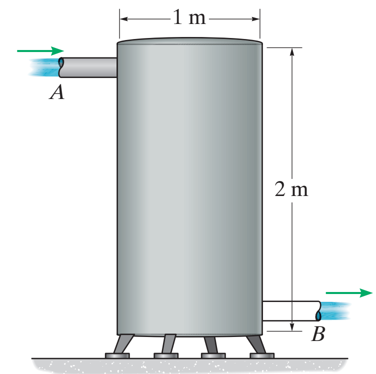
$$\frac{w}{0.02} = e^{-t/450}$$

$$w = \left(0.02e^{-t/450}\right)\text{ m} \qquad \textbf{Ans.}$$



(a)

4–105. The cylindrical tank in a food-processing plant is filled with a concentrated sugar solution having an initial density of $\rho_s = 1400\ \text{kg/m}^3$. Water is piped into the tank at A at $0.03\ \text{m}^3/\text{s}$ and mixes with the sugar solution. If an equal flow of the diluted solution exits at B, determine the amount of water that must be added to the tank so that the density of the sugar solution is reduced by 10% of its original value.



SOLUTION

The control volume considered here is the volume of the tank. It is a fixed control volume since its volume does not change throughout the mixing.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}_{ds} \cdot d\mathbf{A} = 0$$

$$\forall \frac{\partial \rho}{\partial t} + \rho Q - \rho_w Q = 0$$

$$\forall \frac{\partial \rho}{\partial t} = Q(\rho_w - \rho)$$

$$\int_{\rho_s}^{\rho} \frac{\partial \rho}{\rho_w - \rho} = \frac{Q}{\forall}\int_0^t \partial t$$

$$-\ln(\rho_w - \rho)\Big|_{\rho_s}^{\rho} = \frac{Q}{\forall}t$$

$$-\ln\left(\frac{\rho_w - \rho}{\rho_w - \rho_s}\right) = \frac{Q}{\forall}t$$

$$t = \frac{\forall}{Q}\ln\left(\frac{\rho_w - \rho_s}{\rho_w - \rho}\right)$$

Here, $\forall = \pi(0.5\ \text{m})^2(0.2\ \text{m}) = 0.5\pi\ \text{m}^3$, and it is required that $\rho = 0.9\rho_s = 0.9\left(1400\ \text{kg/m}^3\right) = 1260\ \text{kg/m}^3$. Then

$$t = \left(\frac{0.5\pi\ \text{m}^3}{0.03\ \text{m}^3/\text{s}}\right)\ln\left(\frac{1000\ \text{kg/m}^3 - 1400\ \text{kg/m}^3}{1000\ \text{kg/m}^3 - 1260\ \text{kg/m}^3}\right)$$

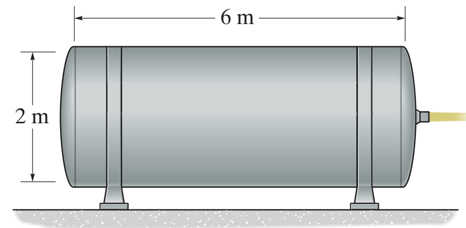
$$= 22.556$$

The amount of water to be added is

$$\forall_w = Qt = \left(0.03\ \text{m}^3/\text{s}\right)(22.556) = 6.77\ \text{m}^3$$ **Ans.**

Ans:
$6.77\ \text{m}^3$

4–106. The cylindrical pressure vessel contains methane at an initial absolute pressure of 2 MPa. If the nozzle is opened, the mass flow depends upon the absolute pressure and is $\dot{m} = 3.5\left(10^{-6}\right)p$ kg/s, where p is in pascals. Assuming the temperature remains constant at 20°C (PASCALs), determine the time required for the pressure to drop to 1.5 MPa.



SOLUTION

From Appendix A, the gas constant for Methane is $R = 518.3\ \mathrm{J/(kg\cdot K)}$. Using the ideal gas law with $T = 20°\mathrm{C} + 273 = 293$ K which is constant throughout,

$$p = \rho RT; \qquad p = \rho\left(518.3\ \mathrm{J/(kg\cdot k)}\right)(293\ \mathrm{k})$$

$$p = 151861.9\rho$$

$$\rho = 6.5849\left(10^{-6}\right)p \qquad \textbf{(1)}$$

The control volume considered is the volume of tank which contains Methane. Since the tank is fully filled at all times, the control volume can be classified as fixed.

$$\frac{d}{dt}\int_{\mathrm{cv}} \rho d\forall + \int_{\mathrm{cs}} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

Here $\int_{\mathrm{cv}} d\forall = \forall$ (fixed control volume) and $\dot{m} = \int_{\mathrm{cv}} \rho\mathbf{V}\cdot\mathbf{A}$. Then

$$\forall\frac{d\rho}{dt} + \dot{m} = 0 \qquad \textbf{(2)}$$

Since, $\forall = \pi(1\ \mathrm{m})^2(6\ \mathrm{m}) = 6\pi\ \mathrm{m}^3$ and $\dot{m} = \left[3.5\left(10^{-6}\right)p\right]$ kg/s, then Eq (1),

$$\frac{d\rho}{dt} = 6.5849\left(10^{-6}\right)\frac{dp}{dt}$$

Substitute these results into Eq. (2),

$$6\pi\left[6.5849\left(10^{-6}\right)\frac{dp}{dt}\right] + 3.5\left(10^{-6}\right)p = 0$$

$$\frac{dp}{dt} = -0.02820p$$

$$\int_{p_0}^{p}\frac{dp}{p} = -0.02820\int_0^t dt$$

$$\ln\left(\frac{p}{p_0}\right) = -0.02820t$$

$$t = -35.46\ln\left(\frac{p}{p_0}\right)$$

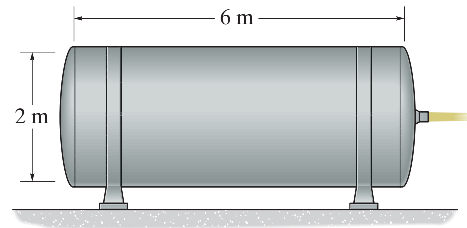
Here $p_0 = 2$ Mpa. Thus when $p = 1.5$ Mpa,

$$t = -35.46\ln\left(\frac{1.5\ \mathrm{MPa}}{2\ \mathrm{MPa}}\right)$$

$$= 10.2\ \mathrm{s} \qquad \textbf{Ans.}$$

Ans:
10.2 s

4–107. The cylindrical pressure vessel contains methane at an initial absolute pressure of 2 MPa. If the nozzle is opened, the mass flow depends upon the absolute pressure and is $\dot{m} = 3.5\left(10^{-6}\right)p$ kg/s, where P is in PASCALS. Assuming the temperature remains constant at 20°C, determine the pressure in the tank as a function of time. Plot this pressure (vertical axis) versus the time for $0 \leq t \leq 15$ s. Give values for increments of $\Delta t = 3$ s.



SOLUTION

From Appendix A, the gas constant for Methane is $R = 518.3$ J/kg·K using the ideal gas law with $T = 20°\text{C} + 273 = 293$ K, which is constant throughout,

$$p = \rho RT; \qquad p = \rho(518.3 \text{ J/kg}\cdot\text{k})(293 \text{ k})$$

$$p = 151861.9\rho$$

$$\rho = 6.5849\left(10^{-6}\right)p \qquad \textbf{(1)}$$

The control volume considered is the volume of tank which contains Methane. Since the tank is fully filled at all times, the control volume can be classified as fixed.

$$\frac{d}{dt}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V}_{ds} \cdot d\mathbf{A} = 0$$

Here $\int_{cv} d\forall = \forall$ (fixed control volume) and $\dot{m} = \int_{cv} \rho \mathbf{V} \cdot d\mathbf{A}$. Then

$$\forall \frac{d\rho}{dt} + \dot{m} = 0 \qquad \textbf{(2)}$$

Since, $\forall = \pi(1 \text{ m})^2(6 \text{ m}) = 6\pi \text{ m}^3$ and $\dot{m} = \left[3.5\left(10^{-6}\right)p\right]$ kg/s, then from Eq (1),

$$\frac{d\rho}{dt} = 6.5849\left(10^{-6}\right)\frac{dp}{dt}$$

Substitute these results into Eq (2),

$$6\pi\left[6.5849\left(10^{-6}\right)\frac{\partial p}{\partial t}\right] + 3.5\left(10^{-6}\right)p = 0$$

$$\frac{dp}{dt} = -0.02820p$$

$$\int_{p_0}^{p} \frac{dp}{p} = -0.02820\int_0^t dt$$

$$\ln\left(\frac{p}{p_0}\right) = -0.02820t$$

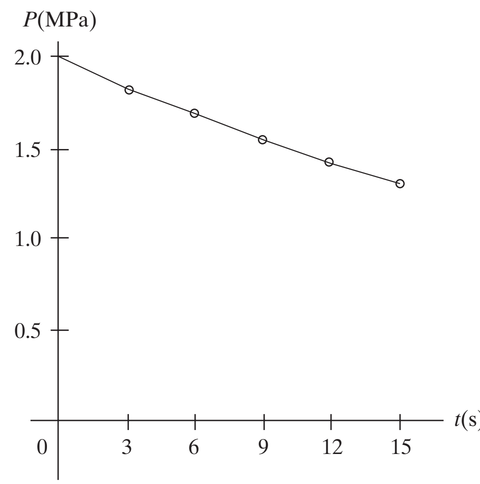
$$\frac{p}{p_0} = e^{-0.02820t}$$

$$p = p_0 e^{-0.02820t}$$

Here $p_0 = 2$MPa, then

$$p = \left(2e^{-0.02820t}\right)\text{Mpa, where } t \text{ is in seconds} \qquad \textbf{Ans.}$$

The plot of p vs t is shown in Fig. a

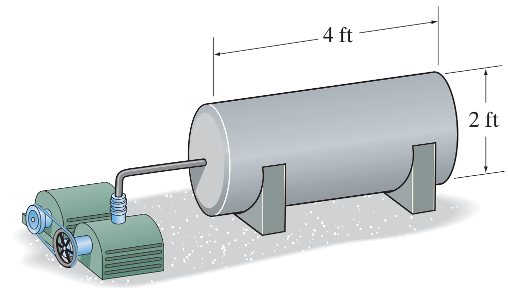


(a)

t(s)	0	3	6	9	12	15
p(MPa)	2.0	1.84	1.69	1.55	1.43	1.31

Ans:
$p = \left(2\, e^{-0.0282t}\right)$ MPa, where t is in seconds

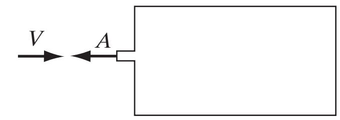
***4–108.** As nitrogen is pumped into the closed cylindrical tank, the mass flow through the tube is $\dot{m} = \left(0.8\rho^{-1/2}\right)$ slug/s. Determine the density of the nitrogen within the tank when $t = 5$ s from the time the pump is turned on. Assume that initially there is 0.5 slug of nitrogen in the tank.



SOLUTION

Control Volume. The fixed control volume is shown in Fig. *a*. This control volume has a constant volume of

$$\forall = \pi(1\text{ ft})^2(4\text{ ft}) = 4\pi\text{ ft}^3$$



(a)

The density of the nitrogen within the control volume changes with time and therefore contributes to local changes.

Continuity Equation. Realizing that $\dot{m} = \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A}$,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$\frac{\partial \rho}{\partial t}\forall - \dot{m} = 0$$

$$4\pi\frac{\partial \rho}{\partial t} - 0.8\rho^{\frac{-1}{2}} = 0$$

$$\frac{\partial \rho}{\partial t} = \frac{0.2}{\pi}\rho^{\frac{-1}{2}}$$

Integrating,

$$\int_0^t dt = \int_{\rho_o}^{\rho} 5\pi\rho^{\frac{1}{2}}\, d\rho$$

$$t = 5\pi\left(\frac{2}{3}\rho^{\frac{3}{2}}\right)\Bigg|_{\rho_o}^{\rho}$$

$$t = \frac{10\pi}{3}\left(\rho^{\frac{3}{2}} - \rho_o^{\frac{3}{2}}\right)$$

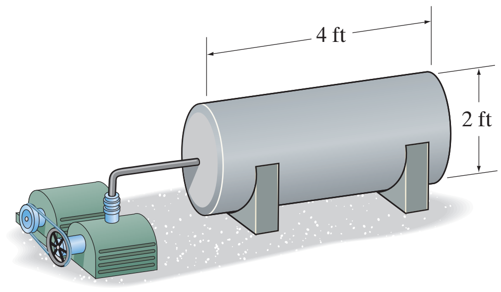
$$\rho = \left(\frac{3t}{10\pi} + \rho_o^{\frac{3}{2}}\right)^{\frac{2}{3}}\text{ slug/ft}^3$$

Here, $\rho_o = \dfrac{0.5\text{ slug}}{4\pi\text{ ft}^3} = \dfrac{0.125}{\pi}$ slug/ft^3. Then, when $t = 5$ s,

$$\rho = \left[\frac{3(5)}{10\pi} + \left(\frac{0.125}{\pi}\right)^{\frac{3}{2}}\right]^{\frac{2}{3}}\text{ slug/ft}^3$$

$$= 0.618\text{ slug/ft}^3 \qquad \textbf{Ans.}$$

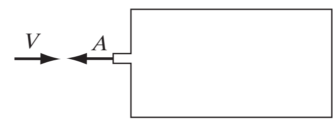
4–109. As nitrogen is pumped into the closed cylindrical tank, the mass flow through the tube is $m = \left(0.8\rho^{-1/2}\right)$ slug/s. Determine the density of the nitrogen within the tank when $t = 10$ s from the time the pump is turned on. Assume that initially there is 0.5 slug of nitrogen in the tank.



SOLUTION

Control Volume. The fixed volume is shown in Fig. *a*. This control volume has a constant volume of

$$\forall = \pi(1\text{ ft})^2(4\text{ ft}) = 4\pi\text{ ft}^3$$



(a)

The density of the nitrogen within the control volume changes with time and therefore contributes to local changes.

Continuity Equation. Realizing that $\dot{m} = \int_{cs} \rho\mathbf{V}\cdot d\mathbf{A}$,

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$\frac{\partial\rho}{\partial t}\forall - \dot{m} = 0$$

$$4\pi\frac{\partial\rho}{\partial t} - 0.8\rho^{\frac{-1}{2}} = 0$$

$$\frac{\partial\rho}{\partial t} = \frac{0.2}{\pi}\rho^{\frac{-1}{2}}$$

Integrating,

$$\int_0^t dt = \int_{\rho_o}^{\rho} 5\pi\rho^{\frac{1}{2}}d\rho$$

$$t = 5\pi\left(\frac{2}{3}\rho^{\frac{3}{2}}\right)\Bigg|_{\rho_o}^{\rho}$$

$$t = \frac{10\pi}{3}\left(\rho^{\frac{3}{2}} - \rho_o^{\frac{3}{2}}\right)$$

$$\rho = \left(\frac{3t}{10\pi} + \rho_o^{\frac{3}{2}}\right)^{\frac{2}{3}}\text{ slug/ft}^3$$

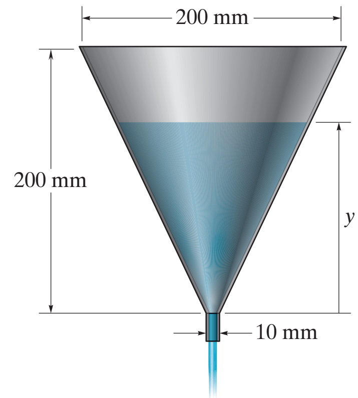
Here, $\rho_o = \dfrac{0.5\text{ slug}}{4\pi\text{ ft}^3} = \dfrac{0.125}{\pi}$ slug/ft^3. Then, when $t = 10$ s,

$$\rho = \left[\frac{3(10)}{10\pi} + \left(\frac{0.125}{\pi}\right)^{\frac{3}{2}}\right]^{\frac{2}{3}}\text{ slug/ft}^3$$

$$= 0.975\text{ slug/ft}^3 \qquad \textbf{Ans.}$$

Ans:
0.975 slug/ft^3

4–110. Water flows out of the stem of the funnel at an average speed of $V = \left(3e^{-0.05t}\right)$ m/s, where t is in seconds. Determine the average speed at which the water level is falling at the instant $y = 100$ mm. At $t = 0$, $y = 200$ mm.



SOLUTION

The control volume is the volume of the water in the funnel. This volume changes with time.

$$\frac{d}{dt}\int_{cv} \rho_w d\forall + \int_{cs} \rho_w \mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_w is constant (in compressible), it can be factored out from the integrals

$$\rho_w \frac{d\forall}{dt} + \rho_w \int_{cs} \mathbf{V}\cdot d\mathbf{A} = 0$$

Here, the average velocity will be used. Then

$$\frac{d\forall}{dt} + VA = 0$$

$$\frac{d\forall}{dt} + \left(3e^{-0.05t}\right)\left[\frac{\pi}{4}(0.01\text{ m})^2\right]$$

$$\frac{d\forall}{dt} + 75\left(10^{-6}\right)\pi e^{-0.05t} = 0 \qquad \textbf{(1)}$$

The volume of the control volume at a particular instant is

$$\forall = \frac{1}{3}\pi\tau^2 y$$

From the geometry shown in Fig. a,

$$\frac{r}{y} = \frac{0.1\text{ m}}{0.2\text{ m}}; \qquad r = \frac{1}{2}y$$

Then

$$\forall = \frac{1}{3}\pi\left(\frac{1}{2}y\right)^2 y = \left(\frac{1}{12}\pi y^3\right)\text{ m}^3$$

$$\frac{d\forall}{dt} = \frac{1}{12}\pi\left(3y^2\frac{dy}{dt}\right)$$

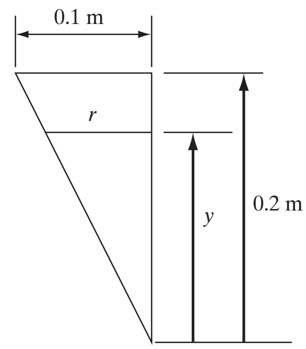
$$\frac{d\forall}{dt} = \frac{1}{4}\pi y^2\frac{dy}{dt}$$

Substitute into Eq. (1),

$$\frac{1}{4}\pi y^2\frac{dy}{dt} + 75\left(10^{-6}\right)\pi e^{-0.05t} = 0$$

$$y^2\frac{dy}{dt} = -0.3\left(10^{-3}\right)e^{-0.05t} \qquad \textbf{(2)}$$

$$\frac{dy}{dt} = -\frac{0.3\left(10^{-3}\right)e^{-0.05t}}{y^2} \qquad \textbf{(3)}$$



(a)

4–110. Continued

Separating the variables of Eq. (2)

$$\int_{0.2\text{ m}}^{0.1\text{ m}} y^2 dy = -0.3\left(10^{-3}\right)\int_0^t e^{-0.05t}dt$$

$$\left.\frac{y^3}{3}\right|_{0.2\text{ m}}^{0.1\text{ m}} = -0.3\left(10^{-3}\right)\left.\left(\frac{e^{-0.05t}}{-0.05}\right)\right|_0^t$$

$$-2.3333\left(10^{-3}\right) = 6\left(10^{-3}\right)e^{-0.05t}\Big|_0^t$$

$$-2.3333\left(10^{-3}\right) = 6\left(10^{-3}\right)\left(e^{-0.05t} - 1\right)$$

$$e^{-0.05t} = 0.6111$$

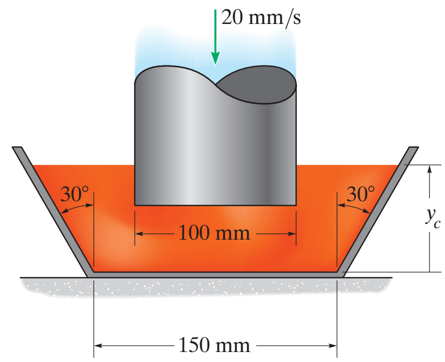
Substitute this value and $y = 0.1$ m into Eq. (3),

$$\frac{dy}{dt} = -\frac{0.3\left(10^{-3}\right)(0.6111)}{0.1^2} = -0.0183\text{ m/s} \qquad \textbf{Ans.}$$

The negative sign indicates that y is decreasing, ie., the water level is falling.

Ans:
-0.0183 m/s

4–111. A part is manufactured by placing molten plastic into the trapezoidal container and then moving the cylindrical die down into it at a constant speed of 20 mm/s. Determine the average speed at which the plastic rises in the form as a function of y_c. The container has a width of 150 mm.



SOLUTION

The control volume segments shown shaded in Fig. *a* can be consider fixed at a particular instant. At this instant, no local changes occur since the molten plastic is incompressible. Also, its density is constant. If we use average velocities. Also then

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 + V_B A_B - Q_A = 0 \qquad \textbf{(1)}$$

Here, $A_B = (2y_c \tan 30° + 0.15\text{ m})(0.15\text{ m}) - \pi(0.05\text{ m})^2$

$$= (0.3 \tan 30° y_c + 0.01465)\text{ m}^2$$

$$V_B = \frac{\partial y_c}{\partial t}$$

The volume of the die submerged in the plastic is

$$\forall_d = \pi(0.05\text{ m})^2 y_d = \left(0.0025\pi y_d\right)\text{ m}^3$$

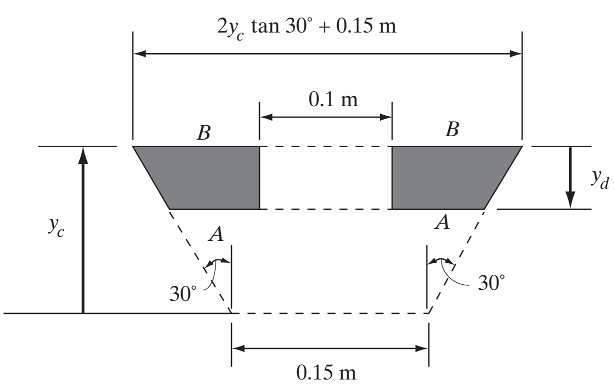
Realizing that $\frac{dy_d}{dt} = V_d = 0.02\text{ m/s}$, then

$$Q_A = \frac{d\forall_2}{dt} = 0.0025\pi \frac{dy_2}{dt} = (0.0025\pi)(0.02\text{ m/s}) = 50\left(10^{-6}\right)\pi\text{ m}^3/\text{s}$$

Substitute these results into Eq (1),

$$\frac{dy_o}{dt}(0.3 \tan 30° y_c + 0.01465) - 50\left(10^{-6}\right)\pi = 0$$

$$\frac{dy_c}{dt} = \left(\frac{0.157\left(10^{-3}\right)}{0.173y_c + 0.0146}\right)\text{ m/s} \qquad \textbf{Ans.}$$

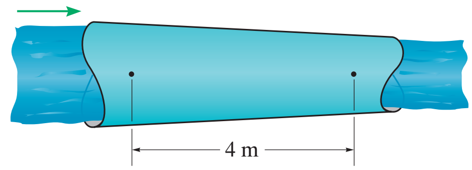


(a)

Ans:

$$\frac{dy_c}{dt} = \left(\frac{0.157\left(10^{-3}\right)}{0.173y_c + 0.0146}\right)\text{ m/s}$$

5–1. Water flows in the horizontal pipe. Determine the average decrease in pressure in 4 m along a horizontal streamline so that the water has an acceleration of 0.5 m/s^2.



SOLUTION

$$\frac{1}{\rho}\frac{dp}{ds} + a_s + g\sin\theta = 0$$

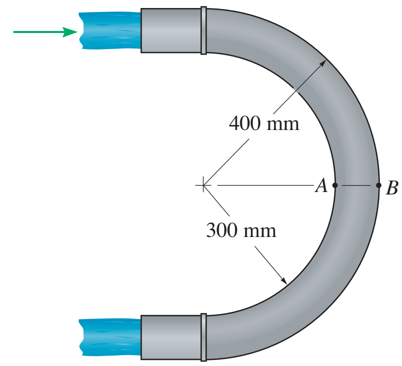
$$\left(\frac{1}{1000\ \text{kg/m}^3}\right)\left(\frac{\Delta p}{4\ \text{m}}\right) + 0.5\ \text{m/s}^2 + 0 = 0$$

$$\Delta p = -2000\ \text{Pa} = -2\ \text{kPa} \qquad \textbf{Ans.}$$

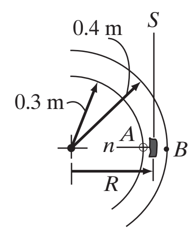
The negative sign indicates that the pressure drops as the water flows from A to B.

Ans:
-2 kPa

5–2. The horizontal 100-mm-diameter pipe is bent so that its inner radius is 300 mm. If the pressure difference between points A and B is $p_B - p_A = 300$ kPa, determine the volumetric flow of water through the pipe.



SOLUTION



(a)

Referring to the coordinate system shown in Fig. a, we notice that $dn = -dR$ since the n and R axes are opposite in sense. Also, $dz = 0$. Since the pipe is lying on the horizontal plane, then

$$-\frac{dp}{dn} - \rho g\frac{dz}{dn} = \frac{\rho V^2}{R}$$

$$\frac{dp}{dR} = \frac{\rho V^2}{R}$$

$$\int_{p_A}^{p_B} dp = \rho V^2 \int_{0.3\text{ m}}^{0.4\text{ m}} \frac{dR}{R}$$

$$p_B - p_A = \rho V^2 \ln\frac{4}{3}$$

$$300\left(10^3\right)\text{ N/m}^2 = \left(1000\text{ kg/m}^3\right)V^2 \ln\frac{4}{3}$$

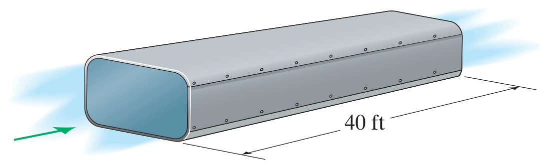
$$V = 32.29\text{ m/s}$$

The volumetric flow is

$$Q = VA = \left(32.29\text{ m/s}\right)\left[\pi(0.05\text{ m})^2\right] = 0.254\text{ m}^3/\text{s}$$ **Ans.**

Ans:
$0.254\text{ m}^3/\text{s}$

5–3. Air at 60°F flows through the horizontal tapered duct. Determine the acceleration of the air if on a streamline the pressure is 14.7 psi and 40 ft away the pressure is 14.6 psi.



SOLUTION

The pressure is approximately 1 atmosphere along the length in question. From Appendix A, the density of air at $T = 60°$ F is $\rho = 0.00237$ slug/ft^3. This density will be used, since ρ will change only slightly with the small change in pressure. Since the duct is level, $\sin\theta = 0$.

$$\frac{1}{\rho}\frac{dp}{ds} + a_s + g\sin\theta = 0$$

$$\left(\frac{1}{0.00237\ \text{slug/ft}^3}\right)\left[\frac{\left[(-0.1\ \text{lb/in}^2)\frac{\text{lb}}{\text{in}^2}\right]\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2}{40\ \text{ft}}\right] + a_s + 0 = 0$$

$$a_s = 151.90\ \text{ft/s}^2 = 152\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Also, realizing that $z_B = z_A = 0$, since the duct is level,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\Delta p = p_B - p_A = \left(\frac{V_A^2 - V_B^2}{2}\right)\rho$$

For constant acceleration $V_B^2 = V_A^2 + 2a_c(s_B - s_A)$ or $\dfrac{V_A^2 - V_B^2}{2} = -a_s(s_B - s_A)$. Then

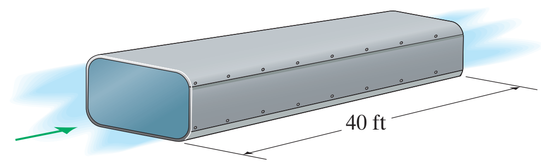
$$\Delta p = -\rho(s_B - s_A)a_s$$

$$a_s = \frac{-\Delta p}{\rho(s_B - s_A)} = \frac{(-0.1\ \text{lb/in}^2)\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^2}{(0.00237\ \text{slug/ft}^3)(40\ \text{ft})}$$

$$a_s = 152\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
152 ft/s^2

***5–4.** Air at 60°F flows through the horizontal tapered duct. Determine the average decrease in pressure in 40 ft, so that the air has an acceleration of 150 $\mathrm{ft/s^2}$.



SOLUTION

From Appendix A, the density of air at $T = 60°\,\mathrm{F}$ is $\rho = 0.00237\ \mathrm{slug/ft^3}$. This density will be used on the assumption that the change in p will be small, leading to only a small change in ρ. Since the duct is level, $\sin\theta = 0$.

$$\frac{1}{\rho}\frac{dp}{ds} + a_s + g\sin\theta = 0$$

$$\left(\frac{1}{0.00237\ \mathrm{slug/ft^3}}\right)\left(\frac{\Delta p}{40\ \mathrm{ft}}\right) + 150\ \mathrm{ft/s^2} + 0 = 0$$

$$\Delta p = -14.22\,\frac{\mathrm{lb}}{\mathrm{ft^2}} = -14.2\ \mathrm{lb/ft^2}$$ **Ans.**

Also, realizing that $z_B - z_A = 0$, since the duct is level,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\Delta p = p_B - p_A = \left(\frac{V_A^2 - V_B^2}{2}\right)\rho$$

For constant acceleration $V_B = V_A^2 + 2a_s(s_B - s_A)$ or $\dfrac{V_A^2 - V_B^2}{2} = -a_s(s_B - s_A)$. Then

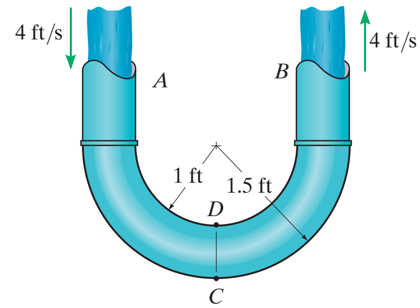
$$\Delta p = -\rho(s_B - s_A)a_s = -(0.00237\ \mathrm{slug/ft^3})(40\ \mathrm{ft})(150\ \mathrm{ft/s^2})$$

$$\Delta p = -14.2\,\frac{\mathrm{lb}}{\mathrm{ft^2}}$$ **Ans.**

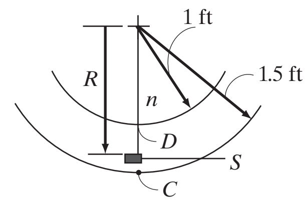
The negative sign indicates that the pressure drops as the air flows from A to B.

Note: 14.2 lb/ft is less then 0.1 psi, so indeed the change in pressure is small compared to $p \approx 1\ \mathrm{atm} = 14.7\ \mathrm{psi}$.

5–6. Water flows through the *horizontal* circular section with a uniform velocity of 4 ft/s. If the pressure at point D is 60 psi, determine the pressure at point C.



SOLUTION



(a)

Referring to the coordinate systems shown in Fig. a, we notice that $dn = -dR$ since the n and R axes are opposite in sense. Also, $dz = 0$ since the pipe is lying on the horizontal plane. Thus,

$$-\frac{dp}{dn} - \rho g\frac{dz}{dn} = \frac{\rho V^2}{R}$$

$$\frac{dp}{dR} = \frac{\rho V^2}{R}$$

$$\int_{p_D}^{p_C} dP = \rho V^2 \int_{1\text{ ft}}^{1.5\text{ ft}} \frac{dR}{R}$$

$$p_C - p_D = \rho V^2 \ln 1.5$$

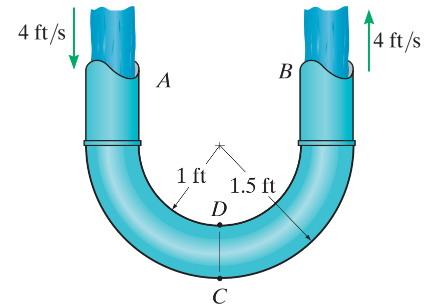
$$p_C = \rho V^2 \ln 1.5 + p_D$$

$$= \left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)(4\text{ ft/s})^2(\ln 1.5) + (60\text{ lb/in}^2)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2$$

$$= \left(8652.57\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = 60.09\text{ psi} = 60.1\text{ psi}$$ **Ans.**

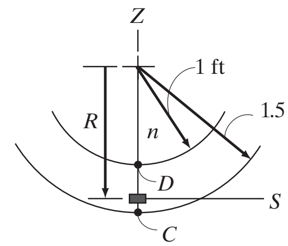
Ans:
60.1 psi

5–7. Solve prob. 5–6 assuming the pipe is *vertical*.



SOLUTION

Referring to the coordinate systems shown in Fig. *a*, we notice that $dn = dz$ and $dn = -dR$. Thus,



(a)

$$-\frac{dp}{dn} - \rho g\frac{dz}{dn} = \frac{\rho V^2}{R}$$

$$\frac{dp}{dR} - \gamma = \frac{\rho V^2}{R}$$

$$\int_{p_D}^{p_C} dp = \rho V^2 \int_{1\text{ ft}}^{1.5\text{ ft}} \frac{dR}{R} + \gamma \int_{1\text{ ft}}^{1.5\text{ ft}} dR$$

$$p_C - p_D = \rho V^2 \ln 1.5 + \gamma(0.5\text{ ft})$$

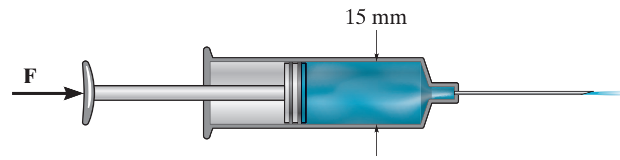
$$p_C = \rho V^2 \ln 1.5 + \gamma(0.5\text{ ft}) + p_D$$

$$= \left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)(4\text{ ft/s})^2(\ln 1.5) + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(0.5\text{ ft}) + \left(60\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2$$

$$= 8683.77\frac{\text{lb}}{\text{ft}^2}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = 60.3\text{ psi}$$ **Ans.**

Ans:
60.3 psi

***5–8.** By applying a force **F**, a saline solution is ejected from the 15-mm-diameter syringe through a 0.6-mm-diameter needle. If the pressure developed within the syringe is 60 kPa, determine the average velocity of the solution through the needle. Take $\rho = 1050 \text{ kg/m}^3$.



SOLUTION

The saline solution can be considered as an ideal fluid (incompressible and inviscid). Also, the flow is steady. Therefore, Bernoulli's equation is applicable. Applying this equation between a point in the syringe and the other point at the tip of the needle of which both points are on the central streamline,

$$\frac{p_s}{\rho} + \frac{V_s^{\,2}}{2} + gz_s = \frac{p_n}{\rho} + \frac{V_n^{\,2}}{2} + gz_n$$

Since $V_s <<<< V_n$, the term $\frac{V_s^{\,2}}{2}$ is negligible. Since the tip of the needle is exposed to the atmosphere, $p_n = 0$. Here, the datum will coincide with the central stream line. Then

$$\frac{60(10^3) \text{ N/m}^2}{1050 \text{ kg/m}^3} + 0 + 0 = 0 + \frac{V_n^{\,2}}{2} + 0$$

$$V_n = 10.69 \text{ m/s} = 10.7 \text{ m/s} \qquad \textbf{Ans.}$$

Using the continuity equation,

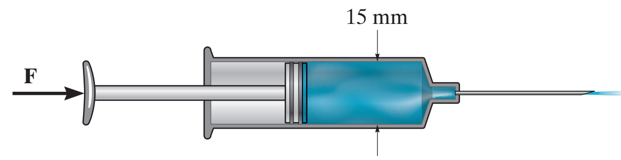
$$-V_s A_s + V_n A_n = 0$$

$$-V_s\left[\pi(0.0075 \text{ m})^2\right] + (10.69 \text{ m/s})\left\{\pi\left[0.3(10^{-3})\text{m}\right]^2\right\} = 0$$

$$V_s = 0.0171 \text{ m/s}$$

This result proves that V_s is indeed very small as compared to V_n. Therefore, the solution is acceptable.

5–9. By applying a force **F**, a saline solution is ejected from the 15-mm-diameter syringe through a 0.6-mm-diameter needle. Determine the average velocity of the solution through the needle as a function of the force F applied to the plunger. Plot this velocity (vertical axis) as a function of the force for $0 \le F \le 20$ N. Give values for increments of $\Delta F = 5$ N. Take $\rho = 1050\ \mathrm{kg/m^3}$.



SOLUTION

The saline solution can be considered as an ideal fluid (incompressible and inviscid). Also, the flow is steady. Therefore, Bernoulli's equation is applicable. Applying this equation between a point in the syringe and the other at the tip of the needle of which both points are on the central streamline,

$$\frac{p_s}{\rho} + \frac{V_s^{\,2}}{2} + gz_s = \frac{p_n}{\rho} + \frac{V_n^{\,2}}{2} + gz_n$$

Since $V_s <<<< V_n$ the term $\frac{V_s^{\,2}}{2}$ is negligible. Since the tip of the needle is exposed to the atmosphere, $p_n = 0$. Here, $P_s = \frac{F}{A_s} = \frac{F}{\pi(0.0075\ \mathrm{m})^2} = 5.659\left(10^3\right)$ F and the datum will coincide with the control streamline. Then

$$\frac{5.659\left(10^3\right)\ \mathrm{F}}{1050\ \mathrm{kg/m^3}} + 0 + 0 = 0 + \frac{V_n^{\,2}}{2} + 0$$

$$V_n = \left(3.2831\ \sqrt{F}\right)\ \mathrm{m/s}$$

$$V_n = \left(3.283\ \sqrt{F}\right)\ \mathrm{m/s} \text{ where } F \text{ is in N} \qquad \textbf{Ans.}$$

The plot of V vs F is shown in Fig. *a*.

Using the continuity equation,

$$-V_sA_s + V_nA_n = 0$$

$$-V_s\left[\pi(0.0075\ \mathrm{m})^2\right] + \left(3.2831\ \sqrt{F}\right)\left\{\pi\left[0.3\left(10^{-3}\right)\mathrm{m}\right]^2\right\} = 0$$

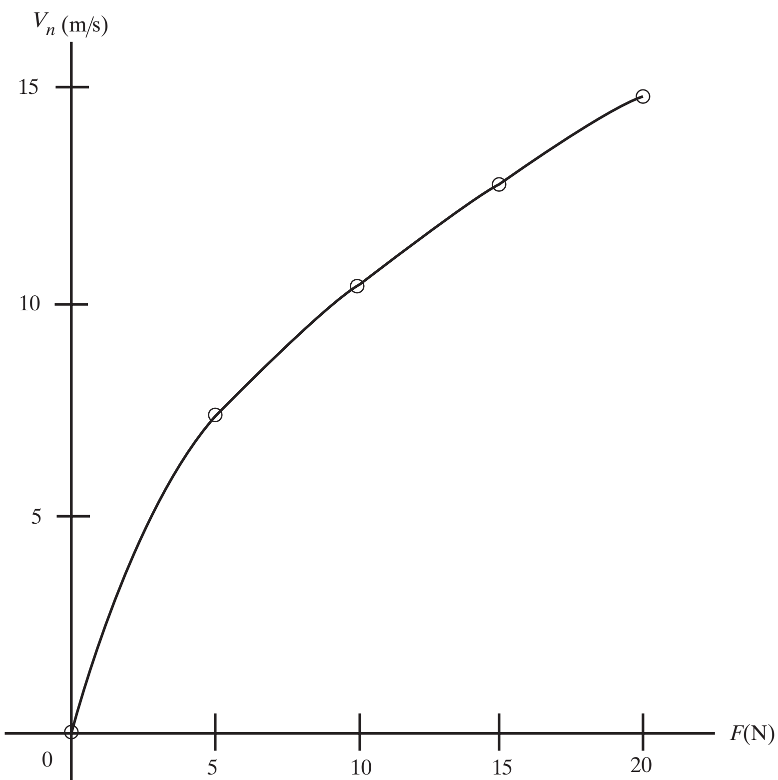
$$V_s = 0.00525\sqrt{F}$$

This result shows that V_s is indeed very small as compared to V_n. Therefore, the solution is acceptable.

5–9. Continued

This result shows that V_s is indeed very small as compared to V_n. Therefore, the solution is acceptable.

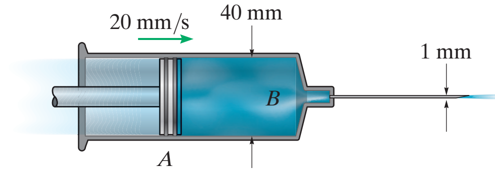
F(N)	0	5	10	15	20
V_n(m/s)	0	7.34	10.4	12.7	14.7



(a)

Ans:
$V_n = \left(3.283\sqrt{F}\right)$ m/s, where F is in N

5–10. An infusion pump produces pressure within the syringe that gives the plunger A a velocity of 20 mm/s. If the saline fluid has a density of $\rho_s = 1050\ \text{kg/m}^3$, determine the pressure developed in the syringe at B.



SOLUTION

The saline solution can be considered as an ideal fluid (incompressible and inviscid). Also, the flow is steady. Therefore, Bernoulli's equation is applicable. Applying this equation between a point in the syringe and the other at the tip of the needle of which both points are on the central streamline,

$$\frac{p_s}{\rho} + \frac{V_s^{\,2}}{2} + gz_s = \frac{p_n}{\rho} + \frac{V_n^{\,2}}{2} + gz_n$$

Since the tip of the needle is exposed to the atmosphere, $p_n = 0$. Here, the datum will coincide with the central streamline.

$$\frac{p_s}{\rho} + \frac{V_s^{\,2}}{2} + 0 = 0 + \frac{V_n^{\,2}}{2} + 0$$

$$p_s = \frac{\rho}{2}\left(V_n^{\,2} - V_s^{\,2}\right) \qquad \textbf{(1)}$$

Applying the continuity equation,

$$-V_s A_s + V_n A_n = 0$$

$$-\left[20\left(10^{-3}\right)\text{ m/s}\right]\left[\pi(0.02\text{ m})^2\right] + V_n\left\{\pi\left[0.5\left(10^{-3}\right)\text{ m}\right]^2\right\} = 0$$

$$V_n = 32.0\text{ m/s}$$

Substituting this value into Eq. (1),

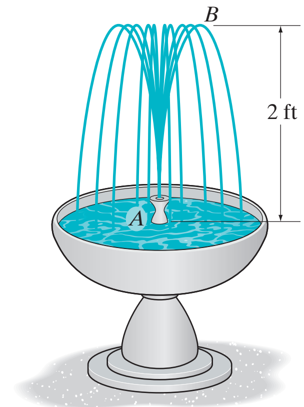
$$p_s = \left(\frac{1050\text{ kg/m}^3}{2}\right)\left\{(32.0\text{ m/s})^2 - \left[20\left(10^{-3}\right)\text{ m/s}\right]^2\right\}$$

$$= 537.60\left(10^3\right)\text{ Pa}$$

$$= 538\text{ kPa} \qquad \textbf{Ans.}$$

Ans:
538 kPa

5–11. If the fountain nozzle sprays water 2 ft into the air, determine the velocity of the water it leaves the nozzle at A.



SOLUTION

Bernoulli Equation. Since the water jet is in the open atmosphere at A and B, $p_A = p_B = 0$. Also, $v_B = 0$ since the jet achieves its maximum height at B. If the datum is set at A, $z_A = 0$ and $z_B = 2$ ft.

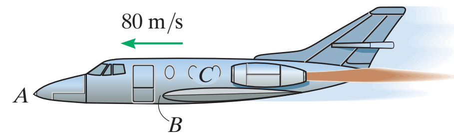
$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A + \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$0 + \frac{V_A^2}{2} + 0 = 0 + 0 + \left(32.2\ \text{ft/s}^2\right)(2\ \text{ft})$$

$$V_A = 11.3\ \text{ft/s} \qquad \textbf{Ans.}$$

Ans:
11.3 ft/s

***5–12.** The jet airplane is flying at 80 m/s in still air, A, at an altitude of 3 km. Determine the absolute stagnation pressure at the leading edge B of the wing.



SOLUTION

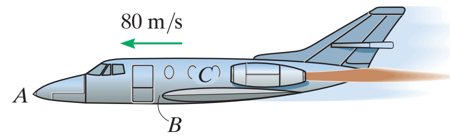
Bernoulli Equation. If the flow of the air is viewed from the plane, it will be a steady flow. If we observe the air from the plane, the still air at A will have $V_A = 80\text{ m/s}$ and the air at B has the same velocity as the plane, $V_B = 0$. From Appendix A, $(p_A)_{\text{abs}} = 70.12\text{ kPa}$ and $\rho = 0.9092\text{ kg/m}^3$ at an altitude of 3 km.

$$\frac{p_A}{\rho} + \frac{V_A^{\,2}}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^{\,2}}{2} + gz_B$$

$$\frac{70.12\left(10^3\right)\text{ N/m}^2}{0.9092\text{ kg/m}^3} + \frac{(80\text{ m/s})^2}{2} + 0 = \frac{(p_B)_{\text{abs}}}{0.9092\text{ kg/m}^3} + 0 + 0$$

$$\left(p_B\right)_{\text{abs}} = 73029.44\text{ Pa} = 73.0\text{ kPa} \qquad \textbf{Ans.}$$

5–13. The jet airplane is flying at 80 m/s in still air, *A*, at an altitude of 4 km. If the air flows past point *C* near the wing at 90 m/s, determine the difference in pressure between the air near the leading edge *B* of the wing and point *C*.



SOLUTION

Bernoulli Equation. If the flow of the air is viewed from the plane, it will be a steady flow. Thus, from the plane, the air at *B* is $V_B = 80$ m/s and at *C*, $V_C = 90$ m/s. From Appendix *A*, $\rho = 0.8194\ \text{kg/m}^3$ at an altitude of 4 km.

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B = \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C$$

$$\frac{p_B}{0.8194\ \text{kg/m}^3} + \frac{(80\ \text{m/s})^2}{2} + 0 = \frac{p_C}{0.8194\ \text{kg/m}^3} + \frac{(90\ \text{m/s})^2}{2} + 0$$

$$p_B - p_C = 696\ \text{Pa} \qquad \textbf{Ans.}$$

Ans:
696 Pa

5–14. A river flows at 12 ft/s and then turns and drops as a waterfall, from a height of 80 ft. Determine the velocity of the water just before it strikes the rocks below the falls.

SOLUTION

Bernoulli Equation. If the datum is set at the rocks, $z_A = 80$ ft (before the flow drops), z_B (just before it strikes the rocks) = 0. Since the flow from A to B in the open atmosphere, $p_A = p_B = 0$. From A to B,

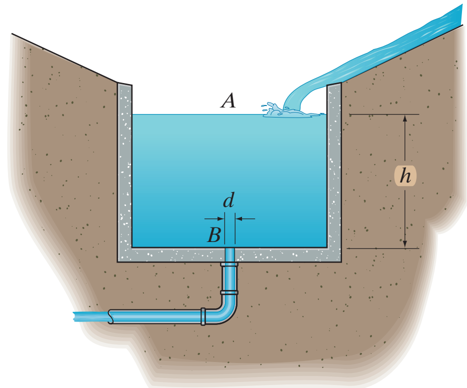
$$\frac{p_A}{\rho} + \frac{V_A{}^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B{}^2}{2} + gz_B$$

$$0 + \frac{(12\text{ ft/s})^2}{2} + (32.2\text{ ft/s}^2)(80\text{ ft}) = 0 + \frac{V_B{}^2}{2} + 0$$

$$V_B = 72.8\text{ ft/s}$$ **Ans.**

Ans:
72.8 ft/s

5–15. Water is discharged through the drain pipe at B from the large basin at $0.03\ \text{m}^3/\text{s}$. If the diameter of the drainpipe is $d = 60$ mm, determine the pressure at B just inside the drain in the drain when the depth of the water is $h = 2$ m.



SOLUTION

$$Q_B = V_B A_B$$

$$0.03\ \text{m}^3/\text{s} = V_B\left[\pi(0.03\ \text{m})^2\right]$$

$$V_B = 10.61\ \text{m/s}$$

Bernoulli Equation. Since the water is discharged from a large source, $V_A \cong 0$. Here, $p_A = 0$ since surface A is exposed to the atmosphere. If we set the datum along the base of the basin, $z_B = 0$ and $z_A = 2$ m.

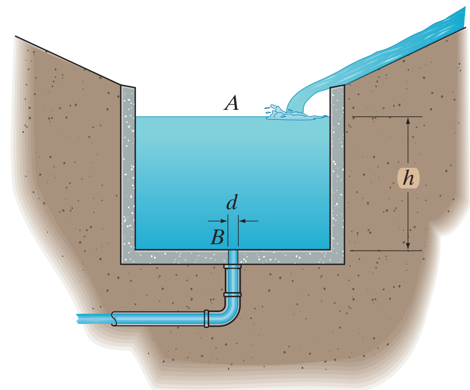
$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$0 + 0 + \left(9.81\ \text{m/s}^2\right)(2\ \text{m}) = \frac{p_B}{1000\ \text{kg/m}^3} + \frac{(10.61\ \text{m/s})^2}{2} + 0$$

$$p_B = -36.67\left(10^3\right)\ \text{Pa} = -36.7\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
-36.7 kPa

***5–16.** Water is discharged through the drain pipe at B from the large basin at $0.03\ \text{m}^3/\text{s}$. Determine the pressure at B just inside the drain as a function of the diameter d of the drainpipe. The height of the water is maintained at $h = 2$ m. Plot the pressure (vertical axis) versus the diameter for $40\ \text{mm} < d < 120\ \text{mm}$. Give values for increments of $\Delta d = 20$ mm.



SOLUTION

The discharge requirement is

$$Q = V_B A_B; \qquad 0.03\ \text{m}^3/\text{s} = V_B\left[\frac{\pi}{4}\left(\frac{d}{1000}\right)^2\right]$$

$$V_B = \left[\frac{38.197(10^3)}{d^2}\right]\ \text{m/s}$$

The water can be considered as an ideal fluid (incompressible and inviscid). Also, the flow is steady. Therefore Bernoulli's equation is applicable. Applying this equation between A and B realizing that $V_A \cong 0$ (the water is discharged from a large reservoir), $p_A = 0$ (surface A is exposed to the atmosphere). Also if we set the datum along the base of the basin $z_B = 0$ and $z_A = 2$ m.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

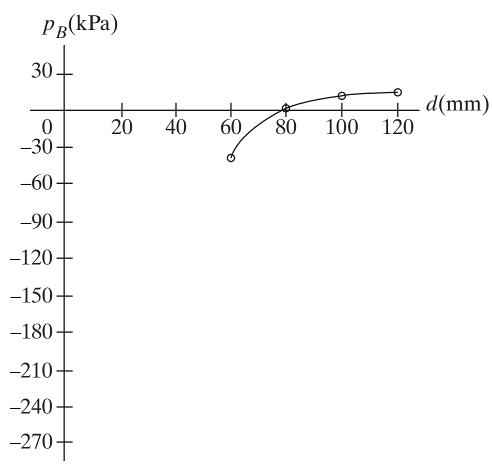
$$0 + 0 + (9.81\ \text{m/s}^2)(2\ \text{m}) = \frac{p_B}{1000\ \text{kg/m}} + \frac{\left[38.197(10^3)/d^2\right]^2}{2} + 0$$

$$p_B = \left(19.62 - \frac{0.7295(10^9)}{d^4}\right)(10^3)$$

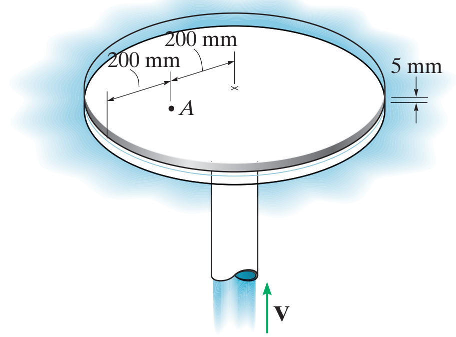
The plot of p_B vs. d is shown in Fig. a

$$p_A = \left[19.6 - \frac{0.730(10^9)}{d^4}\right]\ \text{kPa where } d \text{ is in mm} \qquad \textbf{Ans.}$$

d(mm)	60	80	100	120
p_B(kPa)	–36.7	1.79	12.3	16.1



5–17. A fountain is produced by water that flows up the tube at $Q = 0.08\ \text{m}^3/\text{s}$ and then radially through two cylindrical plates before exiting to the atmosphere. Determine the velocity and pressure of the water at point A.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's Equation is applicable. Writing this equation between points A and B on the radial streamline,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Since point B is exposed to the atmosphere, $p_B = p_{\text{atm}} = 0$. Here, points A and B have the same elevation since the cylindrical plates are in the horizontal plane. Thus, $z_A = z_B = z$.

$$\frac{p_A}{1000\ \text{kg/m}^3} + \frac{V_A^2}{2} + gz = 0 + \frac{V_0^2}{2} + gz$$

$$p_A = 500\left(V_B^2 - V_A^2\right) \qquad \textbf{(1)}$$

Continuity requires that

$Q = V_A A_A;$ $\qquad 0.08\ \text{m}^3/\text{s} = V_A\,[2\pi(0.2\ \text{m})(0.005\ \text{m})]$

$$V_A = 12.73\ \text{m/s} = 12.7\ \text{m/s} \qquad \textbf{Ans.}$$

$Q = V_B A_B;$ $\qquad 0.08\ \text{m}^3/\text{s} = V_B[2\pi\,(0.4\ \text{m})(0.005\ \text{m})]$

$$V_B = 6.366\ \text{m/s}$$

Substituting these results into Eq. (1),

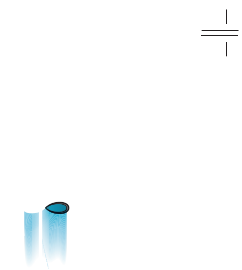
$$p_A = 500\left(6.366^2 - 12.73^2\right)$$

$$= -60.79\left(10^3\right)\ \text{Pa} = -60.8\ \text{kPa} \qquad \textbf{Ans.}$$

The negative sign indicates that the pressure at A is a partial vacuum.

Ans:
$V = 12.7\ \text{m/s}$, $p = -60.8\ \text{kPa}$

5–18. A fountain is produced by water that flows up the tube at $Q = 0.08\ \text{m}^3/\text{s}$ and then radially through two cylindrical plates before exiting to the atmosphere. Determine the pressure of the water as a function of the radial distance r. Plot the pressure (vertical axis) versus r for $200\ \text{mm} \le r \le 400\ \text{mm}$. Give values for increments of $\Delta r = 50\ \text{mm}$.



SOLUTION

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's Equation is applicable. Writing this equation between points A and B on the radial streamline,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Since point B is exposed to the atmosphere, $p_B = p_{\text{atm}} = 0$. Here points A and B have the same elevation since the cylindrical plates are in the horizontal plane. Thus $z_A = z_B = z$.

$$\frac{p}{1000\ \text{kg/m}^3} + \frac{V^2}{2} + gz = 0 + \frac{V_B^2}{2} + gz$$

$$p = \left[500\left(V_B^2 - V^2\right)\right]\ \text{Pa} \qquad \textbf{(1)}$$

Continuity requires that

$$Q = V_A A_A; \qquad 0.08\ \text{m}^3/\text{s} = V\left[2\pi\left(\frac{r}{1000}\right)(0.005\ \text{m})\right]$$

$$V = \left(\frac{2546.48}{r}\right)\ \text{m/s}$$

$$Q = V_B A_B; \qquad 0.08\ \text{m}^3/\text{s} = V_B\left[2\pi\ (0.4\ \text{m})(0.005\ \text{m})\right]$$

$$V_B = 6.366\ \text{m/s}$$

5–18. Continued

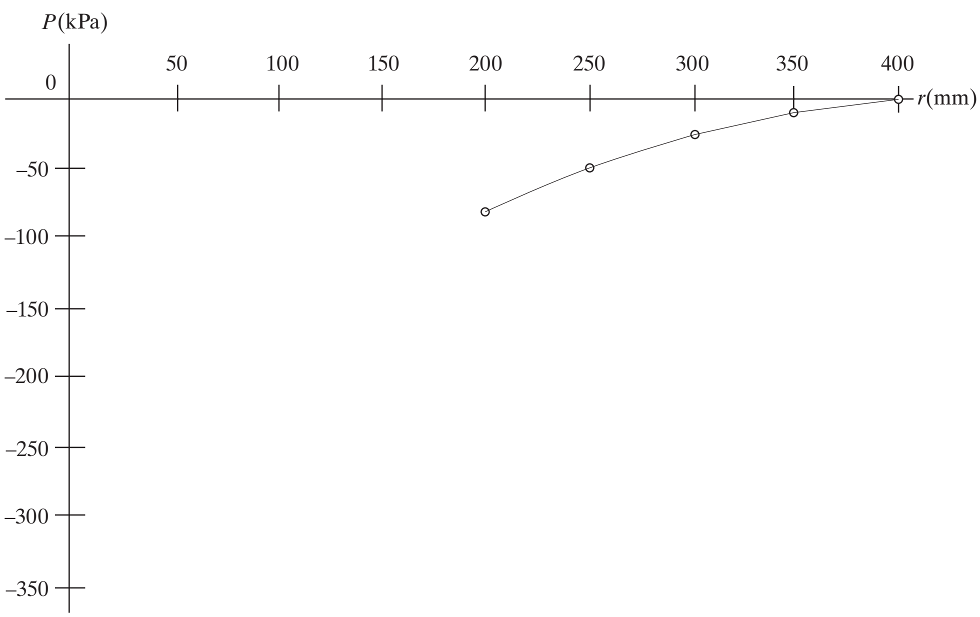
Substituting these results into Eq. (1),

$$p = 500\left[6.366^2 - \left(\frac{2546.48}{r}\right)^2\right] \text{Pa}$$

$$p = 500\left[40.5 - \frac{6.48(10^6)}{r^2}\right] \text{Pa}$$

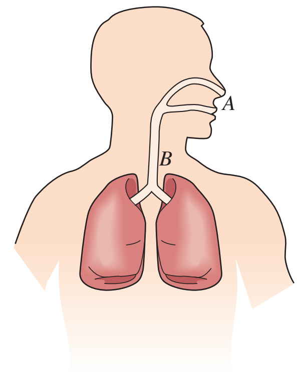
$$p = 0.5\left[40.5 - \frac{6.48(10^6)}{r^2}\right] \text{kPa where } r \text{ is in mm} \qquad \textbf{Ans.}$$

r(mm)	200	250	300	350	400
p(kPa)	−60.8	−31.6	−15.8	−6.20	0



Ans:
$p = 0.5\left[40.5 - \frac{6.48(10^6)}{r^2}\right]$ kPa,
where r is in mm.

5–19. The average human lung takes in about 0.6 liter of air with each inhalation, through the mouth and nose, A. This lasts for about 1.5 seconds. Determine the power required to do this if it occurs through the trachea B having a cross-sectional area of 125 mm^2. Take $\rho_a = 1.23\ \mathrm{kg/m^3}$. *Hint*: Recall that power is force F times velocity V, where $F = pA$.



SOLUTION

Assume that air is incompressible and inviscid and the flow is steady. Then, Bernoulli's equation can be applied between points A and B on the central streamline along the trachea shown in Fig. *a*.

$$\frac{p_A}{\rho_a} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_a} + \frac{V_B^2}{2} + gz_B$$

Since the density of air is small, the elevation terms can be neglected. Since the air is taken in from the atmosphere, which is a large reservoir, $p_A = 0$ and $V_A = 0$. Then

$$0 + 0 = \frac{p_B}{\rho_a} + \frac{V_B^2}{2}$$

$$V_B = \sqrt{-\frac{2p_B}{\rho_a}}$$

Using this result, the volumetric flow is

$Q = V_B A_B;$ $$Q = \left(\sqrt{-\frac{2p_B}{\rho_a}}\right) A_B$$

$$p_B = -\frac{\rho_a}{2}\left(\frac{Q}{A_B}\right)^2$$

The negative sign indicates that the pressure in the trachea is in partial vacuum.

Then

$$F = p_B A_B = \frac{\rho_a}{2}\left(\frac{Q_B}{A_B}\right)^2 A_B$$

The power of F is

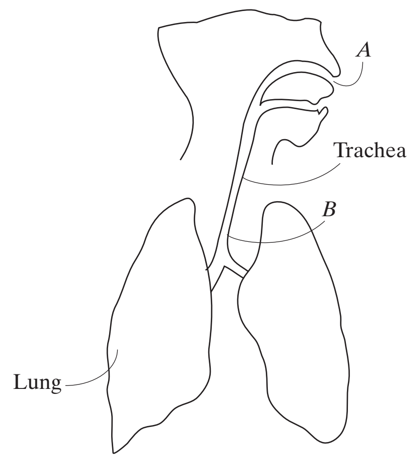
$$P = FV_B = \frac{\rho_a}{2}\left(\frac{Q}{A_B}\right)^2 (A_B V_B)$$

$$= \frac{1}{2}\left(\frac{\rho_a Q^3}{A_B^2}\right)$$

Here $\rho_a = 1.23\ \mathrm{kg/m^3}$, $A_B = \left(125\ \mathrm{mm^2}\right)\left(\frac{1\ \mathrm{m}}{1000\ \mathrm{mm}}\right)^2 = 0.125\left(10^{-3}\right)\ \mathrm{m^2}$ and

$$Q = \left(\frac{0.6\ \mathrm{L}}{1.5\ \mathrm{s}}\right)\left(\frac{1\ \mathrm{m^3}}{1000\ \mathrm{L}}\right) = 0.4\left(10^{-3}\right)\ \mathrm{m^3/s}.\ \text{Then}$$

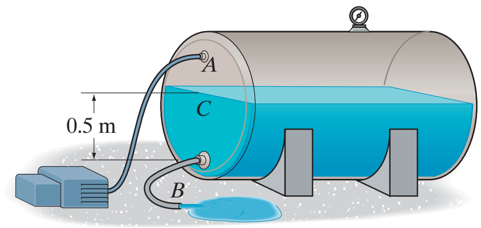
$$p = \frac{1}{2}\left[\frac{\left(1.23\ \mathrm{kg/m^3}\right)\left[0.4\left(10^{-3}\right)\ \mathrm{m^3/s}\right]^3}{\left[0.125\left(10^{-3}\right)\ \mathrm{m^2}\right]^2}\right] = 0.00252\ \mathrm{W} = 2.52\ \mathrm{mW}$$ **Ans.**



(a)

Ans:
2.52 mW

***5–20.** Water flows from the hose at B at the rate of 4 m/s when the water level in the large tank is 0.5 m. Determine the pressure of air that has been pumped into the top of the tank at A.



SOLUTION

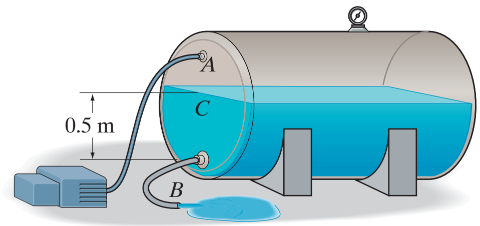
Bernoulli Equation. Since the water is discharged from a large tank, $V_C = 0$. Also, the water is discharged into the atmosphere at B, thus $p_B = 0$. If the datum is at B, $z_C = 0.5$ m and $z_B = 0$.

$$\frac{p_C}{\rho} + \frac{V_C{}^2}{2} + gz_C = \frac{p_B}{\rho} + \frac{V_B{}^2}{2} + gz_B$$

$$\frac{p_C}{1000\ \text{kg/m}^3} + 0 + \left(9.81\ \text{m/s}^2\right)(0.5\ \text{m}) = 0 + \frac{(4\ \text{m/s})^2}{2} + 0$$

$$p_C = 3095\ \text{Pa} = 3.10\ \text{kPa} \qquad \textbf{Ans.}$$

5–21. If the hose at A is used to pump air into the tank with a pressure of 150 kPa, determine the discharge of water at the end of the 15-mm-diameter hose at B when the water level is 0.5 m.



SOLUTION

Bernoulli Equation. Since the water is discharged from a large tank, $V_C = 0$. Also, the water is discharged into the atmosphere at B, thus $p_B = 0$. If the datum is set at B, $z_C = 0.5$ m and B, $z_B = 0$.

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{150\left(10^3\right)\dfrac{\text{N}}{\text{m}^2}}{1000\text{ kg/m}^3} + 0 + \left(9.81\text{ m/s}^2\right)(0.5\text{ m}) = 0 + \frac{V_B^2}{2} + 0$$

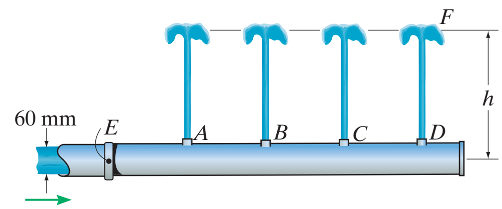
$$V_B = 17.601\text{ m/s}$$

$$Q = V_B A_B = \left(17.601\text{ m/s}\right)\left[\pi(0.0075\text{ m})^2\right]$$

$$= 3.11\left(10^{-3}\right)\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$3.11\left(10^{-3}\right)\text{ m}^3/\text{s}$

***5–24.** A fountain ejects water through the four nozzles, which have inner diameters of 10 mm. Determine the maximum height h of the water stream passing through the nozzles as a function of the volumetric flow rate into the 60-mm-diameter pipe at E. Also what is the corresponding pressure at E as a function of h?



SOLUTION

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Since the water stream DF flow in the open atmosphere, $p_D = p_F = 0$. Also, when the stream achieves its maximum height at F, $V_F = 0$. If we set the datum along the streamline coinciding with the centerline of the pipe, $z_D = z_E = 0$ and $z_F = h$. Writing between points D and F,

$$\frac{p_D}{\rho} + \frac{V_D^2}{2} + gz_D = \frac{p_F}{\rho} + \frac{V_F^2}{2} + gz_F$$

$$0 + \frac{V_D^2}{2} + 0 = 0 + 0 + \left(9.81 \text{ m/s}^2\right) h$$

$$V_D = \left(\sqrt{19.62\,h}\right) \text{ m/s}$$

Between E and F,

$$\frac{p_E}{\rho} + \frac{V_E^2}{2} + gz_E = \frac{p_F}{\rho} + \frac{V_F^2}{2} + gz_F$$

$$\frac{p_E}{1000 \text{ kg/m}^3} + \frac{V_E^2}{2} + 0 = 0 + 0 + \left(981 \text{ m/s}^2\right)h$$

$$p_E = \left(9.81\,h - \frac{V_E^2}{2}\right)\left(10^3\right) \text{ Pa} \qquad \textbf{(1)}$$

Take the fixed control volume to be the water contained in the pipe, since there are four nozzles,

$$\frac{\partial}{\partial t}\int_{cv} \rho d\maltese + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_E A_E + 4V_D A_D = 0$$

$$-V_E\left[\pi(0.03 \text{ m})^2\right] + 4\left(\sqrt{19.62\,h}\right)\left[\pi(0.005 \text{ m})^2\right] = 0$$

$$V_E = 0.4922\sqrt{h}$$

The discharge is

$$Q = V_E A_E; \quad Q = \left(0.4922\sqrt{h}\right)\left[\pi(0.03 \text{ m})^2\right]$$

$$h = \left[516\left(10^3\right)Q^2\right] \text{ m where } Q \text{ is in m}^3/\text{s} \qquad \textbf{Ans.}$$

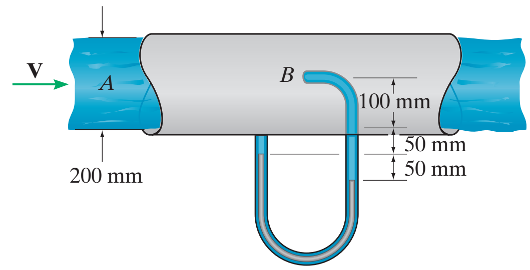
Substituting the result of V_E into Eq. (1),

$$p_E = \left[9.81h - \frac{\left(0.4922\sqrt{h}\right)^2}{2}\right]\left(10^3\right)$$

$$p_E = (9.69h)\left(10^3\right) \text{ Pa}$$

$$p_E = (9.69h) \text{ kPa where } h \text{ is in meters} \qquad \textbf{Ans.}$$

5–25. Determine the velocity of water through the pipe if the manometer contains mercury held in the position shown. Take $\rho_{Hg} = 13\,550 \text{ kg/m}^3$.



SOLUTION

Bernoulli Equation. Since point B is a stagnation point, $V_B = 0$. If the datum is along the horizontal streamline connecting A and B, $z_A = z_B = 0$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{1000 \text{ kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{p_B}{1000 \text{ kg/m}^3} + 0 + 0$$

$$p_A - p_B = -500V_A^2$$

Manometer Equation. Referring to Fig. *a*,

$$p_A + \rho_w g h_{AC} + \rho_{Hg} g h_{CD} - \rho_w g h_{BD} = p_B$$

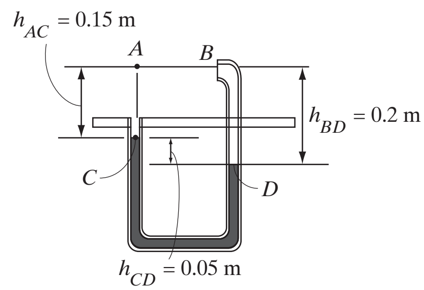
$$p_A + \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(0.15 \text{ m}) +$$

$$\left(13\,550 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(0.05 \text{ m}) - \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(0.2 \text{ m}) = p_B$$

$$p_A - p_B = 6155.78$$

Solving Eqs. (1) and (2)

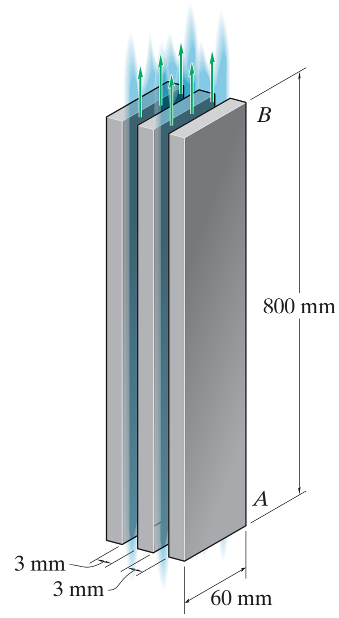
$$V_A = 3.51 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Ans:
3.51 m/s

5–26. A water-cooled nuclear reactor is made with plate fuel elements that are spaced 3 mm apart and 800 mm long. During an initial test, water enters at the bottom of the reactor (plates) and flows upwards at 0.8 m/s. Determine the pressure difference in the water between A and B. Take the average water temperature to be 80°C.



SOLUTION

The water can be considered as an ideal fluid (incompressible and inviscid). Also, the flow is steady. Therefore, Bernoulli's equation is applicable. Applying this equation between A and B, where both points are on the central streamline,

$$\frac{p_A}{\rho_W} + \frac{V_A{}^2}{2} + gz_A = \frac{p_B}{\rho_W} + \frac{V_B{}^2}{2} + gz_B$$

Since the inlet (A) and outlet (B) control surfaces have the same cross-sectional area, continuity requires that $V_A = V_B = V$. Here, the datum will be set through point A. Then,

$$\frac{p_A}{\rho_W} + \frac{V^2}{2} + 0 = \frac{p_B}{\rho_W} + \frac{V^2}{2} + gh$$

$$p_A - p_B = \rho_w gh$$

From Appendix A, $\rho_w = 971.6\ \text{kg/m}^3$ at $T = 80°\ \text{C}$.

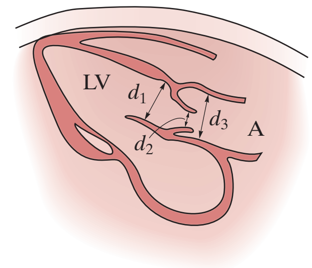
Here, $h = 0.8$ m. Then

$$p_A - p_B = (971.6\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.8\ \text{m})$$

$$= 7.625(10^3)\ \text{Pa} = 7.63\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
7.63 kPa

5–27. Blood flows from the left ventricle (LV) of the heart which has an exit diameter of $d_1 = 16$ mm, through the stenotic aortic valve of diameter $d_2 = 8$ mm, and then into the aorta A having a diameter of $d_3 = 20$ mm. If the cardiac output is 4 liters per minute, the heart rate is 90 beats per minute, and each ejection of blood lasts 0.31 s, determine the pressure drop over the valve. Take $\rho_b = 1060\ \text{kg/m}^3$.



SOLUTION

The volume of blood pumped per heartbeat is

$$\forall = \frac{4\ \text{L/min}}{90\ \text{beat/min}} = (0.04444\ \text{L/beat})\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right) = 44.44(10^{-6})\ \text{m}^3/\text{beat}$$

Thus, the discharge of blood by LV is

$$Q = \frac{\forall}{t} = \frac{44.44(10^{-6})\ \text{m}^3/\text{beat}}{0.31\ \text{s/beat}} = 0.1434(10^{-3})\ \text{m}^3/\text{s}$$

Then, the average velocities of the blood flow from the LV and into the Aorta; V_1 and V_3, respectively are

$$Q = V_1A_1; \qquad 0.1434(10^{-3})\ \text{m}^3/\text{s} = V_1\left[\pi(0.008\ \text{m})^2\right]$$

$$V_1 = 0.7131\ \text{m/s}$$

$$Q = V_3A_3: \qquad 0.1434(10^{-3})\ \text{m}^3/\text{s} = V_3\left[\pi(0.01\ \text{m})^2\right]$$

$$V_3 = 0.4564\ \text{m/s}$$

Writing Bernoulli's equation between the two points,

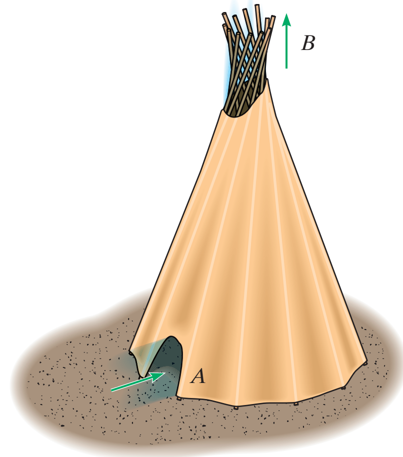
$$\frac{p_1}{\rho_b} + \frac{V_1^2}{2} + gz_1 = \frac{p_3}{\rho_b} + \frac{V_3^2}{2} + gz_3$$

$$\frac{p_1}{1060\ \text{kg/m}^3} + \frac{(0.7131\ \text{m/s})^2}{2} + 0 = \frac{p_3}{1060\ \text{kg/m}^3} + \frac{(0.4564\ \text{m/s})^2}{2} + 0$$

$$\Delta_p = p_3 - p_1 = 159\ \text{Pa} \qquad \textbf{Ans.}$$

Ans:
159 Pa

***5–28.** Air enters the tepee door at A with an average speed of 2 m/s and exits at the top B. Determine the pressure difference between these two points and find the average speed of the air at B. The areas of the openings are $A_A = 0.3\ \text{m}^2$ and $A_B = 0.05\ \text{m}^2$. The density of the air is $\rho_a = 1.20\ \text{kg/m}^3$.



SOLUTION

Since the air can be considered as an ideal fluid (incompressible and inviscids) and the flow is steady, the Bernoulli's equation is applicable. Consider the control volume to be the air within the nozzle. Continuity requires

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(2\ \text{m/s})(0.3\ \text{m}^2) + V_B(0.05\ \text{m}^2) = 0$$

$$V_B = 12\ \text{m/s} \qquad \textbf{Ans.}$$

Applying the Bernoulli equation between points A and B,

$$\frac{p_A}{\rho_a} + \frac{V_A{}^2}{2} + gz_A = \frac{p_B}{\rho_a} + \frac{V_B{}^2}{2} + gz_B$$

Since the density of the air is small, the elevation terms can be neglected.

$$\frac{p_A}{\rho_a} + \frac{V_A{}^2}{2} = \frac{p_B}{\rho_a} + \frac{V_B{}^2}{2}$$

$$\Delta p = p_B - p_A = \frac{\rho_a}{2}\left(V_B{}^2 - V_A{}^2\right)$$

$$\Delta p = \frac{(1.20\ \text{kg/m}^3)}{2}\left[(2\ \text{m/s})^2 - (12\ \text{m/s})^2\right]$$

$$= -84.0\ \text{Pa} \qquad \textbf{Ans.}$$

5–29. One method of producing energy is to use a tapered channel (TAPCHAN), which diverts sea water into a reservoir as shown in the figure. As a wave approaches the shore through the closed tapered channel at A, its height will begin to increase until it begins to spill over the sides and into the reservoir. The water in the reservoir then passes through a turbine in the building at C to generate power and is returned to the sea at D. If the speed of the water at A is $V_A = 2.5\text{ m/s}$, and the water depth is $h_A = 3\text{ m}$, determine the minimum height of the channel to prevent water from entering the reservoir.

SOLUTION

The sea water can be considered as an ideal fluid (incompressible and inviscid). Also, the flow is steady. Therefore, Bernoulli's equation is applicable. Apply this equation between points A and B along the streamline on the water surface,

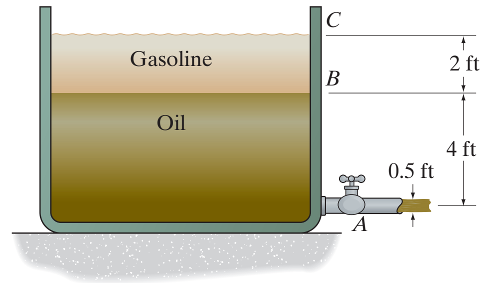
$$\frac{p_A}{\rho_{sw}} + \frac{V_A{}^2}{2} + gz_A = \frac{p_B}{\rho_{sw}} + \frac{V_B{}^2}{2} + gz_B$$

The datum is set along the base of the channel, then $z_A = h_A = 3\text{ m}$, $z_B = h_B$. Since points A and B are on the water surface, $p_A = p_B = p_{\text{atm}} = 0$. We require $V_B = 0$.

$$0 + \frac{(2.5\text{ m/s})^2}{2} + (9.81\text{ m/s}^2)(3\text{ m}) = 0 + 0 + (9.81\text{ m/s}^2)\,h_B$$

$$h_B = 3.32\text{ m} \qquad \textbf{Ans.}$$

5–30. The large tank is filled with gasoline and oil to the depth shown. If the valve at A is opened, determine the initial discharge from the tank. Take $\rho_g = 1.41\ \text{slug/ft}^3$ and $\rho_o = 1.78\ \text{slug/ft}^3$.



SOLUTION

Bernoulli Equation. Since the oil is discharged from a larger tank, $V_B = 0$. The pressure at B is $p_B = \rho_g g h_{BC} = (1.41\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(2\ \text{ft}) = 90.804\ \dfrac{\text{lb}}{\text{ft}^2}$. Since the oil is discharged to the atmosphere, $p_A = 0$. If the datum is at A, $z_A = 0$ and $z_B = 4$ ft.

$$\frac{p_B}{\rho_o} + \frac{V_B{}^2}{2} + gz_B = \frac{p_A}{\rho_o} + \frac{V_A{}^2}{2} + gz_A$$

$$\frac{90.804\ \dfrac{\text{lb}}{\text{ft}^3}}{1.78\ \text{slug/ft}^3} + 0 + (32.2\ \text{ft/s}^2)(4\ \text{ft}) = 0 + \frac{V_A{}^2}{2} + 0$$

$$V_A = 18.96\ \text{ft/s}$$

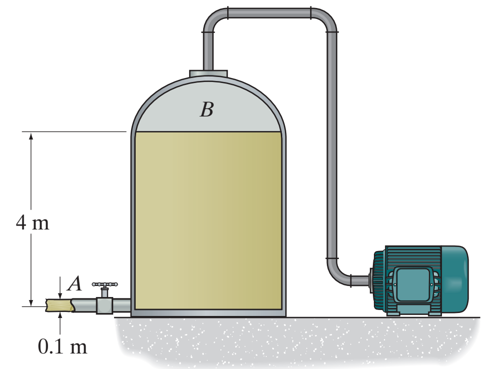
Discharge.

$$Q = V_A A_A = (18.96\ \text{ft/s})\left[\pi(0.25\ \text{ft})^2\right]$$

$$= 3.72\ \text{ft}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$3.72\ \text{ft}^3/\text{s}$

5–31. Determine the air pressure that must be exerted at the top of the kerosene in the large tank at B so that the initial discharge through the drain pipe at A is $0.1\ \text{m}^3/\text{s}$ once the valve at A is opened.



SOLUTION

Since the kerosene can be considered as an ideal fluid (incompressible and inviscids) and the flow is steady, Bernoulli's equation is applicable. Applying this equation between points B and A,

$$\frac{p_B}{\rho_{kc}} + \frac{V_B^2}{2} + gz_B = \frac{p_A}{\rho_{kc}} + \frac{V_A^2}{2} + gz_A$$

Since the kerosene tank is a large reservoir, $V_B \simeq 0$. Also, the drain pipe is exposed to the atmosphere, $p_A = p_{\text{atm}} = 0$. From Appendix A, $\rho_{kc} = 814\ \text{kg/m}^3$. Here the datum is set through point A and so $z_B = 4$ m and $z_A = 0$.

$$\frac{p_B}{814\ \text{kg/m}^3} + 0 + \left(9.81\ \text{m/s}^2\right)(4\ \text{m}) = 0 + \frac{V_A^2}{2} + 0$$

$$p_B = 407\left(V_A^2 - 78.48\right)\ \text{Pa} \qquad \textbf{(1)}$$

From the given discharge,

$$Q = V_A A_A; \qquad 0.1\ \text{m}^3/\text{s} = V_A\left[\pi(0.05\ \text{m})^2\right]$$

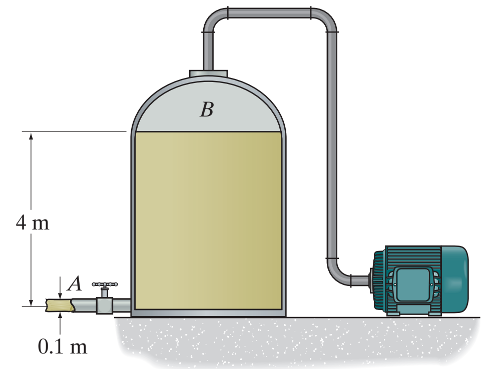
$$V_A = 12.73\ \text{m/s}$$

Substituting this result into Eq. (1),

$$p_B = 407\left(12.73^2 - 78.48\right)$$
$$= 34.04\left(10^3\right)\ \text{Pa}$$
$$= 34.0\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
34.0 kPa

***5–32.** If air pressure at the top of the kerosene in the large tank is 80 kPa, determine the initial discharge through the drainpipe at A once the valve is opened.



SOLUTION

Since the kerosene can be considered as an ideal fluid (incompressible and inviscids) and the flow is steady, Bernoulli's equation is applicable. Applying this equation between points B and A,

$$\frac{p_B}{\rho_{kc}} + \frac{V_B^{\,2}}{2} + gz_B = \frac{p_A}{\rho_{kc}} + \frac{V_A^{\,2}}{2} + gz_A$$

Since the kerosene tank is a large reservoir, $V_B = 0$. Also the drain pipe is exposed to the atmosphere $p_A = p_{\text{atm}} = 0$. From the Appendix A, $\rho_{kc} = 814\ \text{kg/m}^3$. Here, the datum is set to pass through point A and so $z_B = 4$ m and $z_A = 0$.

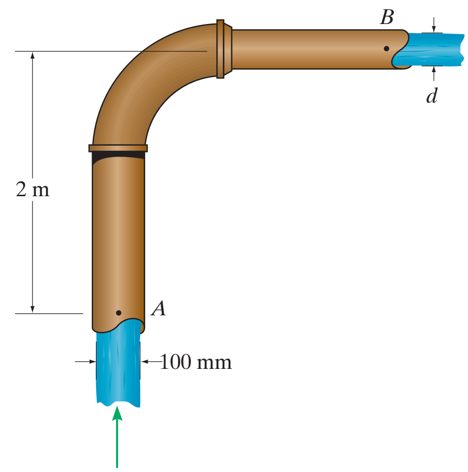
$$\frac{80(10^3)\ \text{N/m}^2}{814\ \text{kg/m}^3} + 0 + (9.81\ \text{m/s}^2)(4\ \text{m}) = 0 + \frac{V_A^{\,2}}{2} + 0$$

$$V_A = 16.58\ \text{m/s}$$

Thus, the discharge through the drain pipe is

$$Q = V_A A_A = (16.58\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right] = 0.130\ \text{m}^3/\text{s} \qquad \textbf{Ans.}$$

5–33. Water flows up through the *vertical pipe* such that when it is at A, it is subjected to a pressure of 150 kPa and has a velocity of 3 m/s. Determine the pressure and its velocity at B. Set $d = 75$ mm.



SOLUTION

Continuity Equation.

$$\frac{d}{dt}\int_{cv} e\, d\forall + \int_{cs} e\mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(3\text{ m/s})\left[\pi(0.05\text{ m})^2\right] + V_B\left[\pi(0.0375\text{ m})^2\right] = 0$$

$$V_B = 5.333\text{ m/s} = 5.33\text{ m/s} \qquad \textbf{Ans.}$$

Bernoulli Equation. If we set the datum at point A, $Z_A = 0$ and $z_B = 2$ m.

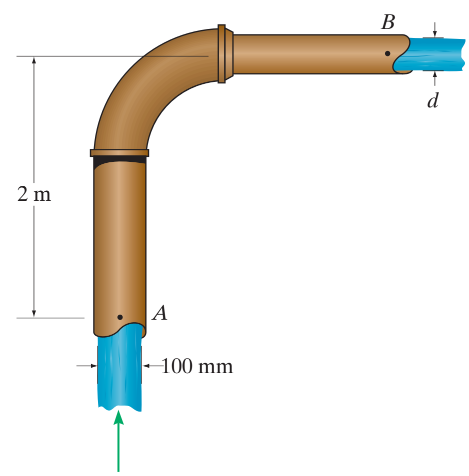
$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\left[\frac{150(10^3)\text{ N/m}^2}{1000\text{ kg/m}^3}\right] + \frac{(3\text{ m/s})^2}{2} + 0 = \frac{p_B}{(1000\text{ kg/m}^3)} + \frac{(5.333\text{ m/s})^2}{2} + (9.81\text{ m/s}^2)(2\text{ m})$$

$$p_B = 120.66(10^3)\text{ Pa} = 121\text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$V_B = 5.33$ m/s
$p_B = 121$ kPa

5–34. Water flows through the *vertical pipe* such that when it is at A it is subjected to a pressure of 150 kPa and has a velocity of 3 m/s. Determine the pressure and velocity at B as a function of the diameter d of the pipe at B. Plot the pressure and velocity (vertical axis) versus the diameter for $50 \text{ mm} \le d \le 100 \text{ mm}$. Give values for increments of $\Delta d = 25$ mm. If $d_B = 25$ mm, what is the pressure at B? Is this reasonable? Explain.



SOLUTION

The fixed control volume contains the water in the pipe,

$$\frac{d}{dt}\int_{cv} \rho d V\!\!\!\!- = \int_{cs} \rho\, \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

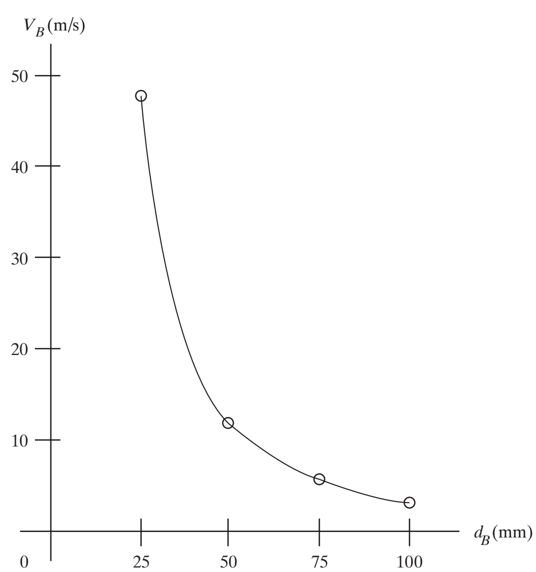
$$-(3 \text{ m/s})\left[\pi(0.05 \text{ m})^2\right] + V_B\left[\frac{\pi}{4}\left(\frac{d_B}{1000}\right)^2\right] = 0$$

$$V_B = \left[\frac{30(10^3)}{d_B^{\,2}}\right] \text{m/s where } d_B \text{ is in mm} \qquad \textbf{Ans.}$$

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points A and B. If we set the datum through point A, $z_A = 0$ and $z_B = 2$ m, then

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

d_B(mm)	25	50	75	100
V_B(m/s)	48.0	12.0	5.33	3.00



(a)

5–34. Continued

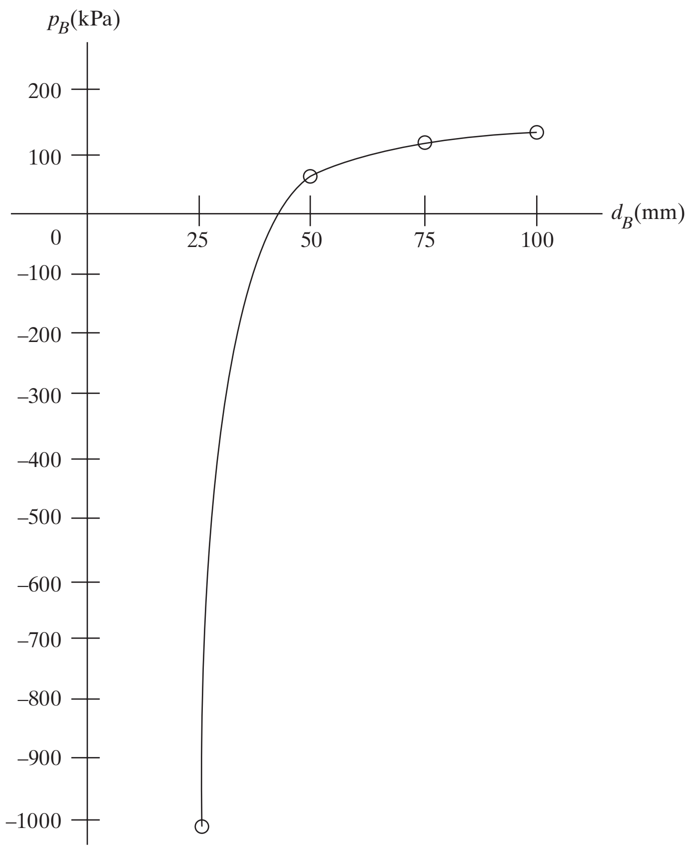
$$\left[\left(\frac{150(10^3)\ \text{N/m}^2}{1000\ \text{kg/m}^3}\right)\right] + \frac{(3\ \text{m/s})^2}{2} + 0 = \frac{p_B}{1000\ \text{kg/m}^3} + \frac{[300(10^3)/d_B^2]^2}{2} + (9.81\ \text{m/s}^2)(2\ \text{m})$$

$$p_B = \left[134.88 - \frac{450(10^6)}{d_B^4}\right](10^3)\ \text{Pa}$$

$$p_B = \left[135 - \frac{450(10^6)}{d_B^4}\right]\ \text{kPa where } d_B \text{ is in mm} \qquad \textbf{Ans.}$$

The plot of V_B V_S d_B and P_B V_S d_B are shown in Fig. *a* and *b* respectively. **Realistically, gage pressures less than −101 kPa are physically impossible, and water will cavitate a few kPa above that.**

d_B(mm)	25	50	75	100
p_B(kPa)	−1017	62.9	121	130



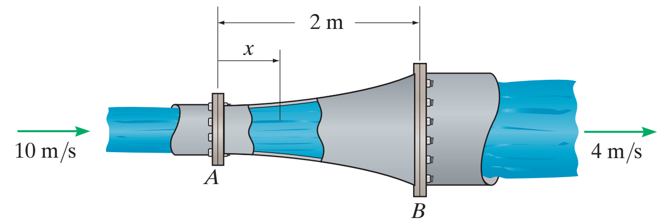
(b)

Ans:

$$V_B = \left[\frac{30(10^3)}{d_B^2}\right]\ \text{m/s, where } d_B \text{ is in mm.}$$

$$p_B = \left[135 - \frac{450(10^6)}{d_B^4}\right]\ \text{kPa, where } d_B \text{ is in mm.}$$

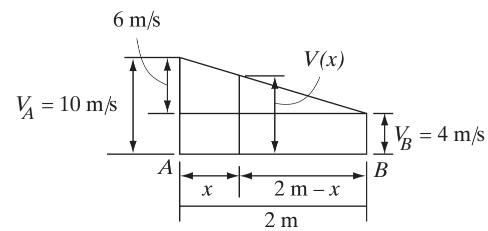
5–35. If the velocity of water changes uniformly along the transition from $V_A = 10\text{ m/s}$ to $V_B = 4\text{ m/s}$, determine the pressure difference between A and x.



SOLUTION

Bernoulli Equation. Referring to Fig. *a*,

$$V(x) = 4\text{ m/s} + \left(\frac{6\text{ m/s}}{2\text{ m}}\right)(2\text{ m} - x) = (10 - 3x)\text{ m/s}$$



(a)

If the datum is set along the horizontal streamline, $z_A = z(x) = 0$.

$$\frac{p_A}{\rho} + \frac{V_A{}^2}{2} + gz_A = \frac{p(x)}{\rho} + \frac{V^2(x)}{2} + gzx$$

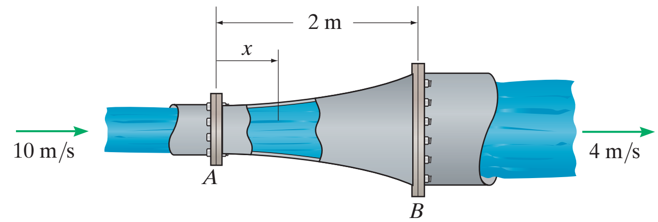
$$\frac{p_A}{1000\text{ kg/m}^3} + \frac{(10\text{ m/s})^2}{2} + 0 = \frac{p(x)}{(1000\text{ kg/m}^3)} + \frac{[(10 - 3x)\text{ m/s}]^2}{2} + 0$$

$$p(x) - p_A = (30x - 4.5x^2)(10^3)\text{ Pa}$$

$$= (30x - 4.5x^2)\text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$p(x) - p_A = (30x - 4.5x^2)\text{ kPa}$

***5–36.** If the velocity of water changes uniformly along the transition from $V_A = 10$ m/s to $V_B = 4$ m/s, find the pressure difference between A and $x = 1.5$ m.



SOLUTION

Bernoulli Equation. Referring to Fig. *a*,

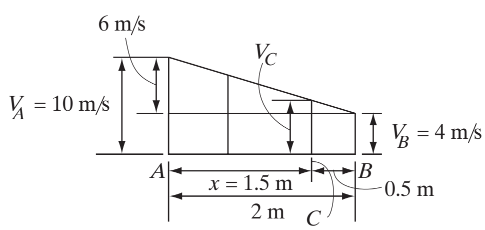
$$V_C = 4\text{ m/s} + \frac{0.5\text{ m}}{2\text{ m}}(6\text{ m/s}) = 5.5\text{ m/s}$$

If the datum is set along the horizontal streamline, $z_A = z_C = 0$.

$$\frac{p_A}{\rho} + \frac{V_A{}^2}{2} + gz_A = \frac{p_C}{\rho} + \frac{V_C{}^2}{2} + gz_C$$

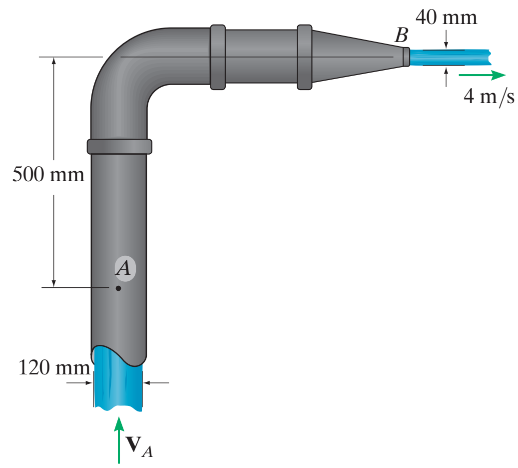
$$\frac{P_A}{1000\text{ kg/m}^3} + \frac{(10\text{ m/s})^2}{2} + 0 = \frac{P_C}{1000\text{ kg/m}^3} + \frac{(5.5\text{ m/s})^2}{2} + 0$$

$$p_C - p_A = 34.875(10^3)\text{ Pa} = 34.9\text{ kPa} \qquad \textbf{Ans.}$$



(a)

5–37. Water flows up through the *vertical pipe*. Determine the pressure at A if the average velocity at B is $4\ \text{m/s}$.



SOLUTION

Since the water can be considered as ideal fluid (incompressible and inviscids) and the flow is steady, the Bernoulli Equation is applicable. Writing this equation between points A and B on the central streamline,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Since point B is exposed to the atmosphere, $P_B = P_{\text{atm}} = 0$. Here the datum is set through A, then $z_A = 0$ and $z_B = 0.5$ m.

$$\frac{p_A}{1000\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = 0 + \frac{(4\ \text{m/s})^2}{2} + (9.81\ \text{m/s}^2)(0.5\ \text{m})$$

$$P_A = \left[500(25.81 - V_A^2)\right]\ \text{Pa} \qquad \textbf{(1)}$$

Consider the control volume to be the water in the pipe. The continuity requires that

$$\frac{d}{dt}\int_{cv} \rho d\forall + \int_{cs} e\mathbf{V}\cdot\mathbf{dA} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi(0.06\ \text{m})^2\right] + (4\ \text{m/s})\left[\pi(0.02\ \text{m})^2\right] = 0$$

$$V_A = 0.4444\ \text{m/s}$$

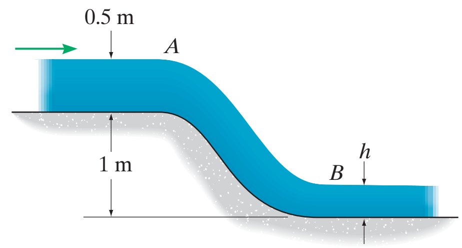
Substitute this result into Eq. (1),

$$P_A = \left[500(25.81 - 0.4444^2)\right]\ \text{Pa}$$

$$= 12.81(10^3)\ \text{Pa} = 12.8\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
12.8 kPa

5–38. Water flows along the rectangular channel such that after it falls to the lower elevation, the depth becomes $h = 0.3$ m. Determine the volumetric discharge through the channel. The channel has a width of 1.5 m.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscids) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points A and B on the streamline along the water surface,

$$\frac{p_A}{p_w} + \frac{V_A}{2} + gz_A = \frac{p_B}{p_w} + \frac{V_B^2}{2} + gz_B$$

Since the surface of the water is exposed to the atmosphere, $P_A = P_B = P_{atm} = 0$. Here the datum is set through points B, then $z_A = 1\text{ m} + 0.5\text{ m} - 0.3\text{ m} = 1.2\text{ m}$ and $z_B = 0$

$$0 + \frac{V_A^2}{2} + \left(9.81\text{ m/s}^2\right)(1.2\text{ m}) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B^2 - V_A^2 = 23.544 \qquad \textbf{(1)}$$

Consider the control volume to be the water from A to B. Continuity requires that

$$\frac{\partial}{\partial t}\int_{cv} e\,d\forall + \int_{cs} e\mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A[(1.5\text{ m})(0.5\text{ m})] + V_B[(1.5\text{ m})(0.3\text{ m})] = 0$$

$$V_A = 0.6V_B \qquad \textbf{(2)}$$

solving Eqs. 1 and 2,

$$V_B = 6.065\text{ m/s} \qquad V_A = 3.639\text{ m/s}$$

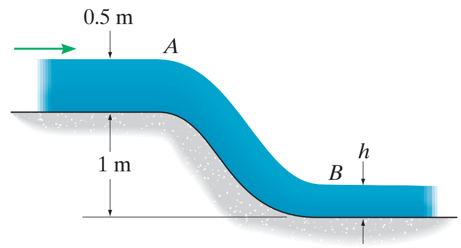
Thus, the discharge is

$$Q = V_B A_B = \left(6.065\text{ m/s}\right)[(1.5\text{ m})(0.3\text{ m})]$$

$$= 2.73\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$2.73\text{ m}^3/\text{s}$

5–39. Water flows at 3 m/s at A along the rectangular channel that has a width of 1.5 m. If the depth at A is 0.5 m, determine the depth at B.



SOLUTION

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points A and B on the streamline along the water surface,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_W} + \frac{V_B^2}{2} + gz_B$$

Since the water surface is exposed to the atmosphere, $P_A = P_B = P_{\text{atm}} = 0$. Here the datum is through points B, then $Z_A = 1\text{ m} + 0.5\text{ m} - h = (1.5-h)\text{ m}$ and $z_B = 0$.

$$0 + \frac{(3\text{ m/s})^2}{2} + \left(9.81\text{ m/s}^2\right)(1.5-h) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B^2 = 38.43 - 19.62h \qquad \textbf{(1)}$$

The control volume considered contains the water from section A and B which is fixed. Continuity requires that

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho\, \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(3\text{ m/s})[1.5\text{ m}(0.5\text{ m})] + V_B[(1.5\text{ m})h] = 0$$

$$V_B = \frac{1.5}{h} \qquad \textbf{(2)}$$

substituting Eq. 2 into 1,

$$\left(\frac{1.5}{h}\right)^2 = 38.43 - 19.62h$$

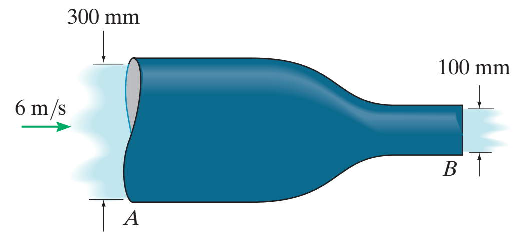
$$38.43h^2 - 19.62h^3 - 2.25 = 0$$

solving numerically,

$$h = 0.2598\text{ m} = 0.260\text{ m} \qquad \textbf{Ans.}$$

Ans:
0.260 m

***5–40.** Air at a temperature of 40°C flows into the nozzle at 6 m/s and then exits to the atmosphere at B, where the temperature is 0°C. Determine the pressure at A.



SOLUTION

Assume that air is an ideal fluid (incompressible and inviscids) and the flow is steady. Then Bernoulli's equation is applicable. Writing this equation between points A and B on the central streamline,

$$\frac{p_a}{(p_a)_A} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{(p_a)_B} + \frac{V_B^2}{2} + gz_B$$

From Appendix A, $(P_a)_A = 1.127\ \text{kg/m}^3\ (T = 40°c)$ and $(p_a)_B = 1.292\ \text{kg/m}^3$ $(T = 0°c)$. Since point B is exposed to the atmosphere $P_B = P_{\text{atm}} = 0$. Here the datum coincides with central streamline. Then $Z_A = Z_B = 0$.

$$\frac{P_A}{1.127\ \text{kg/m}^3} + \frac{(6\ \text{m/s})^2}{2} + 0 = 0 + \frac{V_B^2}{2} + 0$$

$$p_A = \left[0.5635\,(V_B^2 - 36)\right] p_a \qquad \textbf{(1)}$$

Consider the control volume to be the air within the nozzle. For steady flow, the continuity condition requires

$$\frac{\partial}{\partial t}\int_{cv} e\,d\forall + \int_{cs} e\mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - (P_a)_A V_A A_A + (P_a)_B V_B A_B = 0$$

$$-\left(1.127\ \text{kg/m}^3\right)\left(6\ \text{m/s}\right)\left[\pi(0.15\ \text{m})^2\right] + \left(1.292\ \text{kg/m}^3\right)(V_B)\left[\pi(0.05\ \text{m})^2\right] = 0$$

$$V_B = 47.10\ \text{m/s}$$

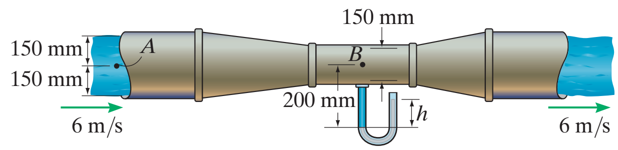
substituting this result into Eq. 1

$$P_A = 0.5635\left(47.10^2 - 36\right)$$

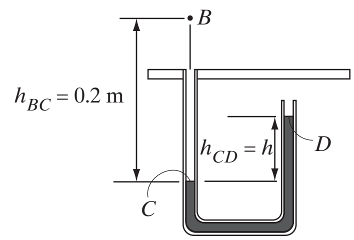
$$= 1229.98\ \text{Pa}$$

$$= 1.23\ \text{kPa} \qquad \textbf{Ans.}$$

5–41. Water flows through the pipe at A with a velocity of 6 m/s and at a pressure of 280 kPa. Determine the velocity of the water at B and the difference in elevation h of the mercury in the manometer.



SOLUTION



(a)

Continuity Equation. Consider the water within the pipe and transition.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(6 \text{ m/s})\left[\pi(0.15 \text{ m})^2\right] + V_B\left[\pi(0.075 \text{ m})^2\right] = 0$$

$$V_B = 24.0 \text{ m/s}$$

Bernoulli Equation. If we set the datum to coincide with the horizontal streamline connecting A and B, $z_A = z_B = 0$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{280(10^3)\text{N/m}^2}{1000 \text{ kg/m}^3} + \frac{(6 \text{ m/s})^2}{2} + 0 = \frac{p_B}{1000 \text{ kg/m}^3} + \frac{(24.0 \text{ m/s})^2}{2} + 0$$

$$p_B = 10(10^3) \text{ Pa}$$

Manometer Equation. Referring to Fig. *a*,

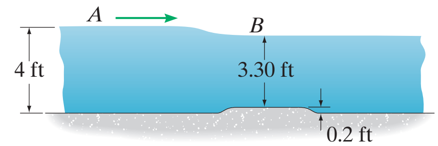
$$p_B + \rho_w g h_{BC} - \rho_{Hg} g h_{CD} = 0$$

$$10(10^3) \text{ Pa} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.2 \text{ m}) - (13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2)h = 0$$

$$h = 0.090 \text{ m} = 90 \text{ mm}$$ **Ans.**

Ans:
90 mm

5–42. In order to determine the flow in a rectangular channel, a 0.2-ft-high bump is added on its bottom surface. If the measured depth of flow at the bump is 3.30 ft, determine the volumetric discharge. The flow is uniform, and the channel has a width of 2 ft.



SOLUTION

Bernoulli Equation. Since surfaces A and B are exposed to the atmosphere, $p_A = p_B = 0$. If we set the datum at the base of the channel, $z_A = 4$ ft and $z_B = 3.30$ ft $+ 0.2$ ft $= 3.5$ ft.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$0 + \frac{V_A^2}{2} + (32.2\text{ ft/s}^2)(4\text{ ft}) = 0 + \frac{V_B^2}{2} + (32.2\text{ ft/s}^2)(3.50\text{ ft})$$

$$V_B^2 - V_A^2 = 32.2 \qquad \textbf{(1)}$$

Continuity Equation. Consider the water from A to B as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho\, \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A[(4\text{ ft})(2\text{ ft})] + V_B(3.30\text{ ft})(2\text{ ft}) = 0$$

$$V_A = 0.825 V_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$V_A = 8.284\text{ ft/s}$$

$$V_B = 10.04\text{ ft/s}$$

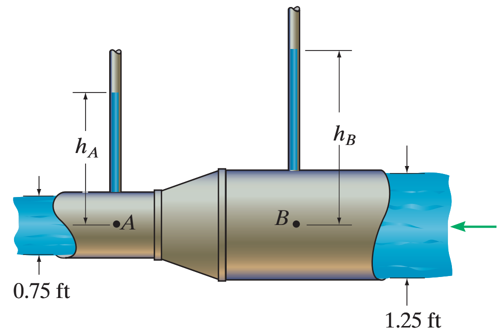
Discharge.

$$Q = V_A A_A = (8.284\text{ ft/s})[(4\text{ ft})(2\text{ ft})]$$

$$= 66.3\text{ ft}^3\text{/s} \qquad \textbf{Ans.}$$

Ans:
$66.3\text{ ft}^3/\text{s}$

5–43. As water flows through the pipes, it rises within the piezometers at A and B to the heights $h_A = 1.5$ ft and $h_B = 2$ ft. Determine the volumetric flow.



SOLUTION

Continuity Equation. Consider the water within the transition from A to B to be the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_B A_B + V_A A_A = 0$$

$$-V_B\left[\pi\left(\frac{1.25\text{ ft}}{2}\right)^2\right] + V_A\left[\pi\left(\frac{0.75\text{ ft}}{2}\right)^2\right] = 0$$

$$V_A = 2.778 V_B \qquad \textbf{(1)}$$

Bernoulli Equation. The static pressure at A and B is given by

$$p_A = \rho_w g h_A = \rho_w(32.2\text{ ft/s}^2)(1.5\text{ ft}) = 48.3\rho_w$$

$$p_B = \rho_w g h_B = \rho_w(32.2\text{ ft/s}^2)(2\text{ ft}) = 64.4\rho_w$$

If we set the datum to coincide with the horizontal streamline connecting A and B, $z_A = z_B = 0$.

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

$$\frac{64.4\rho_w}{\rho_w} + \frac{V_B^2}{2} + 0 = \frac{48.3\rho_w}{\rho_w} + \frac{V_A^2}{2} + 0$$

$$V_A^2 - V_B^2 = 32.2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$V_A = 6.082\text{ ft/s}$$

$$V_B = 2.190\text{ ft/s}$$

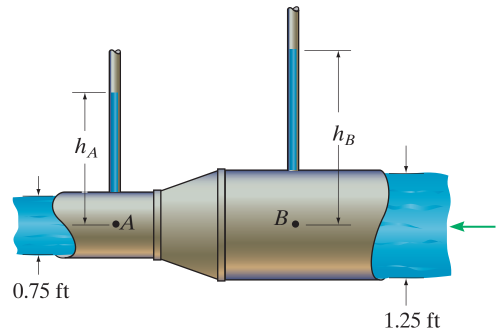
Discharge.

$$Q = V_A A_A = (6.082\text{ ft/s})\left[\pi\left(\frac{0.75\text{ ft}}{2}\right)^2\right]$$

$$= 2.69\text{ ft}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$2.69\text{ ft}^3/\text{s}$

***5–44.** The volumetric flow of water through the transition is 3 ft^3/s. Determine the height it rises in the piezometer at A if $h_B = 2$ ft.



SOLUTION

The control volume considered contains the water within the transition from A to B which is fixed. Using the discharge,

$$Q = V_A A_A; \qquad 3\ \text{ft}^3/\text{s} = V_A\left[\pi\left(\frac{0.75\ \text{ft}}{2}\right)^2\right] \qquad V_A = 6.7906\ \text{ft/s}$$

$$Q = V_B A_B; \qquad 3\ \text{ft}^3/\text{s} = V_B\left[\pi\left(\frac{1.25\ \text{ft}}{2}\right)^2\right] \qquad V_B = 2.4446\ \text{ft/s}$$

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points B and A,

$$\frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B = \frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A$$

The static pressures at A and B are

$$p_A = \rho_w g h_A = \rho_w(32.2\ \text{ft/s}^2)h_A = (32.2\ \rho_w h_A)\ \text{lb/ft}^2$$

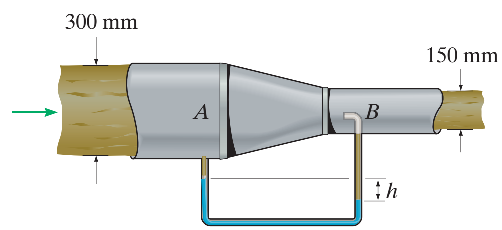
$$p_B = \rho_w g h_B = \rho_w\left(32.2\ \text{ft/s}^2\right)(2\ \text{ft}) = (64.4\ \rho_w)\ \text{lb/ft}^2$$

If we set the datum to coincide with the horizontal streamline connecting A and B, then $z_A = z_B = 0$. Therefore,

$$\frac{64.4\ \rho_w}{\rho_w} + \frac{(2.4446\ \text{ft/s})^2}{2} + 0 = \frac{32.2\ \rho_w h_A}{\rho_w} + \frac{(6.7906\ \text{ft/s})^2}{2} + 0$$

$$h_A = 1.377\ \text{ft} = 1.38\ \text{ft} \qquad \textbf{Ans.}$$

5–45. Determine the flow of oil through the pipe if the difference in height of the water column in the manometer is $h = 100$ mm. Take $\rho_o = 875\ \text{kg/m}^3$.



SOLUTION

Bernoulli Equation. Since point B is a stagnation point, $V_B = 0$. If we set the datum to coincide with the horizontal line connecting A and B, $z_A = z_B = 0$.

$$\frac{p_A}{\rho_o} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_o} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{875\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{p_B}{875\ \text{kg/m}^3} + 0 + 0$$

$$p_B - p_A = 437.5V_A^2 \qquad \textbf{(1)}$$

Manometer Equation. Referring to Fig. a,

$$p_A + \rho_o g h_{AC} + \rho_w g h_{CD} - \rho_o g h_{BD} = p_B$$

$$p_A + (875\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(a) + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) - (875\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(a + 0.1\ \text{m}) = p_B$$

$$p_B - p_A = 122.625 \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$437.5\,V_A^2 = 122.625$$

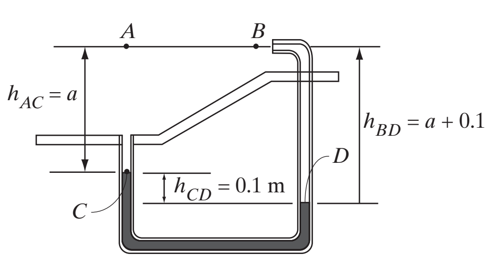
$$V_A = 0.5294\ \text{m/s}$$

Discharge.

$$Q = V_A A_A$$

$$= (0.5294\ \text{m/s})\left[\pi(0.150\ \text{m})^2\right]$$

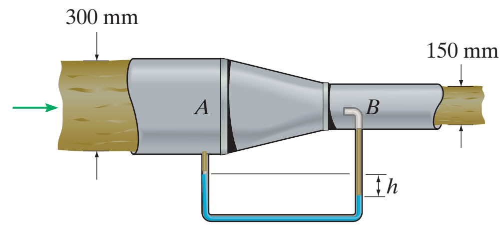
$$= 0.0374\ \text{m}^3/\text{s} \qquad \textbf{Ans.}$$



(a)

Ans:
$0.0374\ \text{m}^3/\text{s}$

5–46. Determine the difference in height h of the water column in the manometer if the flow of oil through the pipe is $0.04\ \text{m}^3/\text{s}$. Take $\rho_o = 875\ \text{kg/m}^3$.



SOLUTION

Since oil can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points A and B,

$$\frac{p_A}{\rho_o} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_o} + \frac{V_B^2}{2} + gz_B.$$

Since point B is a stagnation point, $V_B = 0$. If we set the datum to coincide with the horizontal streamline connecting A and B, then $z_A = z_B = 0$. Also, from the discharge

$$Q = V_A A_A; \qquad 0.04\ \text{m}^3/\text{s} = V_A\left[\pi(0.15\ \text{m})^2\right] \qquad V_A = 0.5659\ \text{m/s}$$

Therefore

$$\frac{p_A}{\left(875\ \text{kg/m}^3\right)} + \frac{\left(0.5659\ \text{m/s}\right)^2}{2} + 0 = \frac{p_B}{\left(875\ \text{kg/m}^3\right)} + 0 + 0$$

$$p_B - p_A = 140.10 \qquad \textbf{(1)}$$

Writing the manometer equation with reference to Fig. a,

$$p_A + \rho_o g h_{AC} + \rho_w g h_{CD} - \rho_o g h_{BD} = p_B$$

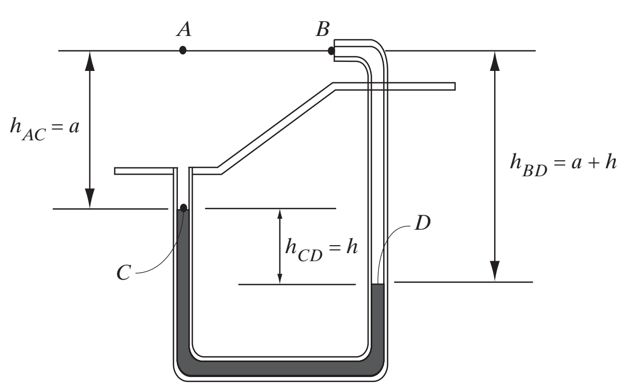
$$p_A + \left(875\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(a) + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(h)$$
$$- \left(875\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(a + h) = p_B$$

$$p_B - p_A = 1226.25\,h \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

$$1226.25\,h = 140.10$$

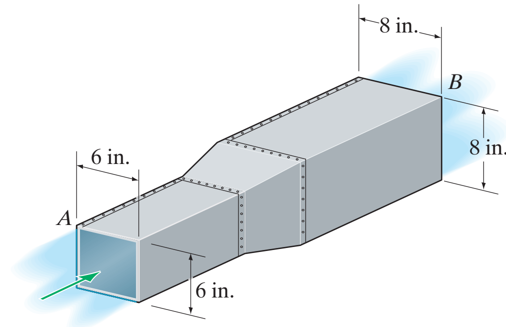
$$h = 0.1142\ \text{m} = 114\ \text{mm} \qquad \textbf{Ans.}$$



(a)

Ans:
114 mm

5–47. Air at 60°F flows through the duct such that the pressure at A is 2 psi and at B it is 2.6 psi. Determine the volumetric discharge through the duct.



SOLUTION

Assume that air is an ideal fluid (incompressible and inviscid) and the flow is steady. Then, the Bernoulli equation is applicable. Writing this equation between points A and B on central streamline,

$$\frac{p_A}{\rho_a} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_a} + \frac{V_B^2}{2} + gz_B$$

Here, $p_A = \left(2\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)^2 = 288\text{ lb/ft}^2$ and $p_B = \left(2.6\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)^2 = 374.4\text{ lb/ft}^2$

From Appendix A, at 60°F, $\rho_a = 0.00237\text{ slug/ft}^3$. Here, the datum coincides with the central streamline. Then, $z_A = z_B = 0$

$$\frac{288\text{ lb/ft}^2}{0.00237\text{ slug/ft}^3} + \frac{V_A^2}{2} + 0 = \frac{374.4\text{ lb/ft}^2}{0.00237\text{ slug/ft}^3} + \frac{V_B^2}{2}$$

$$V_A^2 - V_B^2 = 72.911(10^3) \qquad \textbf{(1)}$$

Consider the air within the dust as the control volume. Continuity condition requires that

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\left(\frac{6}{12}\text{ft}\right)\left(\frac{6}{12}\text{ft}\right)\right] + V_B\left[\left(\frac{8}{12}\text{ft}\right)\left(\frac{8}{12}\text{ft}\right)\right] = 0$$

$$V_B = 0.5625V_A \qquad \textbf{(2)}$$

Solving Eqs 1 and 2

$$V_A = 326.59\text{ ft/s} \quad V_B = 183.71\text{ ft/s}$$

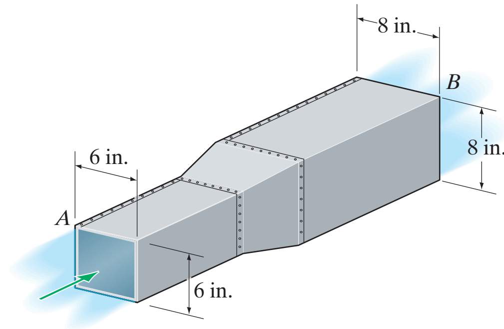
The discharge is

$$Q = V_A A_A = (326.59\text{ ft/s})\left[\left(\frac{6}{12}\text{ft}\right)\left(\frac{6}{12}\text{ft}\right)\right]$$

$$= 81.6\text{ ft}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$81.6\text{ ft}^3/\text{s}$

***5–48.** Air at 100°F flows through the duct at A at 200 ft/s under a pressure of 1.50 psi. Determine the pressure at B.



SOLUTION

Assume that air is an ideal fluid (incompressible and inviscid) and the flow is steady. Then, the Bernoulli equation is applicable. Writing this equation between points A and B on the central streamline,

$$\frac{p_A}{\rho_a} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_a} + \frac{V_B^2}{2} + gz_B$$

Here, $p_A = \left(1.50\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)^2 = 216\text{ lb/ft}^2$. From Appendix A, at $100\,°F$, $\rho_a =$ 0.00220 slug/ft^3. Here, the datum coincides with the central streamline. Then, $z_A = z_B = 0$.

$$\frac{216\text{ lb/ft}^2}{0.00220\text{ slug/ft}^3} + \frac{(200\text{ ft/s})^2}{2} + 0 = \frac{p_B}{0.00220\text{ slug/ft}^3} + \frac{V_B^2}{2} + 0$$

$$p_B = 0.00110\left[236.36(10^3) - V_B^2\right]\text{ lb/ft}^2 \quad \textbf{(1)}$$

Consider the air within the dust as the control volume. Continuity requires that

$$\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall + \int_{cs}\rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(200\text{ ft/s})\left[\left(\frac{6}{12}\text{ft}\right)\left(\frac{6}{12}\text{ft}\right)\right] + V_B\left[\left(\frac{8}{12}\text{ft}\right)\left(\frac{8}{12}\text{ft}\right)\right] = 0$$

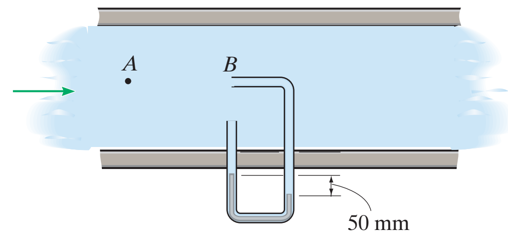
$$V_B = 112.5\text{ ft/s}$$

Substituting this result into Eq 1

$$p_B = 0.001100\left[236.36(10^3) - 112.5^2\right]$$

$$= \left(246.08\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)^2 = 1.71\text{ Psi} \qquad \textbf{Ans.}$$

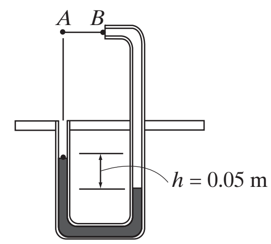
5–49. Carbon dioxide at 20°C flows past the Pitot tube B such that mercury within the manometer is displaced 50 mm as shown. Determine the mass flow if the duct has a cross-sectional area of 0.18 m^2.



SOLUTION

Assume that carbon dioxide is an ideal fluid (incompressible and inviscid) and the flow is steady. Thus, the Bernoulli equation is applicable. Writing this equation between points A and B on the central streamline

$$\frac{p_A}{\rho_{CO_2}} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_{CO_2}} + \frac{V_B^2}{2} + gz_B.$$



(a)

Here $V_B = 0$ since it is a stagnation point. From Appendix A, $\rho_{CO_2} = 1.84\ \text{kg/m}^3$ at $T = 20°\text{C}$. Here, the datum coincides with the central streamline. Then $z_A = z_B = 0$.

$$\frac{p_A}{1.84\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{p_B}{1.84\ \text{kg/m}^3} + 0 + 0$$

$$p_B - p_A = 0.92\ V_A^2 \qquad \textbf{(1)}$$

Write the manometer equation, where the pressure contribution of the CO_2 is negligible compared to that of the mercury.

$$p_A + \rho_{Hg} g h_{Hg} = p_B$$

$$p_B - p_A = \rho_{Hg} g h_{Hg}$$

$$p_B - p_A = \left(13550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.05\ \text{m})$$

$$p_B - p_A = 6646.275 \qquad \textbf{(2)}$$

Substitute Eq 2 into 1

$$0.92\ V_A^2 = 6646.275$$

$$V_A = 84.995\ \text{m/s}$$

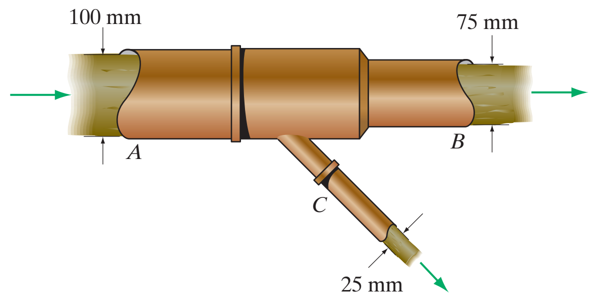
The mass flow rate is

$$\dot{m} = \rho_{CO_2} V_A A_A = \left(1.84\ \text{kg/m}^3\right)(84.995\ \text{m/s})\left(0.18\ \text{m}^2\right)$$

$$= 28.2\ \text{kg/s} \qquad \textbf{Ans.}$$

Ans:
28.2 kg/s

5–50. Oil flows through the horizontal pipe under a pressure of 400 kPa and at a velocity of 2.5 m/s at A. Determine the pressure in the pipe at B if the pressure at C is 150 kPa. Neglect any elevation difference. Take $\rho_o = 880\ \text{kg/m}^3$.



SOLUTION

Bernoulli Equations. Since the flow occurs in the horizontal plane, the elevation terms can be ignored. From A to C,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C$$

$$\frac{400(10^3)\text{N/m}^3}{880\ \text{kg/m}^3} + \frac{(2.5\ \text{m/s})^2}{2} + 0 = \frac{150(10^3)\ \text{N/m}^3}{880\ \text{kg/m}^3} + \frac{V_C^2}{2} + 0$$

$$V_C = 23.97\ \text{m/s}$$

From A to B,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + z_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + z_B$$

$$\frac{400(10^3)\text{N/m}^3}{880\ \text{kg/m}^3} + \frac{(2.5\ \text{m/s})^2}{2} + 0 = \frac{p_B}{880\ \text{kg/m}^3} + \frac{V_B^2}{2} + 0$$

$$p_B = 880(457.67 - 0.5V_B^2) \qquad \textbf{(1)}$$

Continuity Equation. Consider the oil in the pipe as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B + V_C A_C = 0$$

$$-(2.5\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right] + (23.97\ \text{m/s})\left[\pi(0.0125\ \text{m})^2\right] + V_B\left[\pi(0.0375\ \text{m})^2\right] = 0$$

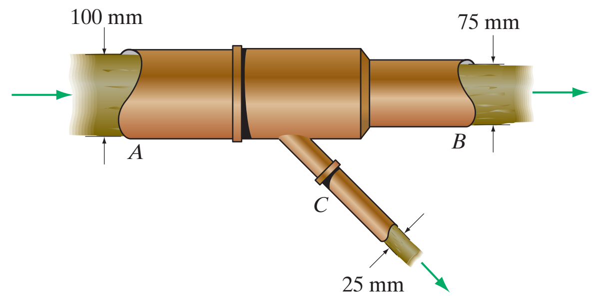
$$V_B = 1.781\ \text{m/s}$$

Substituting the result of V_B into Eq. (1),

$$p_B = 401.35(10^3)\ \text{Pa} = 401\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
401 kPa

5–51. Oil flows through the horizontal pipe under a pressure of 100 kPa and a velocity of 2.5 m/s at A. Determine the pressure in the pipe at C if the pressure at B is 95 kPa. Take $\rho_o = 880\ \text{kg/m}^3$.



SOLUTION

Bernoulli Equations. Since the flow occurs in the horizontal plane, the elevation terms can be ignored. From A to B,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{100(10^3)\ \text{N/m}^2}{880\ \text{kg/m}^3} + \frac{(2.5\ \text{m/s})^2}{2} + 0 = \frac{95(10^3)\ \text{N/m}^2}{880\ \text{kg/m}^3} + \frac{V_B^2}{2} + 0$$

$$V_B = 4.197\ \text{m/s}$$

From A to C,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C$$

$$\frac{100(10^3)\ \text{N/m}^3}{880\ \text{kg/m}^3} + \frac{(2.5\ \text{m/s})^2}{2} + 0 = \frac{p_C}{880\ \text{kg/m}^3} + \frac{V_C^2}{2} + 0$$

$$p_C = 880(116.76 - 0.5V_C^2) \qquad \textbf{(1)}$$

Continuity Equation. Consider the oil in the pipe are the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B + V_C A_C = 0$$

$$-(2.5\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right] + (4.197\ \text{m/s})\left[\pi(0.0375\ \text{m})^2\right] + V_C\left[\pi(0.0125\ \text{m})^2\right] = 0$$

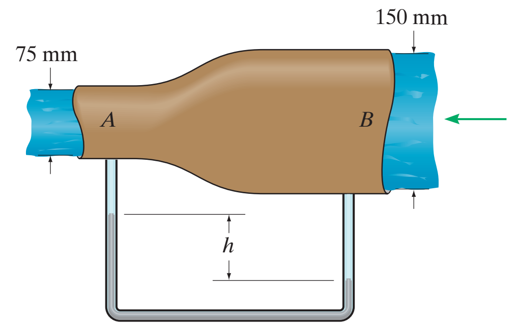
$$V_C = 2.228\ \text{m/s}$$

Substituting the result of V_C into Eq. (1),

$$p_C = 100.57(10^3)\ \text{Pa} = 101\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
101 kPa

***5–52.** Water flows through the pipe transition at a rate of 6 m/s at A. Determine the difference in the level of mercury within the manometer. Take $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.



SOLUTION

Continuity Equation. Consider the water in the pipe as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 + V_A A_A - V_B A_B = 0$$

$$(6\ \text{m/s})\left[\pi(0.0375\ \text{m})^2\right] - V_B\left[\pi(0.075\ \text{m})^2\right] = 0$$

$$V_B = 1.5\ \text{m/s}$$

Bernoulli Equation. If we set the datum to coincide with the horizontal line connecting points A and B, $z_A = z_B = 0$.

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{1000\ \text{kg/m}^3} + \frac{(6\ \text{m/s})^2}{2} + 0 = \frac{p_B}{1000\ \text{kg/m}^3} + \frac{(1.5\ \text{m/s})^2}{2} + 0$$

$$p_B - p_A = 16\,875\ \text{Pa} \qquad \textbf{(1)}$$

Manometer Equation. Referring to Fig. a,

$$p_A + \rho_w g h_{AC} + \rho_{Hg} g h_{CD} - \rho_w g h_{BD} = p_B$$

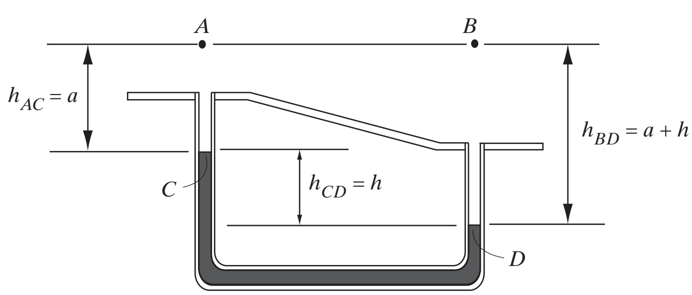
$$p_A + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(a) + (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h) - (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(a + h) = p_B$$

$$p_B - p_A = 123\,115.5h \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

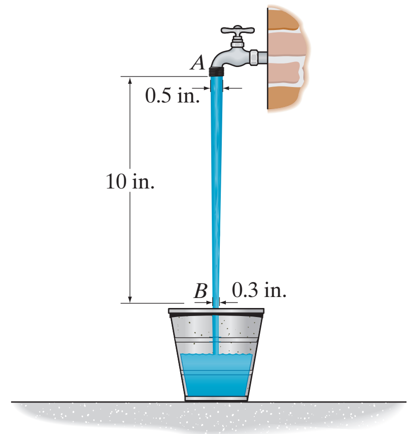
$$16\,875 = 123\,115.5h$$

$$h = 0.1371\ \text{m} = 137\ \text{mm} \qquad \textbf{Ans.}$$



(a)

5–53. Due to the effect of surface tension, water from a faucet tapers from a diameter of 0.5 in. to 0.3 in. after falling 10 in. Determine the average velocity of the water at A and at B.



SOLUTION

Continuity Equation. Consider the water stream as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\frac{\pi}{4}(0.5 \text{ in.})^2\right] + V_B\left[\frac{\pi}{4}(0.3 \text{ in.})^2\right] = 0$$

$$V_A = 0.36 V_B \qquad \textbf{(1)}$$

Bernoulli Equation. Since the water flows in the open atmosphere, $p_A = p_B = 0$. If we set the datum at B, $z_A = \frac{10}{12}$ ft and $z_B = 0$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + g z_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + g z_B$$

$$0 + \frac{V_A^2}{2} + \left(32.2 \text{ ft/s}^2\right)\left(\frac{10}{12} \text{ ft}\right) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B^2 - V_A^2 = 53.667 \qquad \textbf{(2)}$$

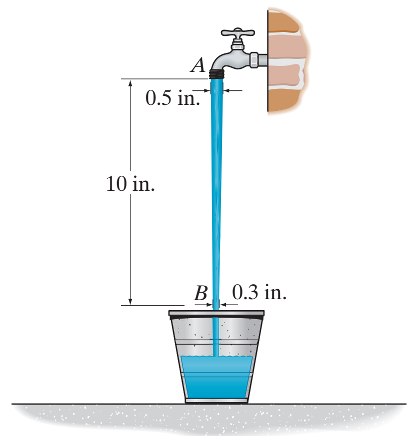
Solving Eqs. (1) and (2) yields

$$V_A = 2.83 \text{ ft/s} \qquad \textbf{Ans.}$$

$$V_B = 7.85 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$V_A = 2.83$ ft/s
$V_B = 7.85$ ft/s

5–54. Due to the effect of surface tension, water from a faucet tapers from a diameter of 0.5 in. to 0.3 in. after falling 10 in. Determine the mass flow in slug/s.



SOLUTION

Continuity Equation. Consider the water stream as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d V + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\frac{\pi}{4}(0.5 \text{ in.})^2\right] + V_B\left[\frac{\pi}{4}(0.3 \text{ in.})^2\right] = 0$$

$$V_A = 0.36 V_B \qquad \textbf{(1)}$$

Bernoulli Equation. Since the water flow in the open atmosphere, $p_A = p_B = 0$. If we set the datum at B, $z_A = \frac{10}{12}$ ft and $z_B = 0$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + g z_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + g z_B$$

$$0 + \frac{V_A^2}{2} + (32.2 \text{ ft/s}^2)\left(\frac{10}{12}\text{ ft}\right) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B^2 - V_A^2 = 53.667 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$V_A = 2.827 \text{ ft/s}$$

$$V_B = 7.852 \text{ ft/s}$$

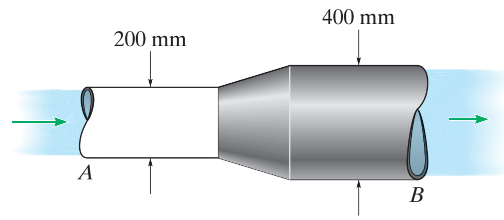
Mass Flow Rate.

$$\dot{m} = \rho V_A A_A = \left(\frac{62.4}{32.2}\text{ slug/ft}^3\right)(2.827 \text{ ft/s})\left[\frac{\pi}{4}\left(\frac{0.5}{12}\text{ ft}\right)^2\right]$$

$$= 0.00747 \text{ slug/s} \qquad \textbf{Ans.}$$

Ans:
0.00747 slug/s

5–55. Air at 15°C and absolute pressure of 275 kPa flows through the 200-mm-diameter duct at $V_A = 4\text{ m/s}$. Determine the absolute pressure of the air after it passes through the transition and into the 400-mm-diameter duct B. The temperature of the air remains constant.



SOLUTION

Here, the flow is steady. The control volume can be classified as fixed since its volume does not change with time (contains the air in the 200-mm diameter duct, the transition and 400-mm diameter duct).

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

Since the flow is steady and the control volume is fixed, there are no local changes. Thus, the above equation becomes

$$\int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

Using the average velocities, and treating the air as incompressible so that $\rho_A = \rho_B$,

$$-V_A A_A + V_B A_B = 0 \qquad \textbf{(1)}$$

Using the ideal gas law with $R = 286.9\text{ J/kg}\cdot\text{K}$ for air (Appendix A)

$$p_A = \rho_A R T_A; \qquad 275(10^3)\frac{\text{N}}{\text{m}^2} = \rho_A(286.9\text{ J/kg}\cdot\text{k})(15 + 273)\text{ K}$$

$$\rho_A = 3.3282\text{ kg/m}^3$$

Substitue into Eq 1

$$-(4\text{ m/s})\left[\frac{\pi}{4}(0.2\text{ m})^2\right] + (V_B)\left[\frac{\pi}{4}(0.4\text{ m})^2\right] = 0$$

$$V_B = 1\text{ m/s}$$

Applying the Bernoulli equation since the air is assumed incompressible and no temperature change occurs, we have

$$\frac{p_A}{\rho_A} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_B} + \frac{V_B^2}{2} + gz_B$$

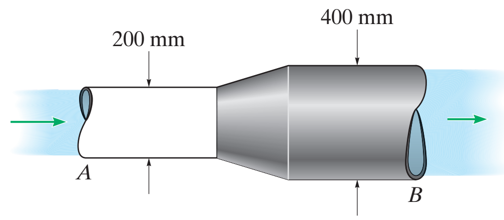
$$\frac{275(10^3)\text{ N/m}^2}{3.3282\text{ kg/m}^3} + \frac{(4\text{ m/s})^2}{2} + 0 = \frac{p_B}{3.3282\text{ kg/m}^3} + \frac{(1\text{ m/s})^2}{2} + 0$$

$$p_B = 275.025\text{ kPa} \qquad \textbf{Ans.}$$

The very small change in pressure is consistent with our assumption that the density remains constant.

Ans:
275.025 kPa

***5–56.** Air at 15°C and absolute pressure of 250 kPa flows through the 200-mm-diameter duct at $V_A = 20\text{ m/s}$. Determine the rise in pressure ratio $\Delta\rho = p_B - p_A$ when the air passes through the transition and into the 400 mm diameter duct. The temperature of the air remains constant.



SOLUTION

Assume the flow is steady and the control volume can be classified as fixed since its volume does not change with time. Applying the continuity equation,

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - \rho_A V_A A_A + \rho_B V_B A_B = 0$$

But we can assume $\rho_A = \rho_B$, so that

$$V_A A_A = V_B A_B$$

$$(20\text{ m/s})\left[\frac{\pi}{4}(0.2\text{ m})^2\right] = V_B\left[\frac{\pi}{4}(0.4\text{ m})^2\right]$$

$$V_B = 5\text{ m/s}$$

The density of the air is determined from the ideal gas law

$$p_A = \rho_A R T_A; \qquad 250\left(10^3\right)\frac{\text{N}}{\text{m}^2} = \rho_A\left(286.9\text{ J/kg}\cdot\text{k}\right)(15° + 273)\text{ K}$$

$$\rho_A = 3.0256\text{ kg/m}^3$$

Applying Bernoulli's equation since the air is assumed incompressible and no temperature change occur, we have

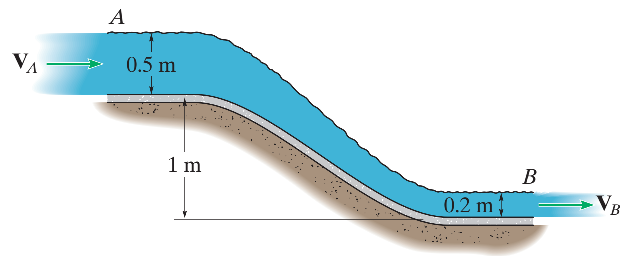
$$\frac{p_A}{\rho_A} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_B} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{3.0256\text{ kg/m}^3} + \frac{(20\text{ m/s})^2}{2} + 0 = \frac{p_B}{3.0256\text{ kg/m}^3} + \frac{(5\text{ m/s})^2}{2} + 0$$

$$\Delta p = p_B - p_A$$

$$= 567\text{ Pa} \qquad \textbf{Ans.}$$

5–57. Water flows in a rectangular channel over the 1-m drop. If the width of the channel is 1.5 m, determine the volumetric flow in the channel.



SOLUTION

Continuity Equation. Consider the water from A to B as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$\Sigma \mathbf{V} \cdot \mathbf{A} = 0;$ $\quad 0 - V_A A_A + V_B A_B = 0$

$$-V_A[0.5\text{ m}(1.5\text{ m})] + V_B[0.2\text{ m}(1.5\text{ m})] = 0$$

$$V_A = 0.4V_B \qquad \textbf{(1)}$$

Bernoulli Equation. Since surfaces A and B are exposed to the open atmosphere, $p_A = p_B = 0$. If we set the datum at the lower base of the channel, $z_A = (1\text{ m} + 0.5\text{ m}) = 1.5\text{ m}$ and $z_B = 0.2\text{ m}$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$0 + \frac{V_A^2}{2} + (9.81\text{ m/s}^2)(1.5\text{ m}) = 0 + \frac{V_B^2}{2} + (9.81\text{ m/s}^2)(0.2\text{ m})$$

$$V_B^2 - V_A^2 = 25.506 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

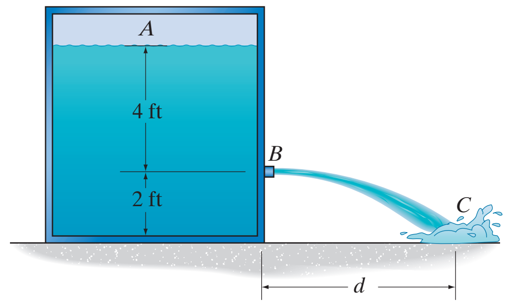
$$V_A = 2.204\text{ m/s} \qquad V_B = 5.510\text{ m/s}$$

Discharge.

$$Q = V_A A_A = (2.204\text{ m/s})[0.5\text{ m}(1.5\text{ m})] = 1.65\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$1.65\text{ m}^3/\text{s}$

5–58. Air at the top A of the water tank has a pressure of 60 psi. If water issues from the nozzle at B, determine the velocity of the water as it exits the hole, and the average distance d from the opening to where it strikes the ground.



SOLUTION

Bernoulli Equation. Since water is discharged into atmosphere at B, $p_B = 0$. Also, it is discharged from a large source, $V_A \cong 0$. If we set the datum at B, $z_A = 4$ ft and $z_B = 0$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{\left(60\,\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)} + 0 + \left(32.2\text{ ft/s}^2\right)(4\text{ ft}) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B = 95.78\text{ ft/s}$$

Vertical Motion.

$$(+\downarrow) \qquad (s_C)_y = (s_B)_y + (V_B)_y t + \frac{1}{2}a_c t^2$$

$$2 = 0 + 0 + \frac{1}{2}\left(32.2\text{ ft/s}^2\right)t^2$$

$$t = 0.3525\text{ s}$$

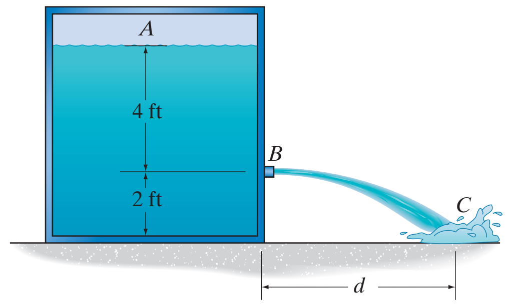
Horizontal Motion.

$$(\overset{+}{\rightarrow}) \qquad (s_C)_x = (s_B)_x + (V_B)_x t$$

$$d = 0 + (95.78\text{ ft/s})(0.3525\text{ s}) = 33.8\text{ ft} \qquad \textbf{Ans.}$$

Ans:
33.8 ft

5–59. Air is pumped into the top A of the water tank, and water issues from the small hole at B. Determine the distance d where the water strikes the ground as a function of the gage pressure at A. Plot this distance (vertical axis) versus the pressure p_A for $0 \le p_A \le 100$ psi. Give values for increments of $\Delta p_A = 20$ psi.



SOLUTION

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points A and B,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Since the water is discharged into the atmosphere at B, $p_B = 0$. Also, it is discharged from a large reservoir, $V_A = 0$. If we set the datum at B, $z_A = 4$ ft and $z_B = 0$.

$$\frac{p_A\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)^2}{\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)} + 0 + (32.2 \text{ ft/s}^2)(4 \text{ ft}) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B = \left(\sqrt{148.62\, p_A + 257.6}\right) \text{ ft/s}$$

Vertical Motion,

$$(+\downarrow) \qquad (s_C)_y = (s_B)_y + (V_B)_y t + \frac{1}{2} a_C t^2$$

$$2 \text{ ft} = 0 + 0 + \frac{1}{2}(32.2 \text{ ft/s}^2)t^2$$

$$t = 0.3525 \text{ s}$$

Horizontal Motion,

$$(\overset{+}{\rightarrow}) \qquad (s_C)_x = (s_B)_x + (V_B)_x t$$

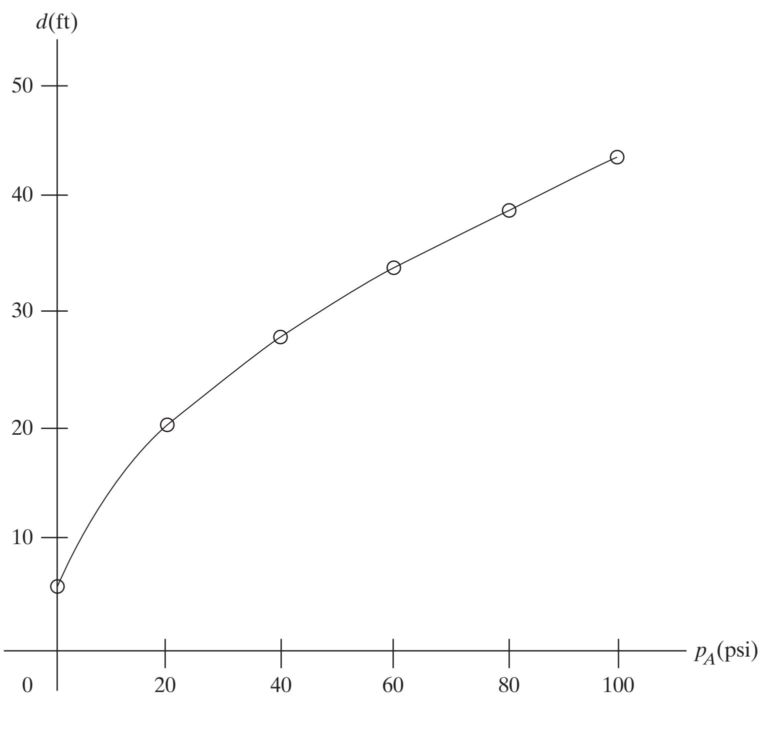
5–59. Continued

$$d = 0 + \left(\sqrt{148.62\, p_A + 257.6}\right)(0.3525\ \text{s})$$

$$d = \left(0.352\sqrt{149.6 p_A + 258.}\right)\ \text{ft where } p_A \text{ is in psi} \qquad \textbf{Ans.}$$

The plot of d vs p_A is shown in Fig. *a*

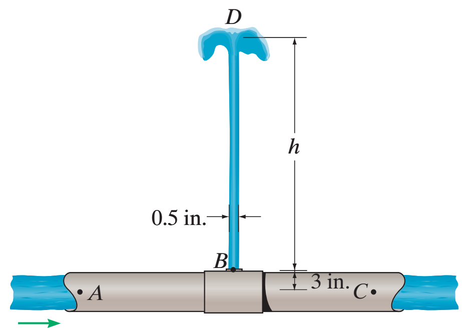
p_A(psi)	0	20	40	60	80	100
d(ft)	5.66	20.0	27.8	33.8	38.8	43.3



(a)

Ans:
$d = \left(0.352\sqrt{149.6\, p_A + 258}\right)$ ft,
where p_A is in psi.

***5–60.** Determine the height h of the water column and the average velocity at C if the pressure of the water in the 6-in.-diameter pipe at A is 10 psi and water flows past this point at 6 ft/s.



SOLUTION

Bernoulli Equation. Since the water column achieves a maximum height at D, $V_D = 0$. Here, B and D are open to the atmosphere, $p_B = p_D = 0$. If the datum is set horizontally at A, $z_A = 0$, $z_B = \frac{3}{12}\text{ ft} = 0.25\text{ ft}$, and $z_D = h + \frac{3}{12}\text{ ft} = h + 0.25\text{ ft}$. From A to D,

$$\frac{p_A}{\rho} + \frac{V_D^2}{2} + gz_A = \frac{p_D}{\rho} + \frac{V_D^2}{2} + gz_D$$

$$\frac{\left(10\,\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}} + \frac{(6\text{ ft/s})^2}{2} = 0 + 0 + (32.2\text{ ft/s}^2)(h + 0.25\text{ ft})$$

$$h = 23.39\text{ ft} = 23.4\text{ ft} \qquad \textbf{Ans.}$$

From A to B,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{\left(10\,\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}} + \frac{(6\text{ ft/s})^2}{2} + 0 = 0 + \frac{V_B^2}{2} + (32.2\text{ ft/s}^2)(0.25\text{ ft})$$

$$V_B = 38.81\text{ ft/s} \qquad \textbf{Ans.}$$

Continuity Equation. Consider the water in the pipe from A to B to be the control volume,

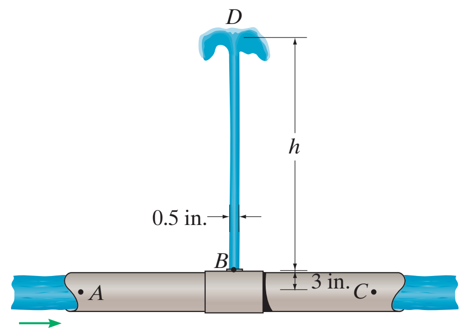
$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B + V_C A_C = 0$$

$$-(6\text{ ft/s})\left[\pi\left(\frac{3}{12}\text{ ft}\right)^2\right] + (38.81\text{ ft/s})\left[\pi\left(\frac{0.25}{12}\text{ ft}\right)^2\right] + V_C\left[\pi\left(\frac{3}{12}\text{ ft}\right)^2\right] = 0$$

$$V_C = 5.73\text{ ft/s} \qquad \textbf{Ans.}$$

5–61. If the pressure in the 6-in.-diameter pipe at A is 10 psi, and the water column rises to a height of $h = 30$ ft, determine the pressure and velocity in the pipe at C.



SOLUTION

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady the Bernoulli's equation is applicable. Writting this equation between points A and D, realizing that $p_D = 0$ (D is open to atmosphere) and $V_D = 0$ (the water column achives a maximum height at D). If the datum coincides with the Central Stream line, $z_A = 0$, and $z_D = \left(\frac{3}{12}\text{ ft}\right) + 30\text{ ft} = 30.25\text{ ft}$.

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_D}{\rho_w} + \frac{V_D^2}{2} + gz_D$$

$$\frac{\left(10\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)^2}{\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)} + \frac{V_A^2}{2} + 0 = 0 + 0 + (32.2\text{ ft/s}^2)(30.25\text{ ft})$$

$$\frac{V_A^2}{2} = 230.97 \qquad V_A = 21.49\text{ ft/s}$$

Between points B and D where $p_B = 0$ (B is open to atmosphere) and $z_B = \frac{3}{12}\text{ft} = 0.25\text{ ft}$,

$$\frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B = \frac{p_D}{\rho_w} + \frac{V_D^2}{2} + gz_D$$

$$0 + \frac{V_B^2}{2} + (32.2\text{ ft/s}^2)(0.25\text{ ft}) = 0 + 0 + (32.2\text{ ft/s}^2)(30.25\text{ ft})$$

$$\frac{V_B^2}{2} = 966 \qquad V_B = 43.95\text{ ft/s}$$

Consider the fixed control volume that contains the water in the pipe from A to C, continuity requires that

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\, \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B + V_C A_C = 0$$

$$(-21.49\text{ ft/s})\left[\pi\left(\frac{3}{12}\text{ ft}\right)^2\right] + (43.95\text{ ft/s})\left[\pi\left(\frac{0.25}{12}\text{ ft}\right)^2\right] + V_C\left[\pi\left(\frac{3}{12}\text{ ft}\right)^2\right] = 0$$

$$V_C = 21.188 = 21.2\text{ m/s} \qquad \textbf{Ans.}$$

5–61. Continued

Between points A and C,

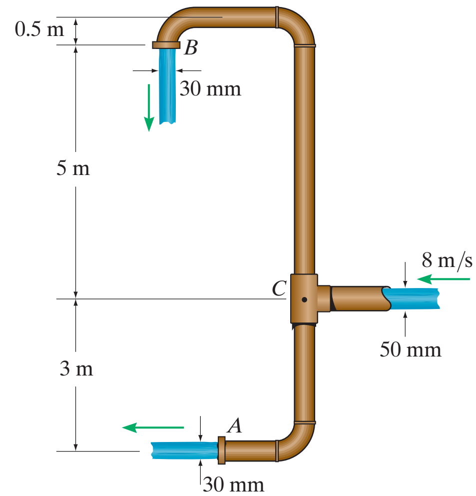
$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + 0 = \frac{p_C}{\rho_w} + \frac{V_C^2}{2} = 0$$

$$\frac{\left(10\,\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)} + \frac{(21.49\text{ ft/s})^2}{2} = \frac{p_C}{\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)} + \frac{(21.188\text{ ft/s})^2}{2}$$

$$p_C = \left(1452.62\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = 10.1\text{ psi} \qquad \textbf{Ans.}$$

Ans:
$V_C = 21.2\text{ m/s}$
$p_C = 10.1\text{ psi}$

5–62. Determine the velocity of the flow out of the vertical pipes at A and B, if water flows into the Tee at 8 m/s and under a pressure of 40 kPa.



SOLUTION

Continuity Equation. Consider the water within the pipe to be the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_C A_C + V_B A_B + V_A A_A = 0$$

$$-(8\text{ m/s})\left[\pi(0.025\text{ m})^2\right] + V_B\left[\pi(0.015\text{ m})^2\right] + V_A\left[\pi(0.015\text{ m})^2\right] = 0$$

$$V_A + V_B = 22.22 \qquad \textbf{(1)}$$

Bernoulli Equation. Since the water discharged into the atmosphere at A and B, $p_A = p_B = 0$. If we set the datum horizontally through point C, $z_B = 5$ m and $z_A = -3$ m.

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + g z_B = \frac{p_A}{\rho} + \frac{V_A^2}{2} + g z_A$$

$$0 + \frac{V_B^2}{2} + (9.81\text{ m/s}^2)(5\text{ m}) = 0 + \frac{V_A^2}{2} + (9.81\text{ m/s}^2)(-3\text{ m})$$

$$V_A^2 - V_B^2 = 156.96 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

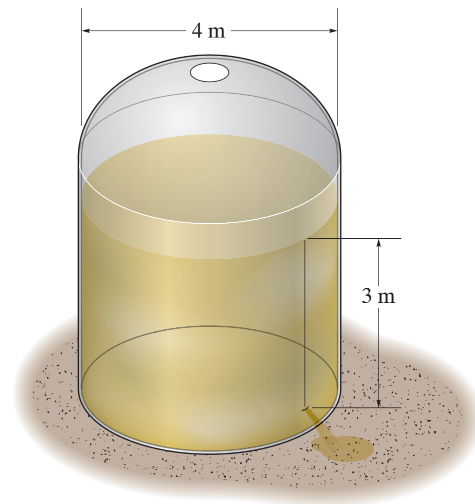
$$V_A = 14.6\text{ m/s} \qquad \textbf{Ans.}$$

$$V_B = 7.58\text{ m/s} \qquad \textbf{Ans.}$$

Note: Treating A and B as if they lie on the same streamline is a harmless shortcut. Officially, the solution process should proceed by considering *two* streamlines that each run through C.

Ans:
$V_A = 14.6$ m/s
$V_B = 7.58$ m/s

5–63. The open cylindrical tank is filled with linseed oil. A crack having a length of 50 mm and average height of 2 mm occurs at the base of the tank. How many liters of oil will slowly drain from the tank in eight hours? Take $\rho_o = 940\ \text{kg/m}^3$.



SOLUTION

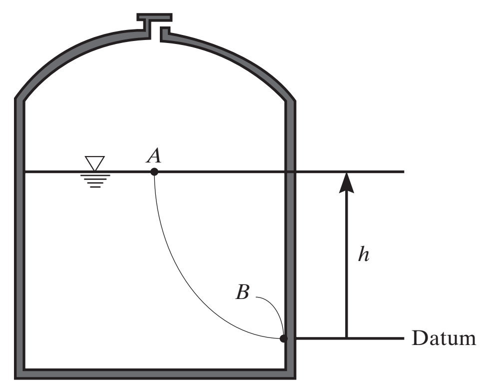
The linseed oil can be considered as an ideal fluid (incompressible and inviscid). Here, we assume that the flow is steady. Therefore, Bernoulli's equation is applicable. Applying this equation between A and B Fig. a, where both are on the streamline shown,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

Since the tank is a large reservoir, $V_A \cong 0$. Therefore, V_A^2 is negligible. Since A and B are exposed to the atmosphere, $p_A = p_B = 0$. Here, the datum is set through point B. Then

$$0 + 0 + gh = 0 + \frac{V_B^2}{2} + 0$$

$$V_B = \sqrt{2gh} = \sqrt{2\left(9.81\ \text{m/s}^2\right)h} = \sqrt{19.62\,h} \qquad \textbf{(1)}$$



(a)

The control volume changes with time. The volume of the control volume at a particular instant is

$$\forall = \pi r^2 h = \pi(2\ \text{m})^2 h = (4\pi h)\ \text{m}^3$$

$$\frac{\partial \forall}{\partial t} = 4\pi\,\frac{\partial h}{\partial t} \qquad \textbf{(2)}$$

Thus,

$$\frac{\partial}{\partial t}\int_{cv} \rho_o\, d\forall + \int_{cs} \rho_o\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$\rho_o\,\frac{\partial \forall}{\partial t} + \rho_o V_B A_B = 0$$

$$\frac{\partial \forall}{\partial t} + V_B A_B = 0$$

Using Eq 1 and 2

$$4\pi\,\frac{\partial h}{\partial t} = -\left(\sqrt{19.62\,h}\right)[(0.05\ \text{m})(0.002\ \text{m})]$$

$$\frac{\partial h}{\partial t} = -35.2484\left(10^{-6}\right)\sqrt{h}$$

When $h = 3$ m, $V_A = -\dfrac{\partial h}{\partial t} = 35.2484\left(10^{-6}\right)\sqrt{3\ \text{m}} = 61.1\left(10^{-6}\right)\ \text{m/s}$, which indeed is very small as compare to $V_B = \sqrt{19.62\,(3\ \text{m})} = 7.67\ \text{m/s}$. Thus, the assumption of $V_A \cong 0$ is quite reasonable. Integrating the above equation

5–63. Continued

$$\int_{3\text{ m}}^{h} \frac{\partial h}{\sqrt{h}} = -\int_{0}^{8(3600\text{ s})} 35.2484\left(10^{-6}\right)\partial t$$

$$2h^{\frac{1}{2}}\Big|_{3\text{ m}}^{h} = -35.2484\left(10^{-6}\right)t\Big|_{0}^{8(3600\text{ s})}$$

$$2\left(h^{\frac{1}{2}} - 1.732\right) = -1.0152$$

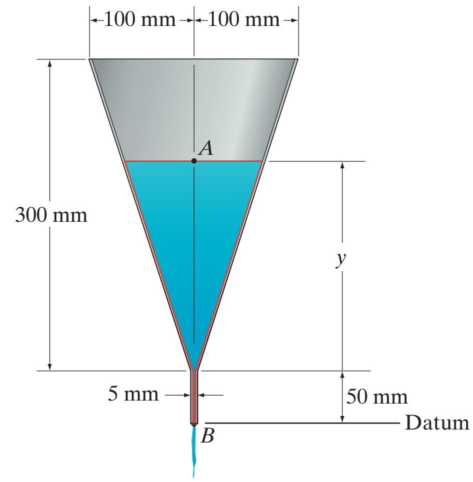
$$h = 1.4993\text{ m}$$

Thus, the volume of the leakage is

$$V_{le} = \pi(2\text{ m})^2(3\text{ m} - 1.4993\text{ m}) = \left(18.858\text{ m}^3\right)\left(\frac{1000\text{ L}}{1\text{ m}^3}\right) = 18.9\left(10^3\right)\text{ liters} \quad \textbf{Ans.}$$

Ans:
$18.9\left(10^3\right)$ liters

***5–64.** At the instant shown, the level of water in the conical funnel is $y = 200$ mm. If the stem has an inner diameter of 5 mm, determine the rate at which the surface level of the water is dropping.



SOLUTION

We will select the vertical streamline containing the points A and B. Here the flow is steady since the stem opening is small in relation to the volume of water in the funnel. In other words, the water level in the funnel will drop at a very slow rate, which can be considered constant for a large value of y. The pressures $p_A = p_B = 0$ (the static gauge pressure) and the gravity datum is at B.

Bernoulli Equation.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$0 + \frac{V_A^2}{2} + \left(9.81\ \text{m/s}^2\right)(0.250\ \text{m}) = 0 + \frac{V_B^2}{2} + 0 \qquad \textbf{(1)}$$

Continuity Equation. The velocities at A and B can be related by applying the continuity equation. A fixed control volume contains the water within the funnel at the instant shown. When $y = 200$ mm, the surface area at A has a radius which is determined by proportion.

$$\frac{r_A}{200\ \text{mm}} = \frac{100\ \text{mm}}{300\ \text{mm}}$$

$$r_A = 66.67\ \text{mm}$$

Thus

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi(0.06667\ \text{m})^2\right] + V_B\left[\pi(0.0025\ \text{m})^2\right] = 0$$

or

$$V_B = 711.1 V_A$$

Substituting into Eq. (1) and solving, yields

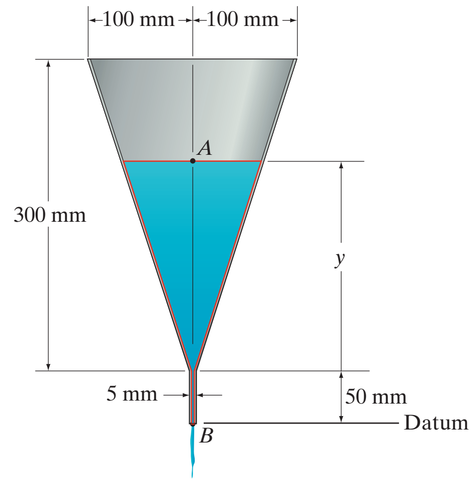
$$V_A = 0.00311\ \text{m/s} = 3.11\ \text{mm/s}$$

$$V_B = 2.21\ \text{m/s}$$

$$\frac{dy}{dt} = -V_A = -3.11\ \text{mm/s} \qquad \textbf{Ans.}$$

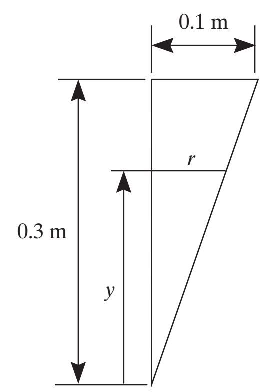
By comparison, note that the steady flow assumption is appropriate since $V_A \approx 0$ for these larger values of y_a

5–65. If the stem of the conical funnel has a diameter of 5 mm, determine the rate at which the surface level of water is dropping as a function of the depth y. Assume steady flow. *Note*: For a cone, $\forall = \frac{1}{3}\pi r^2 h$.



SOLUTION

The control volume considered is deformable. It contains the water in the funnel and stem. Here, the volume of the water $\forall_0$ in the stem is constant. Referring to the geometry shown in Fig. *a*



$$\frac{r}{y} = \frac{0.1}{0.3}; \qquad r = \frac{1}{3}y$$

Thus, the volume of the control volume is

$$\forall = \frac{1}{3}\pi r^2 h + \forall_0 = \frac{1}{3}\pi\left(\frac{1}{3}y\right)^2(y) + \forall_0 = \frac{1}{27}\pi y^3 + \forall_0$$

$$\frac{d\forall}{dt} = \frac{1}{9}\pi y^2 \frac{dy}{dt}$$

Continuity requires that

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ is constant for water,

$$\rho\left(\frac{d\forall}{dt} + V_B A_B\right) = 0$$

$$\frac{1}{9}\pi y^2 \frac{dy}{dt} + V_B\left[\pi(0.0025\text{ m})^2\right] = 0$$

$$\frac{dy}{dt} = -\left[\frac{56.25\left(10^{-6}\right)}{y^2}\right]V_B \qquad \textbf{(1)}$$

The Negative sign indicates that y is decreasing.

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points A and B, realizing that $V_A = -\frac{dy}{dt}$, $p_A = p_B = 0$ (A and B are open to atmosphere) $z_B = 0$ and $z_A = y + 0.05$ m (datum is set through B), Then

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

$$0 + \frac{(dy/dt)^2}{2} + \left(9.81\text{ m/s}^2\right)(y + 0.05\text{ m}) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B^2 = \left(\frac{dy}{dt}\right)^2 + 19.62(y + 0.05)$$

$$V_B = \sqrt{\left(\frac{dy}{dt}\right)^2 + 19.62(y + 0.05)} \qquad \textbf{(2)}$$

5–65. Continued

Substituting this result into Eq. 1,

$$\frac{dy}{dt} = -\left[\frac{56.25\left(10^{-6}\right)}{y^2}\right]\sqrt{\left(\frac{dy}{dt}\right)^2 + 19.62(y + 0.05)}$$

$$\frac{dy}{dt} = \left(-\sqrt{\frac{19.62(y + 0.05)}{0.3160(10^9)y^4 - 1}}\right)\ \text{m/s}$$

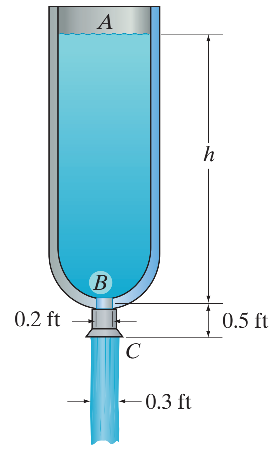
$$\frac{dy}{dt} = \left(-\sqrt{\frac{19.6(y + 0.05)}{0.316(10^9)y^4 - 1}}\right)\ \text{m/s where } y \text{ is in meters} \quad \textbf{Ans.}$$

Ans:

$$\frac{dy}{dt} = \left(-\sqrt{\frac{19.6(y + 0.05)}{0.316\left(10^9\right)y^4 - 1}}\right)\text{m/s,}$$

where y is in meters.

5–66. Water flows from the large container through the nozzle at B. If the absolute vapor pressure for the water is 0.65 psi, determine the maximum height h of the contents so that cavitation will not occur at B.



SOLUTION

Bernoulli Equation. Since the water is discharged through B from a large source, $V_A \cong 0$. Here, A and C are open to the atmosphere, $p_A = p_C = 0$. Also, at B, the pressure is required to be equal to the vapor pressure (cavitation). Then,

$$(p_B)_{\text{abs}} = (p_B)_g + p_{\text{atm}}$$

$$0.65 \text{ psi} = (p_B)_g + 14.7 \text{ psi}$$

$$(p_B)_g = \left(-14.05 \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 = -2023.2 \frac{\text{lb}}{\text{ft}^2}$$

If we set the datum at C, $z_C = 0$, $z_B = 0.5$ ft, and $z_A = 0.5 \text{ ft} + h$. From A to B,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$0 + 0 + (32.2 \text{ ft/s}^2)(0.5 \text{ ft} + h) = \frac{-2023.2 \dfrac{\text{lb}}{\text{ft}^2}}{\left(\dfrac{62.4 \text{ lb/ft}^3}{32.2 \text{ lb/ft}^2}\right)} + \frac{V_B^2}{2} + (32.2 \text{ ft/s}^2)(0.5 \text{ ft})$$

$$V_B^2 = 64.4\,h + 2088.05 \qquad \textbf{(1)}$$

From A to C,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C$$

$$0 + 0 + (32.2 \text{ lb/ft}^2)(0.5 \text{ ft} + h) = 0 + \frac{V_C^2}{2} + 0$$

$$V_C^2 = 64.4\,h + 32.2 \qquad \textbf{(2)}$$

Continuity Equation. Consider the water within the container of the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_B A_B + V_C A_C = 0$$

$$-V_B\left[\pi(0.1 \text{ ft})^2\right] + V_C\left[\pi(0.15 \text{ ft})^2\right] = 0$$

$$V_B = 2.25 V_C \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

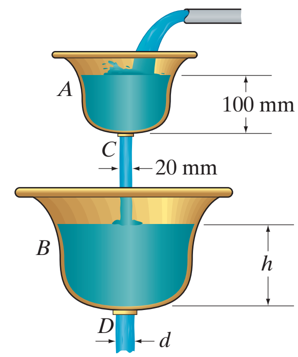
$$V_B = 50.6 \text{ ft/s}$$

$$V_C = 22.5 \text{ ft/s}$$

$$h = 7.36 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
7.36 ft

5–67. Water drains from the fountain cup A to cup B. Determine the depth h of the water in B in order for steady flow to be maintained. Take $d = 25$ mm.



SOLUTION

Bernoulli Equation. Since A, C, B, and D are exposed to the atmosphere, $p_A = p_C = p_B = p_D = 0$. To maintain steady flow, the level of water in cups A and B must be constant. Thus, from A to C with the datum set at C, $z_C = 0$ and $z_A = 0.1$ m,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C$$

$$0 + 0 + \left(9.81 \text{ m/s}^2\right)(0.1 \text{ m}) = 0 + \frac{V_C^2}{2} + 0$$

$$V_C = 1.401 \text{ m/s}$$

Continuity Equation. Consider the units within the cup B as the control volume. To meet the continuity requirement at C and D,

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_C A_C + V_D A_D = 0$$

$$-\left(1.401 \text{ m/s}\right)\left[\pi(0.01 \text{ m})^2\right] + V_D\left[\pi(0.0125 \text{ m})^2\right] = 0$$

$$V_D = 0.8964 \text{ m/s}$$

Bernoulli Equation. From B to D with datum set at D, $z_D = 0$ and $z_B = h$,

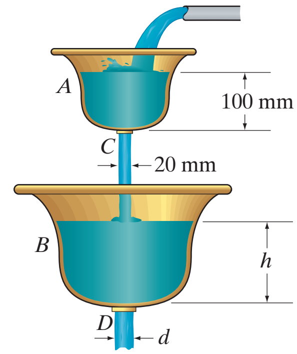
$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B = \frac{p_D}{\rho} + \frac{V_D^2}{2} + gz_D$$

$$0 + 0 + \left(9.81 \text{ m/s}^2\right)h = 0\frac{\left(0.8964 \text{ m/s}\right)^2}{2} + 0$$

$$h = 0.04096 \text{ m} = 41.0 \text{ mm}$$ **Ans.**

Ans:
41.0 mm

***5–68.** Water drains from the fountain cup A to cup B. If the depth in cup B is $h = 50$ mm, determine the velocity of the water at C and the diameter d of the opening at D so that steady flow occurs.



SOLUTION

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is required to be steady, Bernoulli's equation is applicable. Since A, B, C and D are exposed to the atmosphere, $p_A = p_B = p_C = p_D = 0$. To maintain the steady flow, the level of water in cups A and B must be constant. Thus, $V_A = V_B = 0$. Between A and C with the datum at C, $z_C = 0$ and $z_A = 0.1$ m,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho_w} + \frac{V_C^2}{2} + gz_C$$

$$0 + 0 + \left(9.81\text{ m/s}^2\right)(0.1\text{ m}) = 0 + \frac{V_C^2}{2} + 0$$

$$V_C = 1.401\text{ m/s} = 1.40\text{ m/s} \qquad \textbf{Ans.}$$

Between B and D with the datum at D, $z_B = 0.05$ m and $z_D = 0$.

$$\frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B = \frac{p_D}{\rho_w} + \frac{V_D^2}{2} + gz_D$$

$$0 + 0 + \left(9.81\text{ m/s}^2\right)(0.05\text{ m}) = 0 + \frac{V_D^2}{2} + 0$$

$$V_D = 0.9904\text{ m/s} = 0.990\text{ m/s}$$

The fixed control volume that contains the water in cup B will be considered. Continuity requires that

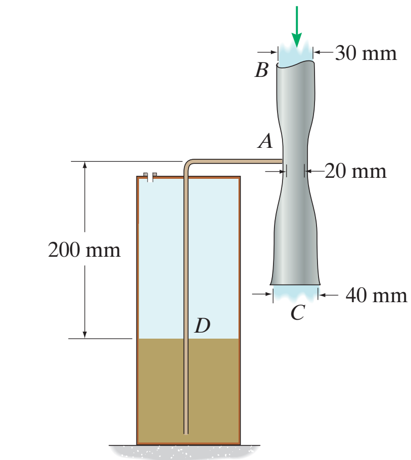
$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_C A_C + V_D A_D = 0$$

$$-(1.401\text{ m/s})\left[\pi(0.01\text{ m})^2\right] + (0.9904\text{ m/s})\left(\frac{\pi}{4}d_D^2\right) = 0$$

$$d_D = 0.02378\text{ m} = 23.8\text{ mm} \qquad \textbf{Ans.}$$

5–69. As air flows downward through the venturi constriction, it creates a low pressure a A that causes ethyl alcohol to rise in the tube and be drawn into the air stream. If the air is then discharged to the atmosphere at C, determine the smallest volumetric flow of air required to do this. Take $\rho_{ea} = 789\ \text{kg/m}^3$ and $\rho_a = 1.225\ \text{kg/m}^3$.



SOLUTION

Manometer Equation. Since D is exposed to the atmosphere, $p_D = 0$. Referring to Fig. a,

$$p_D + \rho_{ea} g h_{AD} = p_A$$

$$0 - (789\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.2\ \text{m}) = p_A$$

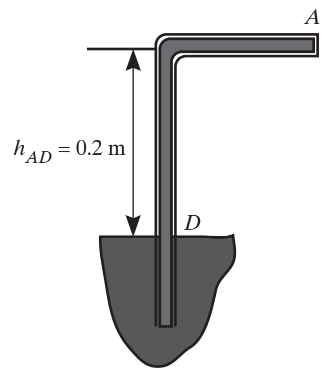
$$p_A = -1548.02\ \text{Pa}$$

Bernoulli Equation. Since air is discharged into the atmosphere at C, $p_C = 0$. Neglecting the elevation terms,

$$\frac{p_A}{\rho_a} + \frac{V_A^2}{2} + g z_A = \frac{p_C}{\rho_a} + \frac{V_C^2}{2} + g z_C$$

$$\frac{-1548.02\ \text{Pa}}{1.225\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = 0 + \frac{V_C^2}{2} + 0$$

$$V_A^2 - V_C^2 = 2527.38 \quad \textbf{(1)}$$



(a)

Continuity Equation. Consider the air within the tube of the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_C A_C = 0$$

$$-V_A\left[\pi(0.01\ \text{m})^2\right] + V_C\left[\pi(0.02\ \text{m})^2\right] = 0$$

$$V_A = 4V_C \quad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$V_A = 51.92\ \text{m/s}$$

$$V_C = 12.98\ \text{m/s}$$

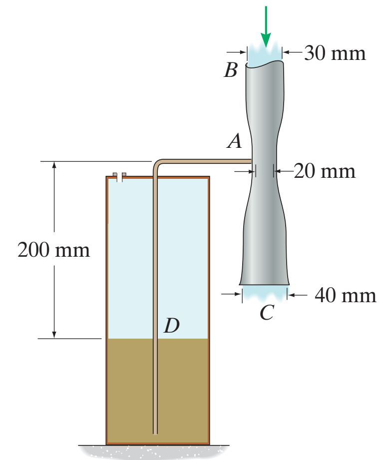
Discharge.

$$Q = V_C A_C = (12.98\ \text{m/s})\left[\pi(0.02\ \text{m})^2\right]$$

$$= 0.0163\ \text{m}^3/\text{s} \quad \textbf{Ans.}$$

Ans:
$0.0163\ \text{m}^3/\text{s}$

5–70. As air flows downward through the Venturi constriction, it creates a low pressure at A that causes ethyl alcohol to rise in the tube and be drawn into the air stream. Determine the velocity of the air as it passes through the tube at B in order to do this. The air is discharged to the atmosphere at C. Take $\rho_{ea} = 789\ \text{kg/m}^3$ and $\rho_a = 1.225\ \text{kg/m}^3$.



SOLUTION

Manometer Equation. Since D is exposed to the atmosphere, $p_D = 0$. Referring to Fig. a,

$$p_D + \rho_{ea} g h_{AD} = p_A$$

$$0 - \left(789\ \text{kg/m}^2\right)\left(9.81\ \text{m/s}^2\right)(0.2\ \text{m}) = p_A$$

$$p_A = -1548.02\ \text{Pa}$$

Bernoulli Equation. Since air is discharged into the atmosphere at C, $p_C = 0$. Neglecting the elevation terms,

$$\frac{p_A}{\rho_a} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho_a} + \frac{V_C^2}{2} + gz_C$$

$$\frac{-1548.02\ \text{Pa}}{1.225\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = 0 + \frac{V_C^2}{2} + 0$$

$$V_A^2 - V_C^2 = 2527.38 \qquad \textbf{(1)}$$

Continuity Equation. Consider the air in the tube of the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_C A_C = 0$$

$$-V_A\left[\pi(0.01\ \text{m})^2\right] + V_C\left[\pi(0.02\ \text{m})^2\right] = 0$$

$$V_A = 4V_C \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$V_A = 51.92\ \text{m/s}$$

$$V_C = 12.98\ \text{m/s}$$

Also, consider the air in the tube the control volume.

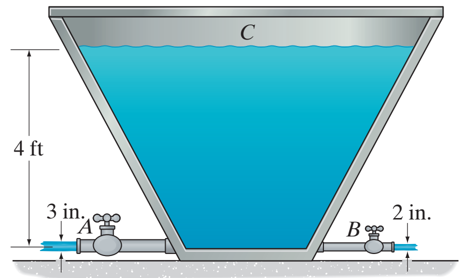
$$0 - V_B A_B + V_C A_C = 0$$

$$-V_B\left[\pi(0.015\ \text{m})^2\right] + \left(12.98\ \text{m/s}\right)\left[\pi(0.02\ \text{m})^2\right] = 0$$

$$V_B = 23.1\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
23.1 m/s

5–71. Water from the large *closed tank* is to be drained through the lines at A and B. When the valve at B is opened, the initial discharge is $Q_B = 0.8\ \text{ft}^3/\text{s}$. Determine the pressure at C and the initial volumetric discharge at A if this valve is also opened.



SOLUTION

$$Q_B = V_B A_B$$

$$0.8\ \text{ft}^3/\text{s} = V_B\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right]$$

$$V_B = 36.67\ \text{ft/s}$$

and

$$Q_A = V_A A_A = V_A\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right] = 0.04909\ V_A \qquad \textbf{(1)}$$

Bernoulli Equation. Since the water is discharged from a large source, $V_C \cong 0$. Also, the water is discharged into the atmosphere at A and B, $p_A = p_B = 0$. If the datum coincides with the horizontal line joining A and B, $z_A = z_B = 0$ and $z_C = 4$ ft. From C to B,

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_C}{\left(\dfrac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)} + 0 + \left(32.2\ \text{ft/s}^2\right)(4\ \text{ft}) = 0 + \frac{(36.67\ \text{ft/s})^2}{2} + 0$$

$$p_C = 1053.28\ \text{lb/ft}^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 7.31\ \text{psi} \qquad \textbf{Ans.}$$

From C to A,

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

$$\frac{\left(1053.28\ \text{lb/ft}^2\right)}{\left(\dfrac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)} + 0 + \left(32.2\ \text{ft/s}^2\right)(4\ \text{ft}) = 0 + \frac{V_A^2}{2} + 0$$

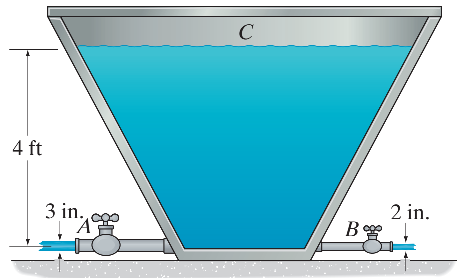
$$V_A = 36.67\ \text{ft/s}$$

Substituting the results of V_A into Eq. (1),

$$Q_A = 0.04909(36.67) = 1.80\ \text{ft}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$p_C = 7.31$ psi
$Q_A = 1.80\ \text{ft}^3/\text{s}$

***5–72.** Water from the large *closed tank* is to be drained through the lines at A and B. When the valve at A is opened, the initial discharge is $Q_A = 1.5\ \text{ft}^3/\text{s}$. Determine the pressure at C and the initial volumetric discharge at B when this valve is also opened.



SOLUTION

$$Q_A = V_A A_A$$

$$1.5\ \text{ft}^3/\text{s} = V_A\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right]$$

$$V_A = 30.558\ \text{ft/s}$$

and

$$Q_B = V_B A_B = V_B\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] = 0.02182\ V_B \qquad \textbf{(1)}$$

Bernoulli Equation. Since the water is discharged from a large source, $V_C \cong 0$. Also, the water is discharged into the atmosphere at A and B, $p_A = p_B = 0$. If the datum coincides with the horizontal line joining A and B, $z_A = z_B = 0$ and $z_C = 4$ ft. From C to A,

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

$$\frac{p_C}{\left(\dfrac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)} + 0 + \left(32.2\ \text{ft/s}^2\right)(4\ \text{ft}) = 0 + \frac{\left(30.558\ \text{ft/s}\right)^2}{2} + 0$$

$$p_C = 655.17\ \text{lb/ft}^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 4.55\ \text{psi} \qquad \textbf{Ans.}$$

From C to B,

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

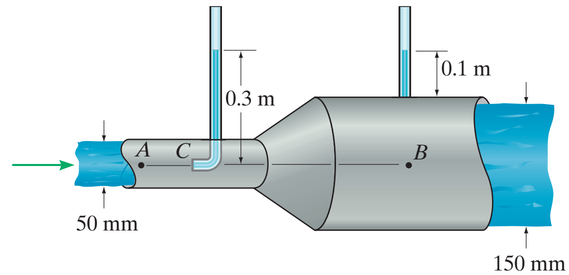
$$\frac{655.17\ \text{lb/ft}^2}{\left(\dfrac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)} + 0 + \left(32.2\ \text{ft/s}^2\right)(4\ \text{ft}) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B = 30.56\ \text{ft/s}$$

Substituting the result of V_B into Eq. (1),

$$Q_B = 0.02182(30.56) = 0.667\ \text{ft}^3/\text{s} \qquad \textbf{Ans.}$$

5–73. Determine the volumetric flow and the pressure in the pipe at A if the height of the water column in the Pitot tube is 0.3 m and the height in the piezometer is 0.1 m.



SOLUTION

Continuity Equation. Consider the water in the pipe from A to B as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi(0.025\text{ m})^2\right] + V_B\left[\pi(0.075\text{ m})^2\right] = 0$$

$$V_A = 9V_B \qquad \textbf{(1)}$$

Bernoulli Equation. Since point C is a stagnation point, $V_C = 0$. If we set the datum to coincide with the horizontal line connecting points A, B, and C, $z_A = z_B = z_C = 0$. The pressures at B and C are

$$p_B = \rho_w g h_B = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(0.1\text{ m} + 0.075\text{ m}) = 1.71675\left(10^3\right)\text{ Pa}$$

$$p_C = \rho_w g h_C = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(0.3\text{ m}) = 2.943\left(10^3\right)\text{ Pa}$$

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C$$

$$\frac{p_A}{1000\text{ kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{2.943\left(10^3\right)\text{ Pa}}{1000\text{ kg/m}^3} + 0 + 0$$

$$p_A + 500V_A^2 = 2.943\left(10^3\right) \qquad \textbf{(2)}$$

From C to B,

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{2.943\left(10^3\right)\text{ Pa}}{1000\text{ kg/m}^3} + 0 + 0 = \frac{1.71675\left(10^3\right)\text{ Pa}}{1000\text{ kg/m}^3} + \frac{V_B^2}{2} + 0$$

$$V_B = 1.566\text{ m/s}$$

Using this result to solve Eqs. (1) and (2),

$$V_A = 14.094\text{ m/s}$$

$$p_A = -96.38\left(10^3\right)\text{ Pa} = -96.4\text{ kPa} \qquad \textbf{Ans.}$$

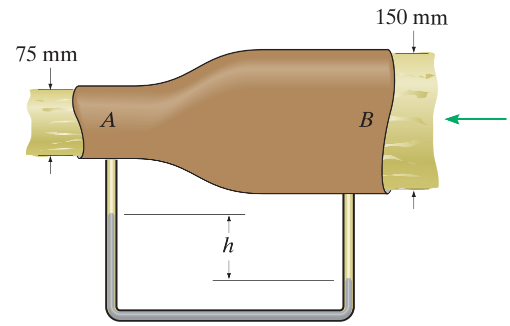
Thus

$$Q = V_B A_B = (1.566\text{ m/s})\left[\pi(0.075\text{ m})^2\right]$$

$$Q = 0.0277\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$p_A = -96.4$ kPa
$Q = 0.0277\text{ m}^3/\text{s}$

5–74. The mercury in the manometer has a difference in elevation of $h = 0.15$ m. Determine the volumetric discharge of gasoline through the pipe. Take $\rho_{gas} = 726\ \text{kg/m}^3$.



SOLUTION

Since gasoline can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable writing this equation between points A and B on the central streamline,

$$\frac{p_A}{\rho_{ga}} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_{ga}} + \frac{V_B^2}{2} + gz_B$$

With reference to the datum set on the central streamline, $z_A = z_B = 0$. From Appendix A, $\rho_{ga} = 726\ \text{kg/m}^3$. Then

$$\frac{p_A}{726\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{p_B}{726\ \text{kg/m}^3} + \frac{V_B^2}{2} + 0$$

$$p_B - p_A = 363\left(V_A^2 - V_B^2\right) \qquad \textbf{(1)}$$

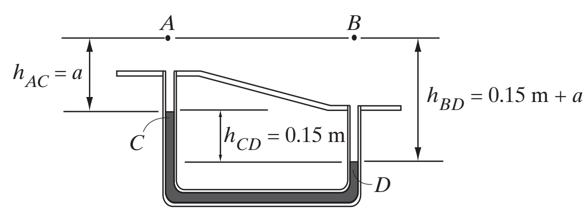
Consider the gasoline in the pipe as the control volume. Continuity requires that

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi(0.0375\ \text{m})^2\right] + V_B\left[\pi(0.075\ \text{m})^2\right] = 0$$

$$V_A = 4V_B$$



(a)

Substitute this result into Eq. (1)

$$p_B - p_A = 363\left[(4V_B)^2 - V_B^2\right]$$

$$p_B - p_A = 5445\ V_B^2 \qquad \textbf{(2)}$$

Write the Manometer equation from point A to B,

$$p_A + \rho_{ga} ga + \rho_{Hg} gh - \rho_{gas} g(a + h) = p_B$$

$$p_B - p_A = \left(\rho_{Hg} - \rho_{gas}\right)gh$$

From Appendix A, $\rho_{Hg} = 13550\ \text{kg/m}^3$. Thus

$$p_B - p_A = \left(13550\ \text{kg/m}^3 - 726\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.15\ \text{m})$$

$$p_B - p_A = 18.871\left(10^3\right)\ \text{Pa}$$

Substitute this result into Eq. (2),

$$18.871\left(10^3\right) = 5445\ V_B^2$$

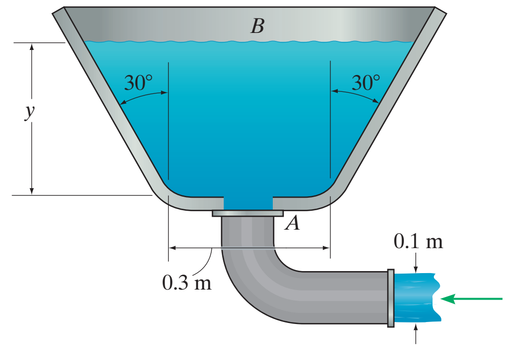
$$V_B = 1.862\ \text{m/s}$$

Thus, the discharge through the pipe is

$$Q = V_B A_B = (1.862\ \text{m/s})\left[\pi(0.075\ \text{m})^2\right] = 0.0329\ \text{m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$0.0329\ \text{m}^3/\text{s}$

5–75. If water flows into the pipe at a constant rate of 30 kg/s, determine the pressure acting at the inlet A when $y = 0.5$ m. Also, what is the rate at which the water surface at B is rising when $y = 0.5$ m? The container is circular.



SOLUTION

The volume of the control volume changes with time. Since it contains the water in the container. Referring to the geometry shown in Fig. a, the volume is

$$\forall = 2\pi\Sigma\gamma A = 2\pi\left\{(0.075\text{ m})(0.15\text{ m})(y) + \left(0.15\text{ m} + \frac{y}{3}\tan 30°\right)\left[\frac{1}{2}(y\tan 30°)(y)\right]\right\}$$

$$= 2\pi\left(0.05556y^3 + 0.075\tan 30° y^2 + 0.01125y\right)\text{ m}^3$$

Then

$$\frac{\partial\forall}{\partial t} = 2\pi\left(0.1667y^2\frac{\partial y}{\partial t} + 0.15\tan 30° y\frac{\partial y}{\partial t} + 0.01125\frac{\partial y}{\partial t}\right)$$

$$= 2\pi\left(0.1667y^2 + 0.15\tan 30° y + 0.01125\right)\frac{\partial y}{\partial t}$$

Thus

$$\frac{\partial}{\partial t}\int_{cv}\rho_w d\forall + \int_{cs}\rho_w \mathbf{V}\cdot d\mathbf{A} = 0$$

$$\rho_w\frac{\partial\forall}{\partial t} - \rho_w V_A A_A = 0$$

$$(1000\text{ kg})\left[2\pi\left(0.1667y^2 + 0.15\tan 30° y + 0.01125\right)\right]\frac{\partial y}{\partial t} - 30\text{ kg/s} = 0$$

$$\frac{\partial y}{\partial t} = \frac{3}{200\pi\left(0.1667y^2 + 0.15\tan 30° y + 0.01125\right)}$$

At the instant $y = 0.5$ m,

$$V_B = \frac{\partial y}{\partial t} = \frac{3}{200\pi\left[0.1667\left(0.5^2\right) + 0.15\tan 30°(0.5) + 0.01125\right]}$$

$$= 0.04962\text{ m/s} = 0.0496\text{ m/s} \qquad \textbf{Ans.}$$

Since water can be considered an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable writing this equation between points A and B on the central streamline,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Here, $p_B = p_{atm} = 0$ since point B is exposed to the atmosphere. With reference to the datum set through point A, $z_A = 0$ and $z_A = 0.5$ m. Also, from the mass flow rate

$$\dot{m} = \rho_w V_A A_A; \qquad 30\text{ kg/s} = \left(1000\text{ kg/m}^3\right)\left(V_A\right)\left[\pi(0.05\text{ m})^2\right]$$

$$V_A = 3.8197\text{ m/s}$$

Thus,

$$\frac{p_A}{1000\text{ kg/m}^3} + \frac{(3.8197\text{ m/s})^2}{2} + 0 = 0 + \frac{(0.04962\text{ m/s})^2}{2} + \left(9.81\text{ m/s}^2\right)(0.5\text{ m})$$

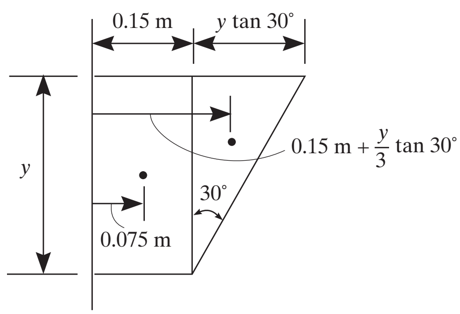
5–75. Continued

$$p_A = -2.389\left(10^3\right)\text{ Pa} = -2.39\text{ kPa}$$ **Ans.**

Note: Since V_B is very small, then the term $\frac{V_B^2}{2}$ can be neglected.

$$\frac{p_A}{1000\text{ kg/m}^3} + \frac{(3.8197\text{ m/s})^2}{2} + 0 = 0 + 0 + \left(9.81\text{ m/s}^2\right)(0.5\text{ m})$$

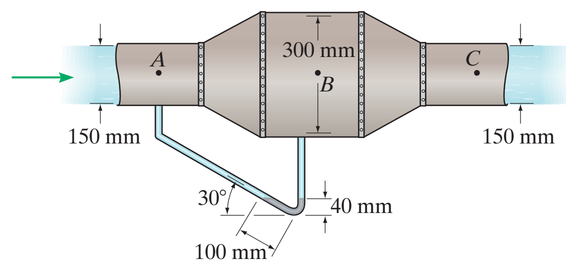
$$p_A = -2.390\left(10^3\right)\text{ Pa} = -2.39\text{ kPa}$$



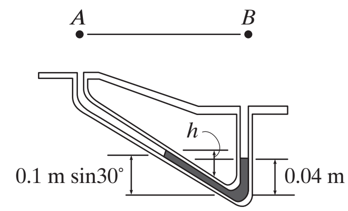
(a)

Ans:
$dy/dt = 0.496\text{ m/s}$, $p = -2.39\text{ kPa}$

***5–76.** Carbon dioxide at 20°C passes through the expansion chamber, which causes mercury in the manometer to settle as shown. Determine the velocity of the gas at A. Take $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.



SOLUTION



(a)

Bernoulli Equation. From Appendix A, $\rho_{CO_2} = 1.84\ \text{kg/m}^2$ at $T = 20°\,\text{C}$. If we set the datum to coincide with the horizontal line connecting points A and B, $z_A = z_B = 0$.

$$\frac{p_A}{\rho_{CO_2}} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_{CO_2}} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{1.84\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{p_B}{1.84\ \text{kg/m}^3} + \frac{V_B^2}{2} + 0$$

$$p_B - p_A = 0.920\left(V_A^2 - V_B^2\right) \qquad \textbf{(1)}$$

Continuity Equation. Consider the gas from A to B to the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi(0.075\ \text{m})^2\right] + V_B\left[\pi(0.15\ \text{m})^2\right] = 0$$

$$V_B = 0.25\,V_A \qquad \textbf{(2)}$$

Manometer Equation. Referring to Fig. a, $h = 0.1\ \text{m} \sin 30° - 0.04\ \text{m} = 0.01\ \text{m}$. Then, neglecting the weight of the CO_2,

$$p_A + \rho_{Hg} gh = p_B$$

$$p_A + \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.01\ \text{m}) = p_B$$

$$p_B - p_A = 1329.255 \qquad \textbf{(3)}$$

Equating Eqs. (1) and (3),

$$0.920\left(V_A^2 - V_B^2\right) = 1329.255$$

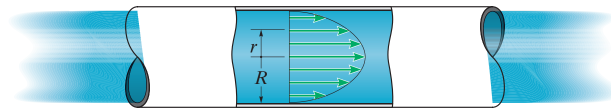
Substituting Eq. (2) into this equation,

$$0.9375 V_A^2 = 1444.84$$

Thus,

$$V_A = 39.3\ \text{m/s} \qquad \textbf{Ans.}$$

5–77. Determine the kinetic energy coefficient α if the velocity distribution for laminar flow in a smooth pipe has a velocity profile defined by $u = U_{\max}\left(1 - (r/R)^2\right)$.



SOLUTION

$$\alpha = \frac{1}{\dot{m}\overline{V}^2}\int_{cs} V^2 \rho \mathbf{V} \cdot d\mathbf{A}$$

$$\alpha = \frac{\rho}{(\rho \overline{V} A)\overline{V}^2}\int_{cs} V^3 dA = \frac{\int_{cs} V^3 dA}{\overline{V}^3 A}$$

$$\alpha = \frac{1}{\overline{V}^3 A}\int_0^R u^3(2\pi r dr)$$

$$\alpha = \frac{2u_{\max}^3}{R^2\overline{V}^3}\int_0^R \left(1 - \frac{r^2}{R^2}\right)^3 r dr$$

$$\alpha = \frac{2u_{\max}^3}{R^8\overline{V}^3}\int_0^R (R^2 - r^2)^3 r dr$$

$$\alpha = \frac{2u_{\max}^3}{R^8\overline{V}^3}\int_0^R (R^6 r - 3R^4 r^3 + 3R^2 r^5 - r^7) dr$$

$$\alpha = \frac{2u_{\max}^3}{R^8\overline{V}^3}\left(\frac{R^8}{8}\right) = \frac{1}{4}\left(\frac{u_{\max}}{\overline{V}}\right) \qquad \textbf{(1)}$$

$$\overline{V} = \frac{1}{A}\int_{cs} u dA = \frac{u_{\max}}{\pi R^2}\int_0^R \left(1 - \frac{r^2}{R^2}\right)(2\pi r dr)$$

$$\overline{V} = \frac{2u_{\max}}{R^4}\int_0^R (R^2 r - r^3) dr$$

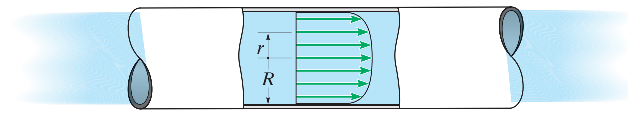
$$\overline{V} = \frac{1}{2}u_{\max}$$

Substitute into Eq. (1),

$$\alpha = 2 \qquad \textbf{Ans.}$$

Ans:
2

5–78. Determine the kinetic energy coefficient α if the velocity distribution for turbulent flow in a smooth pipe is assumed to have a velocity profile defined by Prandtl's one-seventh power law, $u = U_{\max}\left(1 - r/R\right)^{1/7}$.



SOLUTION

$$\alpha = \frac{1}{\dot{m}\overline{V}^2}\int_{cs} V^2\rho\,\mathbf{V}\cdot d\mathbf{A}$$

$$\alpha = \frac{\rho}{\rho\overline{V}A\overline{V}^2}\int_{cs} V^3 dA = \frac{\displaystyle\int_{cs} V^3 dA}{\overline{V}^3 A}$$

$$\alpha = \frac{1}{\overline{V}^3 A}\int_0^R u^3(2\pi r dr)$$

$$\alpha = \frac{2u_{\max}^3\pi}{R^3 h\overline{V}^3 A}\int_0^R (R-r)^{\frac{3}{7}} r dr$$

Set $y = R - r$

$$dy = -dr$$

$$\alpha = \frac{2u_{\max}^3\pi}{R^{\frac{3}{7}}\overline{V}^3 A}\int_R^0 y^{\frac{3}{7}}\,(R-y)(-dy)$$

$$\alpha = \frac{2u_{\max}^3\pi}{R^{\frac{3}{7}}\overline{V}^3 A}\left[\frac{7}{10}R^{\frac{17}{7}} - \frac{7}{17}R^{\frac{17}{7}}\right]$$

$$\alpha = \frac{49u_{\max}^3 R^2\pi}{85\overline{V}^3 A} = \frac{49u_{\max}^3}{85\overline{V}^3} \qquad \textbf{(1)}$$

$$\overline{V} = \frac{1}{A}\int_{cs} u dA = \frac{u_{\max}}{R^{\frac{1}{7}}A}\int_0^R (R-r)^{\frac{1}{7}}(2\pi r\,dr)$$

$$\overline{V} = \frac{2\pi u_{\max}}{R^{\frac{1}{7}}A}\int_R^0 y^{\frac{1}{7}}(R-y)(-dy)$$

$$\overline{V} = \frac{2\pi u_{\max}}{R^{\frac{1}{7}}A}\left[\frac{7}{8}R^{\frac{15}{7}} - \frac{7}{15}R^{\frac{15}{7}}\right]$$

$$\overline{V} = \frac{\pi R^2 u_{\max}}{\pi R^2}\left[\frac{49}{60}\right]$$

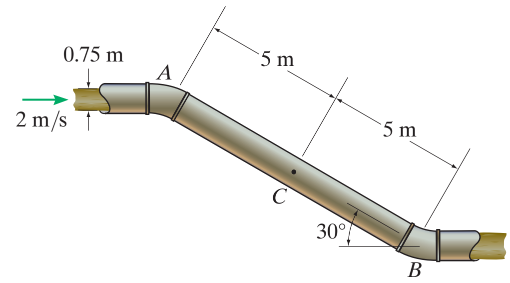
$$\overline{V} = \frac{49}{60}u_{\max}$$

Substitute into Eq. (1),

$$\alpha = 1.058 = 1.06 \qquad \textbf{Ans.}$$

Ans:
1.06

5–79. Oil flows through the constant-diameter pipe such that at A the pressure is 50 kPa, and the velocity is 2 m/s. Determine the pressure and velocity at B. Draw the energy and hydraulic grade lines for AB using a datum at B. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

Bernoulli Equation. Since the pipe has a constant diameter,

$$V_B = V_A = 2\ \text{m/s} \qquad \textbf{Ans.}$$

With reference to the datum through B, $z_A = (10\ \text{m})\sin 30° = 5\ \text{m}$ and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$\frac{50(10^3)\dfrac{\text{N}}{\text{m}^2}}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 5\ \text{m} = \frac{p_B}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_B = 94.145(10^3)\ \text{Pa} = 94.1\ \text{kPa} \qquad \textbf{Ans.}$$

El and HGL. EL will have a constant value of

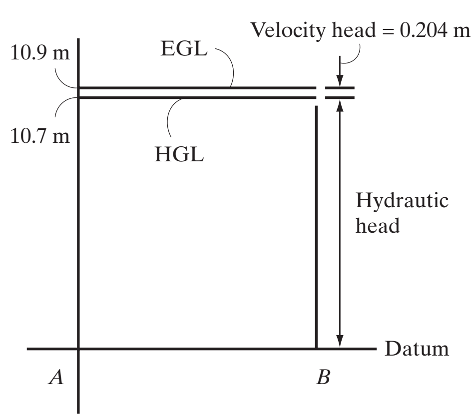
$$H = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A$$

$$= \frac{50(10^3)\dfrac{\text{N}}{\text{m}^2}}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 5\ \text{m} = 10.9\ \text{m}$$

Here, the velocity head is constant, with a value of

$$\frac{V_A^2}{2g} = \frac{(2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 0.204\ \text{m}$$

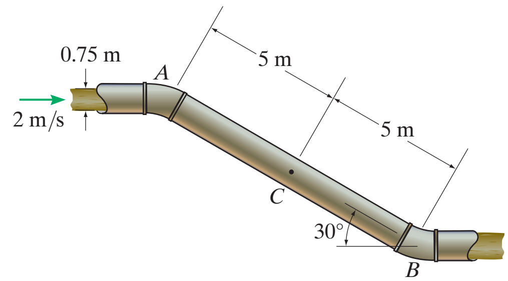
Thus, the HGL will be 0.204 m below and parallel to the EL. A plot of the EL and HGL is shown in Fig. a.



(a)

Ans:
$V_B = 2\ \text{m/s}$
$p_B = 94.1\ \text{kPa}$

***5–80.** Oil flows through the constant-diameter pipe such that at A the pressure is 50 kPa, and the velocity is 2 m/s. Plot the pressure head and the gravitational head for AB using a datum at B. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

Bernoulli Equation. Since the pipe has a constant diameter, $V_B = V_A = 2\ \text{m/s}$. With reference to the datum through B, $z_A = (10\ \text{m})\sin 30° = 5\ \text{m}$ and $z_B = 0$.

$$\frac{pA}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$\frac{50(10^3)\dfrac{\text{N}}{\text{m}^2}}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 5\ \text{m} = \frac{p_B}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

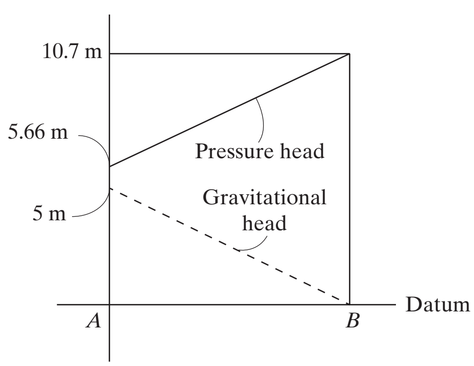
$$p_B = 94.145(10^3)\ \text{Pa}$$

Therefore, the pressure head at A and B is

$$\frac{pA}{\gamma} = \frac{50(10^3)\dfrac{\text{N}}{\text{m}^2}}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 5.66\ \text{m}$$

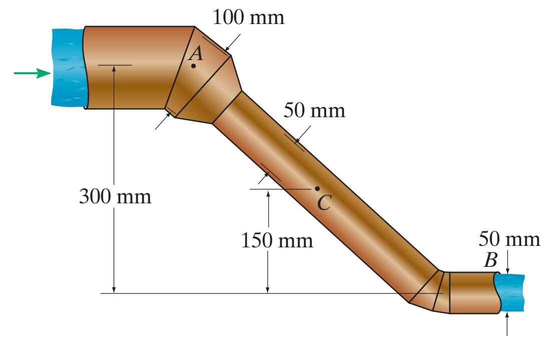
$$\frac{p_B}{\gamma} = \frac{94.145(10^3)\dfrac{\text{N}}{\text{m}^2}}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 10.66\ \text{m}$$

The gravitational head coincides with the centerline of the pipe. A plot of the pressure head and gravitational head is shown in Fig. *a*.



(a)

5–81. Water at a pressure of 80 kPa and a velocity of 2 m/s at A flows through the transition. Determine the velocity and the pressure at B. Draw the energy and hydraulic grade lines using a datum at B.



SOLUTION

Continuity Equation. Consider the water in the pipe as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(2\text{ m/s})\left[\pi(0.05\text{ m})^2\right] + V_B\left[\pi(0.025\text{ m})^2\right] = 0$$

$$V_B = 8\text{ m/s} \qquad \textbf{Ans.}$$

Bernoulli Equation. With reference to the datum through B, $z_A = 0.3$ m and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$\frac{80(10^3)\dfrac{\text{N}}{\text{m}^2}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{(2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.3\text{ m}$$

$$= \frac{p_B}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{(8\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0$$

$$p_B = 52.943(10^3)\text{ Pa} = 52.9\text{ kPa} \qquad \textbf{Ans.}$$

EL and HGL. EL will have a constant value of

$$H = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A$$

$$= \frac{80(10^3)\dfrac{\text{N}}{\text{m}^2}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{(2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.3\text{ m} = 8.66\text{ m}$$

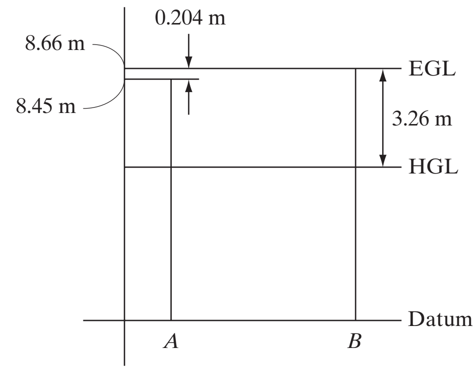
The velocity head before A is

$$\frac{V_A^2}{2g} = \frac{(2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.204\text{ m}$$

The velocity head after A is

$$\frac{V_A^2}{2g} = \frac{(8\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 3.26\text{ m}$$

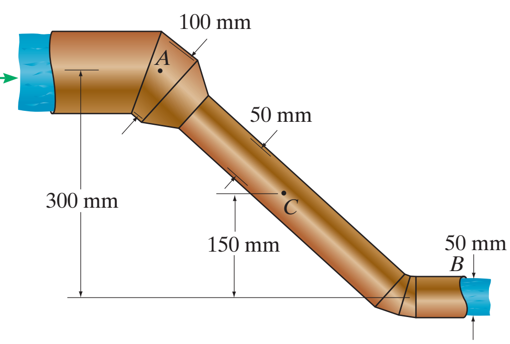
The EL and HGL are plotted as shown in Fig. a.



(a)

Ans:
$V_B = 8$ m/s
$p_B = 52.9$ kPa

5–82. Water at a pressure of 80 kPa and a velocity of 2 m/s at A flows through the transition. Determine the velocity and the pressure at C. Plot the pressure head and the gravitational head for AB using a datum at B.



SOLUTION

Continuity Equation. Consider the water in the pipe as the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(2\text{ m/s})\left[\pi(0.05\text{ m})^2\right] + V_B\left[\pi(0.025\text{ m})^2\right] = 0$$

$$V_B = 8\text{ m/s} \qquad \textbf{Ans.}$$

Also, $v_C = v_B = 8\text{ m/s}$ **Ans.**

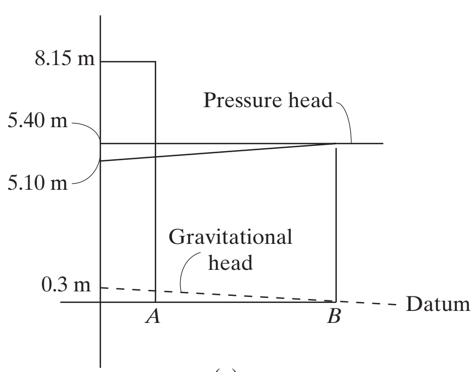
Bernoulli Equation. With reference to the datum through C, $z_A = 0.3\text{ m} - 0.15\text{ m} = 0.15\text{ m}$ and $z_C = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A{}^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C{}^2}{2g} + z_C$$

$$\frac{80(10^3)\dfrac{\text{N}}{\text{m}^2}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{(2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.15\text{ m}$$

$$= \frac{p_C}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{(8\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0$$

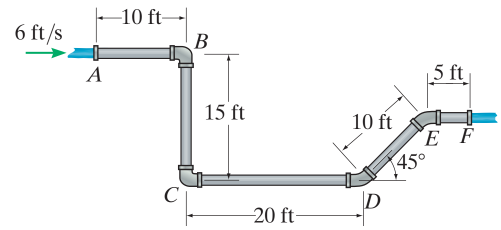
$p_C = 51.47(10^3)\text{ Pa} = 51.5\text{ kPa}$ **Ans.**



(a)

Ans:
$V_C = 8\text{ m/s}$
$p_C = 51.5\text{ kPa}$

5–83. Water flows through the constant-diameter pipe such that at A it has a velocity of 6 ft/s and a pressure of 30 psi. Draw the energy and hydraulic grade lines for the flow from A to F using a datum through CD.



SOLUTION

EL and HGL. With reference to the datum through CD, $z_A = 15$ ft. Thus, EL will have a constant value of

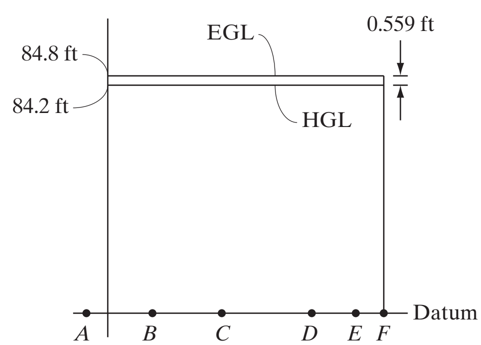
$$H = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A$$

$$= \frac{\left(30\,\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{62.4\text{ lb/ft}^3} + \frac{(6\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 15\text{ ft} = 84.8\text{ ft}$$

Since the pipe has a constant diameter, the velocity head will have a constant value of

$$\frac{V_A^2}{2g} = \frac{(6\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 0.559\text{ ft}$$

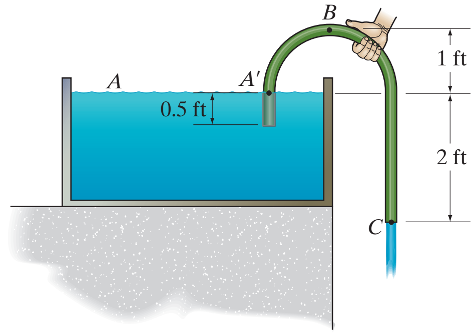
A plot of the EL and HGL is shown in Fig. a.



(a)

Ans:
EGL at 84.8 ft, HGL at 84.2 ft

***5–84.** The hose is used to siphon water from the tank. Determine the smallest pressure in the tube and the volumetric discharge at C. The hose has an inner diameter of 0.75 in. Draw the energy and hydraulic grade lines for the hose using a datum at C.



SOLUTION

Bernoulli Equation. Since the tank is a large source, $V_A \cong 0$. Here, A and C are exposed to the atmosphere, $p_A = p_C = 0$. If the datum coincides with the horizontal line through C, $z_A = 2$ ft, $z_B = 3$ ft, and $z_C = 0$. From A to C,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C$$

$$0 + 0 + 2\text{ ft} = 0 + \frac{V_C^2}{2(32.2\text{ ft/s}^2)} + 0$$

$$V_C = 11.35\text{ ft/s}$$

The smallest pressure occurs at the maximum height of the hose, point B. Since the hose has a constant diameter, $V_B = V_C = 11.35$ ft/s. From A to B,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + 0 + 2\text{ ft} = \frac{p_B}{62.4\text{ lb/ft}^3} + \frac{(11.35\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 3\text{ ft}$$

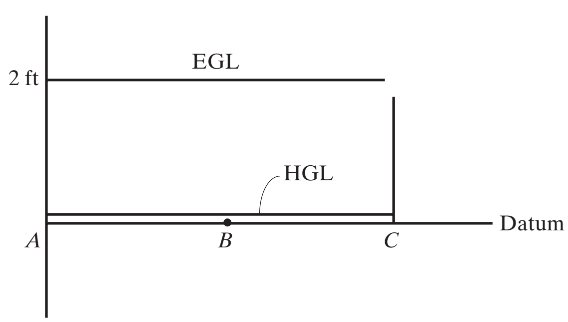
$$p_B = \left(-187.2\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = -1.30\text{ psi} \qquad \textbf{Ans.}$$

Discharge.

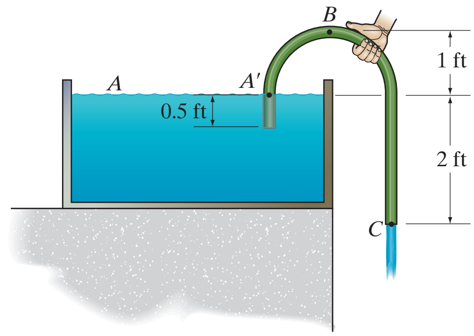
$$Q = V_C A_C = (11.35\text{ ft/s})\left[\pi\left(\frac{0.375}{12}\text{ ft}\right)^2\right]$$

$$= 0.0348\text{ ft}^3/\text{s} \qquad \textbf{Ans.}$$

$$\frac{V_B^2}{2g} = \frac{V_C^2}{2g} = 2\text{ ft}$$



5–85. The hose is used to siphon water from the tank. Determine the pressure in the hose at points A' and B. The hose has an inner diameter of 0.75 in. Draw the energy and hydraulic grade lines for the hose using a datum at B.



SOLUTION

Bernoulli Equation. Since the tank is a large source, $V_A \cong 0$. Here, A and C are exposed to the atmosphere, $p_A = p_C = 0$. If the datum coincides with the horizontal line through C, $z_A = 2$ ft, $z_B = 3$ ft, and $z_C = 0$. From A to C,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C$$

$$0 + 0 + 2\text{ ft} = 0 + \frac{V_C^2}{2(32.2\text{ ft/s}^2)} + 0$$

$$V_C = 11.35\text{ ft/s}$$

Since the hose has a constant diameter, $V_A' = V_B = V_C = 11.35$ ft/s. From A to A'

and A to B,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_{A'}}{\gamma} + \frac{V_{A'}^2}{2g} + z_{A'}$$

$$0 + 0 + 2\text{ ft} = \frac{p_{A'}}{62.4\text{ lb/ft}^3} + \frac{(11.35\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 2\text{ ft}$$

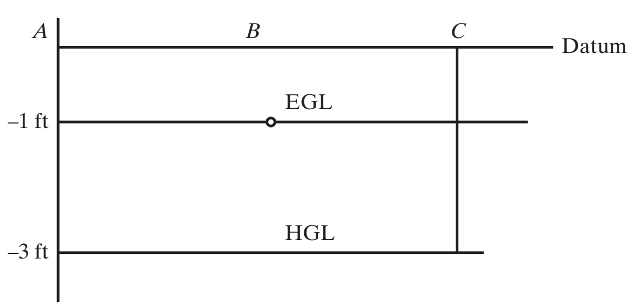
$$p_{A'} = \left(-124.8\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = -0.867\text{ psi} \qquad \textbf{Ans.}$$

and

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

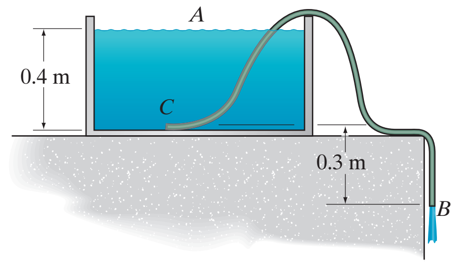
$$0 + 0 + 2\text{ ft} = \frac{p_B}{62.4\text{ lb/ft}^3} + \frac{(11.35\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 3\text{ ft}$$

$$p_B = \left(-187.2\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = -1.30\text{ psi} \qquad \textbf{Ans.}$$



Ans:
$p_{A'} = -0.867$ psi
$p_B = -1.30$ psi

5–86. Water is siphoned from the open tank. Determine the volumetric discharge from the 20-mm-diameter hose. Draw the energy and hydraulic grade lines for the hose using a datum at B.



SOLUTION

Bernoulli Equation. Since the tank is a large source, $V_A = 0$. Here, A and B are exposed to the atmosphere, $p_A = p_B = 0$. If the datum is set to coincide with the horizontal line through B, $z_A = 0.3\text{ m} + 0.4\text{ m} = 0.7\text{ m}$ and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

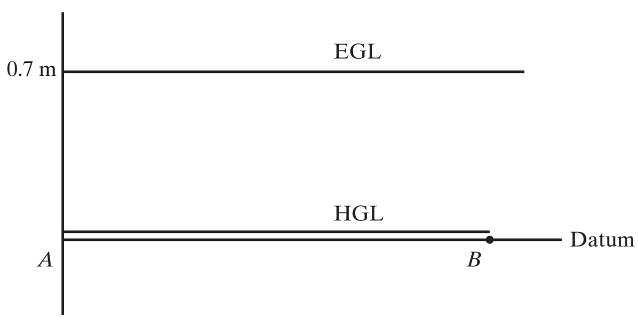
$$0 + 0 + 0.7\text{ m} = 0 + \frac{V_B^2}{2\left(9.81\text{ m/s}^2\right)} + 0$$

$$V_B = 3.706\text{ m/s}$$

Discharge.

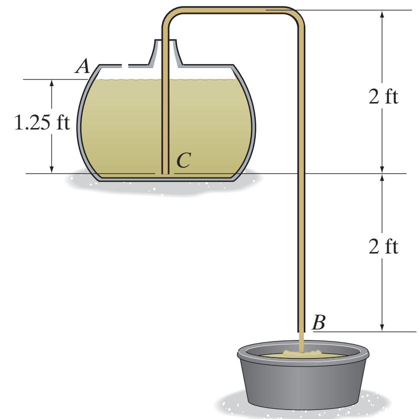
$$Q = V_B A_B = \left(3.706\text{ m/s}\right)\left[\pi(0.01\text{ m})^2\right]$$

$$= 0.00116\text{ m}^3\text{/s} \qquad \textbf{Ans.}$$



Ans:
$0.00116\text{ m}^3\text{/s}$

5–87. Gasoline is siphoned from the large open tank. Determine the volumetric discharge from the 0.5-in.-diameter hose at B. Draw the energy and hydraulic grade lines for the hose using a datum at B.



SOLUTION

Bernoulli Equation. Since the tank is a large source, $V_A = 0$. Here, A and B are exposed to the atmosphere, $p_A = p_B = 0$. If the datum is set to coincide with the horizontal line through B, $z_A = 2\text{ ft} + 1.25\text{ ft} = 3.25\text{ ft}$ and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

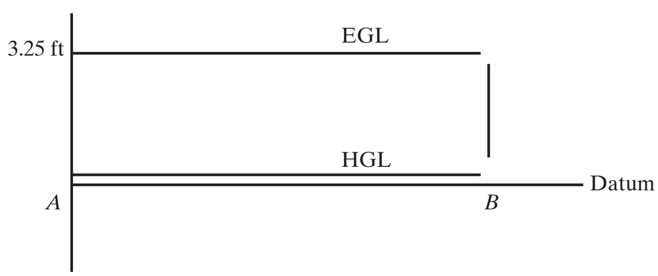
$$0 + 0 + 3.25\text{ ft} = 0 + \frac{V_B^2}{2(32.2\text{ ft/s}^2)} + 0$$

$$V_B = 14.47\text{ ft/s}$$

Discharge.

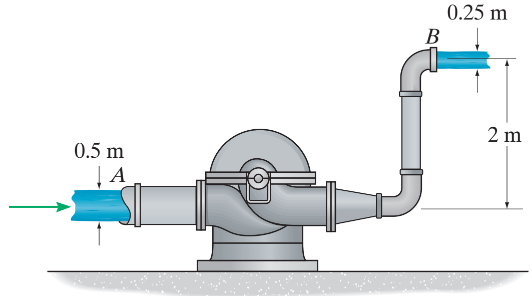
$$Q = V_B A_B = (14.47\text{ ft/s})\left[\pi\left(\frac{0.25}{12}\text{ ft}\right)^2\right]$$

$$= 0.0197\text{ ft}^3/\text{s}$$ **Ans.**



Ans:
$0.0197\text{ ft}^3/\text{s}$

***5–88.** The pump discharges water at B at $0.05\ \text{m}^3/\text{s}$. If the friction head loss between the intake at A and the outlet at B is 0.9 m, and the power input to the pump is 8 kW, determine the difference in pressure between A and B. The efficiency of the pump is $e = 0.7$.



SOLUTION

Discharge.

$$Q = V_A A_A; \qquad 0.05\ \text{m}^3/\text{s} = V_A\left[\pi(0.25\ \text{m})^2\right]$$

$$V_A = 0.2546\ \text{m/s}$$

$$Q = V_B A_B; \qquad 0.05\ \text{m}^3/\text{s} = V_B\left[\pi(0.125\ \text{m})^2\right]$$

$$V_B = 1.019\ \text{m/s}$$

Here, $(\dot{W}_s)_{\text{out}} \equiv P_{\text{out}} = \varepsilon P_{\text{in}} = 0.7(8\ \text{kW}) = 5.6\ \text{kW}$.

$$P_{\text{out}} = Q\gamma h_{\text{pump}}$$

$$5.6(10^3)\ \text{W} = (0.05\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h_{\text{pump}})$$

$$h_{\text{pump}} = 11.42$$

Energy Equation. Take the water in the system from A to B as the control volume. If the datum coincides with the horizontal line through A, $z_A = 0$ and $z_B = 2$ m.

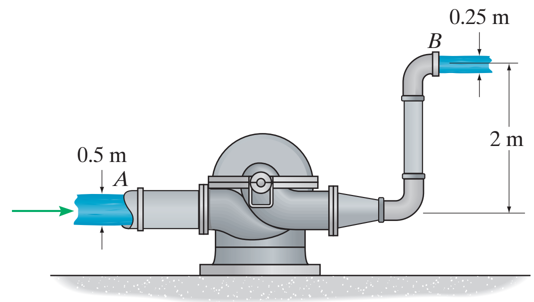
$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B = V + h_{\text{turb}} + h_L$$

$$\frac{p_A}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(0.2546\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 + (11.42\ \text{m}) =$$

$$\frac{p_B}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(1.019\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 2\ \text{m} + 0.9\ \text{m}$$

$$p_B - p_A = 83.06(10^3)\ \text{Pa} = 83.1\ \text{kPa}$$

5–89. The power input of the pump is 10 kW and the friction head loss between A and B is 1.25 m. If the pump has an efficiency of $e = 0.8$, and the increase in pressure from A to B is 100 kPa, determine the volumetric flow of water through the pump.



SOLUTION

From the discharge,

$Q = V_A A_A;\qquad Q = V_A\left[\pi(0.25\text{ m})^2\right]$

$$V_A = \left(\frac{16Q}{\pi}\right)\text{ m/s}$$

$Q = V_B A_B;\qquad Q = V_B\left[\pi(0.125\text{ m})^2\right]$

$$V_B = \left(\frac{64Q}{\pi}\right)\text{ m/s}$$

Here, $(\dot{W}_s)_{\text{out}} = \rho(\dot{W}_s)_{\text{in}} = 0.8(10\text{ kW}) = 8\text{ kW}$. Then

$(\dot{W}_s)_{\text{out}} = Q\gamma h_{\text{pump}}\ 8(10^3)\text{W} = Q(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)h_{\text{pump}}$

$$h_{\text{pump}} = \left(\frac{0.8155}{Q}\right)\text{ m}$$

Consider the fixed control volume that contains water in the system from A to B. Writing the energy equation with the datum set through A, $z_A = 0$ and $z_B = 2$ m,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{p_A}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{\left(\frac{16Q}{\pi}\right)^2}{2(9.81\text{ m/s}^2)} + 0 + \frac{0.8155}{Q} =$$

$$\frac{p_B}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{\left(\frac{64Q}{\pi}\right)^2}{2(9.81\text{ m/s}^2)} + 2\text{ m} + 0 + 1.25\text{ m}$$

$$p_B - p_A = \frac{8000}{Q} - 194.54(10^3)Q^2 - 31.88(10^3)$$

$$100(10^3) = \frac{8000}{Q} - 194.54(10^3)Q^2 - 31.88(10^3)$$

$$100 = \frac{8}{Q} - 194.54Q^2 - 31.88$$

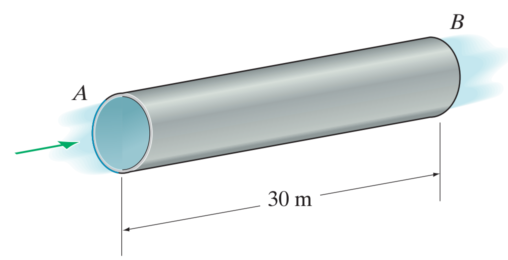
$$194.54Q^3 + 131.88Q - 8 = 0$$

Solving numerically,

$$Q = 0.06024\text{ m}^3/\text{s} = 0.0603\text{ m}^3/\text{s}\qquad\textbf{Ans.}$$

Ans:
$0.0603\text{ m}^3/\text{s}$

5–90. As air flows through the duct, its absolute pressure changes from 220 kPa at A to 219.98 kPa at B. If the temperature remains constant at $T = 60°\text{C}$, determine the head loss between these points. Assume the air is incompressible.



SOLUTION

Energy Equation. Take the air from A to B to be the control volume. From Appendix A, $\rho_a = 1.060\ \text{kg/m}^3$ at $T = 60°\ \text{C}$. The continuity condition requires that $V_A = V_B = V$ since the duct has a constant diameter. Also, the pipe is level, so that $z_A = z_B = z$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

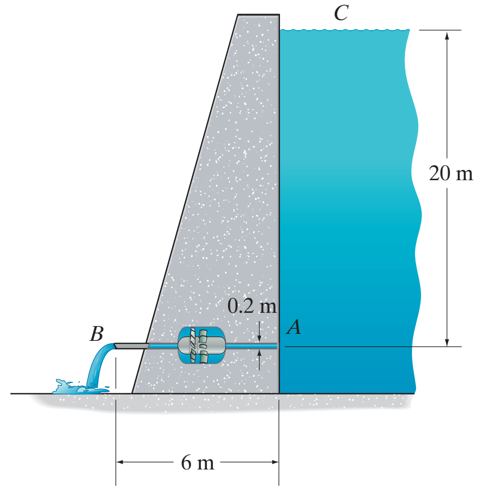
$$\frac{220(10^3)\ \text{N/m}^2}{(1.060\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{V^2}{2g} + z + 0 = \frac{219.98(10^3)\ \text{N/m}^2}{(1.060\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}$$

$$+ \frac{V^2}{2g} + z + 0 + h_L$$

$$h_L = 1.92\ \text{m} \qquad \textbf{Ans.}$$

Ans:
1.92 m

5–91. Water in the reservoir flows through the 0.2-m-diameter pipe at A into the turbine. If the discharge at B is $0.5\ \mathrm{m^3/s}$, determine the power output of the turbine. Assume the turbine runs with an efficiency of 65%. Neglect frictional losses in the pipe.



SOLUTION

$$Q = V_B A_B$$

$$0.5\ \mathrm{m^3/s} = V_B\left[\pi(0.1\ \mathrm{m})^2\right]$$

$$V_B = 15.92\ \mathrm{m/s}$$

Energy Equation. Take the water in the pipe-turbine system from A to B to be the control volume. Since B and C are exposed to the atmosphere, $p_B = p_C = 0$. Also, the discharge is drawn from a large source, so $V_C = 0$. If we set the datum through B, $z_C = 20\ \mathrm{m}$ and $z_B = 0$.

$$\frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 20\ \mathrm{m} + 0 = 0 + \frac{(15.92\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} + 0 + h_{\text{turb}} + 0$$

$$h_{\text{turb}} = 7.090\ \mathrm{m}$$

$$(\dot{W}_s)_{\text{in}} = Q\gamma_w h_{\text{turb}} = (0.5\ \mathrm{m^3/s})(1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(7.090\ \mathrm{m})$$

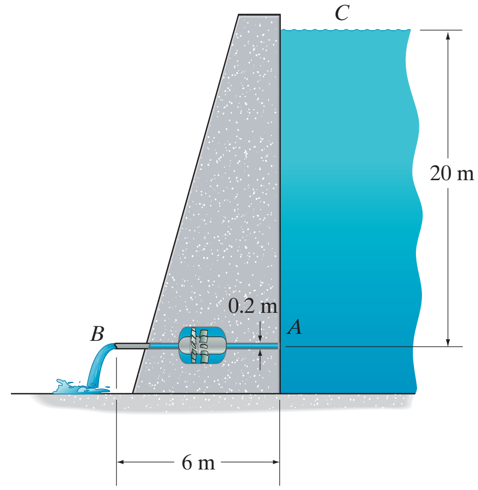
$$= 34.77(10^3)\mathrm{W} = 34.77\ \mathrm{kW}$$

Thus,

$$P_{\text{out}} = \varepsilon P_{\text{in}} = 0.65\ (34.77) = 22.6\ \mathrm{kW} \qquad \textbf{Ans.}$$

Ans:
22.6 kW

***5–92.** Water in the reservoir flows through the 0.2-m-diameter pipe at A into the turbine. If the discharge at B is $0.5\ \text{m}^3/\text{s}$, determine the power output of the turbine. Assume the turbine runs with an efficiency of 65%, and there is a head loss of 0.5 m through the pipe.



SOLUTION

$$Q = V_B A_B$$

$$0.5\ \text{m}^3/\text{s} = V_B\left[\pi(0.1\ \text{m})^2\right]$$

$$V_B = 15.92\ \text{m/s}$$

Energy Equation. Take the water in the pipe-turbine system from A to B to be the control volume. Since B and C are exposed to the atmosphere, $p_B = p_C = 0$. Also, the discharge is drawn from a large source, so $V_C = 0$. If we set the datum through B, $z_C = 20$ m and $z_B = 0$.

$$\frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 20\ \text{m} + 0 = 0 + \frac{(15.92\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 + h_{\text{turb}} + 0.5\ \text{m}$$

$$h_{\text{turb}} = 6.590\ \text{m}$$

$$\dot{W}_s = Q\gamma_w h_{\text{turb}} = (0.5\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6.590\ \text{m})$$

$$= 32.32(10^3)\text{W} = 32.32\ \text{kW}$$

$$\dot{W}_s = 32.32\ \text{kW}(0.65) = 21.0\ \text{kW} \qquad \textbf{Ans.}$$

5–93. A 300-mm-diameter horizontal oil pipeline extends 8 km connecting two large open reservoirs having the same level. If friction in the pipe creates a head loss of 3 m for every 200 m of pipe length, determine the power that must be supplied by a pump to produce a flow of $6\ \mathrm{m^3/min}$ through the pipe. The ends of the pipe are submerged in the reservoirs. Take $\rho_o = 880\ \mathrm{kg/m^3}$.

SOLUTION

Energy Equation. Take the oil in the pipes system to be the control volume. Here, $p_{\text{in}} = p_{\text{out}} = 0$ since the reservoirs are opened to the atmosphere. Also, $V_{\text{in}} = V_{\text{out}} = 0$ since the reservoirs are large. Since both reservoirs are at the same level $z_{\text{in}} = z_{\text{out}} = z$.

$$\frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + h_L$$

$$0 + 0 + z + h_{\text{pump}} = 0 + 0 + z + 0 + (8000\ \text{m})\left(\frac{3\ \text{m}}{200\ \text{m}}\right)$$

$$h_{\text{pump}} = 120\ \text{m}$$

$$\dot{W}_S = Q\gamma h_{\text{pump}}$$

$$= \left(6\ \text{m}^3/\ \text{min.}\right)\left(\frac{1\ \text{min.}}{60\ \text{s}}\right)\left(880\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(120\ \text{m})$$

$$= 103.59\left(10^3\right)\text{W} = 104\ \text{kW}$$ **Ans.**

Ans:
104 kW

5–94. A pump is used to deliver water from a large reservoir to another large reservoir that is 20 m higher. If the friction head loss in the 200-mm-diameter, 4-km-long pipeline is 2.5 m for every 500 m of pipe length, determine the required power output of the pump so the flow is $0.8 \text{ m}^3/\text{s}$. The ends of the pipe are submerged.

SOLUTION

Consider the fixed control volume as the water contained in the piping system. Here, $p_{\text{in}} = p_{\text{out}} = 0$, since the reservoirs are opened to the atmosphere. Also, $V_{\text{in}} = V_{\text{out}} = 0$ since the reservoirs are large. If the datum is set at the surface of the lower reservoir, $z_{\text{in}} = 0$ and $z_{\text{out}} = 20 \text{ m}$.

$$\frac{p_{\text{in}}}{\gamma_w} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + 0 + 20 \text{ m} + 0 + \left(\frac{2.5 \text{ m}}{500 \text{ m}}\right)(4000 \text{ m})$$

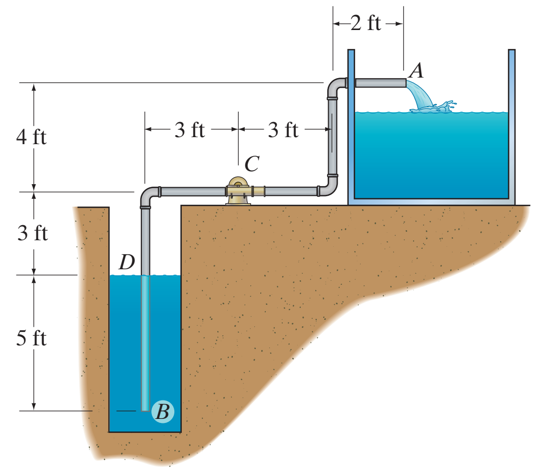
$$h_{\text{pump}} = 40 \text{ m}$$

Thus, the required power output of the pump is

$$\begin{aligned} \dot{W}_S &= Q\gamma h_{\text{pump}} \\ &= \left(0.8 \text{ m}^3/\text{s}\right)\left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(40 \text{ m}) \\ &= 313.92\left(10^3\right) \text{ W} \\ &= 314 \text{ kW} \end{aligned}$$ **Ans.**

Ans:
314 kW

5–95. Water is drawn from the well at B through the 3-in.-diameter suction pipe and discharged through the pipe of the same size at A. If the pump supplies 1.5 kW of power to the water, determine the velocity of the water when it exits at A. Assume the frictional head loss in the pipe system is $1.5V^2/2g$. Note that 746 W = 1 hp and 1 hp = 550 ft · lb/s.



SOLUTION

$$P = Q\gamma h_{\text{pump}}$$

$$1500\,\text{W}\left(\frac{1\text{ hp}}{746\text{ W}}\right)\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = \left[V\left[\pi\left(\frac{1.5}{12}\text{ft}\right)^2\right]\right]\left(62.4\text{ lb/ft}^3\right)h_{\text{pump}}$$

$$h_{\text{pump}} = \frac{361.0}{V}$$

Energy Equation. Take the water from D to A to be the control volume. Since A and D are exposed to the atmosphere, $p_A = p_D = 0$. Also, $V_D = 0$ since the water is drawn from a large reservoir. Since the pipe has a constant diameter, the continuity condition requires that the water flows with a constant velocity of V into the pipe. With reference to the datum through B, $z_D = 5$ ft and $z_A = 5\text{ ft} + 3\text{ ft} + 4\text{ ft} = 12\text{ ft}$.

$$\frac{p_D}{\gamma} + \frac{V_D^2}{2g} + z_D + h_{\text{pump}} = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{turb}} + h_L$$

$$0 + 0 + 5\text{ ft} + \left(\frac{361.04}{V}\right) = 0 + \frac{V^2}{2\left(32.2\text{ ft/s}^2\right)} + 12\text{ ft} + 0 + \frac{1.5\,V^2}{2\left(32.2\text{ ft/s}^2\right)}$$

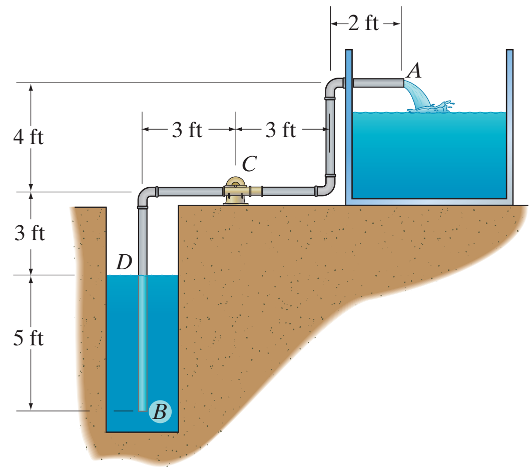
$$V^3 + 180.32\,V - 9300.50 = 0$$

Solving numerically,

$$V = 18.19\text{ ft/s} = 18.2\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
18.2 ft/s

***5–96.** Draw the energy and hydraulic grade lines for the pipe *BCA* in Prob. 5–95 using a datum at point *B*. Assume that the head loss is constant along the pipe at $1.5V^2/2g$.



SOLUTION

$$\dot{W}_s = Q\gamma h_S$$

$$1500W\left(\frac{1\text{ hp}}{746\text{ W}}\right)\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = \left[V\left[\pi\left(\frac{1.5}{12}\text{ft}\right)^2\right]\right](62.4\text{ lb/ft}^3)h_s$$

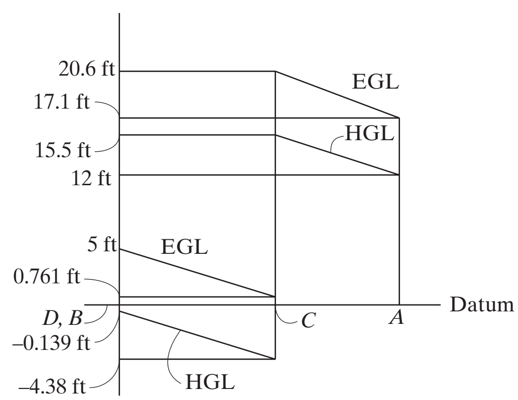
$$h_{\text{pump}} = \frac{361.0}{V}$$

Energy Equation. Take the water from D to A to be the control volume. Since A and D are exposed to the atmosphere, $p_A = p_D = 0$. Also, $V_D = 0$ since the water is drawn from a large reservoir. Since the pipe has a constant diameter, the continuity condition requires that the water flows with a constant velocity of V into the pipe. With reference to the datum through B, $z_D = 5$ ft and $z_A = 5\text{ ft} + 3\text{ ft} + 4\text{ ft} = 12\text{ ft}$.

$$\frac{p_D}{\gamma} + \frac{V_D^2}{2g} + z_D + h_{\text{pump}} = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{turb}} + h_L$$

$$0 + 0 + 5\text{ ft} + \left(\frac{361.04}{V}\right) = 0 + \frac{V^2}{2(32.2\text{ ft/s}^2)} + 12\text{ ft} + 0 + \frac{1.5\,V^2}{2(32.2\text{ ft/s}^2)}$$

$$V^3 + 180.32V - 9300.50 = 0$$



Solving,

$$V = 18.19\text{ ft/s}$$

EGL and HGL. A + B,

$$H_B = H_D = \frac{p_D}{\gamma} + \frac{V_D^2}{2g} + z_D = 0 + 0 + 5\text{ ft} = 5\text{ ft}$$

The total head loss from B to A is

$$(h_L)_{\text{Tot}} = \frac{1.5V^2}{2g} = \frac{1.5(18.19\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 7.708\text{ ft}$$

Thus, the head loss per foot length of the pipe is $\dfrac{7.708\text{ ft}}{(5 + 3 + 3 + 3 + 4 + 2)\text{ ft}}$ $= 0.3854$ ft/ft. And, the total head just before the pump is

$$(H_{C^-}) = 5\text{ ft} - (0.3854\text{ ft/ft})(11\text{ ft}) = 0.761\text{ ft}$$

Just after the pump, a head of $h_{\text{pump}} = \dfrac{361.04}{18.19} = 19.85$ ft is added. Thus,

$$(H_{C^+}) = 0.761\text{ ft} + 19.85\text{ ft} = 20.61\text{ ft}$$

5–96. Continued

Then, the total head at A is

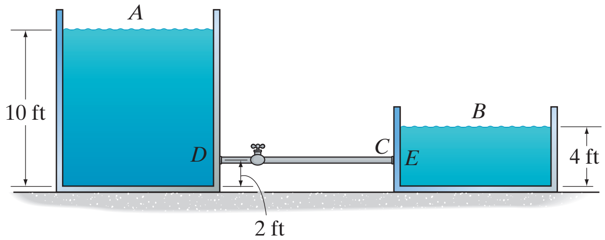
$$H_A = 20.6\text{ ft} - (0.3854\text{ ft/ft})(9\text{ ft}) = 17.1\text{ ft}$$

Since the pipe has a constant diameter, the velocity head has a constant value of

$$\frac{V^2}{2g} = \frac{(18.19\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 5.14\text{ ft}$$

Therefore, the HGL will always be 5.14 ft below and parallel to the EGL. Both are plotted as shown in Fig. a.

5–97. Determine the initial volumetric flow of water from tank A into tank B, and the pressure at end C of the pipe when the valve is opened. The pipe has a diameter of 0.25 ft. Assume that friction losses within the pipe, valve, and connections can be expressed as $1.28V^2/2g$, where V is the average velocity of flow through the pipe.



SOLUTION

Energy Equation. We will write the energy equation between points A and B since these points are exposed to the atmosphere, $p_A = p_B = p_{atm} = 0$. Take the water from A to B to be the control volume. Also, the tanks can be considered as large reservoirs, $V_A = V_B \simeq 0$ with reference to the datum set through the base of the tank, $z_A = 10$ ft and $z_B = 4$ ft.

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 10\text{ ft} + 0 = 0 + 0 + 4\text{ ft} + 0 + h_L$$

$$h_L = 6\text{ ft}$$

Then

$$h_L = 1.28\,\frac{V_C^2}{2g}; \quad 6\text{ ft} = (1.28)\left[\frac{V_C^2}{2(32.2\text{ ft/s}^2)}\right]$$

$$V_C = 17.37\text{ ft/s}$$

Thus, the discharge is

$$Q = V_C A_C = (17.37\text{ ft/s})\left[\pi(0.125\text{ ft})^2\right] = 0.853\text{ ft}^3/\text{s} \qquad \textbf{Ans.}$$

Now take the water from A to C to be the control volume. With reference to the datum set through the base of the tank, $z_A = 10$ ft and $z_C = 2$ ft. Thus,

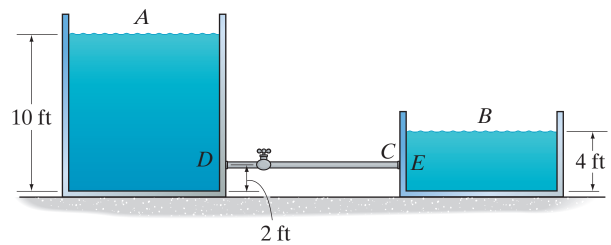
$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C + h_{\text{turb}} + h_L$$

$$0 + 0 + 10\text{ ft} + 0 = \frac{p_C}{62.4\text{ lb/ft}^3} + \frac{(17.37\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 2\text{ ft} + 0 + 6\text{ ft}.$$

$$p_C = (-167.70\text{ lb/ft}^2)\left(\frac{1\text{ ft}}{12\text{ in}}\right)^2 = -1.16\text{ psi} \qquad \textbf{Ans.}$$

Ans:
$Q = 0.853\text{ ft}^3/\text{s}$, $p = -1.16$ psi

5–98. Draw the energy and hydraulic grade lines between points A and B using a datum set at the base of both tanks. The valve is opened. Assume that friction losses within the pipe, valve, and connections can be expressed as $1.28V^2/2g$, where V is the average velocity of flow through the 0.25-ft-diameter pipe.



SOLUTION

We will write the energy equation between points A and B. So take that water from A to B to be the control volume. Since these points are exposed to the atmosphere, $p_A = p_B = p_{atm} = 0$. Also, the tank can be considered as large reservoir, $V_A = V_B \simeq 0$. With reference to the datum set through the base of the tank, $z_A = 10$ ft and $z_B = 4$ ft.

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 10\text{ ft} + 0 = 0 + 0 + 4\text{ ft} + 0 + h_L$$

$$h_L = 6\text{ ft}$$

Then

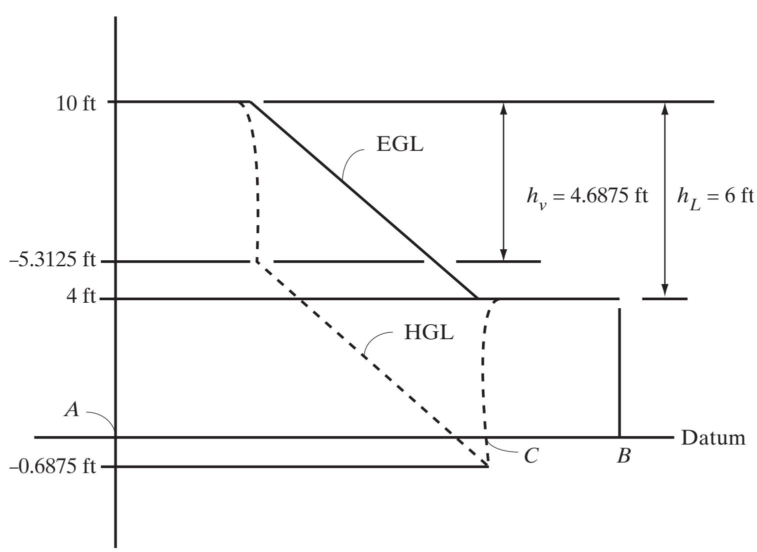
$$h_L = 1.28\frac{V_C^2}{2g};\quad 6\text{ ft} = (1.28)\left[\frac{V_C^2}{2(32.2\text{ ft/s}^2)}\right]$$

$$V_C = 17.37\text{ ft/s}$$

Thus, the velocity head is

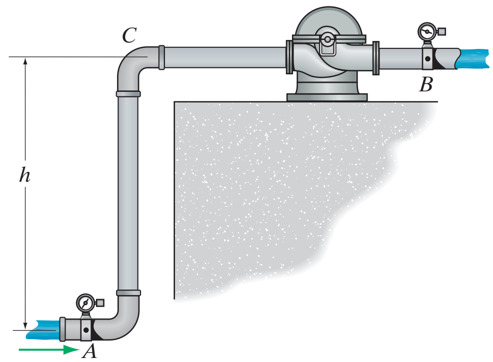
$$h_v = \frac{V_C^2}{2g} = \frac{(17.37\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 4.6875\text{ ft}$$

Based on these results, the energy and hydraulic grade lines are plotted in Fig. *a*



Ans:
$V_C = 17.4$ ft/s

5–99. Water is drawn into the pump, such that the pressure at the inlet A is -35 kPa and the pressure at B is 120 kPa. If the discharge at B is 0.08 m^3/s, determine the power output of the pump. Neglect friction losses. The pipe has a constant diameter of 100 mm. Take $h = 2$ m.



SOLUTION

Energy Equation. Take the water from A to B to be the control volume. Since the pipe has a constant diameter, $V_A = V_B = V$. If the datum is set through A, $z_A = 0$ and $z_B = 2$ m. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{-35(10^3)\,\text{N/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{V^2}{2g} + 0 + h_{\text{pump}} = \frac{120(10^3)\,\text{N/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{V^2}{2g} + 2\text{ m} + 0 + 0$$

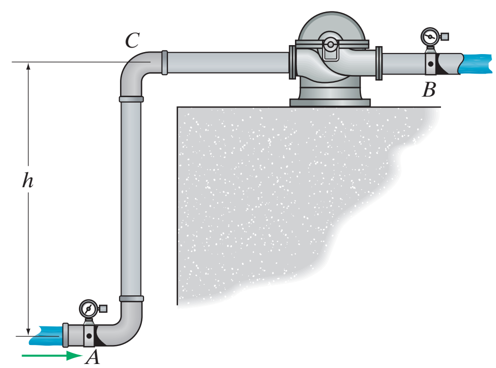
$$h_{\text{pump}} = 17.80\text{ m}$$

$$\dot{W}_s = Q\gamma h_{\text{pump}} = (0.08\text{ m}^3/\text{s})(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(17.80\text{ m})$$

$$= 13.97\,(10^3)\text{ W} = 14.0\text{ kW}$$ **Ans.**

Ans:
14.0 kW

5–100. Draw the energy and hydraulic grade lines for the pipe *ACB* in Prob. 5–99 using a datum at *A*.



SOLUTION

Discharge. Since the pipe has a constant diameter, the velocity in the pipe is constant throughout the pipe as required by the continuity condition.

$$Q = VA; \quad 0.08\ \text{m}^3/\text{s} = V\left[\pi(0.05\ \text{m})^2\right]$$

$$V = 10.19\ \text{m/s}$$

Energy Equation. Take the water from A to B to be the control volume. With reference to the datum through A, $z_A = 0$ and $z_B = 2$ m. With $h_L = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$= \frac{-35(10^3)\frac{\text{N}}{\text{m}^2}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{V^2}{2g} + 0 +$$

$$h_{\text{pump}} = \frac{120(10^3)\frac{\text{N}}{\text{m}^2}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{V^2}{2g} + 2\ \text{m} + 0 + 0$$

$$h_{\text{pump}} = 17.80\ \text{m}$$

EGL and HGL. Since no losses occur, the total head before the pump is

$$H = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A$$

$$= \frac{-35(10^3)\frac{\text{N}}{\text{m}^2}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(10.19\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = 1.72\ \text{m}$$

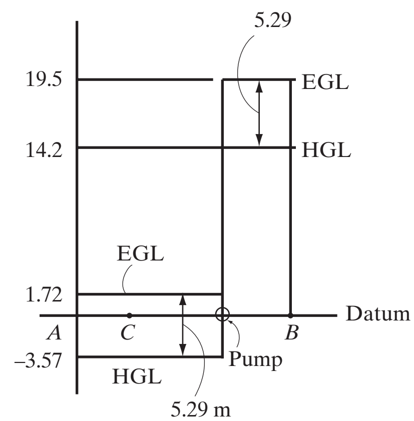
After the pump, a head of 17.80 m is added to the water and becomes

$$H = 1.72\ \text{m} + 17.80\ \text{m} = 19.5\ \text{m}$$

The velocity head has a constant value of

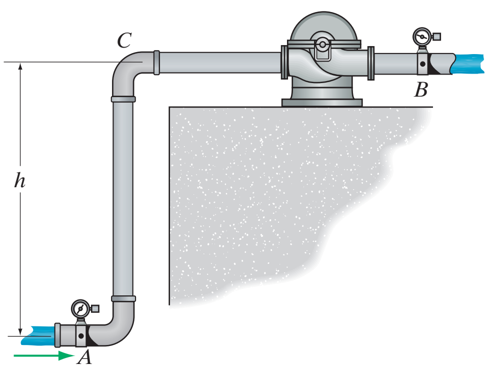
$$\frac{V^2}{2g} = \frac{(10.19\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 5.29\ \text{m}$$

The HGL is always 5.29 m below and parallel to the EL. Both are plotted as shown in Fig. *a*.



(a)

5–101. Water is drawn into the pump, such that the pressure at A is $-6\ \mathrm{lb/in^2}$ and the pressure at B is $20\ \mathrm{lb/in^2}$, If the volumetric flow at B is $4\ \mathrm{ft^3/s}$, determine the power output of the pump. The pipe has a diameter of 4 in. Take $h = 5$ ft and $\rho_w = 1.94\ \mathrm{slug/ft^3}$.



SOLUTION

Energy Equation. Take the water from A to B to be the control volume. Since the pipe has a constant diameter, $V_A = V_B = V$. If we set the datum through A, $z_A = 0$ and $z_B = 5$ ft. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{-6\dfrac{\text{lb}}{\text{in}^2}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)^2}{1.94\dfrac{\text{slug}}{\text{ft}^3}\left(32.2\dfrac{\text{ft}}{\text{s}^2}\right)} + \frac{V^2}{2g} + 0 + h_{\text{pump}} = \frac{20\dfrac{\text{lb}}{\text{in}^2}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)^2}{1.94\dfrac{\text{slug}}{\text{ft}^3}\left(32.2\dfrac{\text{ft}}{\text{s}^2}\right)}$$

$$+ \frac{V^2}{2g} + 5\text{ ft} + \text{h} + 0 + 0$$

$$h_{\text{pump}} = 64.93$$

$$\dot{W}_s = Q_{\text{pump}}\,\gamma h_{\text{pump}}$$

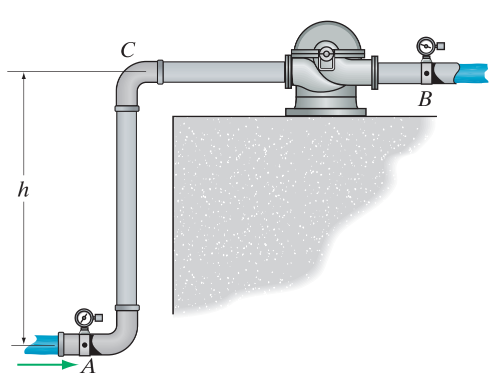
$$= \left(4\ \text{ft}^3/\text{s}\right)\left(1.94\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(64.93\ \text{ft})$$

$$= 16225.36\ \text{ft.lb/s}\left(\frac{1\ \text{hp}}{550\ \text{ft.lb/s}}\right)$$

$$= 29.5\ \text{hp}$$ **Ans.**

Ans:
29.5 hp

5–102. Draw the energy and hydraulic grade lines for the pipe ACB in Prob. 5–101 with reference to the datum at A.



SOLUTION

Discharge. Since the pipe has a constant diameter, the water velocity in the pipe is constant throughout the pipe as required by the continuity condition.

$$Q = VA; \quad 4\text{ft}^3/\text{s} = V\left[\pi\left(\frac{2}{12}\text{ft}\right)^2\right]$$

$$V = 45.84 \text{ ft/s}$$

Energy Equation. Take the water from A to B to be the control volume. With reference to the datum through A, $z_A = 0$ and $z_B = 5$ ft. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$= \frac{\left(-6\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{(1.904\text{ slug/ft}^3)(32.2\text{ ft/s}^2)} + \frac{V^2}{2g} + 0 + h_{\text{pump}} = \frac{\left(20\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{(1.904\text{ slug/ft}^3)(32.2\text{ ft/s}^2)}$$

$$+ \frac{V^2}{2g} + 5\text{ ft} + 0 + 0$$

$$h_{\text{pump}} = 66.07\text{ ft}$$

EGL and HGL. Since no losses occur, the total head before the pump is

$$H = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A$$

$$\frac{\left(-6\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{(1.904\text{ slug/ft}^3)(32.2\text{ ft/s}^2)} + \frac{(45.84\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0 = 18.53\text{ ft} = 18.5\text{ ft}$$

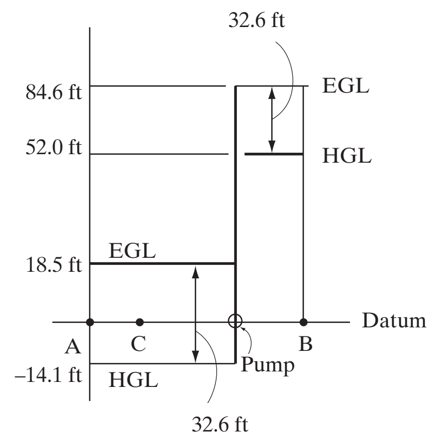
After the pump, a head of 66.07 ft is added to the water and becomes

$$H = 18.53\text{ ft} + 66.07\text{ ft} = 84.6\text{ ft}$$

The velocity head has a constant value of

$$\frac{V^2}{2g} = \frac{(45.84\text{ft/s})^2}{2(32.2\text{ ft/s}^2)} = 32.6\text{ ft}$$

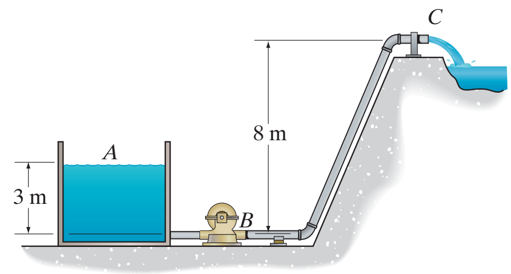
The HGL is always 32.6 ft below and parallel to the EL. Both are plotted as shown in Fig. a.



(a)

Ans:
$V = 45.8$ ft/s, $h_{\text{pump}} = 66.1$ ft

5–103. The pump draws water from the large reservoir A and discharges it at $0.2\ \text{m}^3/\text{s}$ at C. If the diameter of the pipe is 200 mm, determine the power the pump delivers to the water. Neglect friction losses. Construct the energy and hydraulic grade lines for the pipe using a datum at B.



SOLUTION

$$Q = V_C A_C$$

$$0.2\ \text{m}^3/\text{s} = V\left[\pi(0.1\ \text{m})^2\right]$$

$$V_C = 6.366\ \text{m/s}$$

Energy Equation. Take the water from A to C to be the control volume. Since A and C are exposed to the atmosphere, $p_A = p_C = 0$. Also, since the water is drawn from a large reservoir, $V_A = 0$. If we set the datum through B, $z_A = 3$ m and $z_C = 8$ m. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + h_{\text{turb}} + h_L$$

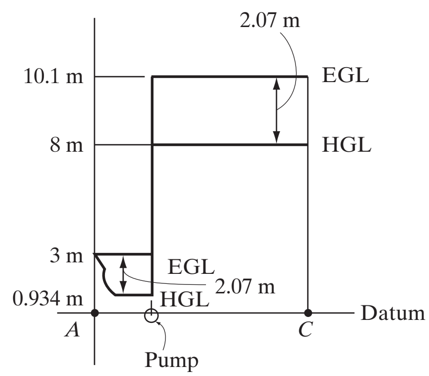
$$0 + 0 + 3\ \text{m} + h_{\text{pump}} = 0 + \frac{(6.366\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 8\ \text{m} + 0 + 0$$

$$h_{\text{pump}} = 7.066\ \text{m}$$

$$\dot{W}_s = Q_{\text{pump}}\, \gamma h_{\text{pump}}$$

$$= (0.2\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(7.066\ \text{m})$$

$$= 13.86(10^3)\text{W} = 13.9\ \text{kW}$$ **Ans.**



(a)

EGL and HGL. Since no loss occurs, the total head before the pump is

$$H = \frac{p_A}{\gamma_A} + \frac{V_A^2}{2g} + z_A = 0 + 0 + 3\ \text{m} = 3\ \text{m}$$

After the pump, a head of $h_{\text{pump}} = 7.066$ m is added to the water and becomes

$$H = 3\ \text{m} + 7.066\ \text{m} = 10.1\ \text{m}$$

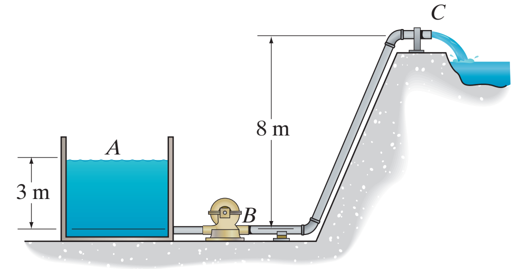
The velocity head has a constant value of

$$\frac{V^2}{2g} = \frac{(6.366\ \text{m/s})^2}{2(9.81\ \text{m/s})} = 2.07\ \text{m}$$

The *HGL* will always be 2.07 m below and parallel with *EGL*. Both are plotted as shown in Fig. *a*.

Ans:
13.9 kW

***5–104.** Solve Prob. 5–103, but include a friction head loss in the pump of 0.5 m, and a friction loss of 1 m for every 5 m length of pipe. The pipe extends 3 m from the reservoir to B, then 12 m from B to C.



SOLUTION

From the discharge

$$Q = V_C A_C; \quad 0.2\ \text{m}^3/\text{s} = V_C\left[\pi(0.1\ \text{m})^2\right]$$

$$V_C = 6.366\ \text{m/s}$$

Consider the fixed control volume as the water contained in the piping system. Since A and C are exposed to the atmosphere $p_A = p_C = 0$. Also, since the water is drawn from a large reservoir $V_A = 0$. If we set the datum through B, $z_A = 3$ m and $z_C = 8$ m.

$$\frac{p_A}{\gamma_W} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_C}{\gamma_W} + \frac{V_C^2}{2g} + z_C + h_{\text{turb}} + h_L$$

$$0 + 0 + 3\ \text{m} + h_{\text{pump}} = 0 + \frac{(6.366\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 8\ \text{m} + 0 + 0.5\ \text{m} + \left(\frac{1\ \text{m}}{5\ \text{m}}\right)(15\ \text{m})$$

$$h_{\text{pump}} = 10.57\ \text{m}$$

Thus, the power the pump delivers to the water is

$$\dot{W}_s = Q\gamma_W h_{\text{pump}}$$

$$= (0.2\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(10.57\ \text{m})$$

$$= 20.73(10^3)\text{W}$$

$$= 20.7\ \text{kw}$$ **Ans.**

The total hand at A is

$$H = \frac{p_A}{\gamma_A} + \frac{V_A^2}{2g} + z_A = 0 + 0 + 3\ \text{m} = 3\ \text{m}$$

The total head before the pump is

$$H = 3\ \text{m} - \left(\frac{1\ \text{m}}{5\ \text{m}}\right)(3\ \text{m}) = 2.40\ \text{m}$$

After the pump, a loss in head of 0.5 m and a head of $h_{\text{pump}} = 10.57$ m is added to the water.

$$H = 2.40\ \text{m} - 0.5\ \text{m} + 10.57\ \text{m} = 12.47\ \text{m} = 12.5\ \text{m}$$

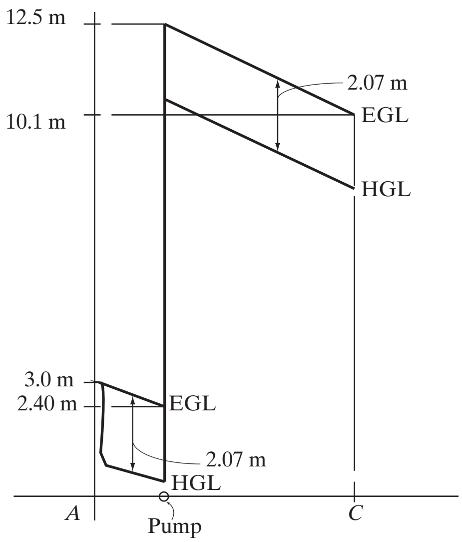
The total head at C is

$$H = 12.47\ \text{m} - \left(\frac{1\ \text{m}}{5\ \text{m}}\right)(12\ \text{m}) = 10.07\ \text{m} = 10.1\ \text{m}$$

The velocity head has a constant value of

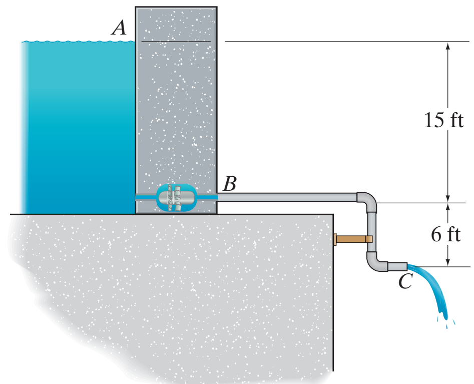
$$\frac{V^2}{2g} = \frac{(6.366\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 2.066\ \text{m} = 2.07\ \text{m}$$

The HGL will always be 2.07 m below and parallel with EGL. Both are plotted as shown in Fig. a.



(a)

5–105. The turbine removes potential energy from the water in the reservoir such that it has a discharge of 20 ft^3/s through the 2-ft-diameter pipe. Determine the horsepower delivered to the turbine. Construct the energy and hydraulic grade lines for the pipe using a datum at point C. Neglect friction losses.



SOLUTION

$$Q = V_C A_C$$
$$20\ \text{ft}^3/\text{s} = V_C\left[\pi(1\ \text{ft})^2\right]$$
$$V_C = 6.366\ \text{ft/s}$$

Energy Equation. Take the water from A to C to be the control volume. Since A and C are exposed to the atmosphere, $p_A = p_C = 0$. Also, since the water is drawn form a large reservoir, $V_A = 0$. With reference to the datum through C, $z_A = 15\text{ ft} + 6\text{ ft} = 21\text{ ft}$ and $z_C = 0$. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + h_{\text{turb}} + h_L$$

$$0 + 0 + 21\text{ ft} + 0 = 0 + \frac{(6.366\text{ft/s})^2}{2(32.2\text{ft/s}^2)} + h_{\text{turb}} + 0$$

$$h_{\text{turb}} = 20.37\text{ ft}$$

$$\dot{W}_s = Q\gamma h_{\text{turb}} = (20\ \text{ft}^3/\text{s})(62.4\ \text{lb/ft}^3)(20.37\ \text{ft})$$
$$= 25423\ \text{ft.lb/s}\left(\frac{1\ \text{hp}}{550\ \text{ft.lb/s}}\right) = 46.2\ \text{hp} \qquad \textbf{Ans.}$$

EGL and HGL. Since no loss occurs, the total head before the pump is

$$H = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = 0 + 0 + 21\text{ ft} = 21\text{ ft}$$

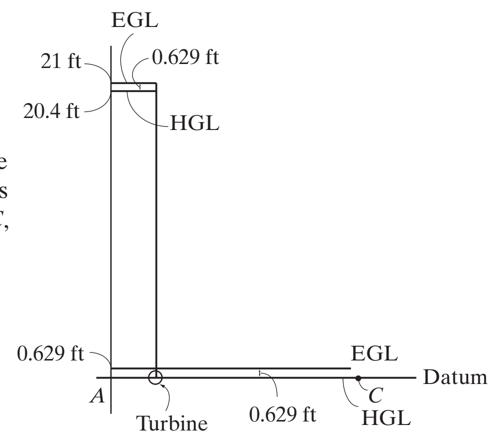
After the turbine, a head of $h_{\text{turb}} = 20.37$ ft is drawn from the water and becomes

$$H = 21\text{ ft} - 20.37\text{ ft} = 0.629\text{ ft}$$

The velocity head has a constant value of

$$\frac{V^2}{2g} = \frac{(6.366\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = 0.629\ \text{ft}$$

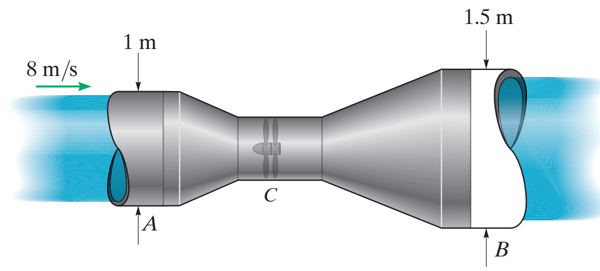
The HGL will always be 0.629 ft below and parallel with the EGL. Both are plotted as shown in Fig. a.



(a)

Ans:
46.2 hp

5–106. The turbine C removes 300 kW of power from the water that passes through it. If the pressure at the intake A is $p_A = 300$ kPa and the velocity is 8 m/s, determine the pressure and velocity of the water at the exit B. Neglect the frictional losses between A and B.



SOLUTION

The control volume considered contains the water in the turbine case from A to B. Since the flow is steady, the continuity condition become

$$\frac{\partial}{\partial t}\int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(8 \text{ m/s})\left[\pi(0.5 \text{ m})^2\right] + V_B\left[\pi(0.75 \text{ m})^2\right] = 0$$

$$V_B = 3.556 \text{ m/s} = 3.56 \text{ m/s} \qquad \textbf{Ans.}$$

Here, $Q = V_A A_A = (8 \text{ m/s})\left[\pi(0.5 \text{ m})^2\right] = 2\pi \text{ m}^3/\text{s}$. The turbine head can be determined from

$$\dot{W}_s = Q\gamma_w h_{\text{turb}}; \qquad 300(10^3)\text{W} = (2\pi \text{ m}^3/\text{s})(9810 \text{ N/m}^3)h_{\text{turb}}$$

$$h_{\text{turb}} = 4.8671 \text{ m}$$

With reference to the datum that contain points A and B, $Z_A = Z_B = 0$. The energy equation between points A and B is

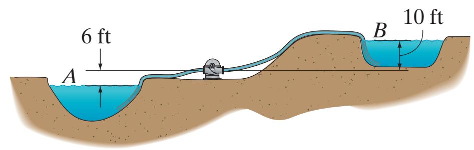
$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{300(10^3) \text{ N/m}^2}{9810 \text{ N/m}^3} + \frac{(8 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 + 0 = \frac{p_B}{9810 \text{ N/m}^3} + \frac{(3.556 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 + 4.8671 \text{ m} + 0$$

$$p_B = 277.93(10^3) \text{ Pa} = 278 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$V = 3.56$ m/s, $p = 278$ kPa

5–107. The pump has a volumetric flow of $0.3\ \text{ft}^3/\text{s}$ as it moves water from the pond at A to the one at B. If the hose has a diameter of 0.25 ft, and friction losses within it can be expressed as $5\,V^2/g$, where V is the average velocity of the flow, determine the horsepower the pump supplies to the water.



SOLUTION

$$Q = V_A$$

$$0.3\ \text{ft}^3/\text{s} = V\left[\pi(0.125\ \text{ft})^2\right]$$

$$V = 6.112\ \text{ft/s}$$

Energy Equation. Take the water from A to B to be the control volume. Since A and B are both free surfaces, $p_A = p_B = 0$. Also, $V_A = V_B = 0$ since both ponds are large reservoirs. If the datum passes through A, $z_A = 0$ and $z_B = 6\ \text{ft} + 10\ \text{ft} = 16\ \text{ft}$.

$$\frac{p_A}{\gamma} + \frac{V_A^{\,2}}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^{\,2}}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + 0 + 16\ \text{ft} + 0 + 5\left[\frac{(6.111\ \text{ft/s})^2}{(32.2\ \text{ft/s}^2)}\right]$$

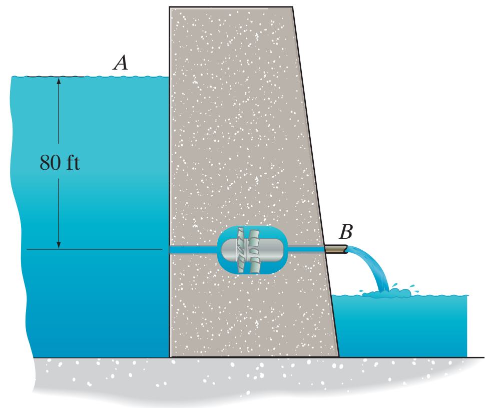
$$h_{\text{pump}} = 21.80\ \text{ft}$$

$$\dot{W}_s = Q\gamma h_{\text{pump}} = (0.3\ \text{ft}^3/\text{s})(62.4\ \text{lb/ft}^3)(21.80\ \text{ft})$$

$$= 408.1\frac{\text{ft}\cdot\text{lb}}{\text{s}}\left(\frac{1\ \text{hp}}{550\text{ft}\cdot\text{lb/s}}\right) = 0.742\ \text{hp}$$ **Ans.**

Ans:
0.742 hp

***5–108.** Water from the reservoir passes through a turbine at the rate of 18 ft^3/s. If it is discharged at B with a velocity of 15 ft/s, and the turbine withdraws 100 hp, determine the head loss in the system.



SOLUTION

$$\dot{W}_s = Q\gamma h_s$$

$$(100\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = \left(18\text{ ft}^3/\text{s}\right)\left(62.4\text{ lb/ft}^3\right)h_s$$

$$h_s = 48.97\text{ ft}$$

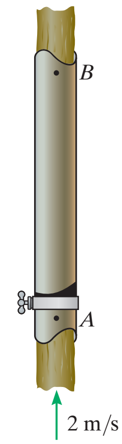
Energy Equation. Take the water from A to B to be the control volume. Since A and B are both free surfaces, $p_A = p_B = 0$. Also, due to the large source at the reservoir, $V_A = 0$. If the datum passes through B, $z_A = 80$ ft and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 80\text{ ft} + 0 = 0 + \frac{(15\text{ ft/s})^2}{2\left(32.2\text{ ft/s}^2\right)} + 0 + 48.97\text{ ft} + h_L$$

$$h_L = 27.5\text{ ft} \qquad \textbf{Ans.}$$

5–109. The vertical pipe is filled with oil. When the valve at A is closed, the pressure at A is 160 kPa, and at B it is 90 kPa. When the valve is open, the oil flows at 2 m/s, and the pressure at A is 150 kPa and at B it is 70 kPa. Determine the head loss in the pipe between A and B. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

Static Pressure. When the valve is closed

$$p_A = p_B + \rho_o g z_B$$

$$160(10^3)\ \text{N/m}^2 = 90(10^3)\ \text{N/m}^2 + (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(z_B)$$

$$z_B = 7.928\ \text{m}$$

Energy Equation. Take the oil from A to B to be the control volume. Since the pipe has a constant diameter, continuity requires $V_A = V_B = V$. If the datum passes through A, $z_A = 0$ and $z_B = 7.928\ \text{m}$. With $h_S = 0$,

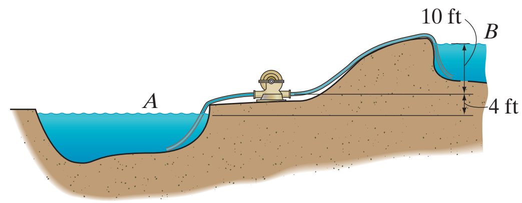
$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{150(10^3)\ \text{N/m}^2}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{V^2}{2g} + 0 + 0 = \frac{70(10^3)\ \text{N/m}^2}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{V^2}{2g} + 7.928\ \text{m} + h_L$$

$$h_L = 1.13\ \text{m}$$ **Ans.**

Ans:
1.13 m

5–110. It is required that a pump be used to discharge water at 80 gal/min from a river to a pond, at B. If the frictional head loss through the hose is 3 ft, and the hose has a diameter of 0.25 ft, determine the required power output of the pump. Note that 7.48 gal = 1 ft^3.



SOLUTION

Energy Equation. Take the water from A to B to be the control volume. Since A and B are free surfaces, $p_A = p_B = 0$. Also, both the river and pond are large reservoirs, $V_A = V_B = 0$. If the datum is at the free surface at A, $z_A = 0$ and $z_B = 4\text{ ft} + 10\text{ ft} = 14\text{ ft}$. With $h_L = 3$ ft,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + 0 + 14\text{ ft} + 0 + 3\text{ ft}$$

$$h_{\text{pump}} = 17.0\text{ ft}$$

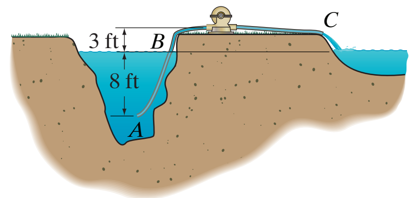
$$\dot{W}_s = Q\gamma h_{\text{pump}}$$

$$\dot{W}_s = \left(\frac{80\text{ gal}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{\text{ft}^3}{7.48\text{ gal}}\right)(62.4\text{ lb/ft}^3)(17\text{ ft})$$

$$= 189.1\text{ ft}\cdot\text{lb/s}\left(\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}}\right) = 0.344\text{ hp} \qquad \textbf{Ans.}$$

Ans:
0.344 hp

5–111. *A* 6-hp pump with a 3-in-diameter hose is used to drain water from a large cavity at *B*. Determine the discharge at *C*. Neglect friction losses and the efficiency of the pump. 1 hp = 550 ft · lb/s.



SOLUTION

$$\dot{W}_s = Q\gamma h_{\text{pump}}$$

$$(6\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = V\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right](62.4\text{ lb/ft}^3)(h_{\text{pump}})$$

$$h_{\text{pump}} = \frac{1077.4}{V}$$

Energy Equation. Take the water in the cavity B and in the base to C to be the control volume. Since B and C are exposed to the atmosphere, $p_B = p_C = 0$. Also, the water is drawn from a large cavity, so that $V_B = 0$. If we set the datum through B, $z_B = 0$ and $z_C = 3$ ft. Since the pump supplies a head of water, h_s is a negative quantity.

$$\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{pump}} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + \left(\frac{1077.4}{V}\right) = 0 + \frac{V^2}{2(32.2\text{ ft/s}^2)} + 3\text{ ft} + 0 + 0$$

$$V^3 + 193.2V = 69381.76$$

Solving numerically,

$$V = 39.52\text{ ft/s}$$

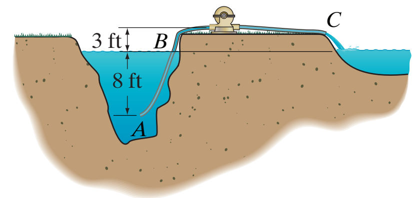
Discharge.

$$Q = VA = (39.52\text{ ft/s})\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right]$$

$$= 1.94\text{ ft}^3/\text{s}$$ **Ans.**

Ans:
$1.94\text{ ft}^3/\text{s}$

***5–112.** The pump is used with a 3-in.-diameter hose to draw water from the cavity. If the discharge is $1.5\ \text{ft}^3/\text{s}$, determine the required power developed by the pump. Neglect friction losses.



SOLUTION

From the discharge,

$$Q = V_C A_C; \qquad 1.5\ \text{ft}^3/\text{s} = V_C\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right]$$

$$V_C = 30.56\ \text{ft/s}$$

The fixed control volume contains the water in the system. Since B and C are exposed to the atmosphere, $p_B = p_C = 0$. Also, the water is drawn form a large reservoir, $V_B = 0$. If we set the datum through B, $z_B = 0$ and $z_C = 3\ \text{ft}$.

$$\frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B + h_{\text{pump}} = \frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + \frac{(30.56\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 3\ \text{ft} + 0 + 0$$

$$h_{\text{pump}} = 17.50\ \text{ft}$$

The required power output of the pump is

$$\dot{W}_s = Q\gamma_w h_{\text{pump}}$$

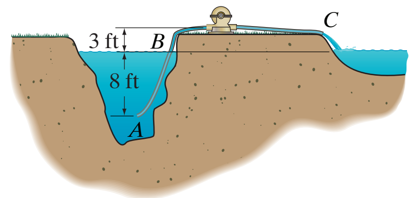
$$= (1.5\ \text{ft}^3/\text{s})(62.4\ \text{lb/ft}^3)(17.50\ \text{ft})$$

$$= \left(1637.97\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}\right)\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right)$$

$$= 2.98\ \text{hp}$$

Ans.

5–113. Solve Prob. 5–112 by including frictional head losses in the hose of 1.5 ft for every 20 ft of hose. The hose has a total length of 130 ft.



SOLUTION

From the discharge,

$$Q = V_C A_C; \qquad 1.5\ \text{ft}^3/\text{s} = V_C\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right]$$

$$V_C = 30.56\ \text{ft/s}$$

The fixed control volume contains the water in the system. Since B and C are exposed to the atmosphere, $p_B = p_C = 0$. Also, the water is drawn from a large reservoir, $V_B = 0$. If we set the datum through B, $z_B = 0$ and $z_C = 3$ ft.

$$\frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B + h_{\text{pump}} = \frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + \frac{(30.56\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 3\ \text{ft} + 0 + \left(\frac{1.5\ \text{ft}}{20\ \text{ft}}\right)(130\ \text{ft})$$

$$h_{\text{pump}} = 27.25\ \text{ft}$$

The required power output of the pump is

$$\dot{W}_s = Q\gamma_w h_{\text{pump}}$$

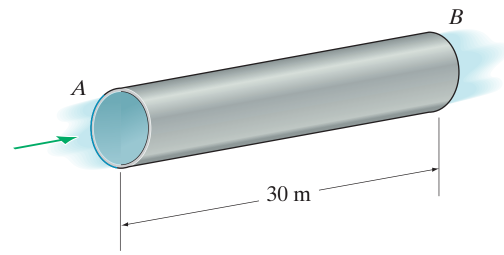
$$= (1.5\ \text{ft}^3/\text{s})(62.4\ \text{lb/ft}^3)(27.25\ \text{ft})$$

$$= \left(2550.57\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}\right)\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right)$$

$$= 4.64\ \text{hp}$$ **Ans.**

Ans:
4.64 hp

5–114. The flow of air through a 200-mm-diameter duct has an absolute inlet pressure of 180 kPa, a temperature of 15°C, and a velocity of 10 m/s. Farther downstream a 2-kW exhaust system increases the outlet velocity to 25 m/s. Determine the density of the air at the outlet, and the change in enthalpy of the air. Neglect heat transfer through the pipe.



SOLUTION

Ideal Gas Law. Referring to Appendix A, $R = 286.9\ \text{J/kg}\cdot\text{K}$.

$$p_{in} = \rho_{in} R T_{in}$$

$$180(10^3)\ \text{N/m}^2 = \rho_{in}(286.9\ \text{J/kg}\cdot\text{K})(15° + 273)\ \text{K}$$

$$\rho_{in} = 2.178\ \text{kg/m}^3$$

$$\dot{m} = \rho_{in} V_{in} A_{in} = (2.178\ \text{kg/m}^3)(10\ \text{m/s})\left[\pi(0.1\ \text{m})^2\right]$$

$$= 0.6844\ \text{kg/s}$$

Energy Equation. With $\left(\dfrac{dQ}{dt}\right)_{in} = \left(\dfrac{dW_{turb}}{dt}\right) = 0$ and $z_{in} = z_{out} = z$,

$$\left(\frac{dQ}{dt}\right)_{in} - \left(\frac{dW_{turb}}{dt}\right) + \left(\frac{dW_{pump}}{dt}\right) = \left[\left(h_{out} + \frac{V_{out}^{\,2}}{2} + gz_{out}\right) - \left(h_{in} + \frac{V_{in}^{\,2}}{2} + gz_{in}\right)\right]\dot{m}$$

$$0 - 0 + \left(\frac{dW_{pump}}{dt}\right) = \left[\left(h_{out} + \frac{V_{out}^{\,2}}{2} + gz\right) - \left(h_{in} + \frac{V_{in}^{\,2}}{2} + gz\right)\right]\dot{m}$$

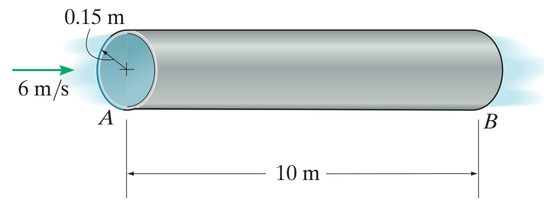
$$\Delta h = h_{out} - h_{in} = \left(\frac{V_{in}^{\,2}}{2} - \frac{V_{out}^{\,2}}{2}\right) + \left(\frac{dW_{pump}}{dt}\right)\frac{1}{\dot{m}}$$

$$= \left[\frac{(10\ \text{m/s})^2}{2} - \frac{(25\ \text{m/s})^2}{2}\right] + \frac{2000\ \text{N}\cdot\text{m/s}}{0.6844\ \text{kg/s}}$$

$$= 2660\ \text{J/kg} = 2.66\ \text{kJ/kg}$$ **Ans.**

Ans:
2.66 kJ/kg

5–115. Nitrogen gas having an enthalpy of 250 J/kg is flowing at 6 m/s into the 10-m-long pipe at A. If the heat loss from the walls of the duct is 60 W, determine the enthalpy of the gas at the exit B. Assume that the gas is incompressible with a density of $\rho = 1.36\ \text{kg/m}^3$.



SOLUTION

Mass Flow Rate.

$$\dot{m} = \rho V A = \left(1.36\ \text{kg/m}^3\right)\left(6\ \text{m/s}\right)\left[\pi(0.15\ \text{m})^2\right] = 0.5768\ \text{kg/s}$$

Since the density and diameter of the duct are constant, continuity requires

$$V_{\text{in}} = V_{\text{out}} = V$$

Energy Equation. The flow is steady. Take the nitrogen on the pipe to be the control volume.

Here, $\left(\dfrac{dQ}{dt}\right)_{\text{in}} = -60\ \text{J/s}$ and $\left(\dfrac{dW_{\text{turb}}}{dt}\right) = \left(\dfrac{dW_{\text{pump}}}{dt}\right) = 0$. With $z_{\text{in}} = z_{\text{out}} = z$,

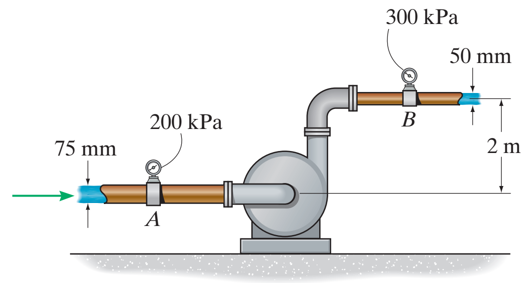
$$\left(\frac{dQ}{dt}\right)_{\text{in}} - \left(\frac{dW_{\text{turb}}}{dt}\right) + \left(\frac{dW_{\text{pump}}}{dt}\right) = \left[\left(h_{\text{out}} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}}\right) - \left(h_{\text{in}} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}}\right)\right]\dot{m}$$

$$-60\ \text{J/s} - 0 + 0 = \left[\left(h_{\text{out}} + \frac{V^2}{2} + gz\right) - \left(250\ \text{J/kg} + \frac{V^2}{2} + gz\right)\right]\left(0.5768\ \text{kg/s}\right)$$

$$h_{\text{out}} = 145.98\ \text{J/kg} = 146\ \text{J/kg} \qquad \textbf{Ans.}$$

Ans:
146 J/kg

***5–116.** The measured water pressure at the inlet and exit portions of the pipe are indicated for the pump. If the flow is $0.1\ \text{m}^3/\text{s}$, determine the power that the pump supplies to the water. Neglect friction losses.



SOLUTION

Flow:

$$Q = V_A A_A; \qquad 0.1\ \text{m}^3/\text{s} = V_A\left[\pi(0.0375\ \text{m})^2\right]$$

$$V_A = 22.64\ \text{m/s}$$

$$Q = V_B A_B; \qquad 0.1\ \text{m}^3/\text{s} = V_B\left[\pi(0.025\ \text{m})^2\right]$$

$$V_B = 50.93\ \text{m/s}$$

Energy Equation. Take the water from A to B to be the control volume. If we set the datum through A, $z_A = 0$ and $z_B = 2\ \text{m}$. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{200(10^3)\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(22.64\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 + h_{\text{pump}}$$

$$= \frac{300(10^3)\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(50.93\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 2\ \text{m} + 0 + 0$$

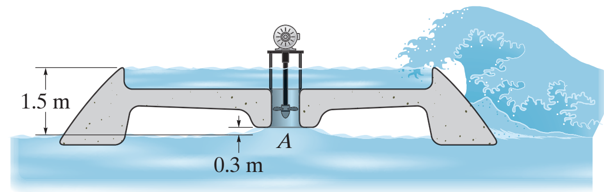
$$h_{\text{pump}} = 118.3\ \text{m}$$

$$\dot{W}_s = Q\gamma h_{\text{pump}}$$

$$= (0.1\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(118.3\ \text{m})$$

$$= 116.04(10^3)\ \text{W} = 116\ \text{kW} \qquad \textbf{Ans.}$$

5–117. The wave overtopping device consists of a floating reservoir that is continuously filled by waves, so that the water level in the reservoir is always higher than that of the surrounding ocean. As the water drains out at A, the energy is drawn by the low-head hydroturbine, which then generates electricity. Determine the power that can be produced by this system if the water level in the reservoir is always 1.5 m above that in the ocean, The waves add $0.3\ \text{m}^3/\text{s}$ to the reservoir, and the diameter of the tunnel containing the turbine is 600 mm. The head loss through the turbine is 0.2 m. Take $\rho_w = 1050\ \text{kg/m}^3$.



SOLUTION

From the discharge,

$$Q = V_{\text{out}} A_{\text{out}}; \qquad 0.3\ \text{m}^3/\text{s} = V_{\text{out}}\left[\pi(0.3\ \text{m})^2\right]$$

$$V_{\text{out}} = 1.061\ \text{m/s}$$

Take the water within the turbine to be the control volume. We will apply the energy equation between the inlet and the outlet.

$$\frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + h_L$$

Here, $V_{\text{in}} = 0$ since the inlet is the surface of a large reservoir. $p_{\text{in}} = p_{\text{out}} = p_{\text{atm}} = 0$ since the inlet and outlet are exposed to the atmosphere. Here, the datum is set at the ocean water level. Then, $z_{\text{in}} = 1.5$ m and $z_{\text{out}} = 0.3$ m.

$$0 + 0 + 1.5\ \text{m} + 0 = 0 + \frac{(1.061\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.3\ \text{m} + h_{\text{turb}} + 0.2\ \text{m}$$

$$h_{\text{turb}} = 0.9426\ \text{m}$$

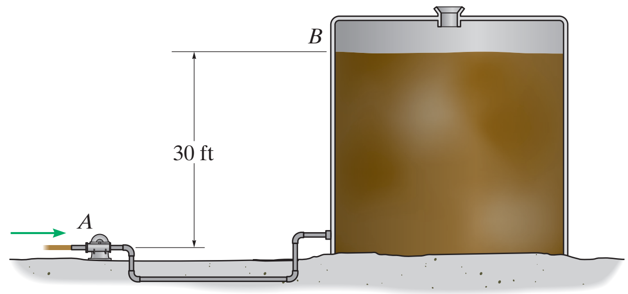
The turbine is

$$\begin{aligned}\dot{W}_s &= Q\gamma_{sw}h_{\text{turb}}\\ &= (0.3\ \text{m}^3/\text{s})\left[(1050\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\right](0.9426\ \text{m})\\ &= 2912.83\ \text{W}\\ &= 2.91\ \text{kW}\end{aligned}$$

Ans.

Ans:
2.91 kW

5–118. Crude oil is pumped from the test separator at A to the stock tank using a galvanized iron pipe that has a diameter of 4 in. If the total pipe length is 180 ft, and the volumetric flow at A is 400 gal/min, determine the required horsepower supplied by the pump. The pressure at A is 4 psi, and the stock tank is opened to the atmosphere. The frictional head loss in the pipe is 0.25 in./ft, and the head loss for each of the four bends is $K(V^2/2g)$, where $K = 0.09$ and V is the velocity of the flow in the pipe. Take $\gamma_o = 55\ \text{lb/ft}^3$. Note that $1\ \text{ft}^3 = 7.48$ gal.



SOLUTION

The discharge is

$$Q = \left(400\,\frac{\text{gal}}{\text{min}}\right)\left(\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 0.8913\ \text{ft}^3/\text{s}$$

Thus,

$$Q = V_A A_A; \qquad 0.8913\ \text{ft}^3/\text{s} = V_A\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right]$$

$$V_A = 10.21\ \text{ft/s}$$

Energy Equation.

Take the water from A to B on the control volume. Then

$$\frac{p_A}{\gamma_o} + \frac{V_A^{\,2}}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_o} + \frac{V_B^{\,2}}{2g} + z_B + h_{\text{turb}} + h_L$$

Since point B is exposed to atmosphere, $p_B = p_{\text{atm}} = 0$. Also,

$$p_A = \left(4\,\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^2 = 576\ \text{lb/ft}^2$$

Since the stock tank is a large reservoir $V_B \simeq 0$. The frictional head loss is

$$(h_L)_f = \left[\frac{(0.25/12)\ \text{ft}}{\text{ft}}\right](180\ \text{ft}) = 3.75\ \text{ft}$$

There are four bends between point A and B. Thus, the head losses due to the bends is

$$(h_L)_M = 4\left(k\,\frac{V^2}{2g}\right) = 4\left\{(0.09)\left[\frac{(10.21\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}\right]\right\} = 0.5831\ \text{ft}$$

5–118. Continued

With reference to the datum set through point A, $z_A = 0$ and $z_B = 30$ ft. Substituting these results into the energy equation,

$$\frac{576\ \text{lb/ft}^2}{55.0\ \text{lb/ft}^3} + \frac{(10.21\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 + h_{\text{pump}} = 0 + 0 + 30\ \text{ft} + 0 + (3.75\ \text{ft} + 0.5831\ \text{ft})$$

$$h_{\text{pump}} = 22.24\ \text{ft}$$

The required output power can be determined from

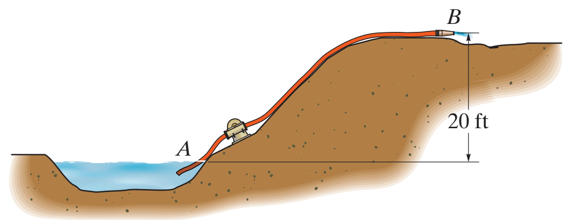
$$\dot{W}_s = Q\gamma_{co}h_{\text{pump}} = (0.8913\ \text{ft}^3/\text{s})(55.0\ \text{lb/ft}^3)(22.24\ \text{ft})$$

$$= (1090.23\ \text{ft}\cdot\text{lb/s})\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right)$$

$$= 1.98\ \text{hp}$$ **Ans.**

Ans:
1.98 hp

5-119. The pump is used to transport water at 90 ft^3/min from the stream up the 20-ft embankment. If frictional head losses in the 3-in.-diameter pipe are $h_L = 1.5$ ft, determine the power output of the pump.



SOLUTION

Energy Equation. Take the water from A to B to be the control volume. Then we will apply the energy equation between point A on the surface of water in the stream and point B at the pipe's exit. Here, points A and B are exposed to the atmosphere, $p_A = p_B = p_{atm} = 0$. Since the stream can be considered as a large reservoir, $V_A \simeq 0$. Here the discharge is

$$Q = \left(90\ \frac{\text{ft}^3}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 1.5\ \text{ft}^3/\text{s}$$

Then,

$$Q = V_B A_B; \qquad 1.5\ \text{ft}^3/\text{s} = V_B\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right]$$

$$V_B = 30.56\ \text{ft/s}$$

With reference to the datum set through point A, $z_A = 0$ and $z_B = 20$ ft.

$$\frac{p_A}{\gamma_W} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_W} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + \frac{(30.56\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 20\ \text{ft} + 0 + 1.5\ \text{ft}$$

$$h_{\text{pump}} = 36.00\ \text{ft}$$

The power output of the pump can be determined from

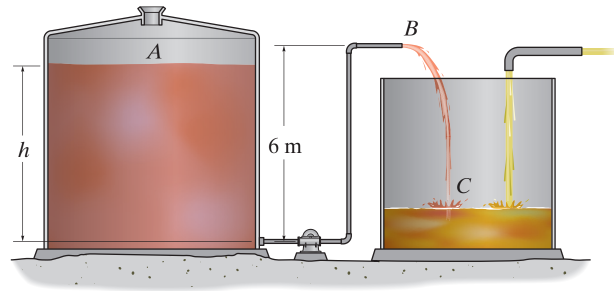
$$\dot{W}_s = Q\gamma_W h_{\text{pump}} = (1.5\ \text{ft}^3/\text{s})(62.4\ \text{lb/ft}^2)(36.00\ \text{ft})$$

$$= (3369.57\ \text{ft}\cdot\text{lb/s})\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right)$$

$$= 6.13\ \text{hp}$$ **Ans.**

Ans:
6.13 hp

***5–120.** The pump is used to transfer carbon tetrachloride in a processing plant from a storage tank A to the mixing tank C. If the total head loss due to friction and the pipe fittings in the system is 1.8 m, and the diameter of the pipe is 50 mm, determine the power developed by the pump when $h = 3$ m. The velocity at the pipe exit is 10 m/s, and the storage tank is opened to the atmosphere $\rho_{ct} = 1590 \text{ kg/m}^3$.



SOLUTION

Take the carbon tetrachloride in the tank and pipe to point B to be the control volume. Then we will apply the energy equation between point A on the surface of carbon tetrachloride in the storage tank and point B at the pipe's exit. Here points A and B are exposed to atmosphere, $p_A = p_B = p_{atm} = 0$. Since the storage tank can be considered as a large reservoir, $V_A \simeq 0$. With reference to the datum set through point A, $z_A = 0$ and $z_B = 6\text{ m} - 3\text{ m} = 3\text{m}$. Also, $\gamma_{ct} = \rho_{ct}\, g = (1590 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 15597.9 \text{ N/m}^3$.

$$\frac{p_A}{\gamma_{ct}} + \frac{V_A^2}{2g} + z_A + h_{pump} = \frac{p_B}{\gamma_{ct}} + \frac{V_B^2}{2g} + z_B + h_{turb} + h_L$$

$$0 + 0 + 0 + h_{pump} = 0 + \frac{(10 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 3\text{ m} + 0 + 1.8\text{ m}$$

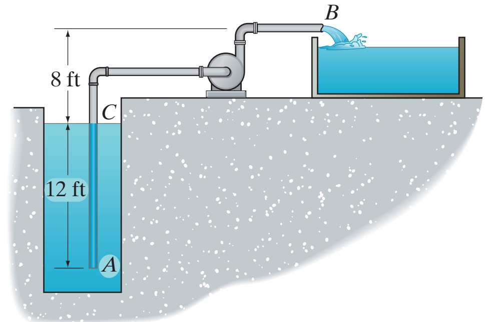
$$h_{pump} = 9.8968 \text{ m}$$

Here, $Q = V_B A_B = (10 \text{ m/s})\left[\pi(0.025 \text{ m})^2\right] = 6.25\pi\left(10^{-3}\right) \text{m}^3/\text{s}$. Then, the power output of the pump can be determined from

$$\dot{W}_S = Q\gamma_{ct}h_{pump} = \left[6.25\,\pi\left(10^{-3}\right)\text{m}^3/\text{s}\right](15597.9 \text{ N/m}^3)(9.8968 \text{ m})$$

$$= 3031.04 \text{ W} = 3.03 \text{ kW}$$ **Ans.**

5–121. The pump takes in water from the large reservoir at A and discharges it at B at 0.8 ft^3/s through a 6-in.-diameter pipe. If the frictional head loss is 3 ft, determine the power output of the pump.



SOLUTION

$$Q = V_B A_B$$

$$0.8\ \text{ft}^3/\text{s} = V_B\left[\pi\left(\frac{3}{12}\text{ft}\right)^2\right]$$

$$V_B = 4.074\ \text{ft/s}$$

Energy Equation. Takes the water in the reservoir and pipe system to B at the control volume. Since B and C are exposed to the atmosphere, $p_B = p_C = 0$. Also, $V_C = 0$ since the water is drawn from the large reservior. If the datum passes through C, $z_C = 0$ and $z_B = 8$ ft. With $h_L = 3$ ft,

$$= \frac{p_C}{\gamma} + \frac{V_C{}^2}{2g} + z_C + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B{}^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + \frac{(4.074\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 8\ \text{ft} + 0 + 3\ \text{ft}$$

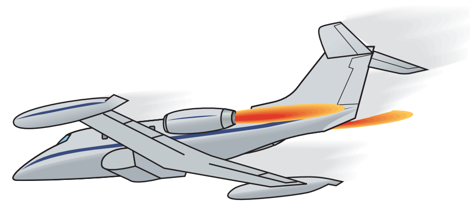
$$h_{\text{pump}} = 11.26\ \text{ft}$$

$$\dot{W}_S = Q\gamma h_{\text{pump}} = (0.8\ \text{ft}^3/\text{s})(62.4\ \text{lb/ft}^3)(11.26\ \text{ft})$$

$$= 562.0\ \text{ft}\cdot\text{lb/s}\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right) = 1.02\ \text{hp} \qquad \textbf{Ans.}$$

Ans:
1.02 hp

5–122. Air and fuel enter a turbojet engine (turbine) having an enthalpy of 800 kJ/kg and a relative velocity of 15 m/s. The mixture exits with a relative velocity of 60 m/s and an enthalpy of 650 kJ/kg. If the mass flow is 30 kg/s, determine the power output of the jet. Assume no heat transfer occurs.



SOLUTION

Energy Equation. Take the Air and fuel and engine the control volume. Here, $\left(\frac{dQ}{dt}\right)_{\text{in}} = 0$ and $z_{\text{in}} = z_{\text{out}} = z$.

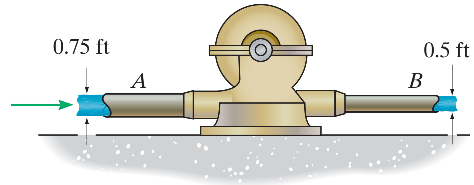
$$\left(\frac{dQ}{dt}\right)_{\text{in}} - \left(\frac{dW_{\text{turb}}}{dt}\right) + \left(\frac{dW_{\text{pump}}}{dt}\right) = \left[\left(h_{\text{out}} + \frac{V_{out}^2}{2} + gz_{\text{out}}\right) - \left(h_{\text{in}} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}}\right)\right]\dot{m}$$

$$0 - \left(\frac{dW_{\text{turbine}}}{dt}\right) + 0 = \left[\left(650(10^3)\,\text{J/kg} + \frac{(60\text{ m/s})^2}{2} + gz\right) - \left(800(10^3)\,\text{J/kg} + \frac{(15\text{ m/s})^2}{2} + gz\right)\right](30\text{ kg/s})$$

$$\left(\frac{dW_{\text{turbine}}}{dt}\right) = 4.449(10^6)\,\text{W} = 4.45\text{ MW}$$ **Ans.**

Ans:
4.45 MW

5–123. Water flows into the pump at 600 gal/min and has a pressure of 4 psi. It exits the pump at 18 psi. Determine the power output of the pump. Neglect friction losses. Note that $1\ \text{ft}^3 = 7.48$ gal.



SOLUTION

$$Q = (600\ \text{gal/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\right) = 1.337\ \text{ft}^3/\text{s}$$

$Q = V_A A_A;$ $$1.337\ \text{ft}^3/\text{s} = V_A\left[\pi(0.375\ \text{ft})^2\right]$$

$$V_A = 3.026\ \text{ft/s}$$

$Q = V_B A_B;$ $$1.337\ \text{ft}^3/\text{s} = V_B\left[\pi(0.25\ \text{ft})^2\right]$$

$$V_B = 6.809\ \text{ft/s}$$

Energy Equation. Take the water from A to B as the control volume. Since the centerline of the pipe lies on the same horizontal line, $z_A = z_B = z$. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{4\ \text{lb/in}^2\left(\dfrac{12\ \text{in.}}{1\ \text{ft}}\right)^2}{62.4\ \text{lb/ft}^3} + \frac{(3.026\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + z + h_{\text{pump}} = \frac{18\ \text{lb/in}^2\left(\dfrac{12\ \text{in.}}{1\ \text{ft}}\right)^2}{62.4\ \text{lb/ft}^3} + \frac{(6.809\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + z + 0 + 0$$

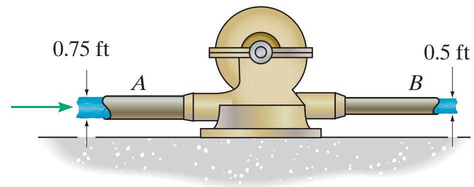
$$h_{\text{pump}} = 32.89\ \text{ft}$$

$$\dot{W}_S = Q\gamma h_{\text{pump}} = (1.337\ \text{ft}^3/\text{s})(62.4\ \text{lb/ft}^3)(32.89\ \text{ft})$$

$$= 2743\ \text{ft}\cdot\text{lb/s}\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right) = 4.99\ \text{hp}$$ **Ans.**

Ans:
4.99 hp

***5–124.** The 5-hp pump has an efficiency of $e = 0.8$ and produces a flow velocity of 3 ft/s through the pipe at A. If the frictional head loss within the system is 8 ft, determine the difference in the water pressure between A and B.



SOLUTION

From the discharge,

$$Q = V_A A_A; \qquad Q = (3\text{ ft/s})\left[\pi(0.375\text{ ft})^2\right] = 0.421875\pi\text{ ft}^3/\text{s}$$

$$Q = V_B A_B; \qquad 0.421875\pi\text{ ft}^3/\text{s} = V_B\left[\pi(0.25\text{ ft})^2\right]$$

$$V_B = 6.75\text{ ft/s}$$

The power output of the pump is given by

$$\varepsilon = \frac{\dot{W}_{s_{\text{out}}}}{\dot{W}_{s_{\text{in}}}}; \qquad 0.8 = \frac{\dot{W}_{s_{\text{out}}}}{5h_p} \qquad \dot{W}_{s_{\text{out}}} = 4h_p$$

Thus, the pump head is

$$\dot{W}_{s_{\text{out}}} = Q\gamma_w h_{\text{pump}}; \qquad (4h_p)\left(\frac{550\text{ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = \left(0.421875\pi\text{ ft}^3/\text{s}\right)\left(62.4\text{ lb/ft}^3\right)h_{\text{pump}}$$

$$h_{\text{pump}} = 26.60\text{ ft}$$

The fixed control volume contains the water in the system from A to B. Since the centerline of the pipe lies on the same elevation,

$$Z_A = Z_B = Z.$$

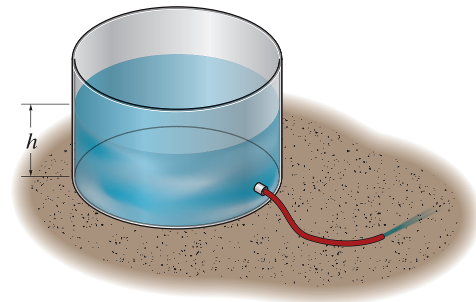
$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{p_A}{62.4\text{ lb/ft}^3} + \frac{(3\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + z + 26.60\text{ ft}$$

$$= \frac{p_B}{62.4\text{ lb/ft}^3} + \frac{(6.75\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + z + 0 + 8\text{ ft}$$

$$p_B - p_A = \left(1125.30\text{ lb/ft}^2\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)^2 = 7.814\text{ psi} = 7.81\text{ psi} \qquad \textbf{Ans.}$$

5–125. The water tank is being drained using the 1-in.-diameter hose. If the flow out of the hose is 5 ft^3/min, determine the head loss in the hose when the water depth is $d = 6$ ft.



SOLUTION

Energy Equation: Take the water in the tank and fill the hose print B as the central volume. We will apply the energy equation between point A on the surface of water in the tank and point B at the hose's exit. Here, points A and B are exposed to atmosphere, $p_A = p_B = p_{\text{atm}} = 0$. Since the tank can be considered as a large reservoir, $V_A \cong 0$. Here the discharge is

$$Q = \left(5\frac{\text{ft}^3}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 0.08333\text{ ft}^3/\text{s}$$

Then

$$Q = V_B A_B; \qquad 0.08333\text{ ft}^3/\text{s} = V_B\left[\pi\left(\frac{0.5}{12}\text{ft}\right)^2\right]$$

$$V_B = 15.28\text{ ft/s}$$

With reference to the datum through point B, $z_A = 6$ ft and $z_B = 0$.

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

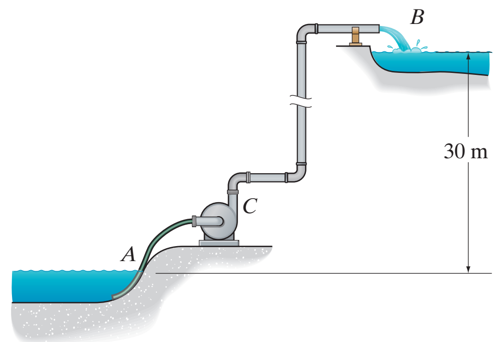
Then

$$0 + 0 + 6\text{ ft} + 0 = 0 + \frac{(15.28\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0 + h_L$$

$$h_L = 2.38\text{ ft} \qquad \textbf{Ans.}$$

Ans:
2.38 ft

5–126. The pump at C produces a discharge of water at B of $0.035\ \mathrm{m^3/s}$. If the pipe at B has a diameter of 50 mm and the hose at A has a diameter of 30 mm, determine the power output supplied by the pump. Assume frictional head losses within the pipe system are determined from $3V_B^2/2g$.



SOLUTION

$$Q = V_B A_B$$

$$0.035\ \mathrm{m^3/s} = V_B\left[\pi(0.025\ \mathrm{m})^2\right]$$

$$V_B = 17.825\ \mathrm{m/s}$$

Energy Equation. Take the water in the lower reservoir and in the pipe system to B as the control volume. Since A and B are exposed to the atmosphere, $p_A = p_B = 0$. Also, $V_A = 0$ since water is drawn from a large reservoir. If the datum coincides with the free surface A, $z_A = 0$ and $z_B = 30\ \mathrm{m}$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 0 + h_{\text{pump}} = 0 + \frac{(17.825\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} + 30\ \mathrm{m} + 0 + \frac{3(17.825\ \mathrm{m/s})^2}{(9.81\ \mathrm{m/s^2})}$$

$$h_{\text{pump}} = 143.36\ \mathrm{m}$$

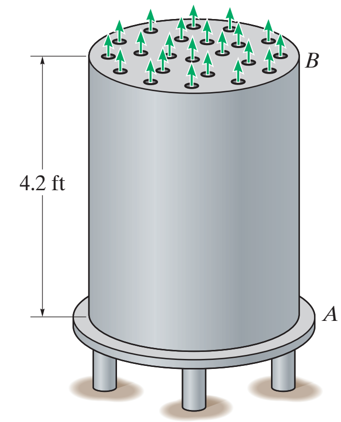
$$\dot{W}_S = Q\gamma h_{\text{pump}}$$

$$= (0.035\ \mathrm{m^3/s})(1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(143.36\ \mathrm{m})$$

$$= 49.22(10^3)\ \mathrm{W} = 49.2\ \mathrm{kW} \qquad \textbf{Ans.}$$

Ans:
49.2 kW

5–127. Determine the power output required to pump sodium coolant at 3 ft^3/s through the core of a liquid metal fast-breeder reactor if the piping system consists of 23 stainless steel pipes, each having a diameter of 1.25 in. and length of 4.2 ft. The pressure at the inlet A is 47.5 lb/ft^2 and at the outlet B it is 15.5 lb/ft^2. The frictional head loss for each pipe is 0.75 in. $\gamma_{Na} = 57.9\ lb/ft^3$.



SOLUTION

Take the sodium passing through the water as the control volume. Then we will apply the energy equation between point A (inlet) and point B (outlet). Since the pipes have constant diameter, the continuity condition requires that $V_A = V_B = V$. With reference to the datum set through point A, $z_A = 0$ and $z_B = 4.2$ ft. Here, the head loss is $h_L = 23(0.75\text{ in}) = (17.75\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right) = 1.4375\text{ ft}$

$$\frac{p_A}{\gamma_{NA}} + \frac{V_A{}^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_{NA}} + \frac{V_B{}^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{47.5\text{ lb/ft}^2}{57.9\text{ lb/ft}^3} + \frac{V^2}{2g} + 0 + h_{\text{pump}} = \frac{15.5\text{ lb/ft}^2}{57.9\text{ lb/ft}^3} + \frac{V^2}{2g} + 4.2 + D + 1.4375$$

$$h_{\text{pump}} = 5.085\text{ ft}$$

Thus, the power output required can be determined from

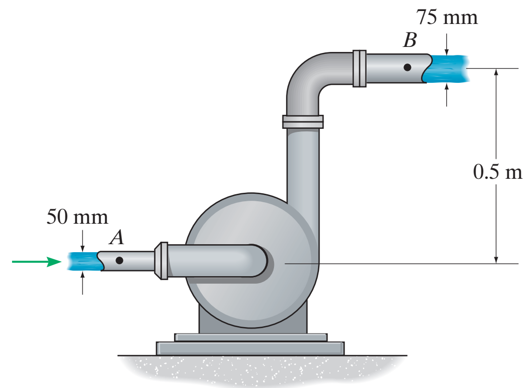
$$\dot{W}_S = Q\gamma_{NA}h_{\text{pump}} = (3\text{ ft}^3/\text{s})(57.9\text{ lb/ft}^3)(5.085\text{ ft})$$

$$= (883.234\text{ ft}\cdot\text{lb/s})\left(\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}}\right)$$

$$= 1.61\text{ hp} \qquad \textbf{Ans.}$$

Ans:
1.61 hp

***5–128.** If the pressure at A is 60 kPa, and the pressure at B is 180 kPa, determine the power output supplied by the pump if the water flows at $0.02\ \text{m}^3/\text{s}$. Neglect friction losses.



SOLUTION

$$Q = V_A A_A; \qquad 0.02\ \text{m}^3/\text{s} = V_A\left[\pi(0.025\ \text{m})^2\right]$$

$$V_A = 10.186\ \text{m/s}$$

$$Q = V_B A_B; \qquad 0.02\ \text{m}^3/\text{s} = V_B\left[\pi(0.0375\ \text{m})^2\right]$$

$$V_B = 4.527\ \text{m/s}$$

Energy Equation. Take the water from A to B as the control volume. If we set the datum through B, $z_A = 0$ and $z_C = 0.5\ \text{m}$. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A{}^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B{}^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{60\left(10^3\right)\ \text{N/m}^2}{\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)} + \frac{(10.186\ \text{m/s})^2}{2\left(9.81\ \text{m/s}^2\right)} + 0 + h_{\text{pump}} = \frac{180\left(10^3\right)\ \text{N/m}^2}{\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)}$$

$$+ \frac{(4.527\ \text{m/s})^2}{2\left(9.81\ \text{m/s}^2\right)} + 0.5\ \text{m} + 0 + 0$$

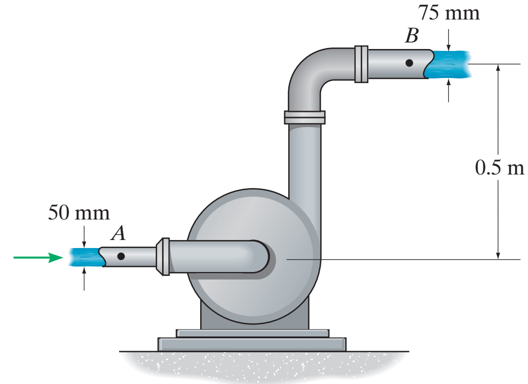
$$h_{\text{pump}} = 8.489\ \text{m}$$

$$\dot{W}_S = Q\gamma h_{\text{pump}} = \left(0.02\ \text{m}^2/\text{s}\right)\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(8.489\ \text{m})$$

$$= 1.666\ \left(10^3\right)\ \text{W} = 1.67\ \text{kW}$$

Ans.

5–129. The pump supplies a power of 1.5 kW to the water producing a volumetric flow of $0.015\ \text{m}^3/s$. If the total frictional head loss within the system is 1.35 m, determine the pressure difference between the inlet A and outlet B of the pipes.



SOLUTION

From the discharge

$Q = V_A A_A;$ $\qquad 0.015\ \text{m}^3/\text{s} = V_A\left[\pi(0.025\ \text{m})^2\right]$

$$V_A = 7.639\ \text{m/s}$$

$Q = V_B A_B;$ $\qquad 0.015\ \text{m}^3/\text{s} = V_B\left[\pi(0.0375\ \text{m})^2\right]$

$$V_B = 3.395\ \text{m/s}$$

From the power supplied to the water,

$$(\dot{W}_s)_{\text{out}} = Q\gamma h_{\text{pump}};$$

$$1.5(10^3)W = (0.015\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)h_{\text{pump}}$$

$$h_{\text{pump}} = 10.19\ \text{m}$$

The fixed control volume contains the water in the system from A to B. If we set the datum through A, $z_A = 0$ and $z_B = 0.5$ m.

$$\frac{p_A}{\gamma_W} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_W} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

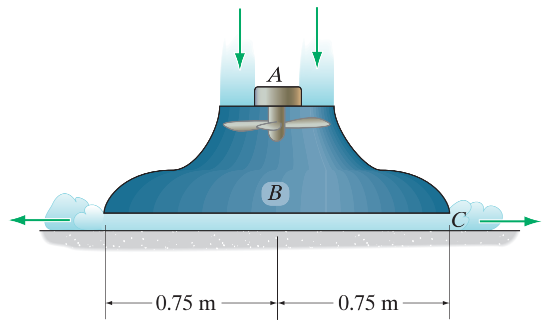
$$\frac{p_A}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(7.639\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 + 10.19\ \text{m} = \frac{p_B}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(3.395\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.5\ \text{m} + 0 + 1.35\ \text{m}$$

$$p_B - p_A = 105.27(10^3)\ \text{Pa} = 105\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:

$p_B - p_A = 105$ kPa

5–130. The circular hovercraft draws in air through the fan A and discharges it through the bottom B near the ground, where it produces a pressure of 1.50 kPa on the ground. Determine the average velocity of the air entering at A that is needed to lift the hovercraft 100 mm off the ground. The open area at A is $0.75\ \text{m}^2$. Neglect friction losses. Take $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION

Between B and C.

$$\frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + h_L$$

$$\frac{1.50(10^3)}{1.22(9.81)} + 0 + 0 + 0 = 0 + \frac{V_C^2}{2(9.81)} + 0 + 0 + 0$$

$$V_C = 49.59\ \text{m/s}$$

Between A and C,

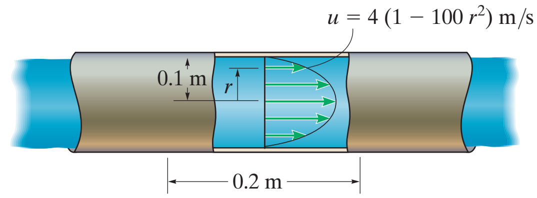
$$\frac{\partial}{\partial t}\int_{cv} \rho \forall dA + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 + V_A(0.75\ \text{m}^2) - (49.59\ \text{m/s})(2\pi)(0.75\ \text{m})(0.1\ \text{m}) = 0$$

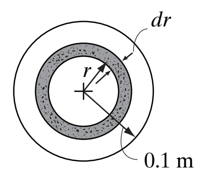
$$V_A = 31.2\ \text{m/s}$$ **Ans.**

Ans:
31.2 m/s

6–1. Determine the linear momentum of a mass of fluid in a 0.2-m length of pipe if the velocity profile for the fluid is a paraboloid as shown. Compare this result with the linear momentum of the fluid using the average velocity of flow. Take $\rho = 800\ \text{kg/m}^3$.



SOLUTION



(a)

The shell differential element that has a thickness dr and length 0.2 m shown shaded in Fig. a has a volume of $d\forall = (2\pi r dr)(0.2\text{ m}) = 0.4\pi r dr$. Thus, the mass of this element is $dm = \rho d\forall = \left(800\ \text{kg/m}^3\right)(0.4\pi r dr) = 320\pi r dr$. The linear momentum of the fluid is

$$
\begin{aligned}
L &= \int_m v\, dm \\
&= \int_0^{0.1\text{ m}} 4\left(1 - 100r^2\right)(320\pi r\, dr) \\
&= 1280\pi \int_0^{0.1\text{ m}} \left(r - 100r^3\right)dr \\
&= 1280\pi\left(\frac{r^2}{2} - 25r^4\right)\Bigg|_0^{0.1\text{ m}} \\
&= 10.05\ \text{kg}\cdot\text{m/s} = 10.1\ \text{kg}\cdot\text{m/s} \qquad \textbf{Ans.}
\end{aligned}
$$

The ring differential element shown shaded in Fig. a has an area of $dA = 2\pi r dr$. Therefore

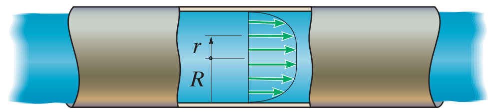
$$
\begin{aligned}
V_{\text{avg}} &= \int \frac{v\, dA}{A} \\
&= \frac{\displaystyle\int_0^{0.1\text{ m}} 4\left(1 - 100r^2\right)(2\pi r\, dr)}{\pi(0.1\text{ m})^2} \\
&= \frac{8\pi \displaystyle\int_0^{0.1\text{ m}} \left(r - 100r^3\right)dr}{\pi(0.1\text{ m})^2} \\
&= \frac{8\pi\left(\dfrac{r^2}{2} - 25r^4\right)\Bigg|_0^{0.1\text{ m}}}{\pi(0.1\text{ m})^2} \\
&= 2\ \text{m/s}
\end{aligned}
$$

The mass of the fluid is $m\rho\forall = \left(800\ \text{kg/m}^3\right)\left[\pi(0.1\text{ m})^2\right](0.2\text{ m}) = 1.6\pi\ \text{kg}$. Thus,

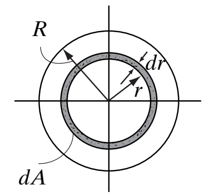
$$
\begin{aligned}
L &= mV_{\text{avg}} = \rho\forall V_{\text{avg}} = (1.6\pi\ \text{kg})(2\ \text{m/s}) \\
&= 10.05\ \text{kg}\cdot\text{m/s} = 10.1\ \text{kg}\cdot\text{m/s} \qquad \textbf{Ans.}
\end{aligned}
$$

Ans:
$L = 10.1\ \text{kg}\cdot\text{m/s}$ by either method.

6–2. Flow through the circular pipe is turbulent, and the velocity profile can be modeled using Prandtl's one-seventh power law, $v = V_{max}(1 - r/R)^{1/7}$. If ρ is the density, show that the momentum of the fluid per unit time passing through the pipe is $(49/72)\pi R^2 \rho V_{max}^2$. Then show that $V_{max} = (60/49)V$, where V is the average velocity of the flow. Also, show that the momentum per unit time is $(50/49)\pi R^2 \rho V^2$.



SOLUTION



(a)

The amount of mass per unit time passing through a differential ring element of area dA (shown shaded in Fig. a) on the cross-section is

$$d\dot{m} = \rho V dA$$

Then the momentum per unit time passing through this element is

$$d\dot{L} = (d\dot{m})V = (\rho V dA)V = \rho V^2 dA$$

Thus, for the entire cross-section,

$$\dot{L} = \int_A d\dot{L} = \int_A \rho V^2 dA$$

Here $dA = 2\pi r dr$. Then

$$\dot{L} = \int_0^R \rho\left[V_{max}\left(1 - \frac{r}{R}\right)^{\frac{1}{7}}\right]^2 (2\pi r dr)$$

$$= 2\pi\rho V_{max}^2 \int_0^R r\left(1 - \frac{r}{R}\right)^{\frac{2}{7}} dr$$

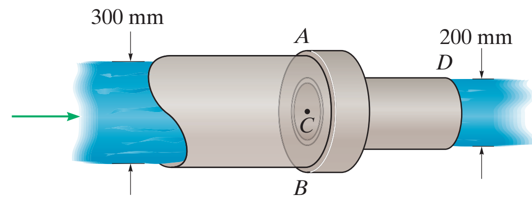
Let $u = 1 - \dfrac{r}{R}$, then $r = R(1 - u)$ and $dr = -Rdu$. Also, the integration limits are $r = 0, u = 1$ and $r = R, u = 0$. Thus,

$$\dot{L} = 2\pi\rho V_{max}^2 \int_1^0 R(1 - u)\left(u^{\frac{2}{7}}\right)(-Rdu)$$

$$= 2\pi R^2 \rho V_{max}^2 \int_1^0 \left(u^{\frac{9}{7}} - u^{\frac{2}{7}}\right) du$$

$$= 2\pi R^2 \rho V_{max}^2 \left(\frac{7}{16} u^{\frac{16}{7}} - \frac{7}{9} u^{\frac{9}{7}}\right)\Bigg|_1^0 = \frac{49}{72}\pi R^2 \rho V_{max}^2 \qquad \textbf{(Q.E.D.)}$$

6–3. Oil flows at $0.05 \text{ m}^3/\text{s}$ through the transition. If the pressure at the transition C is 8 kPa, determine the resultant horizontal shear force acting along the seam AB that holds the cap to the larger pipe. Take $\rho_o = 900 \text{ kg/m}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

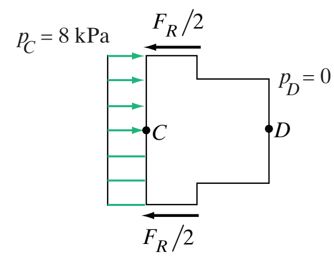
$Q = V_C A_C;$ $\quad 0.05 \text{ m}^3/\text{s} = V_C\left[\pi(0.15 \text{ m})^2\right]$

$V_C = 0.7074 \text{ m/s}$

$Q = V_D A_D;$ $\quad 0.05 \text{ m}^3/\text{s} = V_D\left[\pi(0.1 \text{ m})^2\right]$

$V_D = 1.592 \text{ m/s}$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since D is open to the atmosphere, $p_D = 0$.



(a)

Linear Momentum. Since the flow is steady and incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

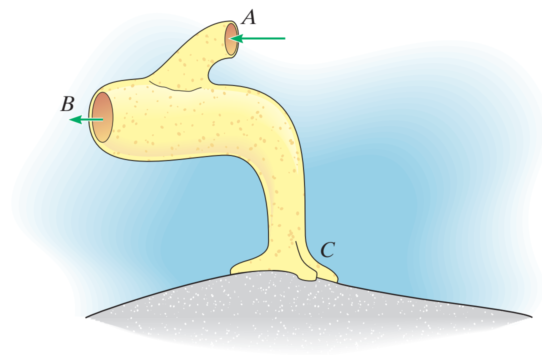
$\overset{+}{\rightarrow} \Sigma F = \rho Q(V_D - V_C);$

$$\left[8\left(10^3\right) \text{ N/m}^2\right]\left[(\pi)(0.15 \text{ m})^2\right] - F_R = \left(900 \text{ kg/m}^3\right)\left(0.05 \text{ m}^3/\text{s}\right)\left(1.592 \text{ m/s} - 0.7074 \text{ m/s}\right)$$

$F = 526 \text{ N}$ **Ans.**

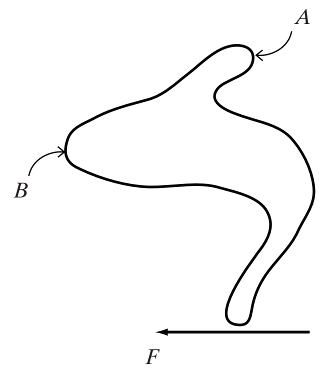
Ans:
526 N

***6–4.** A small marine ascidian called a styela fixes itself on the sea floor and then allows moving water to pass through it in order to feed. If the opening at A has a diameter of 2 mm, and at the exit B the diameter is 1.5 mm, determine the horizontal force needed to keep this organism attached to the rock at C when the water is moving at 0.2 m/s in to the opening at A. Take $\rho = 1050\ \text{kg/m}^3$.



SOLUTION

The flow is steady and the sea water can be considered as an ideal fluid (incompressible and inviscid) such that average velocities can be used and $\rho_{sw} = 1050\ \text{kg/m}^3$. The control volume considered contains the sea water in "styela", Fig. a. Since the depth of points A and B are almost the same, the pressure forces acting on opened control surfaces A and B can be considered the same and an assumed to cancel each other. Continuity requires



(a)

$$\frac{\partial}{\partial t}\int_{cv} \rho_{sw} d\forall + \int_{cs} \rho_{sw} \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(0.2\ \text{m/s})\left[\pi(0.001\ \text{m})^2\right] + V_B\left[\pi(0.00075\ \text{m})^2\right] = 0$$

$$V_B = 0.3556\ \text{m/s}$$

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho_{sw} d\forall + \int_{cs} \mathbf{V}\rho_{sw}\mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation along x axis by referring to the FBD of the control volume, Fig. a

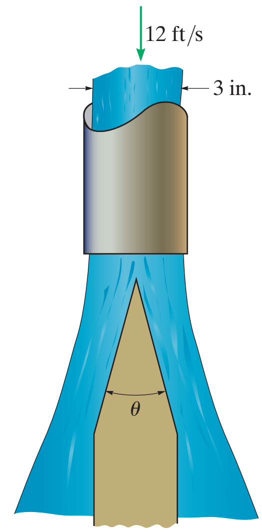
$$(\overset{+}{\rightarrow})\ \Sigma F_x = 0 + (-V_A)\rho_{sw}(-V_A A_A) + (-V_B)\rho_{sw}(V_B A_B)$$

$$-F = (-0.2\ \text{m/s})(1050\ \text{kg/m}^2)\left\{-(0.2\ \text{m/s})\left[\pi(0.001\ \text{m})^2\right]\right\}$$
$$+ (-0.3556\ \text{m/s})(1050\ \text{kg/m}^3)\left\{(0.3556\ \text{m/s})\left[\pi(0.00075\ \text{m})^2\right]\right\}$$

$$F = 0.103(10^{-3})\ \text{N} = 0.103\ \text{mN}$$ **Ans.**

Note: The direction of F implies that if the styela were detached from the rock, it would drift upstream. In reality, it would drift downstream due to forces on its closed surface, which were not considered.

6–5. Water exits the 3-in.-diameter pipe at a velocity of 12 ft/s and is split by the wedge diffuser. Determine the force the flow exerts on the diffuser. Take $\theta = 30°$.



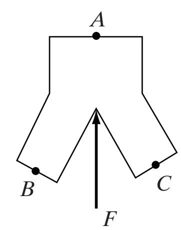
SOLUTION

We consider steady flow of an ideal fluid.

$$Q_A = V_A A_A$$

$$= (12 \text{ ft/s})\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right]$$

$$= 0.5890 \text{ ft}^3/\text{s}$$



(a)

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since this is free flow, $p_A = p_B = p_C = 0$.

Linear Momentum. Since the change in elevation is negligible and the pressure at A, B, and C is zero gauge, $V_A = V_B = V_C = 12$ ft/s (Bernoulli equation). The flow is steady and incompressible.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\,\rho d\forall + \int_{cs} \mathbf{V}\,\rho \mathbf{V}\cdot d\mathbf{A}$$

or

$$+\uparrow \Sigma F_y = \rho Q_B (V_B)_y + \rho Q_C (V_C)_y - \rho Q_A (V_A)_y$$

$$F = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left[Q_B(-12\cos 15° \text{ ft/s}) + Q_C(-12\cos 15° \text{ ft/s}) - (0.5890 \text{ ft}^3/\text{s})(-12 \text{ ft/s})\right]$$

$$= 1.9379\left[7.0686 - 12\cos 15°(Q_B + Q_C)\right]$$

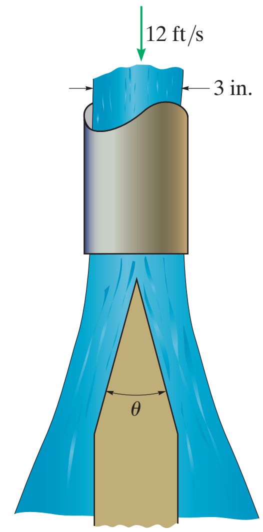
However, $Q_B + Q_C = Q_A = 0.5891 \text{ ft}^3/\text{s}$. Then,

$$F = 1.9379[7.0686 - 12\cos 15°(0.5890)]$$

$$= 0.4668 \text{ lb} = 0.467 \text{ lb}$$ **Ans.**

Ans:
0.467 lb

6–6. Water exits the 3-in.-diameter pipe at a velocity of 12 ft/s, and is split by the wedge diffuser. Determine the force the flow exerts on the diffuser as a function of the diffuser angle θ. Plot this force (vertical axis) versus θ for $0 \le \theta \le 30°$. Give values for increments of $\Delta\theta = 5°$.

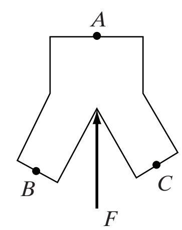


SOLUTION

The discharge is

$$Q_A = V_A A_A = (12 \text{ ft/s})\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right] = 0.1875\pi \text{ ft}^3/\text{s}$$

The free-body diagram of the control volume is shown in Fig. *a*. Since this is a free flow, $p_A = p_B = p_C = 0$. Also, since the change in elevation is negligible, $V_A = V_B = V_C = 12$ ft/s. The flow is steady and incompressible. Thus



(a)

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

The vertical component of this equation gives

$$+\uparrow\Sigma F_y = 0 + [-(V_A)_y]\rho(-V_A A_A) + [-(V_B)_y]\rho(V_B A_B) + [-(V_C)_y]\rho(V_C A_C)$$

$$F = \left(\frac{624 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)[(-12 \text{ ft/s})(-0.1875\pi \text{ ft}^3/\text{s}) + (-12 \cos\theta/2 \text{ ft/s})Q_B + (-12\cos\theta/2 \text{ ft/s})Q_C]$$

$$F = 23.25[0.1875\pi - (Q_B + Q_C)\cos\theta/2]$$

However, continuity requires that $Q_A = Q_B + Q_C$. Then

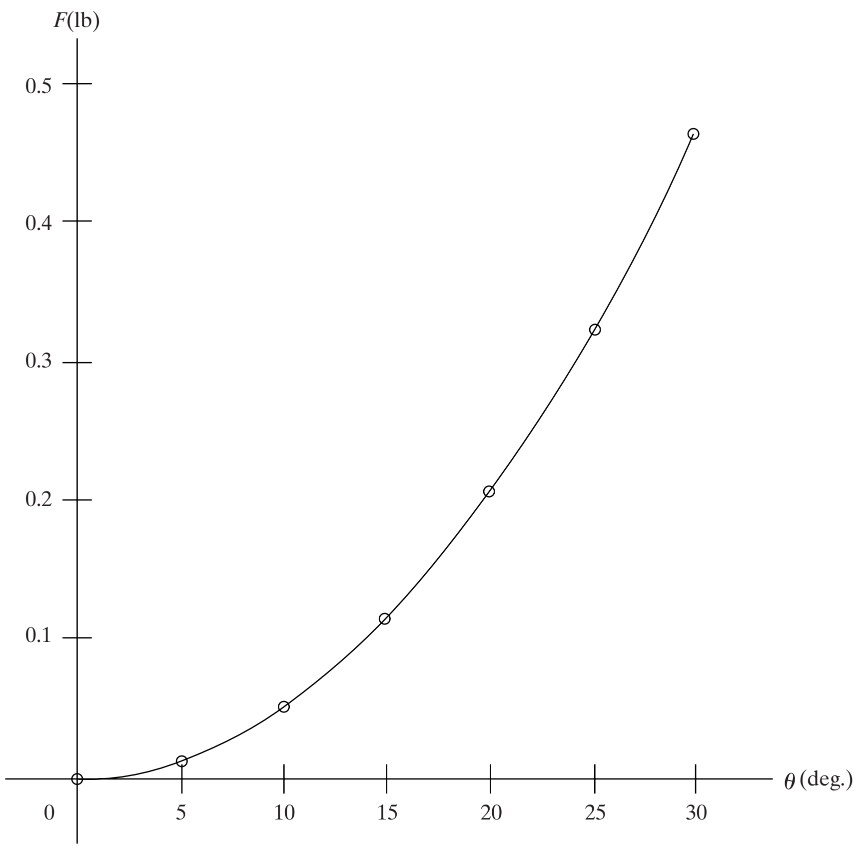
$$F = 23.25(0.1875\pi - 0.1875\pi\cos\theta/2)$$

$$F = [13.7(1 - \cos\theta/2)] \text{ lb} \qquad \textbf{Ans.}$$

The plot of F vs θ is shown in Fig. *b*.

6–6. Continued

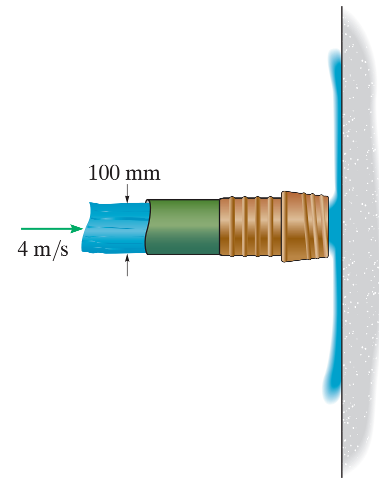
θ(deg.)	0	5	10	15	20	25	30
F(lb)	0	0.0130	0.0521	0.117	0.208	0.325	0.467



(b)

Ans:
$F = [13.7(1 - \cos\theta/2)]$ lb

6–7. Water flows through the hose with a velocity of 4 m/s. Determine the force that the water exerts on the wall. Assume the water does not splash back off the wall.

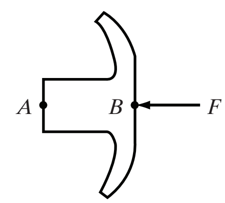


SOLUTION

We consider steady flow of an ideal fluid.

$$Q = VA = (4\text{ m/s})\left[\pi(0.05\text{ m})^2\right] = 0.03142\text{ m}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since the flow is free, $p_A = p_B = 0$.



(a)

Linear Momentum. The horizontal component of flow velocity is zero when the water jet hits the wall, $(V_{\text{out}})_x = 0$. Since the flow is steady and incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d V\!\!\!\!-\; + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

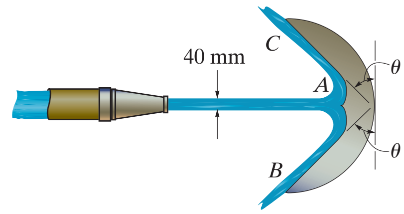
$$\overset{+}{\rightarrow}\Sigma F_x = 0 + (V_A)(\rho)(-Q_A)$$

$$-F = (4\text{ m/s})(1000\text{ kg/m}^3)(-0.03142\text{ m}^3/\text{s})$$

$$F = 126\text{ N}$$ **Ans.**

Ans:
126 N

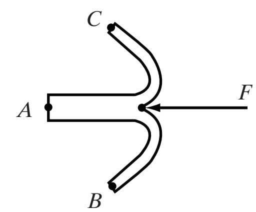
***6–8.** The nozzle has a diameter of 40 mm. If it discharges water with a velocity of 20 m/s against the fixed blade, determine the horizontal force exerted by the water on the blade. The blade divides the water evenly at an angle of $\theta = 45°$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q_A = V_A A_A = (20 \text{ m/s})\left[\pi(0.02 \text{ m})^2\right] = 0.02513 \text{ m}^3/\text{s}$$



(a)

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since this is a free flow, $p_A = p_B = p_C$.

Linear Momentum. Since the change in elevation is negligible and the pressure at A, B, and C is zero gauge, $V_A = V_B = V_C = 20$ m/s (Bernoulli equation). Since the flow is steady and incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

or

$$\overset{+}{\leftarrow} \Sigma F_x = (-V_B)_x \rho Q_B - (V_C)_x \rho Q_C + (V_A)_x \rho(-Q_A)$$

$$-F = (1000 \text{ kg/m}^3)\left[Q_B(-20 \text{ m/s})(\cos 45°) + Q_C(-20 \text{ m/s})(\cos 45°) - (20 \text{ m/s})(0.02513 \text{ m}^3/\text{s})\right]$$

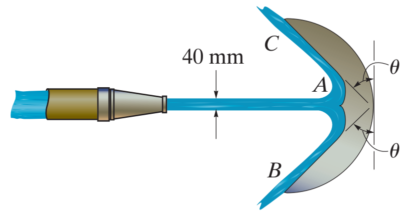
$$F = 1000\left[(Q_B + Q_C)(20 \cos 45°) + 0.5027\right]$$

However, $Q_B + Q_C = Q_A = 0.02513 \text{ m}^3/\text{s}$. Then

$$F = 1000[0.02513(20 \cos 45°) + 0.5027]$$

$$= 858.09 \text{ N} = 858 \text{ N} \qquad \textbf{Ans.}$$

6–9. The nozzle has a diameter of 40 mm. If it discharges water with a velocity of 20 m/s against the fixed blade, determine the horizontal force exerted by the water on the blade as a function of the blade angle θ. Plot this force (vertical axis) versus θ for $0 \leq \theta \leq 75°$. Give values for increments of $\Delta\theta = 15°$. The blade divides the water evenly.

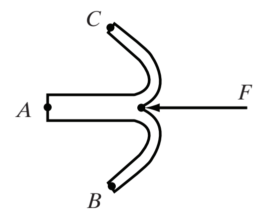


SOLUTION

The discharge is

$$Q = V_A A_A = (20\text{ m/s})\left[\pi(0.02\text{ m})^2\right] = 0.008\pi\text{ m}^3/\text{s}$$

The free-body diagram of the control volume is shown in Fig. *a*. Since this is a free flow, $p_A = p_B = p_C = 0$. Also, since the change in elevation is negligible, $V_A = V_B = V_C = 20\text{ m/s}$ (Bernoulli's equation). The flow is steady and incompressible. Thus



(a)

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

The horizontal component of this equation gives

$$\overset{+}{\leftarrow} \Sigma F_x = 0 + \left[-(V_A)_x\right]\rho(-V_A A_A) + (V_B)_x\rho(V_B A_B) + (V_C)_x\rho(V_C A_C)$$

$$F = \left(1000\text{ kg/m}^3\right)\left[(20\text{ m/s})\left(0.008\pi\text{ m}^3/\text{s}\right) + (20\sin\theta\text{ m/s})Q_B + (20\sin\theta\text{ m/s})Q_C\right]$$

$$F = 20\left(10^3\right)\left[0.008\pi + (Q_B + Q_C)\sin\theta\right]$$

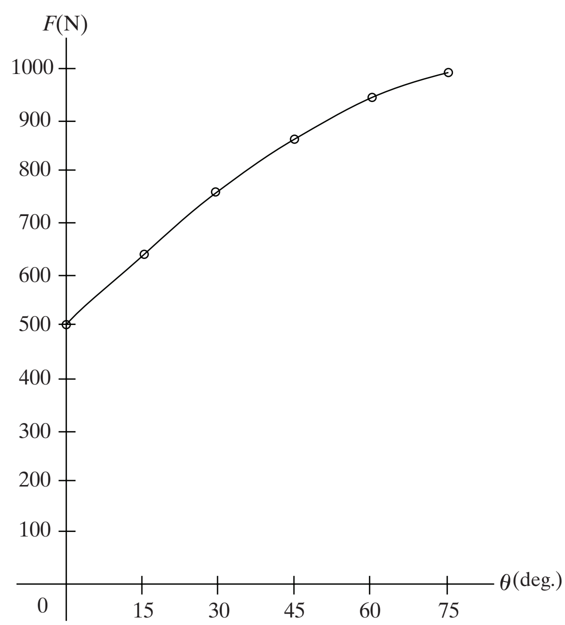
However continuity requires that $Q_A = Q_B + Q_C$. Then,

$$F = 20\left(10^3\right)[0.008\pi + (0.008\pi)\sin\theta]$$

$$F = [160\pi(1 + \sin\theta)]\text{ N where } \theta \text{ is in deg.}$$ **Ans.**

The plot of F vs θ is shown in Fig. *b*.

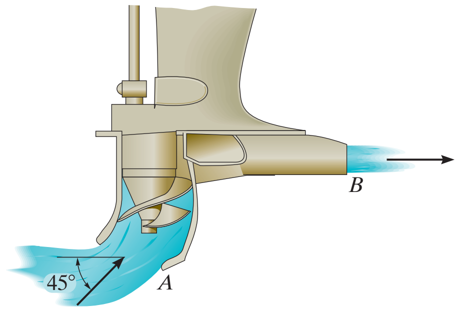
θ(deg.)	0	15	30	45	60	75
F(N)	503	633	754	858	938	988



(b)

Ans:
$F = \left[160\pi(1 + \sin\theta)\right]$ N

6–10. A speedboat is powered by the jet drive shown. Seawater is drawn into the pump housing at the rate of $20\ \text{ft}^3/\text{s}$ through a 6-in.-diameter intake A. An impeller accelerates the water and forces it out horizontally through a 4-in.-diameter nozzle B. Determine the horizontal and vertical components of thrust exerted on the speedboat. The specific weight of seawater is $\gamma_{sw} = 64.3\ \text{lb/ft}^3$.

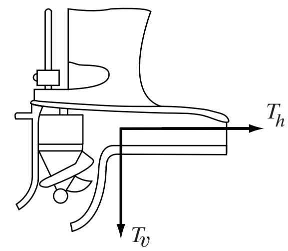


SOLUTION

Consider the control volume to be the jet drive and the water it contains, Fig. *a*. From the discharge

$$Q = V_A A_A; \qquad 20\ \text{ft}^3/\text{s} = V_A\left[\pi\left(\frac{3}{12}\ \text{ft}\right)^2\right] \qquad V_A = 101.86\ \text{ft/s}$$

$$Q = V_B A_B; \qquad 20\ \text{ft}^3/\text{s} = V_B\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right] \qquad V_B = 229.18\ \text{ft/s}$$



(a)

Here the flow is steady. Applying the Linear Momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

Writing the horizontal and vertical scalar components of this equation by referring to the *FBD* of the control volume, Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (V_A \cos 45°)\rho(-V_A A_A) + V_B \rho(V_B A_B)$$

$$T_h = \left[(101.86\ \text{ft/s})\cos 45°\right]\left(\frac{64.3\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(-20\ \text{ft}^3/\text{s}) + (229.18\ \text{ft/s})\left(\frac{64.3\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(20\ \text{ft}^3/\text{s})$$

$$= 6276.55\ \text{lb} = 6.28\ \text{kip} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + (V_A \sin 45°)\rho(-V_A A_A)$$

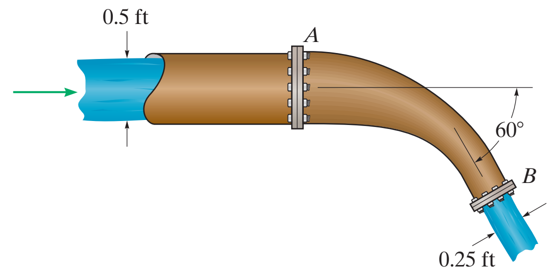
$$-T_v = \left[(101.86\ \text{ft/s})\sin 45°\right]\left(\frac{64.3\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(-20\ \text{ft}^3/\text{s})$$

$$T_v = 2876.54\ \text{lb} = 2.88\ \text{kip} \qquad \textbf{Ans.}$$

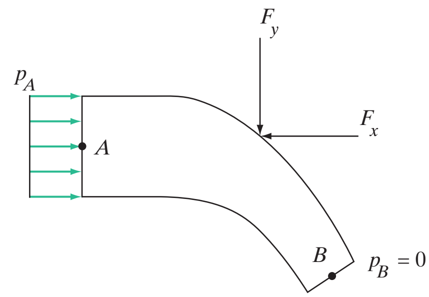
The thrust components on the speedboat are equal and opposite to those exerted on the water.

Ans:
$T_h = 6.28$ kip
$T_v = 2.88$ kip

6–11. Water flows out of the reducing elbow at $0.4 \text{ ft}^3/\text{s}$. Determine the horizontal and vertical components of force that are necessary to hold the elbow in place at A. Neglect the size and weight of the elbow and the water within it. The water is discharged to the atmosphere at B.



SOLUTION



(a)

$$Q = V_A A_A; \qquad 0.4 \text{ ft}^3/\text{s} = V_A\left[\pi(0.25 \text{ ft})^2\right]$$

$$V_A = 2.0372 \text{ ft/s}$$

Continuity equation

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-0.4 \text{ ft}^3/\text{s} + V_B(\pi)(0.125 \text{ ft})^2 = 0$$

$$V_B = 8.149 \text{ ft/s}$$

Bernoulli equation. Neglecting elevation change

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{\left(\dfrac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)} + \frac{(2.037 \text{ ft/s})^2}{2} + 0 = 0 + \frac{(8.149 \text{ ft/s})^2}{2} + 0$$

$$p_A = 60.3234 \text{ lb/ft}^2$$

The free-body diagram is shown in Fig. *a*.
Linear Momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + \rho Q\left(V_{B_x} - V_{A_x}\right)$$

$$-F_x + \left(60.3234 \text{ lb/ft}^2\right)\left[(\pi)(0.25 \text{ ft})^2\right] = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left(0.4 \text{ ft}^3/\text{s}\right)\left[8.149 \text{ ft/s}(\cos 60^\circ) - 2.0372 \text{ ft/s}\right]$$

$$F_x = 10.3 \text{ lb} \qquad \textbf{Ans.}$$

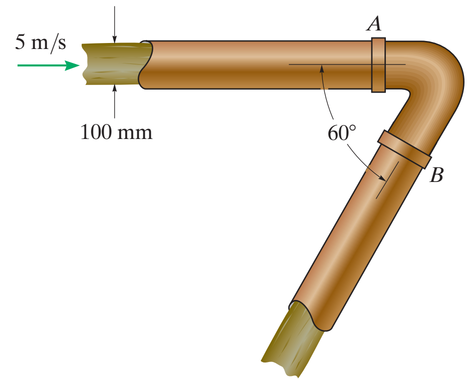
$$+\uparrow\Sigma F_y = \rho Q\left[-V_{B_y} + 0\right]$$

$$-F_y = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left(0.4 \text{ ft}^3/\text{s}\right)\left(-8.149 \text{ ft/s}\right)(\sin 60^\circ)$$

$$F_y = 5.47 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$F_x = 10.3$ lb
$F_y = 5.47$ lb

***6–12.** Oil flows through the 100-mm-diameter pipe with a velocity of 5 m/s. If the pressure in the pipe at A and B is 80 kPa, determine the x and y components of force the flow exerts on the elbow. The flow occurs in the horizontal plane. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = VA = (5\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right]$$
$$= 0.03927\ \text{m}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Here, $p_A = p_B = 80\ \text{kPa}$.

Linear Momentum. Since the flow is steady incompressible

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\mathcal{V} + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

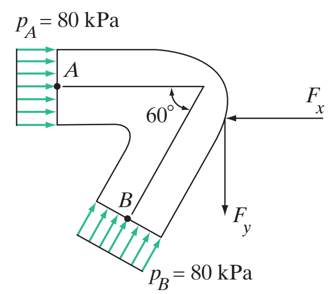
or

$$\xrightarrow{+} \Sigma F_x = 0 + (V_A)_x\rho(-Q) + (V_B)_x\rho Q$$

$$-F_x + \left[80(10^3)\ \text{N/m}^2\right]\left[\pi(0.05\ \text{m})^2\right] + \left[80(10^3)\ \text{N/m}^2\right]\left[\pi(0.05\ \text{m})^2\right]\cos 60°$$
$$= (900\ \text{kg/m}^3)(0.03927\ \text{m}^3/\text{s})(-5\ \text{m/s}\cos 60° - 5\ \text{m/s})$$
$$F_x = 1207.55 = 1.21\ \text{kN} \qquad \textbf{Ans.}$$

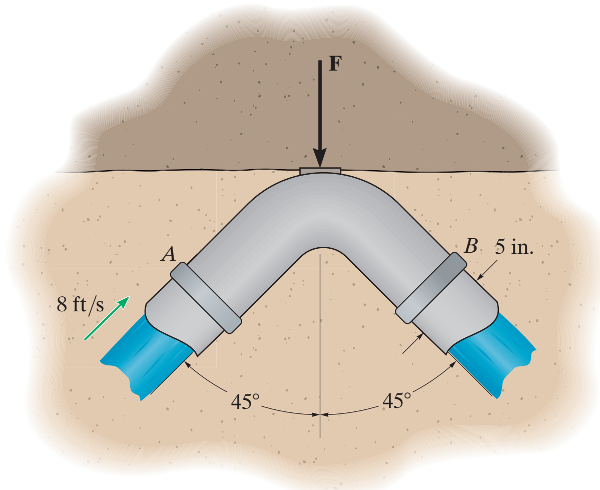
$$+\uparrow \Sigma F_y = 0 + 0 + (V_B)_y\rho(-Q)$$

$$-F_y + \left[80(10^3)\ \text{N/m}^2\right]\left[\pi(0.05\ \text{m})^2\right]\sin 60° = (5\ \text{m/s}\sin 60°)(900\ \text{kg/m}^3)(-0.03927\ \text{m}^3/\text{s})$$
$$F_y = 697\ \text{N} \qquad \textbf{Ans.}$$

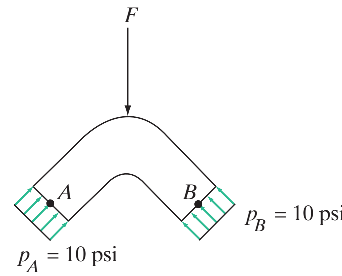


(a)

6–13. The speed of water passing through the elbow on a buried pipe is $V = 8\text{ ft/s}$. Assuming that the pipe connections at A and B do not offer any force resistance on the elbow, determine the resultant horizontal force $\mathbf{F}$ that the soil must exert on the elbow in order to hold it in equilibrium. The pressure within the pipe at A and B is 10 psi.



SOLUTION



(a)

We consider steady flow of an ideal fluid.

$$Q = VA = (8\text{ ft/s})\left[\pi\left(\frac{2.5}{12}\text{ ft}\right)^2\right]$$

$$= 1.091\text{ ft}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*.

Linear Momentum. Since the flow is steady and incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d V\!\!\!\!/ + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

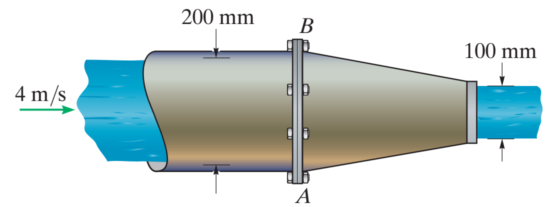
$$+\uparrow\Sigma F_y = 0 + (V_A)_y(\rho)(-Q) + (-V_B)_y\rho Q$$

$$2\left[(10\text{ lb/in}^2)\left(\frac{12\text{ in}}{1\text{ ft}}\right)^2\cos 45°\left[\pi\left(\frac{2.5}{12}\text{ ft}\right)^2\right]\right] - F = \left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)(1.091\text{ ft}^3/\text{s})\left[-8\text{ ft/s}\cos 45° - 8\text{ ft/s}\cos 45°\right]$$

$F = 301.60\text{ lb} = 302\text{ lb}$ **Ans.**

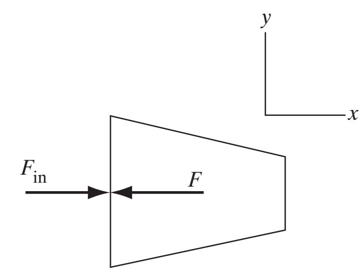
Ans:
302 lb

6–14. Water flows through the 200-mm-diameter pipe at 4 m/s. If it exits into the atmosphere through the nozzle, determine the resultant force the bolts must develop at the connection AB to hold the nozzle onto the pipe.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$ average velocities will be used. The control volume contains the water in the nozzle as shown in Fig. *a*. Continuity requires



(a)

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_{in}A_{in} + V_{out}A_{out} = 0$$

$$-(4 \text{ m/s})\left[\pi(0.1 \text{ m})^2\right] + V_{out}\left[\pi(0.05 \text{ m})^2\right] = 0$$

$$V_{out} = 16 \text{ m/s}$$

Applying the Bernoulli's equation between two points on the control streamline with $p_{out} = p_{atm} = 0$,

$$\frac{p_{in}}{\gamma_w} + \frac{V_{in}^2}{2g} + z_{in} = \frac{p_{out}}{\gamma_w} + \frac{V_{out}^2}{2g} + z_{out}$$

$$\frac{p_{in}}{9810 \text{ N/m}^3} + \frac{(4 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 = 0 + \frac{(16 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$p_{in} = 120(10^3) \text{ N/m}^2$$

Thus, the pressure force acting on the inlet control surface on the *FBD* of the control volume is

$$F_{in} = p_{in}A_{in} = \left[120(10^3) \text{ N/m}^2\right]\left[\pi(0.1 \text{ m})^2\right] = 3769.91 \text{ N}$$

Applying the linear momentum equation,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho_w \mathbf{V} \cdot d\mathbf{A}$$

Write the scalar component of this equation along x axis, referring to Fig. *a*

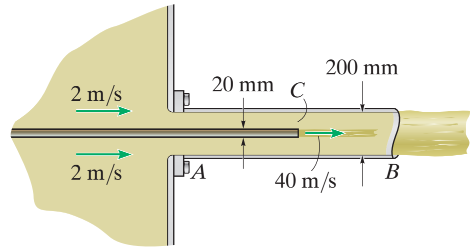
$$(\xrightarrow{+})\ \Sigma F_x = 0 + V_{out}\rho_w(V_{out}A_{out}) + V_{in}\rho_w(-V_{in}A_{in})$$

$$3769.91 \text{ N} - F = (16 \text{ m/s})(1000 \text{ kg/m}^3)(16 \text{ m/s})\left[\pi(0.05 \text{ m})^2\right] + (4 \text{ m/s})(1000 \text{ kg/m}^3)(-4 \text{ m/s})\left[\pi(0.1 \text{ m})^2\right]$$

$$F = 2261.95 \text{ N} = 2.26 \text{ kN}$$ **Ans.**

Ans:
2.26 kN

6–15. The apparatus or "jet pump" used in an industrial plant is constructed by placing the tube within the pipe. Determine the increase in pressure $P_B - P_A$ that occurs between the back A and front B of the pipe if the velocity of the flow within the 200-mm-diameter pipe is 2 m/s, and the velocity of the flow through the 20-mm-diameter tube is 40 m/s. The fluid is ethyl alcohol having a density of $\rho_{ea} = 790\ \text{kg/m}^3$. Assume the pressure at each cross section of the pipe is uniform.



SOLUTION

The flow is steady and the ethyl alcohol can be considered an ideal fluid (incompressible and inviscid) Such that $\rho_{\text{ea}} = 790\ \text{kg/m}^3$. Average velocities will be used. The control volume considered is shown in Fig. *a*. Continuity requires

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - (V_A)_t(A_A)_t - (V_A)_p(A_A)_p + (V_B)_p(A_B)_p = 0$$

$$-(40\ \text{m/s})\left[\pi(0.01\ \text{m})^2\right] - (2\ \text{m/s})\left\{\pi\left[(0.1\ \text{m})^2 - (0.01\ \text{m})^2\right]\right\} + (V_B)_p\left[\pi(0.1\ \text{m})^2\right] = 0$$

$$(V_B)_p = 2.38\ \text{m/s}$$

Within the tube, $z_C = z_A$ and $V_C = V_A$, so by Bernoulli's equation, $p_C = p_A$. Furthermore, because p_C at the tube exit equals p_C in the surrounding pipe flow, which again by Bernoulli's equation equals p_A in the pipe, it follows that p_A is the same inside and outside the tube.

The pressure forces on the inlet and outlet control surfaces are

$$F_A = p_A A_A = p_A\left[\pi(0.1\ \text{m})^2\right] = 0.01\ \pi p_A$$

$$F_B = p_B A_B = p_A\left[\pi(0.1\ \text{m})^2\right] = 0.01\ \pi p_B$$

Applying the linear momentum equation,

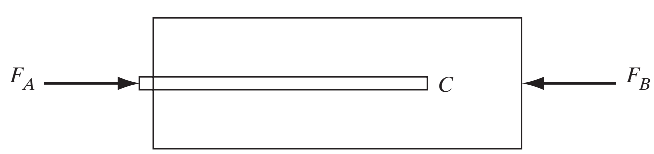
$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho_{\text{ea}} d\forall + \int_{\text{cs}} \mathbf{V}\rho_{\text{ea}}\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along the x axis by referring to Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + (V_B)_p\rho_{\text{ea}}(V_B)_p(A_B)_p + (V_A)_t\rho_{\text{ea}}\left[-(V_A)_t(A_A)_t\right] + (V_A)_p\rho_{\text{ea}}\left[-(V_A)_p(A_A)_p\right]$$

$$0.01\pi p_A - 0.01\pi p_B = (2.38\ \text{m/s})^2(790\ \text{kg/m}^3)\left[\pi(0.1\ \text{m})^2\right] - (40\ \text{m/s})^2(790\ \text{kg/m}^3)\left[\pi(0.01\ \text{m})^2\right]$$
$$-(2\ \text{m/s})^2(790\ \text{kg/m}^3)\left\{\pi\left[(0.1\ \text{m})^2 - (0.01\ \text{m})^2\right]\right\}$$

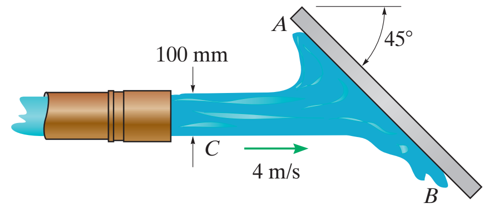
$$\Delta P = p_B - p_A = 11.29(10^3)p_a = 11.3\ \text{kPa} \qquad \textbf{Ans.}$$



(a)

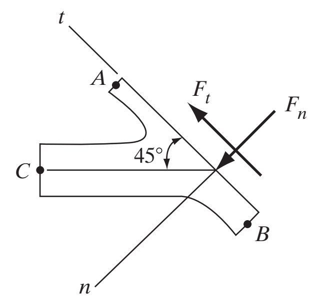
Ans:
11.3 kPa

***6–16.** The jet of water flows from the 100-mm-diameter pipe at $4\ \text{m/s}$. If it strikes the fixed vane and is deflected as shown, determine the normal force the jet exerts on the vane.



SOLUTION

We consider steady flow of an ideal fluid.



(a)

Bernoulli Equation. Since the water jet is a free flow, $p_A = p_B = p_C = 0$. Also, if we neglect the elevation change in the water jet, the Bernoulli equation gives

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g}$$

$$0 + \frac{V_A^2}{2g} = 0 + \frac{V_B^2}{2g} = 0 + \frac{(4\ \text{m/s})^2}{2g}$$

$$V_A = V_B = 4\ \text{m/s} \qquad \textbf{Ans.}$$

The discharge at C is

$$Q_C = V_C A_C = (4\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right] = 0.03142\ \text{m}^3/\text{s}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Since the flow is steady incompressible.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

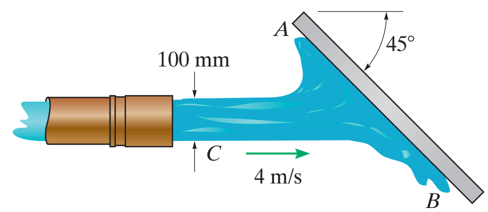
or

$$\Sigma F_n = 0 + (-Q_C)(\rho)(-V_C)_n$$

$$F_n = (-0.03142\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3)(-4\ \text{m/s}\sin 45°)$$

$$F_n = 88.9\ \text{N} \qquad \textbf{Ans.}$$

6–17. The jet of water flows from the 100-mm-diameter pipe at 4 m/s. If it strikes the fixed vane and is deflected as shown, determine the volume flow towards A and towards B if the tangential component of the force that the water exerts on the vane is zero.



SOLUTION

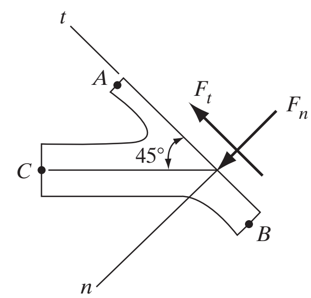
We consider steady flow of an ideal fluid.

Bernoulli Equation. Since the water jet is a free flow, $p_A = p_B = p_C = 0$. Also, if we neglect the elevation change in the water jet, the Bernoulli equation gives

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g}$$

$$0 + \frac{V_A^2}{2g} = 0 + \frac{V_B^2}{2g} = 0 + \frac{(4\text{ m/s})^2}{2g}$$

$$V_A = V_B = 4\text{ m/s}$$



(a)

The discharge at C is

$$Q_C = V_C A_C = (4\text{ m/s})\left[\pi(0.05\text{ m})^2\right] = 0.03142\text{ m}^3/\text{s}$$

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - Q_C + Q_A + Q_B = 0 \qquad Q_A + Q_B = 0.03142 \qquad \textbf{(1)}$$

Control Volume. The free-body diagram of the control volume is shown in Fig. *a*. Here, it is required that $F_t = 0$. Since the flow is steady incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$\overset{+}{\nwarrow}\ \Sigma F_t = \rho\left[Q_A(V_A)_t + Q_B(V_B)_t - Q_C(V_C)_t\right];$$

$$0 = (1000\text{ kg/m}^3)\left[Q_A(4\text{ m/s}) + Q_B(-4\text{ m/s}) - 0.03142\text{ m}^3/\text{s}\,(-4\text{ m/s}\cos 45^\circ)\right]$$

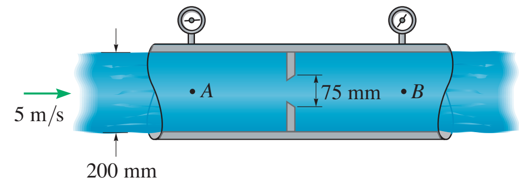
$$Q_A - Q_B = -0.02221 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

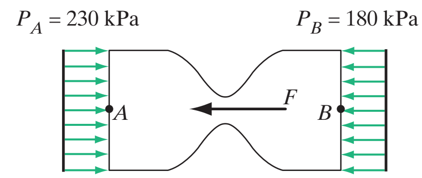
$$Q_A = 0.00460\text{ m}^3/\text{s} \qquad Q_B = 0.0268\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$Q_A = 0.00460\text{ m}^3/\text{s}$
$Q_B = 0.0268\text{ m}^3/\text{s}$

6–18. As water flows through the pipe at a velocity of 5 m/s, it encounters the orifice plate, which has a hole in its center. If the pressure at A is 230 kPa, and at B it is 180 kPa, determine the force the water exerts on the plate.



SOLUTION



(a)

We consider steady flow of an ideal fluid.

Take the water from A to B to be the control volume.

Continuity Equation. Since the diameters of the pipe at A and B are equal, continuity requires

$$V_A = V_B = 5 \text{ m/s}$$

The free-body diagram of the control volume is shown in Fig. *a*.

Linear Momentum. The flow is steady and incompressible since points A and B are selected at a sufficient distance from the gate.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\mathcal{V} + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

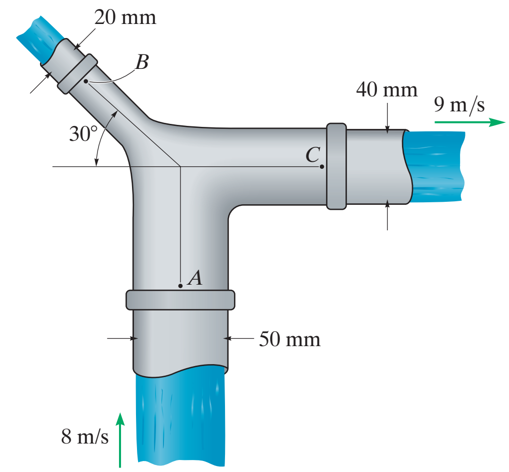
$$\overset{+}{\rightarrow}\Sigma F_x = 0 + (V_A)\rho(-Q) + (V_B)\rho(Q)$$

$$-F + \left[230(10^3)\text{ N/m}^2\right]\left[\pi(0.1\text{ m})^2\right] - \left[180(10^3)\text{ N/m}^2\right]\left[\pi(0.1\text{ m})^2\right] = \rho Q(V - V) = 0$$

$$F = 1570.80\text{ N} = 1.57\text{ kN}$$ **Ans.**

Ans:
1.57 kN

6–19. Water enters A with a velocity of 8 m/s and pressure of 70 kPa. If the velocity at C is 9 m/s, determine the horizontal and vertical components of the resultant force that must act on the transition to hold it in place. Neglect the size of the transition.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000\ \text{kg/m}^3$. The average velocities will be used. The control volume contains the water in the transition as shown in Fig. *a*. The continuity condition requires

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B + V_C A_C = 0$$

$$-(8\ \text{m/s})\left[\pi(0.025\ \text{m})^2\right] + V_B\left[\pi(0.01\ \text{m})^2\right] + (9\ \text{m/s})\left[\pi(0.02\ \text{m})^2\right] = 0$$

$$V_B = 14\ \text{m/s}$$

Write the Bernoulli's equation between A and B, and A and C,

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{70(10^3)\ \text{N/m}^2}{9810\ \text{N/m}^3} + \frac{(8\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = \frac{p_B}{9810\ \text{N/m}^3} + \frac{(14\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_B = 4(10^3)\ \text{N/m}^2$$

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C$$

$$\frac{70(10^3)\ \text{N/m}^2}{9810\ \text{N/m}^3} + \frac{(8\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = \frac{p_C}{9810\ \text{N/m}^3} + \frac{(9\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_C = 61.5(10^3)\ \text{N/m}^3$$

The pressure forces acting on the inlet and outlet control surfaces indicated on the *FBD* of the control volume are

$$F_A = p_A A_A = \left[70(10^3)\ \text{N/m}^2\right]\left[\pi(0.025\ \text{m})^2\right] = 43.75\pi\ \text{N}$$

$$F_B = p_B A_B = \left[4(10^3)\ \text{N/m}^2\right]\left[\pi(0.01\ \text{m})^2\right] = 0.4\pi\ \text{N}$$

$$F_C = p_C A_C = \left[61.5(10^3)\ \text{N/m}^2\right]\left[\pi(0.02\ \text{m})^2\right] = 24.6\pi\ \text{N}$$

6–19. Continued

Applying the Linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along x and y axes,

$$(\overset{+}{\rightarrow})\ \Sigma F_x = 0 + (-V_B \cos 30^\circ)(\rho_w)(V_B A_B) + V_C \rho_w (V_C A_C)$$

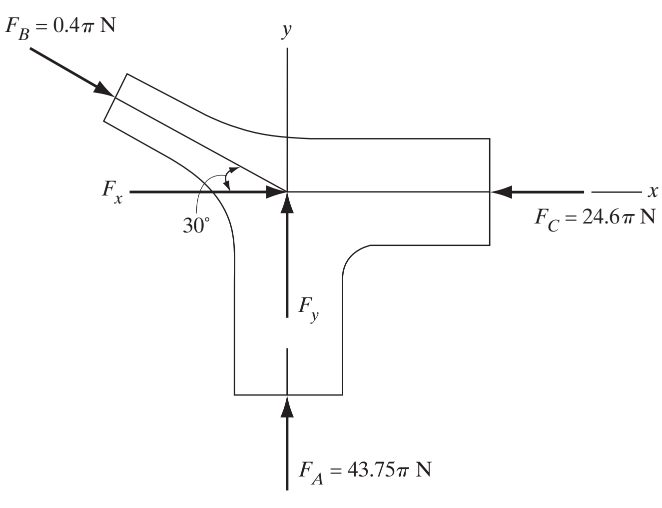
$$F_x + (0.4\pi \text{ N}) \cos 30^\circ - 24.6\pi \text{ N} = -(14 \text{ m/s})(\cos 30^\circ)(1000 \text{ kg/m}^3)(14 \text{ m/s})[\pi(0.01 \text{ m})\ \ ] + (9 \text{ m/s})(1000 \text{ kg/m}^3)(9 \text{ m/s})[\pi(0.02 \text{ m})^2]$$

$$F_x = 125 \text{ N} \rightarrow \qquad \textbf{Ans.}$$

$$(+\uparrow)\ \Sigma F_y = 0 + (V_B \sin 30^\circ)(\rho_w)(V_B A_B) + V_A \rho_w (-V_A A_A)$$

$$F_y + 43.75\pi \text{ N} - (0.4\pi \text{ N}) \sin 30^\circ = (14 \text{ m/s})(\sin 30^\circ)(1000 \text{ kg/m}^3)(14 \text{ m/s})[\pi(0.01 \text{ m})^2] + (8 \text{ m/s})(1000 \text{ kg/m}^3)(-8 \text{ m/s})[\pi(0.025 \text{ m})^2]$$

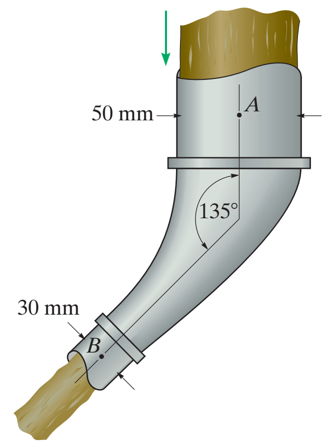
$$F_y = +231.69 \text{ N} = 232 \text{ N}\downarrow \qquad \textbf{Ans.}$$



(a)

Ans:
$F_x = 125$ N
$F_y = 232$ N

***6–20.** Crude oil flows through the horizontal tapered 45° elbow at 0.02 m^3/s. If the pressure at A is 300 kPa, determine the horizontal and vertical components of the resultant force the oil exerts on the elbow. Neglect the size of the elbow.



SOLUTION

The flow is steady and crude oil can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_{co} = 880\ \text{kg/m}^3$ average velocities will be used. The control volume considered contains the crude oil in the elbow as shown in Fig. *a*. From the discharge,

$$Q = V_A A_A; \quad 0.02\ \text{m}^3/\text{s} = V_A\left[\pi(0.025\ \text{m})^2\right] \quad V_A = 10.19\ \text{m/s}$$

$$Q = V_B A_B; \quad 0.02\ \text{m}^3/\text{s} = V_B\left[\pi(0.015\ \text{m})^2\right] \quad V_B = 28.29\ \text{m/s}$$

Applying Bernoulli's equation between A and B,

$$\frac{p_A}{\gamma_{co}} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_{co}} + \frac{V_B^2}{2g} + z_B$$

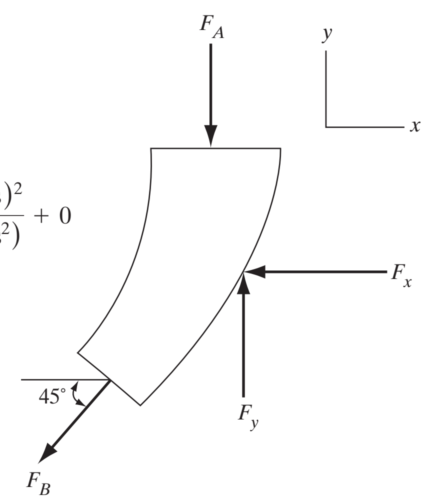
$$\frac{300(10^3)\ \text{N/m}^2}{(880\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(10.19\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = \frac{p_B}{(880\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(28.29\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_B = -6.596(10^3)\ \text{Pa}$$

The negative sign indicates that suction occurs at B. The pressure for acting on the inlet and outlet control surfaces indicated on the *FBD* of the control volume are

$$F_A = p_A A_A = \left[300(10^3)\ \text{N/m}^2\right]\left[\pi(0.025\ \text{m})^2\right] = 589.05\ \text{N}$$

$$F_B = p_B A_B = \left[6.596(10^3)\ \text{N/m}^2\right]\left[\pi(0.015\ \text{m})^2\right] = 4.663\ \text{N}$$



(a)

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho_{co}\,d\forall + \int_{cs} \mathbf{V}\rho_{co}\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along x and y axis by referring to Fig. *a*

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + (-V_B\cos 45°)(\rho_{co})(V_B A_B)$$

$$(-4.663\ \text{N})\cos 45° - F_x = (-28.29\ \text{m/s})\cos 45°(880\ \text{kg/m}^3)(0.02\ \text{m}^3/\text{s})$$

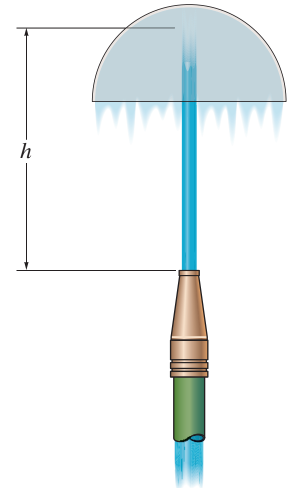
$$F_x = 349\ \text{N} \leftarrow \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0 + (-V_B\sin 45°)\rho_{co}(V_B A_B) + (-V_A)\rho_{co}(-V_A A_A)$$

$$F_y - (4.663\ \text{N})\sin 45° - 589.05\ \text{N} = (-28.29\ \text{m/s})\sin 45°(880\ \text{kg/m}^3)(0.02\ \text{m}^3/\text{s}) + (-10.19\ \text{m/s})(880\ \text{kg/m}^3)(-0.02\ \text{m}^3/\text{s})$$

$$F_y = 419\ \text{N}\uparrow$$

6–21. The hemispherical bowl of mass m is held in equilibrium by the vertical jet of water discharged through a nozzle of diameter d. If the volumetric flow is Q, determine the height h at which the bowl is suspended. The water density is ρ_w.



SOLUTION

The flow is steady and water can be considered as an ideal fluid (incompressible and inviscid) such that its density is constant. Average velocities will be used. From the discharge the velocity of the water leaving the nozzle (point A on the control volume shown in Fig. a) is

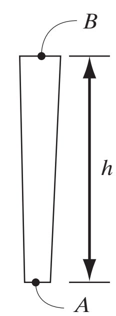
$$Q = V_A A_A; \quad Q = V_A\left(\frac{\pi}{4}d^2\right) \quad V_A = \frac{4Q}{\pi d^2}$$

Applying Bernoulli's equation between points A and B on the central streamline with $p_A = p_B = 0$, $z_A = 0$ and $z_B = h$,

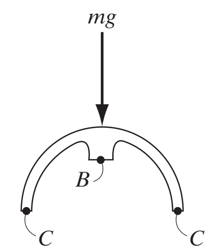
$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{\left(\frac{4Q}{\pi d^2}\right)^2}{2g} + 0 = 0 + \frac{V_B^2}{2g} + h$$

$$V_B = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh} \qquad \textbf{(1)}$$



(a)



(b)

By considering the FBD of the control volume shown in Fig. b, where B and C are the inlet and outlet control surfaces,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho_w d\forall + \int_{cs} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along the y axis realizing that by Bernoulli's equation $V_C = V_B = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$ and $Q = VA$,

$$+\uparrow \Sigma F_y = 0 + V_B\rho_w(-V_B A_B) + (-V_C)\rho_w(V_C A_C)$$

$$-mg = V_B\rho_w(-Q) - V_B\rho_w Q$$

$$mg = 2\rho_w Q V_B$$

Substituting Eg. 1 into this equation

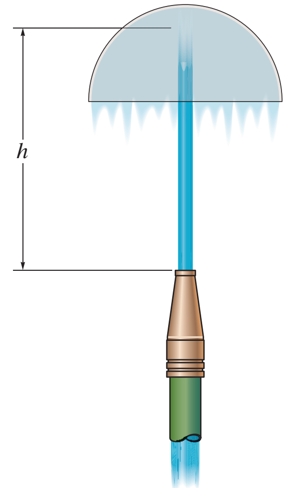
$$mg = 2\rho_w Q\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$$

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w{}^2 Q^2} \qquad \textbf{Ans.}$$

Ans:

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w^2 Q^2}$$

6–22. The 500-g hemispherical bowl is held in equilibrium by the vertical jet of water discharged through the 10-mm-diameter nozzle. Determine the height h of the bowl as a function of the volumetric flow Q of the water through the nozzle. Plot the height h (vertical axis) versus Q for $0.5\left(10^{-3}\right)\ \text{m}^3/\text{s} \le Q \le 1\left(10^{-3}\right)\ \text{m}^3/\text{s}$. Give values for increments of $\Delta Q = 0.1\left(10^{-3}\right)\ \text{m}^3/\text{s}$.

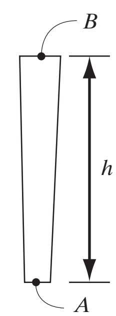


SOLUTION

The flow is steady and water can be considered as an ideal fluid (incompressible and inviscid) such that its density is constant. Average velocities will be used. From the discharge, the velocity of the water leaving the nozzle (point A on the control volume as shown in Fig. a) is

$$Q = V_A A_A; \quad Q = V_A\left[\pi(0.005\ \text{m})^2\right]$$

$$V_A = \left[\frac{40\left(10^3\right)}{\pi}Q\right]\ \text{m/s}$$



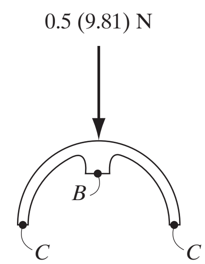
(a)

Applying Bernoulli's equation between points A and B on the central streamline with $p_A = p_B = 0$, $z_A = 0$ and $z_B = h$,

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{\left[\frac{40\left(10^3\right)}{\pi}Q\right]^2}{2\left(9.81\ \text{m/s}^2\right)} + 0 = 0 + \frac{V_B^2}{2\left(9.81\ \text{m/s}^2\right)} + h$$

$$V_B = \sqrt{\frac{1.6\left(10^9\right)}{\pi^2}Q^2 - 19.62\,h} \qquad \textbf{(1)}$$



(b)

By considering the FBD of the fixed control volume shown in Fig. b, where B and C are the inlet and outlet control surfaces,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho_w d\forall + \int_{\text{cs}} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of the equation along y axis realizing that $V_c = V_B = \sqrt{\frac{1.6\left(10^9\right)}{\pi^2}Q^2 - 19.62\,h}$ and $Q = VA$,

$$+\uparrow \Sigma F_y = 0 + V_B\rho_w\left(-V_B A_B\right) + \left(-V_C\right)\rho_w\left(V_C A_C\right)$$

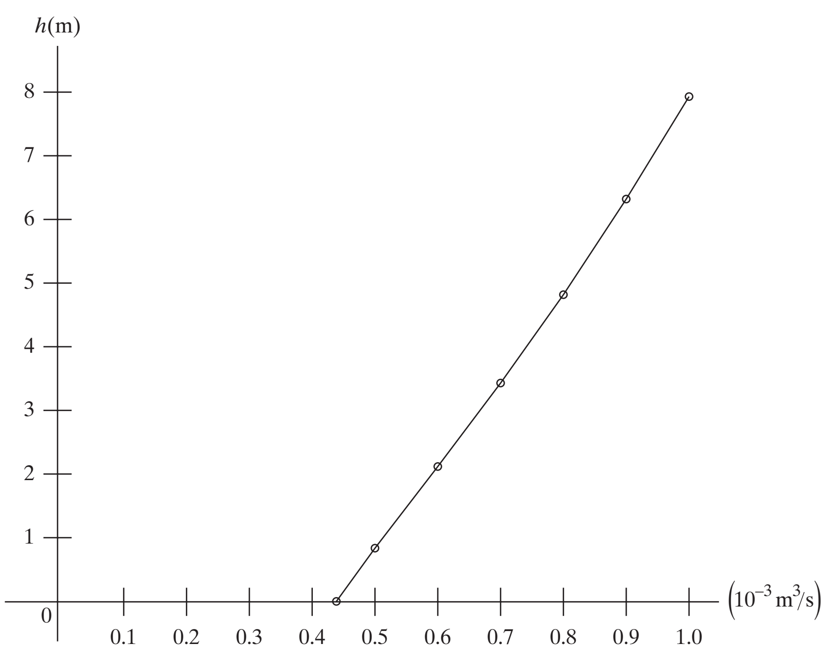
6–22. Continued

$$-0.5(9.81)\text{N} = \left(1000\ \text{kg/m}^3\right)\left[-2\left(\sqrt{\frac{1.6\left(10^9\right)}{\pi^2}Q^2 - 19.62\,h}\right)Q\right]$$

$$h = \left[\frac{8.26\left(10^6\right)Q^4 - 0.307\left(10^{-6}\right)}{Q^2}\right]\ \text{m, where } Q \text{ is in m}^3/\text{s} \qquad \textbf{Ans.}$$

The plot of h vs. Q is shown in Fig. c

$Q\left(10^{-3}\ \text{m}^3/\text{s}\right)$	0.5	0.6	0.7	0.8	0.9	1.0	0.439
h(m)	0.839	2.12	3.42	4.81	6.31	7.96	0

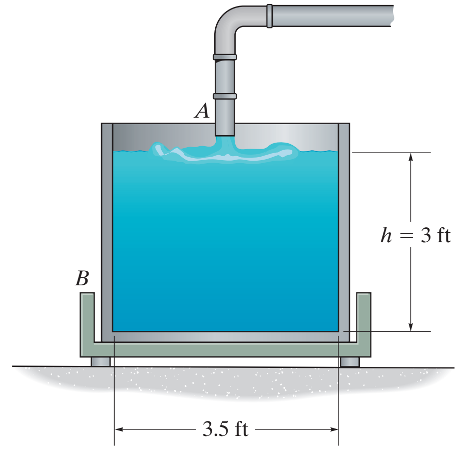


(c)

Ans:

$$h = \left[\frac{8.26\left(10^6\right)Q^4 - 0.307\left(10^{-6}\right)}{Q^2}\right]\ \text{m}$$

6–23. Water flows into the rectangular tank at the rate of $0.5 \text{ ft}^3/\text{s}$ from the 3-in.-diameter pipe at A. If the tank has a width of 2 ft and an empty weight of 150 lb, determine the apparent weight of the tank caused by the flow at the instant $h = 3$ ft.



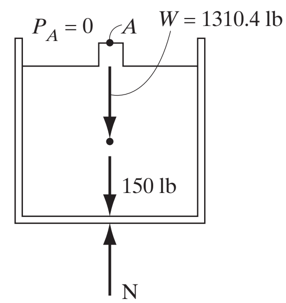
SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_A A_A; \qquad 0.5 \text{ ft}^3/\text{s} = V\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right]$$

$$V_A = 10.186 \text{ ft/s}$$

The control volume is the water in the tank. Its free-body diagram is shown in Fig. a. The weight of the water in the control volume is $W = \gamma_w \forall = \left(62.4 \text{ lb/ft}^3\right)\left[(3.5 \text{ ft})(2 \text{ ft})(3 \text{ ft})\right] = 1310.4$ lb. Here, A is exposed to the atmosphere, $p_A = 0$.



(a)

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

or

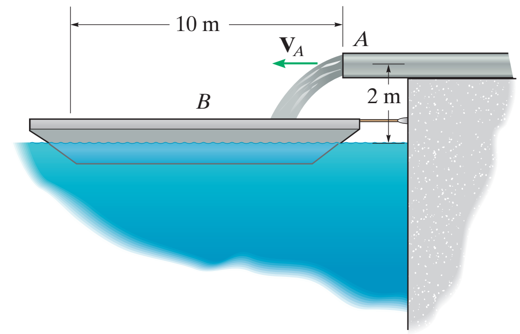
$$+\uparrow \Sigma F_y = 0 + \left(-V_A\right)\rho(-Q)$$

$$\text{N} - 150 \text{ lb} - 1310.4 \text{ lb} = \left(-10.186 \text{ ft/s}\right)\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left(-0.5 \text{ ft}^3/\text{s}\right)$$

$$\text{N} = 1470 \text{ lb} = 1.47 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
1.47 kip

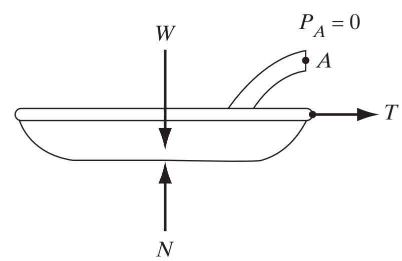
***6–24.** The barge is being loaded with an industrial waste liquid having a density of $1.2\ \text{Mg/m}^3$. If the average velocity of flow out of the 100-mm-diameter pipe is $V_A = 3\ \text{m/s}$, determine the force in the tie rope needed to hold the barge stationary.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_A A_A$$
$$= (3\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right]$$
$$= 0.023562\ \text{m}^3/\text{s}$$



(a)

The control volume is the barge and its contents. Its free-body diagram is shown in Fig. *a*. Since the flow is free, $p_A = 0$.

Linear Momentum. Since the flow is steady and incompressible,

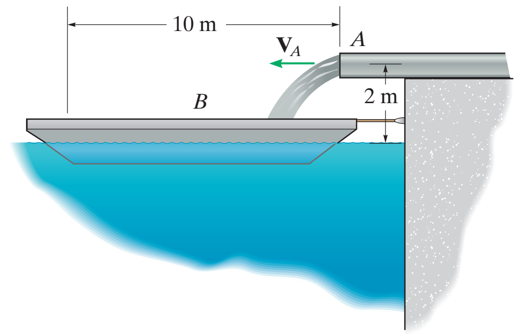
$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\text{V} + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (-V_A)\rho(-Q)$$

$$T = (-3\ \text{m/s})\left(1.2\left(10^3\ \text{kg/m}^3\right)\left(-0.023562\ \text{m}^3\right)\right)$$

$$T = 84.8\ \text{N} \qquad \textbf{Ans.}$$

6–25. The barge is being loaded with an industrial waste liquid having a density of $1.2\ \mathrm{Mg/m^3}$. Determine the maximum force in the tie rope needed to hold the barge stationary. The waste can enter the barge at any point within the 10-m region. Also, what is the speed of the waste exiting the pipe at A when this occurs? The pipe has a diameter of 100 mm.



SOLUTION

The maximum force developed in the tie rope occurs when the velocity **V** of the flow is maximum. This happens when the flow achieves the maximum range, ie, $S_x = 10$ m. Consider the vertical motion by referring to Fig. *a*.

$$(+\downarrow)\ S_y = (S_0)_y + (v_0)_y t + \frac{1}{2}a_c t^2; \quad 2\ \mathrm{m} = 0 + 0 + \frac{1}{2}(9.81\ \mathrm{m/s^2})t^2$$

$$t = 0.6386\ \mathrm{s}$$

The horizontal motion gives

$$(\overset{+}{\leftarrow})\ S_x = (S_0)_x + (v_0)_x t; \quad 10\ \mathrm{m} = 0 + V_A(0.6386\ \mathrm{s})$$

$$V_A = 15.66\ \mathrm{m/s} = 15.7\ \mathrm{m/s} \qquad \textbf{Ans.}$$

The fixed control volume considered is the barge and its contents as shown in Fig. *b*. Since the flow is free, $p_A = 0$. The flow is steady and incompressible. Then

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

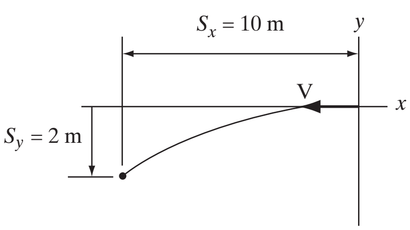
Writing the horizontal scalar component of this equation by referring to the *FBD* of the control volume, Fig. *b*,

$$(\overset{+}{\rightarrow})\ \Sigma F_x = 0 + (-V)\rho(-V_A)$$

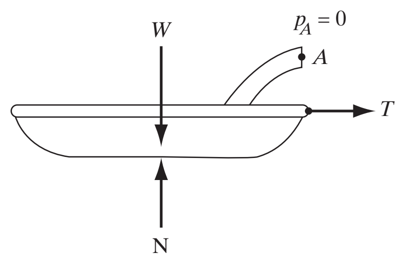
$$T = 0 + (-15.66\ \mathrm{m/s})(1200\ \mathrm{kg/m^3})\left[-(15.66\ \mathrm{m/s})\pi(0.05\ \mathrm{m})^2\right]$$

$$= 2311.43\ \mathrm{N}$$

$$= 2.31\ \mathrm{kN} \qquad \textbf{Ans.}$$



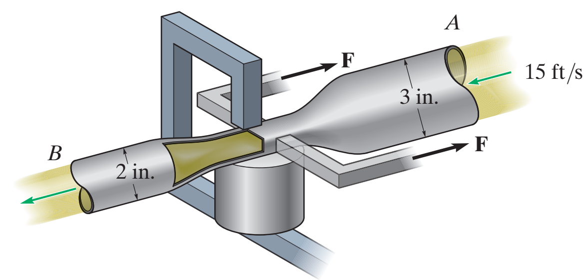
(a)



(b)

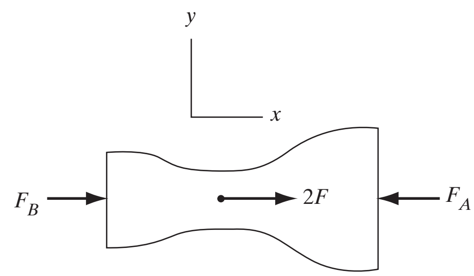
Ans:
$V_A = 15.7\ \mathrm{m/s}$
$T = 2.31\ \mathrm{kN}$

6–26. A nuclear reactor is cooled with liquid sodium, which is transferred through the reactor core using the electromagnetic pump. The sodium moves through a pipe at A having a diameter of 3 in., with a velocity of 15 ft/s and pressure of 20 psi, and passes through the rectangular duct, where it is pumped by an electromagnetic force giving it a 30-ft pumphead. If it emerges at B through a 2-in.-diameter pipe, determine the restraining force **F** on each arm, needed to hold the pipe in place. Take $\gamma_{Na} = 53.2\ \text{lb/ft}^3$.



SOLUTION

The flow is steady and the liquid sodium can be considered as an ideal fluid (incompressible and inviscid) such that $\gamma_{NA} = 53.2\ \text{lb/ft}^3$. Average velocities will be used. The control volume contains the liquid in the pipe and the transition as shown in Fig. *a*.



(a)

Continuity requires

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(15\ \text{ft/s})\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right] + V_B\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] = 0$$

$$V_B = 33.75\ \text{ft/s}$$

Applying the energy equation with $h_s = -30$ ft (negative sign indicates pump head), $p_A = \left(20\ \frac{\text{lb}}{\text{in}}\right)\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^2 = 2880\ \text{lb/ft}^2$ and $h_l = 0$,

$$\frac{p_A}{\gamma_{NA}} + \frac{V_A^2}{2g} + Z_A + h_t + h_l = \frac{p_B}{\gamma_{NA}} + \frac{V_B^2}{2g} + Z_B + h_t + h_l$$

$$\frac{2880\ \text{lb/ft}^2}{53.2\ \text{lb/ft}^3} + \frac{(15\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 + (30\ \text{ft}) - 0 = \frac{p_B}{53.2\ \text{lb/ft}^3} + \frac{(33.75\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0$$

$$p_B = 3720.90\ \text{lb/ft}^2$$

Thus, the pressure force acting on opened control surfaces at A and B are

$$F_A = p_A A_A = (2880\ \text{lb/ft}^2)\left[\pi\left(\frac{1.5}{12}\text{ft}\right)^2\right] = 141.37\ \text{lb}$$

$$F_B = p_B A_B = (3720.90\ \text{lb/ft}^2)\left[\pi\left(\frac{1}{12}\text{ft}\right)^2\right] = 81.18\ \text{lb}$$

Applying the linear momentum equation

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho_{NA} d\forall + \int_{cs} \mathbf{V}\rho_{NA}\mathbf{V}\cdot d\mathbf{A}$$

6–26. Continued

Writing the scalar component of this equation along x axis by referring to Fig. a

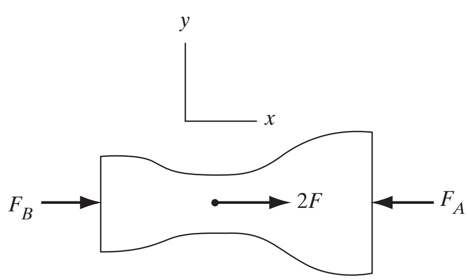
$$\left(\xrightarrow{+}\right)\ \Sigma F_x = 0 + (-V_B)\rho_{NA}(V_B A_B) + (-V_A)\rho_{NA}(-V_A A_A)$$

$$81.18\text{ lb} - 141.37\text{ lb} + 2F = (-33.75\text{ ft/s})\left(\frac{53.2\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)\left\{(33.75\text{ ft/s})\left[\pi\left(\frac{1}{12}\text{ft}\right)^2\right]\right\}$$

$$+ (-15\text{ ft/s})\left(\frac{53.2\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)\left\{(-15\text{ ft/s})\left[\pi\left(\frac{1.5}{12}\text{ft}\right)^2\right]\right\}$$

$$F = 18.7\text{ lb}$$ **Ans.**

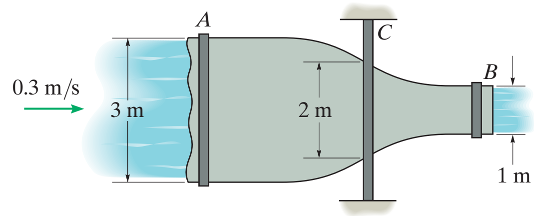
Note: This solution assumes that the electromagnetic pump is mounted on the outside of the duct, so that the EM force of the pump on the liquid is canceled by the equal and opposite reaction force on the pump, transferred to the pipe.



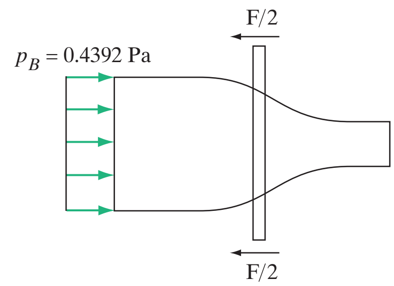
(a)

Ans:
18.7 lb

6–27. Air flows through the closed duct with a uniform velocity of 0.3 m/s. Determine the horizontal force **F** that the strap must exert on the duct to hold it in place. Neglect any force at A due to a slip joint. Take $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION



Assume the air is incompressible and non-viscids.

$$Q = V_A A_A = (0.3\ \text{m/s})(3\ \text{m})(1\ \text{m}) = 0.9\ \text{m}^3/\text{s}$$

Continuity requires

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - 0.9\ \text{m}^3/\text{s} + V_B(1\ \text{m})(1\ \text{m}) = 0$$

$$V_B = 0.9\ \text{m/s}$$

Apply the Bernoulli's equation between A and B.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{1.22\ \text{kg/m}^3} + \frac{(0.3\ \text{m/s})^2}{2} + 0 = 0 + \frac{(0.9\ \text{m/s})^2}{2} + 0$$

$$p_A = 0.4392\ P_a$$

Linear Momentum equation

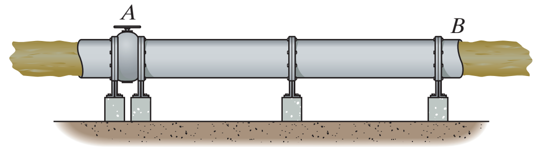
$$\overset{+}{\rightarrow} \Sigma F_x = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

$$-F + (0.4392\ \text{N/m}^2)(3\ \text{m})(1\ \text{m}) = 0 + (0.3\ \text{m/s})(1.22\ \text{kg/m}^3)(-0.9\ \text{m}^3/\text{s}) + (0.9\ \text{m/s})(1.22\ \text{kg/m}^3)(0.9\ \text{m}^3/\text{s})$$

$$F = 0.659\ \text{N}$$ **Ans.**

Ans:
0.659 N

***6–28.** As oil flows through the 20-m-long, 200-mm-diameter pipeline, it has a constant average velocity of 2 m/s. Friction losses along the pipe cause the pressure at B to be 8 kPa less than the pressure at A. Determine the resultant friction force on this length of pipe. Take $\rho_o = 880\ \text{kg/m}^3$.



SOLUTION

Here $\Delta p = 8$ hpa and so the force developed by the pressure difference is

$$F_p = 8\left(10^3\right)\ \text{N/m}^2(\pi)(0.1\ \text{m})^2 = 251.3\ \text{N}$$

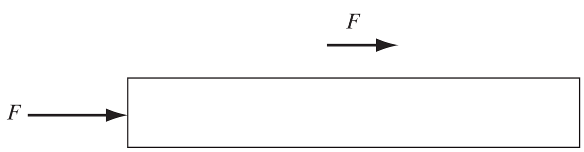
The free-body diagram is shown in Fig. a.

Applying the linear momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho d\maltese + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

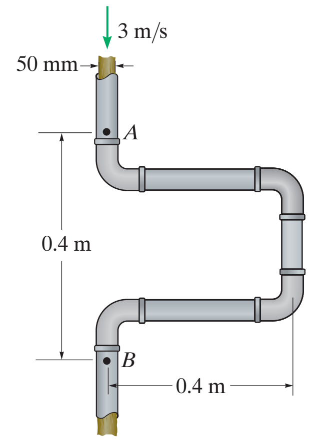
$$251.3 - F = 0 + (2\ \text{m/s})(880\ \text{kg/m}^3)(2\ \text{m/s})(\pi)(0.1\ \text{m}^2) + (2\ \text{m/s})(880\ \text{kg/m}^3) - (2\ \text{m/s})(\pi)(0.1\ \text{m}^2)$$

$$F = 251\ \text{N}$$ **Ans.**



(a)

6–29. Oil flows through the 50-mm-diameter vertical pipe assembly such that the pressure at A is 240 kPa and the velocity is 3 m/s. Determine the horizontal and vertical components of force the oil exerts on the U-section AB of the assembly. The assembly and the oil within it has a weight of 60 N. Take $\rho_o = 900 \text{ kg/m}^3$.



SOLUTION

Bernoulli Equation: Because the diameter is the same at A and B, $V_A = V_B = V$. With the datum at B,

$$\frac{p_A}{\rho} + \frac{V^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V^2}{2} + gz_B$$

$$\frac{240\left(10^3\right)\text{Pa}}{900 \text{ Kg/m}^3} + \left(9.81 \text{ m/s}^2\right)(0.4 \text{ m}) = \frac{p_B}{900 \text{ Kg/m}^3} + 0$$

$$p_B = 243.532\left(10^3\right) \text{ Pa}$$

Linear Momentum:

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

$(\overset{+}{\rightarrow})\ F_x = 0 + 0 = 0$ **Ans.**

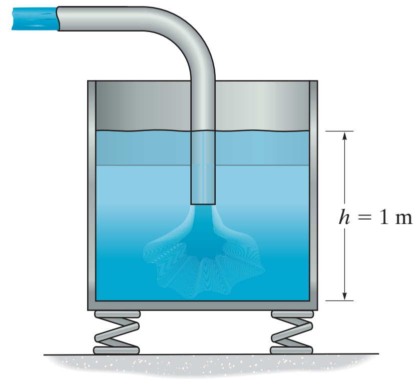
$$(+\uparrow) F_y + \left[243.532\left(10^3\right) \text{Pa}\right]\pi(0.025 \text{ m})^2 - \left[240\left(10^3\right)\text{Pa}\right]\pi(0.025 \text{ m})^2$$

$$-60 \text{ N} = 0 + (-V)\rho(-V_A) + (-V)\rho(V_A)$$

$F_y = 53.1 \text{ N}$ **Ans.**

Ans:
$F_x = 0$
$F_y = 53.1 \text{ N}$

6–30. Water flows into the tank at the rate of $0.05\ \text{m}^3/\text{s}$ from the 100-mm-diameter pipe. If the tank is 500 mm on each side, determine the compression in each of the four springs that support its corners when the water reaches a depth of $h = 1$ m. Each spring has a stiffness of $k = 8$ kN/m. When empty, the tank compresses each spring 30 mm.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid). Hence Average velocities are used and $p_w = 1000\ \text{kg/m}^3$.The control volume contains the water in the pipe and the tank and it is fixed instantaneously, Fig. *a*. From the discharge

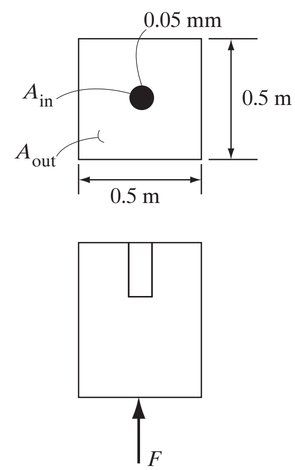
$$Q = V_{\text{in}}A_{\text{in}};\quad -0.05\ \text{m}^3/\text{s} = V_{\text{in}}\left[\pi(0.05\ \text{m})^2\right]\quad V_{\text{in}} = 6.366\ \text{m/s}$$

$$Q = V_{\text{out}}A_{\text{out}};\quad 0.05\text{m}^3/\text{s} = V_{\text{out}}\left[(0.5\ \text{m})^2 - \pi(0.05\ \text{m})^2\right]\quad V_{\text{out}} = 0.2065\ \text{m/s}$$

Applying the linear momentum equation

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar equation along the *y* axis by refering to the *FBD* of the control volume, Fig. *a*,



(a)

$$+\uparrow\ F = 0 + V_{\text{out}}\rho_w V_{\text{out}}A_{\text{out}} + (-V_{\text{in}})\rho_w(-V_{\text{in}}A_{\text{in}})$$

$$F = (0.2065\ \text{m/s})^2(1000\ \text{kg/m}^3)\left[(0.5\ \text{m})^2 - \pi(0.05\ \text{m})^2\right] + (6.366\ \text{m/s})^2(1000\ \text{Kg/m}^3)\left[\pi(0.05\ \text{m})^2\right]$$

$$= 328.63\ \text{N}$$

The weight of the water in the tank at a depth of 1m is

$$W_N = \rho_W g\forall_W = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(0.5\ \text{m})^2(1\ \text{m})\right] = 2452.5\ \text{N}$$

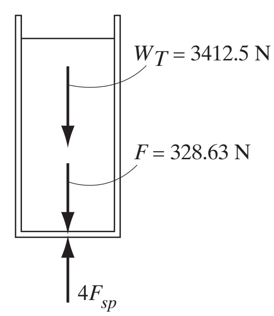
The weight of the empty tank is

$$W_t = 4kx = 4\left[8(10^3)\ \text{N/m}\right](0.03\ \text{m}) = 960\ \text{N}$$

Thus, the total weight is

$$W_T = W_W + W_t = 2452.5\ \text{N} + 960\ \text{N} = 3412.5\ \text{N}$$

Equilibrium of the *FBD* of the tank, Fig. *b*, requires



(b)

$$+\uparrow\ \Sigma F_y = 0;\quad 4F_{sp} - 328.63\ \text{N} - 3412.5\ \text{N} = 0$$

$$F_{sp} = 935.28\ \text{N}$$

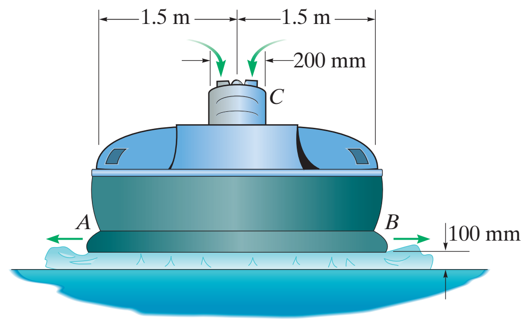
Thus, the compression of the spring is

$$F_{sp} = kx;\quad 935.28\ \text{N} = \left[8(10^3)\ \text{N/m}\right]x$$

$$x = 0.1169\ \text{m} = 117\ \text{mm}$$ **Ans.**

Ans:
117 mm

6–31. The 300-kg circular craft is suspended 100 mm from the ground. For this to occur, air is drawn in at 18 m/s through the 200-mm-diameter intake and discharged to the ground as shown. Determine the pressure that the craft exerts on the ground. Take $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_C A_C = (18\ \text{m/s})\left[\pi(0.1\ \text{m})^2\right] = 0.5655\ \text{m}^3/\text{s}$$

Take the control volume to be the craft and the air inside it. Its free-body diagram is shown in Fig. *a*. Since the flow is open to the atmosphere, $p_C = 0$.

Linear Momentum. Since no air escapes from the hovercraft vertically, $V_{\text{out}} = 0$. Since the flow is steady incompressible,

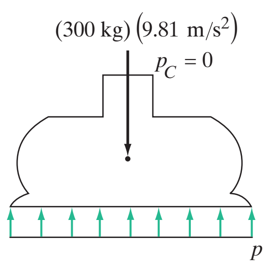
$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\mathcal{V} + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$+\uparrow \Sigma F_y = 0 + (-V_C)\rho(-Q)$$

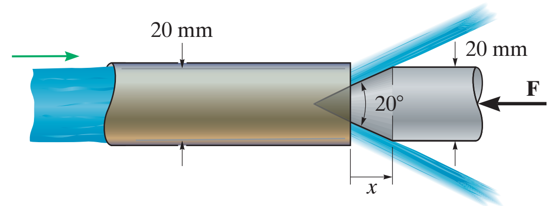
$$p\left[\pi(1.5\ \text{m})^2 - 300\ \text{kg}(9.81\ \text{m/s}^2) = (-18\ \text{m/s})(1.22\ \text{kg/m}^3) - (0.5655\ \text{m}^3/\text{s})\right]$$

$$p = 418\ \text{Pa}$$ **Ans.**



Ans:
418 Pa

***6–32.** The cylindrical needle valve is used to control the flow of $0.003\ \text{m}^3/\text{s}$ of water through the 20-mm-diameter tube. Determine the force **F** required to hold it in place when $x = 10$ mm.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000\ \text{kg/m}^3$. Average velocities will be used. The control volume is shown in Fig. *a*. From the discharge,

$$Q = V_{in}A_{in}; \qquad 0.003\ \text{m}^3/\text{s} = V_{in}\left[\pi(0.01\ \text{m})^2\right] \qquad V_{in} = 9.549\ \text{m/s}$$

From the geometry shown in Fig. *b*,

$$\frac{r}{\dfrac{0.01\ \text{m}}{\tan 10^\circ} - 0.01\ \text{m}} = \frac{0.01\ \text{m}}{\dfrac{0.01\ \text{m}}{\tan 10^\circ}}; \qquad r = 0.008237\ \text{m}$$

Thus,

$$A_{out} = \pi\left[(0.01\ \text{m})^2 - (0.008237\ \text{m})^2\right] = 0.1010\left(10^{-3}\right)\ \text{m}^2$$

Then

$$Q = V_{out}A_{out}; \qquad 0.003\ \text{m}^3/\text{s} = V_{out}\left[0.1010\left(10^{-3}\right)\ \text{m}^2\right]$$

$$V_{out} = 29.70\ \text{m/s}$$

Applying Bernoulli's equation between the center points of the inlet and outlet control surfaces where $p_{out} = p_{atm} = 0$

$$\frac{p_{in}}{\gamma_w} + \frac{V_{in}^2}{2g} + z_{in} = \frac{p_{out}}{\gamma_w} + \frac{V_{out}^2}{2g} + z_{out}$$

$$\frac{p_{in}}{9810\ \text{N/m}^3} + \frac{(9.549\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = 0 + \frac{(29.70\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_{in} = 395.35\left(10^3\right)\ \text{Pa}$$

Thus, the pressure force exerted on the inlet control surface is

$$F_{in} = p_{in}A_{in} = \left[395.35\left(10^3\right)\ \text{N/m}^2\right]\left[\pi(0.01\text{m})^2\right] = 124.20\ \text{N}$$

Applying the linear momentum equation,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho_w dV + \int_{cs}\mathbf{V}\rho_w\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component along x axis by referring to the *FBD* of the control volume shown in Fig. *a*

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + V_{out}\rho_w V_{out}A_{out} + V_{in}\rho_w(-V_{in}A_{in})$$

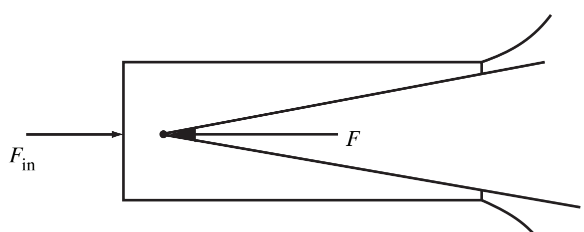
***6–32. Continued**

However, $Q = V_{\text{out}}A_{\text{out}} = V_{\text{in}}A_{\text{in}} = 0.003\ \text{m}^3/\text{s}$

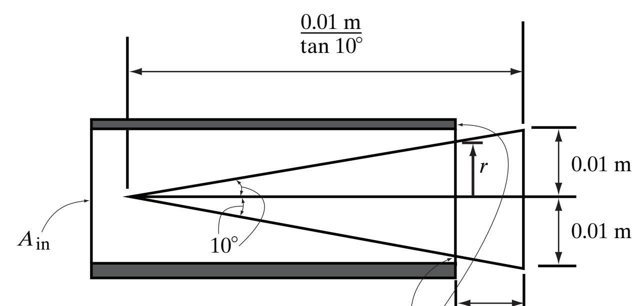
$$124.20\ \text{N} - F = (29.70\ \text{m/s})(1000\ \text{kg/m}^3)(0.003\ \text{m}^3/\text{s}) + (9.549\ \text{m/s})(1000\ \text{kg/m}^3)(-0.003\ \text{m}^3/\text{s})$$

$$F = 63.76\ \text{N} = 63.8\ \text{N} \qquad \textbf{Ans.}$$

Note: For simplicity, the effect of the slight deflection of the stream, away from the central axis, has been neglected. If it were accounted for, F would be slightly ($< 2\%$) larger.

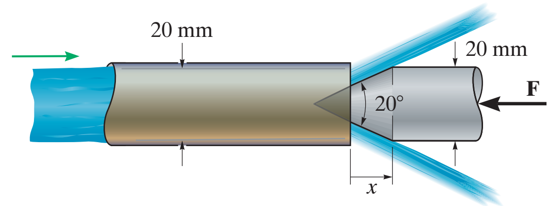


(a)



(b)

6–33. The cylindrical needle valve is used to control the flow of 0.003 m^3/s of water through the 20-mm-diameter tube. Determine the force **F** required to hold it in place for any position x of closure of the valve.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000\ \mathrm{kg/m^3}$. Average velocities will be used. The control volume is shown in Fig. *a*. From the discharge,

$$Q = V_{in} A_{in}; \qquad 0.003\ \mathrm{m^3/s} = V_{in}\left[\pi(0.01\ \mathrm{m})^2\right] \qquad V_{in} = 9.549\ \mathrm{m/s}$$

From the geometry shown in Fig. *b*,

$$\frac{r}{\dfrac{0.01\ \mathrm{m}}{\tan 10^\circ} - x} = \frac{0.01\ \mathrm{m}}{\dfrac{0.01\ \mathrm{m}}{\tan 10^\circ}}; \qquad r = 0.01\ \mathrm{m} - (\tan 10^\circ)x$$

Thus,

$$A_{out} = \pi\left[(0.01\ \mathrm{m})^2 - (0.01\ \mathrm{m} - (\tan 10^\circ)x)^2\right] = 0.01108x - 0.09768x^2$$

Then

$$Q = V_{out}A_{out};\ 0.003\ \mathrm{m^3/s} = V_{out}\left(0.01108x - 0.09768x^2\right)$$

$$V_{out} = \left(\frac{1}{3.693x - 32.559x^2}\right)\mathrm{m/s}$$

Applying the energy equation between the center points of the inlet and outlet control surfaces, where $p_{out} = p_{atm} = 0$.

$$\frac{p_{in}}{\gamma_w} + \frac{V_{in}^2}{2g} + z_{in} + h_{pump} = \frac{p_{out}}{\gamma_w} + \frac{V_{out}^2}{2g} + z_{out} + h_{turb} + h_L$$

$$\frac{p_{in}}{9810\ \mathrm{N/m^3}} + \frac{(9.549\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} + 0 + 0 = 0 + \frac{\left(\dfrac{1}{3.693x - 32.559x^2}\right)^2}{2(9.81\ \mathrm{m/s^2})} + 0 + 0 + 0$$

$$p_{in}\left[\frac{500}{(3.693x - 32.559x^2)^2} - 45.595(10^3)\right]\mathrm{Pa}$$

Thus, the pressure force on the inlet control surface is

$$F_{in} = p_{in}A_{in} = \left[\frac{500}{(3.693x - 32.559x^2)^2} - 45.595(10^3)\right]\left[\pi(0.01\ \mathrm{m})^2\right]$$

$$= \frac{0.05\pi}{(3.693x - 32.559x^2)^2} - 14.324$$

6–33. Continued

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho_w dV + \int_{cs} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component along the x axis by referring to the *FBD* of the control volume shown in Fig. *a*,

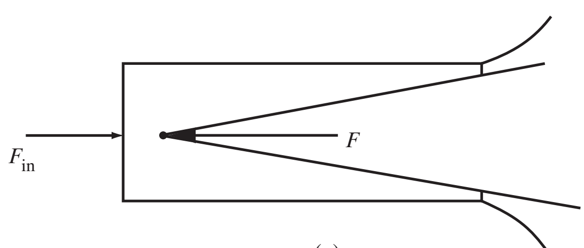
$$\xrightarrow{+} \Sigma F_x = 0 + V_{out}\rho_w V_{out}A_{out} + V_{in}\rho_w(-V_{in}A_{in})$$

However, $Q = V_{out}A_{out} = V_{in}A_{in} = 0.003 \text{ m}^3/\text{s}$

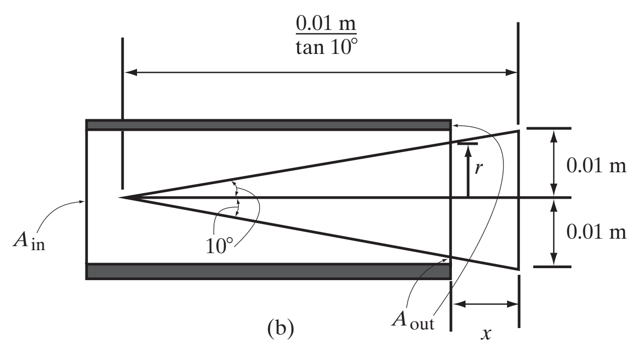
$$\frac{0.05\pi}{(3.693x - 32.559x^2)^2} - 14.324 - F = \left(\frac{1}{3.693x - 32.559x^2}\right)(1000 \text{ kg/m}^3)(0.003 \text{ m}^3/\text{s}) + (9.549 \text{ m/s})(1000 \text{ kg/m}^3)(-0.003 \text{ m}^3/\text{s})$$

$$F = \left[\frac{97.7x^2 - 11.1x + 0.157}{(3.69x - 32.6x^2)^2} + 14.3\right] \text{N} \qquad \textbf{Ans.}$$

Note: As in the preceding problem, the slight effect of the 10° deflection of the stream has be neglected.



(a)

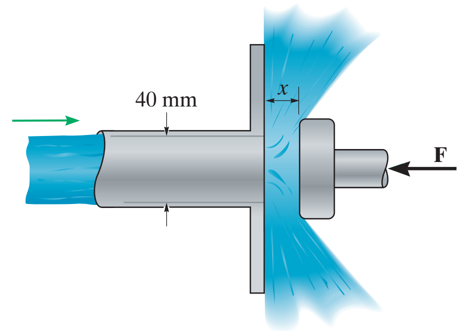


(b)

Ans:

$$F = \left[\frac{97.7x^2 - 11.1x + 0.157}{(3.69x - 32.6x^2)^2} + 14.3\right] \text{N}$$

6–34. The disk valve is used to control the flow of $0.008\ \text{m}^3/\text{s}$ of water through the 40-mm-diameter tube. Determine the force **F** required to hold the valve in place for any position x of closure of the valve.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000\ \text{kg/m}^3$. Average velocities will be used. The control volume is shown in Fig. *a*. From the discharge,

$$Q = V_{in}A_{in}; \qquad 0.008\ \text{m}^3/\text{s} = V_{in}\left[\pi(0.02\ \text{m})^2\right] \qquad V_{in} = 6.366\ \text{m/s}$$

The cross-sectional area of the outlet control surfaces is

$$A_{out} = 2\pi(0.02\ \text{m})x = (0.04\pi x)\ \text{m}^2$$

Then

$$Q = V_{out}A_{out}; \qquad 0.008\ \text{m}^3/\text{s} = V_{out}(0.04\pi x)$$

$$V_{out} = \left(\frac{0.06366}{x}\right)\ \text{m/s}$$

Applying Bernoulli's equation between the center points of the inlet and outlet control surfaces, where $p_{out} = p_{atm} = 0$.

$$\frac{p_{in}}{\gamma_w} + \frac{V_{in}^2}{2g} + z_{in} = \frac{p_{out}}{\gamma_w} + \frac{V_{out}^2}{2g} + z_{out}$$

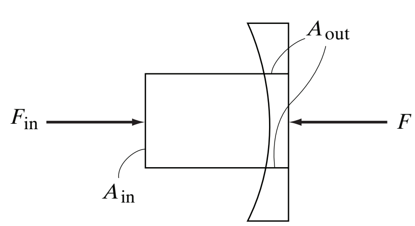
$$\frac{p_{in}}{9810\ \text{N/m}^3} + \frac{(6.366\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = 0 + \frac{\left(\frac{0.06366}{x}\right)^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_{in} = \left[\frac{2.026}{x^2} - 20.264(10^3)\right]\ \text{Pa}$$

Thus, the pressure force on the inlet control surface is

$$F_{in} = p_{in}A_{in} = \left[\frac{2.026}{x^2} - 20.264(10^3)\right]\left[\pi(0.02\ \text{m})^2\right]$$

$$= \left[\frac{2.546(10^{-3})}{x^2} - 25.465\right]\ \text{N}$$



(a)

6–34. Continued

Applying the linear momentum equation

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho_w d\forall + \int_{\text{cs}} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along x axis by referring to the *FBD* of the control volume shown in Fig. *a*

$$\rightarrow \Sigma F_x = 0 + V_{\text{in}}\rho_w(-V_{\text{in}}A_{\text{in}})$$

However, $Q = V_{\text{in}}A_{\text{in}} = 0.008\ \text{m}^3/\text{s}$. Thus

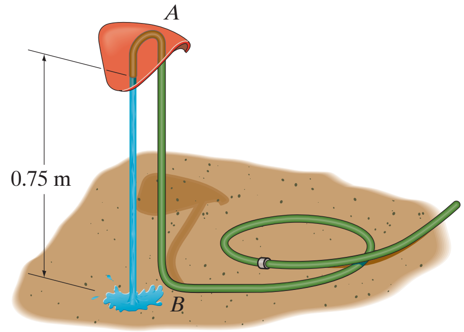
$$\frac{2.546\left(10^{-3}\right)}{x^2} - 25.465 - F = (6.366\ \text{m/s})\left(1000\ \text{kg/m}^3\right)\left(-0.008\ \text{m}^3/\text{s}\right)$$

$$F = \left[\frac{2.55\left(10^3\right)}{x^2} + 25.5\right]\text{N} \qquad \textbf{Ans.}$$

Ans:

$$F = \left[\frac{2.55\left(10^3\right)}{x^2} + 25.5\right]\text{N}$$

6–35. The toy sprinkler consists of a cap and a rigid tube having a diameter of 20 mm. If water flows through the tube at $0.7\left(10^{-3}\right)$ m^3/s, determine the vertical force the tube must support at B. Neglect the weight of the sprinkler head the water within the curved segment of the tube. The weight of the tube and water within the vertical segment AB is 4N.



SOLUTION

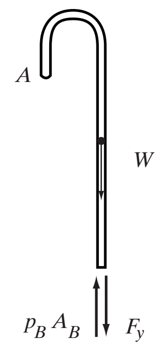
$$Q = VA$$

$$0.7\left(10^{-3}\right)\text{ m}^3/\text{s} = V(\pi)(0.01\text{ m})^2$$

$$V = 2.228\text{ m/s}$$

Since the hose has a constant diameter, continuity requires $V_A = V_B = V = 2.228$ m/s

Applying Bernoulli's equation between A and B, with the datum of B,



(a)

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{V^2}{2g} + 0.75\text{ m} = \frac{p_B}{\left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)} + \frac{V^2}{2g} + 0$$

$$p_B = 7357.5\text{ Pa}$$

The free-body diagram of the control volume is shown in Fig a. Applying the linear momentum equation in the vertical direction, for steady flow

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d\forall + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

$$+\uparrow\Sigma F_y = 0 + \left(-V_A\right)\rho\left(Q_A\right) + V_B\,\rho\left(-Q_B\right)$$

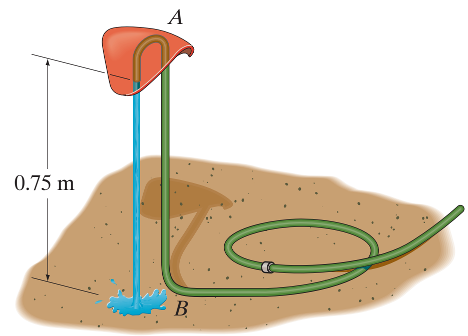
$$\uparrow\Sigma F_y = -2\,V\rho\,Q$$

$$\left(7357.5\text{ N/m}^2\right)(\pi)(0.01\text{ m})^2 - F_y - 4\text{ N} = -2\left(2.228\text{ m/s}\right)\left(1000\text{ kg/m}^3\right)\left(0.7\left(10^{-3}\right)\text{ m}^3/\text{s}\right)$$

$$F_y = 1.43\text{ N} \qquad \textbf{Ans.}$$

Ans:
1.43 N

***6–36.** The toy sprinkler consists of a cap and a rigid tube having a diameter of 20 mm. Determine the flow through the tube such that it creates a vertical force of 6 N in the tube at B. Neglect the weight of the sprinkler head and the water within the curved segment of the tube. The weight of the tube and water within the vertical segment AB is 4N.



SOLUTION

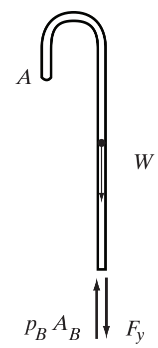
Since the hose has a constant diameter, continuity requires $V_A = V_B = V$ Applying Bernoulli's equation between A and B, with the datum at B,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{V^2}{2g} + 0.75\text{ m} = \frac{p_B}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} + \frac{V^2}{2g} + 0$$

$$p_B = 7357.5\text{ Pa}$$

The free-body diagram of the control volume is shown in Fig. a. Applying the linear momentum equation in the vertical direction for steady flow,



(a)

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

$$+\uparrow \Sigma F_y = 0 + (-V_A)\rho Q_A + V_B\,\rho(-Q_B)$$

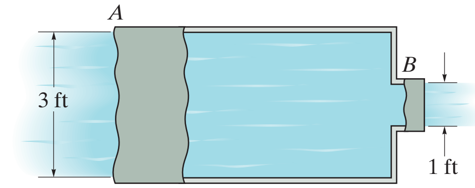
$$+\uparrow \Sigma F_y = -2V\rho Q$$

$$(7357.5\text{ N/m}^2)(\pi)(0.01\text{ m})^2 - 4 - 6 = -2(V)(1000\text{ kg/m}^3)(V)(\pi)(0.01\text{ m})^2$$

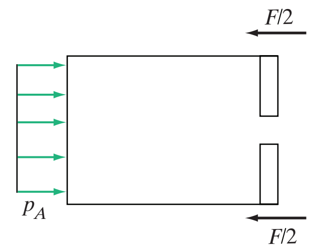
$$V = 3.4981\text{ m/s}$$

$$Q = VA = (3.4981\text{ m/s})(\pi)(0.01\text{ m})^2 = 1.10(10^{-3})\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

6–37. Air flows through the 1.5 ft-wide rectangular duct at 900 ft^3/min. Determine the horizontal force acting on the end plate B of the duct. Take $\rho_a = 0.00240\ \text{slug/ft}^3$.



SOLUTION



$$Q = 900\ \text{ft}^3/\text{min}(1\ \text{min.}/60\ \text{s}) = 15\ \text{ft}^3/\text{s}$$

$$V_A = \frac{15\ \text{ft}^3/\text{s}}{(3\ \text{ft})(1.5\ \text{ft})} = 3.33\ \text{ft/s}$$

$$V_B = \frac{15\ \text{ft}^3/\text{s}}{(1\ \text{ft})(1.5\ \text{ft})} = 10\ \text{ft/s}$$

Apply Bernoulli's equation between A and B.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{0.00240\ \text{slug/ft}^3} + \frac{(3.33\ \text{ft/s})^2}{2} + 0 = 0 + \frac{(10\ \text{ft/s})^2}{2} + 0$$

$$p_A = 0.10667\ \text{lb/ft}^2$$

Using the free-body diagram, Fig. *a* the linear momentum equation becomes

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{\partial}{\partial t}\int_{\text{cv}} V_x \rho d\forall + \int_{\text{cs}} V_x \rho \mathbf{V} \cdot d\mathbf{A}$$

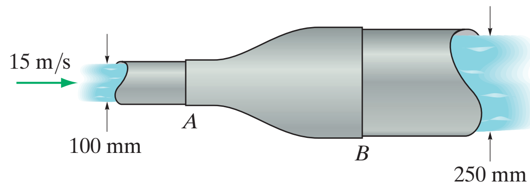
$$(0.10667\ \text{lb/ft}^2)(3\ \text{ft})(1.5\ \text{ft}) - \text{F} =$$

$$0 + (3.33\ \text{ft/s})(0.00240\ \text{slug/ft}^3)(-15\ \text{ft}^3/\text{s}) + (10\ \text{ft/s})(0.00240\ \text{slug/ft}^3)(15\ \text{ft}^3/\text{s})$$

$$F = 0.24\ \text{lb} \qquad \textbf{Ans.}$$

Ans:
0.24 lb

6–38. Air at a temperature of 30°C flows through the expansion fitting such that its velocity at A is 15 m/s and the absolute pressure is 250 kPa. If no heat or frictional loss occurs, determine the resultant force needed to hold the fitting in place.



SOLUTION

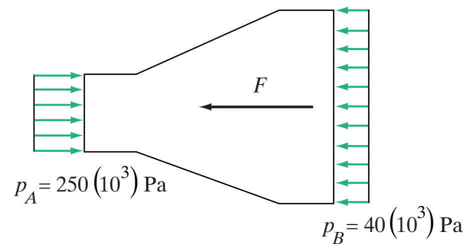
Using the ideal gas law with $R = 286.9\ \text{J/kg}\cdot\text{k}$ for air (Appendix A),

$$p_A = \rho_A R T_A; \qquad 250(10^3)\ \text{N/m}^2 = \rho_A(286.9\ \text{J/kg}\cdot\text{k})(273 + 30)\ \text{k}$$

$$\rho_A = 2.8759\ \text{kg/m}^3$$

$$p_B = \rho_B R T_B; \qquad p_B = \rho_B(286.9\ \text{J/kg}\cdot\text{k})(273 + 30)\ \text{k}$$

$$\rho_B = \left[11.5034(10^{-6})p_B\right]\ \text{kg/m}^3 \qquad \textbf{(1)}$$



(a)

Consider the fixed control volume to be the air contained in the expansion fitting as shown in Fig. a. Continuity requires

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 + \rho_A(-V_A A_A) + \rho_B(V_B A_B) = 0$$

$$(2.8759\ \text{kg/m}^3)\left\{-(15\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right]\right\} + \left[11.5034(10^{-6})p_B\right]\left\{V_B\left[\pi(0.125\ \text{m})^2\right]\right\} = 0$$

$$V_B = \left[\frac{0.6(10^6)}{p_B}\right]\ \text{m/s} \qquad \textbf{(2)}$$

Since the fitting remains horizontal, $z_A = z_B = z$. The energy equation gives

$$\frac{p_A}{\gamma_A} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_B} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{250(10^3)\ \text{N/m}^2}{(2.8759\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + \frac{(15\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + z + 0 = \frac{p_B}{\left[11.0534(10^{-6})p_B\right](9.81\ \text{m/s}^2)} + \frac{\left[\dfrac{0.6(10^6)}{p_B}\right]^2}{2(9.81\ \text{m/s}^2)} + z + 0 + 0$$

$$p_B = 40(10^3)\ \text{Pa}$$

Substituting this result into Eqs. (1) and (2)

$$\rho_B = 0.4601\ \text{kg/m}^3 \qquad V_B = 15\ \text{m/s}$$

Since the flow is steady,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

6–38. Continued

Writing the horizontal scalar component of this equation by referring to the *FBD* of the control volume, Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0 + V_A \rho_A(-V_A A_A) + V_B \rho_B(V_B A_B)$$

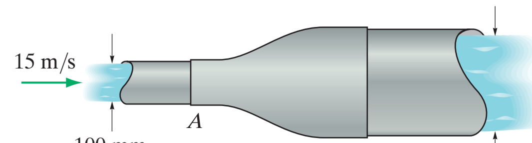
$$\left[250(10^3)\ \text{N/m}^2\right]\left[\pi(0.05\ \text{m})^2\right] - F - \left[40(10^3)\ \text{N/m}^2\right]\left[\pi(0.125\ \text{m})^2\right]$$

$$= (15\ \text{m/s})(2.8759\ \text{kg/m}^3)\left\{-(15\ \text{m/s})\left[\pi(0.05\ \text{m})^2\right]\right\} + (15\ \text{m/s})(0.4601\ \text{kg/m}^3)\left\{(15\ \text{m/s})\left[\pi(0.125\ \text{m})^2\right]\right\}$$

$$F = 0 \qquad \textbf{Ans.}$$

Ans:
$F = 0$

6–39. Air at a temperature of 30°C flows through the expansion fitting such that its velocity at A is 15 m/s and the pressure is 250 kPa. If heat and frictional loss due to the expansion causes the temperature and absolute pressure of the air at D to become 20°C and 7.50 kPa, determine the resultant force needed to hold the fitting in place.



SOLUTION

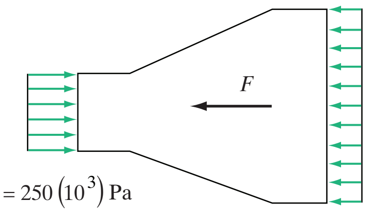
Using the ideal gas law with $R = 286.9\ \text{J/kg}\cdot\text{k}$ for air (Appendix A),

$$p_A = \rho_A R T_A; \qquad 250(10^3)\ \text{N/m}^2 = \rho_A(286.9\ \text{J/kg}\cdot\text{k})(273 + 30)\ \text{k}$$

$$\rho_A = 2.8759\ \text{kg/m}^3$$

$$p_B = \rho_B R T_B; \qquad 7.50(10^3)\ \text{N/m}^2 = \rho_B(286.9\ \text{J/kg}\cdot\text{k})(273 + 20)\ \text{k}$$

$$\rho_B = 0.08922\ \text{kg/m}^3 \qquad \textbf{(1)}$$



(a)

Consider the fixed control volume to be the water contained in the expansion fitting as shown in Fig. a. The continuity requires

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 + \rho_A(-V_A A_A) + \rho_B(V_B A_B) = 0$$

$$(2.8759\ \text{kg/m}^3)\{-(15\ \text{m/s})[\pi(0.05\ \text{m})^2]\} + [0.08922\ \text{kg/m}^3]\{V_B[\pi(0.125\ \text{m})^2]\} = 0$$

$$V_B = 77.36\ \text{m/s}$$

Since the flow is steady,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

Writing the horizontal scalar component of this equation by referring to the FBD of the control volume, Fig. a

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + V_A\rho_A(-V_A A_A) + V_B\rho_B(V_B A_B)$$

$$[250(10^3)\ \text{N/m}^2][\pi(0.05\ \text{m})^2] - F - (7.5(10^3)\ \text{N/m}^2)[\pi(0.125\ \text{m})^2]$$

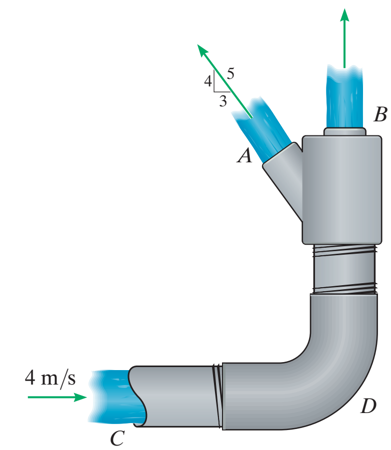
$$= (15\ \text{m/s})(2.8759\ \text{kg/m}^3)\{-(15\ \text{m/s})[\pi(0.05\text{m})^2]\}$$

$$+ (77.36\ \text{m/s})(0.08938\ \text{kg/m}^3)\{(77.36\ \text{m/s})[\pi(0.125\ \text{m})^2]\}$$

$$F = 1.57\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
1.57 kN

***6–40.** Water flows through the pipe C at 4 m/s. Determine the horizontal and vertical components of force exerted by elbow D necessary to hold the pipe assembly in equilibrium. Neglect the size and weight of the pipe and the water within it. The pipe has a diameter of 60 mm at C, and at A and B the diameters are 20 mm.



SOLUTION

Assume water is incompressible. We have steady flow.

$$Q = 4\text{ m/s}\,(\pi)(0.03\text{ m})^2 = 0.011310\text{ m}^3$$

Continuity requires

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - 4\text{ m/s}(\pi)(0.03\text{ m})^2 + V_A(\pi)(0.01\text{ m})^2 + V_B(\pi)(0.01\text{ m})^2 = V_A + V_B = 36 \qquad \textbf{(1)}$$

Bernoulli Equation.

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

$$\frac{p_C}{1000\text{ kg/m}^3} + \frac{(4\text{ m/s})^2}{2} + 0 = 0 + \frac{V_A^2}{2} + 0$$

$$V_A^2 = 16 + 0.002\,p_C \qquad \textbf{(2)}$$

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_C}{1000\text{ kg/m}^3} + \frac{(4\text{ m/s})^2}{2} + 0 = 0 + \frac{V_B^2}{2} + 0$$

$$V_B^2 = 16 + 0.002\,p_C \qquad \textbf{(3)}$$

From Eqs. (2) and (3), $V_A = V_B$. From Eq. (1),

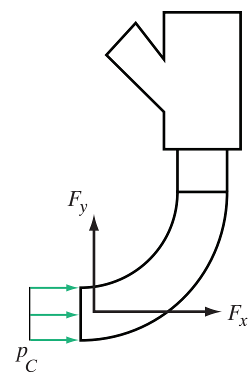
$$V_A = V_B = 18\text{ m/s}$$

Thus

$$(18\text{ m/s})^2 = 16 + 0.002\,p_C$$

$$p_C = 154\text{ kPa}$$

The free-body diagram is shown in Fig. *a*.



(a)

***6–40. Continued**

Linear momentum.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (V_C)(\rho)(-V_C A_C) + \left(-V_A \frac{3}{5}\right)\rho V_A A_A + 0$$

$$F_x + 154\,(10^3)(\pi)(0.03\text{ m})^2 = (4\text{ m/s})(1000\text{ kg/m}^3)(-4\text{ m/s})(\pi)(0.03\text{m})^2$$

$$- (18\text{ m/s})\left(\frac{3}{5}\right)(1000\text{ kg/m}^3)(18\text{ m/s})(\pi)(0.01\text{ m})^2$$

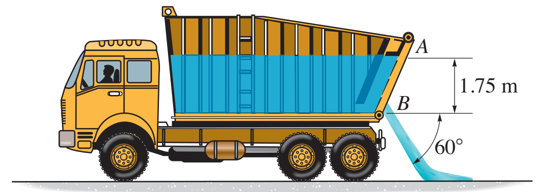
$$F_x = -542\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + V_A\left(\frac{4}{5}\right)\rho\, V_A A_A + V_B \rho V_B A_B$$

$$F_y = 18\text{ m/s}\left(\frac{4}{5}\right)(1000\text{ kg/m}^3)(18\text{ m/s})(\pi)(0.01\text{ m})^2 + 18\text{ m/s}\,(1000\text{ kg/m}^3)(18\text{ m/s})(\pi)(0.01\text{ m})^2$$

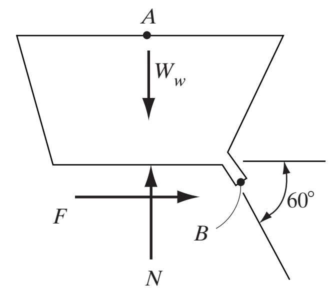
$$F_y = 183\text{ N}\uparrow \qquad \textbf{Ans.}$$

6–41. The truck dumps water on the ground such that it flows from the truck through a 100-mm-wide opening at an angle of 60°. The length of the opening is 2 m. Determine the friction force that all the wheels of the truck must exert on the ground to keep the truck from moving at the instant the water depth in the truck is 1.75 m.



SOLUTION

We consider steady flow of an ideal fluid.



Bernoulli Equation. Since A and B are exposed to the atmosphere, $p_A = p_B = 0$. Since the water discharges from a large reservoir, $V_A \cong 0$. If the datum is at B, $z_A = 1.75$ m and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + 0 + 1.75 \text{ m} = 0 + \frac{V_B^2}{2(9.81 \text{ m/s}^2)} + 0$$

$$V_B = 5.860 \text{ m/s}$$

The discharge at B is

$$Q_B = V_B A_B = (5.860 \text{ m/s})(2 \text{ m})(0.1 \text{ m})$$

$$= 1.172 \text{ m}^3/\text{s}$$

Take the control volume to be the dump truck and its contents. Its free-body diagram is shown in Fig. *a*.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V} \cdot d\mathbf{A}$$

or

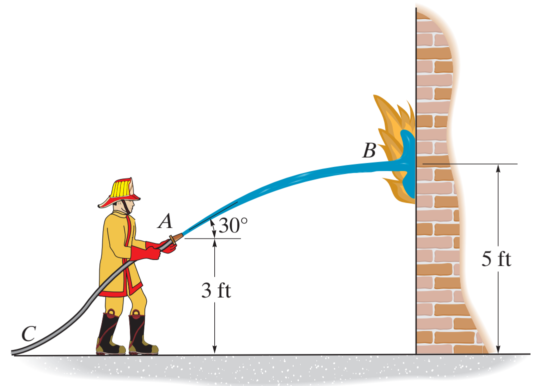
$$\overset{+}{\rightarrow} \Sigma F_x = (V_B)_x \, \rho \, (Q_B)$$

$$F = (5.860 \text{ m/s} \cos 60°)(1000 \text{ kg/m}^3)(1.172 \text{ m}^3/\text{s})$$

$$F = 3.43 \text{ kN}$$ **Ans.**

Ans:
3.43 kN

6–42. The fireman sprays a 2-in.-diameter jet of water from a hose at the burning building. If the water is discharged at 1.5 ft^3/s, determine the magnitude of the velocity of the water when it splashes on the wall. Also, find the normal reaction of both the fireman's feet on the ground. He has a weight of 180 lb. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



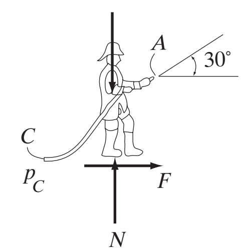
SOLUTION

We consider steady flow of an ideal fluid.

$$Q = V_A A_A$$

$$1.5\ \text{ft}^3/\text{s} = V_A\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right]$$

$$V_A = 68.75\ \text{ft/s}$$



(a)

Bernoulli Equation. Since the water jet from A and B is free flow, $p_A = p_B = 0$. If the datum is at A, $z_A = 0$ and $z_B = 5\ \text{ft} - 3\ \text{ft} = 2\ \text{ft}$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{(68.75\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 = 0 + \frac{V_B^2}{2(32.2\ \text{ft/s}^2)} + 2\ \text{ft}$$

$$V_B = 67.81\ \text{ft/s} = 67.8\ \text{ft/s} \qquad \textbf{Ans.}$$

Take the control volume to be the fireman and hose CA and the water within it. Its free-body diagram is shown in Fig. a. Here, the pressure at C, p_C, acts horizontally.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\mathcal{V} + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

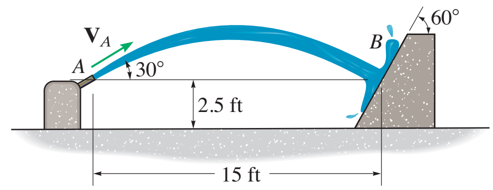
$$+\uparrow \Sigma F_y = 0 + (V_A)_y \rho(Q)$$

$$N - 180\ \text{lb} = 68.75\ \text{ft/s}\sin 30°\left(\frac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(1.5\ \text{ft}^3/\text{s})$$

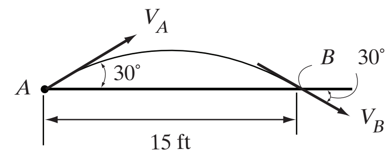
$$N = 279.93\ \text{lb} = 280\ \text{lb} \qquad \textbf{Ans.}$$

Ans:
$V_B = 67.8\ \text{ft/s}$
$N = 280\ \text{lb}$

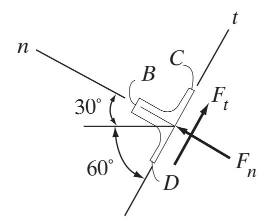
6–43. The fountain sprays water in the direction shown. If the water is discharged at 30° from the horizontal, and the cross-sectional area of the water stream is approximately 2 in^2, determine the normal force the water exerts on the wall at B.



SOLUTION



(a)



(b)

We consider steady flow of an ideal fluid.

Motion of Water Jet. Consider the horizontal motion by referring to Fig. *a*.

$\xrightarrow{+} s_x = (s_o)_x + (v_o)_x t$

$$15\text{ ft} = 0 + (V_A \cos 30°)t \qquad \textbf{(1)}$$

Referring to Fig. *a*, vertical motion gives

$+\uparrow s_y = (s_o)_y + (v_o)_y t + \frac{1}{2}at^2$

$$0 = 0 + (V_A \sin 30°)t + \frac{1}{2}(-32.2\text{ ft/s}^2)t^2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$V_A = 23.62\text{ ft/s} \qquad t = 0.7334\text{ s}$$

Bernoulli Equation. Since the water jet from A and B is free flow, $p_A = p_B = 0$. If the datum passes through A and B, $z_A = z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{(23.62\text{ ft/s})^2}{2g} + 0 = 0 + \frac{V_B^2}{2g} + 0$$

$$V_B = 23.62\text{ ft/s}$$

The discharge of the flow is

$$Q = V_A A_A = (23.62\text{ ft/s})\left[(2\text{ in}^2)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2\right] = 0.3280\text{ ft}^3/\text{s}$$

Take the control volume to be the portion of water striking the wall. Its free-body diagram is shown in Fig. *b*.

Linear Momentum. Here, $\mathbf{V}_B$ is perpendicular to the wall. Since the flow is steady incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\mathcal{V} + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

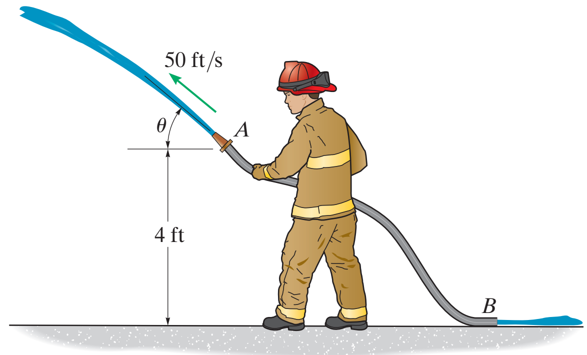
$$\Sigma F_n = 0 + (-V_B)\rho(-Q)$$

$$F_n = (23.62\text{ ft/s})\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)(0.3280\text{ ft}^3/\text{s})$$

$$F_n = 15.0\text{ lb} \qquad \textbf{Ans.}$$

Ans:
15.0 lb

***6–44.** The 150-lb fireman is holding a hose that has a nozzle diameter of 1 in. If the nozzle velocity of the water is 50 ft/s, determine the resultant normal force acting on both the man's feet at the ground when $\theta = 30^\circ$. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



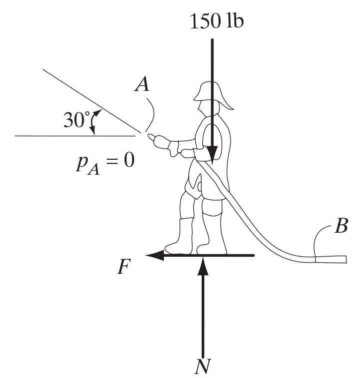
SOLUTION

The discharge of the flow is

$$Q_A = V_A A_A$$

$$Q_A = (50\text{ ft/s})\left[\pi\left(\frac{0.5}{12}\text{ ft}\right)^2\right]$$

$$Q_A = 0.2727\text{ ft}^3/\text{s}$$



The free-body diagram of the control volume is shown in Fig. *a*.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d\mathcal{V} + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

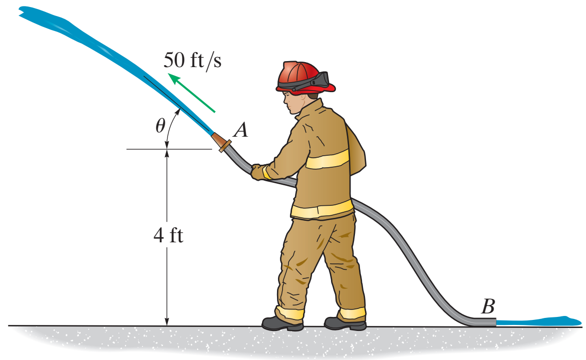
or

$$+\uparrow\Sigma F_y = 0 + (V_A)_y\rho Q$$

$$N - 150\text{ lb} = 50\text{ ft/s}\sin 30^\circ\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)(0.2727\text{ ft}^3/\text{s})$$

$$N = 163\text{ lb}$$ **Ans.**

6–45. The 150-lb fireman is holding a hose that has a nozzle diameter of 1 in. If the velocity of the water is 50 ft/s, determine the resultant normal force acting on both the man's feet at the ground as a function of u. Plot this normal reaction (vertical axis) versus u for 0° 6 θ 6 30°. Give values for increments of $\Delta\theta = 5°$. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



SOLUTION

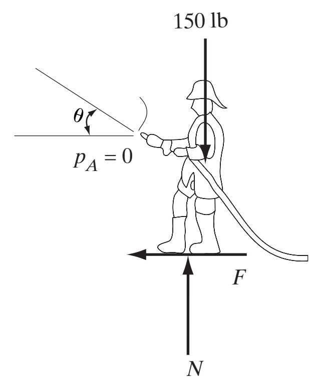
The discharge of the flow is

$$Q = V_A A_A = (50\text{ ft/s})\left[\pi\left(\frac{0.5}{12}\text{ ft}\right)^2\right] = 0.2727\text{ ft}^3/\text{s}$$

Here the flow is steady. Applying linear momentum equation.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

Writing the vertical scalar component of this equation by referring to the *FBD* of the control volume shown in Fig. *a*.



(a)

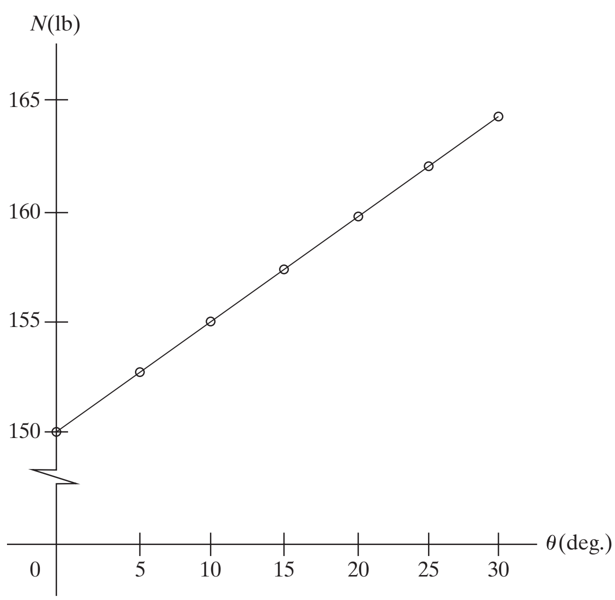
$$+\uparrow \Sigma F_y = 0 + (V_A)_y \rho(V_A A_A) + 0$$

$$N - 150\text{ lb} = [(50\text{ ft/s})\sin\theta]\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)(0.2727\text{ ft}^3/\text{s})$$

$$N = (150 + 26.4\sin\theta)\text{ lb where } \theta \text{ is in deg.}$$ **Ans.**

The plot of N vs θ is shown in Fig. *a*

θ(deg.)	0	5	10	15	20	25	30
N(lb)	150	152.30	154.59	156.84	159.04	161.17	163.21

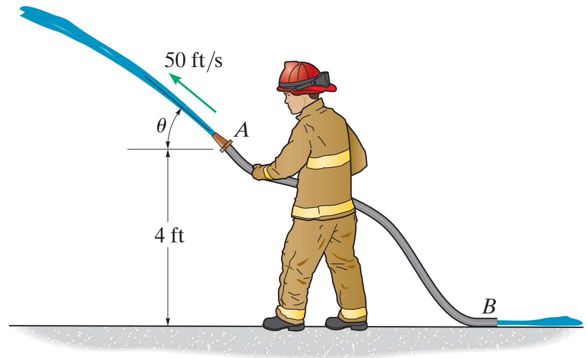


(b)

Note: See solution 6–44 regarding the effects of hose tension.

Ans:
$N = (150 + 26.4\sin\theta)$ lb

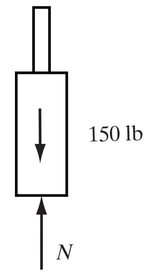
6–46. The 150-lb fireman is holding a hose that has a nozzle diameter of 1 in. If the velocity of the water is 50 ft/s, determine the resultant normal force acting on both the man's feet at the ground if he holds the hose directly over his head at $\theta = 90°$. Neglect the weight of the hose, the water within it, and the normal reaction of the hose on the ground.



SOLUTION

The flow is

$$Q = V_A A_A = (50 \text{ ft/s})\left[\pi\left(\frac{0.5}{12}\text{ ft}\right)^2\right] = 0.2727 \text{ ft}^3/\text{s}$$



Linear momentum

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\mathcal{V} + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

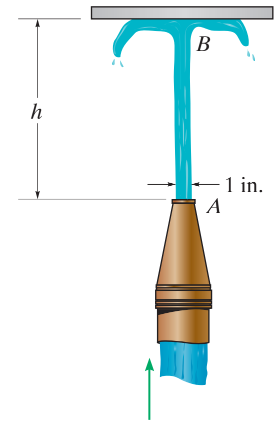
$$+\uparrow \Sigma F_y = 0 + (V_A)_y \rho(Q)$$

$$N - 150 \text{ lb} = (50 \text{ ft/s})\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.2727 \text{ ft}^3/\text{s})$$

$$N = 176 \text{ lb}$$ **Ans.**

Ans:
176 lb

6–47. Water at A flows out of the 1-in.-diameter nozzle at 8 ft/s and strikes the 0.5-lb plate. Determine the height h above the nozzle at which the plate can be supported by the water jet.

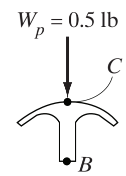


SOLUTION

We consider steady flow of an ideal fluid.

Discharge.

$$Q = V_A A_A = (8\text{ ft/s})\left[\pi\left(\frac{0.5}{12}\text{ ft}\right)^2\right] = 0.04363\text{ ft}^3/\text{s}$$



(a)

Take the control volume of the plate and portion of water striking it. Its free-body diagram is shown in Fig. *a*. Since the jet has free flow, the pressure at any point is zero gauge.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d\forall + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$+\downarrow\Sigma F_y = 0 + (-V_B)\rho(-Q)$$

$$0.5\text{ lb} = (-V_B)\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)(-0.04363\text{ ft}^3/\text{s})$$

$$V_B = 5.913\text{ ft/s}$$

Bernoulli Equation. If the datum coincides with the horizontal line through A, $z_B = h$ and $z_A = 0$.

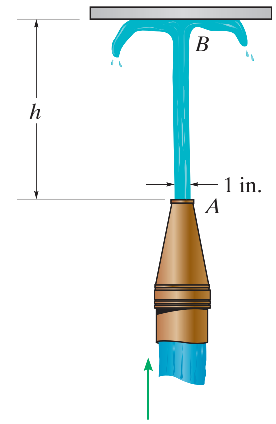
$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{(8\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0 = 0 + \frac{(5.913\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + h$$

$$h = 0.4508\text{ ft} = 0.451\text{ ft}$$ **Ans.**

Ans:
0.451 ft

***6–48.** Water at A flows out of the 1-in.-diameter nozzle at 18 ft/s. Determine the weight of the plate that can be supported by the water jet $h = 2$ ft above the nozzle.



SOLUTION

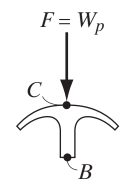
We consider steady flow of an ideal fluid.

Bernoulli Equation. Since the jet is free flow, the pressure at any point is zero gauge. If the datum passes through A, $z_A = 0$ and $z_B = 2$ ft.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$0 + \frac{(18\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0 = 0 + \frac{V_B^2}{2(32.2\text{ ft/s}^2)} + 2\text{ ft}$$

$$V_B = 13.97\text{ ft/s}$$



(a)

The discharge is

$$Q = V_A A_A = (18\text{ ft/s})\left[\pi\left(\frac{0.5}{12}\text{ ft}\right)^2\right] = 0.09817\text{ ft}^3/\text{s}$$

Take the control volume as the plate and a portion of water striking it. Its free-body diagram is shown in Fig. *a*.

Linear Momentum. Since the flow is steady incompressible,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

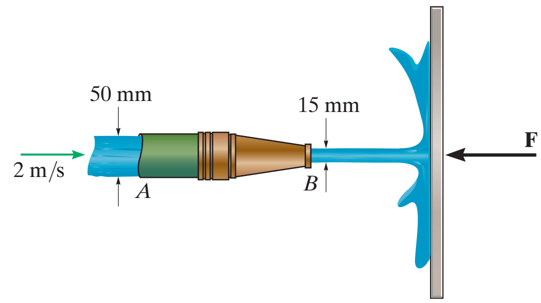
or

$$+\downarrow \Sigma F_y = 0 + (-V_B)\rho(-Q)$$

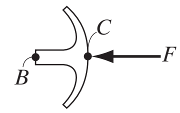
$$W_p = (-13.97\text{ ft/s})\left(\frac{62.4\text{ lb ft}^3}{32.2\text{ ft/s}^2}\right)(-0.09817\text{ ft}^3/\text{s})$$

$$W_p = 2.658\text{ lb} = 2.66\text{ lb} \qquad \textbf{Ans.}$$

6–49. Water flows through the hose with a velocity of 2 m/s. Determine the force **F** needed to keep the circular plate moving to the right at 2 m/s.



SOLUTION



(a)

We consider steady flow of an ideal fluid.

Take the control volume as the plate and a portion of water striking it.

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(2\text{ m/s})\left[\pi(0.025\text{ m})^2\right] + V_B\left[\pi(0.0075\text{ m})^2\right] = 0$$

$$V_B = 22.22\text{ m/s}$$

Relative Velocity. Relative to the control volume, the velocity at B is

$$\overset{+}{\rightarrow} \qquad V_{f/cs} = V_f - V_{cv} = 22.22\text{ m/s} - 2\text{ m/s} = 20.22\text{ m/s}$$

Thus, the flow onto the plate is

$$Q_{f/cs} = V_{f/cs} A_B = (20.22\text{ m/s})\left[\pi(0.0075\text{ m})^2\right] = 0.003574\text{ m}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V} \cdot d\mathbf{A}$$

or

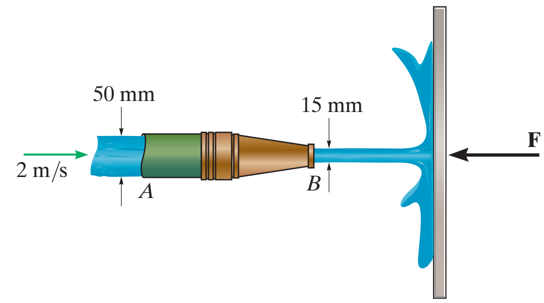
$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (V_{f/cs})_{B_x} \rho(-Q_{f/cs})$$

$$-F = (20.22\text{ m/s})(1000\text{ kg/m}^3)(-0.003574\text{ m}^3/\text{s})$$

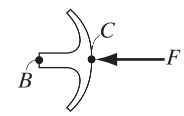
$$F = 72.3\text{ N} \qquad \textbf{Ans.}$$

Ans:
72.3 N

6–50. Water flows through the hose with a velocity of 2 m/s. Determine the force **F** needed to keep the circular plate moving to the left at 2 m/s.



SOLUTION



(a)

We consider steady flow of an ideal fluid.

Take the control volume as the plate and a portion of water striking it.

Continuity Equation.

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(2 \text{ m/s})\left[\pi(0.025 \text{ m})^2\right] + V_B\left[\pi(0.0075 \text{ m})^2\right] = 0$$

$$V_B = 22.22 \text{ m/s}$$

Relative Velocity. Relative to the control volume, the velocity at B is

$$\overset{+}{\rightarrow} (V_{f/cs})_B = V_f - V_{cs} = 22.22 \text{ m/s} - (-2 \text{ m/s}) = 24.22 \text{ m/s}$$

Thus, the relative flow onto the plate is

$$Q_{f/cs} = (V_{f/cs})_C A_B = (24.22 \text{ m/s})\left[\pi(0.0075 \text{ m})^2\right] = 0.004280 \text{ m}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V} \rho d\forall + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}$$

or

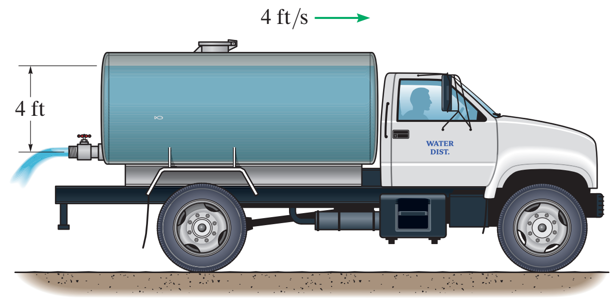
$$-F = 0 + (V_{f/cs})_{B_x} \rho(-Q_{f/cs})$$

$$-F = (24.22 \text{ m/s})(1000 \text{ kg/m}^3)(-0.004280 \text{ m}^3/\text{s})$$

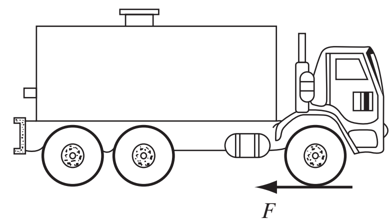
$$F = 104 \text{ N} \qquad \textbf{Ans.}$$

Ans:
104 N

6–51. The large water truck releases water at the rate of $45\ \text{ft}^3/\text{min}$ through the 3-in.-diameter pipe. If the depth of the water in the truck is 4 ft, determine the frictional force the road has to exert on the tires to prevent the truck from rolling. How much force does the water exert on the truck if the truck is moving forward at a constant velocity of 4 ft/s and the flow is maintained at $45\ \text{ft}^3/\text{min}$?



SOLUTION



(a)

We consider steady flow of an ideal fluid.

For the case when the truck is required to be stationary, the control volume is the entire truck and its contents. Here the flow is steady. The *FBD* of the control volume is shown in Fig. *a*.

The discharge is

$$Q = \left(45\frac{\text{ft}^3}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 0.75\ \text{ft}^3/\text{s}$$

Thus, the velocity at the outlet is

$$Q = V_{\text{out}}A_{\text{out}};\qquad 0.75\ \text{ft}^3/\text{s} = V_{\text{out}}\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right]\qquad V_{\text{out}} = 15.28\ \text{ft/s}$$

Applying the linear momentum equation by referring to Fig. *a*,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}}\mathbf{V}\rho_w d\forall + \int_{\text{cs}}\mathbf{V}\rho_w\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along x axis,

$$\overset{+}{\leftarrow}\Sigma F_x = 0 + V_{\text{out}}\rho_w V_{\text{out}}A_{\text{out}}$$

$$F = (15.28\ \text{ft/s})\left(\frac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(0.75\ \text{ft}^3/\text{s}) = 22.2\ \text{lb}\qquad \textbf{Ans.}$$

For the case when the truck is moving with a constant velocity, the same control volume is considered, but it moves with this constant velocity. Then, the flow measured relative to the control volume is steady. From the discharge, the relative velocity at the outlet is

$$Q = (V_{\text{out/cs}})A_{\text{out}};\qquad 0.75\ \text{ft}^3/\text{s} = V_{\text{out/cs}}\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right]$$

$$V_{\text{out/cs}} = 15.28\ \text{ft/s}$$

Applying the linear momentum equation by referring to Fig. *a*, but this time using the relative velocity,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}}\mathbf{V}_{w/\text{cs}}\rho_w d\forall + \int_{\text{cs}}\mathbf{V}_{w/\text{cs}}\rho_w\mathbf{V}_{w/\text{cs}}\cdot d\mathbf{A}$$

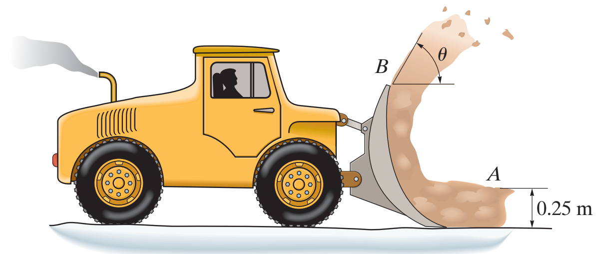
Applying the scalar component of this equation along x axis,

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + (V_{\text{out/cs}})(\rho_w)(V_{\text{out/cs}}A_{\text{out}})$$

$$F = (15.28\ \text{ft/s})\left(\frac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(0.75\ \text{ft}^3/\text{s}) = 22.2\ \text{lb}\qquad \textbf{Ans.}$$

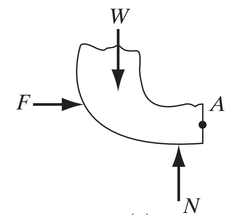
Ans:
22.2 lb

***6–52.** A plow located on the front of a truck scoops up a liquid slush at the rate of $12\ \text{ft}^3/\text{s}$ and throws it off perpendicular to its motion, $\theta = 90°$. If the truck is traveling at a constant speed of $14\ \text{ft/s}$, determine the resistance to motion caused by the shoveling. The specific weight of the slush is $\gamma_s = 5.5\ \text{lb/ft}^3$.



SOLUTION

We consider steady flow of an ideal fluid.



(a)

Take the slush in context with the blade as the control volume.

Relative Velocity. Since the slush is at rest before it enters control volume, then the velocity at A relative to control volume is

$$\overset{+}{\rightarrow} \qquad (V_{f/cs})_A = V_f - V_{cs} = 0 - 14\ \text{ft/s} = 14\ \text{ft/s} \leftarrow$$

Linear Momentum. Here, $Q_{f/cs} = 12\ \text{ft}^3/\text{s}$ and $(V_{f/cs})_B = (V_{f/cs})_A = 14\ \text{ft/s}$. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

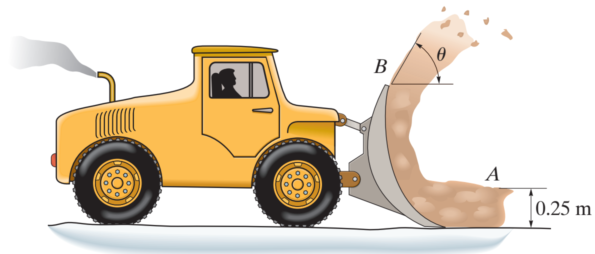
or

$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (-V_A)\rho(-Q)$$

$$-F = (-14\ \text{ft/s})\left(\frac{5.5\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(-12\ \text{ft}^3/\text{s})$$

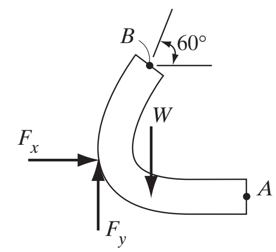
$$F = -28.69\ \text{lb} = 28.7\ \text{lb} \leftarrow \qquad \textbf{Ans.}$$

6–53. The truck is traveling forward at 5 m/s, shoveling a liquid slush that is 0.25 m deep. If the slush has a density of 125 kg/m^3 and is thrown upwards at an angle of $\theta = 60°$ from the 3-m-wide blade, determine the traction force of the wheels on the road necessary to maintain the motion. Assume that the slush is thrown off the shovel at the same rate as it enters the shovel.



SOLUTION

We consider steady flow of an ideal fluid.



(a)

Take the slush in context with the blade as the control volume.

Relative Velocity. Since the slush is at rest before it enters the control volume, then the velocity at A relative to the control volume is

$$\xrightarrow{+} \quad V_{f/cs} = V_f - V_{cs} = 0 - 5\text{ m/s} = 5\text{ m/s} \leftarrow$$

Thus, the flow rate of snow onto the shovel is

$$Q_{f/cs} = V_{f/cs} A_A = (5\text{ m/s})\left[0.25\text{ m}(3\text{ m})\right] = 3.75\text{ m}^3/\text{s}$$

Linear Momentum. Here, $\left(V_{f/cs}\right)_B = \left(V_{f/cs}\right)_A = 5$ ft/s. Referring to the free-body diagram of the control volume in Fig. *a*,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

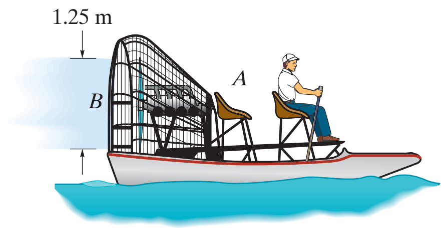
$$\xrightarrow{+} \Sigma F_x = 0 + \left(-V_{f/cs}\right)_{A_x}\rho\left(-Q_{f/cs}\right) + \left(V_{f/cs}\right)_{B_x}\rho\left(Q_{f/cs}\right)$$

$$F_x = 0 + (-5\text{ m/s})\left(125\text{ kg/m}^3\right)\left(-3.75\text{ m}^3/\text{s}\right) + (5\text{ m/s}\cos 60°)\left(125\text{ kg/m}^3\right)\left(3.75\text{ m}^3/\text{s}\right)$$

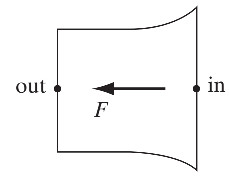
$$F_x = 3.52\text{ kN} \qquad \textbf{Ans.}$$

Ans:
3.52 kN

6–54. The boat is powered by the fan, which develops a slipstream having a diameter of 1.25 m. If the fan ejects air with an average velocity of 40 m/s, measured relative to the boat, and the boat is traveling with a constant velocity of 8 m/s, determine the force the fan exerts on the boat. Assume that the air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$ and that the entering air at A is essentially at rest relative to the ground.



SOLUTION



(a)

We consider steady flow of an ideal fluid.

Relative Velocity. Since the air is at rest before it enters the control volume, then the inlet velocity relative to the control volume is

$$\xrightarrow{+} \qquad (V_{f/\text{cs}})_A = V_f - V_{\text{cs}} = 0 - 8\ \text{m/s} = 8\ \text{m/s} \leftarrow$$

The outlet velocity relative to the control volume is $(V_{f/\text{cv}})_{\text{out}} = 40\ \text{m/s}$. Then, the flow of air in and out of the fan is

$$Q_{f/\text{cs}} = (V_{f/\text{cs}})_B A_B = (40\ \text{m/s})\left[\pi(0.625\ \text{m})^2\right] = 49.09\ \text{m}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\mathcal{V} + \int_{\text{cs}} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

or

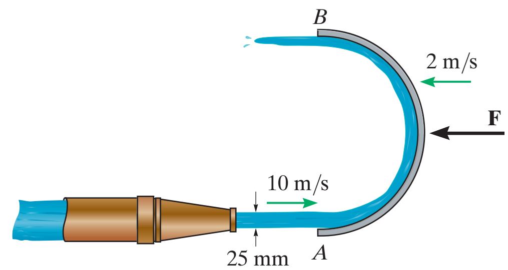
$$\overset{+}{\leftarrow} \Sigma F_x = 0 + (V_{f/\text{cs}})_A \rho(-Q_{f/\text{cs}}) + (V_{f/\text{cs}})_B \rho(Q_{f/\text{cs}})$$

$$= (1.22\ \text{kg/m}^3)\left[(8\ \text{m/s})(-49.09\ \text{m}^3/\text{s}) + (40\ \text{m/s})(49.09\ \text{m}^3/\text{s})\right]$$

$$= 1.92\ \text{kN}$$ **Ans.**

Ans:
1.92 kN

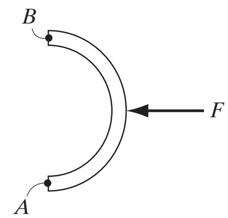
6–55. A 25-mm-diameter stream flows at 10 m/s against the vane and is deflected 180° as shown. If the vane is moving to the left at 2 m/s, determine the horizontal force F of the vane on the water.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the blade.



(a)

Relative Velocity. Relative to the control volume, the velocity at A is

$$\xrightarrow{+} \qquad (V_{f/cs})_A = V_f - V_{cs} = 10\text{ m/s} - (-2\text{ m/s}) = 12\text{ m/s} \rightarrow$$

Thus, the flow rate onto the vane is

$$Q_{f/cs} = (V_{f/cs})_A A_A = (12\text{ m/s})\left[\pi(0.0125\text{ m})^2\right] = 0.005890\text{ m}^3/\text{s}$$

Linear Momentum. Here, $(V_{f/cs})_B = (V_{f/cs})_A = 12\text{ m/s}$ (Bernoulli equation). Referring to the free-body diagram of the control volume in Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

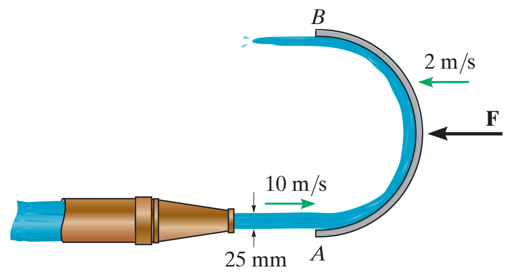
$$\xrightarrow{+} \Sigma F_x = 0 + (V_{f/cs})_A \rho(-Q_{f/cs}) + (-V_{f/cs})_B \rho(Q_{f/cs})$$

$$-F = (1000\text{ kg/m}^3)\left[(12\text{ m/s})(-0.005890\text{ m}^3/\text{s}) + (-12\text{ m/s})(0.005890\text{ m}^3/\text{s})\right]$$

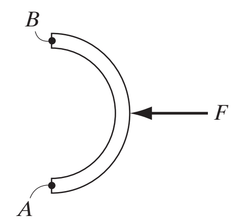
$$F = 141\text{ N} \qquad \textbf{Ans.}$$

Ans:
141 N

***6–56.** Solve Prob. 6–55 if the blade is moving *to the right* at 2 m/s. At what speed must the blade be moving to the right to reduce the force F to zero?



SOLUTION



(a)

Consider the control volume as the water on the blade. The velocity of the water at A relative to the control volume is

$$(\xrightarrow{+})(V_{f/cs})_A = 10\text{ m/s} - 2\text{ m/s} = 8\text{ m/s}\rightarrow$$

To satisfy the Bernoulli's equation, $(V_{f/cs})_B = 8\text{ m/s}\leftarrow$ for small elevations. The flow is steady relative to control volume.

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}_{f/cv}\,\rho\, d\forall + \int_{cs}\mathbf{V}_{f/cs}\,\rho\,\mathbf{V}_{f/cs}\cdot d\mathbf{A}$$

Writing the horizontal scalar component of this equation by referring to the *FBD* of the control volume shown in Fig. *a*

$$\xrightarrow{+}\Sigma F_x = 0 + (V_{f/cs})_A\,\rho\left[-(V_{f/cs})_A A_A\right] + \left[-(V_{f/cs})_B\right]\rho\left[(V_{f/cs})_B A_B\right]$$

However, $Q = (V_{f/cs})_A A_A = (V_{f/cs})_B A_B$ and $(V_{f/cs})_B = (V_{f/cs})_A$. Then

$$-F = -2\rho(V_{f/cs})_A\left[(V_{f/cs})_A A_A\right]$$

$$F = 2\rho(V_{f/cs})_A^2 A_A \qquad \textbf{(1)}$$

$$F = 2(1000\text{ kg/m}^3)(8\text{ m/s})^2\left[\pi(0.0125\text{ m})^2\right]$$

$$= 62.8\text{ N} \qquad \textbf{Ans.}$$

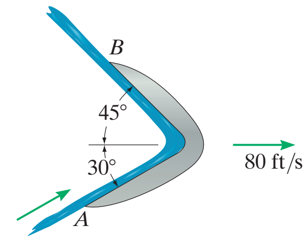
By inspecting Eq (1), $F = 0$ if $(V_{f/cs})_A = 0$. Then

$$\xrightarrow{+}(V_{f/cs})_A = V_w - V_b$$

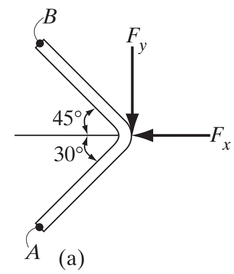
$$0 = 10\text{ m/s} - V_b$$

$$V_b = 10\text{ m/s}\rightarrow \qquad \textbf{Ans.}$$

6–57. The vane is moving at 80 ft/s when a jet of water having a velocity of 150 ft/s enters at A. If the cross-sectional area of the jet is 1.5 in^2, and it is diverted as shown, determine the horsepower developed by the water on the blade. 1 hp = 550 ft·lb/s.



SOLUTION



We consider steady flow of an ideal fluid.

Take the control volume as the water on the blade.

Relative Velocity. Applying the relative velocity equation to determine the velocity relative to the vane, $V_{A/cs}$, and the angle θ, of the jet in a stationary frame,

$$\mathbf{V}_{A/cs} = \mathbf{V}_A - \mathbf{V}_{cs}$$

$(\xrightarrow{+})$ $\quad V_{A/cs}\cos 30° = 150\cos\theta - 80 \qquad$ **(1)**

$(+\uparrow)$ $\quad V_{A/cs}\sin 30° = 150\sin\theta \qquad$ **(2)**

Solving Eqs. (1) and (2),

$$\theta = 14.53° \qquad V_{A/cs} = 75.29 \text{ ft/s}$$

Here, $(V_{f/cs})_A = V_{A/cs} = 75.29$ ft/s. Thus,the relative flow rate at the vane is

$$Q_{f/cs} = (V_{f/cs})_A A = (75.29 \text{ ft/s})\left[1.5 \text{ in}^2\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2\right] = 0.7842 \text{ ft}^3/\text{s}$$

Linear Momentum. Here, $(V_{f/cs})_A = V_{A/cs} = 75.29$ ft/s (Bernoulli equation). Referring to the free-body diagram of the control volume in Fig. *a*,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d\mathcal{V} + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$\xrightarrow{+}\Sigma F_x = 0 + (V_{A/cs})_x\,\rho(-Q_{f/cs}) + (-V_{B/cs})_x\,\rho(Q_{f/cs})$$

$$-F_x = \left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left[(75.29 \text{ ft/s}\cos 30°)(-0.7842 \text{ ft}^3/\text{s}) + (-75.29 \text{ ft/s}\cos 45°)(0.7842 \text{ ft}^3/\text{s})\right]$$

$$F_x = 179.99 \text{ lb}$$

Thus, the power of the water jet can be determined from

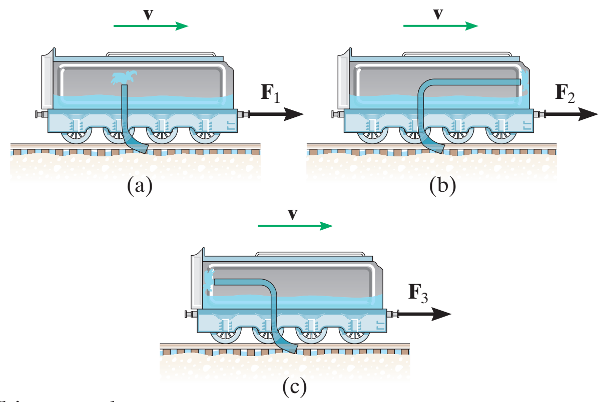
$$\dot{W} = \mathbf{F}\cdot\mathbf{V} = F_x V = (179.99 \text{ lb})(80 \text{ ft/s})$$

$$= \left(14399.40\frac{\text{ft}\cdot\text{lb}}{\text{s}}\right)\left(\frac{1 \text{ hp}}{550 \text{ ft}\cdot\text{lb/s}}\right)$$

$$= 26.2 \text{ hp}$$ **Ans.**

Ans:
26.2 hp

6–58. The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



SOLUTION

The control volume considered consists of the car and the scoop. This control volume has only inlet control surface (the scoop) but no outlet control surface. Since this same control volume can be used for cases a, b, and c, $F_1 = F_2 = F_3 = F$. Here,

$$\dot{m}a = \rho_w VA \quad \dot{m}_f = 0 \quad V_e = 0 \quad \frac{dV_{cv}}{dt} = 0 \text{ (constant velocity)}$$

Along x axis,

$$\overset{+}{\rightarrow} \Sigma F_x = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - \left(\dot{m}_a + \dot{m}_f\right)V_e$$

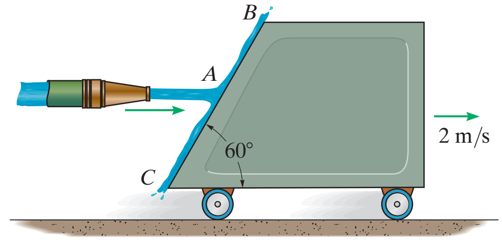
$$F = 0 + \rho_w VAV = \rho_w AV^2$$

Therefore

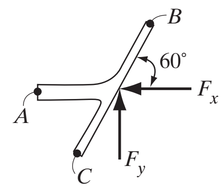
$$F_1 = F_2 = F_3 = \rho_w AV^2 \qquad \textbf{Ans.}$$

Ans:
$F_1 = F_2 = F_3 = \rho_w AV^2$

6–59. Flow from the water stream strikes the inclined surface of the cart. Determine the power produced by the stream if, due to rolling friction, the cart moves to the right with a constant velocity of 2 m/s. The discharge from the 50-mm-diameter nozzle is 0.04 m^3/s. One-fourth of the discharge flows down the incline, and three-fourths flows up the incline.



SOLUTION



(a)

We consider steady flow of an ideal fluid.

Take the control volume as a portion of water striking the cart.

Relative Velocity. The velocity of the jet at A is

$$V_A = \frac{Q}{A_A} = \frac{0.04\ \text{m}^3/\text{s}}{\pi(0.025\ \text{m})^2} = 20.37\ \text{m/s}$$

Thus, the velocity at A relative to the control volume is

$$\overset{+}{\rightarrow} \qquad V_{A/\text{cs}} = V_A - V_{\text{cs}} = 20.37\ \text{m/s} - 2\ \text{m/s} = 18.37\ \text{m/s}$$

Here, $V_{B/\text{cs}} = V_{C/\text{cs}} = V_{A/\text{cs}} = 18.37\ \text{m/s}$ can be determined using the Bernoulli equation and neglecting the elevation change. Thus, the relative flow at A, B, and C are

$$Q_{A/\text{cs}} = V_{A/\text{cs}}A_A = (18.37\ \text{m/s})\left[\pi(0.025\ \text{m})^2\right] = 0.03607\text{m}^3/\text{s}$$

$$Q_{B/\text{cs}} = \frac{3}{4}\left(Q_{A/\text{cs}}\right) = \frac{3}{4}\left(0.03607\ \text{m}^3/\text{s}\right) = 0.02705\ \text{m}^3/\text{s}$$

$$Q_{C/\text{cs}} = \frac{1}{4}\left(Q_{A/\text{cs}}\right) = \frac{1}{4}\left(0.03607\ \text{m}^3/\text{s}\right) = 0.009018\ \text{m}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$\overset{+}{\rightarrow}\Sigma F_x = \rho\left[Q_{B/\text{cs}}(V_{B/\text{cs}})_x + Q_{C/\text{cs}}(V_{C/\text{cs}})_x - Q_{A/\text{cs}}(V_{A/\text{cs}})_x\right]$$

$$-F_x = (1000\ \text{kg/m}^3)\left[\left(0.02705\ \text{m}^3/\text{s}\right)(18.37\ \text{m/s}\cos 60°) + \left(0.009018\ \text{m}^3/\text{s}\right)(-18.37\ \text{m/s}\ \cos\ 60°) - \left(0.03607\ \text{m}^3/\text{s}\right)(18.37\ \text{m/s})\right]$$

$$F_x = 497.04\ \text{N}$$

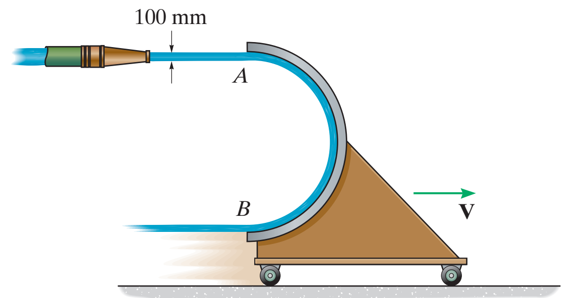
Thus, the power of the jet stream can be determined from

$$\dot{W} = \mathbf{F}\cdot\mathbf{V} = F_xV = (497.04\ \text{N})(2\ \text{m/s})$$

$$= 994.09\ \text{W} = 994\ \text{W} \qquad \textbf{Ans.}$$

Ans:
994 W

***6–60.** Water flows at $0.1\ \text{m}^3/\text{s}$ through the 100-mm-diameter nozzle and strikes the vane on the 150-kg cart, which is originally at rest. Determine the velocity of the cart 3 seconds after the jet strikes the vane.



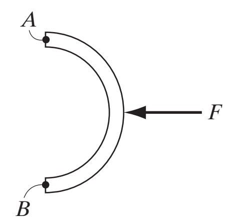
SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the cart.

Relative Velocity. The velocity of the jet at A is

$$V_A = \frac{Q}{A_A} = \frac{0.1\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2} = 12.73\ \text{m/s}$$



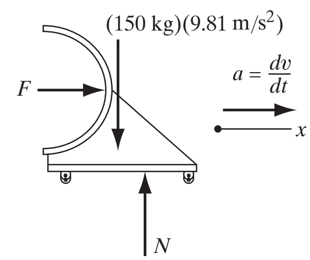
(a)

Thus, the velocity at A relative to the control volume is

$$\overset{+}{\rightarrow} \qquad V_{A/\text{cs}} = V_A - V_{\text{cs}} = 12.73 - V \rightarrow$$

Here, $(V_{f/\text{cs}})_A = (V_{f/\text{cs}})_B = V_{A/\text{cv}}$. Thus, the relative flow rate onto the vane is

$$Q_{f/\text{cs}} = (V_{f/\text{cs}})_A A_A = (12.73 - V)\left[\pi(0.05\ \text{m})^2\right] = 2.5(10^{-3})\pi(12.73 - V)$$



(b)

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho d\Psi + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (-V_{f/\text{cs}})_B\,\rho(Q_{f/\text{cs}}) + (V_{f/\text{cs}})_A\,\rho(-Q_{f/\text{cs}})$$

$$-F = 1000\ \text{kg/m}^3\left[-(12.73 - V)(2.5(10^{-3})\pi(12.73 - V) + (12.73 - V)(-2.5(10^{-3})\pi(12.73 - V)\right]$$

$$F = 5\pi(12.73 - V)^2$$

Equation of Motion. Referring to the free-body diagram of the cart in Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = ma; \qquad 5\pi(12.73 - V)^2 = (150\ \text{kg})\left(\frac{dV}{dt}\right)$$

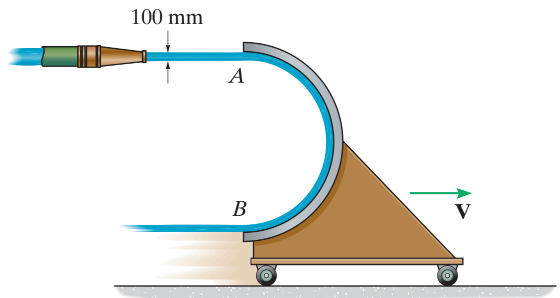
$$\int_0^{3\,\text{s}} dt = \frac{30}{\pi}\int_0^V \frac{dV}{(12.73 - V)^2}$$

$$t\Big|_0^{3\,\text{s}} = \frac{30}{\pi}\left(\frac{1}{12.73 - V}\right)\Bigg|_0^V$$

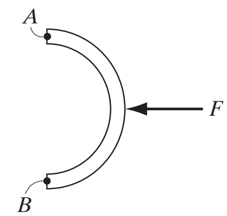
$$3 = \frac{30}{\pi}\left(\frac{1}{12.73 - V} - \frac{1}{12.73}\right)$$

$$V = 10.19\ \text{m/s} = 10.2\ \text{m/s} \qquad \textbf{Ans.}$$

6–61. Water flows at $0.1\ \text{m}^3/\text{s}$ through the 100-mm-diameter nozzle and strikes the vane on the 150-kg cart, which is originally at rest. Determine the acceleration of the cart when it attains a velocity of 2 m/s.



SOLUTION



(a)

The velocity of the jet at A can be determined from the discharge.

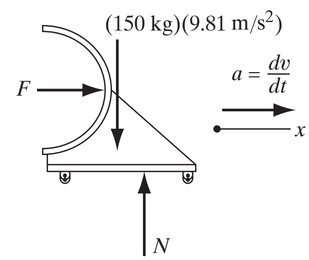
$$Q = V_A A_A; \quad 0.1\ \text{m}^3/\text{s} = V_A\left[\pi(0.05\ \text{m})^2\right] \quad V_A = 12.73\ \text{m/s}$$

The velocity at A relative to the control volume is

$$\xrightarrow{+} \quad (V_{f/\text{cs}})_A = V_A - V_{\text{cs}} = (12.73 - V)\ \text{m/s} \rightarrow$$

To satisfy Bernoulli's equation $(V_{f/\text{cs}})_B = (12.73 - V)\ \text{m/s} \leftarrow$ for small equations The flow is steady relative to control volume.

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}_{f/\text{cv}}\,\rho\, d\forall + \int_{\text{cs}} \mathbf{V}_{f/\text{cs}}\,\rho\,\mathbf{V}_{f/\text{cs}} \cdot d\mathbf{A}$$



(b)

Writing the horizontal scalar component of this equation by referring to the *FBD* of the control volume shown in Fig. *a*,

$$\xrightarrow{+} \qquad \Sigma F_x = 0 + (V_{f/\text{cs}})_A\,\rho\left[-(V_{f/\text{cs}})_A A_A\right] + \left[-(V_{f/\text{cs}})_B\right]\rho\left[(V_{f/\text{cs}})_B A_B\right]$$

However, $Q = (V_{f/\text{cs}})_A A_A = (V_{f/\text{cs}})_B A_B$ and $(V_{f/\text{cs}})_B = (V_{f/\text{cs}})_A$. Then

$$-F = -2\rho(V_{f/\text{cs}})_A\left[(V_{f/\text{cs}})_A A_A\right]$$

$$F = 2\rho(V_{f/\text{cs}})_A^2 A_A$$

$$F = 2(1000\ \text{kg/m}^3)(12.73 - V)^2\left[\pi(0.05\ \text{m})^2\right]$$

$$= \left[5\pi(12.73 - V)^2\right]\ \text{N}$$

Referring to the *FBD* of the cart Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = ma; \qquad 5\pi(12.73 - V)^2 = 150\,a$$

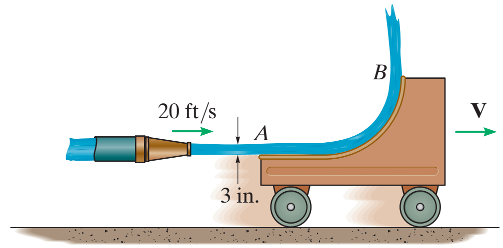
$$a = \left[\frac{\pi}{30}(12.73 - V)^2\right]\ \text{m/s}^2$$

When $V = 2\text{m/s}$,

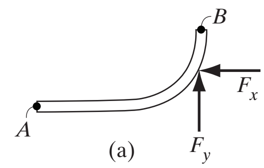
$$a = \frac{\pi}{30}(12.73 - 2)^2 = 12.06\ \text{m/s}^2 = 12.1\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$12.1\ \text{m/s}^2$

6–62. Determine the rolling resistance on the wheels if the cart moves to the right with a constant velocity of $V_c = 4$ ft/s when the vane is struck by the water jet. The jet flows from the nozzle at 20 ft/s and has a diameter of 3 in.



SOLUTION



(a)

We consider steady flow of an ideal fluid.

Take the control volume as the water on the cart.

Relative Velocity. The velocity at A relative to the control volume is

$$\overset{+}{\rightarrow} \quad V_{A/\text{cs}} = V_A - V_{\text{cv}} = 20 \text{ ft/s} - 4 \text{ ft/s} = 16 \text{ ft/s}$$

Here, $(V_{f/\text{cs}})_{\text{in}} = (V_{f/\text{cs}})_B = V_{A/\text{cs}} = 16$ ft/s (Bernoulli equation). Thus, the relative flow rate onto the vane is

$$Q_{f/\text{cs}} = (V_{f/\text{cs}})_A A_A = (16 \text{ ft/s})\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right] = 0.7856 \text{ ft}^3/\text{s}$$

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

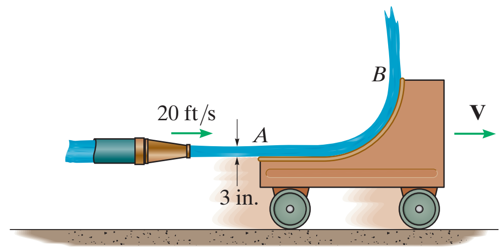
$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (V_{A/\text{cs}})\rho(-Q_{f/\text{cs}})$$

$$-F_x = (16 \text{ ft/s})\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(-0.7854 \text{ ft}^3/\text{s})$$

$$F_x = 24.35 \text{ lb} = 24.4 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
24.4 lb

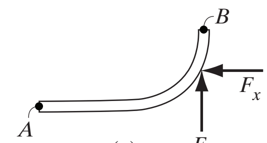
6–63. Determine the velocity of the 50-lb cart in 3 s starting from rest if a stream of water, flowing from the nozzle at 20 ft/s, strikes the vane and is deflected upwards. The stream has a diameter of 3 in. Neglect the rolling resistance of the wheels.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume as the water on the cart.



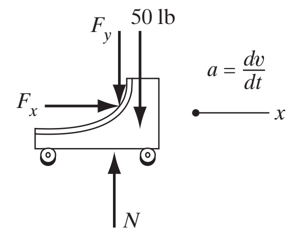
(a)

Relative Velocity. The velocity of the jet at A relative to the control volume is

$$\overset{+}{\rightarrow} \qquad V_{A/cs} = V_A - V_{cs} = (20 - V) \text{ ft/s}$$

Here, $(V_{f/cs})_{in} = (V_{f/cs})_B = V_{A/cs}$. Thus, the relative flow rate onto the vane is

$$Q_{f/cs} = (V_{f/cs})_A A_A = (20 - V)\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right] = 0.015625\pi(20 - V)$$



(b)

Linear Momentum. Referring to the free-body diagram of the control volume in Fig. a,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$\overset{+}{\rightarrow} \Sigma F_x = 0 + (V_{A/cs})\rho(-Q_{f/cs})$$

$$-F_x = (20 - V)\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.015625\,\pi(20 - V))$$

$$F_x = 0.09513(20 - V)^2$$

Equation of Motion. Referring to the free-body diagram of the cart in Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = ma; \qquad 0.09513(20 - V)^2 = \left(\frac{50 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\frac{dV}{dt}$$

$$0.06126\int_0^{3\text{ s}} dt = \int_0^V \frac{dV}{(20 - V)^2}$$

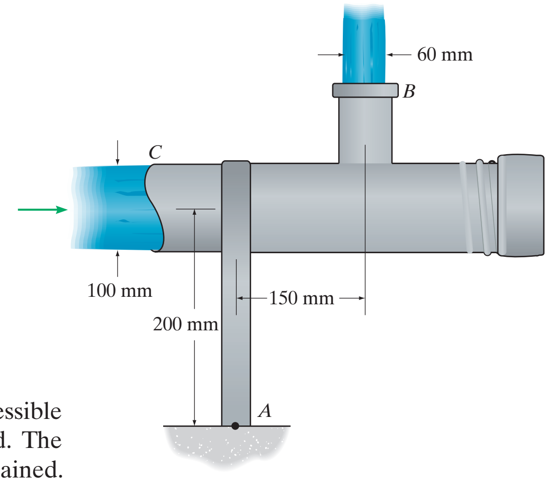
$$0.06126(t)\Big|_0^{3\text{ s}} = \left(\frac{1}{20 - V}\right)\Bigg|_0^V$$

$$0.1838 = \left(\frac{1}{20 - V}\right) - \frac{1}{20}$$

$$V = 15.72 \text{ ft/s} = 15.7 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
15.7 ft/s

***6–64.** Water flows through the Tee fitting at $0.02\ \text{m}^3/\text{s}$. If the water exits the fitting at B to the atmosphere, determine the horizontal and vertical components of force, and the moment that must be exerted on the fixed support at A, in order to hold the fitting in equilibrium. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $p_w = 1000\ \text{kg/m}^3$. Average velocities will be used. The control volume consists of the fitting the fixed support and the water contained. From the discharge,

$$Q = V_C A_C; \qquad 0.02\ \text{m}^3/\text{s} = V_A\left[\pi(0.05\ \text{m})^2\right] \qquad V_C = 2.546\ \text{m/s}$$

$$Q = V_B A_B; \qquad 0.02\ \text{m}^3/\text{s} = V_B\left[\pi(0.03\ \text{m})^2\right] \qquad V_B = 7.074\ \text{m/s}$$

Applying Bernoulli's equation between C and B, with $p_B = p_{\text{atm}} = 0$.

$$\frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{p_C}{9810\ \text{N/m}^3} + \frac{(2.546\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = 0 + \frac{(7.074\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_C = 21.775(10^3)\ \text{N/m}^2$$

Then the pressure force on inlet control surface C is

$$F_C = p_C A_C = \left[21.775(10^3)\ \text{N/m}^2\right]\left[\pi(0.05\ \text{m})^2\right] = 171.02\ \text{N}$$

Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

writing the scalar component of this equation along the x and y axes by referring to the free-body diagram, Fig. *a*

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0 + V_C \rho_w(-V_C A_C)$

$A_x + 171.02\ \text{N} = (2.546\ \text{m/s})(1000\ \text{kg/m}^3)(-0.02\ \text{m}^3/\text{s})$

$A_x = -221.95\ \text{N} = 222\ \text{N} \leftarrow$ **Ans.**

$(+\uparrow)\Sigma F_y = 0 + V_B \rho_w(V_B A_B)$

$A_y = (7.074\ \text{m/s})(1000\ \text{kg/m}^3)(0.02\ \text{m}^3/\text{s})$

$= 141.47\ \text{N} = 141\ \text{N}$ **Ans.**

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{r} \times \mathbf{V})\rho\, d\forall + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V}\cdot d\mathbf{A}$$

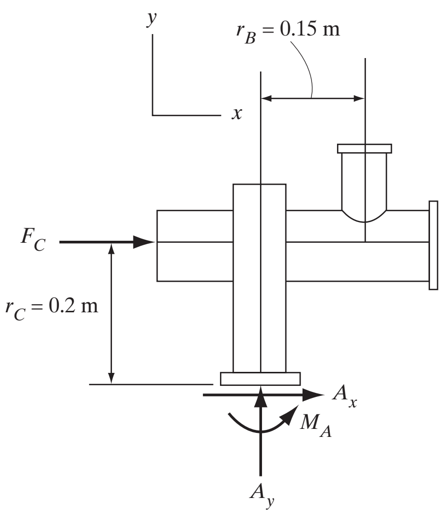
writing the scalar component of this equation about point A by referring to Fig. *a*,

$\circlearrowright + \Sigma M_A = 0 + r_B V_B \rho_w(V_B A_B) - r_C V_C \rho_w(-V_C A_C)$

$M_A - (171.02\ \text{N})(0.2\ \text{m}) = (0.15\ \text{m})(7.074\ \text{m/s})(1000\ \text{kg/m}^3)(0.02\ \text{m}^3/\text{s})$

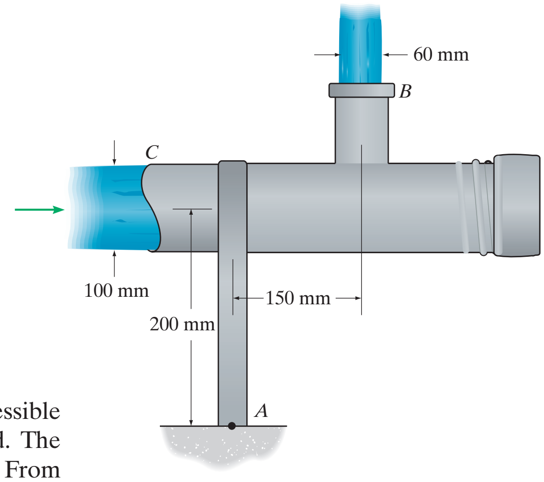
$\qquad -(0.2\ \text{m})(2.546\ \text{m/s})(1000\ \text{kg/m}^3)(-0.02\ \text{m}^3/\text{s})$

$M_A = 65.61\ \text{N}\cdot\text{m} = 65.6\ \text{N}\cdot\text{m}$ **Ans.**



(a)

6–65. Water flows through the Tee fitting at $0.02\ \text{m}^3/\text{s}$. If the pipe at B is extended and the pressure in the pipe at B is 75 kPa, determine the horizontal and vertical components of force, and the moment that must be exerted on the fixed support at A, to hold the fitting in equilibrium. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000\ \text{kg/m}^3$. Average velocities will be used. The control volume consists of the fitting, fixed support and the contained water. From the discharge,

$$Q = V_C A_C; \qquad 0.02\ \text{m}^3/\text{s} = V_C\left[\pi(0.05\ \text{m})^2\right] \qquad V_C = 2.546\ \text{m/s}$$

$$Q = V_B A_B; \qquad 0.02\ \text{m}^3/\text{s} = V_B\left[\pi(0.03\ \text{m})^2\right] \qquad V_B = 7.074\ \text{m/s}$$

Applying Bernoulli's equation between A and B,

$$\frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{p_C}{9810\ \text{N/m}^3} + \frac{(2.546\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0 = \frac{75(10^3)\ \text{N/m}^2}{9810\ \text{N/m}^3} + \frac{(7.074\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0$$

$$p_C = 96.775(10^3)\ \text{N/m}^2$$

Then the pressure forces on the inlet and outlet control surfaces at C and B are

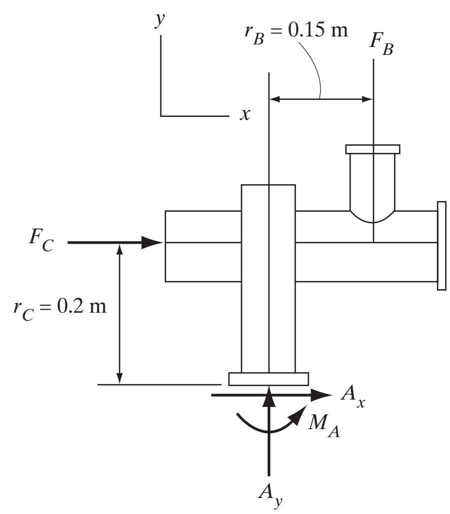
$$F_C = p_C A_C = \left[96.775(10^3)\ \text{N/m}^2\right]\left[\pi(0.05\ \text{m})^2\right] = 760.07\ \text{N}$$

$$F_B = p_B A_B = \left[75(10^3)\ \text{N/m}^2\right]\left[\pi(0.03\ \text{m})^2\right] = 212.06\ \text{N}$$

Applying linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho_w dV + \int_{cs} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

writing the scalar component of this equation along the x and y axes by referring to the free-body diagram, Fig. a,



(a)

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0 + V_A\rho_w(-V_A A_A)$

$A_x + 760.07\ \text{N} = (2.546\ \text{m/s})(1000\ \text{kg/m}^3)(-0.02\ \text{m}^3/\text{s})$

$A_x = -811\ \text{N} = 811\ \text{N} \leftarrow$ **Ans.**

$(+\uparrow)\Sigma F_y = 0 + V_B\rho_w(V_B A_B)$

$A_y - 212.06\ \text{N} = (7.074\ \text{m/s})(1000\ \text{kg/m}^3)(0.02\ \text{m}^3/\text{s})$

$A_y = 353.53\ \text{N} = 354\ \text{N}\uparrow$ **Ans.**

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv} (\mathbf{r} \times \mathbf{V})\rho_w dV + \int_{cs} (\mathbf{r} \times \mathbf{V})\rho_w \mathbf{V}\cdot d\mathbf{A}$$

writing the scalar component of this equation about point A by referring to Fig. a,

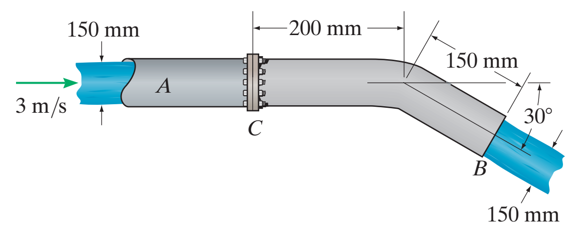
$\circlearrowleft + \Sigma M_A = 0 + r_B V_B \rho_w (V_B A_B) - r_C V_C \rho_w(-V_C A_C)$

$M_A - (760.07\ \text{N})(0.2\ \text{m}) - (212.06\ \text{N})(0.15\ \text{m}) = (0.15\ \text{m})(7.074\ \text{m/s})(1000\ \text{kg/m}^3)(0.02\ \text{m}^3/\text{s})$
$\qquad - (0.2\ \text{m})(2.546\ \text{m/s})(1000\ \text{kg/m}^3)(-0.02\ \text{m}^3/\text{s})$

$M_A = 215.22\ \text{N}\cdot\text{m} = 215\ \text{N}\cdot\text{m}\ \circlearrowleft$ **Ans.**

Ans:
$A_x = 811\ \text{N}$
$A_y = 354\ \text{N}$
$M_A = 215\ \text{N}\cdot\text{m}$

6–66. Water flows into the bend fitting with a velocity of 3 m/s. If the water exists at B into the atmosphere, determine the horizontal and vertical components of force, and the moment at C, needed to hold the fitting in place. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000\ \text{kg/m}^3$. Average velocities will be used. The control volume consists of the bend fitting and the contained water. The discharge is

$$Q = V_C\,A_C = (3\ \text{m/s})\left[\pi(0.075\ \text{m})^2\right] = 0.016875\pi\ \text{m}^3/\text{s}$$

The water exits at B into the atmosphere. Then $p_B = p_{\text{atm}} = 0$. Since the diameter of the bend fitting is constant, $V_B = V_C = 3$ m/s and the elevation change is small. Therefore $p_C = p_B = 0$. As a result, no pressure force acting on the control volume. The *FBD* of the control volume is shown in Fig. *a*. Applying the linear momentum equation,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho_w dV\!\!\!/ + \int_{\text{cs}} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar components of this equation along the x and y axes by referring to the free-body diagram, Fig. *a*,

$$(\overset{+}{\rightarrow})\ \Sigma F_x = 0 + V_B\cos 30^\circ \rho_w(V_B A_B) + V_C\rho_w(-V_C A_C)$$

$$C_x = \left[(3\ \text{m/s})\cos 30^\circ\right](1000\ \text{kg/m}^3)(0.016875\pi\ \text{m}^3/\text{s}) + (3\ \text{m/s})(1000\ \text{kg/m}^3)(-0.016875\pi\ \text{m}^3/\text{s})$$

$$= -21.31\ \text{N} = 21.3\ \text{N}\leftarrow \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + (-V_B \sin 30^\circ)(\rho_w)(V_B A_B)$$

$$-C_y = \left[-(3\ \text{m/s})\sin 30^\circ\right](1000\ \text{kg/m}^3)(0.016875\pi\ \text{m}^3/\text{s})$$

$$C_y = 79.52\ \text{N} = 79.5\ \text{N}\downarrow \qquad \textbf{Ans.}$$

Applying the angular momentum equation,

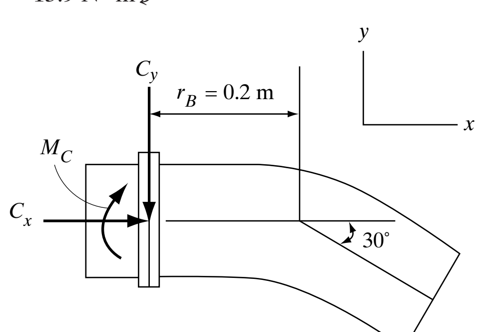
$$\Sigma\mathbf{M} = \frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{r}\times\mathbf{V})\rho_w dV\!\!\!/ + \int_{\text{cs}} (\mathbf{r}\times\mathbf{V})\rho_w \mathbf{V}\cdot d\mathbf{A}$$

writing the scalar component of this equation about point C by referring to Fig. *a*,

$$\circlearrowleft + \Sigma M_C = 0 + (-r_B V_B \sin 30^\circ)\rho_w(V_B A_B)$$

$$-M_C = -(0.2\ \text{m})\left[(3\ \text{m/s})\sin 30^\circ\right](1000\ \text{kg/m}^3)(0.016875\pi\ \text{m}^3/\text{s})$$

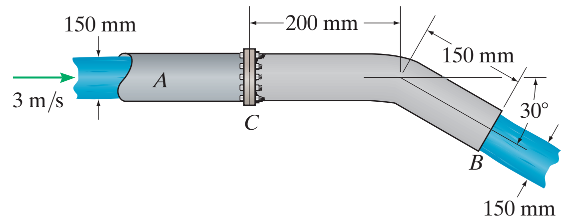
$$M_C = 15.9\ \text{N}\cdot\text{m} \circlearrowright \qquad \textbf{Ans.}$$



(a)

Ans:
$C_x = 21.3$ N
$C_y = 79.5$ N
$M_C = 15.9\ \text{N}\cdot\text{m}$

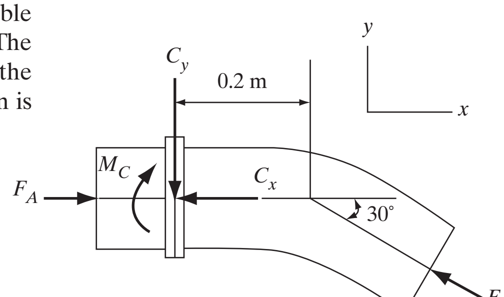
6–67. Water flows into the bend fitting with a velocity of 3 m/s. If the water at B exits into a tank having a gage pressure of 10 kPa, determine the horizontal and vertical components of force, and the moment at C, needed to hold the fitting in place. Neglect the weight of the fitting and the water within it.



SOLUTION

The flow is steady and water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000\ \text{kg/m}^3$. Average velocities will be used. The fixed control volume consists of the bend fitting and the contained water. Since the diameter of the pipe is constant, $V_B = V_A = 3$ m/s. Also the charge in elevation is negligible, $p_A = p_B = 10$ kPa, to satisfy Bernoulli's equation. Then

$$F_A = F_B = \left[10\left(10^3\right)\ \text{N/m}^2\right]\left[\pi\left(0.075\ \text{m}\right)^2\right] = 56.25\pi\ \text{N}$$



(a)

Also, the discharge is

$$Q = V_A A_A = V_B A_B = (3\ \text{m/s})\left[\pi(0.075\ \text{m})^2\right] = 0.016875\pi\ \text{m}^3/\text{s}$$

The *FBD* of the control volume is shown in Fig. *a*. Applying the linear momentum equation,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho_w d\mathcal{V} + \int_{cs} \mathbf{V}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along x and y axes by referring to the *FBD*, Fig. *a*

$$(\overset{+}{\rightarrow})\ \Sigma F_x = 0 + V_B \cos 30°\rho_w(V_B A_B) + V_A\rho_w(-V_A A_A)$$

$$56.25\pi\ \text{N} - [(56.25\pi\ \text{N})\cos 30°] - C_x = [(3\ \text{m/s})\cos 30°](1000\ \text{kg/m}^3)(0.016875\pi\ \text{m}^3/\text{s})$$
$$+(3\ \text{m/s})(1000\ \text{kg/m}^3)(-0.016875\pi\ \text{m}^3/\text{s})$$

$$C_x = 44.98\ \text{N} = 45.0\ \text{N} \leftarrow \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + (-V_B \sin 30°)(\rho_w)(V_B A_B)$$

$$(56.25\pi\ \text{N})\sin 30° - C_y = [-(3\ \text{m/s})\sin 30°]\left(1000\ \text{kg/m}^3\right)\left(0.016875\pi\ \text{m}^3/\text{s}\right)$$

$$C_y = 167.88\text{N} = 168\ \text{N}\downarrow \qquad \textbf{Ans.}$$

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv} (\mathbf{r}\times\mathbf{V})\rho_w d\mathcal{V} + \int_{cs} (\mathbf{r}\times\mathbf{V})\rho_w \mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation about point C by referring to the *FBD*, Fig. *a*,

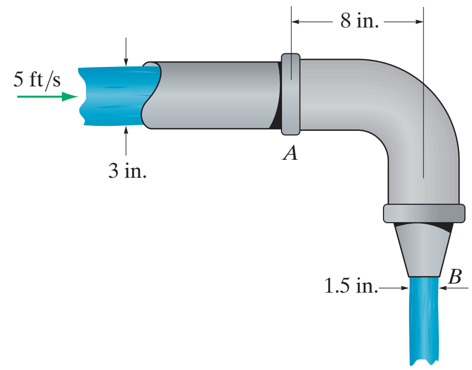
$$\circlearrowleft + \Sigma M_C = 0 + (-r_B V_B \sin 30°)\rho_W(V_B A_B)$$

$$[(56.25\ \pi\ \text{N})\sin 30°](0.2\ \text{m}) - M_C = -(0.2\ \text{m})\left[\left(3\ \text{m/s}\right)\sin 30°\right]\left(1000\ \text{kg/m}^3\right)\left(0.016875\pi\ \text{m}^3/\text{s}\right)$$

$$M_C = 33.58\ \text{N}\cdot\text{m} = 33.6\ \text{N}\cdot\text{m} \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$C_x = 45.0$ N
$C_y = 168$ N
$M_C = 33.6\ \text{N}\cdot\text{m}$

***6–68.** Water flows into the pipe with a velocity of 5 ft/s. Determine the horizontal and vertical components of force, and the moment at A, needed to hold the elbow in place. Neglect the weight of the elbow and the water within it.



SOLUTION

We consider steady flow of an ideal fluid.

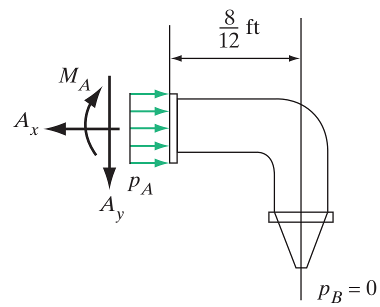
Take the control volume as the elbow and the water within it.

$$Q = V_A A_A$$

$$= (5\text{ ft/s})\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right]$$

$$= 0.2454\text{ ft}^3/\text{s}$$

Continuity Equation.



(a)

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-0.2454\text{ ft}^3/\text{s} + V_B\left[\pi\left(\frac{0.75}{12}\text{ft}\right)^2\right] = 0$$

$$V_B = 20\text{ ft/s}$$

Applying the Bernoulli equation between A and B,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{\left(\dfrac{62.4\text{ lb ft}^3}{32.2\text{ ft/s}^2}\right)} + \frac{(5\text{ ft/s})^2}{2} + 0 = 0 + \frac{(20\text{ ft/s})^2}{2} + 0$$

$$p_A = 363.354\text{ lb/ft}^2 = 2.523\text{ lb/in}^2$$

The free-body diagram of the control volume is shown in Fig. *a*. Here, water is discharged into the atmosphere at B. Therefore, $p_B = 0$.

Linear Momentum. Referring to Fig. *a*,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d\forall + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

$\overset{+}{\rightarrow}\Sigma F_x = \rho Q[(V_B)_x - (V_A)_x];$

$$-A_x + 2.523\text{ lb/in}^2\left[\pi(1.5\text{ in.})^2\right] = \left(\frac{62.4}{32.2}\text{slug/ft}^3\right)(0.2454\text{ ft}^3/\text{s})(0 - 5\text{ ft/s})$$

$$A_x = 20.2\text{ lb} \leftarrow \qquad \textbf{Ans.}$$

$+\uparrow\Sigma F_y = \rho Q[(V_B)_y - (V_A)_y];$

$$-A_y = \left(\frac{62.4}{32.2}\text{slug/ft}^3\right)(0.2454\text{ ft}^3/\text{s})(-20\text{ ft/s} - 0)$$

$$A_y = 9.51\text{ lb}\downarrow \qquad \textbf{Ans.}$$

***6–68. Continued**

Angular Momentum. Referring to Fig. *a*,

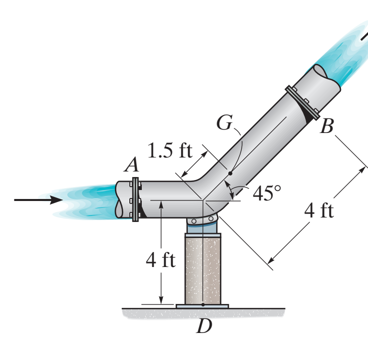
$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv} (\mathbf{r} \times \mathbf{V})\rho d\forall + \int_{cs} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot d\mathbf{A}$$

or

$$\circlearrowleft + \Sigma M_A = \Sigma \rho Q V d; \qquad -M_A = \left(\frac{62.4}{32.2}\ \text{slug/ft}^3\right)(0.2454\ \text{ft}^3/\text{s})\left[\left(-\frac{8}{12}\ \text{ft}\right)(20\ \text{ft/s}) - 0\right]$$

$$M_A = 6.34\ \text{lb.ft} \circlearrowright \qquad \textbf{Ans.}$$

6–69. The bend is connected to the pipe at flanges A and B as shown. If the diameter of the pipe is 1 ft and it carries a volumetric flow of 50 ft^3/s, determine the horizontal and vertical components of force and the moment exerted at the fixed base D of the support. The total weight of the bend and the water within it is 500 lb, with a mass center at point G. The pressure of the water at A is 15 psi. Assume that no force is transferred to the flanges at A and B.



SOLUTION

From the discharge,

$$Q = VA; \quad 50\text{ ft}^3/\text{s} = V\left[\pi(0.5\text{ ft})^2\right]$$

$$V_A = V_B = V = 63.66\text{ ft/s}$$

The flow is steady and water can be considered as an ideal fluid (incompressible and inviscid), such that $\gamma_w = 62.4\text{ lb/ft}^2$. Average velocities will be used.

Bernoulli Equation

$$p_B = 1983.5\text{ lb/ft}^2$$

$$F_A = p_A A_A = 15\text{ lb/in.}^2(\pi)(6\text{ in.})^2 = 1696.46\text{ lb}$$

$$F_B = p_B A_B = 1983.5\text{ lb/ft}^2(\pi)(0.5\text{ ft})^2 = 1557.84\text{ lb}$$

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{P_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{15(144)\text{ lb/ft}^2}{\left(\dfrac{62.4\text{ lb/ft}^2}{32.2\text{ ft/s}^2}\right)} + \frac{V^2}{2} + 0 = \frac{p_B}{\left(\dfrac{62.4\text{ lb/ft}^2}{32.2\text{ ft/s}^2}\right)} + \frac{V^2}{2}\left(32.2\text{ ft/s}^2\right)(4\text{ ft}\sin 45°)$$

Applying the linear momentum equation.

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho_w d\forall + \int_{cs}\mathbf{V}\rho_w\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along x and y axes by referring to the FBD of the control volume, Fig. a

$$\xrightarrow{+}\Sigma F_x = 0 + V_A\rho_w(-V_A A_A) + (V_B\cos 45°)\rho_w(V_B A_B)$$

$$1696.46\text{ lb} - [(1557.84\text{ lb})\cos 45°] - D_x = (63.66\text{ ft/s})\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)\left(-50\text{ ft}^3/\text{s}\right)$$

$$+\left[\left(63.66\text{ ft/s}\right)\cos 45°\right]\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)\left(50\text{ ft}^3/\text{s}\right)$$

$$D_x = 2401.6\text{ lb} = 2.40\text{ kip} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0 + (V_B\sin 45°)\rho_w(V_B A_B)$$

$$D_y - 500 - [(1557.84)\sin 45°] = \left[\left(63.66\text{ ft/s}\right)\sin 45°\right]\left(\frac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\right)\left(50\text{ ft}^3/\text{s}\right)$$

$$D_y = 5963.3\text{ lb} = 5.96\text{ kip} \qquad \textbf{Ans.}$$

6–69. Continued

Applying the Angular Momentum equation

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv} (\mathbf{r} \times \mathbf{V})\rho_w d\forall + \int_{cs} (\mathbf{r} \times \mathbf{V})\rho_w \mathbf{V} \cdot d\mathbf{A}$$

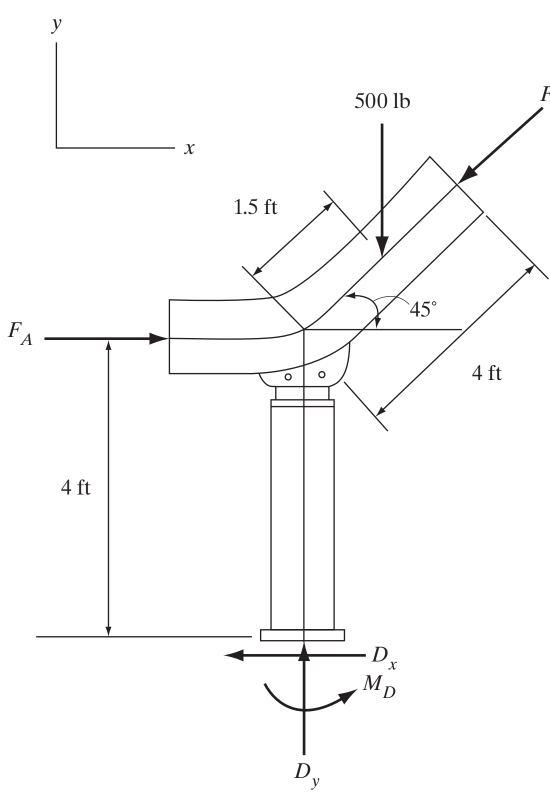
Writing the scalar component of this equation about D by referring to the FBD

$$\circlearrowleft + \Sigma M_D = 0 + (-r_A V_A)\rho_w(-V_A A_A) + (-r_B V_B \cos 45^\circ)\rho_w(V_B A_B)$$

$$M_D + [(1559.84 \text{ lb}) \cos 45^\circ](4 \text{ ft}) - (1696.46 \text{ lb})(4 \text{ ft}) - (500 \text{ lb})[(1.5 \text{ ft}) \cos 45^\circ]$$

$$= -(4 \text{ ft})(63.66 \text{ ft/s})\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(-50 \text{ ft}^3/\text{s}) + (-4 \text{ ft})\left[(63.66 \text{ ft/s})\cos 45^\circ\right]\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(50 \text{ ft}^3/\text{s})$$

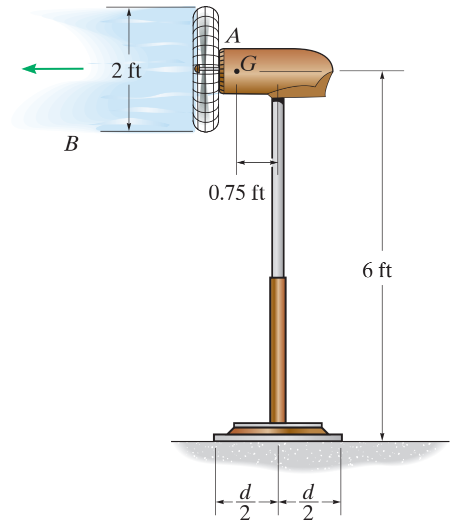
$$M_D = 10136.8 \text{ lb} \cdot \text{ft} = 10.1 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$



(a)

Ans:
$D_x = 2.40$ kip
$D_y = 5.96$ kip
$M_D = 10.1$ kip · ft

6–70. The fan blows air at 6000 ft^3/min. If the fan has a weight of 40 lb and a center of gravity at G, determine the smallest diameter d of its base so that it will not tip over. Assume the airstream through the fan has a diameter of 2 ft. The specific weight of the air is $\gamma_a = 0.076\ \text{lb/ft}^3$.



SOLUTION

We consider steady flow of an ideal fluid.

$$Q = \left(\frac{6000\ \text{ft}^3}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 100\ \text{ft}^3/\text{s}$$

Then,

$$Q = V_B A_B; \qquad 100\ \text{ft}^3/\text{s} = V_B\left[\pi(1\ \text{ft})^2\right]$$

$$V_B = 31.83\ \text{ft/s}$$

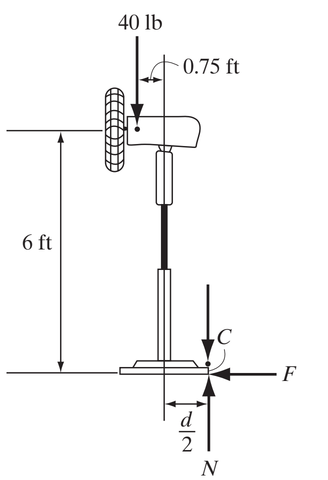
Take the control volume as the fan and air passing through it. The free-body diagram of the control volume is shown in Fig. a. Here, tipping will occur about point C.

Angular Momentum. Air is sucked into the fan at A from a large source of still air, $V_A \cong 0$. Referring to Fig. a,

$$\Sigma \mathbf{M}_c = \frac{\partial}{\partial t}\int_{cv} (\mathbf{r} \times \mathbf{V})\rho d\mathcal{V} + \int_{cs} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot d\mathbf{A}$$

$$\circlearrowleft + 40\ \text{lb}\left(0.75\ \text{ft} + \frac{d}{2}\right) = \left(\frac{0.076\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)\left(100\ \text{ft}^3/\text{s}\right)[6\ \text{ft}(31.83\ \text{ft/s}) - 0]$$

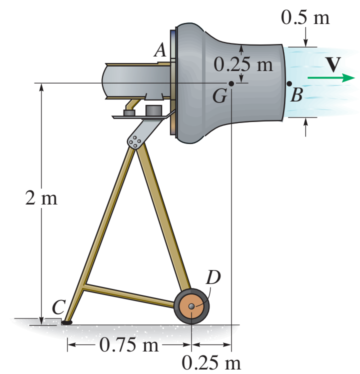
$$d = 0.7539\ \text{ft} = 0.754\ \text{ft}$$ **Ans.**



(a)

Ans:
0.754 ft

6–71. When operating, the air-jet fan discharges air with a speed of $V = 18$ m/s into a slipstream having a diameter of 0.5 m. If the air has a density of 1.22 kg/m^3, determine the horizontal and vertical components of reaction at C, and the vertical reaction at each of the two wheels, D. The fan and motor have a mass of 25 kg and a center of mass at G. Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



SOLUTION

We consider steady flow of an ideal fluid.

Take the control volume to be the fan and the air passing through it.

$$Q = V_B A_B = (18\text{ m/s})\left[\pi(0.25\text{ m})^2\right] = 3.5343\text{ m}^3/\text{s}$$

The free-body diagram of the control volume is shown in Fig. *a*.

Angular Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv}(\mathbf{r}\times\mathbf{V})\rho\, d\forall + \int_{cs}(\mathbf{r}\times\mathbf{V})\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$2N_D(0.75\text{ m}) - 25\text{ kg}\left(9.81\text{ m/s}^2\right)(1\text{m}) = 0 + \left(1.22\text{ kg/m}^3\right)\left(3.5343\text{ m}^3/\text{s}\right)[(-2\text{ m})(18\text{ m/s}) - 0]$$

$$N_D = 60.02\text{ N} = 60.0\text{ N} \qquad \textbf{Ans.}$$

Linear Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho\, d\forall + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot d\mathbf{A}$$

or

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + V_B\rho Q$$

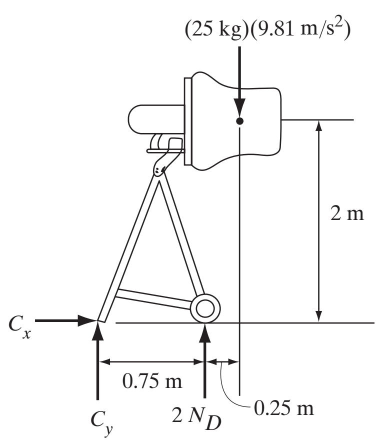
$$C_x = (18\text{ m/s})\left(1.22\text{ kg/m}^3\right)\left(3.5343\text{ m}^3/\text{s}\right)$$

$$C_x = 77.6\text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0 + 0$$

$$C_y + 2(60.02\text{ N}) - 25\text{ kg}\left(9.81\text{ m/s}^2\right) = \left(1.22\text{ kg/m}^3\right)\left(3.5343\text{ m}^3/\text{s}\right)(0 - 0)$$

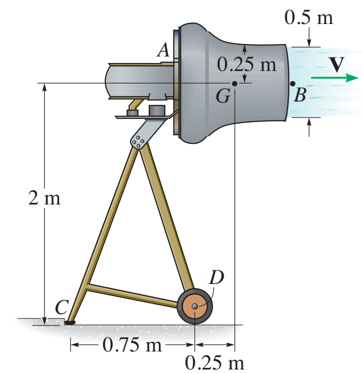
$$C_y = 125.22\text{ N} = 125\text{ N} \qquad \textbf{Ans.}$$



(a)

Ans:
$N_D = 60.0$ N
$C_x = 77.6$ N
$C_y = 125$ N

***6–72.** If the air has a density of $1.22\ \text{kg/m}^3$, determine the maximum speed V that the air-jet fan can discharge air into the slipstream having a diameter of 0.5 m at B so that the fan does not topple over. The fan and motor have a mass of 25 kg and a center of mass at G. Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



SOLUTION

Consider the control volume to be the fan and the air passing through it, Fig. a. Since the inlet A and outlet B are opened to the atmosphere, $p_A = p_B = 0$. The free-body diagram of the control volume is shown in Fig. a. Here, if the fan is about to topple about C, $N_D = 0$. Applying the angular momentum equation

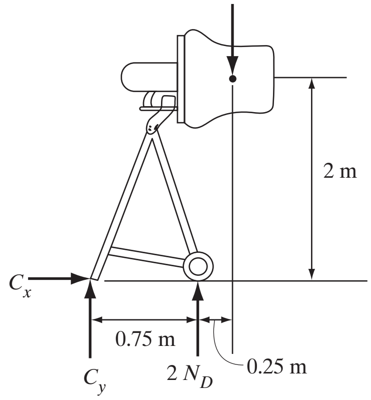
$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv}(\mathbf{r}\times\mathbf{V})\rho\, d\forall + \int_{cs}(\mathbf{r}\times\mathbf{V})\rho\mathbf{V}\cdot d\mathbf{A}$$

And writing the scalar component of the equation about C by referring to the FBD,

$$\circlearrowleft + \Sigma M_C = 0 + (-r_B V_B)(\rho_a)(V_B A_B)$$

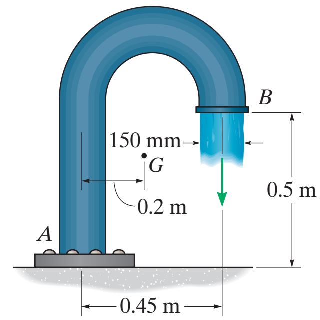
$$-(25\ \text{kg})(9.81\ \text{m/s}^2)(1\ \text{m}) = -\left[(2\text{m})V_B\right](1.22\ \text{kg/m}^3)V_B\left[\pi(0.25\ \text{m})^2\right]$$

$$V_B = 22.63\ \text{m/s} = 22.6\ \text{m/s} \qquad \textbf{Ans.}$$



(a)

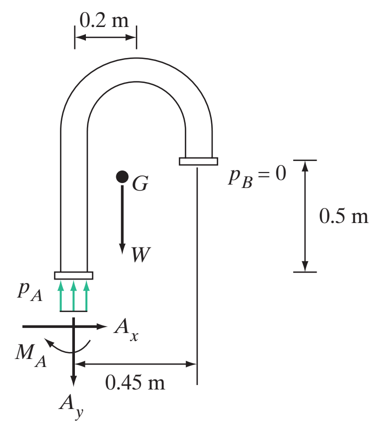
6–73. Water flows through the curved pipe at a speed of 5 m/s. If the diameter of the pipe is 150 mm, determine the horizontal and vertical components of the resultant force, and the moment acting on the coupling at A. The weight of the pipe and the water within it is 450N, having a center of gravity at G.



SOLUTION

Take the control volume as the pipe and the water within it.

$$Q_A = V_A A_A = (5 \text{ m/s})\left[\pi(0.075 \text{ m})^2\right]$$
$$= 0.08836 \text{ m}^3/\text{s}$$



(a)

Bernoulli Equation, where $V_A = V_B$. Datum at A, the free-body diagram of the control volume is shown in Fig. *a*. Here, water is discharged into the atmosphere at B. Therefore, $p_B = 0$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{1000 \text{ kg/m}^3} + \frac{V^2}{2} + 0 = 0 + \frac{V^2}{2} + (9.81 \text{ m/s}^2)(0.5 \text{ m})$$

$$p_A = 4905\ P_a$$

Linear Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V}\cdot d\mathbf{A}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0 + pQ\left[(V_B)_x - (V_A)_x\right];$

$A_x = pQ(0 - 0) = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0 + pQ\left[(V_B)_y - (V_A)_y\right];$

$-A_y + \left[4905 \text{ N/m}^2\right]\left[\pi(0.075 \text{ m})^2\right] - 450 \text{ N} = \left(1000 \text{ kg/m}^3\right)\left(0.08836 \text{ m}^3/\text{s}\right)\left(-5\text{m/s} - 5 \text{ m/s}\right)$

$A_y = 520 \text{ N}$ **Ans.**

Angular Momentum. Referring to Fig. *a*,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv} (\mathbf{r}\times\mathbf{V})\rho d\forall + \int_{cs} (\mathbf{r}\times\mathbf{V})\rho \mathbf{V}\cdot d\mathbf{A}$$

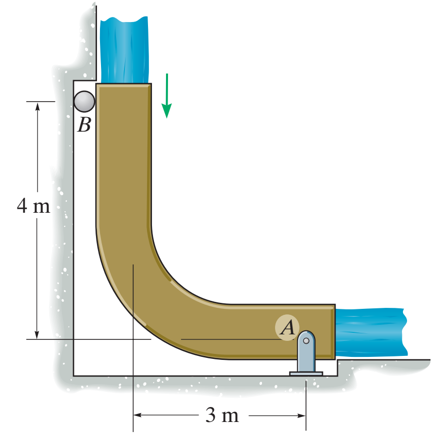
$\circlearrowleft + \Sigma M_D = 0 + \Sigma \rho Q V d;$

$-M_A - (450 \text{ N})(0.2 \text{ m}) = \left(1000 \text{ kg/m}^3\right)\left(0.08836 \text{ m}^3/\text{s}\right)[(-0.45 \text{ m})(5 \text{ m/s}) - 0]$

$M_A = 109 \text{ N}\cdot\text{m}$ **Ans.**

Ans:
$A_x = 0$
$A_y = 520 \text{ N}$
$M_A = 109 \text{ N}\cdot\text{m}$

6–74. The chute is used to divert the flow of water. If the flow is $0.4\ \text{m}^3/\text{s}$ and it has a cross-sectional area of $0.03\ \text{m}^2$, determine the horizontal and vertical force components at the pin A, and the horizontal force at the roller B, necessary for equilibrium. Neglect the weight of the chute and the water on it.



SOLUTION

Take the control volume as the chute and the water on it.

$$Q = VA; \qquad 0.4\ \text{m}^3/\text{s} = V(0.03\ \text{m}^2)$$

$$V = 13.33\ \text{m/s}$$

The free-body diagram of the control volume is shown in Fig. a. Here, $p_A = p_B = 0$ since points A and B are exposed to the atmosphere,

Angular Momentum. Referring to Fig. a,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{cv} \mathbf{r} \times \mathbf{V}\rho d\forall + \int_{cs} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot d\mathbf{A}$$

$$\circlearrowleft + \Sigma M_A = 0 + \Sigma \rho Q V d;$$

$$-B_x(4\ \text{m}) = (1000\ \text{kg/m}^3)(0.4\ \text{m}^3/\text{s})[0 - 3\ \text{m}(13.33\ \text{m/s})]$$

$$B_x = 4000\ \text{N} = 4\ \text{kN} \qquad \textbf{Ans.}$$

Linear Momentum. Referring to Fig. a,

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho d\forall + \int_{cs} \mathbf{V}\rho \mathbf{V} \cdot d\mathbf{A}$$

$$\xrightarrow{+} \Sigma F_x = 0 + (V_A)\rho Q$$

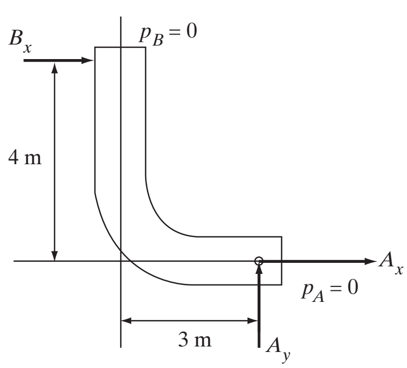
$$4000\ \text{N} + A_x = (13.33\ \text{m/s})(1000\ \text{kg/m}^3)(0.4\ \text{m}^3/\text{s})$$

$$A_x = 1.33\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0 + V_B \rho Q$$

$$A_y = (13.33\ \text{m/s})(1000\ \text{kg/m}^3)(0.4\ \text{m}^3/\text{s})$$

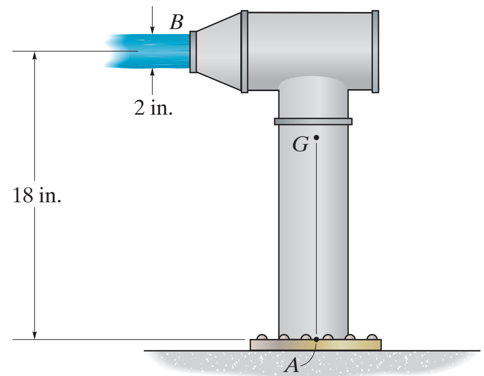
$$A_y = 5.33\ \text{kN} \qquad \textbf{Ans.}$$



(a)

Ans:
$B_x = 4\ \text{kN}$
$A_x = 1.33\ \text{kN}$
$A_y = 5.33\ \text{kN}$

6–75. Water flows through A at 400 gal/min and is discharged to the atmosphere through the reducer at B. Determine the horizontal and vertical components of force, and the moment acting on the coupling at A. The vertical pipe has an inner diameter of 3 in. Assume the assembly and the water within it has a weight of 40 lb and a center of gravity at G. 1 $\text{ft}^3{}_3 = 7.48$ gal.



SOLUTION

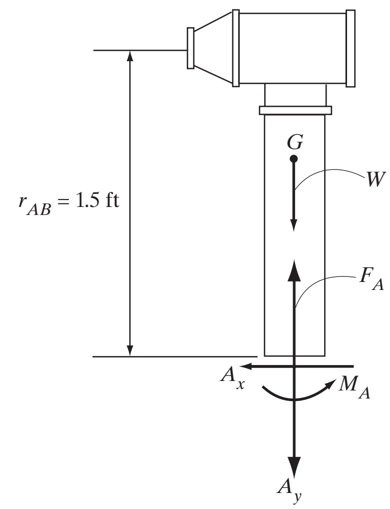
The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\gamma_w = 62.4\ \text{lb/ft}^3$. Average velocities will be used. The control volume consists of the vertical pipe, reducer and the contained water as shown in Fig. *a*. The discharge is

$$Q = \left(400\ \frac{\text{gal}}{\text{min}}\right)\left(\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 0.8913\ \text{ft}^3/\text{s}$$

Thus,

$$Q = V_A A_A; \qquad 0.8913\ \text{ft}^3/\text{s} = V_A\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right] \qquad V_A = 18.16\ \text{ft/s}$$

$$Q = V_B A_B; \qquad 0.8913\ \text{ft}^3/\text{s} = V_B\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] \qquad V_B = 40.85\ \text{ft/s}$$



(a)

Applying Bernoulli's equation between points A and B with $p_B = p_{\text{atm}} = 0$ and $z_B = 1.5$ ft,

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{p_A}{62.4\ \text{lb/ft}^3} + \frac{(18.16\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 = 0 + \frac{(40.85\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 1.5\ \text{ft}$$

$$p_A = 1391.28\ \text{lb/ft}^2$$

Then the pressure force acting on the inlet control surface A, indicated in the *FBD* of the control volume, is

$$F_A = p_A A_A = \left(1391.28\ \text{lb/ft}^2\right)\left[\pi\left(\frac{1.5}{12}\ \text{ft}\right)^2\right] = 68.28\ \text{lb}$$

Applying the linear momentum equation,

$$\Sigma\mathbf{F} = \frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho_w d\forall + \int_{cs}\mathbf{V}\rho_w\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar components of this equation along the x and y axes by referring to Fig. *a*

$$\overset{+}{\rightarrow}\Sigma F_x = 0 + (-V_B)\rho_w(V_B A_B)$$

$$-A_x = (-40.85\ \text{ft/s})\left(\frac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(0.8913\ \text{ft}^3/\text{s})$$

$$A_x = 70.56\ \text{lb} = 70.6\ \text{lb}$$ **Ans.**

6–75. Continued

$$+\uparrow \Sigma F_y = 0 + V_A \rho_w(-V_A A_A)$$

$$-40 \text{ lb } 68.29 \text{ lb} - A_y = (18.16 \text{ ft/s})\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(-0.8913 \text{ ft}^3/\text{s})$$

$$A_y = 59.65 \text{ lb} = 59.7 \text{ lb}$$ **Ans.**

Applying the angular momentum equation,

$$\Sigma \mathbf{M} = \frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{r} \times \mathbf{V})\rho_w d\forall + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho_w \mathbf{V} \cdot d\mathbf{A}$$

Writing the scalar component of this equation about point A,

$$\circlearrowleft + \Sigma M_A = 0 + \text{r}_{AB} V_B (V_B A_B)$$

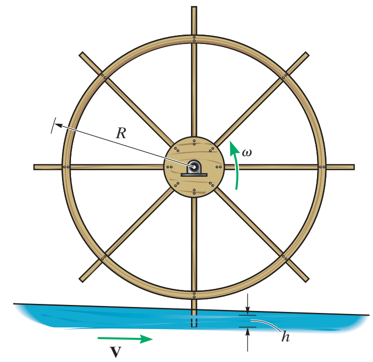
$$M_A = (1.5 \text{ ft})(40.85 \text{ ft/s})\left(\frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}}\right)(0.8913 \text{ ft}^3/\text{s})$$

$$= 105.84 \text{ lb} \cdot \text{ft} = 106 \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:

$A_x = 70.6$ lb
$A_y = 59.7$ lb
$M_A = 106$ lb · ft

***6–76.** The waterwheel consists of a series of flat plates that have a width b and are subjected to the impact of water to a depth h from a stream that has an average velocity of V. If the wheel is turning at ω, determine the power supplied to the wheel by the water.



SOLUTION

Using a fixed control volume, with water entering on the left with velocity V and exiting on the right with (x-component) velocity ωR (the speed of the plates), we apply the angular momentum equation:

$$\circlearrowleft + \Sigma M_{\text{hub}} = \frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{r} \times \mathbf{V})\rho\, d\forall + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot d\mathbf{A}$$

$$-T = 0 + RV\rho_w(-VA) + R\omega R\rho_w(VA)$$

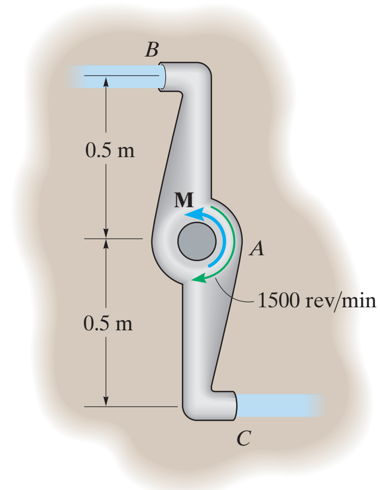
where T is the torque or moment exerted by the water on the wheel and −T is the torque exerted by the wheel on the water. So then, since $A = bh$,

$$T = \rho_w bhRV(V - \omega R)$$

and since $\dot{W} = T\omega$,

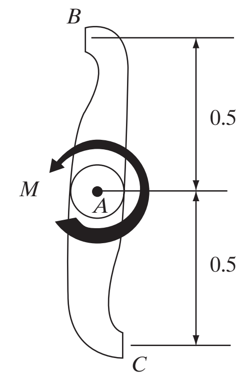
$$P = \rho_w bh\omega RV(V - \omega R) \qquad \textbf{Ans.}$$

6–77. Air enters into the hollow propeller tube at A with a mass flow of 3 kg/s and exits at the ends B and C with a velocity of 400 m/s, measured relative to the tube. If the tube rotates at 1500 rev/min, determine the frictional torque m on the tube.



SOLUTION

The flow is periodic hence it can be considered steady in the mean. The air is assumed to be an ideal fluid (incompressible and inviscid) such that its density is constant. Average velocities will be used. The control volume consists of the hollow propeller and the contained air. Its *FBD* is shown in Fig. *a*



(a)

The velocity of point B (or C) is

$$V_B = \omega r = \left[\left(1500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right](0.5\text{ m}) = 25\pi\text{ m/s}\rightarrow$$

Thus, the velocity of the air ejected from B (or C) is

$$V_a = V_B + V_{a/B}$$

$$(\overset{+}{\leftarrow})\; V_a = (-25\pi\text{ m/s}) + (400\text{ m/s}) = 321.46\text{ m/s}\leftarrow$$

Applying the angular momentum equation,

$$\Sigma\mathbf{M} = \frac{\partial}{\partial t}\int_{cv}(\mathbf{r}\times\mathbf{V})\rho d\forall + \int_{cs}(\mathbf{r}\times\mathbf{V})\rho\mathbf{V}\cdot d\mathbf{A}$$

Writing the scalar component about point A,

$$\circlearrowleft + \Sigma M_A = 0 + 2\left[r_{AB}V_B\rho_a V_B A_B\right]$$

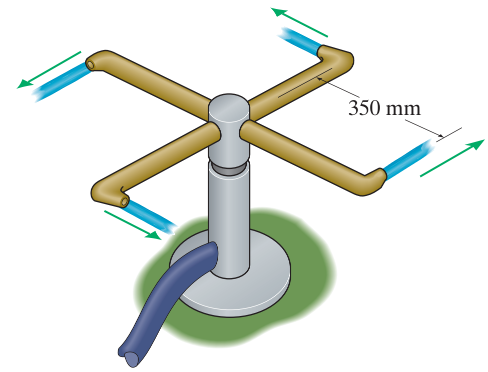
Here $\rho_a V_B A_B = \dot{m}_B = 1.5\text{ kg/s}$. Then

$$M = 2(0.5\text{ m})(321.46\text{ m/s})(1.5\text{ kg/s})$$

$$= 482.19\text{ N}\cdot\text{m} = 482\text{ N}\cdot\text{m}$$ **Ans.**

Ans:
482 N · m

6–78. The lawn sprinkler consists of four arms that rotate in the horizontal plane. The diameter of each nozzle is 10 mm, and the water is supplied through the hose at $0.008\ \text{m}^3/\text{s}$ and is ejected horizontally, through the four arms. Determine the torque required to hold the arms from rotating.



SOLUTION

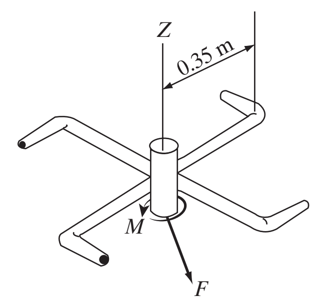
We consider steady flow of an ideal fluid relative to the control volume.

Take the control volume as the sprinkler and the water within it. Due to symmetry and the continuity condition, the discharge from each nozzle is $Q = (0.008\ \text{m}^3/\text{s})/4 = 0.002\ \text{m}^3/\text{s}$.

$$Q = VA; \qquad 0.002\ \text{m}^3/\text{s} = V\left[\pi(0.005\ \text{m}^2)\right]$$

$$V = 25.46\ \text{m/s}$$

The free-body diagram of the control volume is shown in Fig. *a*. Here, water is discharged to the atmosphere at the nozzle, $p = 0$.



(a)

Angular Momentum. Referring to Fig. *a*,

$$\Sigma\mathbf{M} = \frac{\partial}{\partial t}\int_{cv}(\mathbf{r}\times\mathbf{V})\rho d\forall \int_{cs}(\mathbf{r}\times\mathbf{V})\rho\mathbf{V}\cdot d\mathbf{A}$$

or

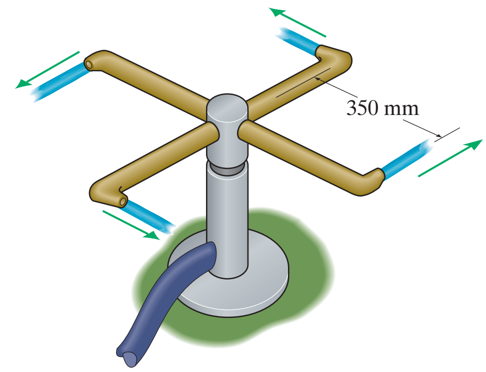
$$\zeta+\Sigma M_A = \Sigma\rho QVd;$$

$$M = 4\left[(1000\ \text{kg/m}^3)(0.002\ \text{m}^3/\text{s})\right]\left[0.35\ \text{m}(25.46\ \text{m/s}) - 0\right]$$

$$= 71.30\ \text{N}\cdot\text{m} = 71.3\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$71.3\ \text{N}\cdot\text{m}$

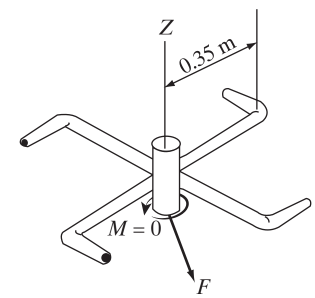
6–79. The lawn sprinkler consists of four arms that rotate in the horizontal plane. The diameter of each nozzle is 10 mm, and the water is supplied through the hose at $0.008\ \text{m}^3/\text{s}$ and is ejected horizontally, through the four arms. Determine the steady-state angular velocity of the arms. Neglect friction.



SOLUTION

We consider steady flow of an ideal fluid relative to the control volume.

Take the control volume as the sprinkler and the water within it. Due to symmetry and the continuity condition, the discharge from each nozzle is $Q = (0.008\ \text{m}^3/\text{s})/4 = 0.002\ \text{m}^3/\text{s}$.



(a)

$$Q = V_{f/n}A; \qquad 0.002\ \text{m}^3/\text{s} = V_{f/n}\left[\pi(0.005\ \text{m}^2)\right]$$

$$V_{f/n} = 25.46\ \text{m/s}$$

The velocity of the nozzle is

$$V_n = \omega r = \omega(0.35\ \text{m}) = 0.35\,\omega$$

Thus, the velocity of the flow can be determined from

$$\mathbf{V}_f = \mathbf{V}_n + \mathbf{V}_{f/n}$$

$$V_f = -0.35\omega + 25.46$$

The free-body diagram of the control volume is shown in Fig. *a*. Here, water is discharged to the atmosphere at the nozzle, $p = 0$.

Angular Momentum. Referring to Fig. *a*,

$$\Sigma\mathbf{M} = \frac{\partial}{\partial t}\int_{cv}(\mathbf{r}\times\mathbf{V})\rho\, d\mathcal{V} + \int_{cs}(\mathbf{r}\times\mathbf{V})\rho\mathbf{V}\cdot d\mathbf{A}$$

or

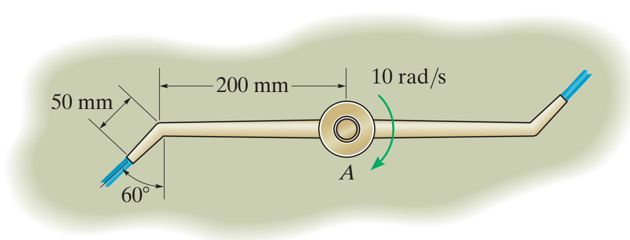
$$\circlearrowleft + \Sigma M_A = \Sigma\rho Q dV;$$

$$0 = 4\left[(1000\ \text{kg/m}^3)(0.002\ \text{m}^3/\text{s})\right][0.35\ \text{m}(-0.35\omega + 25.46)]$$

$$\omega = 72.76\ \text{rad/s} = 72.8\ \text{rad/s}$$ **Ans.**

Ans:
72.8 rad/s

***6–80.** The 5-mm-diameter arms of a rotating lawn sprinkler have the dimensions shown. Water flows out relative to the arms at 6 m/s, while the arms are rotating at 10 rad/s. Determine the frictional torsional resistance at the bearing A, and the speed of the water as it emerges from the nozzles, as measured by a fixed observer.

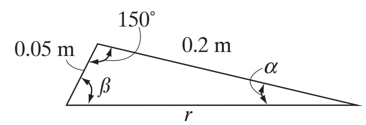


SOLUTION

Referring to the geometry shown in Fig. *a*, the cosine and sine laws give

$$r = \sqrt{50^2 + 200^2 - 2(50)(200)\cos 150^\circ} = 244.6 \text{ mm}$$

$$\frac{\sin\alpha}{0.05 \text{ m}} = \frac{\sin 150^\circ}{0.2446 \text{ m}}; \qquad \alpha = 5.867^\circ$$



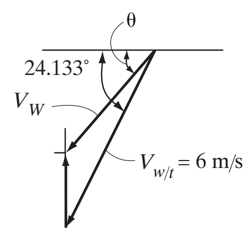
(a)

Then

$$\beta = 180^\circ - 150^\circ - 5.867^\circ = 24.133^\circ$$

Thus, the velocity of the tip of the arm is

$$V_t = \omega r = (10 \text{ rad/s})(0.2446 \text{ m}) = 2.446 \text{ m/s}\uparrow$$



(b)

Referring to the velocity vector diagram shown in Fig. *b*, the relative velocity equation gives

$$\mathbf{V}_w = \mathbf{V}_t + \mathbf{V}_{w/t}$$

$$\begin{bmatrix} (V_w)_x \leftarrow \\ (V_w)_y \downarrow \end{bmatrix} = \begin{bmatrix} 2.446 \text{ m/s} \\ \uparrow \end{bmatrix} + \begin{bmatrix} 6 \text{ m/s} \\ \swarrow 24.133^\circ \end{bmatrix}$$

$(\overset{+}{\rightarrow})\quad -(V_w)_x = -(6 \text{ m/s})\cos 24.133^\circ \qquad (V_w)_x = 5.476 \text{ m/s} \leftarrow$

$(+\uparrow)\quad -(V_w)_y = 2.446 \text{ m/s} - (6 \text{ m/s})\sin 24.133^\circ \qquad (V_w)_y = 0.007339 \text{ m/s}\downarrow$

The magnitude of $\mathbf{V}_w$ is

$$V_w = \sqrt{(V_w)_x^{\,2} + (V_w)_y^{\,2}} = \sqrt{(5.476 \text{ m/s})^2 + (0.007339 \text{ m/s})^2}$$

$$= 5.476 \text{ m/s} = 5.48 \text{ m/s} \qquad \textbf{Ans.}$$

The flow is steady and the water can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_w = 1000 \text{ kg/m}^3$. Average velocity will be used. The control volume consists of the entire arm and the contained water as shown in Fig. *a*. Applying the angular momentum equation,

$$\Sigma\mathbf{M} = \frac{\partial}{\partial t}\int_{cv} (\mathbf{r}\times\mathbf{V})\rho_w d\forall + \int_{cs} (\mathbf{r}\times\mathbf{V})\rho_w \mathbf{V}\cdot d\mathbf{A}$$

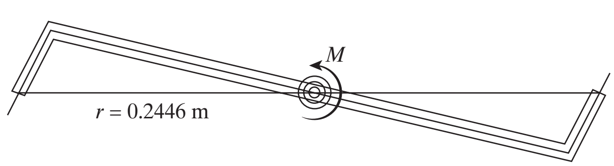
Writing the scalar component of this equation about point A, by referring to the *FBD* of the control volume, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0 + r(V_w)_y \rho_w (V_{w/t} A)$$

$$M = (0.2446 \text{ m})(0.007339 \text{ m/s})(1000 \text{ kg/m}^3)\{(6 \text{ m/s})[\pi(0.0025 \text{ m})^2]\}$$

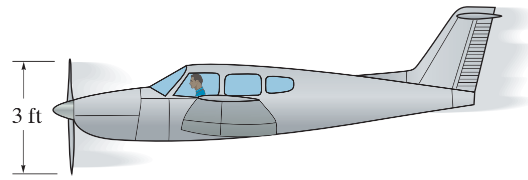
$$= 2.1145(10^{-4}) \text{ N}\cdot\text{m}$$

$$= 0.211 \text{ mN}\cdot\text{m} \qquad \textbf{Ans.}$$



(c)

6–81. The airplane is flying at 250 km/h through still air as it discharges 350 m^3/s of air through its 1.5-m-diameter propeller. Determine the thrust on the plane and the ideal efficiency of the propeller. Take $\rho_a = 1.007\ \text{kg/m}^3$.



SOLUTION

The average velocity of the air flow through the propeller (control volume) is

$$Q = VA; \qquad 350\ \text{m}^3/\text{s} = V\left[\pi(0.75\ \text{m})^2\right]$$

$$V = 198.06\ \text{m/s}$$

$$\text{Here, } V_1 = \left(250\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 69.44\ \text{m/s}$$

$$V = \frac{V_1 + V_2}{2}; \qquad 198.06\ \text{m/s} = \frac{(69.44\ \text{m/s}) + V_2}{2}$$

$$V_2 = 326.67\ \text{m/s}$$

The ideal efficiency is

$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(69.44\ \text{m/s})}{69.44\ \text{m/s} + 326.67\ \text{m/s}} = 0.3506 = 0.351 \qquad \textbf{Ans.}$$

The thrust of the propeller is

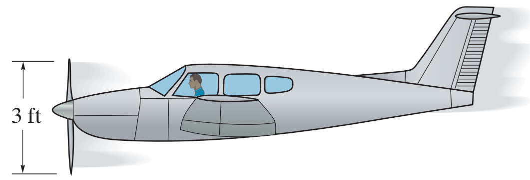
$$F = \frac{\rho\pi R^2}{2}\left(V_2^2 - V_1^2\right)$$

$$= \frac{(1.007\ \text{kg/m}^3)(\pi)(0.75\ \text{m})^2}{2}\left[(326.67\ \text{m/s})^2 - (69.44\ \text{m/s})^2\right]$$

$$= 90.66(10^3)\ \text{N} = 90.7\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$e = 0.351$
$F = 90.7\ \text{kN}$

6–82. The airplane travels at 400 ft/s through still air. If the air flows through the propeller at 560 ft/s, measured relative to the plane, determine the thrust on the plane and the ideal efficiency of the propeller. Take $\rho_a = 2.15\left(10^{-3}\right)\ \text{slug/ft}^3$.



SOLUTION

The propeller and air within it is the control volume. We consider steady flow of an ideal fluid relative to the control volume.

Here, $V_1 = 400$ ft/s and $V = 560$ ft/s.

$$V = \frac{V_1 + V_2}{2}; \qquad 560\ \text{ft/s} = \frac{400\ \text{ft/s} + V_2}{2}$$

$$V_2 = 720\ \text{ft/s}$$

The ideal efficiency is

$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(400\ \text{ft/s})}{400\ \text{ft/s} + 720\ \text{ft/s}} = 0.7143 = 0.714 \qquad \textbf{Ans.}$$

The thrust of the propeller is

$$F = \frac{p\pi R^2}{2}\left(V_2^2 - V_1^2\right)$$

$$= \frac{\left(2.15\left(10^{-3}\right)\ \text{slug/ft}^3\right)(\pi)(1.5\ \text{ft})^2}{2}\left[(720\ \text{ft/s})^2 - (400\ \text{ft/s})^2\right]$$

$$= 2723.38\ \text{lb} = 2.72\ \text{kip} \qquad \textbf{Ans.}$$

Ans:
$e = 0.714$
$F = 2.72$ kip

6–83. A boat has a 250-mm-diameter propeller that discharges 0.6 m^3/s of water as the boat travels at 35 km/h in still water. Determine the thrust developed by the propeller on the boat.

SOLUTION

The propeller and water within is the control volume. The average velocity of the water through the propeller is

$$Q = VA; \qquad 0.6\ \text{m}^3/\text{s} = V\left[\pi(0.125\ \text{m})^2\right]$$

$$V = 12.22\ \text{m/s}$$

Here, $V_1 = \left(35\ \dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000\ \text{m}}{1\ \text{km}}\right)\left(\dfrac{1\ \text{h}}{3600\ \text{s}}\right) = 9.722\ \text{m/s}$

$$V = \frac{V_1 + V_2}{2}; \qquad 12.22\ \text{m/s} = \frac{9.722\ \text{m/s} + V_2}{2}$$

$$V_2 = 14.72\ \text{m/s}$$

The thrust of the propeller is

$$F = \frac{\rho\pi R^2}{2}\left(V_2^2 - V_1^2\right)$$

$$= \frac{\left(1000\ \text{kg/m}^3\right)(\pi)(0.125\ \text{m})^2}{2}\left[\left(14.72\ \text{m/s}\right)^2 - \left(9.722\ \text{m/s}\right)^2\right]$$

$$= 3.001\left(10^3\right)\ \text{N} = 3.00\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
300 kN

***6–84.** A ship has a 2.5-m-diameter propeller with an ideal efficiency of 40%. If the thrust developed by the propeller is 1.5 MN, determine the constant speed of the ship in still water and the power that must be supplied to the propeller to operate it.

SOLUTION

The propeller and water within it is the control volume.

The ideal efficiency is

$$e = \frac{2V_1}{V_1 + V_2}; \qquad 0.4 = \frac{2V_1}{V_1 + V_2} \qquad V_2 = 4V_1 \qquad \textbf{(1)}$$

The thrust of the propeller is

$$F = \frac{\rho\pi R^2}{2}\left(V_2^2 - V_1^2\right); \qquad 1.5\left(10^6\right)\text{ N} = \frac{\left(1000\text{ kg/m}^3\right)(\pi)(1.25\text{ m})^2}{2}\left(V_2^2 - V_1^2\right)$$

$$V_2^2 - V_1^2 = 611.15 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$V_1 = 6.383\text{ m/s} = 6.38\text{ m/s}$$

$$V_2 = 25.53\text{ m/s} \qquad \textbf{Ans.}$$

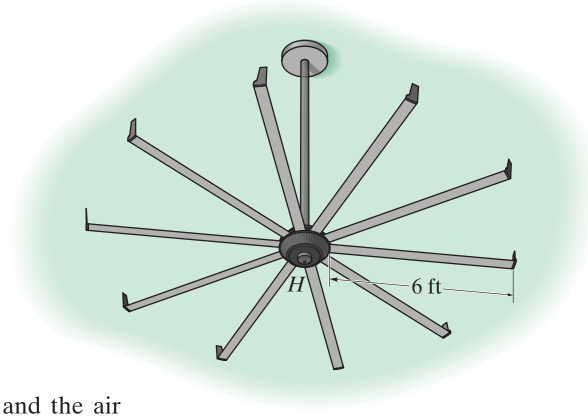
The power output is

$$\dot{W}_{\text{out}} = FV_1 = \left[1.5\left(10^6\right)\text{ N}\right]\left(6.383\text{ m/s}\right) = 9.575\left(10^6\right)\text{ W} = 9.575\text{ MW}$$

Thus, the power supply to the propeller is

$$\dot{W}_{\text{in}} = \frac{P_{\text{out}}}{e} = \frac{9.575\text{ MW}}{0.4} = 23.94\text{ MW} = 23.9\text{ MW} \qquad \textbf{Ans.}$$

6–85. The fan is used to circulate air within a large industrial building. The blade assembly weighs 200 lb and consists of 10 blades, each having a length of 6 ft. Determine the power that must be supplied to the motor to lift the assembly off its bearings and allow it to freely turn without friction. What is the downward air velocity for this to occur? Neglect the size of the hub H. Take $\rho_a = 2.36\left(10^{-3}\right)$ slug/ft^3.



SOLUTION

The blade and air within it is the control volume. The flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 2.36\left(10^{-3}\right)$ slug/ft^3. Average velocities will be used. To lift the blade assembly off the bearings, the thrust must be equal to the weight of the assembly, i.e, $F = 200$ lb. Since the air enters the blade assembly from the surroundings which is at rest, $V_1 = 0$.

$$F = \frac{\rho_a \pi R^2}{2}\left(V_2^2 - V_1^2\right); \qquad 200\text{ lb} = \frac{2.36\left(10^{-3}\right)\text{ slug/ft}^3\left[\pi(6\text{ ft})^2\right]}{2}\left(V_2^2 - 0\right)$$

$$V_2 = 38.71\text{ ft/s} = 38.7\text{ ft/s} \qquad \textbf{Ans.}$$

$$V = \frac{V_1 + V_2}{2} = \frac{0 + 38.71\text{ ft/s}}{2} = 19.36\text{ ft/s}$$

The power required by the motor is

$$\dot{W} = FV = (200\text{ lb})(19.36\text{ ft/s})$$

$$= \left(3871.22\,\frac{\text{ft}\cdot\text{lb}}{\text{s}}\right)\left(\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}}\right)$$

$$= 7.04\text{ hp} \qquad \textbf{Ans.}$$

Ans:
$V_2 = 38.7$ ft/s
$\dot{W} = 7.04$ hp

6–86. The 12-Mg helicopter is hovering over a lake as the suspended bucket collects 5 m^3 of water used to extinguish a fire. Determine the power required by the engine to hold the filled water bucket over the lake. The horizontal blade has a diameter of 14 m. Take $\rho_a = 1.23\ \text{kg/m}^3$.

SOLUTION

The helicopter, bucket, water, and air within the helicopter blade is the control volume. The flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 1.23\ \text{kg/m}^3$. Average velocities will be used. To maintain the hovering, the thrust produced by the rotor blade must be equal to the weight of the helicopter and the water. Thus,

$$F = \left[12\left(10^3\right)\text{ kg}\right]\left(9.81\text{ m/s}^2\right) + \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left(5\text{ m}^3\right)$$

$$= 166.77\left(10^3\right)\text{ N}$$

Since the air enters the blade from the surroundings, which is at rest, $V_1 = 0$.

$$F = \frac{\rho_a \pi R^2}{2}\left(V_2^2 - V_1^2\right); \qquad 166.77\left(10^3\right)\text{ N} = \frac{\left(1.23\text{ kg/m}^3\right)\left[\pi(7\text{ m})^2\right]}{2}\left(V_2^2 - 0\right)$$

$$V_2 = 41.97\text{ m/s}$$

$$V = \frac{V_1 + V_2}{2} = \frac{0 + 41.97\text{ m/s}}{2} = 20.985\text{ m/s}$$

Thus, the power required by the engine is

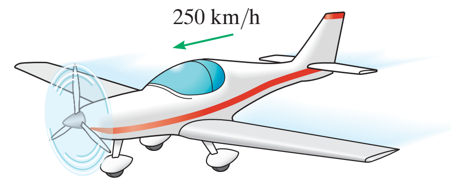
$$\dot{W} = FV = \left[166.77\left(10^3\right)\text{ N}\right]\left(20.985\text{ m/s}\right)$$

$$= 3.4997\left(10^6\right)\text{ W}$$

$$= 3.50\text{ MW} \qquad \textbf{Ans.}$$

Ans:
3.50 MW

6–87. The airplane has a constant speed of 250 km/h in still air. If it has a 2.4-m-diameter propeller, determine the force acting on the plane if the speed of the air behind the propeller, measured relative to the plane, is 750 km/h. Also, what is the ideal efficiency of the propeller, and the power produced by the propeller? Take $\rho_a = 0.910\ \mathrm{kg/m^3}$.



SOLUTION

The airplane moves in the still air and the control volume is attached to the airplane, which is travelling with a constant velocity of $\left(250\ \dfrac{\mathrm{km}}{\mathrm{h}}\right)\left(\dfrac{1000\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\dfrac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) =$ 69.44 m/s. Then the inlet velocity is $V_1 = 69.44\ \mathrm{m/s}$. Relative to the control volume, the flow is steady. The air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 0.910\ \mathrm{kg/m^3}$. Average velocities will be used. The outlet velocity is $V_2 = \left(750\ \dfrac{\mathrm{km}}{\mathrm{h}}\right)\left(\dfrac{1000\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\dfrac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = 208.33\ \mathrm{m/s}$. The thrust on the plane is

$$F = \frac{\rho_a \pi R^2}{2}\left(V_2^2 - V_1^2\right);$$

$$= \frac{\left(0.910\ \mathrm{kg/m^3}\right)\left[\pi(1.2\ \mathrm{m})^2\right]}{2}\left[(208.33\ \mathrm{m/s})^2 - (69.44\ \mathrm{m/s})^2\right]$$

$$= 79.41\left(10^3\right)\ \mathrm{N}$$

The power generated by the propeller is

$$\dot{W}_0 = FV_1 = \left[79.41\left(10^3\right)\ \mathrm{N}\right](69.44\ \mathrm{m/s})$$

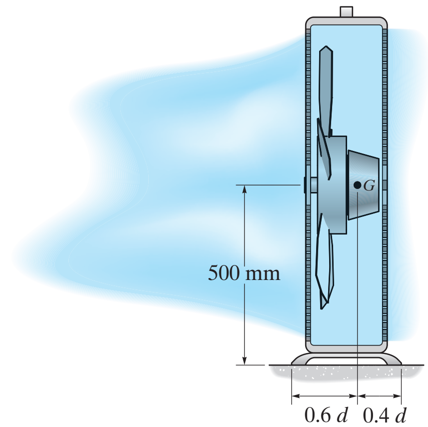
$$= 5.515\left(10^6\right)\ \mathrm{W} = 5.51\ \mathrm{MW} \qquad \textbf{Ans.}$$

The efficiency of the propeller is

$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(69.44\ \mathrm{m/s})}{69.44\ \mathrm{m/s} + 208.33\ \mathrm{m/s}} = 0.5 \qquad \textbf{Ans.}$$

Ans:
$F = 79.4\ \mathrm{kN}$
$\dot{W} = 5.51\ \mathrm{MW}$
$e = 0.5$

***6–88.** The 12-kg fan develops a breeze of 10 m/s using a 0.8-m-diameter blade. Determine the smallest dimension d for the support so that the fan does not tip over. Take $\rho_a = 1.20\ \text{kg/m}^3$.



SOLUTION

Take the fan and air within it as the control volume. The flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that $\rho_a = 1.20\ \text{kg/m}^3$. Average velocities can be used. Since the air enters the blade from the surroundings which is at rest, $V_1 = 0$. Here, $V_2 = 10\ \text{m/s}$.

$$F = \frac{\rho_a \pi R^2}{2}\left(V_2^2 - V_1^2\right)$$

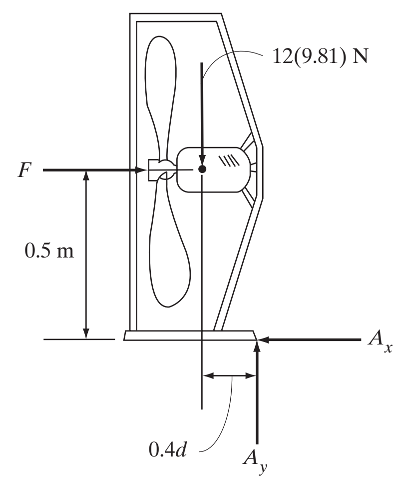
$$= \frac{\left(1.20\ \text{kg/m}^3\right)\left[\pi(0.4\ \text{m})^2\right]}{2}\left[(10\ \text{m/s})^2 - 0\right]$$

$$= 9.6\pi\ \text{N}$$

Referring to the FBD of the fan shown in Fig. a, and writing the moment equation of equilibrium about point A,

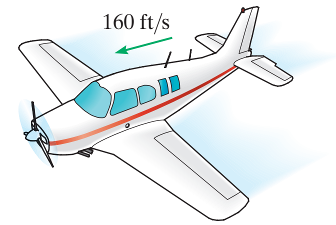
$$\zeta + \Sigma M_A = 0; \qquad [12(9.81)\ \text{N}](0.4\,d) - (9.6\pi\ \text{N})(0.5\ \text{m}) = 0$$

$$d = 0.320\ \text{m} = 320\ \text{mm} \qquad \textbf{Ans.}$$



(a)

6–89. The airplane is flying at 160 ft/s in still air at an altitude of 10 000 ft. The 7-ft-diameter propeller moves the air at 10 000 ft^3/s. Determine the power required by the engine to turn the propeller, and the thrust on the plane.



SOLUTION

Take the propeller and air within it as the control volume. Since the airplane moves in the still air and the control volume is attached to the airplane, which is travelling with a constant velocity of the 160 ft/s, then the inlet velocity is $V_1 = 160$ ft/s.

Relative to the control volume, the flow is steady. The air can be considered as an ideal fluid (incompressible and inviscid) such that at an altitude of 10,000 ft, $\rho_a = 1.754(10^{-3})$ slug/ft^3. Average velocity will be used. From the discharge

$Q = VA$; $\qquad 10\,000 \text{ ft}^3/\text{s} = V\left[\pi(3.5 \text{ ft})^2\right] \qquad V = 259.84 \text{ ft/s}$

$V = \dfrac{V_1 + V_2}{2}$; $\qquad 259.84 \text{ ft/s} = \dfrac{160 \text{ ft/s} + V_2}{2} \qquad V_2 = 359.69 \text{ ft/s}$

The thrust on the plane is

$$F = \frac{\rho_a \pi R^2}{2}\left(V_2^2 - V_1^2\right)$$

$$= \frac{\left[1.754\left(10^{-3}\right) \text{slug/ft}^3\right]\left[\pi(3.5 \text{ ft})^2\right]}{2}\left[(359.69 \text{ ft/s})^2 - (160 \text{ ft/s})^2\right]$$

$$= 3.503\left(10^3\right) \text{ lb} = 3.50 \text{ kip} \qquad \textbf{Ans.}$$

The power required to turn the propeller is

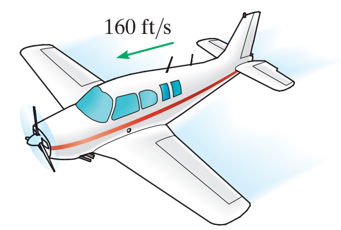
$$\dot{W}_i = FV = \left[3.503\left(10^3\right) \text{ lb}\right](259.84 \text{ ft/s})$$

$$= \left[910.22\left(10^3\right) \text{ ft}\cdot\text{lb/s}\right]\left[\frac{1 \text{ hp}}{550 \text{ ft}\cdot\text{lb/s}}\right]$$

$$= 1655 \text{ hp} \qquad \textbf{Ans.}$$

Ans:
$F = 3.50$ kip
$\dot{W} = 1655$ hp

6–90. The airplane is flying at 160 ft/s in still air at an altitude of 10 000 ft. The 7-ft-diameter propeller moves the air at 10 000 ft^3/s. Determine the propeller's ideal efficiency, and the pressure difference between the front and back of the blades.



SOLUTION

Take the propeller and air within it as the control volume. Since the airplane moves in the still air and the control volume is attached to the airplane, which is travelling with a constant velocity of 160 ft/s, then the inlet velocity is $V_1 = 160$ ft/s.

Relative to the control volume the flow is steady and the air can be considered as an ideal fluid (incompressible and inviscid) such that at an altitude of 10,000 ft, $\rho_a = 1.754(10^{-3})$ slug/ft^3. Average velocities will be used. From the discharge

$$Q = VA; \qquad 10\,000 \text{ ft}^3/\text{s} = V\left[\pi(3.5 \text{ ft})^2\right] \qquad V = 259.84 \text{ ft/s}$$

$$V = \frac{V_1 + V_2}{2}; \qquad 259.84 \text{ ft/s} = \frac{160 \text{ ft/s} + V_2}{2} \qquad V_2 = 359.69 \text{ ft/s}$$

The ideal efficiency of the propeller is

$$e = \frac{2V_1}{V_1 + V_2} = \frac{2(160 \text{ ft/s})}{160 \text{ ft/s} + 359.69 \text{ ft/s}} = 0.616 \qquad \textbf{Ans.}$$

The pressure difference is

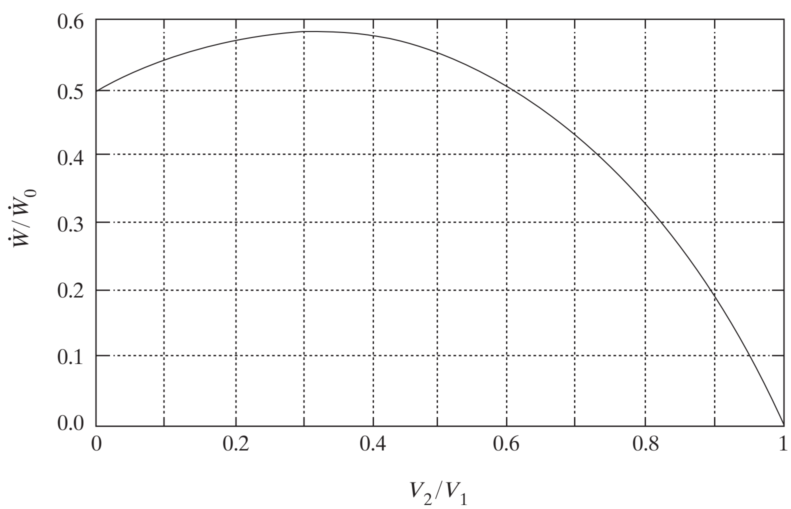
$$\Delta p = p_4 - p_3 = \rho_a V(V_2 - V_1)$$
$$= \left[1.754(10^{-3}) \text{ slug/ft}^3\right](259.84 \text{ ft/s})\left[359.69 \text{ ft/s} - (160 \text{ ft/s})\right]$$
$$= \left(91.01\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^2 = 0.632 \text{ psi} \qquad \textbf{Ans.}$$

Ans:
$e = 0.616$
$\Delta p = 0.632$ psi

6–91. Plot Eq. 6–15 and show that the maximum efficiency of a wind turbine is 59.3% as stated by Betz's law.

SOLUTION

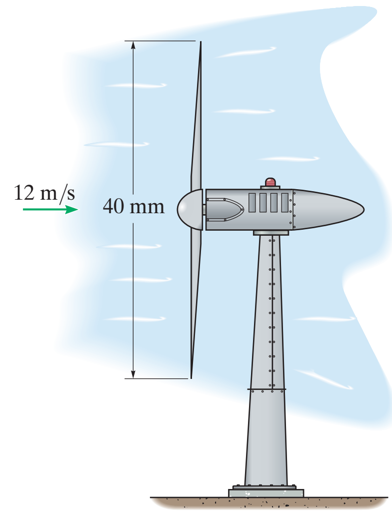
$$e_{\text{turb}} = \frac{1}{2}\left[1 - \left(\frac{V_2^2}{V_1^2}\right)\right]\left[1 + \left(\frac{V_2}{V_1}\right)\right]$$



$$\frac{\dot{W}}{\dot{W}_0} = 0.593 = 59.3\%$$

when $\dfrac{V_2}{V_1} = \dfrac{1}{3}$.

***6–92.** The wind turbine has a rotor diameter of 40 m and an ideal efficiency of 50% in a 12 m/s wind. If the density of the air is $\rho_a = 1.22\ \text{kg/m}^3$, determine the thrust on the blade shaft, and the power withdrawn by the blades.



SOLUTION

$$e = \frac{1}{2}\left[1 - \left(\frac{V_2}{V_1}\right)^2\right]\left[1 + \frac{V_2}{V_1}\right]$$

Solving the cubic equation with $e = 0.5$, we find $V_2/V_1 = 0.6180$ as the nonzero solution. Then $V_2 = 0.6180(12\ \text{m/s}) = 7.416\ \text{m/s}$ and

$$V = \frac{V_1 + V_2}{2} = \frac{12\ \text{m/s} + 7.416\ \text{m/s}}{2} = 9.708\ \text{m/s}$$

The thrust on the blades is

$$F = \frac{\rho_a \pi R^2}{2}(V_2{}^2 - V_1{}^2)$$

$$= \frac{(1.22\ \text{kg/m}^3)\pi(20\ \text{m})^2}{2}\left[(12\ \text{m/s})^2 - (7.416\ \text{m/s})^2\right]$$

$$= 68.220(10^3)\ \text{N} = 68.2\ \text{kN}$$ **Ans.**

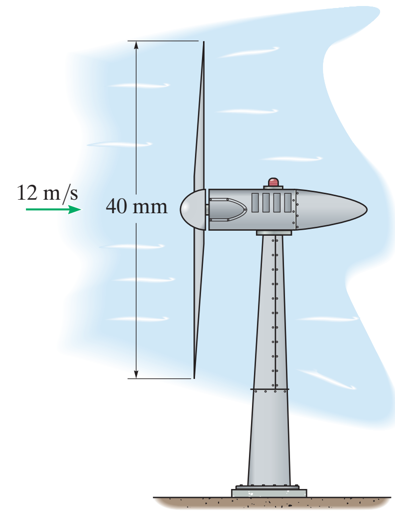
The power withdrawn by the blades is

$$\dot{W} = FV = \left[68.220(10^3)\ \text{N}\right](9.708\ \text{m/s})$$

$$= 662.3(10^3)\ \text{W}$$

$$= 662\ \text{kW}$$ **Ans.**

6–93. The wind turbine has a rotor diameter of 40 m and an efficiency of 50% in a 12 m/s wind. If the density of the air is $\rho_a = 1.22\ \text{kg/m}^3$, determine the difference between the pressure just in front of and just behind the blades. Also find the mean velocity of the air passing through the blades.



SOLUTION

$$e = \frac{1}{2}\left[1 - \left(\frac{V_2}{V_1}\right)^2\right]\left[1 + \frac{V_2}{V_1}\right]$$

Solving the cubic equation with $e = 0.5$, we find $V_2/V_1 = 0.6180$ as the nonzero solution. Then $V_2 = 0.6180(12\ \text{m/s}) = 7.416\ \text{m/s}$ and

$$V = \frac{V_1 + V_2}{2} = \frac{12\ \text{m/s} + 7.416\ \text{m/s}}{2} = 9.71\ \text{m/s} \qquad \textbf{Ans.}$$

The thrust on the blades is

$$F = \frac{\rho_a \pi R^2}{2}\left(V_2^2 - V_1^2\right)$$

$$= \frac{\left(1.22\ \text{kg/m}^3\right)\pi(20\ \text{m})^2}{2}\left[(12\ \text{m/s})^2 - (7.416\ \text{m/s})^2\right]$$

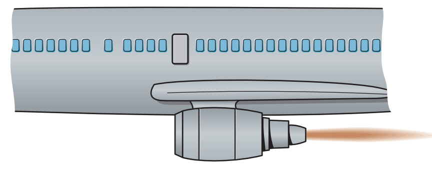
$$= 68.220\left(10^3\right)\ \text{N}$$

The pressure difference is

$$\Delta p = \frac{F}{A} = \frac{68.220\left(10^3\right)\ \text{N}}{\pi(20\ \text{m})^2} = 54.3\ \text{Pa} \qquad \textbf{Ans.}$$

Ans:
$V = 9.71\ \text{m/s}$
$\Delta p = 54.3\ \text{Pa}$

6–94. The jet engine on a plane flying at 160 m/s in still air draws in air at standard atmospheric temperature and pressure through a 0.5-m-diameter inlet. If 2 kg/s of fuel is added and the mixture leaves the 0.3-m-diameter nozzle at 600 m/s, measured relative to the engine, determine the thrust provided by the turbojet.



SOLUTION

From Appendix A, at standard atmospheric pressure and temperature (15° C), the density of air is $\rho_a = 1.23\ \text{kg/m}^3$. Thus,

$$\dot{m}_a = \rho_a VA = \left(1.23\ \text{kg/m}^3\right)\left(160\ \text{m/s}\right)\left[\pi(0.25\ \text{m})^2\right] = 38.64\ \text{kg/s}$$

The thrust of the turbojet is

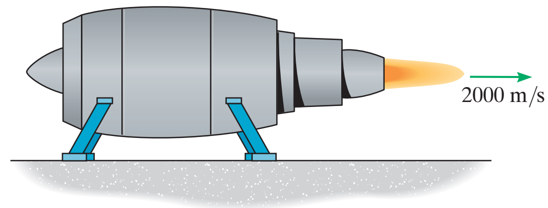
$$T = (\dot{m}_a + \dot{m}_f)V_e - \dot{m}_a V_{\text{cv}}$$

$$= \left(38.64\ \text{kg/s} + 2\ \text{kg/s}\right)\left(600\ \text{m/s}\right) - \left(38.64\ \text{kg/s}\right)\left(160\ \text{m/s}\right)$$

$$= 18.20\left(10^3\right)\ \text{N} = 18.2\ \text{kN}$$ **Ans.**

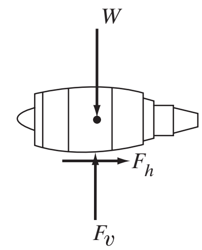
Ans:
18.2 kN

6–95. The jet engine is mounted on the stand while it is being tested. Determine the horizontal force that the engine exerts on the supports, if the fuel–air mixture has a mass flow of 11 kg/s and the exhaust has a velocity of 2000 m/s.



SOLUTION

Take the control volume as the engine and the fluid within it. We consider steady flow of an ideal fluid. Since the turbojet is at rest in still air, $\frac{dV_{cv}}{dt} = 0$, $V_{cv} = 0$, and $\dot{m}_a = 0$. Referring to the free-body diagram of the turbojet in Fig. *a*,



(a)

$$(\overset{+}{\leftarrow})\ \Sigma F_x = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

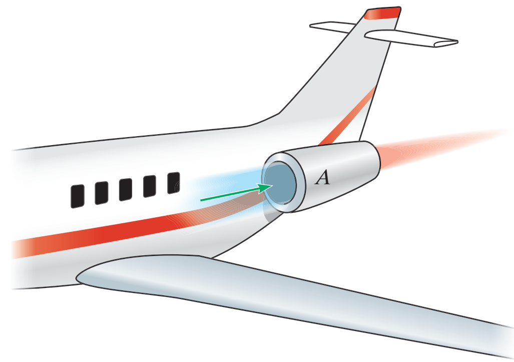
$$-F_h = 0 + 0 - (0 + 11\text{ kg/s})(2000\text{ m/s})$$

$$F_h = 22\text{ kN}$$ **Ans.**

This is the magnitude of the force the supports exert on the engine, and therefore also the magnitude of the equal and opposite force the engine exerts on the supports.

Ans:
22 kN

***6–96.** The jet plane has a constant velocity of 750 km/h. Air enters its engine nacelle at A having a cross-sectional area of 0.8 m^2. Fuel is mixed with the air at $\dot{m}_e = 2.5$ kg/s and is exhausted into the ambient air with a velocity of 900 m/s, measured relative to the plane. Determine the force the engine exerts on the wing of the plane. Take $\rho_a = 0.850$ kg/m^3.



SOLUTION

The control volume is considered to be the entire engine and its contents which move with a constant velocity. The flow, measured relative to the control volume, is steady. Here, $V_{cv} = \left(750\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 208.33$ m/s, $\dot{m}_f = 2.5$ kg/s and $V_e = 900$ m/s.

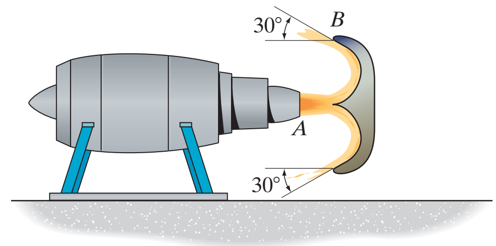
Thus,

$$\dot{m}_a = \rho_a V_{cv} A_A = (0.850\text{ kg/m}^3)(208.33\text{ m/s})(0.8\text{ m}^2) = 141.67\text{ kg/s}$$

The thrust developed is

$$\begin{aligned} T &= -\left[\dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e\right] \\ &= -\left[(141.67\text{ kg/s})(208.33\text{ m/s}) - (141.67\text{ kg/s} + 2.5\text{ kg/s})(900\text{ m/s})\right] \\ &= 100.24(10^3)\text{ N} = 100\text{ kN} \qquad \textbf{Ans.} \end{aligned}$$

6–97. The jet engine is mounted on the stand while it is being tested with the braking deflector in place. If the exhaust has a velocity of 800 m/s and the pressure just outside the nozzle is assumed to be atmospheric, determine the horizontal force that the supports exert on the engine. The fuel–air mixture has a flow of 11 kg/s.



SOLUTION

Under test conditions, with the pressure just outside the nozzle assumed to be atmospheric, the deflector is irrelevant since it is not attached to the engine. Since the engine is at rest in still air, $dV_{cv}/dt = 0$ and $V_{cv} = 0$, so that the support reaction force F, which points rightward, is given by

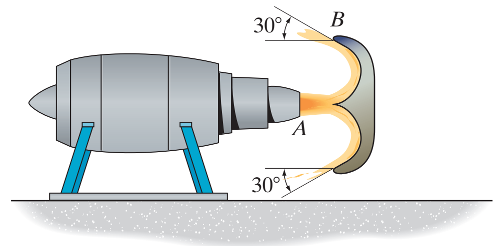
$$(\overset{+}{\leftarrow})\ \Sigma F_x = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$-F = 0 + 0 - (11\text{ kg/s})(800\text{ m/s})$$

$$F = 8800\text{ N} = 8.80\text{ kN}$$ **Ans.**

Ans:
8.80 kN

6–98. If an engine of the type shown in Prob. 6–97 is attached to a jet plane, and it operates the braking deflector with the conditions stated in that problem, determine the speed of the plane in 5 seconds after it lands with a touch-down velocity of 30 m/s. The plane has a mass of 8 Mg. Neglect rolling friction from the landing gear.



SOLUTION

Assume that the fuel is only a small fraction of the fuel-air mixture, so that $\dot{m}_a \approx \dot{m}_a + \dot{m}_f = 11$ kg/s. Then the force equation for the whole plane, of mass m_p, is

$$(\overset{+}{\leftarrow})\ \Sigma F_x = m_p \frac{dV}{dt} + \dot{m}_a V - (\dot{m}_a + \dot{m}_f)V_e$$

$$0 = (8000\text{ kg})\frac{dV}{dt} + (11\text{ kg/s})\,V - (11\text{ kg/s})(-800\text{ m/s} - V)\cos 30°$$

$$-8000\frac{dV}{dt} = 11[V(1 + \cos 30°) + 800\cos 30°]$$

$$-8000\frac{dV}{dt} = 11(1.8660V + 692.82)$$

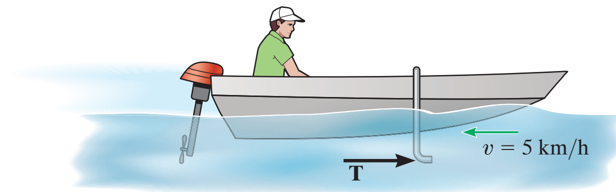
$$-\int_{30\text{ m/s}}^{V} \frac{8000}{1.866\,V + 692.82}\,dV = \int_0^{s} 11\,dt$$

$$-\frac{8000}{1.866}\ln\left(\frac{1.866V + 692.82}{748.80}\right) = 55$$

$V = 24.9$ m/s **Ans.**

Ans:
24.9 m/s

6–99. The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured relative to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection.



SOLUTION

Consider the boat, tube, and water within it as the moving control volume. We consider steady flow of an ideal fluid relative to the control volume.

$$\frac{dm}{dt} = \frac{40}{80} = 0.5 \text{ kg/s}$$

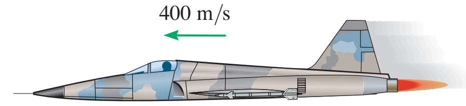
$$v_{D/t} = (70)\left(\frac{1000}{3600}\right) = 19.444 \text{ m/s}$$

$$\Sigma F_x = m\frac{dV}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$T = 0 + 19.444(0.5) = 9.72 \text{ N}$$ **Ans.**

Ans:
9.72 N

***6–100.** The jet is traveling at a constant velocity of 400 m/s in still air, while consuming fuel at the rate of 1.8 kg/s and ejecting it at 1200 m/s relative to the plane. If the engine consumes 1 kg of fuel for every 50 kg of air that passes through the engine, determine the thrust produced by the engine and the efficiency of the engine.



SOLUTION

The control volume considered is the entire airplane and its contents which moves with a constant velocity. We consider steady flow of an ideal fluid. The flow measured relative to the control volume is steady. Here,

$$V_{cv} = 400\text{ m/s},\ \dot{m}_f = 1.8\text{ kg/s},\ \dot{m}_a = 50\left(1.8\text{ kg/s}\right)$$
$$= 90\text{ kg/s and } V_e = 1200\text{ m/s}$$
$$T = -\left[\dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e\right]$$
$$= -\left[\left(90\text{ kg/s}\right)\left(400\text{ m/s}\right) - \left(90\text{ kg/s} + 1.8\text{ kg/s}\right)\left(1200\text{ m/s}\right)\right]$$
$$= 74.16\left(10^3\right)\text{ N} = 74.2\text{ kN} \qquad \textbf{Ans.}$$

The useful power output of the engine is

$$\dot{W}_0 = TV = \left[74.16\left(10^3\right)\text{N}\right]\left(400\text{ m/s}\right) = 29.664\left(10^6\right)\text{ W}$$

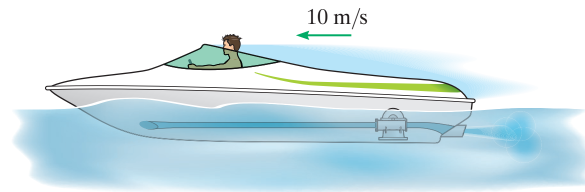
Some of the power produces the kinetic energy per unit time of the exhaust fuel-air mixture. Its velocity relative to the ground is $V_{mix} = V_e - V_{cv} = 1200\text{ m/s} - 400\text{ m/s} = 800\text{ m/s}$. Thus, the power loss is

$$\dot{W}_l = \frac{1}{2}(\dot{m}_a + \dot{m}_f)\,V_{mix}^2$$
$$= \frac{1}{2}\left(90\text{ kg/s} + 1.8\text{ kg/s}\right)\left(800\text{ m/s}\right)^2$$
$$= 29.376\left(10^6\right)\text{ MW}$$

The efficiency of the engine is

$$e = \frac{\dot{W}_0}{\dot{W}_0 + P_l} = \frac{29.664\left(10^6\right)\text{ W}}{29.664\left(10^6\right)\text{ W} + 29.376\left(10^6\right)\text{ W}}$$
$$= 0.502 \qquad \textbf{Ans.}$$

6–101. The jet boat takes in water through its bow at $0.03\ \text{m}^3/\text{s}$, while traveling in still water with a constant velocity of $10\ \text{m/s}$. If the water is ejected from a pump through the stern at $30\ \text{m/s}$, measured relative to the boat, determine the thrust developed by the engine. What would be the thrust if the $0.03\ \text{m}^3/\text{s}$ of water were taken in along the sides of the boat, perpendicular to the direction of motion? If the efficiency is defined as the work done per unit time divided by the energy supplied per unit time, then determine the efficiency for each case.



SOLUTION

The control volume considered is the entire boat and its contents, which moves with a constant velocity. The flow, measured relative to the control volume, is steady. Water is considered to be incompressible. Here, $V_{cv} = 10\ \text{m/s}$, $\dot{m}_f = 0$, $\dot{m}_w = \rho Q = (1000\ \text{kg/m}^3)(0.03\ \text{m}^3/\text{s}) = 30\ \text{kg/s}$. and $V_e = 30\ \text{m/s}$. The thrust is

$$T_1 = -\left[\dot{m}_w V_{cv} - (\dot{m}_w + \dot{m}_f)V_e\right]$$
$$= -\left[(30\ \text{kg/s})(10\ \text{m/s}) - (30\ \text{kg/s} + 0)(30\ \text{m/s})\right]$$
$$= 600\ \text{N} \qquad \textbf{Ans.}$$

If the intake of water is perpendicular to the direction of motion, $V_{cv} = 0$. Then

$$T_2 = \left[\dot{m}_w V_{cv} - (\dot{m}_w + \dot{m}_f)V_e\right]$$
$$= -\left[(30\ \text{kg/s})(0) - (30\ \text{kg/s} + 0)(30\ \text{m/s})\right]$$
$$= 900\ \text{N} \qquad \textbf{Ans.}$$

The power output for both cases can be determined from

$$(\dot{W}_o)_1 = T_1 V = (600\ \text{N})(10\ \text{m/s}) = 6000\ \text{W}$$
$$(\dot{W}_o)_2 = T_2 V = (900\ \text{N})(10\ \text{m/s}) = 9000\ \text{W}$$

Some of the power produces the kinetic energy per unit time of the ejected water. Its velocity relative to ground is $V = V_e - V_{cv} = 30\ \text{m/s} - 10\ \text{m/s} = 20\ \text{m/s}$. For both cases, the power loss in the same and is

$$\dot{W}_l = \frac{1}{2}(\dot{m}_w + \dot{m}_f)V^2 = \frac{1}{2}(30\ \text{kg/s} + 0)(20\ \text{m/s})^2 = 6000\ \text{W} \qquad \textbf{Ans.}$$

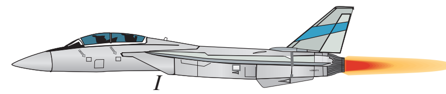
Thus, the efficiency for each case is

$$e_1 = \frac{(\dot{W}_o)_1}{(\dot{W}_o)_1 + (\dot{W}_o),} = \frac{6000\ \text{W}}{6000\ \text{W} + 6000\ \text{W}} = 0.5 \qquad \textbf{Ans.}$$

$$e_2 = \frac{(\dot{W}_o)_2}{(\dot{W}_o)_2 + (\dot{W}_o),} = \frac{9000\ \text{W}}{9000\ \text{W} + 6000\ \text{W}} = 0.6 \qquad \textbf{Ans.}$$

Ans:
$T_1 = 600\ \text{N}$
$T_2 = 900\ \text{N}$
$e_1 = 0.5$
$e_2 = 0.6$

6–102. The 10-Mg jet plane has a constant speed of 860 km/h when it is flying horizontally. Air enters the intake I at the rate of 40 m^3/s. If the engine burns fuel at the rate of 2.2 kg/s, and the gas (air and fuel) is exhausted relative to the plane with a speed of 600 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that the air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION

Take the plane and its contents as the control volume. We consider steady flow of an ideal fluid.

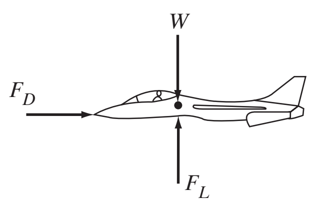
$$V_{cv} = \left(860\frac{\text{km}}{\text{h}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right) = 238.89\ \text{m/s and}\ \dot{m}_a = \rho Q = \left(1.22\ \text{kg/m}^3\right)\left(40\ \text{m}^3/\text{s}\right) = 48.8\ \text{kg/s}$$

Since the airplane is traveling with constant speed, $\dfrac{dV_{cv}}{dt} = 0$. Referring to the free-body diagram of the jet plane in Fig. *a*,

$$(\overset{+}{\leftarrow})\ \Sigma F_x = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$-F_D = 0 + (48.8\ \text{kg/s})(238.89\ \text{m/s}) - (48.8\ \text{kg/s} + 2.2\ \text{kg/s})(600\ \text{m/s})$$

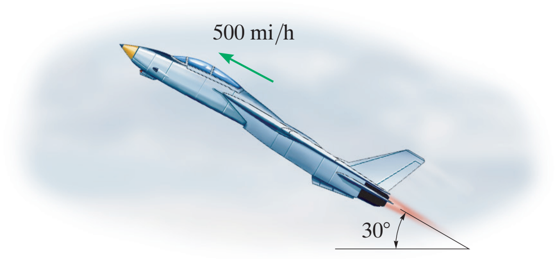
$$F_D = 18.94(10^3)\ \text{N} = 18.9\ \text{kN}$$ **Ans.**



(a)

Ans:
18.9 kN

6–103. The jet is traveling at a speed of 500 mi/h, 30° above the horizontal. If the fuel is being spent at 10 lb/s, and the engine takes in air at 900 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 4000 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.07v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. Take 1 mi = 5280 ft.



SOLUTION

The control volume considered is the entire jet and its contents as shown in Fig. *a* which is accelerating. We consider steady flow of an ideal fluid relative to the control volume. Here,

$$V_{cv} = \left(500\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 733.33\text{ ft/s}$$

$$F_D = 0.07V_{cv}^2 = 0.07(733.33^2) = 37\,644.44\text{ lb}$$

$$\dot{m}_a = \frac{900\text{ lb/s}}{32.2\text{ ft/s}^2} = 27.9503\text{ slug/s}$$

$$\dot{m}_f = \frac{10\text{ lb/s}}{32.2\text{ ft/s}^2} = 0.3106\text{ slug/s}$$

$$V_e = 4000\text{ ft/s}$$

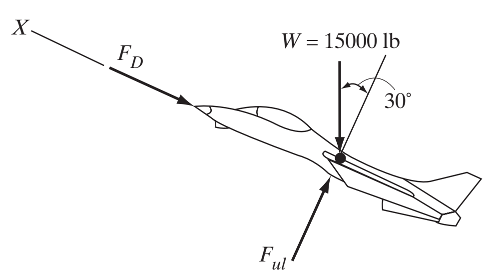
Referring to the FBD of the control volume, Fig. *a*,

$$\overset{+}{\leftarrow}\Sigma F_x = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$-(15000\text{ lb})\sin 30° - 37644.44\text{ lb} = \left(\frac{15000\text{ lb}}{32.2\text{ ft/s}^2}\right)a_{cv} + (27.9503\text{ slug/s})(733.33\text{ ft/s})$$

$$-(27.9503\text{ slug/s} + 0.3106\text{ slug/s})(4000\text{ ft/s})$$

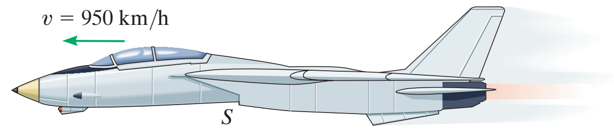
$$a_{cv} = 101.76\text{ ft/s}^2 = 102\text{ ft/s}^2$$ **Ans.**



(a)

Ans:
102 ft/s²

***6–104.** The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s, and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3.



SOLUTION

The control volume considered is the entire jet and its contents as shown in Fig. *a*. We consider steady flow of an ideal fluid relative to the control volume. Here

$$V_{cv} = \left(950\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 263.89\text{ m/s}$$

$$\dot{m}_a = \rho_a Q = \left(1.22\text{ kg/m}^3\right)\left(50\text{ m}^3/\text{s}\right) = 61\text{ kg/s}$$

$$\dot{m}_f = 0.4\text{ kg/s}$$

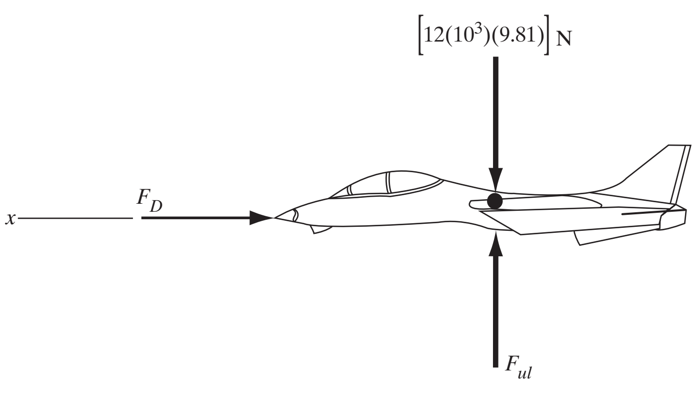
$$V_e = 450\text{ m/s}$$ **Ans.**

Referring to the FBD of the control volume, Fig. *a*, with $\frac{dV_{cv}}{dt} = 0$, since the jet travels with a constant velocity, we have

$$\overset{+}{\leftarrow} \Sigma F_x = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - \left(\dot{m}_a + \dot{m}_f\right)V_e$$

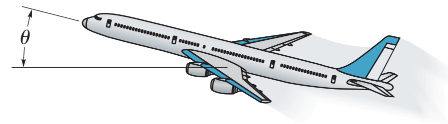
$$-F_D = 0 + (61\text{ kg/s})(263.89\text{ m/s}) - (61\text{ kg/s} + 0.4\text{ kg/s})(450\text{ m/s})$$

$$F_D = 11.53\left(10^3\right)\text{ N} = 11.5\text{ kN}$$ **Ans.**



(a)

6–105. A commercial jet aircraft has a mass of 150 Mg and is cruising at a constant speed of 850 km/h in level flight ($\theta = 0°$). If each of the two engines draws in air at a rate of 1000 kg/s and ejects it with a velocity of 900 m/s relative to the aircraft, determine the maximum angle θ at which the aircraft can fly with a constant speed of 750 km/h. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cV^2$, where c is a constant to be determined. The engines are operating with the same power in both cases. Neglect the amount of fuel consumed.



SOLUTION

The control volume considered is the entire plane and its contents as shown in Fig. a, which is accelerating. We consider steady flow of an ideal fluid relative to the control volume. Here,

$$V_{cv} = \left(850\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 236.11\text{ m/s } (\theta = 0°)$$

$$V_{cv} = \left(750\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 208.33\text{ m/s}$$

$$\dot{m}_a = 2(1000\text{ kg/s}) = 2000\text{ kg/s}$$

$$\dot{m}_f = 0\text{ (negligible)}$$

$$V_e = 900\text{ m/s}$$

Referring to the FBD of the control volume, Fig. a, along the x axis with $\dfrac{dV_{cv}}{dt} = 0$ (constant velocity), we have

$$\Sigma F_x = m\frac{dV_{cv}}{dt} + m_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$\left\{-\left[150(10^3)\ (9.81)\text{ N}\right]\right\}\sin\theta - c(208.33\text{ m/s}^2)^2 = 0 + (2000\text{ kg/s})(208.33\text{ m/s}) - (2000\text{ kg/s} + 0)(900\text{ m/s})$$

$$1.4715(10^6)\sin\theta + 43.403(10^3)c = 1.3833(10^6) \qquad \textbf{(1)}$$

For level flight, $\theta = 0°$. Then

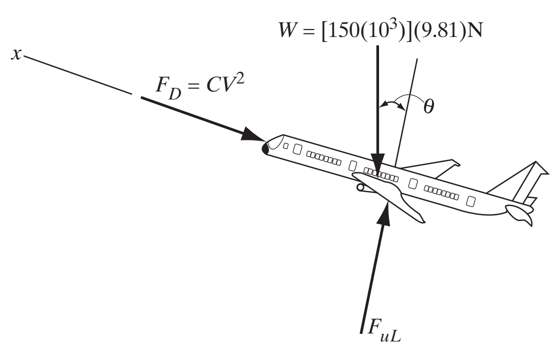
$$-c(236.11\text{ m/s})^2 = 0 + (2000\text{ kg/s})(236.11\text{ m/s}) - (2000\text{ kg/s} + 0)(900\text{ m/s})$$

$$c = 23.817$$

Substituting this result into Eq. (1),

$$1.4715(10^6)\sin\theta + 43.403(10^3)(23.817) = 1.3833(10^6)$$

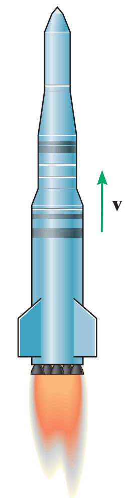
$$\theta = 13.74° = 13.7° \qquad \textbf{Ans.}$$



(a)

Ans:
13.7°

6–106. A missile has a mass of 1.5 Mg (without fuel). If it consumes 500 kg of solid fuel at a rate of 20 kg/s and ejects it with a velocity of 2000 m/s relative to the missile, determine the velocity and acceleration of the missile at the instant all the fuel has been consumed. Neglect air resistance and the variation of gravity with altitude. The missile is launched vertically starting from rest.

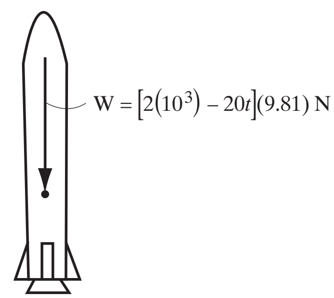


SOLUTION

The control volume consists of the missile and its contents as shown in Fig. *a*, which is accelerating upward. We consider steady flow of an ideal fluid relative to the control volume. The mass of the control volume as a function of time t is

$$M = M_o - \dot{m}_f t = \left[(1.5 + 0.5)(10^3)\text{ kg}\right] - (20\text{ kg/s})t$$

$$= \left[2(10^3) - 20t\right]\text{ kg}$$



(a)

Referring to the FBD of the control volume, Fig. *a*, with $\dot{m}_a = 0$ and $V_e = 2000$ m/s,

$$+\uparrow \Sigma F_y = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$-\left[2(10^3) - 20\,t\right](9.81)\text{ N} = \left\{\left[2(10^3) - 20\,t\right]\text{ kg}\right\}\frac{dV}{dt} + 0 - (0 + 20\text{ kg/s})(2000\text{ m/s})$$

$$\frac{dV}{dt} = \frac{40(10^3)}{2(10^3) - 20t} - 9.81 \qquad \textbf{(1)}$$

Integrating this equation with the initial condition $V = 0$ at $t = 0$,

$$\int_0^V dV = \int_0^t \left(\frac{40(10^3)}{2(10^3) - 20t} - 9.81\right)dt$$

$$V = \left[-2(10^3)\ln\left[2(10^3) - 20t\right] - 9.81t\right]\Big|_0^t$$

$$V = 2(10^3)\ln\left[\frac{2(10^3)}{2(10^3) - 20\,t}\right] - 9.81\,t \qquad \textbf{(2)}$$

The time required to consume all the fuel is

$$t = \frac{m_f}{\dot{m}_f} = \frac{500\text{ kg}}{20\text{ kg/s}} = 25\text{ s}$$

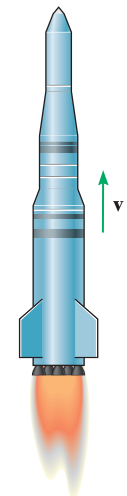
Substituting this result into Eqs. (1) and (2)

$$a = \frac{40(10^3)}{2(10^3) - 20(25)} - 9.81 = 16.9\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$V = 2(10^3)\ln\left[\frac{2(10^3)}{2(10^3) - 20(25)}\right] - 9.81(25) = 330\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
330 m/s

6–107. The rocket has a weight of 65 000 lb, including the solid fuel. Determine the constant rate at which the fuel must be burned, so that its thrust gives the rocket a speed of 200 ft/s in 10 s starting from rest. The fuel is expelled from the rocket at a speed of 3000 ft/s relative to the rocket. Neglect air resistance and the variation of gravity with altitude.



SOLUTION

The control volume considered consists of the rocket and its contents as shown in Fig. *a*, which is accelerating upwards. We consider steady flow of an ideal fluid relative to the control volume. The mass of the control volume as a function of time t is

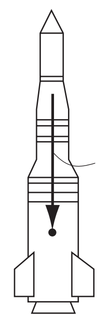
$$M = M_o - \dot{m}_f t = \frac{65000 \text{ lb}}{32.2 \text{ ft/s}^2} - \dot{m}_f t = (2018.63 - \dot{m}_f t) \text{ slug}$$

Referring to the *FBD* of the control volume, Fig. *a* with $\dot{m}_a = 0$ and $V_e = 3000$ ft/s,

$$+\uparrow \Sigma F_y = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$-(2018.63 - \dot{m}_f t)(32.2) = (2018.63 - \dot{m}_f t)\frac{dV}{dt} + 0 - (0 + \dot{m}_f)(3000 \text{ ft/s})$$

$$\frac{dV}{dt} = \frac{3000\,\dot{m}_f}{2018.63 - \dot{m}_f t} - 32.2$$



(a)

Integrating this equation with the initial condition $V = 0$ at $t = 0$ and the requirement $V = 200$ ft/s at $t = 10$ s,

$$\int_0^{200 \text{ ft/s}} dV = \int_0^{10 \text{ s}} \left(\frac{3000\,\dot{m}_f}{2018.63 - \dot{m}_f t} - 32.2\right)dt$$

$$200 = \left[-3000 \ln(2018.63 - \dot{m}_f t) - 32.2\,t\right]\Big|_0^{10 \text{ s}}$$

$$200 = 3000 \ln\left(\frac{2018.63}{2018.63 - 10\,\dot{m}_f}\right) - 322$$

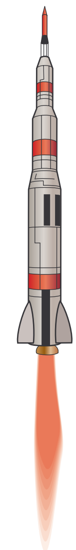
$$\ln\left(\frac{2018.63}{2018.63 - 10\,\dot{m}_f}\right) = 0.174$$

$$\frac{2018.63}{2018.63 - 10\,\dot{m}_f} = e^{0.174}$$

$$\dot{m}_f = 32.2 \text{ slug/s}$$ **Ans.**

Ans:
32.2 slug/s

***6–108.** The rocket is traveling upwards at 300 m/s and discharges 50 kg/s of fuel with a velocity of 3000 m/s measured relative to the rocket. If the exhaust nozzle has a cross-sectional area of 0.05 m^2, determine the thrust of the rocket.



SOLUTION

Take the rocket and its contents as the control volume.

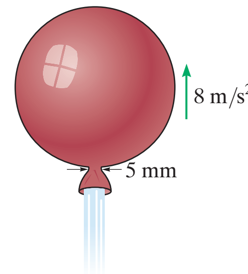
The thrust **T** needed to overcome W, F_D, and $m\dfrac{dV_{cv}}{dt}$ is

$$T = \dot{m}_f V_e$$

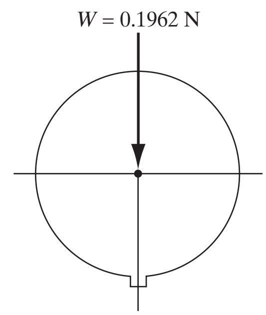
$$= (50\ \text{kg/s})(3000\ \text{m/s})$$

$$= 150(10^3)\ \text{N} = 150\ \text{kN} \quad \textbf{Ans.}$$

6–109. The balloon has a mass of 20 g (empty) and it is filled with air having a temperature of 20°C. If it is released, it begins to accelerate upwards at 8 m/s². Determine the initial mass flow of air from the stem. Assume the balloon is a sphere having a radius of 300 mm.



SOLUTION



(a)

The control volume considered is the balloon and the air contained within it, Fig. *a*. The initial flow measured relative to the accelerated control volume is treated as approximately steady. At $T = 20\,°\mathrm{C}$, $\rho_a = 1.202\ \mathrm{kg/m^3}$. The initial mass and weight of the balloon are

$$m = m_b + m_a = 0.02\ \mathrm{kg} + \left(1.202\ \mathrm{kg/m^3}\right)\left[\frac{4}{3}\pi(0.3\ \mathrm{m})^3\right]$$

$$= 0.1559\ \mathrm{kg}$$

$$W = m_b g = (0.02\ \mathrm{kg})\left(9.81\ \mathrm{m/s^2}\right) = 0.1962\ \mathrm{N}$$

We neglect the weight of the air inside because it is counter-acted by buoyancy. Thus,

$$\Sigma \mathbf{F} = m\frac{dV_{cv}}{dt} + \frac{\partial}{\partial t}\int_{cv} \mathbf{V}_{f/cv}\rho_a d\forall + \int_{cs} \mathbf{V}_{f/cs}(\rho_a \mathbf{V}_{f/cs}\cdot d\mathbf{A})$$

Writing the scalar components of this equation along the *y* axis by referring to the FBD of the control volume, Fig. *a*.

$$+\uparrow \Sigma F_y = m\frac{dV_{cv}}{dt} + 0 + (-V_e)\rho_a(V_e A_e)$$

$$-0.1962\ \mathrm{N} = (0.1559\ \mathrm{kg})\left(8\ \mathrm{m/s^2}\right) - \left(1.202\ \mathrm{kg/m^3}\right)\left[\pi(0.0025\ \mathrm{m})^2\right]V_e^2$$

$$V_e = 247.33\ \mathrm{m/s}$$

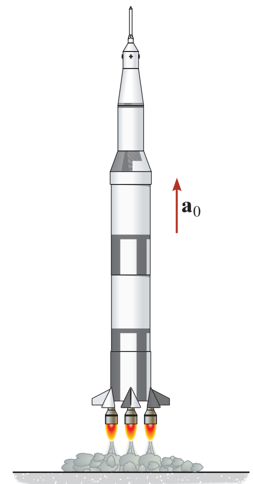
Thus, the initial mass flow is

$$\dot{m}_e = \rho_a V_e A_e = \left(1.202\ \mathrm{kg/m^3}\right)(247.33\ \mathrm{m/s})\left[\pi(0.0025\ \mathrm{m})^2\right]$$

$$= 0.00584\ \mathrm{kg/s} \qquad \textbf{Ans.}$$

Ans:
0.00584 kg/s

6–110. The rocket has an initial total mass m_0, including the fuel. When it is fired, it ejects a mass flow of $\dot{m}_e$ with a velocity of v_e measured relative to the rocket. As this occurs, the pressure at the nozzle, which has a cross-sectional area A_e, is p_e. If the drag force on the rocket is $F_D = ct$, where t is the time and c is a constant, determine the velocity of the rocket if the acceleration due to gravity is assumed to be constant.



SOLUTION

The control volume considered is the entire rocket and its contents, which accelerates upward. We consider steady flow of an ideal fluid relative to the control volume. The FBD of the control volume is shown in Fig. *a*. Here, the mass of the rocket as a function of time t is $m = m_0 - \dot{m}_e t$. Thus, the weight of the rocket as a function of time t is $W = mg = (m_0 - \dot{m}_e t)g$. The gage pressure force on the nozzle is $F_e = p_e A_e$.

$$\Sigma \mathbf{F} = m\frac{d\mathbf{V}_{cv}}{dt} + \frac{\partial}{\partial t}\int_{cv} \mathbf{V}_{f/cs}\,\rho\, d\forall + \int_{cv} \mathbf{V}_{f/cs}\,\rho\,\mathbf{V}_{f/cs}\cdot d\mathbf{A}$$

Writing the scalar component of this equation along the y axis by referring to Fig. *a*,

$$+\uparrow \Sigma F_y = (m_0 - \dot{m}_e t)\frac{dV}{dt} + 0 + (-V_e)(\rho_e V_e A_e)$$

Here, $\dot{m}_e = \rho_e V_e A_e$. Then

$$\rho_e A_e - ct - (m_0 - \dot{m}_e t)g = (m_0 - \dot{m}_e t)\frac{dV}{dt} - \dot{m}_e V_e$$

$$\frac{dV}{dt} = \frac{\dot{m}_e V_e}{m_0 - \dot{m}_e t} + \frac{p_e A_e}{m_0 - \dot{m}_e t} - \frac{ct}{m_0 - \dot{m}_e t} - g$$

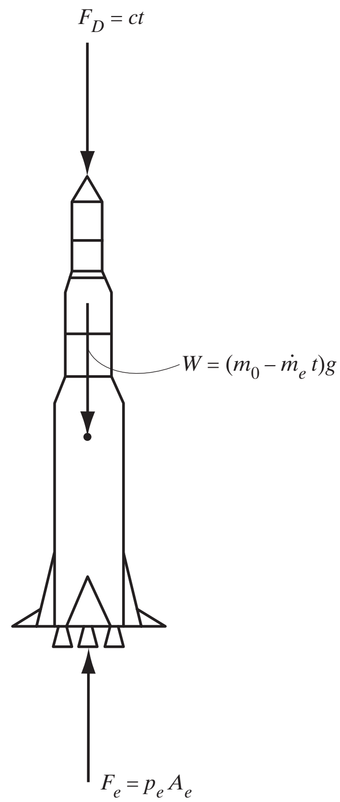
Integrating this equation with the initial condition $V = 0$ at $t = 0$,

$$\int_0^V dV = \int_0^t \left(\frac{\dot{m}_e V_e}{m_0 - \dot{m}_e t} + \frac{p_e A_e}{m_0 - \dot{m}_e t} - \frac{ct}{m_0 - \dot{m}_e t} - g\right)dt$$

$$V = \left\{-V_e \ln(m_0 - \dot{m}_e t) - \frac{p_e A_e}{\dot{m}_e}\ln(m_0 - \dot{m}_e t) - \left[-\frac{ct}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2}\ln(m_0 - \dot{m}_e t)\right] - gt\right\}\Bigg|_0^t$$

$$= V_e \ln\left(\frac{m_0}{m_0 - \dot{m}_e t}\right) + \frac{p_e A_e}{\dot{m}_e}\ln\left(\frac{m_0}{m_0 - \dot{m}_e t}\right) + \frac{ct}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2}\ln\frac{m_0}{m_0 - \dot{m}_e t} - gt$$

$$= \left(V_e + \frac{p_e A_e}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2}\right)\ln\left(\frac{m_0}{m_0 - \dot{m}_e t}\right) + \left(\frac{c}{\dot{m}_e} - g\right)t \qquad \textbf{Ans.}$$

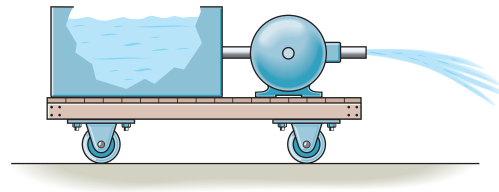


(a)

Ans:

$$V = \left(V_e + \frac{p_e A_e}{\dot{m}_e} - \frac{m_0 c}{\dot{m}_e^2}\right)\ln\left(\frac{m_0}{m_0 - \dot{m}_e t}\right) + \left(\frac{c}{\dot{m}_e} - g\right)t$$

6–111. The cart has a mass M and is filled with water that has an initial mass m_0. If a pump ejects the water through a nozzle having a cross-sectional area A, at a constant rate of v_0 relative to the cart, determine the velocity of the cart as a function of time. What is the maximum speed of the cart, assuming all the water can be pumped out? The frictional resistance to forward motion is F. The density of the water is ρ.



SOLUTION

The control volume considered is the entire cart assembly as shown in Fig. *a* which is accelerating. Here, the mass flow rate of the water is

$$\dot{m}_f = \rho V_e A$$

Thus, the mass of the control volume as a function of time t is

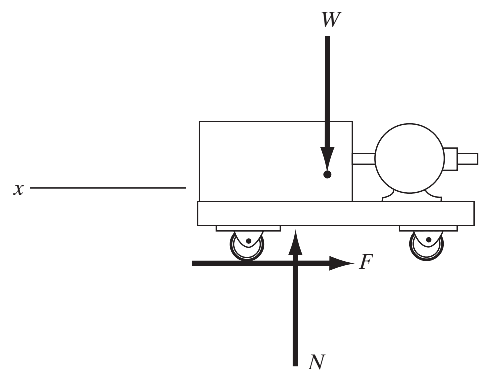
$$m = (M + m_0) - \dot{m}_e t = m + m_0 - \rho V_e A t$$

Referring to the FBD of the control volume, Fig. *a* with $\dot{m}_a = 0$,

$$\xrightarrow{+} \Sigma F_x = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$-F = (M + m_0 - \rho V_e A t)\frac{dV}{dt} + 0 - (0 + \rho V_e A)V_e$$

$$\frac{dV}{dt} = \frac{\rho V_e^2 A - F}{(M + m_0) - \rho V_e A t}$$



(a)

Integrating this equation with the initial condition $V = 0$ at $t = 0$,

$$\int_0^V dV = \int_0^t \left[\frac{\rho V_e^2 A - F}{(M + m_0) - \rho V_e A t}\right] dt$$

$$V = -\frac{\rho V_e^2 A - F}{\rho V_e A}\left[\ln(M + m_0 - \rho V_e A t)\right]\Bigg|_0^t$$

$$= \frac{\rho V_e^2 A - F}{\rho V_e A}\ln\left(\frac{M + m_0}{M + m_0 - \rho V_e A t}\right) \qquad \textbf{Ans.}$$

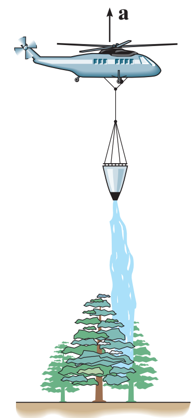
$$t_{\text{empty}} = \frac{m_0}{\dot{m}_e} = \frac{m_0}{\rho V_e A}, \text{ so}$$

$$V_{\max} = \frac{\rho V_e^2 A - F}{\rho V_e A}\ln\left(\frac{M + m_0}{M}\right) \qquad \textbf{Ans.}$$

Ans:

$$V_{\max} = \frac{\rho V_e^2 A - F}{\rho V_e A}\ln\left(\frac{M + m_0}{M}\right)$$

***6–112.** The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s measured relative to the helicopter, determine the initial upward acceleration of the helicopter as the water is being released.



SOLUTION

The control volume considered consists of the helicopter and the bucket containing water as shown in Fig. *a*, which is accelerating upward. We consider steady flow of an ideal fluid relative to the control volume. The initial mass of the control volume is

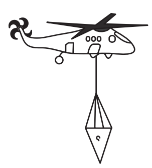
$$M_0 = 10\left(10^3\right)\text{ kg} + 0.5\left(10^3\right)\text{ kg} = 10.5\left(10^3\right)\text{ kg}$$

Since the helicopter is hovering before the water is released, its weight and the water's initial weight are balanced by the uplift generated by the rotor blade. Therefore, they are not shown in the FBD of the control volume, Fig. *a*. Referring to the FBD of the control volume with $\dot{m}_a = 0$, $\dot{m}_f = 50\text{ kg/s}$, $V_e = 10\text{ m/s}$,

$$+\uparrow \Sigma F_y = m\frac{dV_{\text{cv}}}{dt} + \dot{m}_a V_{\text{cv}} - \left(\dot{m}_a + \dot{m}_f\right)V_e$$

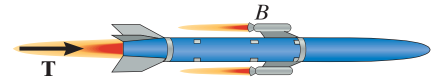
$$0 = \left[10.5\left(10^3\right)\text{ kg}\right]\frac{dV}{dt} + 0 - \left(0 + 50\text{ kg/s}\right)\left(10\text{ m/s}\right)$$

$$a_0 = \frac{dV}{dt} = 0.0476\text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$



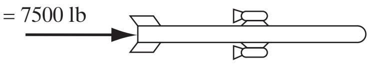
(a)

6–113. The missile has an initial total weight of 8000 lb. The constant horizontal thrust provided by the jet engine is $T = 7500$ lb. Additional thrust is provided by *two* rocket boosters B. The propellant in each booster is burned at a constant rate of 80 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the jet engine can be neglected, determine the velocity of the missile after the 3-s burn time of the boosters. The initial velocity of the missile is 375 ft/s. Neglect drag resistance.



SOLUTION

Take the missile and its contents as the control volume. We consider steady flow of an ideal fluid relative to the control volume.
At any instant t, the total mass of the missile is $m = m_0 = \dot{m}_f t$. Referring to the free-body diagram of the missile in Fig. a.



(a)

$$\xrightarrow{+} \Sigma F = m\frac{dV_{cv}}{dt} - \dot{m}V_e$$

$$T = (m_0 - \dot{m}_f\mathrm{t})\frac{dV}{dt} - \dot{m}_f V_e$$

$$\frac{dV}{dt} = \frac{T + \dot{m}_f V_e}{m_0 - \dot{m}_f t}$$

Integrating gives

$$\int_{V_0}^{V} dV = \int_0^t \left(\frac{T + \dot{m}_f V_e}{m_0 - \dot{m}_f t}\right) dt$$

$$V\Big|_{V_0}^{V} = -\frac{T + \dot{m}_f V_e}{\dot{m}_f}\ln(m_0 - \dot{m}_f t)\Big|_0^t$$

$$V = \frac{T + \dot{m}_f V_e}{\dot{m}_f}\left[\ln\left(\frac{m_0}{m_0 - \dot{m}_f t}\right)\right] + V_0$$

Here, $m_0 = \dfrac{8000\text{ lb}}{32.2\text{ ft/s}^2} = 248.45\text{ slug}$

$$\dot{m}_f = 2\left(\frac{80\text{ lb/s}}{32.2\text{ ft/s}^2}\right) = 4.969\text{ slug/s}$$

$$V_e = 3000\text{ ft/s}$$

$$t = 3\text{ s} \qquad T = 7500\text{ lb} \qquad V_0 = 375\text{ ft/s}$$

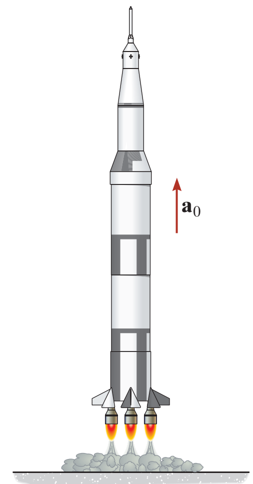
Substituting these values into the expression of V,

$$V = \left(\frac{7500\text{ lb} + (4.969\text{ slug/s})(3000\text{ ft/s})}{4.969\text{ slug/s}}\right)\ln\left(\frac{248.45\text{ slug}}{248.45\text{ slug} - 4.969\text{ slug/s}(3\text{ s})}\right) + 375\text{ ft/s}$$

$V = 654.02\text{ ft/s} = 654\text{ ft/s}$ **Ans.**

Ans:
654 ft/s

6–114. The rocket has an initial mass m_0, including the fuel. For the comfort of the crew, it must maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed v_e, determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.



SOLUTION

The control volume considered is the entire rocket and its contents as shown in Fig. *a*, which accelerates upward. We consider steady flow of an ideal fluid relative to the control volume. The FBD of the control volume is shown in Fig. *a*. Here, $\dot{m}_f$ is a function of time *t*. Also, $\dot{m}_f$ is the negative of the rate of change of the rocket's mass m.

Thus, $\dot{m}_f = -\dfrac{dm}{dt}$. Applying Eq. (6–16) with $\dot{m}_a = 0$,

$$+\uparrow \Sigma F_y = m\frac{dV_{cv}}{dt} + \dot{m}_a V_{cv} - (\dot{m}_a + \dot{m}_f)V_e$$

$$-mg = ma_0 + 0 - \left(0 - \frac{dm}{dt}V_e\right)$$

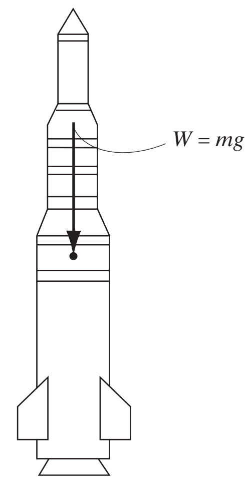
$$\frac{dm}{dt} = -\frac{m(a_0 + g)}{V_e} \qquad \textbf{(1)}$$

$$\int_{m_o}^{m} \frac{dm}{m} = -\frac{a_0 + g}{V_e}\int_0^t dt$$

$$\ln\frac{m}{m_0} = -\left(\frac{a_0 + g}{V_e}\right)t$$

$$\frac{m}{m_0} = e - \left(\frac{a_0 + g}{V_e}\right)t$$

$$m = m_0 e - \left(\frac{a_0 + g}{V_e}\right)t$$



(a)

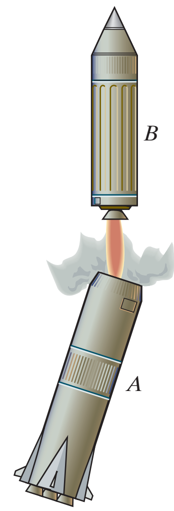
Substitute this result into Eq (1)

$$\dot{m}_f = -\frac{dm}{dt} = \frac{m_0}{V_e}(a_0 + g)e - \left(\frac{a_0 + g}{V_e}\right)t \qquad \textbf{Ans.}$$

Ans:

$$\dot{m}_f = -\frac{dm}{dt} = \frac{m_0}{V_e}(a_0 + g)e^{-(a_0+g)t/V_e}$$

6–115. The second stage B of the two-stage rocket weighs 2500 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 800 lb. If it is consumed at the rate of 75 lb/s, and ejected with a relative velocity of 6000 ft/s, determine the acceleration of the second stage B just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravity and air resistance.



SOLUTION

Take the second stage of the rocket and its contents as the control volume. We consider steady flow of an ideal fluid relative to the control volume. When second stage is fired, the total mass is $m = \dfrac{2500\text{ lb} + 800\text{ lb}}{32.2\text{ ft/s}^2} = 102.48$ slug. Since the effect of gravity and air resistance can be neglected, $\Sigma F_y = 0$.

$$\Sigma F_y = m\frac{dV_{cv}}{dt} - \dot{m}_f V_e$$

$$0 = (102.48\text{ slug})\frac{dV}{dt} - \left(\frac{75}{32.2}\text{ slug/s}\right)(6000\text{ ft/s})$$

$$a = \frac{dV}{dt} = 136.36\text{ ft/s}^2 = 136\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Just before all the fuel is consumed, $m = \dfrac{2500\text{ lb}}{32.2\text{ ft/s}^2} = 77.64$ slug

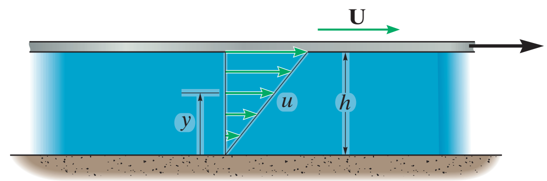
$$\Sigma F_y = m\frac{dV}{dt} - \dot{m}_f V_e$$

$$0 = (77.64\text{ slug})\frac{dV}{dt} - \left(\frac{75}{32.2}\text{ slug/s}\right)(6000\text{ ft/s})$$

$$a = \frac{dV}{dt} = 180\text{ ft/s}^2 \qquad \textbf{Ans.}$$

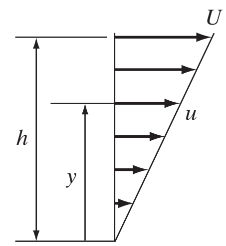
Ans:
When second stage is fired, $a = 136\text{ ft/s}^2$.
Just before all the fuel is consumed, $a = 180\text{ ft/s}^2$.

7–1. As the top plate is pulled to the right with a constant velocity **U**, the fluid between the plates has a linear velocity distribution as shown. Determine the rate of rotation of a fluid element and the shear-strain rate of the element located at *y*.



SOLUTION

We consider steady flow of an ideal fluid.
Referring to the velocity profile shown in Fig. *a*,



(a)

$$\frac{u}{y} = \frac{U}{h}; \qquad u = \frac{U}{h}y$$

And

$$v = 0$$

The rate of rotation or average angular velocity of the fluid element is

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(0 - \frac{u}{h}\right) = -\frac{U}{2h} = \frac{U}{2h} \circlearrowright$$ **Ans.**

The rate of shear strain is

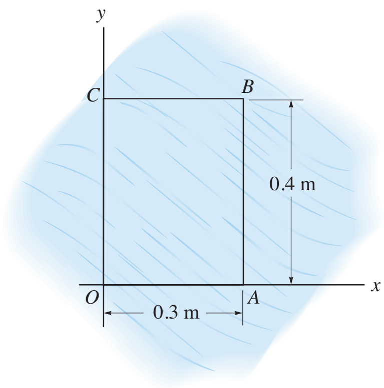
$$\dot{\gamma}_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{U}{h}$$ **Ans.**

Ans:

$$\omega_z = \frac{U}{2h}$$

$$\dot{\gamma}_{xy} = \frac{U}{h}$$

7–2. A flow is defined by its velocity components $u = \left(4x^2 + 4y^2\right)$ m/s and $v = (-8xy)$ m/s, where x and y are in meters. Determine if the flow is irrotational. What is the circulation around the block?



SOLUTION

We consider ideal fluid flow.

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(-8xy) = -8y$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(4x^2 + 4y^2\right) = 8y$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(-8y - 8y) = -8y$$

Since $\omega_z \neq 0$, the flow is rotational. Along edge OA, $y = 0$. Then

$$u_{OA} = 4x^2 + 4\left(0^2\right) = \left(4x^2\right) \text{ m/s}$$

Along edge AB, $x = 0.3$ m. Then

$$v_{AB} = -8(0.3)y = (-2.4y) \text{ m/s}$$

Along edge BC, $y = 0.4$ m. Then

$$u_{BC} = 4x^2 + 4\left(0.4^2\right) = \left(4x^2 + 0.64\right) \text{ m/s}$$

Along edge CO, $x = 0$. Then

$$v_{CO} = -8(0)y = 0$$

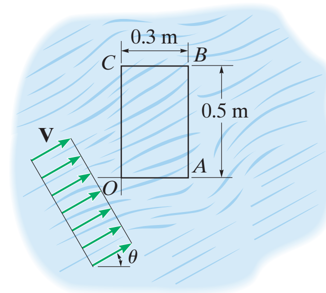
Here, $\mathbf{u}_{OA}$ and $\mathbf{v}_{AB}$ are directed in the positive sense of dx and dy, respectively. Thus, $\oint \mathbf{u}_{OA} \cdot d\mathbf{x}$ and $\oint \mathbf{v}_{AB} \cdot d\mathbf{y}$, are positive. However, $\oint \mathbf{u}_{BC} \cdot d\mathbf{x}$ and $\oint \mathbf{v}_{CO} \cdot dy$ are negative since $\mathbf{u}_{BC}$ and $\mathbf{v}_{CO}$ are directed in the negative sense of $d\mathbf{x}$ and $d\mathbf{y}$, respectively.

$$\begin{aligned}
\Gamma &= \oint \mathbf{V} \cdot d\mathbf{s} = \oint \mathbf{u}_{OA} \cdot d\mathbf{x} + \oint \mathbf{v}_{AB} \cdot d\mathbf{y} - \oint \mathbf{u}_{BC} \cdot d\mathbf{x} - \oint \mathbf{v}_{CO} \cdot d\mathbf{y} \\
&= \int_0^{0.3\text{ m}} 4x^2 dx + \int_0^{0.4\text{ m}} (-2.4y)dy - \int_0^{0.3\text{ m}} \left(4x^2 + 0.64\right)dx - 0 \\
&= \frac{4}{3}x^3\Big|_0^{0.3\text{ m}} - 1.2y^2\Big|_0^{0.4\text{ m}} - \left(\frac{4}{3}x^3 + 0.64x\right)\Big|_0^{0.3\text{ m}} \\
&= -0.384 \text{ m}^2/\text{s}
\end{aligned}$$

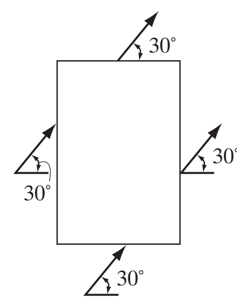
Ans.

Ans:
-0.384 m^2/s

7–3. A uniform flow **V** is directed at an angle θ to the horizontal as shown. Determine the circulation around the rectangular block.



SOLUTION



We consider ideal fluid flow.
The component of V along edges OA and BC is

$$u = V\cos\theta$$

and the component of U along edges AB and CO is

$$v = V\sin\theta$$

Here, **u** and **v** are directed in the same sense as ds_{OA} and d_{AB}, respectively. Thus, $\oint \mathbf{u}\cdot d\mathbf{s}_{BC}$ and $\oint \mathbf{v}\cdot d\mathbf{s}_{CO}$ are negative since **u** and **v** are directed in the opposite sense to that of $d\mathbf{s}_{BC}$ and $d\mathbf{s}_{CO}$, respectively. Thus, the circulation can be determined as

$$\Gamma = \oint \mathbf{V}\cdot d\mathbf{s} = \oint \mathbf{u}\cdot d\mathbf{s}_{OA} + \oint \mathbf{v}\cdot d\mathbf{s}_{AB} - \oint \mathbf{u}\cdot d\mathbf{s}_{BC} - \oint \mathbf{v}\cdot d\mathbf{s}_{CO}$$

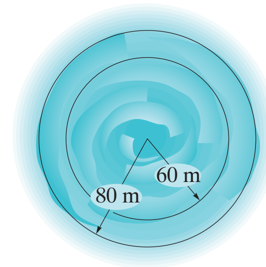
$$= \int_0^{0.3\text{ m}} V\cos\theta\, ds_{OA} + \int_0^{0.5\text{ m}} V\sin\theta\, ds_{AB} - \int_0^{0.3\text{ m}} V\cos\theta\, ds_{BC} - \int_0^{0.5\text{ m}} V\sin\theta\, ds_{CO}$$

$$= 0$$ **Ans.**

Note: The irrotational flow always produces $\Gamma = 0$. In this case, this result is to be expected since the flow is irrotational.

Ans:
$\Gamma = 0$

***7–4.** The velocity within the eye of a tornado is defined by $v_r = 0$, $v_\theta = (0.2r)$ m/s, where r is in meters. Determine the circulation at $r = 60$ m and at $r = 80$ m.



SOLUTION

We consider ideal fluid flow.

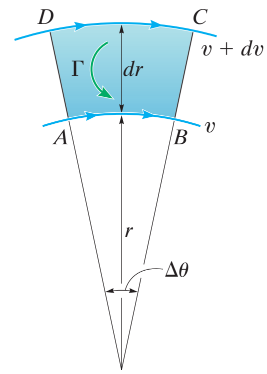
Since $\mathbf{v}_\theta$ is always tangent to the circle, $\mathbf{v} \cdot d\mathbf{s} = v_\theta ds$. For $r = 60$ m, $v_\theta = 0.2(60)$ m/s $= 12$ m/s and $ds = rd\theta = 60d\theta$.

$$\Gamma_{r = 60\text{ m}} = \oint \mathbf{V} \cdot \mathbf{ds} = \int_0^{2\pi} v_\theta ds = \int_0^{2\pi} 12(60d\theta) = 720\theta\big|_0^{2\pi} = 1440\pi \text{ m}^2/\text{s} \quad \textbf{Ans.}$$

For $r = 80$ m, $v_\theta = 0.2(80)$ m/s $= 16$ m/s, and $ds = rd\theta = 80d\theta$.

$$\Gamma_{r = 80\text{ m}} = \oint \mathbf{V} \cdot \mathbf{ds} = \int_0^{2\pi} v_\theta ds = \int_0^{2\pi} 16(80d\theta) = 1280\theta\big|_0^{2\pi} = 2560\pi \text{ m}^2/\text{s} \quad \textbf{Ans.}$$

7–5. Consider the fluid element that has dimensions in polar coordinates as shown and whose boundaries are defined by the streamlines with velocities v and $v + dv$. Show that the vorticity for the flow is given by $\zeta = -\left(v/r + dv/dr\right)$.



SOLUTION

We consider ideal fluid flow.
The circulation of the flow around element $ABCD$ can be determined from

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s}$$

$$= vS_{AB} + (v + dv)(-S_{CD})$$

$$= v(r\Delta\theta) + (v + dv)\left[-(r + dr)\Delta\theta\right]$$

$$= -vdr\Delta\theta - rdv\Delta\theta - dvdr\Delta\theta$$

Neglect the second order terms

$$\Gamma = vdr\Delta\theta - rdv\Delta\theta = -\Delta\theta(vdr + rdv)$$

The area of the element, again, neglecting higher-order terms, is

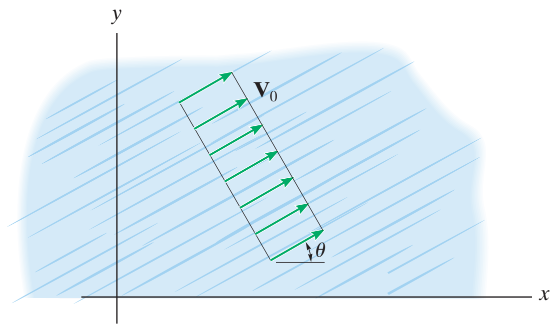
$$A = (r\Delta\theta)dr$$

Thus, the vorticity is

$$\zeta = \frac{\Gamma}{A} = \frac{-\Delta\theta(vdr + rdv)}{(r\Delta\theta)dr}$$

$$= -\left(\frac{v}{r} + \frac{dv}{dr}\right) \qquad \textbf{(Q.E.D)}$$

7–6. Determine the stream and potential functions for the two-dimensional flow field if $\mathbf{V}_0$ and θ are known.



SOLUTION

We consider ideal fluid flow.
The velocity components are

$$u = V_0 \cos\theta_0 \qquad v = V_0 \sin\theta_0$$

Since the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$ is satisfied, the establishment of a stream function is possible using the velocity components,

$$\frac{\partial \psi}{\partial y} = u; \qquad \frac{\partial \psi}{\partial y} = V_0 \cos\theta_0$$

Integrating this equation with respect to y,

$$\psi = (V_0 \cos\theta_0)y + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \qquad -\frac{\partial}{\partial x}\left[\left(V_0 \cos\theta_0\right)y + f(x)\right] = V_0 \sin\theta_0$$

$$\frac{\partial}{\partial x}\left[f(x)\right] = -V_0 \sin\theta_0$$

Integrating this equation with respect to x,

$$f(x) = (-V_0 \sin\theta_0)x + C$$

Setting $C = 0$ and substituting this result into Eq. (1),

$$\psi = \left(V_0 \cos\theta_0\right)y - \left(V_0 \sin\theta_0\right)x$$

$$\psi = V_0\left[\left(\cos\theta_0\right)y - \left(\sin\theta_0\right)x\right] \qquad \textbf{Ans.}$$

Since, $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0$, the flow is indeed irrotational. Thus, the potential function exists.

Using the velocity components,

$$\frac{\partial \phi}{\partial x} = u; \qquad \frac{\partial \phi}{\partial x} = V_0 \cos\theta_0$$

Integrating this equation with respect to x,

$$\phi = \left(V_0 \cos\theta_0\right)x + f(y) \qquad \textbf{(1)}$$

Also,

$$\frac{\partial \phi}{\partial y} = v; \qquad \frac{\partial}{\partial y}\left[V_0 \cos\theta_0 x + f(y)\right] = V_0 \sin\theta_0$$

$$\frac{\partial}{\partial y}\left[f(y)\right] = V_0 \sin\theta_0$$

Integrating this equation with respect to y,

$$f(y) = \left(V_0 \sin\theta_0\right)y + C$$

Setting $C = 0$ and substituting this result into Eq. 1,

$$\phi = \left(V_0 \cos\theta_0\right)x + \left(V_0 \sin\theta_0\right)y$$

$$\phi = V_0\left[\left(\cos\theta_0\right)x + \left(\sin\theta_0\right)y\right] \qquad \textbf{Ans.}$$

Ans:
$\psi = V_0\left[(\cos\theta_0)y - (\sin\theta_0)x\right]$
$\phi = V_0\left[(\cos\theta_0)x + (\sin\theta_0)y\right]$

7–7. A two-dimensional flow is described by the stream function $\psi = \left(xy^3 - x^3y\right)\ \mathrm{m^2/s}$, where x and y are in meters. Show that the continuity condition is satisfied and determine if the flow is rotational or irrotational.

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}\left(xy^3 - x^3y\right) = \left(3xy^2 - x^3\right)\ \mathrm{m/s}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}\left(xy^3 - x^3y\right) = -\left(y^3 - 3x^2y\right)\ \mathrm{m/s} = \left(3x^2y - y^3\right)\ \mathrm{m/s}$$

Then,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(3xy^2 - x^3\right) = \left(3y^2 - 3x^2\right)\ \mathrm{s^{-1}}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left(3x^2y - y^3\right) = \left(3x^2 - 3y^2\right)\ \mathrm{s^{-1}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(3xy^2 - x^3\right) = 6xy\ \mathrm{s^{-1}}$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(3x^2y - y^3\right) = 6xy\ \mathrm{s^{-1}}$$

This gives,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 3y^2 - 3x^2 + 3x^2 - 3y^2 = 0$$

Thus, the flow field satisfies the continuity condition,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left(6xy - 6xy\right) = 0$$

The **flow field is irrotational** since $\omega_z = 0$. **Ans.**

Ans:
irrotational

***7–8.** If the stream function for a flow is $\psi = (3x + 2y)$, where x and y are in meters, determine the potential function and the magnitude of the velocity of a fluid particle at point (1 m, 2 m).

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(3x + 2y) = 2\text{ m/s}$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(3x + 2y) = -3\text{ m/s}$$

Since u and v are constant, the magnitude of the flow velocity at any point in the flow field is the same and is given by

$$V = \sqrt{u^2 + v^2} = \sqrt{(2\text{ m/s})^2 + (-3\text{ m/s})^2}$$

$$= 3.606\text{ m/s} = 3.61\text{ m/s} \qquad \textbf{Ans.}$$

Applying,

$$u = \frac{\partial \phi}{\partial x}; \qquad 2 = \frac{\partial \phi}{\partial x}$$

Integrating with respect to x,

$$\phi = 2x + f(y)$$

Substituting this result into,

$$v = \frac{\partial \phi}{\partial y}; \qquad -3 = \frac{\partial}{\partial y}[2x + f(y)]$$

$$-3 = 0 + \frac{\partial}{\partial y}[f(y)]$$

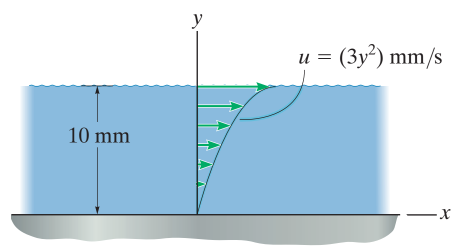
Integrating with respect to y,

$$f(y) = -3y + C$$

Setting $C = 0$, thus

$$\phi = 2x - 3y \qquad \textbf{Ans.}$$

7–9. The velocity profile of a very thick liquid flowing along the channel of constant width is approximated as, $u = (3y^2)$ mm/s, where y is in millimeters. Determine the stream function for the flow and plot the streamlines for $\psi_0 = 0$, $\psi_1 = 1\ \text{mm}^2/\text{s}$, and $\psi_2 = 2\ \text{mm}^2/\text{s}$.



SOLUTION

We consider ideal fluid flow.
The x and y components of the constant flow velocity are

$$u = (3y^2)\ \text{mm/s} \qquad v = 0$$

$$u = \frac{\partial \psi}{\partial y}; \quad 3y^2 = \frac{\partial \psi}{\partial y}$$

Integrating with respect to y,

$$\psi = y^3 + f(x)$$

$$v = -\frac{\partial \psi}{\partial x}; \qquad 0 = -\frac{\partial}{\partial x}\left[y^3 + f(x)\right]$$

$$0 = \frac{\partial}{\partial x}[f(x)]$$

Integrating with respect to x

$$f(x) = C$$

Thus,

$$\psi = y^3 + C$$

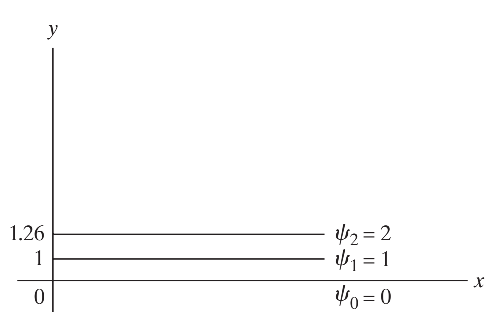
Setting $C = 0$,

$$\psi = y^3 \qquad \textbf{Ans.}$$

$$\psi_0 = 0, y = 0$$

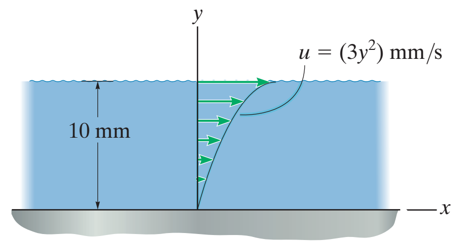
$$\psi_1 = 1, y = 1$$

$$\psi_2 = 2, y = 1.26$$



Ans:
$\psi = y^3$

7–10. The velocity profile of a very thick liquid flowing along the channel of constant width is approximated as $u = (3y^2)$ mm/s, where y is in millimeters. Is it possible to determine the potential function for the flow? If so, what is it?



SOLUTION

We consider ideal fluid flow.
The x and y components of flow velocity are

$$u = (3y^2)\ \text{mm/s} \qquad v = 0$$

Here,

$$\frac{\partial v}{\partial x} = 0$$

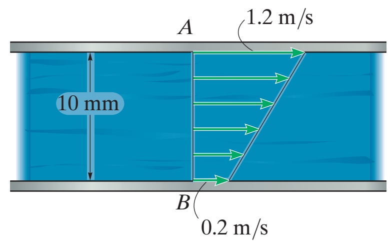
$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(3y^2) = (6y)\ \text{rad/s}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 6y) = -3y$$

Since $\omega_z \neq 0$, the flow is rotational. Thus, **the potential function cannot be established** since it requires the flow to be irrotational.

Ans:
ϕ cannot be established.

7–11. The liquid confined between two plates is assumed to have a linear velocity distribution as shown. Determine the stream function. Does the potential function exist?



SOLUTION

We consider ideal fluid flow.
From the geometry of Fig. *a*, the *x* component of velocity is

$$\frac{u - 0.2}{y} = \frac{1.2 - 0.2}{0.01}; \quad u = (100y + 0.2)\text{ m/s}$$

Also, since the velocity distribution is directed along the *x* axis, $v = 0$.

$$u = \frac{\partial \psi}{\partial y}; \quad 100y + 0.2 = \frac{\partial \psi}{\partial y}$$

Integrating with respect to y,

$$\psi = 50y^2 + 0.2y + f(x)$$

Substituting this result into,

$$v = -\frac{\partial \psi}{\partial x}; \quad 0 = -\frac{\partial}{\partial x}\left[50y^2 + 0.2y + f(x)\right]$$

$$\frac{\partial}{\partial x}[f(x)] = 0$$

Integrating with respect to x,

$$f(x) = C$$

Setting this constant equal to zero,

$$\psi = 50y^2 + 0.2y \qquad \textbf{Ans.}$$

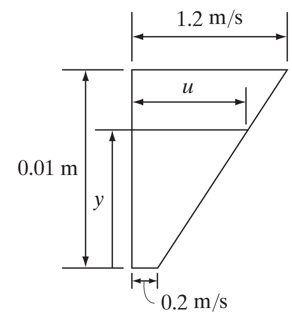
Here

$$\frac{\partial v}{\partial x} = 0; \quad \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(100y + 0.2) = 100\text{ rad/s}$$

Thus,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 100) = -50\text{ rad/s}$$

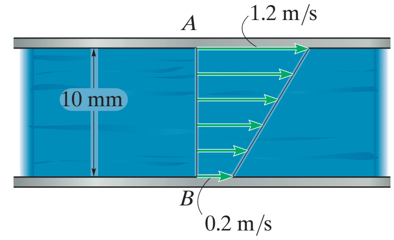
Since $\omega_z \neq 0$, the flow is rotational. Therefore, **it is not possible to establish the potential function.**



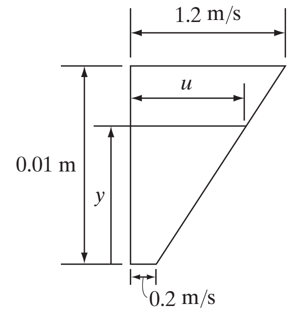
(a)

Ans:
$\psi = 50y^2 + 0.2y$, ϕ cannot be established.

***7–12.** The liquid confined between two plates is assumed to have a linear velocity distribution as shown. If the pressure at the top surface of the bottom plate is $600\ \text{N/m}^2$, detemine the pressure at the bottom surface of the top plate. Take $\rho = 1.2\ \text{Mg/m}^3$.



SOLUTION



(a)

We consider ideal fluid flow.
From the geometry of Fig. *a*, the *x* component of velocity is

$$\frac{u - 0.2}{y} = \frac{1.2 - 0.2}{0.01}; \quad u = (100y + 0.2)\ \text{m/s}$$

Also, since the velocity distribution is directed along the x axis, $v = 0$. Here,

$$\frac{\partial v}{\partial x} = 0; \quad \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(100y + 0.2) = 100\ \text{rad/s}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 100) = -50\ \text{rad/s}$$

Since $\omega_z \neq 0$, the flow is rotational. Thus, the Bernoulli equation can not be applied at points A and B. Instead, we will first apply the Euler equation along the x axis, with $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}100y + 0.2 = 0$ and $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(100y + 0.2) = 100\ \text{rad/s}$,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$-\frac{1}{\rho}\frac{\partial}{\partial x}[-\rho g y + f(x)] = 0 + 0 = 0$$

$$\frac{\partial}{\partial x}[f(x)] = 0$$

Integrating this equation with respect to x,

$$f(x) = C$$

Thus,

$$p = -\rho g y + C$$

At point B, $y = 0$ and $p = 600\ \text{N/m}^2$. Then,

$$600\frac{\text{N}}{\text{m}^2} = -1.2(10^3)(9.81\ \text{m/s}^2)(0) + C$$

$$C = 600\frac{\text{N}}{\text{m}^2}$$

Thus,

$$p = (-\rho g y + 600)\frac{\text{N}}{\text{m}^2}$$

At point A, $y = 0.01$ m. Then,

$$p_A = \left[-1.2(10^3)(9.81\ \text{m/s}^2)(0.01\ \text{m}) + 600\right]\frac{\text{N}}{\text{m}^2}$$

$$= 482.28\frac{\text{N}}{\text{m}^2} = 482\ \text{Pa}$$ **Ans.**

7–13. A two-dimensional flow has a y component of velocity of $v = (4y)$ ft/s, where y is in feet. If the flow is ideal, determine the x component of velocity and find the magnitude of the velocity at the point $x = 4$ ft, $y = 3$ ft. The velocity of the flow at the origin is zero.

SOLUTION

We consider ideal fluid flow.
In order to satisfy the continuity condition,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Here,

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(4y) = 4\ \mathrm{s}^{-1}$$

Then,

$$\frac{\partial u}{\partial x} + 4 = 0$$

$$\frac{\partial u}{\partial x} = -4$$

Integrating with respect to x,

$$u = -4x + f(y)$$

Since ideal flow is irrotational,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

and since $\dfrac{\partial v}{\partial x} = \dfrac{\partial(4y)}{\partial x} = 0$,

$$\frac{\partial u}{\partial y} = 0$$

$$u = g(x)$$

Thus,

$$u = (-4x)\ \mathrm{ft/s} \qquad \textbf{Ans.}$$

At $x = 4$ ft and $y = 3$ ft,

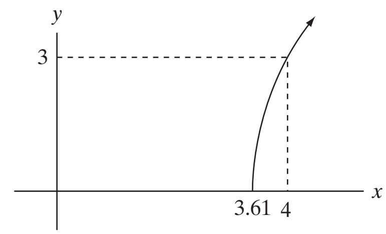
$$u = -4(4) = -16\ \mathrm{ft/s} \qquad v = 4(3) = 12\ \mathrm{ft/s}$$

$$V = \sqrt{u^2 + v^2} = \sqrt{(-16\ \mathrm{ft/s})^2 + (12\ \mathrm{ft/s})^2} = 20\ \mathrm{ft/s} \qquad \textbf{Ans.}$$

Ans:
$u = (-4x)$ ft/s
$V = 20$ ft/s

7–14. A two-dimensional flow field is defined by its components $u = (3y)$ m/s and $v = (9x)$ m/s, where x and y are in meters. Determine if the flow is rotational or irrotational, and show that the continuity condition for the flow is satisfied. Also, find the stream function and the equation of the streamline that passes through point (4 m, 3 m). Plot this streamline.

SOLUTION



We consider ideal fluid flow.

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(9x) = 9 \text{ rad/s}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(3y) = 3 \text{ rad/s}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(3y) = 0$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(9x) = 0$$

Thus,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(9 - 3) = 3 \text{ rad/s}$$

Since $\omega_z \neq 0$, the flow is **rotational**. Also, **Ans.**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$$

The flow **satisfies the continuity condition**. Thus,

$$u = \frac{\partial \psi}{\partial y}; \quad 3y = \frac{\partial \psi}{\partial y}$$

Integrating with respect to y,

$$\psi = \frac{3}{2}y^2 + f(x)$$

Substituting this result into

$$v = -\frac{\partial \psi}{\partial x}; \qquad 9x = -\frac{\partial}{\partial x}\left[\frac{3}{2}y^2 + f(x)\right]$$

$$-9x = 0 + \frac{\partial}{\partial x}[f(x)]$$

Integrating with respect to x,

$$f(x) = -\frac{9}{2}x^2 + C$$

Thus, setting $C = 0$,

$$\psi = \frac{3}{2}y^2 + \left(-\frac{9}{2}x^2 + C\right) = \frac{1}{2}(3y^2 - 9x^2) \qquad \textbf{Ans.}$$

From the slope of the stream function,

$$\frac{dy}{dx} = \frac{v}{u} = \frac{9x}{3y} = \frac{3x}{y}$$

$$\int_{3\text{ m}}^{y} y\,dy = \int_{4\text{ m}}^{x} 3x\,dx$$

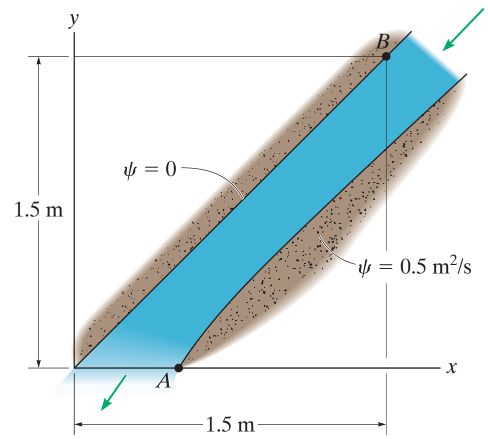
$$\left.\frac{y^2}{2}\right|_{3\text{ m}}^{y} = \left.\frac{3}{2}x^2\right|_{4\text{ m}}^{x}$$

$$y^2 = 3x^2 - 39$$

$$y = \sqrt{3x^2 - 39} \qquad \textbf{Ans.}$$

Ans:
rotational
$\psi = \frac{1}{2}(3y^2 - 9x^2)$
$y = \sqrt{3x^2 - 39}$

7–15. Water flow through the horizontal channel is defined by the stream function $\psi = 2\left(x^2 - y^2\right)$ m²/s. If the pressure at B is atmospheric, determine the pressure at point (0.5 m, 0) and the flow per unit depth in m²/s.



SOLUTION

We consider ideal fluid flow.
The velocity components are

$$u = \frac{\partial \psi}{\partial y} = (-4y)\text{ m/s} \qquad v = -\frac{\partial \psi}{\partial x} = (-4x)\text{ m/s}$$

The continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$ is indeed satisfied.

At point A (0.5 m, 0),

$$u_A = 0 \qquad v_A = -4(0.5) = -2\text{ m/s}$$

Thus,

$$V_A = v_A = -2\text{ m/s}$$

At point B (1.5 m, 1.5 m),

$$u_B = -4(1.5) = -6\text{ m/s} \quad v_B = -4(1.5) = -6\text{ m/s}$$

Thus,

$$V_B = \sqrt{u_B^2 + v_B^2} = \sqrt{(-6\text{ m/s})^2 + (-6\text{ m/s})^2} = \sqrt{72}\text{ m/s} = 8.485\text{ m/s}$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}[-4 - (-4)] = 0$, the flow is irrotational. Thus, Bernoulli's equation can be applied between two points on the different streamlines such as points A and B.

$$\frac{p_A}{\rho_w} = \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} = \frac{V_B^2}{2} + gz_B$$

Since the flow occurs in the horizontal plane, $z_A = z_B$. Also, $p_B = p_{\text{atm}} = 0$.

$$\frac{p_A}{1000\text{ kg/m}^3} + \frac{(2\text{ m/s})^2}{2} = 0 + \frac{(8.485\text{ m/s})^2}{2}$$

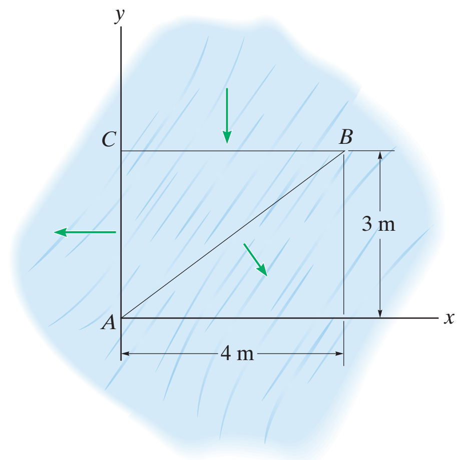
$$p_A = 34\left(10^3\right)\text{ Pa} = 34\text{ kPa} \qquad \textbf{Ans.}$$

The flow per unit depth is

$$\psi_2 - \psi_1 = 0.5\text{ m}^2/\text{s} - 0 = 0.5\text{ m}^2/\text{s} \qquad \textbf{Ans.}$$

Ans:
$p_A = 34$ kPa
$\psi_2 - \psi_1 = 0.5\text{ m}^2/\text{s}$

***7–16.** A flow field is defined by the stream function $\psi = 2\left(x^2 - y^2\right)\ \mathrm{m^2/s}$, where x and y are in meters. Determine the flow per unit depth in $\mathrm{m^2/s}$ that occurs through AB, CB, and AC as shown.



SOLUTION

We consider ideal fluid flow.
At point A, $x = 0$, $y = 0$. Thus,

$$\psi_A = 2\left(0^2 - 0^2\right) = 0$$

At point B, $x = 4$ m, $y = 3$ m. Thus

$$\psi_B = 2\left(4^2 - 3^2\right) = 14\ \mathrm{m^2/s}$$

At point C, $x = 0$, $y = 3$ m

$$\psi_C = 2\left(0^2 - 3^2\right) = -18\ \mathrm{m^2/s}$$

The flow rates per unit depth through AB, BC, and AC are

$$q_{AB} = \psi_B - \psi_A = 14\ \mathrm{m^2/s} - 0 = 14\ \mathrm{m^2/s} \qquad \textbf{Ans.}$$

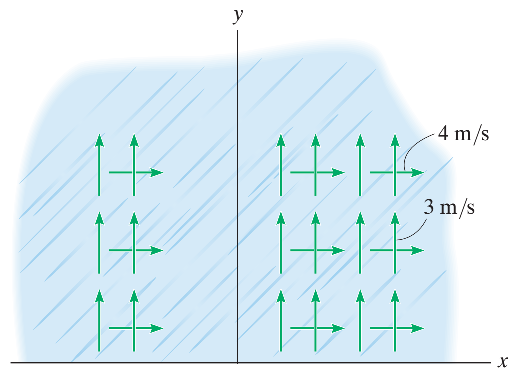
$$q_{BC} = \psi_B - \psi_C = 14\ \mathrm{m^2/s} - \left(-18\ \mathrm{m^2/s}\right) = 32\ \mathrm{m^2/s} \qquad \textbf{Ans.}$$

$$q_{AC} = \psi_A - \psi_C = 0 - \left(-18\ \mathrm{m^2/s}\right) = 18\ \mathrm{m^2/s} \qquad \textbf{Ans.}$$

Note that the flow satisfies the continuity condition through ABC since

$$\Sigma \mathbf{V}\cdot\mathbf{A} = 0 - q_{BC} + q_{AB} + q_{AC} = -32\ \mathrm{m^2/s} + 14\ \mathrm{m^2/s} + 18\ \mathrm{m^2/s} = 0$$

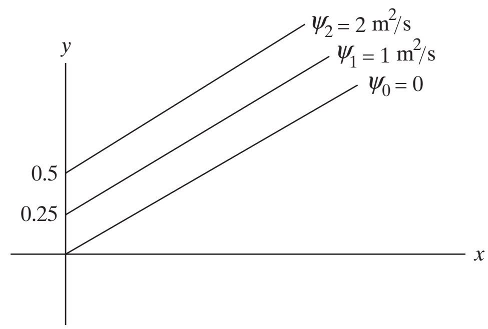
7–17. A fluid has the velocity components shown. Determine the stream and potential functions. Plot the streamline for $\psi_0 = 0$, $\psi_1 = 1\ \text{m}^2/\text{s}$, and $\psi_2 = 2\ \text{m}^2/\text{s}$.



SOLUTION

We consider ideal fluid flow.
Here, the flow velocity has constant x and y components.

$$u = 4\ \text{m/s} \qquad v = 3\ \text{m/s}$$



Applying

$$u = \frac{\partial\psi}{\partial y}; \qquad 4 = \frac{\partial\psi}{\partial y}$$

Integrating with respect to y,

$$\psi = 4y + f(x)$$

Substituting this result into

$$v = -\frac{\partial\psi}{\partial x}; \qquad 3 = \frac{\partial}{\partial x}[4y + f(x)]$$

$$3 = -0 - \frac{\partial}{\partial x}[f(x)]$$

$$\frac{\partial}{\partial x}[f(x)] = -3$$

Integrating with respect to x,

$$f(x) = -3x + C$$

Setting $C = 0$, we get

$$\psi = 4y + (-3x)$$

$$\psi = 4y - 3x \qquad \textbf{Ans.}$$

Applying

$$u = \frac{\partial\phi}{\partial x}; \qquad 4 = \frac{\partial\phi}{\partial x}$$

Integrating with respect to x,

$$\phi = 4x + f(y)$$

Substituting this result into

$$v = \frac{\partial\phi}{\partial y}; \qquad 3 = \frac{\partial}{\partial y}[4x + f(y)]$$

$$3 = 0 + \frac{\partial}{\partial y}[f(y)]$$

Integrating with respect to y,

$$f(y) = 3y + C$$

Setting $C = 0$, we get

$$\phi = 4x + 3y \qquad \textbf{Ans.}$$

Ans:
$\psi = 4y - 3x$
$\phi = 4x + 3y$

7–18. A two-dimensional flow field is defined by its components $u = \left(2x^2\right)$ ft/s and $v = \left(-4xy + x^2\right)$ ft/s, where x and y are in feet. Determine the stream function, and plot the streamline that passes through point (3 ft, 1 ft).

SOLUTION

We consider ideal fluid flow.

Since the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4x + (-4x) = 0$ is satisfied, then the establishment of stream function is possible.

Using the definition of velocity components with respect to the stream function,

$$\frac{\partial \psi}{\partial y} = u; \qquad \frac{\partial \psi}{\partial y} = 2x^2$$

Integrating this equation with respect to y,

$$\psi = 2x^2y + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \qquad -\frac{\partial}{\partial x}\left[2x^2y + f(x)\right] = -4xy + x^2$$

$$-4xy - \frac{\partial}{\partial x}[f(x)] = -4xy + x^2$$

$$\frac{\partial}{\partial x}[f(x)] = -x^2$$

Integrating this equation with respect to x,

$$f(x) = -\frac{1}{3}x^3 + C$$

Substituting this result into Eq. (1),

$$\psi = 2x^2y - \frac{1}{3}x^3 + C$$

Here, C is an arbitary constant that we will set equal to zero. The stream function can be expressed as

$$\psi = 2x^2y - \frac{1}{3}x^3 \qquad \textbf{Ans.}$$

For the streamline passing through point (3 ft, 1 ft),

$$\psi = 2(3)^2(1) - \frac{1}{3}(3)^3$$

Thus,

$$2x^2y - \frac{1}{3}x^3 = 9$$

$$y = \frac{x^3 + 27}{6x^2}$$

7–18. Continued

The plot of stream function is shown in Fig. *a*.

x(m)	0	1	2	3	4	5	6	7	8
y(m)	∞	4.67	1.46	1	0.948	1.01	1.125	1.26	1.40

$$\frac{dy}{dx} = \frac{6x^2(3x^2) - (x^3 + 27)(12x)}{(6x^2)^2} = 0$$

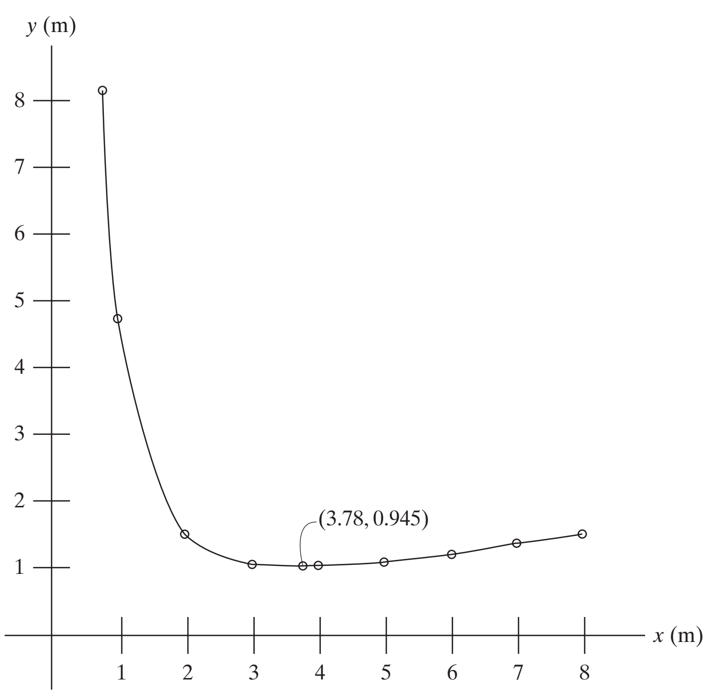
$$6x^4 - 324x = 0$$

$$6x(x^3 - 54) = 0$$

$$x = 3.780 \text{ ft}$$

The corresponding

$$y = \frac{3.780^3 + 27}{6(3.780^2)} = 0.945$$



(a)

Ans:

$\psi = 2x^2y - \frac{1}{3}x^3$

7–19. The stream function for a flow field is defined by $\psi = (4/r^2)\sin 2\theta$. Show that continuity of the flow is satisfied, and determine the r and θ velocity components of fluid particles at point $r = 2$ m, $\theta = (\pi/4)$ rad. Plot the streamline that passes through this point.

SOLUTION

We consider ideal fluid flow.
Using the r and θ velocity components with respect to stream function

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left(\frac{4}{r^2}\right)(2\cos 2\theta) = \frac{8}{r^3}\cos 2\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -\left(-\frac{8}{r^3}\sin 2\theta\right) = \frac{8}{r^3}\sin 2\theta$$

The continuity equation $\dfrac{v_r}{r} + \dfrac{\partial v_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial v_\theta}{\partial\theta} = \dfrac{8}{r^4}\cos 2\theta + \left(-\dfrac{24}{r^4}\cos 2\theta\right)$ $+ \dfrac{16}{r^4}\cos 2\theta = 0$ is indeed satisfied.

At point $r = 2$ m and $\theta = \dfrac{\pi}{4}$,

$$v_r = \frac{8}{2^3}\cos\left[2\left(\frac{\pi}{4}\right)\right] = 0 \qquad \textbf{Ans.}$$

$$v_\theta = \frac{8}{2^3}\sin\left[2\left(\frac{\pi}{4}\right)\right] = 1 \text{ m/s} \qquad \textbf{Ans.}$$

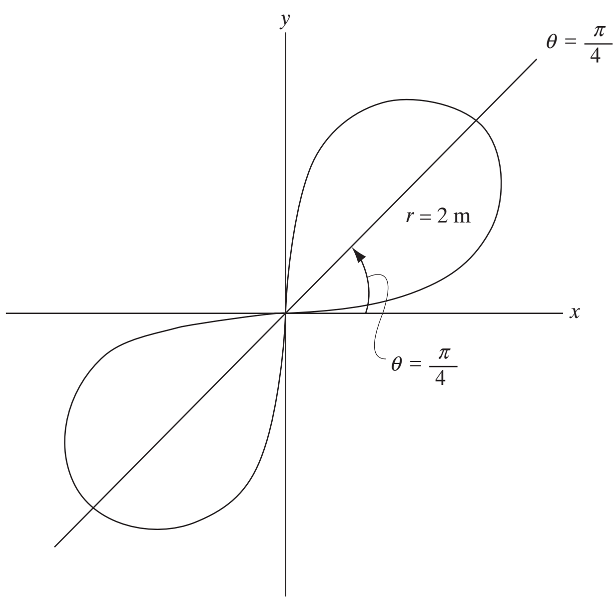
$$\psi = \frac{4}{2^2}\sin\left[2\left(\frac{\pi}{4}\right)\right] = 1$$

Therefore, the stream function that passes through this point is

$$1 = \frac{4}{r^2}\sin 2\theta$$

$$r^2 = 4\sin 2\theta$$

The plot of this streamline is shown in Fig. *a*.



(a)

Ans:
$v_r = 0$
$v_\theta = 1$ m/s

***7–20.** A flow field has velocity components $u = (x - y)$ ft/s and $v = -(x + y)$ ft/s, where x and y are in feet. Determine the stream function, and plot the streamline that passes through the origin.

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial \psi}{\partial y}; \quad x - y = \frac{\partial \psi}{\partial y}$$

Integrating this equation with respect to y,

$$\psi = xy - \frac{y^2}{2} + f(x)$$

Substituting this result into

$$v = -\frac{\partial \psi}{\partial x}; \qquad -(x + y) = -\frac{\partial}{\partial x}\left[xy - \frac{y^2}{2} + f(x)\right]$$

$$x + y = y - 0 + \frac{\partial}{\partial x}[f(x)]$$

$$\frac{\partial}{\partial x}[f(x)] = x$$

Integrating with respect to x,

$$f(x) = \frac{x^2}{2} + C$$

Thus, setting $C = 0$,

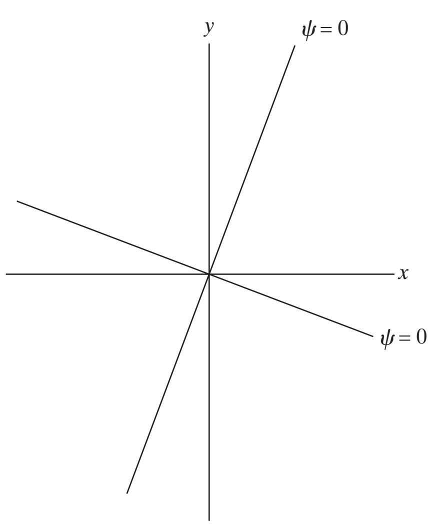
$$\psi = xy - \frac{y^2}{2} + \frac{x^2}{2} \qquad \textbf{Ans.}$$

Evaluate $\psi(x, y)$ at the origin, $x = y = 0$. This equation gives $\psi = 0 - 0 + 0 = 0$
Then, for $\psi = 0$,

$$\frac{x^2}{2} - \frac{y^2}{2} + xy = 0$$

$$x^2 - y^2 + 2xy = 0$$

The plot of this equation is shown in Fig. *a*.



(a)

7–21. A flow is described by the stream function $\psi = (8x - 4y)\ \text{m}^2/\text{s}$, where x and y are in meters. Determine the potential function, and show that the continuity condition is satisfied and that the flow is irrotational.

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(8x - 4y) = -4\ \text{m/s}$$

$$v = \frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(8x - 4y) = -8\ \text{m/s}$$

Applying

$$u = \frac{\partial \phi}{\partial x};\ -4 = \frac{\partial \phi}{\partial x}$$

Integrating with respect to x,

$$\phi = -4x + f(y)$$

Substituting this result into

$$v = \frac{\partial \phi}{\partial y};\ -8 = \frac{\partial}{\partial y}[-4x + f(y)]$$

$$-8 = 0 + \frac{\partial}{\partial y}f(y)$$

$$\frac{\partial}{\partial y}[f(y)] = -8$$

Integrating with respect to y,

$$f(y) = -8y + C$$

Thus,

$$\phi = -4x + (-8y + C)$$

Omitting the integration constant,

$$\phi = -4x - 8y$$ **Ans.**

Here,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(-4) = 0$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(-8) = 0$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(-4) = 0$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(-8) = 0$$

Then,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$$

The flow field **satisfies the continuity condition**. Also,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0$$

The flow field is **irrotational** since $\omega_z = 0$.

Ans:
$\phi = -4x - 8y$

7–22. The stream function for a flow field is defined by $\psi = 2r^3 \sin 2\theta$. Determine the magnitude of the velocity of fluid particles at point $r = 1$ m, $\theta = (\pi/3)$ rad, and plot the streamlines for $\psi_1 = 1\ \text{m}^2/\text{s}$ and $\psi_2 = 2\ \text{m}^2/\text{s}$.

SOLUTION

We consider ideal fluid flow.
The velocity components are

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}; \qquad v_r = \frac{1}{r}\left[2r^3(2\cos 2\theta)\right] = \left(4r^2\cos 2\theta\right)\ \text{m/s}$$

$$v_\theta = -\frac{\partial\psi}{\partial r}; \qquad v_\theta = \left(-6r^2\sin 2\theta\right)\ \text{m/s}$$

The continuity equation $\dfrac{v_r}{r} + \dfrac{\partial vr}{\partial r} + \dfrac{1}{r}\dfrac{\partial v_\theta}{\partial\theta} = 4r\cos 2\theta + 8r\cos 2\theta$

$+ (-12r\cos 2\theta) = 0$ is indeed satisfied.

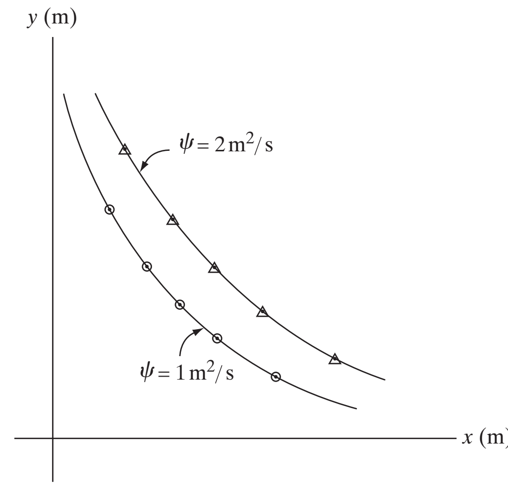
At point $r = 1$ m, $\theta = \pi/3$ rad,

$$v_r = 4(1^2)\cos\left[2\left(\frac{\pi}{3}\right)\right] = -2\ \text{m/s}$$

$$v_\theta = -6(1^2)\sin\left[2\left(\frac{\pi}{3}\right)\right] = -5.196\ \text{m/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-2\ \text{m/s})^2 + (-5.196\ \text{m/s})^2} = 5.57\ \text{m/s} \qquad \textbf{Ans.}$$



(a)

For $\psi = 1\ \text{m}^2/\text{s}$,

$$1 = 2r^3\sin 2\theta \qquad r^3 = \frac{1}{\sin 2\theta}$$

θ(rad)	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
r(m)	∞	1.00	0.833	0.794	0.833	1.00	∞

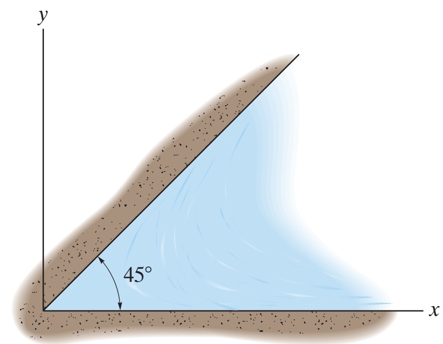
For $\psi = 2\ \text{m}^2/\text{s}$

$$2 = 2r^3\sin 2\theta \qquad r^3 = \frac{1}{2\sin 2\theta}$$

θ(rad)	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
r(m)	∞	1.260	1.049	1.00	1.049	1.260	∞

Ans:
5.57 m/s

7–23. An ideal fluid flows into the corner formed by the two walls. If the stream function for this flow is defined by $\psi = \left(5\,r^4 \sin 4\theta\right)$ m²/s, show that continuity for the flow is satisfied. Also, plot the streamline that passes through point $r = 2$ m, $\theta = (\pi/6)$ rad, and find the magnitude of the velocity at this point.



SOLUTION

We consider ideal fluid flow.

For the stream f unction passing through point $r = 2$ m, $\theta = \dfrac{\pi}{6}$ rad,

$$\psi = 5(2^4)\sin\left[4\left(\frac{\pi}{6}\right)\right] = 40\sqrt{3}\ \mathrm{m^2/s}$$

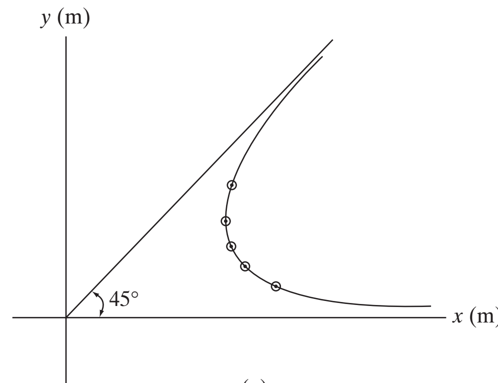
Thus, the stream function passing through this point is

$$40\sqrt{3} = 5r^4 \sin 4\theta$$

$$r^4 \sin 4\theta = 8\sqrt{3}$$

The plot of this streamline is shown in Fig. *a*

θ(rad)	0	$\frac{\pi}{24}$	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{5\pi}{24}$	$\frac{\pi}{4}$
r(m)	∞	± 2.29	± 2.0	± 1.93	± 2.0	± 2.29	∞



(a)

The radial and transverse components of velocity are

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r}\left[5r^4(4\cos 4\theta)\right] = 20r^3 \cos 4\theta$$

$$v_\theta = -\frac{\partial \psi}{\partial r} = -20r^3 \sin 4\theta$$

The continuity equation

$$\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$= 20r^2 \cos 4\theta + 60r^2 \cos 4\theta + (-80r^2 \cos 4\theta)$$

$$= 0 \text{ is indeed satisfied}$$

At the point $r = 2$ m, $\theta = \pi/6$ rad,

$$v_r = 20(2^3)\cos\left[4\left(\frac{\pi}{6}\right)\right] = -80\ \mathrm{m/s}$$

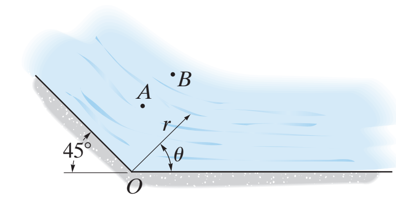
$$v_\theta = -20(2^3)\sin\left[4\left(\frac{\pi}{6}\right)\right] = -138.56\ \mathrm{m/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-80\ \mathrm{m/s})^2 + (-138.56\ \mathrm{m/s})^2} = 160\ \mathrm{m/s} \qquad \textbf{Ans.}$$

Ans:
160 m/s

***7–24.** The horizontal flow confined by the walls is defined by the stream function $\psi = 4r^{4/3} \sin (4/3)\theta \text{ m}^2/\text{s}$, where r is in meters. Determine the magnitude of the velocity at point $r = 2 \text{ m}, \theta = 45°$. Is the flow rotational or irrotational? Can the Bernoulli equation be used to determine the difference in pressure between the two points A and B?



SOLUTION

We consider ideal fluid flow.

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(4r^{\frac{4}{3}}\sin\frac{4}{3}\theta\right) = \frac{1}{r}\left[4r^{\frac{4}{3}}\left(\frac{4}{3}\cos\frac{4}{3}\theta\right)\right] = \frac{16}{3}r^{\frac{1}{3}}\cos\frac{4}{3}\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -\frac{\partial}{\partial r}\left(4r^{\frac{4}{3}}\sin\frac{4}{3}\theta\right) = -\frac{16}{3}\text{r}^{\frac{1}{3}}\sin\frac{4}{3}\theta$$

At the point $r = 2 \text{ m}, \theta = 45°$.

$$v_r = \frac{16}{3}\left(2^{\frac{1}{3}}\right)\cos\left[\frac{4}{3}(45°)\right] = 3.360 \text{ m/s}$$

$$v_\theta = -\frac{16}{3}\left(2^{\frac{1}{3}}\right)\sin\left[\frac{4}{3}(45°)\right] = -5.819 \text{ m/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(3.360 \text{ m/s})^2 + (-5.819 \text{ m/s})^2}$$

$$= 6.72 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_r = \frac{\partial\phi}{\partial r};\ \frac{16}{3}r^{\frac{1}{3}}\cos\frac{4}{3}\theta = \frac{\partial\phi}{\partial r}$$

Integrating with respect to r,

$$\phi = 4r^{\frac{4}{3}}\cos\frac{4}{3}\theta + f(\theta)$$

Substituting this result into,

$$v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}; \qquad -\frac{16}{3}r^{\frac{1}{3}}\sin\frac{4}{3}\theta = \frac{1}{r}\frac{\partial}{\partial\theta}\left[4r^{\frac{4}{3}}\cos\frac{4}{3}\theta + f(\theta)\right]$$

$$-\frac{16}{3}r^{\frac{1}{3}}\sin\frac{4}{3}\theta = \left(4r^{\frac{1}{3}}\right)\left[-\frac{4}{3}\sin\frac{4}{3}\theta\right] + \frac{1}{r}\frac{\partial}{\partial\theta}[f(0)]$$

$$\frac{\partial}{\partial\theta}[f(\theta)] = 0$$

Integrating with respect to θ,

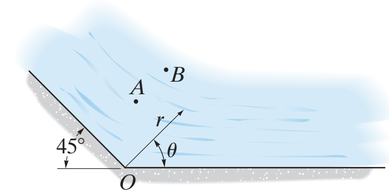
$$f(\theta) = C$$

Setting the constant equal to zero,

$$\phi = 4r^{\frac{4}{3}}\cos\frac{4}{3}\theta$$

Since the potential function can be established, the flow is **irrotational**. Therefore, **the Bernoulli equation is applicable between *any* two points in the flow, including points *A* and *B*.**

7–25. The horizontal flow between the walls is defined by the stream function $\psi = 4r^{4/3}\sin(4/3)\theta\ \text{m}^2/\text{s}$, where r is in meters. If the pressure at the origin O is 20 kPa, determine the pressure at $r = 2$ m, $\theta = 45°$. Take $\rho = 950\ \text{kg/m}^3$.



SOLUTION

We consider ideal fluid flow.

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(4r^{\frac{4}{3}}\sin\frac{4}{3}\theta\right) = \frac{1}{r}\left[4r^{\frac{4}{3}}\left(\frac{4}{3}\cos\frac{4}{3}\theta\right)\right] = \frac{16}{3}r^{\frac{1}{3}}\cos\frac{4}{3}\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -\frac{\partial}{\partial r}\left(4r^{\frac{4}{3}}\sin\frac{4}{3}\theta\right) = -\frac{16}{3}r^{\frac{1}{3}}\sin\frac{4}{3}\theta$$

At point A, where $r = 2$ m $\theta = 45°$,

$$v_r = \frac{16}{3}\left(2^{\frac{1}{3}}\right)\cos\left[\frac{4}{3}(45°)\right] = 3.360\ \text{m/s}$$

$$v_\theta = -\frac{16}{3}\left(2^{\frac{1}{3}}\right)\sin\left[\frac{4}{3}(45°)\right] = -5.819\ \text{m/s}$$

At the origin O, where $r = 0$,

$$v_r = v_\theta = 0$$

Thus, the magnitude of the velocity at these two points is

$$Vo = V_A = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(3.360\ \text{m/s})^2 + (-5.819\ \text{m/s})^2}$$

$$= 6.720\ \text{m/s}$$

$$v_r = \frac{\partial\phi}{\partial r};\qquad \frac{16}{3}r^{\frac{1}{3}}\cos\frac{4}{3}\theta = \frac{\partial\phi}{\partial r}$$

Integrating with respect to r,

$$\phi = 4r^{\frac{4}{3}}\cos\frac{4}{3}\theta + f(\theta),$$

Substituting this result into,

$$v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta};\quad -\frac{16}{3}r^{\frac{1}{3}}\sin\frac{4}{3}\theta = \frac{1}{r}\frac{\partial}{\partial\theta}\left[4r^{\frac{4}{3}}\cos\frac{4}{3}\theta + f(\theta)\right]$$

$$-\frac{16}{3}r^{\frac{1}{3}}\sin\frac{4}{3}\theta = \left(4r^{\frac{1}{3}}\right)\left[-\frac{4}{3}\sin\frac{4}{3}\theta\right] + \frac{1}{r}\frac{\partial}{\partial\theta}[f(\theta)]$$

$$\frac{\partial}{\partial\theta}[f(\theta)] = 0$$

Integrating with respect to θ,

$$f(\theta) = C$$

Setting this constant equal to zero,

$$\phi = 4r^{\frac{4}{3}}\cos\frac{4}{3}\theta$$

Since the potential function can be established, the flow is irrotational. Therefore, the Bernoulli equation is applicable between *any* two points in the flow, including points A and O.

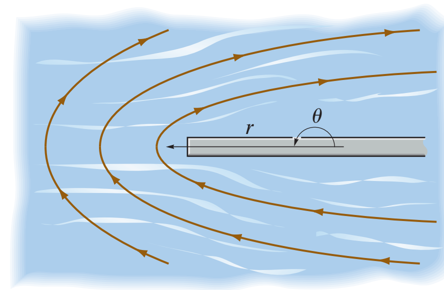
$$\frac{p}{\rho} + \frac{V^2}{2} = \frac{p_O}{\rho} + \frac{V_O^2}{2}$$

$$\frac{p}{950\ \text{kg/m}^3} + \frac{(6.720\ \text{m/s})^2}{2} = \frac{20\left(10^3\right)\frac{\text{N}}{\text{m}^2}}{950\ \text{kg/m}^3} + 0$$

$$p = -1.448\left(10^3\right)\ \text{Pa} = -1.45\ \text{kPa}$$ **Ans.**

Ans:
-1.45 kPa

7–26. The flat plate is subjected to the flow defined by the stream function $\psi = \left[8r^{1/2}\sin(\theta/2)\right]\ \mathrm{m^2/s}$. Sketch the streamline that passes through point $r = 4$ m, $\theta = \pi$ rad, and determine the magnitude of the velocity at this point.



SOLUTION

We consider ideal fluid flow.

For the stream function passing point $r = 4$ m and $\theta = \pi$ rad,

$$\psi = 8\left(4^{\frac{1}{2}}\right)\sin\frac{\pi}{2} = 16$$

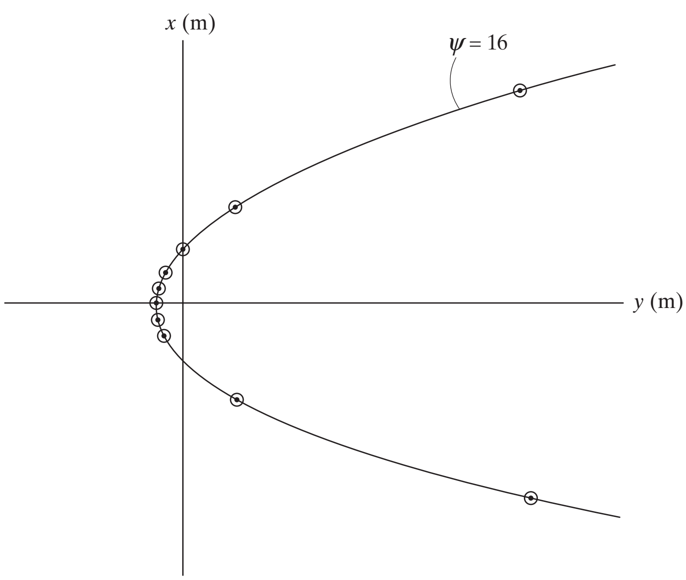
Thus, the stream function passing through this point is

$$16 = 8r^{\frac{1}{2}}\sin\frac{\theta}{2}$$

$$r^{\frac{1}{2}}\sin\frac{\theta}{2} = 2$$

The plot of this function is shown in Fig. *a*

θ(rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r(m)	∞	59.71	16.0	8.00	5.33	4.29	4.00	4.29	5.33	8.00	16.0	59.71	∞



(a)

7–26. Continued

The radial and transverse components of velocity are

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left[8r^{\frac{1}{2}}\left(\frac{1}{2}\cos\frac{\theta}{2}\right)\right] = \frac{4\cos\frac{\theta}{2}}{r^{\frac{1}{2}}}$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -\frac{1}{2}\left(8r^{-\frac{1}{2}}\sin\frac{\theta}{2}\right) = -\frac{4\sin\frac{\theta}{2}}{r^{\frac{1}{2}}}$$

The continuity equation $\dfrac{v_r}{r} + \dfrac{\partial v_r}{\partial_r} + \dfrac{1}{r}\dfrac{\partial v_\theta}{\partial\theta} = \dfrac{4\cos\frac{\theta}{2}}{r^{\frac{3}{2}}} + \left(\dfrac{-2\cos\frac{\theta}{2}}{r^{\frac{3}{2}}}\right) + \left(-\dfrac{2\cos\frac{\theta}{2}}{r^{\frac{3}{2}}}\right) = 0$

is indeed satisfied.

At point $r = 4$ m, $\theta = \pi$ rad,

$$v_r = \frac{4\cos\frac{\pi}{2}}{4^{\frac{1}{2}}} = 0$$

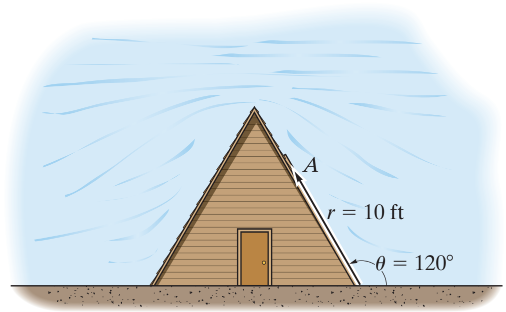
$$v_\theta = -\frac{4\sin\frac{\pi}{2}}{4^{\frac{1}{2}}} = -2\text{ m/s}$$

Thus, the magnitude of the velocity is

$$V = v_\theta = 2\text{ m/s}$$ **Ans.**

Ans:
2 m/s

7–27. An A-frame house has a window A on its right side. If the stream function that models the flow as this side is defined as $\psi = (2r^{1.5}\sin 1.5\,\theta)\ \mathrm{ft^2/s}$, show that continuity of the flow is satisfied, and then determine the wind speed past the window located at $r = 10$ ft, $\theta = (\pi/3)$ rad. Sketch the streamline that passes through this point.



SOLUTION

We consider ideal fluid flow.
For the stream function passing through point $r = 10$ ft, $\theta = \pi/3$ rad,

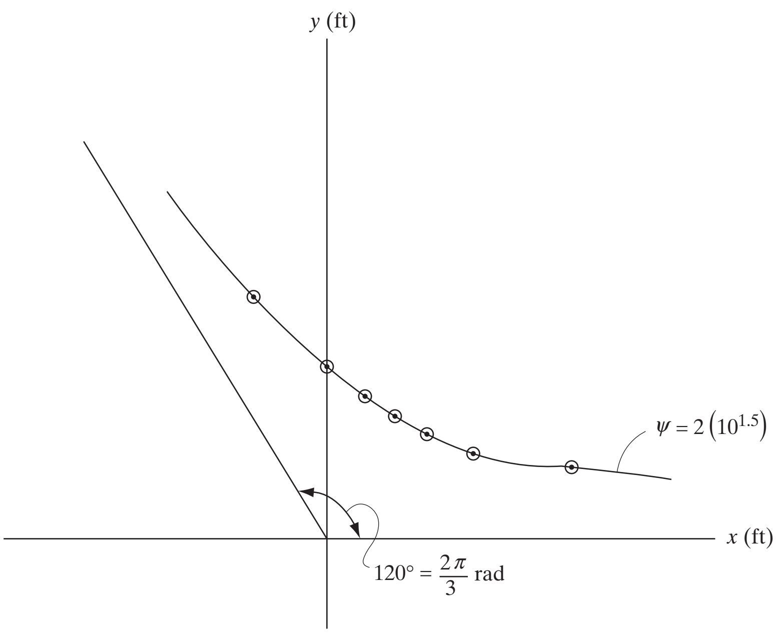
$$\psi = 2(10^{1.5})\sin\left[1.5(\pi/3)\right] = 2(10^{1.5})$$

Thus, the stream function passing through this point is

$$2(10^{1.5}) = 2r^{1.5}\sin 1.5\theta$$
$$r^{1.5}\sin 1.5\theta = 10^{1.5}$$

The plot of this stream function is shown in Fig. *a*

θ(rad)	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$
r(ft)	∞	18.97	12.60	10.54	10	10.54	12.60	18.97	∞



(a)

7–27. Continued

The radial and transverse components of velocity are

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left[2r^{1.5}(1.5\cos 1.5\theta)\right] = 3r^{\frac{1}{2}}\cos 1.5\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -3r^{\frac{1}{2}}\sin 1.5\theta$$

The continuity equation $\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} = \frac{3\cos 1.5\theta}{r^{\frac{1}{2}}} + \frac{1.5\cos 1.5\theta}{r^{\frac{1}{2}}}$

$+\left(-\frac{4.5\cos 1.5\theta}{r^{\frac{1}{2}}}\right) = 0$ is indeed satisfied.

At the window, where $r = 10$ ft, $\theta = 120° = \frac{2}{3}\pi$ rad,

$$v_r = 3\left(10^{\frac{1}{2}}\right)\cos\left[1.5\left(\frac{2\pi}{3}\right)\right] = -9.4868 \text{ ft/s}$$

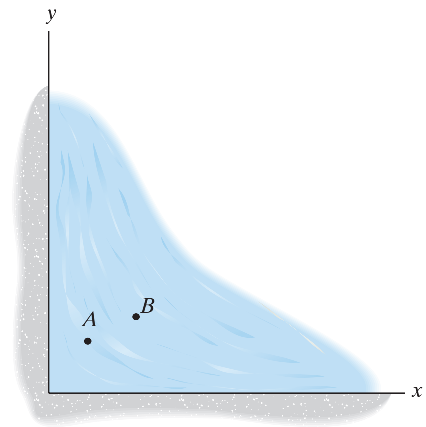
$$v_\theta = -3\left(10^{\frac{1}{2}}\right)\sin\left[1.5\left(\frac{2\pi}{3}\right)\right] = 0$$

Thus, the magnitude of the wind velocity is

$$V = v_r = 9.49 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
9.49 ft/s

***7–28.** The stream function for a horizontal flow near the corner is $\psi = (8xy)\ \text{m}^2/\text{s}$, where x and y are in meters. Determine the x and y components of the velocity and the acceleration of fluid particles passing through point (1 m, 2 m). Show that it is possible to establish the potential function. Plot the streamlines and equipotential lines that pass through point (1 m, 2 m).



SOLUTION

We consider ideal fluid flow.
For the stream function passing through point (1 m, 2 m),

$$\psi = 8(1)(2) = 16$$

Thus,

$$16 = 8xy \qquad y = \frac{2}{x}$$

Using the velocity components,

$$u = \frac{\partial \psi}{\partial y} = (8x)\ \text{m/s} \qquad v = -\frac{\partial \psi}{\partial x} = (-8y)\ \text{m/s}$$

The continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 8 + (-8) = 0$ is indeed satisfied.

The acceleration components are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + 8x(8) + (-8y)(0) = (64x)\ \text{m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 8x(0) + (-8y)(-8) = (64y)\ \text{m/s}^2$$

At point (1 m, 2 m),

$$u = 8(1) = 8\ \text{m/s} \rightarrow v = -8(2) = -16\ \text{m/s} = 16\ \text{m/s}\downarrow \qquad \textbf{Ans.}$$

$$a_x = 64(1) = 64\ \text{m/s}^2 \rightarrow a_y = 64(2) = 128\ \text{m/s}^2\uparrow \qquad \textbf{Ans.}$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0$, the flow is irrotational. Therefore, it is possible to establish potential function. Using the velocity components,

$$\frac{\partial \phi}{\partial x} = u; \qquad \frac{\partial \phi}{\partial x} = 8x$$

Integrating this equation with respect to x

$$\phi = 4x^2 + f(y) \qquad \textbf{(1)}$$

7–28. Continued

Also

$$\frac{\partial \phi}{\partial y} = v; \quad \frac{\partial}{\partial y}\left[4x^2 + f(y)\right] = -8y$$

$$\frac{\partial}{\partial y}\left[f(y)\right] = -\,8y$$

Integrating this equation with respect to y

$$f(y) = -4y^2 + c$$

Setting $C = 0$ and substituting this result into Eq. 1

$$\phi = 4x^2 - 4y^2$$

$$\phi = 4(x^2 - y^2) \qquad \textbf{Ans.}$$

The potential function passing through point (1 m, 2 m),

Then

$$\phi = 4\left(1^2 - 2^2\right) = -12$$

Thus

$$-12 = 4\left(x^2 - y^2\right) \qquad y^2 = x^2 + 3$$

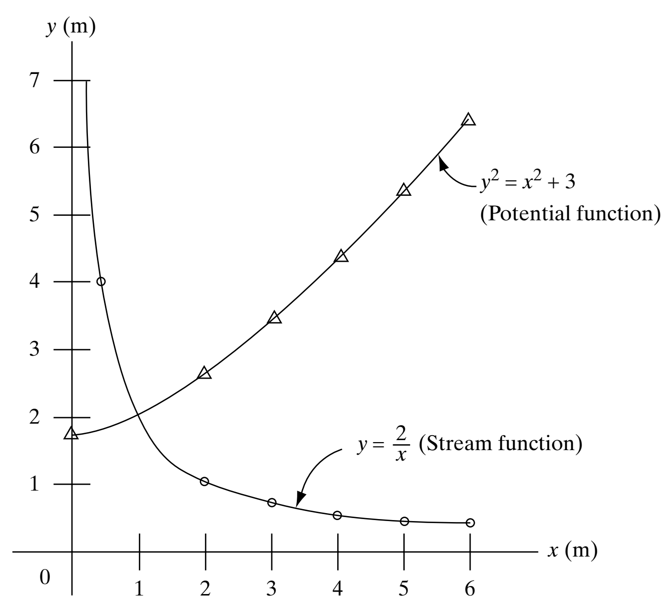
The plots of the stream and potential functions are shown in Fig. *a*.

For the stream function

x(m)	0	1	2	3	4	5	6
y(m)	∞	2	1	0.667	0.5	0.4	0.333

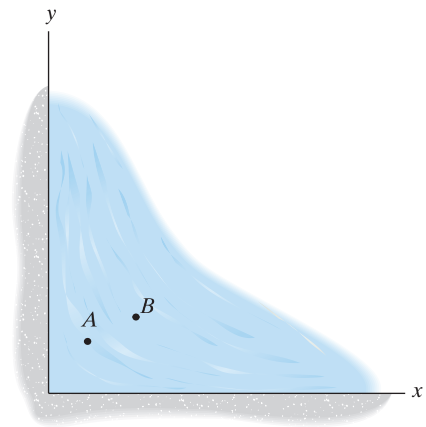
For the potential function

x(m)	0	1	2	3	4	5	6
y(m)	1.73	2	2.65	3.46	4.36	5.29	6.24



(a)

7–29. The stream function for horizontal flow near the corner is defined by $\psi = (8xy)\ \text{m}^2/\text{s}$, where x and y are in meters. Show that the flow is irrotational. If the pressure at point A (1 m, 2 m) is 150 kPa, determine the pressure at point B (2 m, 3 m). Take $\rho = 980\ \text{kg/m}^3$.



SOLUTION

We consider ideal fluid flow.
Using the velocity components,

$$u = \frac{\partial \psi}{\partial y} = (8x)\ \text{m/s} \qquad v = -\frac{\partial \psi}{\partial x} = -(-8y)\ \text{m/s}$$

The continuity equation $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 8 + (-8) = 0$ is indeed satisfied

At point $A(1\,\text{m}, 2\,\text{m})$,

$$u_A = 8(1) = 8\ \text{m/s} \qquad v_A = -8(2) = -16\ \text{m/s}$$

Thus,

$$V_A^2 = u_A^2 + v_A^2 = (8\ \text{m/s})^2 + (-16\,\text{m/s})^2 = 320\ \text{m}^2/\text{s}^2$$

At point $B(2\ \text{m}, 3\ \text{m})$

$$u_B = 8(2) = 16\ \text{m/s} \qquad v_B = -8(3) = -24\ \text{m/s}$$

Thus,

$$V_B^2 = u_B^2 + v_B^2 = (16\ \text{m/s})^2 + (-24\ \text{m/s})^2 = 832\ \text{m}^2/\text{s}^2$$

Since $\omega_z = \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) = \dfrac{1}{2}(0 + 0) = 0$, the flow is irrotational. Therefore, Bernoulli's equation is applicable to two points located on the different streamlines such as points A and B.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B$$

Since the flow is in the horizontal plane, $z_A = z_B = z$.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} + gz = \frac{p_B}{\rho} + \frac{V_B^2}{2} + gz$$

$$p_B = p_A + \frac{\rho}{2}\left(V_A^2 - V_B^2\right)$$

$$p_B = 150\left(10^3\right)\ \text{N/m}^2 + \frac{980\ \text{kg/m}^3}{2}\left(320\ \text{m}^2/\text{s}^2 - 832\ \text{m}^2/\text{s}^2\right)$$

$$= -100.88\left(10^3\right)\ \text{N/m}^2$$

$$= -101\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
-101 kPa

7–30. A flow has velocity components $u = \left(2x^2\right)$ ft/s and $v = (-4xy + 8)$ ft/s, where x and y are in feet. Determine the magnitude of the acceleration of a particle located at point (3 ft, 2 ft). Is the flow rotational or irrotational? Also, show that continuity of flow is satisfied.

SOLUTION

We consider ideal fluid flow.

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(-4xy + 8) = -4y$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(2x^2\right) = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(2x^2\right) = 4x$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(-4xy + 8) = -4x$$

Thus,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(-4y - 0) = -2y$$

Since $\omega_z \neq 0$, the flow is **rotational**. Also

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4x + (-4x) = 0$$

The flow **satisfies the continuity** condition.

Since $\dfrac{\partial u}{\partial t} = \dfrac{\partial v}{\partial t} = 0$ (steady flow)

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + \left(2x^2\right)\left(4x\right) + (-4xy + 8)(0) = 8x^3$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 2x^2(-4y) + (-4xy + 8)(-4x) = -8x^2y + 16x^2y - 32x = 8x^2y - 32x$$

Thus, at $x = 3$ ft, $y = 2$ ft

$$a_x = 8\left(3^3\right) = 216 \text{ ft/s}^2$$

$$a_y = 8(3^2)(2) - 32(3) = 48 \text{ ft/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(216 \text{ ft/s}^2)^2 + \left(48 \text{ ft/s}^2\right)^2}$$

$$= 221.27 \text{ ft/s}^2 = 221 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
rotational
$a = 221 \text{ ft/s}^2$

7–31. The potential function for a flow is $\phi = \left(x^2 - y^2\right)\ \text{ft}^2/\text{s}$, where x and y are in feet. Determine the magnitude of the velocity of fluid particles at point A (3 ft, 1 ft). Show that continuity is satisfied, and find the streamline that passes through point A.

SOLUTION

We consider ideal fluid flow.

From the velocity components

$$u = \frac{\partial \phi}{\partial x} = 2x \quad v = \frac{\partial \phi}{\partial y} = -2y$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0$, the flow is indeed irrotational.

At point (3 ft, 1 ft),

$$u = 2(3) = 6\ \text{ft/s} \quad v = -2(1) = -2\ \text{ft/s}$$

Then the magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(6\ \text{ft/s})^2 + (-2\ \text{ft/s})^2} = 6.32\ \text{ft/s} \qquad \textbf{Ans.}$$

Here $\frac{\partial u}{\partial x} = 2$ and $\frac{\partial v}{\partial y} = -2$. Since $\frac{\partial u}{\partial x} + \frac{\partial y}{\partial y} = 2 + (-2) = 0$, the potential function satisfies the continuity condition. Using the velocity components.

$$\frac{\partial \psi}{\partial y} = u; \quad \frac{\partial \psi}{\partial y} = 2x$$

Integrating this equation with respect to y,

$$\psi = 2xy + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \quad -\frac{\partial}{\partial x}[2xy + f(x)] = -2y$$

$$2y + \frac{\partial}{\partial x}[f(x)] = 2y$$

$$\frac{\partial}{\partial x}[f(x)] = 0$$

Integrating this equation with respect to x,

$$f(x) = C$$

Setting $C = 0$, and substituting this result into Eq. 1

$$\psi = 2xy$$

For the streamline passing through point (3 ft, 1 ft),

$$\psi = 2(3)(1) = 6$$

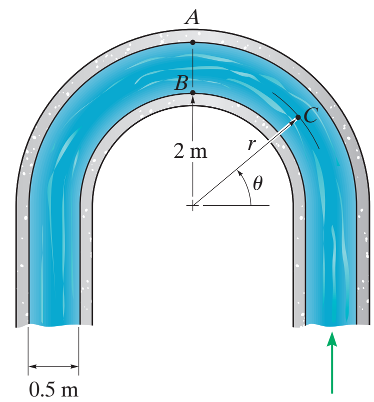
Thus,

$$6 = 2xy$$

$$xy = 3 \qquad \textbf{Ans.}$$

Ans:
$V = 6.32\ \text{ft/s}$
$xy = 3$

***7–32.** The flow around the bend in the horizontal channel can be described as a free vortex for which $v_r = 0$, $v_\theta = (8/r)$ m/s, where r is in meters. Show that the flow is irrotational. If the pressure at point A is 4 kPa, determine the pressure at point B. Take $\rho = 1100\ \text{kg/m}^3$.



SOLUTION

We consider ideal fluid flow.

$$v_r = \frac{\partial \phi}{\partial r}; \quad 0 = \frac{\partial \phi}{\partial r}$$

Integrating with respect to r,

$$\phi = f(\theta)$$

Substituting this result into

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}; \quad \frac{8}{r} = \frac{1}{r}\frac{\partial}{\partial \theta}[f(\theta)]$$

$$\frac{\partial}{\partial \theta}[f(\theta)] = 8$$

Integrating with respect to θ,

$$\phi = f(\theta) = 8\theta + C$$

Since the potential function can be established, the *flow is irrotational* and Bernoulli's equation can be applied between points A and B. The magnitude of velocity at A and B is

$$V_A = (v_\theta)_A = \frac{8}{r_A} = \frac{8}{2.5} = 3.2\ \text{m/s}$$

$$V_B = (v_\theta)_B = \frac{8}{r_B} = \frac{8}{2} = 4\ \text{m/s}$$

Applying the Bernoulli equation,

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} = \frac{p_A}{\rho} + \frac{V_A^2}{2}$$

$$\frac{p_B}{1100\ \text{kg/m}^3} + \frac{(4\ \text{m/s})^2}{2} = \frac{4(10^3)\dfrac{\text{N}}{\text{m}^2}}{1100\ \text{kg/m}^3} + \frac{(3.2\ \text{m/s})^2}{2}$$

$$p_B = 832\ \text{Pa}$$ **Ans.**

7–33. The velocity components for a two-dimensional flow are $u = (8y)$ ft/s and $v = (8x)$ ft/s, where x and y are in feet. Determine if the flow is rotational or irrotational, and show that continuity of flow is satisfied.

SOLUTION

We consider ideal fluid flow.

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(8x) = 8 \text{ rad/s}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(8y) = 8 \text{ rad/s}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(8y) = 0$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(8x) = 0$$

Thus,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(8 - 8) = 0$$

Since $\omega_z = 0$, the flow is **irrotational.** Also, **Ans.**

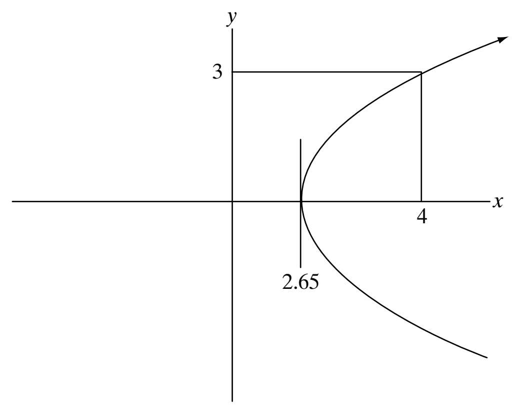
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$$

The flow **satisfies the continuity condition**. **Ans.**

Ans:
irrotational

7–34. The velocity components for a two-dimensional flow are a $u = (8y)$ ft/s and $v = (8x)$ ft/s where x and y are in feet. Find the stream function and the equation of the streamline that passes through point (4 ft, 3 ft). Plot this streamline.

SOLUTION



We consider ideal fluid flow.

$$u = \frac{\partial \psi}{\partial y}; \quad 8y = \frac{\partial \psi}{\partial y}$$

Integrating with respect to y,

$$\psi = 4y^2 + f(x)$$

Then,

$$v = -\frac{\partial \psi}{\partial x}; \quad 8x = -\frac{\partial}{\partial x}\left[4y^2 + f(x)\right]$$

$$-8x = 0 + \frac{\partial}{\partial x}[f(x)]$$

Integrating with respect to x,

$$f(x) = -4x^2 + C$$

Thus,

$$\psi = 4y^2 + \left(-4x^2 + C\right) = 4\left(y^2 - x^2\right) + C$$

Omitting the constant, C,

$$\psi = 4\left(y^2 - x^2\right) \qquad \textbf{Ans.}$$

From the slope of the stream function,

$$\frac{dy}{dx} = \frac{v}{u} = \frac{8x}{8y} = \frac{x}{y}$$

$$\int_{3\text{ ft}}^{y} y\,dy = \int_{4\text{ ft}}^{x} x\,dx$$

$$\left.\frac{y^2}{2}\right|_{3\text{ ft}}^{y} = \left.\frac{x^2}{2}\right|_{4\text{ ft}}^{x}$$

$$y^2 = x^2 - 7$$

$$y = \pm\sqrt{x^2 - 7} \qquad \textbf{Ans.}$$

Also, at (4 ft, 3 ft),

$$\psi = 4\left((3)^2 - (4)^2\right) = -28$$

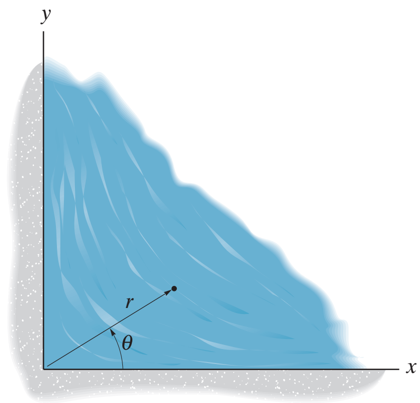
Then,

$$4\left(y^2 - x^2\right) = -28$$

$$y = \pm\sqrt{x^2 - 7}$$

Ans:
$\psi = 4\left(y^2 - x^2\right)$
$y = \pm\sqrt{x^2 - 7}$

7–35. The stream function for the flow field around the 90° corner is $\psi = 8r^2 \sin 2\theta$. Show that the continuity of flow is satisfied. Determine the r and θ velocity components of a fluid particle located at $r = 0.5$ m, $\theta = 30°$, and plot the streamline that passes through this point. Also, determine the potential function for the flow.



SOLUTION

We consider ideal fluid flow.
From the r and θ velocity components to,

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left(8r^2\right)(2\cos 2\theta) = 16r\cos 2\theta$$

$$v_\theta = \frac{\partial\psi}{\partial r} = -(16r\sin 2\theta) = -16r\sin 2\theta$$

The continuity equation $\dfrac{v_r}{r} + \dfrac{\partial v_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial v_\theta}{\partial\theta} = 16\cos 2\theta + 16\cos 2\theta + (-32\cos 2\theta) = 0$ is indeed satisfied.

At point $r = 0.5$ m, $\theta = 30°$,

$$v_r = 16(0.5)\cos\left[2(30°)\right] = 4 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_\theta = -16(0.5)\sin\left[2(30°)\right] = -6.93 \text{ m/s} \qquad \textbf{Ans.}$$

Since $\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, then $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\dfrac{y}{r}\right)\left(\dfrac{x}{r}\right) = \dfrac{2xy}{r^2}$.
Therefore,

$$\psi = 8r^2\left(\frac{2xy}{r^2}\right) = 16xy$$

At point $r = 0.5$ m, $\theta = 30°$,

$$x = r\cos\theta = (0.5\text{ m})\cos 30° = \frac{\sqrt{3}}{4}\text{ m}$$

$$y = r\sin\theta = (0.5\text{ m})\sin 30° = \frac{1}{4}\text{ m}$$

Then

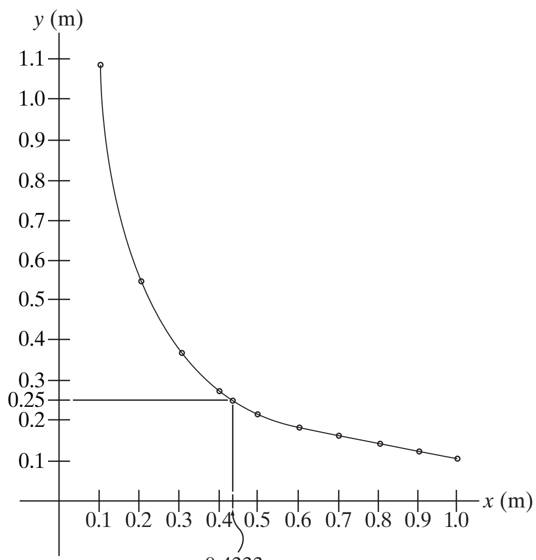
$$\psi = 16\left(\frac{\sqrt{3}}{4}\right)\left(\frac{1}{4}\right) = \sqrt{3}$$

Thus, the streamline passing through this point is

$$\sqrt{3} = 16xy$$

$$y = \frac{\sqrt{3}}{16x}$$

7–35. Continued



(a)

The plot of this streamline is shown in Fig. *a*

x(m)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y(m)	∞	1.08	0.541	0.361	0.271	0.217	0.180	0.155	0.135	0.120	0.108

The velocity components with respect to stream function are

$$u = \frac{\partial\psi}{\partial y} = 16x \quad v = -\frac{\partial\psi}{\partial x} - 16y$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0$, the flow is irrotational. Therefore, it is possible to established the potential function using the velocity components,

$$\frac{\partial\phi}{\partial x} = u; \quad \frac{\partial\phi}{\partial x} = 16x$$

Integrating this equation with respect to x,

$$\phi = 8x^2 + f(y) \qquad \textbf{(1)}$$

Also,

$$\frac{\partial\phi}{\partial y} = v; \quad \frac{\partial}{\partial y}\left[8x^2 + f(y)\right] = -16y$$

$$\frac{\partial}{\partial y}[f(y)] = -16y$$

Integrating this equation with respect to y

$$f(y) = -8y^2 + C$$

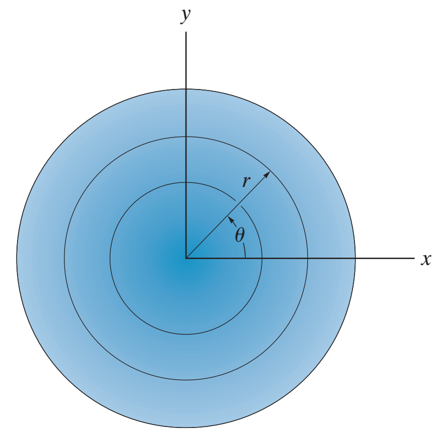
Setting $C = 0$, and substituting this result in Eq. 1

$$\phi = 8x^2 - 8y^2 + C$$

$$\phi = 8\left(x^2 - y^2\right) \qquad \textbf{Ans.}$$

Ans:
$v_r = 4\ \text{m/s}$
$v_\theta = -6.93\ \text{m/s}$
$\phi = 8\left(x^2 - y^2\right)$

***7–36.** The stream function for a concentric flow is defined by $\psi = -4r^2$. Determine the velocity components v_r and v_θ, and v_x and v_y. Can the potential function be established? If so, what is it?



SOLUTION

We consider ideal fluid flow.
Using the r, θ velocity components

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}(0) = 0 \qquad \textbf{Ans.}$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -[-4(2r)] = 8r \qquad \textbf{Ans.}$$

Since $r^2 = x^2 + y^2$, then $\psi = -4(x^2 + y^2)$. Using the velocity components,

$$v_x = u = \frac{\partial\psi}{\partial y} = -4(2y) = -8y \qquad \textbf{Ans.}$$

$$v_y = v = -\frac{\partial\psi}{\partial x} = -[-4(2x)] = 8x \qquad \textbf{Ans.}$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}[8 - (-8)] = 16 \neq 0$, the flow is rotational.
Therefore, the potential function **cannot be established**.

7–37. A fluid has velocity components $u = (x - y)$ ft/s and $v = -(x + y)$ ft/s, where x and y are in feet. Determine the stream and potential functions. Show that the flow is irrotational.

SOLUTION

We consider ideal fluid flow.

Since the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + (-1) = 0$ is satisfied, then the establishment of a stream function is possible. Using the velocity components,

$$\frac{\partial \psi}{\partial y} = u; \quad \frac{\partial \psi}{\partial x} = x - y$$

Integrating this equation with respect to y,

$$\psi = xy - \frac{1}{2}y^2 + f(x) \qquad (1)$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \quad -\frac{\partial}{\partial x} = \left[xy - \frac{1}{2}y^2 + f(x)\right] = -(x + y)$$

$$y - 0 + \frac{\partial}{\partial x}[f(x)] = x + y$$

$$\frac{\partial}{\partial x}[f(x)] = x$$

Integrating this equation with respect to x,

$$f(x) = \frac{1}{2}x^2 + C$$

Setting $C = 0$ and substituting this result into Eq. 1,

$$\psi = xy - \frac{1}{2}y^2 + \frac{1}{2}x^2$$

$$\psi = \frac{1}{2}(x^2 - y^2 + 2xy) \qquad \textbf{Ans.}$$

Using the definition of velocity components with respect to the potential function,

$$\frac{\partial \phi}{\partial x} = u; \quad \frac{\partial \phi}{\partial x} = x - y$$

Integrating this equation with respect to x,

$$\phi = \frac{1}{2}x^2 - xy + f(y) \qquad \textbf{(2)}$$

Also,

$$\frac{\partial \phi}{\partial y} = v; \quad \frac{\partial}{\partial y}\left[\frac{1}{2}x^2 - xy + f(y)\right] = -(x + y)$$

$$-x + \frac{\partial}{\partial y}[f(y)] = -x - y$$

$$\frac{\partial}{\partial y}[f(y)] = -y$$

7–37. Continued

Integrating this equation with respect to y

$$f(y) = \frac{1}{2}y^2 + C$$

Setting $C = 0$ and substituting this result into Eq. 2,

$$\phi = \frac{1}{2}x^2 - xy - \frac{1}{2}y^2$$

$$\phi = \frac{1}{2}\left(x^2 - y^2 - 2xy\right) \qquad \textbf{Ans.}$$

Here

$$\frac{\partial v}{\partial x} = -1 \text{ and } \frac{\partial u}{\partial y} = -1$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}[-1 - (-1)] = 0$, the flow is **irrotational**.

Ans:

$$\psi = \frac{1}{2}\left(x^2 - y^2 + 2xy\right)$$

$$\phi = \frac{1}{2}\left(x^2 - y^2 - 2xy\right)$$

7–38. A fluid has velocity components $u = (2y)$ ft/s and $v = (2x - 10)$ ft/s, where x and y are in feet. Determine the stream and potential functions.

SOLUTION

We consider ideal fluid flow.

Since the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$ is satisfied, then the establishment of stream function is possible.

Using the definition of velocity components with respect to stream function,

$$\frac{\partial \psi}{\partial y} = u; \quad \frac{\partial \psi}{\partial y} = 2y$$

Integrating this equation with respect to y,

$$\psi = y^2 + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \quad -\frac{\partial}{\partial x} = \left[y^2 + f(x)\right] = 2x - 10$$

$$\frac{\partial}{\partial x}[f(x)] = 10 - 2x$$

Integrating this equation with respect to x,

$$f(x) = 10x - x^2 + C$$

Setting $C = 0$, and substituting this result into Eq. 1,

$$\psi = y^2 + 10x - x^2$$

$$\psi = y^2 - x^2 + 10x \qquad \textbf{Ans.}$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(2 - 2) = 0$, the flow is irrotational. Therefore the potential function exists.

Using the definition of velocity components with respect to potential function,

$$\frac{\partial \phi}{\partial x} = u; \quad \frac{\partial \phi}{\partial x} = 2y$$

Integrating this equation with respect to x,

$$\phi = 2xy + f(y) \qquad \textbf{(1)}$$

Also,

$$\frac{\partial \phi}{\partial y} = v; \quad \frac{\partial}{\partial y} = [2xy + f(y)] = 2x - 10$$

$$2x + \frac{\partial}{\partial y}[f(y)] = 2x - 10$$

$$\frac{\partial}{\partial y}[f(y)] = -10$$

Integrating this equation with respect to y,

$$f(y) = -10y + C$$

Setting $C = 0$, and substituting this result into Eq. 1,

$$\phi = 2xy - 10y$$

$$\phi = 2y(x - 5) \qquad \textbf{Ans.}$$

Ans:
$\psi = y^2 - x^2 + 10x$
$\phi = 2y(x - 5)$

7–39. A fluid has velocity components $u = (x - 2y)$ ft/s and $v = -(y + 2x)$ ft/s, where x and y are in feet. Determine the stream and potential functions.

SOLUTION

We consider ideal fluid flow.

Since the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + (-1) = 0$ is satisfied, then the establishment of the stream function is possible using the velocity components,

$$\frac{\partial \psi}{\partial y} = u; \quad \frac{\partial \psi}{\partial y} = x - 2y$$

Integrating this equation with respect to y,

$$\psi = xy - y^2 + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \quad -\frac{\partial}{\partial x}\left[xy - y^2 + f(x)\right] = -y - 2x$$

$$-y - \frac{\partial}{\partial x}[f(x)] = -y - 2x$$

$$\frac{\partial}{\partial x}[f(x)] = 2x$$

Integrating this equation with respect to x,

$$f(x) = x^2 + C$$

Setting $C = 0$ and substituting this result into Eq. 1,

$$\psi = x^2 - y^2 + xy \qquad \textbf{Ans.}$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}[-2 - (-2)] = 0$, the flow is irrotational. Therefore, the potential function exists.

Using the definition of velocity components with respect to potential function,

$$\frac{\partial \phi}{\partial x} = u; \quad \frac{\partial \phi}{\partial x} = x - 2y$$

Integrating this equation with respect to x

$$\phi = \frac{1}{2}x^2 - 2xy + f(y) \qquad \textbf{(1)}$$

Also,

$$\frac{\partial \phi}{\partial y} = v; \quad \frac{\partial}{\partial y} = \left[\frac{1}{2}x^2 - 2xy + f(y)\right] = -y - 2x$$

$$-2x + \frac{\partial}{\partial y}[f(y)] = -y - 2x$$

$$\frac{\partial}{\partial y}[f(y)] = -y$$

Integrating this equation with respect to y

$$f(y) = -\frac{1}{2}y^2 + C$$

Setting $C = 0$ and substituting this result into Eq. 1,

$$\phi = \frac{1}{2}x^2 - 2xy - \frac{1}{2}y^2$$

$$\phi = \frac{1}{2}(x^2 - y^2) - 2xy \qquad \textbf{Ans.}$$

Ans:
$\psi = x^2 - y^2 + xy$
$\phi = \frac{1}{2}(x^2 - y^2) - 2xy$

***7–40.** A fluid has velocity components $u = 2\left(x^2 - y^2\right)$ m/s and $v = (-4xy)$ m/s, where x and y are in meters. Determine the stream function. Also show that the potential function exists, and find this function. Plot the streamlines and equipotential lines that pass through point (1 m, 2 m).

SOLUTION

Since the continuity equation $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 4x + (-4x) = 0$ is satisfied, the stream function can be established.

Using the velocity components

$$\frac{\partial \psi}{\partial y} = u; \quad \frac{\partial \psi}{\partial y} = 2x^2 - 2y^2$$

Integrating this equation with respect to y,

$$\psi = 2x^2 y - \frac{2}{3}y^3 + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \quad -\frac{\partial}{\partial x}\left[2x^2 y - \frac{2}{3}y^3 + f(x)\right] = -4xy$$

$$4xy + \frac{\partial}{\partial x}[f(x)] = 4xy$$

$$\frac{\partial}{\partial x}[f(x)] = 0$$

Integrating this equation with respect to x,

$$f(x) = C$$

Setting $C = 0$ and substituting this result into Eq. 1,

$$\psi = 2x^2 y - \frac{2}{3}y^3$$

$$\psi = \frac{2}{3}y\left(3x^2 - y^2\right) \qquad \textbf{Ans.}$$

Since $\omega_z = \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right) = \dfrac{1}{2}[-4y - (-4y)] = 0$, the flow is irrotational. Thus, the potential function exists. Using the velocity components,

$$\frac{\partial \phi}{\partial x} = u; \quad \frac{\partial \phi}{\partial x} = 2x^2 - 2y^2$$

Integrating this equation with respect to x

$$\phi = \frac{2}{3}x^3 - 2xy^2 + f(y) \qquad \textbf{(2)}$$

***7–40. Continued**

Also,

$$\frac{\partial \phi}{\partial y} = v; \quad \frac{\partial}{\partial y}\left[\frac{2}{3}x^3 - 2xy^2 + f(y)\right] = -4xy$$

$$-4xy + \frac{\partial}{\partial y}[f(y)] = -4xy$$

$$\frac{\partial}{\partial y}[f(y)] = 0$$

Integrating this equation with respect to y,

$$f(y) = C$$

Setting $C = 0$ and substituting this result into Eq. 2,

$$\phi = \frac{2}{3}x^3 - 2xy^2$$

$$\phi = \frac{2}{3}x\left(x^2 - 3y^2\right) \qquad \textbf{Ans.}$$

For stream function and potential functions passing through point (1 m, 2 m),

$$\psi = \frac{2}{3}(2)\left[3(1)^2 - 2^2\right] = -\frac{4}{3}$$

$$\phi = \frac{2}{3}(1)\left[1^2 - 3\left(2^2\right)\right] = -\frac{22}{3}$$

Thus, the streamline is

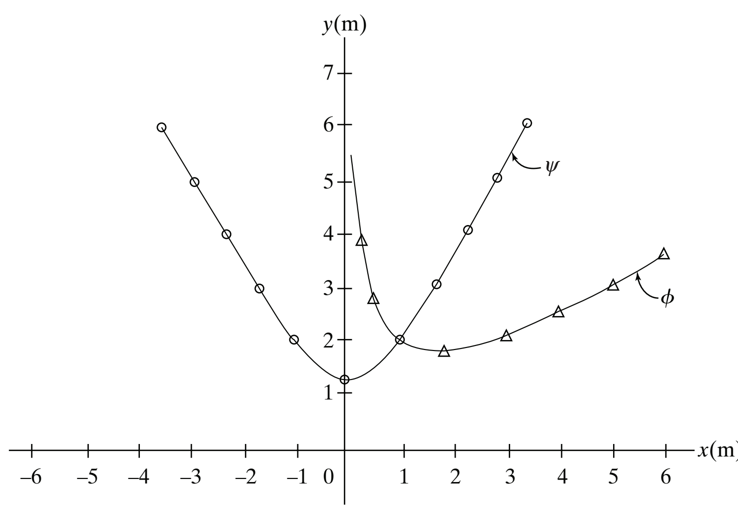
$$-\frac{4}{3} = \frac{2}{3}y\left(3x^2 - y^2\right)$$

$$x^2 = \frac{y^3 - 2}{3y}$$

and the equipotential line is

$$-\frac{22}{3} = \frac{2}{3}x\left(x^2 - 3y^2\right)$$

$$y^2 = \frac{x^3 + 11}{3x}$$



For the streamline

y(m)	1.26	2	3	4	5	6
x(m)	0	±1	±1.67	±2.27	±2.86	±3.45

For the equipotential line

x(m)	0	1	1.77	3	4	5	6	0.25	0.50
y(m)	∞	±2	±1.77	±2.05	±2.50	±3.01	±3.55	±3.83	±2.72

The plot of these two functions is shown in Fig. *a*

7–41. If the potential function for a two-dimensional flow is $\phi = (xy)\ \mathrm{m^2/s}$, where x and y are in meters, determine the stream function and plot the streamline that passes through the point (1 m, 2 m). What are the velocity and acceleration of fluid particles that pass through this point?

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}(xy) = y$$

$$v = \frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y}(xy) = x$$

$$u = \frac{\partial \psi}{\partial y}; \quad y = \frac{\partial \psi}{\partial y}$$

Integrating with respect to y,

$$\psi = \frac{y^2}{2} + f(x)$$

Substituting this result into,

$$v = -\frac{\partial \psi}{\partial x}; \quad x = -\frac{\partial}{\partial x}\left[\frac{y^2}{2} + f(x)\right]$$

$$x = -0 - \frac{\partial}{\partial x}[f(x)]$$

$$\frac{\partial}{\partial x}[f(x)] = -x$$

Integrating with respect to x,

$$f(x) = -\frac{x^2}{2} + C$$

Setting $C = 0$,

$$\psi = \frac{y^2}{2} + \left(-\frac{x^2}{2}\right)$$

$$\psi = \frac{1}{2}(y^2 - x^2) \qquad \textbf{Ans.}$$

When $x = 1$ m, and $y = 2$ m. Then,

$$\psi = \frac{1}{2}(2^2 - 1^2) = 1.5$$

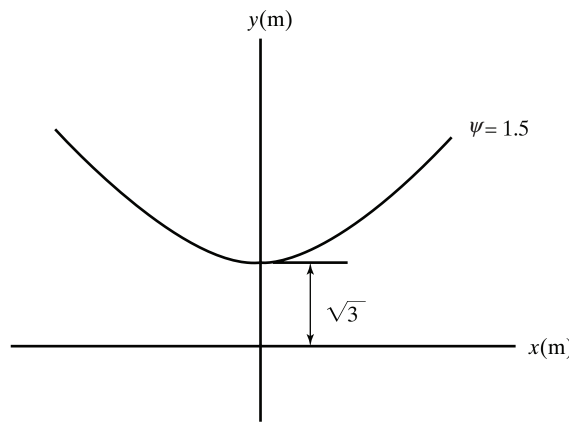
For the streamline defined by $\psi = 1.5$, its equation is

$$\frac{1}{2}(y^2 - x^2) = 1.5$$

$$y^2 = x^2 + 3$$

$$y = \sqrt{x^2 + 3}$$

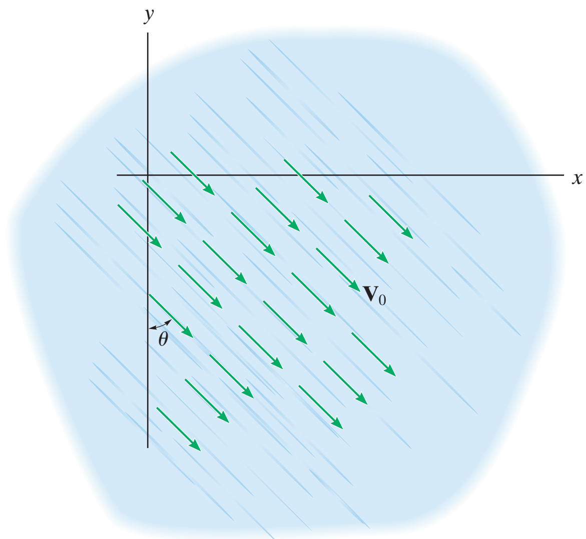
The plot of this streamline is shown in Fig. a.



(a)

Ans:
$\psi = \frac{1}{2}(y^2 - x^2)$

7–42. Determine the potential function for the two-dimensional flow field if $\mathbf{V}_0$ and θ are known.



SOLUTION

We consider ideal fluid flow.
The velocity components are

$$u = V_0 \sin \theta_0 \qquad v = -V_0 \cos \theta_0$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0$, the flow is indeed irrotational. Thus, the potential function exists.

Using the velocity components,

$$\frac{\partial \phi}{\partial x} = u; \qquad \frac{\partial \phi}{\partial x} = V_0 \sin \theta_0$$

Integrating this equation with respect to x,

$$\phi = V_0 \sin \theta_0 x + f(y) \qquad \textbf{(1)}$$

Also,

$$\frac{\partial \phi}{\partial y} = v; \qquad \frac{\partial}{\partial y}\left[(V_0 \sin \theta_0)x + f(y)\right] = -V_0 \cos \theta_0$$

$$\frac{\partial}{\partial y}[f(y)] = -V_0 \cos \theta_0$$

Integrating this equation with respect to y

$$f(y) = -(V_0 \cos \theta_0)y + C$$

Setting $C = 0$, and substituting this result into Eq. 1,

$$\phi = (V_0 \sin \theta_0)x - (V_0 \cos \theta_0)y$$

$$\phi = V_0\left[(\sin \theta_0)x - (\cos \theta_0)y\right] \qquad \textbf{Ans.}$$

Ans:
$\phi = V_0\left[(\sin \theta_0)x - (\cos \theta_0)y\right]$

7–43. The potential function for a flow is $\phi = \left(x^2 - y^2\right)\ \text{ft}^2/\text{s}$, where x and y are in feet. Determine the magnitude of the velocity of fluid particles at point (3 ft, 1 ft). Show that continuity is satisfied, and find the streamline that passes through this point.

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}\left(x^2 - y^2\right) = (2x)\ \text{ft/s}$$

$$v = \frac{\partial\phi}{\partial y} = \frac{\partial}{\partial y}\left(x^2 - y^2\right) = (-2y)\ \text{ft/s}$$

Thus, at $x = 3$ ft, $y = 1$ ft,

$$u = 2(3) = 6\ \text{ft/s}$$

$$v = -2(1) = -2\ \text{ft/s}$$

Then, the magnitude of the flow velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(6\ \text{ft/s})^2 + (-2\ \text{ft/s})^2} = 6.32\ \text{ft/s} \qquad \textbf{Ans.}$$

Applying

$$u = \frac{\partial\psi}{\partial y}; \qquad 2x = \frac{\partial\psi}{\partial y}$$

Integrating with respect to y,

$$\psi = 2xy + f(x)$$

Substituting this result into the second of Eq. (8–8),

$$v = -\frac{\partial\psi}{\partial x}; \qquad -2y = -\frac{\partial}{\partial x}\left[2xy + f(x)\right]$$

$$2y = 2y + \frac{\partial}{\partial x}\left[f(x)\right]$$

$$\frac{\partial}{\partial x}\left[f(x)\right] = 0$$

Integrating with respect to x,

$$f(x) = C$$

Setting $C = 0$, then

$$\psi = 2xy$$

At $x = 3$ ft, $y = 1$ ft,

$$\psi = 2(3)(1) = 6$$

So the streamline through (3 ft, 1 ft) is

$$6 = 2xy$$

$$y = \frac{3}{x} \qquad \textbf{Ans.}$$

7–43. Continued

Here,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(2x) = 2 \text{ ft/s}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(-2y) = -2 \text{ ft/s}$$

Then,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 2 + (-2) = 0$$

Thus, the flow field **satisfies the continuity condition** as required.

Ans:
$V = 6.32 \text{ ft/s}$
$y = \frac{3}{x}$

***7–44.** A fluid has velocity components of $u = (10xy)$ m/s and $v = 5\left(x^2 - y^2\right)$ m/s, where x and y are in meters. Determine the stream function, and show that the continuity condition is satisfied and that the flow is irrotational. Plot the streamlines for $\psi_0 = 0$, $\psi_1 = 1\ \text{m}^2/\text{s}$, and $\psi_2 = 2\ \text{m}^2/\text{s}$.

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial \psi}{\partial y}; \qquad 10xy = \frac{\partial \psi}{\partial y}$$

Integrating with respect to y,

$$\psi = 5xy^2 + f(x)$$

Substituting this result into

$$v = -\frac{\partial \psi}{\partial x}; \qquad 5\left(x^2 - y^2\right) = -\frac{\partial}{\partial x}\left[5xy^2 + f(x)\right]$$

$$5x^2 - 5y^2 = -5y^2 - \frac{\partial}{\partial x}\left[f(x)\right]$$

$$\frac{\partial}{\partial x}\left[f(x)\right] = -5x^2$$

Integrating with respect to x,

$$f(x) = -\frac{5}{3}x^3 + C$$

Setting $C = 0$, then

$$\psi = 5xy^2 + \left(-\frac{5}{3}x^3\right)$$

$$= \frac{5}{3}x\left(3y^2 - x^2\right) \qquad \textbf{Ans.}$$

Here,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(10xy) = (10y)\ \text{s}^{-1}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left[5\left(x^2 - y^2\right)\right] = (-10y)\ \text{s}^{-1}$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left[5\left(x^2 - y^2\right)\right] = (10x)\ \text{s}^{-1}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(10xy) = (10x)\ \text{s}^{-1}$$

Then,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 10y + (-10y) = 0$$

The flow field **satisfies the continuity condition**. Applying,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(10x - 10x) = 0$$

7–44. Continued

The flow field is **irrotational** since $\omega_z = 0$.

When $\psi = 0$,

$$\frac{5}{3}x\left(3y^2 - x^2\right) = 0$$

$$y = \pm\frac{1}{\sqrt{3}}x$$

or

$$x = 0$$

When $\psi = 1$,

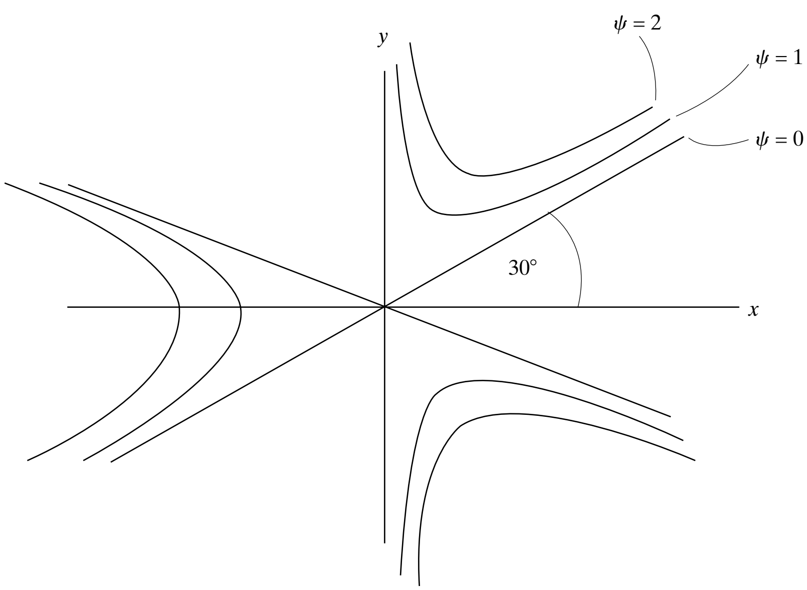
$$\frac{5}{3}x\left(3y^2 - x^2\right) = 1$$

$$y = \pm\sqrt{\frac{1}{5x} + \frac{x^2}{3}}$$

When $\psi = 2$,

$$\frac{5}{3}x\left(3y^2 - x^2\right) = 2$$

$$y = \pm\sqrt{\frac{2}{5x} + \frac{x^2}{3}}$$



7–45. A fluid has velocity components of $u = (y^2 - x^2)$ m/s and $v = (2xy)$ m/s, where x and y are in meters. If the pressure at point A (3 m, 2 m) is 600 kPa, determine the pressure at point B (1 m, 3 m). Also what is the potential function for the flow? Take $\gamma = 8\ \text{kN/m}^3$.

SOLUTION

We consider ideal fluid flow.
Applying

$$u = \frac{\partial \phi}{\partial x}; \qquad y^2 - x^2 = \frac{\partial \phi}{\partial x}$$

Integrating with respect to x,

$$\phi = xy^2 - \frac{x^3}{3} + f(y)$$

Substituting this result into,

$$v = \frac{\partial \phi}{\partial y}; \qquad 2xy = \frac{\partial}{\partial y}\left[xy^2 - \frac{x^3}{3} + f(y)\right]$$

$$2xy = 2xy - 0 + \frac{\partial}{\partial y}[f(y)]$$

$$\frac{\partial}{\partial y}[f(y)] = 0$$

Integrating with respect to y,

$$f(y) = C$$

Setting $C = 0$, we have

$$\phi = xy^2 - \frac{x^3}{3} \qquad \textbf{Ans.}$$

Since the potential function can be established, the flow is irrotational. Thus, the Bernoulli equation can be applied from point A to B. The x and y components of the velocity at these points are

$$u_A = (2^2 - 3^2)\ \text{m/s} = -5\ \text{m/s} \qquad v_A = [2(3)(2)]\ \text{m/s} = 12\ \text{m/s}$$

$$u_B = (3^2 - 1^2)\ \text{m/s} = 8\ \text{m/s} \qquad v_B = [2(1)(3)]\ \text{m/s} = 6\ \text{m/s}$$

Thus, the magnitude of the velocity at these two points is

$$V_A = \sqrt{u_A^2 + v_A^2} = \sqrt{(-5\ \text{m/s})^2 + (12\ \text{m/s})^2} = 13\ \text{m/s}$$

$$V_B = \sqrt{u_B^2 + v_B^2} = \sqrt{(8\ \text{m/s}^2) + (6\ \text{m/s}^2)} = 10\ \text{m/s}$$

Applying the Bernoulli equation for ideal fluid from A to B,

$$\frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_A}{\gamma} + \frac{V_A^2}{2g}$$

$$\frac{p_B}{8(10^3)\ \text{N/m}^3} + \frac{(10\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \frac{600(10^3)\frac{\text{N}}{\text{m}^2}}{8(10^3)\ \text{N/m}^3} + \frac{(13\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}$$

$$p_B = 628.13(10^3)\frac{\text{N}}{\text{m}^2} = 628\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:

$\phi = xy^2 - \dfrac{x^3}{3}$

$p_B = 628\ \text{kPa}$

7–46. The potential function for a horizontal flow is $\phi = \left(x^3 - 5xy^2\right)\ \mathrm{m^2/s}$, where x and y are in meters. Determine the magnitude of the velocity at point A (5 m, 2 m). What is the difference in pressure between this point and the origin? Take $\rho = 925\ \mathrm{kg/m^3}$.

SOLUTION

We consider ideal fluid flow.
Since the flow is described by the potential function, the flow is definitely irrotational. Therefore, the Bernoulli equation can be applied between any two points.

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(x^3 - 5xy^2\right) = 3x^2 - 5y^2$$

$$v = \frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y}\left(x^3 - 5xy^2\right) = -10xy$$

At point A, $x = 5$ m, $y = 2$ m. Thus,

$$u_A = 3\left(5^2\right) - 5\left(2^2\right) = 55\ \mathrm{m/s} \qquad v_A = -10(5)(2) = -100\ \mathrm{m/s}$$

At the origin O, $x = 0$ and $y = 0$. Thus,

$$u_0 = 3\left(0^2\right) - 5\left(0^2\right) = 0 \qquad v_0 = -10(0)(0) = 0$$

The magnitude of the velocity at A and O is

$$V_A = \sqrt{u_A^2 + v_A^2} = \sqrt{(55\ \mathrm{m/s})^2 + (-100\ \mathrm{m/s})^2} = 114.13\ \mathrm{m/s} = 114\ \mathrm{m/s} \quad \textbf{Ans.}$$

$$V_0 = 0$$

Since the flow occurs in the horizontal plane, no change in elevation takes place. Thus, the elevation term can be excluded. Applying the Bernoulli equation for an ideal fluid from O to A,

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} = \frac{p_O}{\rho} + \frac{V_O^2}{2}$$

$$p_O - p_A = \frac{\rho}{2}\left(V_A^2 - V_O^2\right)$$

$$= \left(\frac{925\ \mathrm{kg/m^3}}{2}\right)\left[(114.13\ \mathrm{m/s})^2 - 0^2\right]$$

$$= 6.024\left(10^6\right)\ \mathrm{Pa} = 6.02\ \mathrm{MPa} \quad \textbf{Ans.}$$

Ans:
$V_A = 114\ \mathrm{m/s}$
$p_O - p_A = 6.02\ \mathrm{MPa}$

7–47. A fluid has velocity components of $u = (10xy)$ m/s and $v = 5\left(x^2 - y^2\right)$ m/s, where x and y are in meters. Determine the potential function, and show that the continuity condition is satisfied and that the flow is irrotational.

SOLUTION

We consider ideal fluid flow.

$$u = \frac{\partial \phi}{\partial x}; \qquad 10xy = \frac{\partial \phi}{\partial x}$$

Integrating with respect to x,

$$\phi = 5x^2 y + f(y)$$

Substituting this result into the second of Eq. (8–12),

$$v = \frac{\partial \phi}{\partial y}; \qquad 5\left(x^2 - y^2\right) = \frac{\partial}{\partial y}\left[5x^2 y + f(y)\right]$$

$$5x^2 - 5y^2 = 5x^2 + \frac{\partial}{\partial y}\left[f(y)\right]$$

$$\frac{\partial}{\partial y}\left[f(y)\right] = -5y^2$$

Integrating with respect to x,

$$f(y) = -\frac{5}{3}y^3 + C$$

Setting $C = 0$, then

$$\phi = 5x^2 y + \left(-\frac{5}{3}y^3\right)$$

$$= \frac{5}{3}y\left(3x^2 - y^2\right) \qquad \textbf{Ans.}$$

Here,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(10xy) = (10y)\ \mathrm{s}^{-1}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left[5\left(x^2 - y^2\right)\right] = (-10y)\ \mathrm{s}^{-1}$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left[5\left(x^2 - y^2\right)\right] = (10x)\ \mathrm{s}^{-1}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(10xy) = (10x)\ \mathrm{s}^{-1}$$

Then,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 10y + (-10y) = 0$$

The flow field **satisfies the continuity condition.** Applying,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(10x - 10x) = 0$$

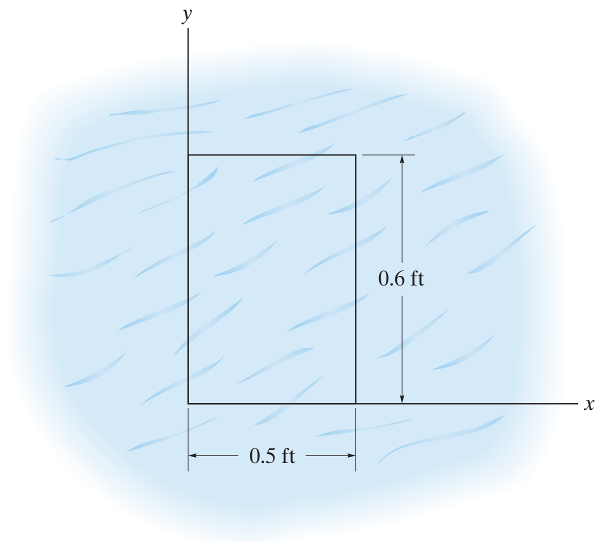
The flow field is **irrotational** since $\omega_z = 0$.

Ans:

$\phi = \frac{5}{3}y\left(3x^2 - y^2\right)$

$\psi = \frac{1}{2}\left(y^2 - x^2\right)$

***7–48.** A velocity field is defined as $u = 2(x^2 + y^2)$ ft/s, $v = (-4xy)$ ft/s. Determine the stream function and the circulation around the rectangle shown. Plot the streamlines for $\psi_0 = 0$, $\psi_1 = 1\ \text{ft}^2/\text{s}$, and $\psi_2 = 2\ \text{ft}^2/\text{s}$.



SOLUTION

We consider ideal fluid flow.

Since the continuity equation $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} = 4x + (-4x) = 0$ is satisfied, the stream function can be established. Using the definition of the velocity components, with respect to stream function,

$$\frac{\partial \psi}{\partial y} = u; \qquad \frac{\partial \psi}{\partial y} = 2(x^2 + y^2)$$

Integrating this equation with respect to y,

$$\psi = 2\left(x^2 y + \frac{1}{3}y^3\right) + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \qquad -\frac{\partial \psi}{\partial x} = -4xy$$

$$-\frac{\partial}{\partial x}\left[2\left(x^2 y + \frac{1}{3}y^3\right) + f(x)\right] = -4xy$$

$$4xy + \frac{\partial}{\partial x}[f(x)] = 4xy$$

$$\frac{\partial}{\partial x}[f(x)] = 0$$

Integrating this equation with respect to x,

$$f(x) = C$$

Substituting this result into Eq. (1),

$$\psi = 2\left(x^2 y + \frac{1}{3}y^3\right) + C$$

C is an arbitary constant. If we set it equal to zero then the stream function can be expressed as

$$\psi = 2y\left(x^2 + \frac{1}{3}y^2\right) \qquad \textbf{Ans.}$$

***7–48. Continued**

For $\psi = 0$, $\quad 0 = 2y\left(x^2 + \dfrac{y^2}{3}\right)$ $\quad$ since $x^2 + \dfrac{y^2}{3} \neq 0$, then

$$y = 0$$

For $\psi = 1\ \mathrm{ft}^2/\mathrm{s}$,

$$1 = 2y\left(x^2 + \frac{y^2}{3}\right)$$

$$x^2 = \frac{3 - 2y^3}{6y} \qquad 0 < y < 1.145$$

For $\psi = 2\ \mathrm{ft}^2/\mathrm{s}$,

$$2 = 2y\left(x^2 + \frac{y^2}{3}\right)$$

$$x^2 = \frac{6 - 2y^3}{6y} \qquad 0 < y < 1.442$$

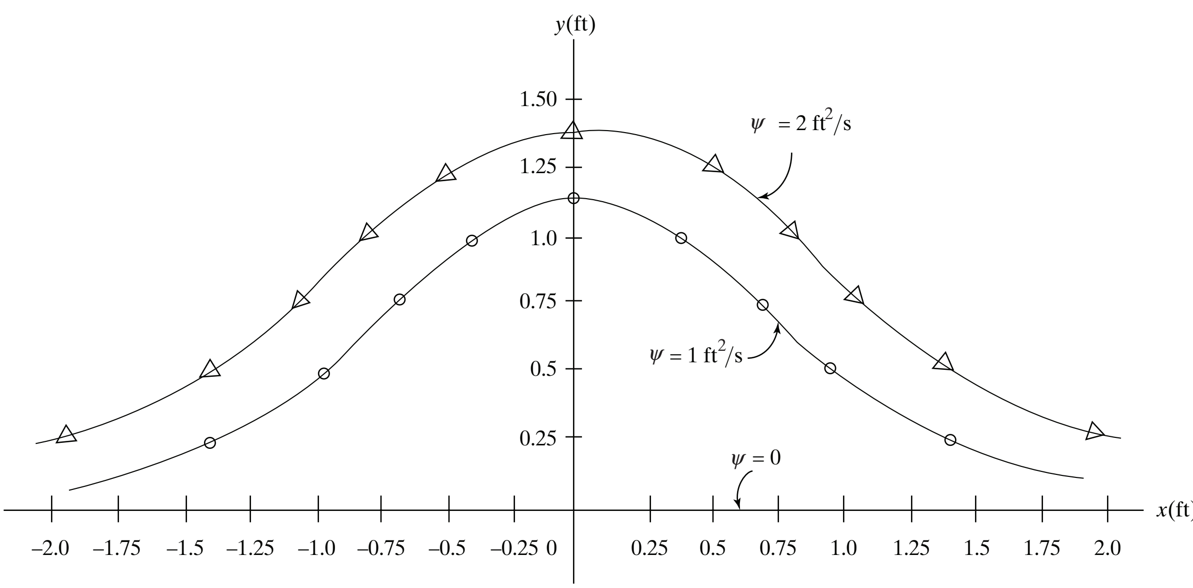
The plot of these streamlines are shown in Fig. *a*.

For $\psi = 1\ \mathrm{ft}^2/\mathrm{s}$

y(ft)	0	0.25	0.50	0.75	1.00
x(ft)	$\pm\infty$	± 1.407	± 0.957	± 0.692	± 0.408
y(ft)	1.145				
x(ft)	0				

For $\psi = 2\ \mathrm{ft}^2/\mathrm{s}$

y(ft)	0	0.25	0.50	0.75	1.00	1.25
x(ft)	$\pm\infty$	± 1.995	± 1.384	± 1.070	± 0.816	± 0.528
y(ft)	1.442					
x(ft)	0					



(a)

***7–48. Continued**

The circulation can be determined using

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s}$$

$$= \int_0^{0.5\text{ ft}} u\,dx + \int_0^{0.6\text{ ft}} v\,dy + \int_0^{0.5\text{ ft}} u(-dx) + \int_0^{0.5\text{ ft}} v(-dy)$$

$$= \int_0^{0.5\text{ ft}} 2\left(x^2 + 0\right)dx + \int_0^{0.6\text{ ft}} -4(0.5)y\,dy + \int_0^{0.5\text{ ft}} 2\left(x^2 + 0.6^2\right)(-dx)$$

$$+ \int_0^{0.6\text{ ft}} -4(0)y(-dy)$$

$$= \frac{2}{3}x^3\Big|_0^{0.5\text{ ft}} - y^2\Big|_0^{0.6\text{ ft}} - \left(\frac{2}{3}x^3 + 0.72x\right)\Big|_0^{0.5\text{ ft}} + 0$$

$$= -0.72\ \text{ft}^2/\text{s}$$ **Ans.**

7–49. If the potential function for a two-dimensional flow is $\phi = (xy)\ \text{m}^2/\text{s}$, where x and y are in meters, determine the stream function, and plot the streamline that passes through the point (1 m, 2 m). What are the x and y components of the velocity and acceleration of fluid particles that pass through this point?

SOLUTION

We consider ideal fluid flow.
Using the velocity components,

$$u = \frac{\partial \phi}{\partial x} = y \quad v = \frac{\partial \phi}{\partial y} = x$$

Since $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(1 - 1) = 0$, the flow is indeed irrotational. Also, since the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$ is satisfied, the establishment of a stream function is possible,

$$\frac{\partial \psi}{\partial y} = u; \qquad \frac{\partial \psi}{\partial y} = y$$

Integrating this equation with respect to y,

$$\psi = \frac{1}{2}y^2 + f(x) \qquad \textbf{(1)}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \qquad -\frac{\partial}{\partial x}\left[\frac{1}{2}y^2 + f(x)\right] = x$$

$$\frac{\partial}{\partial x}[f(x)] = -x$$

Integrating this equation with respect to x

$$f(x) = -\frac{1}{2}x^2 + C$$

Setting $C = 0$ and substituting this result into Eq. 1,

$$\psi = \frac{1}{2}y^2 - \frac{1}{2}x^2$$

$$\psi = \frac{1}{2}\left(y^2 - x^2\right) \qquad \textbf{Ans.}$$

For the streamline passing through point (1 m, 2 m)

$$\psi = \frac{1}{2}\left(2^2 - 1^2\right) = \frac{3}{2}$$

Thus,

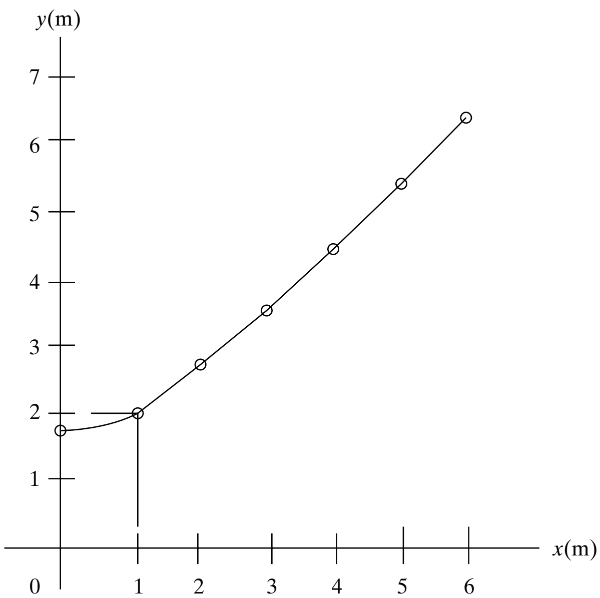
$$\frac{3}{2} = \frac{1}{2}\left(y^2 - x^2\right)$$

$$y^2 = x^2 + 3$$

7–49. Continued

The plot of this streamline is shown in Fig. *a*

x(m)	0	1	2	3	4	5	6
y(m)	1.73	2	2.65	3.46	4.36	5.29	6.24



(a)

At point (1 m, 2 m), the velocity components are

$$u = 2\ \text{m/s} \quad v = 1\ \text{m/s}$$ **Ans.**

The acceleration components are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + y(0) + x(1) = x = 1\ \text{m/s}^2$$ **Ans.**

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + y(1) + x(0) = y = 2\ \text{m/s}^2$$ **Ans.**

Ans:
$u = 2\ \text{m/s}, v = 1\ \text{m/s}$
$a_x = 1\ \text{m/s}^2$
$a_y = 2\ \text{m/s}^2$

7–50. A two-dimensional flow is described by the potential function $\phi = \left(8x^2 - 8y^2\right)\ \mathrm{m^2/s}$, where x and y are in meters. Show that the continuity condition is satisfied, and determine if the flow is rotational or irrotational. Also, establish the stream function for this flow, and plot the streamline that passes through point (1 m, 0.5 m).

SOLUTION

We consider ideal fluid flow.
Here

$$\frac{\partial \phi}{\partial x} = 16x \qquad \frac{\partial^2 \phi}{\partial x^2} = 16$$

$$\frac{\partial \phi}{\partial y} = -16y \qquad \frac{\partial^2 \phi}{\partial y^2} = -16$$

Since $\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} = 16+(-16) = 0$, the potential function ϕ satisfies the continuity condition.

The velocity components can be determined using

$$u = \frac{\partial \phi}{\partial x}; \qquad u = (16x)\ \mathrm{m/s}$$

$$v = \frac{\partial \phi}{\partial y}; \qquad v = (-16y)\ \mathrm{m/s}$$

Then

$$\frac{\partial v}{\partial x} = 0 \qquad \frac{\partial u}{\partial y} = 0$$

Thus

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\right) = 0$$

Since $\omega_z = 0$, the flow is indeed irrotational since all flows that can be described by a potential function are irrotational. Using the definition of velocity components with respect to the stream function,

$$\frac{\partial \psi}{\partial y} = u; \qquad \frac{\partial \psi}{\partial y} = 16x$$

Integrating this equation with respect to y,

$$\psi = 16xy + f(x) \tag{1}$$

Also,

$$-\frac{\partial \psi}{\partial x} = v; \qquad -\frac{\partial}{\partial x}\left[16xy + f(x)\right] = -16y$$

$$-16y - \frac{\partial}{\partial x}\left[f(x)\right] = -16y$$

$$\frac{\partial}{\partial x}\left[f(x)\right] = 0$$

7–50. Continued

Integrating this equation with respect to x

$$f(x) = C$$

Setting $C = 0$ and substituting this result into Eq. 1,

$$\psi = 16xy \qquad \textbf{Ans.}$$

For the streamline passing through point (1 m, 0.5 m),

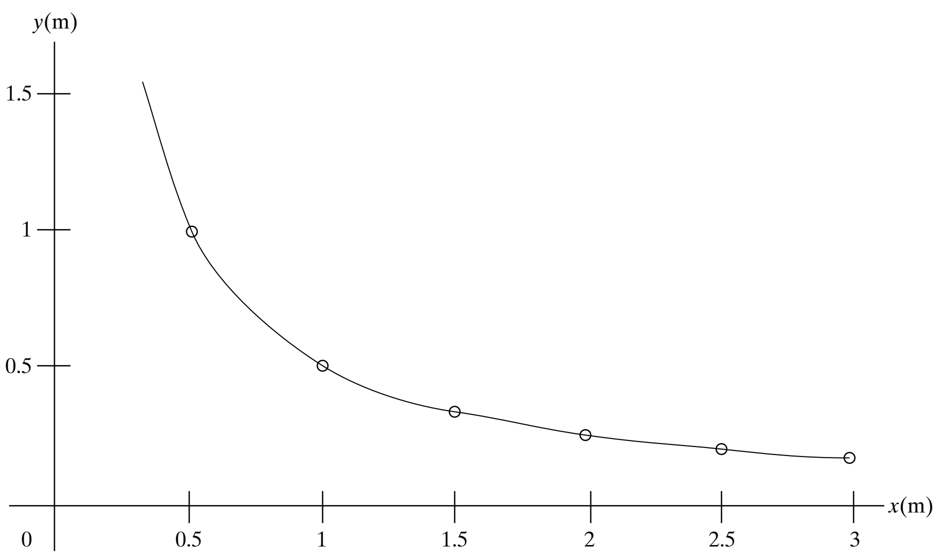
$$\psi = 16(1)(0.5) = 8$$

Thus,

$$8 = 16xy; \qquad y = \frac{1}{2x}$$

The plot of this stream function is shown in Fig. *a*

x(m)	0	0.5	1	1.5	2	2.5	3
y(m)	∞	1	0.5	0.333	0.25	0.2	0.167



(a)

Ans:
$\psi = 16xy$

7–51. The y component of velocity of a two-dimensional irrotational flow that satisfies the continuity condition is $v = \left(4x + x^2 - y^2\right)$ ft/s, where x and y are in feet. Find the x component of velocity if $u = 0$ at $x = y = 0$.

SOLUTION

We consider ideal fluid flow.
In order for the flow to be irrotational, $\omega_z = 0$.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

Here,

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(4x + x^2 - y^2\right) = (4 + 2x) \text{ rad/s}$$

Thus,

$$\frac{1}{2}\left[(4 + 2x) - \frac{\partial u}{\partial y}\right] = 0$$

$$\frac{\partial u}{\partial y} = 4 + 2x$$

Integrating with respect to y,

$$u = 4y + 2xy + f(x)$$

In order to satisfy the continuity condition

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Here,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}[4y + 2xy + f(x)] = 2y + \frac{\partial}{\partial x}[f(x)]$$

$$\frac{\partial v}{\partial y} + \frac{\partial}{\partial y}\left(4x + x^2 - y^2\right) = -2y$$

Then,

$$2y + \frac{\partial}{\partial x}[f(x)] - 2y = 0$$

$$\frac{\partial}{\partial x}[f(x)] = 0$$

Integrating with respect to x,

$$f(x) = C$$

Thus,

$$u = 4y + 2xy + C$$

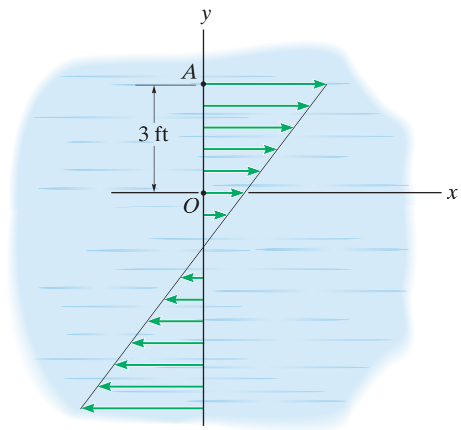
$$= 2y(2 + x) + C$$

At $y = x = 0, u = 0$. Then $C = 0$, and so

$$u = 2y(2 + x) \qquad \textbf{Ans.}$$

Ans:
$u = 2y(2 + x)$

***7–52.** The flow has a velocity of $\mathbf{V} = \{(3y + 8)\mathbf{i}\}$ ft/s, where y is vertical and is in feet. Determine if the flow is rotational or irrotational. If the pressure at point A is 6 lb/ft^2, determine the pressure at the origin. Take $\gamma = 70$ lb/ft^3.



SOLUTION

We consider ideal fluid flow.
The x and y components of velocity are

$$u = (3y + 8)\text{ ft/s} \qquad v = 0$$

Here,

$$\frac{\partial u}{\partial x} = 0; \qquad \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(3y + 8) = 3\text{ rad/s}$$

Thus,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 3) = -1.5$$

Since $\omega_z \neq 0$, the flow is rotational. Thus, the Bernoulli equation can not be applied from O to A. Instead, we will first apply Euler's equation along the y axis. Here, $\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 0$.

Then,

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} - g = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0$$

$$\frac{\partial p}{\partial y} = -\rho g = -\gamma$$

Integrating with respect to y,

$$p = -\gamma y + f(x)$$

Substituting this result into the Euler equation along the x axis, with $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(3y + 8) = 0$ and $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(3y + 8) = 3$ rad/s,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$-\frac{1}{\rho}\frac{\partial}{\partial x}[-\gamma y + f(x)] = 0 + 0$$

$$\frac{\partial}{\partial x}[f(x)] = 0$$

7–52. Continued

Integrating with respect to x,

$$f(x) = C$$

Thus,

$$p = -\gamma y + C$$

At point A, $y = 3$ ft and $p = 6\dfrac{\text{lb}}{\text{ft}^2}$. Then,

$$6\frac{\text{lb}}{\text{ft}^2} = \left(-70\frac{\text{lb}}{\text{ft}^2}\right)(3\text{ft}) + C$$

$$C = 216\ \text{lb/ft}^2$$

Thus,

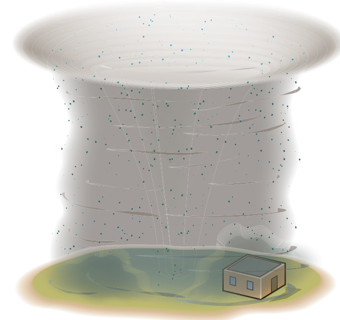
$$p = (-\gamma y + 216)\ \text{lb/ft}^2$$

At point O, $y = 0$ Thus,

$$p_O = [-(70)(0) + 216]\frac{\text{lb}}{\text{ft}^2}$$

$$= 216\frac{\text{lb}}{\text{ft}^2} \qquad \textbf{Ans.}$$

7–53. A tornado has a measured wind speed of 12 m/s a distance of 50 m from its center. If a building has a flat roof and is located 10 m from the center, determine the uplift pressure on the roof. The building is within the free vortex of the tornado. The density of the air is $\rho_a = 1.20\ \text{kg/m}^3$.



SOLUTION

We consider ideal fluid flow.
Since the tornado is a free vortex flow, its velocity components are

$$v_r = 0 \qquad v_\theta = \frac{k}{r}$$

Thus

$$V = v_\theta = \frac{k}{r}$$

It is required that at $r = 50$ m, $V = 12$ m/s. Therefore

$$12\ \text{m/s} = \frac{\text{k}}{50\ \text{m}}; \qquad k = 600\ \text{m}^2/\text{s}$$

Then

$$V = \left(\frac{600}{r}\right)\ \text{m/s}$$

At $r = 10$ m,

$$V = \frac{600}{10} = 60\ \text{m/s}$$

Since free vortex flow is irrotational, Bernoulli's equation can be applied between two points on the different streamlines such as two points on two circular streamlines of radius $r = \infty$ and $r = 10$ m. At $r = \infty$, $V_\infty = 0$ and $p_B = 0$. Since the flow occurs in the horizontal plane, the gravity term can be excluded.

$$\frac{p_\infty}{\rho_a} + \frac{V_\infty^{\ 2}}{2} = \frac{p}{\rho_a} + \frac{V^2}{2}$$

$$0 + 0 = \frac{p}{1.20\ \text{kg/m}^3} + \frac{(60\ \text{m/s})^2}{2}$$

$$p = 2160\ \text{Pa} = -2.16\ \text{kPa}$$ **Ans.**

The negative sign indicates that suction develops.

Ans:
-2.16 kPa

7–54. Show that the equation that defines a sink will satisfy continuity, which in polar coordinates is written as $\frac{\partial(v_r r)}{\partial r} + \frac{\partial(v_\theta)}{\partial \theta} = 0$.

SOLUTION

We consider ideal fluid flow.

For sink flow, $v_r = -\frac{q}{2\pi r}$ and $v_\theta = 0$. Then,

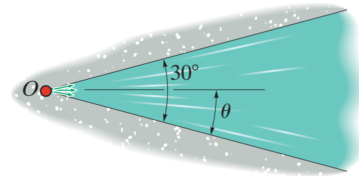
$$\frac{\partial(v_r r)}{\partial r} = \frac{\partial}{\partial r}\left[\left(-\frac{q}{2\pi r}\right)(r)\right] = \frac{\partial}{\partial r}\left(-\frac{q}{2\pi}\right) = 0$$

$$\frac{\partial v_\theta}{\partial \theta} = 0$$

Thus,

$$\frac{\partial(v_r r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = 0 + 0 = 0 \qquad \textbf{(Q.E.D)}$$

7–55. A source at O creates a flow from point O that is described by the potential function $\phi = (8 \ln r)\ \text{m}^2/\text{s}$, where r is in meters. Determine the stream function, and specify the velocity at point $r = 5$ m, $\theta = 15°$.



SOLUTION

We consider ideal fluid flow.
The r and θ components of velocity are

$$v_r = \frac{\partial \phi}{\partial r} = \frac{\partial}{\partial r}(8 \ln r) = \left(\frac{8}{r}\right) \text{m/s}$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial \theta}(8 \ln r) = 0$$

Applying,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}; \qquad \frac{8}{r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

Integrating this equation with respect to θ,

$$\psi = 8\theta + f(r)$$

Substituting this result into the second of Eq. 8–10,

$$v_\theta = -\frac{\partial \psi}{\partial r}; \qquad 0 = -\frac{\partial}{\partial r}[8\theta + f(r)]$$

$$0 = 0 - \frac{\partial}{\partial r}[f(r)]$$

Integrating this equation with respect to r,

$$f(r) = C$$

Thus,

$$\psi = 8\theta + C$$

Setting $C = 0$,

$$\psi = 8\theta \qquad \textbf{Ans.}$$

At $r = 5$ m, $\theta = 15°$,

$$v_r = \frac{8}{5} = 1.6 \text{ m/s}$$

$$v_\theta = 0$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(1.6 \text{ m/s})^2 + (0)^2} = 1.60 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\psi = 8\theta$
$V = 1.60 \text{ m/s}^2$

***7–56.** Combine a source of strength q with a free counterclockwise vortex, and sketch the resultant streamline for $\psi = 0$.

SOLUTION

We consider ideal fluid flow.
Superimposing the streamlines of a source and a free vortex,

$$\psi = \frac{q}{2\pi}\theta - k\ln r$$

For $\psi = 0$,

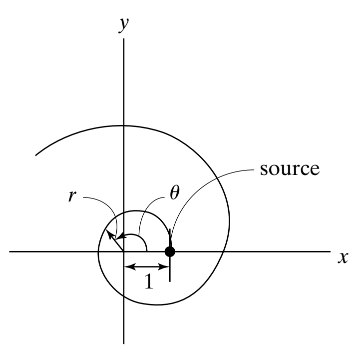
$$0 = \frac{q}{2\pi}\theta - k\ln r$$

$$\ln r = \frac{q}{2\pi k}\theta$$

$$e^{\ln r} = e^{\frac{q}{2\pi k}\theta}$$

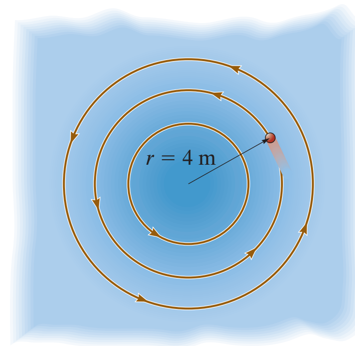
$$r = e^{\frac{q}{2\pi k}\theta}$$

This equation represents a logarithmic spiral from the source and its plot is shown in Fig. *a*.



(a)

7–57. A free vortex is defined by its stream function $\psi = (-240 \ln r)\ \mathrm{m^2/s}$, where r is in meters. Determine the velocity of a particle at $r = 4$ m and the pressure at the points on this streamline. Take $\rho = 1.20\ \mathrm{kg/m^3}$.



SOLUTION

We consider ideal fluid flow.
The velocity components are

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}; \qquad v_r = 0$$

$$v_\theta = -\frac{\partial\psi}{\partial r}; \qquad v_\theta = \left(\frac{240}{r}\right)\ \mathrm{m/s}$$

Thus, the velocity is

$$V = v_\theta = \left(\frac{240}{r}\right)\ \mathrm{m/s}$$

At $r = 4$ m,

$$V = \left(\frac{240}{4}\right)\ \mathrm{m/s} = 60.0\ \mathrm{m/s} \qquad \textbf{Ans.}$$

Since a free vortex flow is irrotational, Bernoulli's equation can be applied between two points on the different streamlines. In this case, the two points are on the circular streamlines $r = \infty$ where $V_0 = 0$ and $p_0 = 0$ and $r = 4$ m where $V = 60.0$ m/s. Since the flow occurs in the horizontal plane, $z_0 = z$.

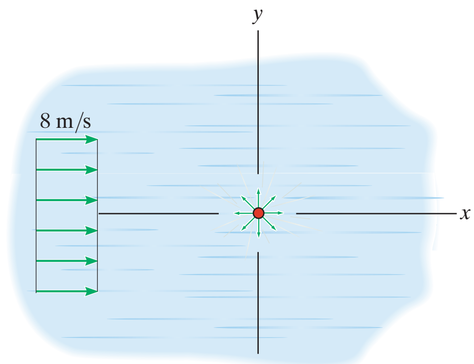
$$\frac{p_0}{\rho} + \frac{V_0^2}{2} + gz_0 = \frac{p}{\rho} + \frac{V^2}{2} + gz$$

$$0 + 0 + gz = \frac{p}{1.20\ \mathrm{kg/m^3}} + \frac{(60.0\ \mathrm{m/s})^2}{2} + gz$$

$$p = -2160\ \mathrm{Pa} = -2.16\ \mathrm{kPa} \qquad \textbf{Ans.}$$

Ans:
$V = 60.0$ m/s
$p = -2.16$ kPa

7–58. Determine the location of the stagnation point for a combined uniform flow of 8 m/s and a source having a strength of 3 m^2/s. Plot the streamline passing through the stagnation point.



SOLUTION

We consider ideal fluid flow.
This is a case of flow past a half body. The location of the stagnation point P is at

$$\theta = \pi \qquad \textbf{Ans.}$$

Using

$$r = r_0 = \frac{q}{2\pi U} = \frac{3\ \text{m}^2/\text{s}}{2\pi(8\ \text{m/s})} = \frac{3}{16\pi}\ \text{m} \qquad \textbf{Ans.}$$

The equation of the streamline (boundary of a half body) that passes through the stagnation point P can be determined by applying.

$$r = \frac{r_0(\pi - \theta)}{\sin\theta}$$

$$r = \frac{\frac{3}{16\pi}(\pi - \theta)}{\sin\theta}$$

$$r = \frac{3(\pi - \theta)}{16\pi \sin\theta}$$

This equation can be written in the form

$$r\sin\theta = \frac{3}{16\pi}(\pi - \theta)$$

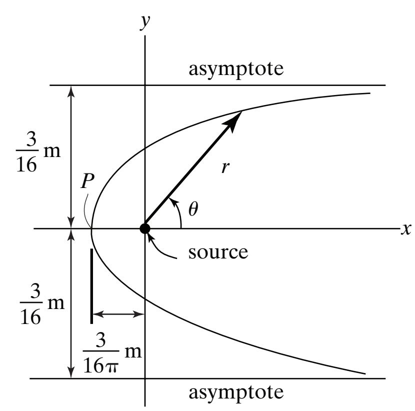
Since $y = r\sin\theta$, this equation becomes

$$y = \frac{3}{16\pi}(\pi - \theta)$$

The half width h of the half body can be determined by setting $y = h$ as θ approaches 0 or 2π. Thus,

$$h = \frac{3}{16\pi}(\pi - \theta) = \frac{3}{16}\ \text{m}$$

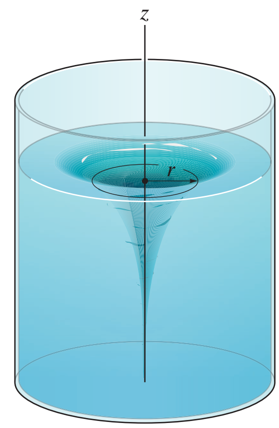
The plot of the half body is shown in Fig. a.



(a)

Ans:
$\theta = \pi$
$r = \frac{3}{16\pi}$ m

7–59. As water drains from the large cylindrical tank, its surface forms a free vortex having a circulation of Γ. Assuming water to be an ideal fluid, determine the equation $z = f(r)$ that defines the free surface of the vortex. *Hint*: Use the Bernoulli equation applied to two points on the surface.



SOLUTION

We consider ideal fluid flow.
For a free vortex, the radial and transverse components of velocity are

$$v_r = 0 \qquad \text{and} \qquad v_\theta = \frac{k}{r}$$

Then

$$V = v_\theta = \frac{k}{r}$$

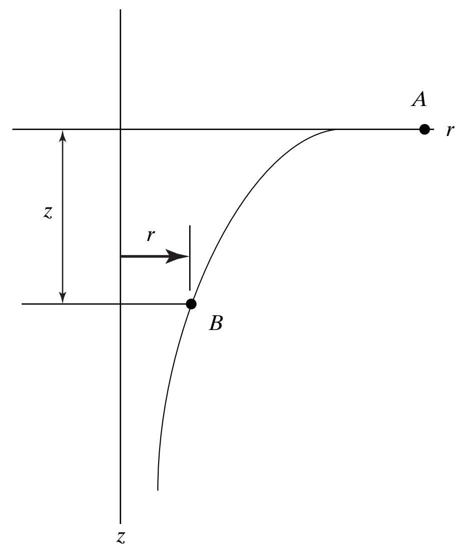
For a circulation Γ,

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s} = \int_0^{2\pi} \frac{\text{k}}{r}(r d\theta) = 2\pi \text{ k}$$

$$k = \frac{\Gamma}{2\pi}$$

Thus,

$$V = \frac{\Gamma}{2\pi r}$$



(a)

Since a free vortex is irrotational flow, Bernoulli's equation can be applied between two points on different streamlines, such as point A and B shown in Fig. *a*. Point A is located at $(r = \infty, 0)$ where $p_A = p_{\text{atm}} = 0$ and $V_A = 0$, and point B is located at (r, z) where $p_B = p_{\text{atm}} = 0$ and $V_B = \dfrac{\Gamma}{2\pi r}$. Establish the datum through point A,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

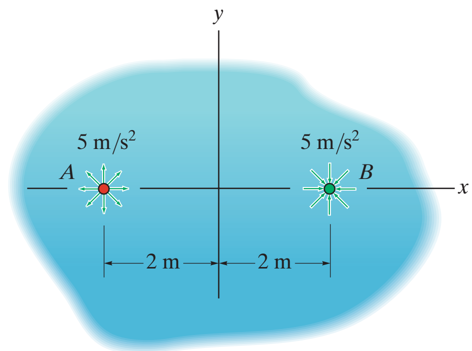
$$0 + 0 + g(0) = 0 + \frac{\left(\dfrac{\Gamma}{2\pi r}\right)^2}{2} + g(-z)$$

$$gz = \frac{\Gamma^2}{8\pi^2 r^2}$$

$$z = \frac{\Gamma^2}{8\pi^2 g r^2} \qquad \textbf{Ans.}$$

Ans:
$z = \dfrac{\Gamma^2}{8\pi^2 g r^2}$

***7–60.** Pipe A provides a source flow of 5 m^2/s, whereas the drain, or sink, at B removes 5 m^2/s. Determine the stream function between AB, and show the streamline for $\psi = 0$.

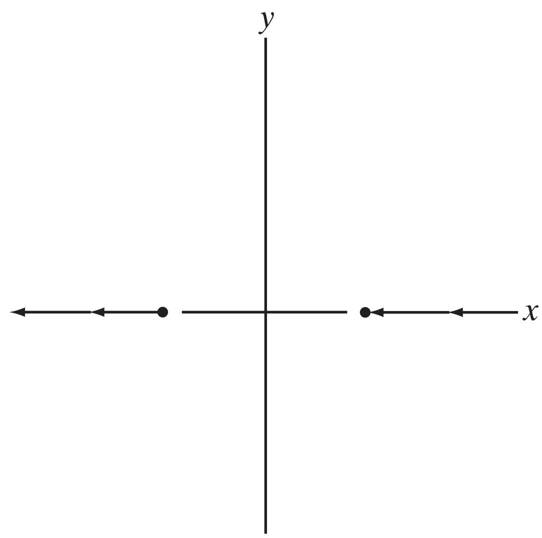


SOLUTION

We consider ideal fluid flow.
When the source and sink are superimposed, Fig. a, the resultant stream function is

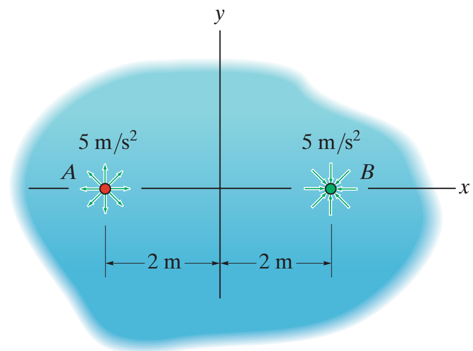
$$\psi = \frac{q}{2\pi}\theta_2 - \frac{q}{2\pi}\theta_1 = \frac{q}{2\pi}(\theta_2 - \theta_1) = \frac{5\ \mathrm{m}^2/\mathrm{s}}{2\pi}(\theta_2 - \theta_1) \qquad \textbf{Ans.}$$

Staying in polar coordinates, $\psi = 0$ implies $\theta_2 - \theta_1 = 0$ or $\theta_2 = \theta_1$. The graph of the corresponding points is shown in Fig. a, with the direction of flow indicated. Note that "the" steamline has two distinct segments.



(a)

7–61. Pipe A provides a source flow of 5 m^2/s, whereas the drain at B removes 5 m^2/s. Determine the potential function between AB, and show the equipotential line for $\phi = 0$.



SOLUTION

We consider ideal fluid flow.
When the source and sink are superimposed, the resultant potential function is

$$\phi = \frac{q}{2\pi}\ln r_2 - \frac{q}{2\pi}\ln r_1 = \frac{q}{2\pi}\ln\frac{r_2}{r_1} = \frac{5\ \mathrm{m^2/s}}{2\pi}\ln\frac{r_2}{r_1} \qquad \textbf{Ans.}$$

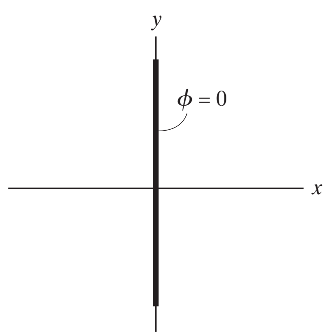
Staying in polar coordinates, $\phi = 0$ implies

$$\ln\frac{r_2}{r_1} = 0$$

$$\frac{r_2}{r_1} = 1$$

$$r_1 = r_2$$

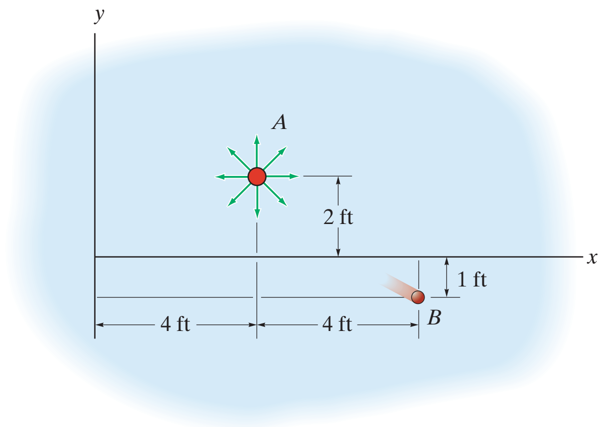
Thus, the equipotential line for $\phi = 0$ is along the y axis as shown in Fig. *a*.



(a)

Ans:
$\phi = \dfrac{5\ \mathrm{m^2/s}}{2\pi}\ln\dfrac{r_2}{r_1}$
The equipotential line for $\phi = 0$ is along the y axis.

7–62. A source having a strength of $q = 80\ \text{ft}^2/\text{s}$ is located at point A (4 ft, 2 ft). Determine the magnitudes of the velocity and acceleration of fluid particles at point B (8 ft, −1 ft).



SOLUTION

We consider ideal fluid flow.
The radial and transverse components of the velocity are

$$v_r = \frac{q}{2\pi r} \qquad v_\theta = 0$$

Thus, the magnitude of the velocity is

$$V = v_r = \frac{q}{2\pi r}$$

Here, $r = \sqrt{(8\text{ ft} - 4\text{ ft})^2 + (-1\text{ ft} - 2\text{ ft})^2} = 5$ ft. Then

$$V = \frac{80\ \text{ft}^2/\text{s}}{2\pi\ (5\text{ ft})} = 2.546\ \text{ft/s} = 2.55\ \text{ft/s}$$ **Ans.**

x any y components of the velocity are

$$u = v_r \cos\theta \qquad v = v_r \sin\theta$$

Here $\cos\theta = \dfrac{x}{r}$ and $\sin\theta = \dfrac{y}{r}$,

$$u = \frac{q}{2\pi r}\left(\frac{x}{r}\right) = \frac{q}{2\pi}\left(\frac{x}{r^2}\right) \quad v = \frac{q}{2\pi r}\left(\frac{y}{r}\right) = \frac{q}{2\pi}\left(\frac{y}{r^2}\right)$$

However, $r^2 = x^2 + y^2$. Then

$$u = \frac{q}{2\pi}\left(\frac{x}{x^2 + y^2}\right) \quad v = \frac{q}{2\pi}\left(\frac{y}{x^2 + y^2}\right)$$

x any y components of the acceleration are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + \frac{q}{2\pi}\left(\frac{x}{x^2 + y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{y^2 - x^2}{(x^2 + y^2)^2}\right]\right\} + \frac{q}{2\pi}\left(\frac{y}{x^2 + y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{-2xy}{(x^2 + y^2)^2}\right]\right\}$$

$$= -\frac{q^2}{4\pi^2}\left[\frac{x^3 + xy^2}{(x^2 + y^2)^3}\right]$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + \frac{q}{2\pi}\left(\frac{x}{x^2 + y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{-2xy}{(x^2 + y^2)^2}\right]\right\} + \frac{q}{2\pi}\left(\frac{y}{x^2 + y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{x^2 - y^2}{(x^2 + y^2)^2}\right]\right\}$$

$$= -\frac{q^2}{4\pi^2}\left[\frac{y^3 + x^2y}{(x^2 + y^2)^3}\right]$$

7–62. Continued

With respect to point A, the coordinates of point B are $B[(8-4)\text{ ft}, (-1-2)\text{ ft}] = B(4\text{ ft}, -3\text{ ft})$. Then

$$a_x = -\frac{\left(80\text{ ft}^2/\text{s}\right)^2}{4\pi^2}\left\{\frac{(4\text{ ft})^3 + (4\text{ ft})(-3\text{ ft})^2}{\left[(4\text{ ft})^2 + (-3\text{ ft})^2\right]^3}\right\} = -1.038\text{ ft/s}^2$$

$$a_y = -\frac{\left(80\text{ ft}^2/\text{s}\right)^2}{4\pi^2}\left\{\frac{(-3\text{ ft})^3 + (4\text{ ft})^2(-3\text{ ft})}{\left[(4\text{ ft})^2 + (-3\text{ ft})^2\right]^3}\right\} = 0.7781\text{ ft/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(-1.038\text{ ft/s}^2\right)^2 + \left(0.7781\text{ ft/s}^2\right)^2} = 1.297\text{ ft/s}^2 = 1.30\text{ ft/s}^2 \qquad \textbf{Ans.}$$

As an alternative solution,

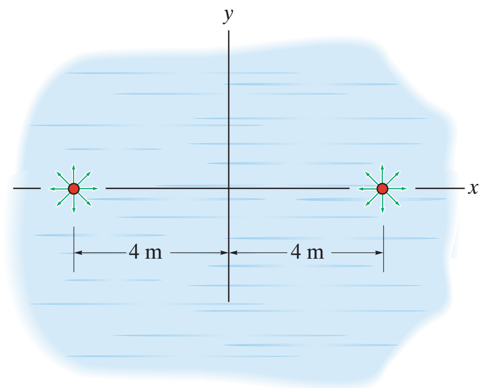
$$a = \frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r}$$

$$= 0 + \frac{q}{2\pi r}\left(\frac{q}{2\pi}\left(-\frac{1}{r^2}\right)\right)$$

$$= \left|\frac{q^2}{(2\pi)^2 r^3}\right| = \frac{\left(80\text{ ft}^2/\text{s}\right)^2}{(2\pi)^2(5\text{ ft})^3} = 1.30\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$V = 2.55\text{ ft/s}$
$a = 1.30\text{ ft/s}^2$

7–63. Two sources, each having a strength of 2 m^2/s, are located as shown. Determine the x and y components of the velocity of fluid particles that pass point (x, y). What is the equation of the streamline that passes through point (0, 8 m) in Cartesian coordinates? Is the flow irrotational?



SOLUTION

We consider ideal fluid flow.
When sources (1) and (2) are superimposed, Fig. *a*, the resultant stream function is

$$\psi = \frac{q}{2\pi}\theta_1 + \frac{q}{2\pi}\theta_2 = \frac{q}{2\pi}\left(\theta_1 + \theta_2\right)$$

From the geometry shown in Fig. *a*,

$$\theta_1 = \tan^{-1}\left(\frac{y}{x-4}\right) \quad \theta_2 = \tan^{-1}\left(\frac{y}{x+4}\right)$$

Then,

$$\psi = \frac{q}{2\pi}\left[\tan^{-1}\left(\frac{y}{x-4}\right) + \tan^{-1}\left(\frac{y}{x+4}\right)\right] \qquad \textbf{(1)}$$

The x and y components of velocity are

$$u = \frac{\partial\psi}{\partial y} = \frac{q}{2\pi}\left[\frac{1}{1+\left(\frac{y}{x-4}\right)^2}\left(\frac{1}{x-4}\right) + \frac{1}{1+\left(\frac{y}{x+4}\right)^2}\left(\frac{1}{x+4}\right)\right]$$

$$= \frac{q}{2\pi}\left[\frac{x-4}{(x-4)^2+y^2} + \frac{x+4}{(x+4)^2+y^2}\right] \qquad \textbf{Ans.}$$

$$v = -\frac{\partial\psi}{\partial y} = -\frac{q}{2\pi}\left[\frac{1}{1+\left(\frac{y}{x-4}\right)^2}\left[-\frac{y}{(x-4)^2}\right] + \frac{1}{1+\left(\frac{y}{x+4}\right)^2}\left[-\frac{y}{(x+4)^2}\right]\right]$$

$$= \frac{q}{2\pi}\left[\frac{y}{(x-4)^2+y^2} + \frac{y}{(x+4)^2+y^2}\right] \qquad \textbf{Ans.}$$

Here,

$$\frac{dv}{dx} = \frac{q}{2\pi}\left[\frac{2y(x-4)}{\left[(x-4)^2+y^2\right]^2} + \frac{2y(x+4)}{\left[(x+4)^2+y^2\right]^2}\right]$$

$$\frac{du}{dy} = \frac{q}{2\pi}\left[\frac{2y(x-4)}{\left[(x-4)^2+y^2\right]^2} + \frac{2y(x+4)}{\left[(x+4)^2+y^2\right]^2}\right]$$

Substituting these results into

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

7–63. Continued

Since $\omega_z = 0$, the flow is **irrotational**. For $q = 2\ \text{m}^2/\text{s}$, the streamline that passes through point $x = 0$ and $y = 8$ m can be determined using Eq. 1,

$$\psi = \frac{2}{2\pi}\left[\tan^{-1}\left(\frac{8}{0-4}\right) + \tan^{-1}\left(\frac{8}{0+4}\right)\right] = 0$$

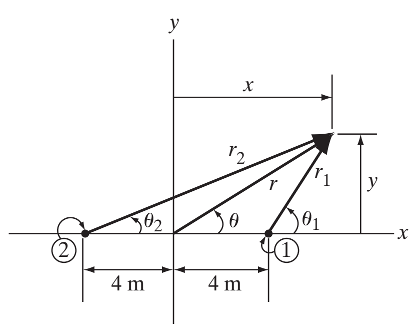
Thus, the equation of this streamline is

$$\tan^{-1}\left(\frac{y}{x-4}\right) + \tan^{-1}\left(\frac{y}{x+4}\right) = 0$$

$$\tan^{-1}\left(\frac{y}{x-4}\right) = -\tan^{-1}\left(\frac{y}{x+4}\right)$$

$$\frac{y}{x-4} = -\frac{y}{x+4}$$

$$x = 0 \qquad \textbf{Ans.}$$



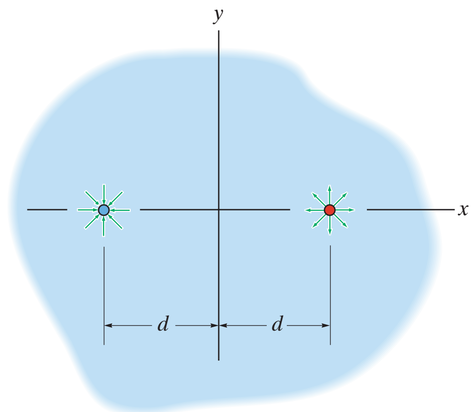
Ans:

$$u = \frac{q}{2\pi}\left[\frac{x-4}{\left(x-4^2\right)+y^2} + \frac{x+4}{\left(x+4^2\right)+y^2}\right]$$

$$v = \frac{q}{2\pi}\left[\frac{y}{\left(x-4^2\right)+y^2} + \frac{y}{\left(x+4^2\right)+y^2}\right]$$

The flow is irrotational.
$x = 0$

***7–64.** The source and sink of equal strength q are located a distance d from the origin as indicated. Determine the stream function for the flow, and draw the streamline that passes through the origin.



SOLUTION

We consider ideal fluid flow.
When the source and sink are superimposed, Fig. *a*, the resultant stream function is

$$\psi = \frac{q}{2\pi}\theta_1 - \frac{q}{2\pi}\theta_2 = \frac{q}{2\pi}\left(\theta_1 - \theta_2\right) \qquad \textbf{Ans.}$$

Staying in polar coordinates, at the origin $\theta_1 = \pi$ and $\theta_2 = 0$.

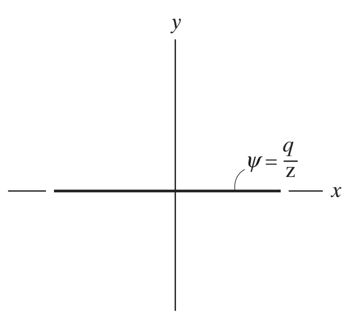
$$\psi = \frac{q}{2\pi}\left(\pi - 0\right) = \frac{q}{2}$$

So the streamline satisfies

$$\frac{q}{2} = \frac{q}{2\pi}\left(\theta_1 - \theta_2\right)$$

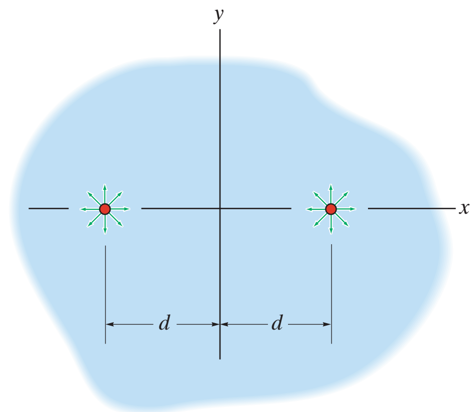
$$\theta_1 = \theta_2 + \pi$$

Thus, the streamline passing through the origin is the straight segment between the source and the sink, as shown in Fig. *a*.



(a)

7–65. Two sources, each having a strength q, are located as shown. Determine the stream function, and show that this is the same as having a single source with a wall along the y axis.

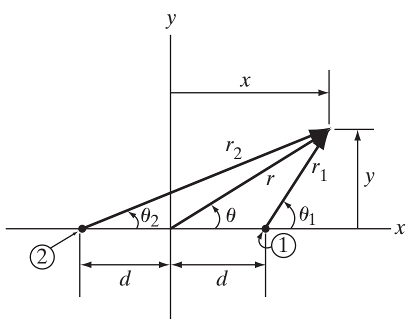


SOLUTION

We consider ideal fluid flow.
When sources (1) and (2) are superimposed, Fig. a, the resultant stream function is

$$\psi = \frac{q}{2\pi}\theta_1 + \frac{q}{2\pi}\theta_2 = \frac{q}{2\pi}(\theta_1 + \theta_2) \qquad \textbf{Ans.}$$

In order for the stream function to be the same as that of a single source and a wall along the y axis, a streamline must exist along the y axis. However, by geometry, along the y axis it is always true that $\theta_1 + \theta_2 = \pm\pi$, so that the value of the stream function is $\psi = \frac{q}{2\pi}(\pm\pi) \pm \frac{q}{2}$, where $+\frac{q}{2}$ corresponds to the $+y$ axis and $-\frac{q}{2}$ corresponds to the $-y$ axis.

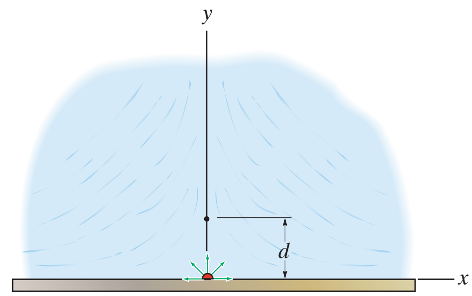


(a)

Ans:

$\psi = \frac{q}{2\pi}(\theta_1 + \theta_2)$

7–66. A source q is emitted from the wall while a flow occurs towards the wall. If the stream function is described as $\psi = (4xy + 8\theta)\ \mathrm{m^2/s}$, where x and y are in meters, determine the distance d from the wall where the stagnation point occurs along the y axis. Plot the streamline that passes through this point.



SOLUTION

We consider ideal fluid flow.
Here, $x = r\cos\theta$ and $y = r\sin\theta$. Then in terms of r and θ coordinates,

$$\psi = 4(r\cos\theta)(r\sin\theta) + 8\theta$$

$$\psi = 2r^2\sin 2\theta + 8\theta \qquad \textbf{(1)}$$

The velocity components are

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left[2r^2(2\cos 2\theta) + 8\right] = \frac{1}{r}\left(4r^2\cos 2\theta + 8\right)$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -(4r\sin 2\theta)$$

At stagnation point p, it is required that these velocity components are equal to zero.

$$v_\theta = -4r\sin 2\theta = 0$$

$$\sin 2\theta = 0 \quad (\text{since } r \neq 0)$$

$$2\theta = 0, \quad \pi\ \text{rad}$$

$$\theta = 0, \quad \frac{\pi}{2}\ \text{rad}$$

$\theta = \dfrac{\pi}{2}$ rad is chosen and it gives the direction r of the stagnation point.

$$v_r = \frac{1}{r}\left(4r^2\cos 2\theta + 8\right) = 0$$

Since $\dfrac{1}{r} \neq 0$, then

$$4r^2\cos 2\theta + 8 = 0$$

Substituting $\theta = \dfrac{\pi}{2}$ rad and $r = d$ into this equation,

$$4d^2\cos\left[2\left(\frac{\pi}{2}\right)\right] + 8 = 0$$

$$d = \sqrt{2}\ \text{m} \qquad \textbf{Ans.}$$

Substituting $\theta = \dfrac{\pi}{2}$ rad and $r = \sqrt{2}$ m into Eq 1,

$$\psi = 2(\sqrt{2})^2\sin\left[2\left(\frac{\pi}{2}\right)\right] + 8\left(\frac{\pi}{2}\right) = 4\pi$$

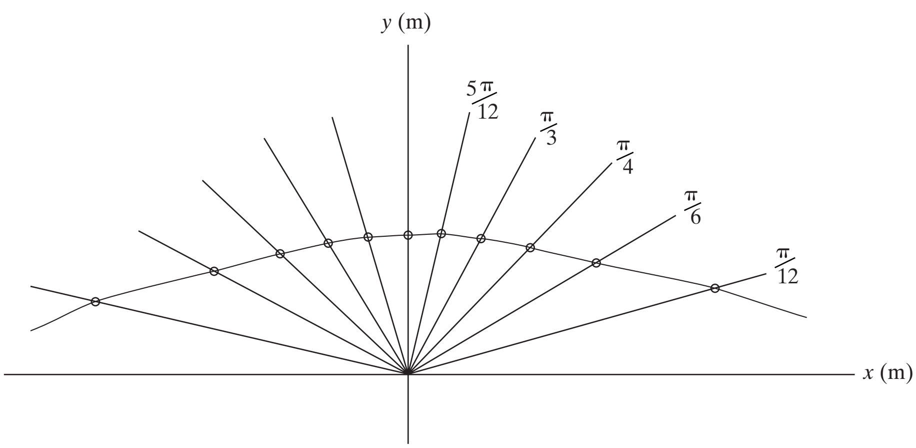
7–66. Continued

Therefore, the streamline passing through the stagnation point is given by

$$4\pi = 2r^2 \sin 2\theta + 8\theta \qquad r^2 = \frac{4\pi - 8\theta}{2 \sin 2\theta}$$

The plot of this stream function is shown in Fig. *a*

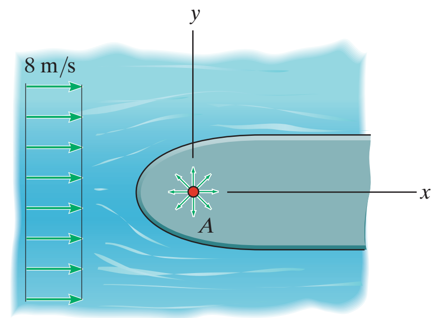
θ(rad)	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
r(m)	3.236	2.199	1.772	1.555	1.447	undef.



(a)

Ans:
$\sqrt{2}$ m

7–67. Determine the equation of the boundary of the half body formed by placing a source of 0.5 m^2/s in the uniform flow of 8 m/s. Express the result in Cartesian coordinates.



SOLUTION

We consider ideal fluid flow.
For the flow past a half body,

$$r_0 = \frac{q}{2\pi U} = \frac{0.5\ \text{m}^2/\text{s}}{2\pi(8\ \text{m/s})} = \frac{0.03125}{\pi}\ \text{m}$$

The equation of the boundary of a half body is given by

$$r = \frac{r_0(\pi - \theta)}{\sin\theta} = \frac{\frac{0.03125}{\pi}(\pi - \theta)}{\sin\theta}$$

$$r\sin\theta = \frac{0.03125}{\pi}(\pi - \theta)$$

Here, $y = r\sin\theta$ and $\theta = \tan^{-1}\frac{y}{x}$. Then, this equation becomes

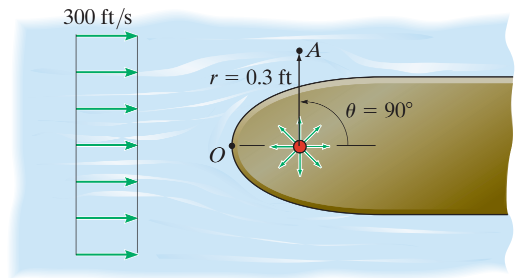
$$32\pi y = \pi - \tan^{-1}\frac{y}{x}$$

$$\tan^{-1}\frac{y}{x} = \pi(1 - 32y)$$

$$\frac{y}{x} = \tan\left[\pi(1 - 32y)\right] \qquad \textbf{Ans.}$$

Ans:
$y = x\tan\left[\pi(1 - 32y)\right]$

***7–68.** The leading edge of a wing is approximated by the half body. It is formed from the superposition of the uniform air flow of 300 ft/s and a source. Determine the required strength of the source so that the width of the half body is 0.4 ft.



SOLUTION

We consider ideal fluid flow.
For the flow past a half body,

$$r_0 = \frac{q}{2\pi U} = \frac{q}{2\pi(300\text{ ft/s})} = \frac{q}{600\pi}\text{ ft}$$

The equation of the boundary of a half body is given by

$$r = \frac{r_0(\pi - \theta)}{\sin\theta}$$

$$r = \frac{\frac{q}{600\pi}(\pi - \theta)}{\sin\theta} = \frac{q(\pi - \theta)}{600\pi\sin\theta}$$

$$r\sin\theta = \frac{q(\pi - \theta)}{600\pi}$$

Since $y = r\sin\theta$, this equation becomes

$$y = \frac{q(\pi - \theta)}{600\pi}$$

The half width h of the half body can be determined by setting $y = h$ as θ approaches 0 or 2π. Thus,

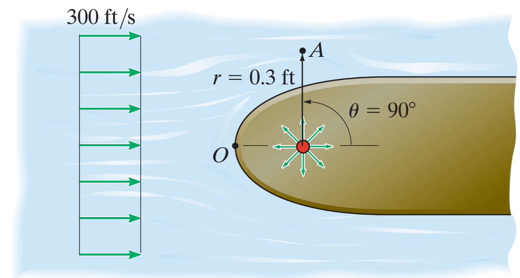
$$h = \frac{q(\pi - \theta)}{600\pi} = \frac{q}{600}$$

Here, $h = \dfrac{0.4\text{ ft}}{2} = 0.2\text{ ft}$. Then

$$0.2\text{ ft} = \frac{q}{600}$$

$$q = 120\text{ ft}^2/\text{s} \qquad \textbf{Ans.}$$

7–69. The leading edge of a wing is approximated by the half body. It is formed from the superposition of the uniform air flow of 300 ft/s and a source having a strength of 100 ft^2/s. Determine the width of the half body and the difference in pressure between the stagnation point O and point A, where $r = 0.3$ ft, $\theta = 90°$. Take $\rho = 2.35(10^{-3})$ slug/ft^3.



SOLUTION

We consider ideal fluid flow.
For the flow past a half body,

$$r_0 = \frac{q}{2\pi U} = \frac{100 \text{ ft}^2/\text{s}}{2\pi(300 \text{ ft/s})} = \frac{1}{6\pi} \text{ ft}$$

The equation of the boundary of a half body is given by

$$r = \frac{r_0(\pi - \theta)}{\sin\theta}$$

$$r = \frac{\frac{1}{6\pi}(\pi - \theta)}{\sin\theta}$$

$$r\sin\theta = \frac{1}{6\pi}(\pi - \theta)$$

Since $y = r\sin\theta$, this equation becomes

$$y = \frac{1}{6\pi}(\pi - \theta)$$

The half width h of the half body can be determined by setting $y = h$ as θ approaches 0 or 2π. Thus,

$$h = \frac{1}{6\pi}(\pi - \theta) = \frac{1}{6} \text{ ft}$$

Here, $2h = 2\left(\frac{1}{6} \text{ ft}\right) = 0.333$ ft **Ans.**

At the stagnation point O, $V_O = 0$. The r and θ components of velocity at point A can be determined using

$$v_r = \frac{q}{2\pi r} + U\cos\theta = \frac{100 \text{ ft}^2/\text{s}}{2\pi(0.3 \text{ ft})} + (300 \text{ ft/s})\cos 90° = 53.05 \text{ ft/s}$$

$$v_\theta = -U\sin\theta = -(300 \text{ ft/s})\sin 90° = -300 \text{ ft/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(53.05 \text{ ft/s})^2 + (-300 \text{ ft/s})^2} = 304.65 \text{ ft/s}$$

The flow past a half body is irrotational. Thus, the Bernoulli equation for an ideal fluid is applicable from point O at A. Neglecting the elevation term,

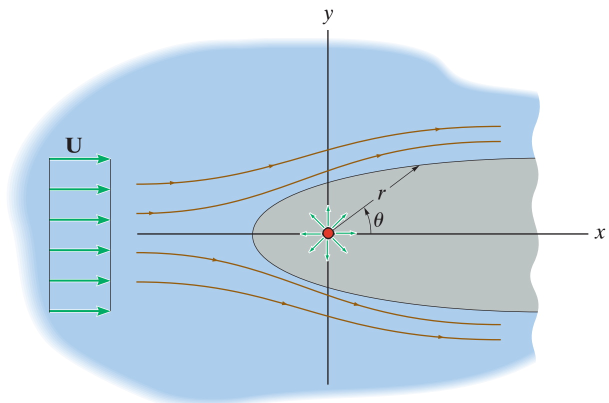
$$\frac{p_O}{\rho} + \frac{V_O^2}{2} = \frac{p_A}{\rho} + \frac{V_A^2}{2}$$

$$\frac{p_O}{2.35(10^{-3}) \text{ slug/ft}^3} + 0 = \frac{p_A}{2.35(10^{-3}) \text{ slug/ft}^3} + \frac{(304.65 \text{ ft/s})^2}{2}$$

$$\Delta p = p_O - p_A = 109.06 \frac{\text{lb}}{\text{ft}^2}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 0.757 \text{ psi}$$ **Ans.**

Ans:
0.333 ft
$\Delta p = 0.757$ psi

7–70. The half body is defined by a combined uniform flow having a velocity of U and a point source of strength q. Determine the pressure distribution along the top boundary of the half body as a function of θ, if the pressure within the uniform flow is p_0. Neglect the effect of gravity. The density of the fluid is ρ.



SOLUTION

We consider ideal fluid flow.
For the flow past a half body,

$$r_0 = \frac{q}{2\pi U}$$

The equation of the boundary of a half body is given by

$$r = \frac{r_0(\pi - \theta)}{\sin\theta} = \frac{\frac{q}{2\pi U}(\pi - \theta)}{\sin\theta} = \frac{q(\pi - \theta)}{2\pi U \sin\theta}$$

The r and θ components of velocity at any point on the boundary can be determined using

$$v_r = \frac{q}{2\pi r} + U\cos\theta = \frac{q}{2\pi\left[\frac{q(\pi - \theta)}{2\pi U \sin\theta}\right]} + U\cos\theta = \frac{U\sin\theta}{\pi - \theta} + U\cos\theta$$

$$v_\theta = -U\sin\theta$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{\left(\frac{U\sin\theta}{\pi - \theta} + U\cos\theta\right)^2 + (-U\sin\theta)^2}$$

$$= \frac{U}{\pi - \theta}\sqrt{\sin^2\theta + (\pi - \theta)\,\sin 2\theta + (\pi - \theta)^2}$$

Since the potential function exists, the flow past a half body is irrotational. The Bernoulli equation is applicable between any two points in the flow. If point A is an arbitrary point on the boundary where $V_A = V$ and $p_A = p$, and point O is a point remote from the body where $V_O = U$, then

$$\frac{p_0}{\rho} + \frac{V_O^2}{2} = \frac{p_A}{\rho} + \frac{V_A^2}{2}$$

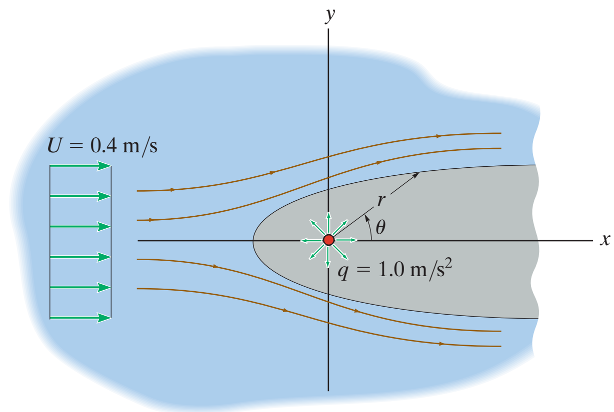
$$\frac{p_0}{\rho} + \frac{U^2}{2} + \frac{p}{\rho} + \frac{\frac{U^2}{(\pi - \theta)^2}\left[\sin^2\theta + (\pi - \theta)\sin 2\theta + (\pi - \theta)^2\right]}{2}$$

$$p = p_0 - \frac{\rho U^2}{2(\pi - \theta)^2}\left[\sin^2\theta + (\pi - \theta)\sin 2\theta\right] \qquad \textbf{Ans.}$$

Ans:

$$p = p_0 - \frac{\rho U^2}{2(\pi - \theta)^2}\left[\sin^2\theta + (\pi - \theta)\sin 2\theta\right]$$

7–71. A fluid flows over a half body for which $U = 0.4$ m/s and $q = 1.0$ m²/s. Plot the half body, and determine the magnitudes of the velocity and pressure in the fluid at the point $r = 0.8$ m and $\theta = 90°$. The pressure within the uniform flow is 300 Pa. Take $\rho = 850$ kg/m³.



SOLUTION

We consider ideal fluid flow.
The location of the stagnation point p can be determined from

$$r_0 = \frac{q}{2\pi U} = \frac{1.0\ \text{m}^2/\text{s}}{2\pi(0.4\ \text{m/s})} = 0.3979\ \text{m} = 0.398\ \text{m}$$

The half width of the half body is

$$h = \pi r_0 = \pi(0.3979\ \text{m}) = 1.25\ \text{m}$$

The resulting half body is shown in Fig. *a*. The velocity components of the flow passing a half body at point B where $r = 0.8$ m and $\theta = 90°$ are

$$(v_r)_B = \frac{q}{2\pi r} + U\cos\theta = \frac{1.0\ \text{m}^2/\text{s}}{2\pi(0.8\ \text{m})} + (0.4\ \text{m/s})\cos 90° = 0.1989\ \text{m/s}$$

$$(v_\theta)_B = -U\sin\theta = -(0.4\ \text{m/s})\sin 90° = -0.4\ \text{m/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{(v_r)_B^{\,2} + (v_\theta)_B^{\,2}} = \sqrt{(0.1989\ \text{m/s})^2 + (-0.4\ \text{m/s})^2} = 0.4467\ \text{m/s} = 0.447\ \text{m/s} \qquad \textbf{Ans.}$$

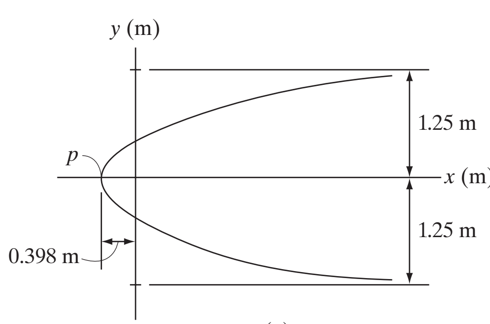
Since the flow passing a half body is irrotational, Bernoulli's equation can be applied between two points on the different streamlines such as point A within the uniform flow and point B. Since the flow occurs in the horizontal plane, the gravity term can be excluded. Here $V_A = U = 0.4$ m/s.

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} = \frac{p_B}{\rho} + \frac{V_B^2}{2}$$

$$p_B = p_A + \frac{\rho}{2}\left(V_A^2 - V_B^2\right)$$

$$= 300\ \text{Pa} + \left(\frac{850\ \text{kg/m}^3}{2}\right)\left[(0.4\ \text{m/s})^2 - (0.4467\ \text{m/s})^2\right]$$

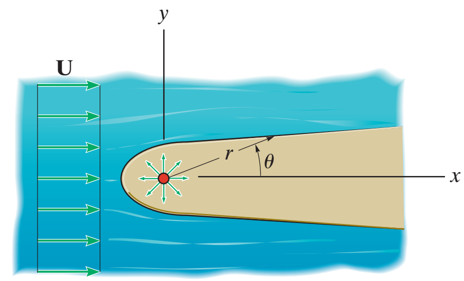
$$= 283\ \text{Pa} \qquad \textbf{Ans.}$$



(a)

Ans:
$r_0 = 0.398$ m
$h = 1.25$ m
$V = 0.447$ m/s
$p = 283$ Pa

***7–72.** The half body is defined by a combined uniform flow having a velocity of U and a point source of strength q. Determine the location θ on the boundary of the half body where the pressure p is equal to the pressure p_0 within the uniform flow. Neglect the effect of gravity.



SOLUTION

We consider ideal fluid flow.
For the flow past a half body,

$$r_0 = \frac{q}{2\pi U}$$

The equation of the boundary of a half body is given by

$$r = \frac{r_0(\pi - \theta)}{\sin\theta} = \frac{\frac{q}{2\pi U}(\pi - \theta)}{\sin\theta} = \frac{q(\pi - \theta)}{2\pi U \sin\theta}$$

The r and θ components of velocity at any point on the boundary can be determined using

$$v_r = \frac{q}{2\pi r} + U\cos\theta = \frac{q}{2\pi\left[\frac{q(\pi - \theta)}{2\pi U \sin\theta}\right]} + U\cos\theta = \frac{U\sin\theta}{\pi - \theta} + U\cos\theta$$

$$v_\theta = -U\sin\theta$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{\left(\frac{U\sin\theta}{\pi - \theta} + U\cos\theta\right)^2 + (-U\sin\theta)^2}$$

$$= \frac{U}{\pi - \theta}\sqrt{\sin^2\theta + (\pi - \theta)\sin 2\theta + (\pi - \theta)^2}$$

Since the potential function exists, the flow past a half body is irrotational. The Bernoulli equation is applicable between any two points in the flow. If point A is an arbitrary point on the boundary where $V_A = V$ and $p_A = p_O$, and point O is a point remote from the body where $V_O = U$, then

$$\frac{p_O}{\rho} + \frac{V_O^2}{2} = \frac{p_A}{\rho} + \frac{V_A^2}{2}$$

$$\frac{p_O}{\rho} + \frac{U^2}{2} = \frac{p_A}{\rho} + \frac{\frac{U^2}{(\pi - \theta)^2}\left[\sin^2\theta + (\pi - \theta)\sin 2\theta + (\pi - \theta)^2\right]}{2}$$

$$p_O - p_A = \frac{\rho U^2}{2(\pi - \theta)^2}\left[\sin^2\theta + (\pi - \theta)\sin 2\theta\right] = 0$$

Since $\dfrac{\rho U^2}{2(\pi - \theta)^2} \neq 0$, then

$$\sin^2\theta + (\pi - \theta)\sin 2\theta = 0$$

$$\sin^2\theta + 2(\pi - \theta)\sin\theta\cos\theta = 0$$

$$\sin\theta[\sin\theta + 2(\pi - \theta)\cos\theta] = 0$$

Since $\sin\theta \neq 0$, then

$$\sin\theta + 2(\pi - \theta)\cos\theta = 0$$

$$\tan\theta + 2(\pi - \theta) = 0$$

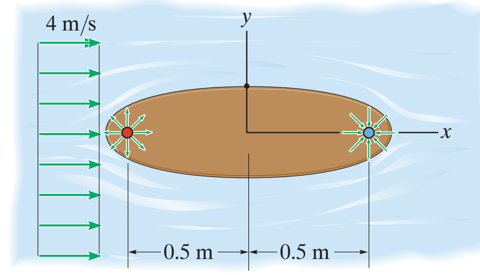
Solving by trial and error,

$$\theta = 1.9760\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right) = 113.22^\circ = 113^\circ \qquad \textbf{Ans.}$$

or

$$\theta = 4.3072\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right) = 246.8^\circ = 247^\circ \qquad \textbf{Ans.}$$

7–73. The Rankine body is defined by the source and sink, each having a strength of $0.2\ \text{m}^2/\text{s}$. If the velocity of the uniform flow is $4\ \text{m/s}$, determine the longest and shortest dimensions of the body.



SOLUTION

We consider ideal fluid flow.
The half length of the Rankine oval is

$$b = \left(\frac{q}{U\pi}a + a^2\right)^{\frac{1}{2}}$$

$$b = \left[\left(\frac{0.2\ \text{m}^2/\text{s}}{(4\ \text{m/s})\pi}\right)(0.5\ \text{m}) + (0.5\ \text{m})^2\right]^{\frac{1}{2}} = 0.5079\ \text{m}$$

Thus, the length of the Rankine oval is

$$L = 2b = 2(0.5079\ \text{m}) = 1.02\ \text{m} \qquad \textbf{Ans.}$$

The half width of the Rankine oval can be determined using

$$h = \frac{h^2 - a^2}{2a}\tan\left(\frac{2\pi U h}{q}\right)$$

$$h = \frac{h^2 - (0.5\ \text{m})^2}{2(0.5\ \text{m})}\tan\left[\frac{2\pi(4\ \text{m/s})h}{0.2\ \text{m}^2/\text{s}}\right]$$

$$h = (h^2 - 0.25)\tan(40\pi h)$$

Solving numerically, and noting that $q/2U = 0.2/[2(4)] = 0.025$, we start with $h = 0.024$ and adjust this until we find that

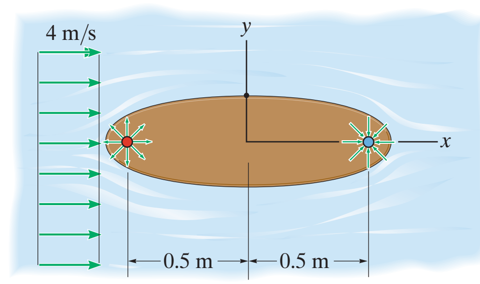
$$h = 0.02423\ \text{m}$$

Thus, the width of the Rankine oval is

$$W = 2h = 2(0.02423) = 0.0485\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$L = 1.02$ m
0.0485 m

7–74. The Rankine body is defined by the source and sink, each having a strength of 0.2 m^2/s. If the velocity of the uniform flow is 4 m/s, determine the equation in Cartesian coordinates that defines the boundary of the body.



SOLUTION

We consider ideal fluid flow.
The stream function of the flow around the Rankine oval is given by

$$\psi = Uy - \frac{q}{2\pi}\tan^{-1}\left(\frac{2ay}{x^2 + y^2 - a^2}\right)$$

Since the boundary of the Rankine oval contains the stagnation point where $y = 0$, then this equation gives

$$\psi = U(0) - \frac{q}{2\pi}\tan^{-1}\left(\frac{2(a)(0)}{x^2 + 0^2 - a^2}\right) = 0$$

Thus, the equation that describes the boundary of the Rankine oval is

$$Uy - \frac{q}{2\pi}\tan^{-1}\left(\frac{2ay}{x^2 + y^2 - a^2}\right) = 0$$

Here, $U = 4\ \text{m/s}$, $q = 0.2\ \text{m}^2/\text{s}$ and $a = 0.5$ m.

$$4y - \frac{0.2}{2\pi}\tan^{-1}\left[\frac{2(0.5)y}{x^2 + y^2 - 0.5^2}\right] = 0$$

$$4y - \frac{1}{10\pi}\tan^{-1}\left[\frac{y}{x^2 + y^2 - 0.25}\right] = 0$$

$$\frac{y}{x^2 + y^2 - 0.25} = \tan 40\pi y \qquad \textbf{Ans.}$$

Ans:

$$\frac{y}{x^2 + y^2 - 0.25} = \tan 40\pi y$$

7–75. A fluid has a uniform velocity of $U = 10$ m/s. A source $q = 15\ \text{m}^2/\text{s}$ is at $x = 2$ m, and a sink $q = -15\ \text{m}^2/\text{s}$ is at $x = -2$ m. Graph the Rankine body that is formed, and determine the magnitudes of the velocity and the pressure at point (0, 2 m). The pressure within the uniform flow is 40 kPa. Take $\rho = 850\ \text{kg/m}^3$.

SOLUTION

We consider ideal fluid flow.
The major and minor axes of the Rankine oval can be determined from

$$b = \left(\frac{q}{U\pi}a + a^2\right)^{\frac{1}{2}}$$

$$= \left\{\left[\frac{15\ \text{m}^2/\text{s}}{(10\ \text{m/s})\pi}\right](2\ \text{m}) + (2\ \text{m})^2\right\}^{\frac{1}{2}}$$

$$= 2.23\ \text{m}$$

$$h = \frac{h^2 - a^2}{2a}\tan\left(\frac{2\pi Uh}{q}\right)$$

$$h = \frac{h^2 - (2\ \text{m})^2}{2(2\ \text{m})}\tan\left[\frac{2\pi(10\ \text{m/s})h}{15\ \text{m}^2/\text{s}}\right]$$

$$h = \frac{h^2 - 4}{4}\tan\left(\frac{4\pi}{3}h\right)$$

Solving numerically, and noting that $q/2U = 15/[2(10)] = 0.75$, we start with $h = 0.74$ and adjust this until we find that

$$h = 0.609\ \text{m}$$

The resulting Rankine oval is shown in Fig. *a*

Since the flow around a Rankine oval is irrotational, Bernoulli's equation can be applied between two points on different streamlines where these two points are point A, within the uniform flow and point B (0, 2 m). Since the flow occurs in the horizontal plane, the gravity term can be excluded.

Here, $V_A = U = 10$ m/s and the velocity components at B are

$$u_B = U + \frac{q}{2\pi}\left[\frac{x + a}{(x + a)^2 + y^2} - \frac{x - a}{(x - a)^2 + y^2}\right]$$

$$= 10\ \text{m/s} + \left(\frac{15\ \text{m}^2/\text{s}}{2\pi}\right)\left[\frac{0 + 2\ \text{m}}{(0 + 2\ \text{m})^2 + (2\ \text{m})^2} - \frac{0 - 2\ \text{m}}{(0 - 2\ \text{m})^2 + (2\ \text{m})^2}\right]$$

$$= 11.19\ \text{m/s}$$

$$v_B = \frac{q}{2\pi}\left[\frac{y}{(x + a)^2 + y^2} - \frac{y}{(x - a)^2 + y^2}\right]$$

$$= \frac{15\ \text{m}^2/\text{s}}{2\pi}\left[\frac{2\ \text{m}}{(0 + 2\ \text{m})^2 + (2\ \text{m})^2} - \frac{2\ \text{m}}{(0 - 2\ \text{m})^2 + (2\ \text{m})^2}\right]$$

$$= 0$$

7–75. Continued

Thus,

$$V_B = u_B = 11.19 \text{ m/s} = 11.2 \text{ m/s} \qquad \textbf{Ans.}$$

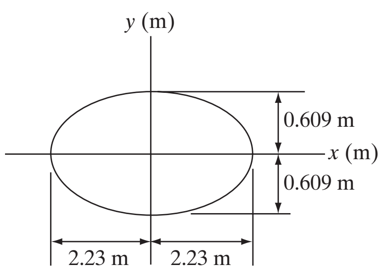
Bernoulli's equation written between points A and B is

$$\frac{p_A}{\rho} + \frac{V_A^2}{2} = \frac{p_B}{\rho} + \frac{V_B^2}{2}$$

$$p_B = p_A + \frac{\rho}{2}\left(V_A^2 - V_B^2\right)$$

$$= 40\left(10^3\right) \text{ Pa} + \left(\frac{850 \text{ kg/m}^3}{2}\right)\left[\left(10 \text{ m/s}\right)^2 - \left(11.19 \text{ m/s}\right)^2\right]$$

$$= 29.24\left(10^3\right) \text{ Pa} = 29.2 \text{ kPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$h = 0.609$ m
$V = 11.2$ m/s
$p = 29.2$ kPa

***7–76.** Integrate the pressure distribution, Eq. 7–67, over the surface of the cylinder in Fig. 7–33*b*, and show that the resultant force is equal to zero.

SOLUTION

We consider ideal fluid flow.
The pressure distribution around a cylinder is given by

$$p = p_0 + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$$

The force that the pressure exerts on the differential area $dA = (ad\theta)L = aLd\theta$ is

$$dF = pdA = \left[p_0 + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)\right](aLd\theta) = p_0 aLd\theta + \frac{1}{2}\rho aLU^2\left(1 - 4\sin^2\theta\right)d\theta$$

Equating the resultant forces along the *x* and *y* axes shown in Fig. *a*,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -\int_0^{2\pi} dF\cos\theta$$

$$= -\int_0^{2\pi}\left[p_0 aLd\theta + \frac{1}{2}\rho aLU^2\left(1 - 4\sin^2\theta\right)d\theta\right]\cos\theta$$

$$= -\int_0^{2\pi} p_0 aL\cos\theta d\theta - \int_0^{2\pi}\frac{1}{2}\rho aLU^2\left(\cos\theta - 4\sin^2\theta\cos\theta\right)d\theta$$

$$= -p_0 aL(\sin\theta)\Big|_0^{2\pi} - \frac{1}{2}\rho aLU^2\left(\sin\theta - \frac{4\sin^3\theta}{3}\right)\Bigg|_0^{2\pi}$$

$$= 0$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -\int_0^{2\pi} dF\sin\theta$$

$$= -\int_0^{2\pi}\left[p_0 aLd\theta + \frac{1}{2}\rho aLU^2\left(1 - 4\sin^2\theta\right)d\theta\right]\sin\theta$$

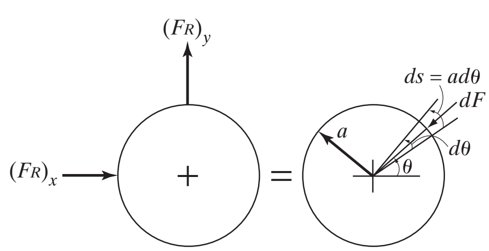
$$= -\int_0^{2\pi} p_0 aL\sin\theta d\theta - \int_0^{2\pi}\frac{1}{2}\rho aLU^2\left(\sin\theta - 4\sin^3\theta\right)d\theta$$

$$= -p_0 aL\left(\cos\theta\right)\Big|_0^{2\pi} - \frac{1}{2}\rho aLU^2\left[-\cos\theta - \left[-\frac{4}{3}\cos\theta\left(\sin^2\theta + 2\right)\right]\right]\Bigg|_0^{2\pi}$$

$$= 0$$

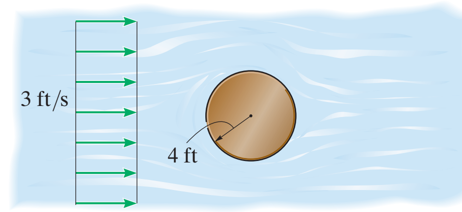
Therefore,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = 0 \qquad \textbf{(Q.E.D.)}$$



(a)

7–77. The tall rotating cylinder is subjected to a uniform horizontal airflow of 3 ft/s. If the radius of the cylinder is 4 ft, determine the location of the stagnation points and the lift per unit length. The circulation around the cylinder is 18 ft^2/s. Take $\rho = 2.35\left(10^{-3}\right)$ slug/ft^3.



SOLUTION

We consider ideal fluid flow.
The lift can be determined by using

$$F_y = \rho U \Gamma = \left[2.35\left(10^{-3}\right)\text{ slug/ft}^3\right](3\text{ ft/s})(18\text{ ft}^2\text{/s})$$
$$= 0.127\text{ lb/ft} \qquad \textbf{Ans.}$$

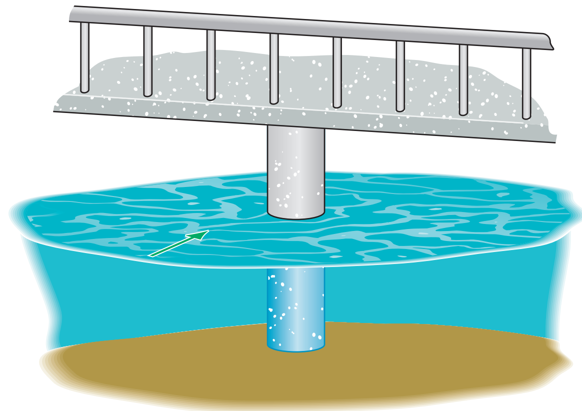
For this case,

$$\sin\theta = \frac{\Gamma}{4\pi U a} = \frac{18\text{ ft}^2\text{/s}}{4\pi(3\text{ ft/s})(4\text{ ft})} = 0.1194$$
$$\theta = 6.86° \text{ and } 173° \qquad \textbf{Ans.}$$

Since the solution has two roots, there are two stagnation points on the surface of the cylinder.

Ans:
$F_y = 0.127$ lb/ft
$\theta = 6.86°$ and $173°$

7–78. The 0.5-m-diameter bridge pier is subjected to the uniform flow of water at 4 m/s. Determine the maximum and minimum pressures exerted on the pier at a depth of 2 m.



SOLUTION

We consider ideal fluid flow.
For the flow around a cylinder, the pressure at any point on the boundary can be determined by using

$$p = p_0 + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$$

At the depth of $h = 2$ m,

$$p_0 = \rho g h = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m}) = 19.62\left(10^3\right)\ \text{Pa}$$

The pressure extremes occur when $\frac{dp}{d\theta} = 0$. Thus,

$$\frac{dp}{d\theta} = 0 + \frac{1}{2}\rho U^2(0 - 8\sin\theta\cos\theta) = 0$$

$$8\sin\theta\cos\theta = 0$$

$$4\sin 2\theta = 0$$

Solving,

$$\theta = 0, 90°, 180°...$$

The maximum pressure occurs when $\theta = 0°$ or $180°$. Thus,

$$p_{\max} = 19.62\left(10^3\right)\ \text{Pa} + \frac{1}{2}\left(1000\ \text{kg/m}^3\right)\left(4\ \text{m/s}\right)^2\left[1 - 4\sin^2 0°\right]$$

$$= 27.62\left(10^3\right)\ \text{Pa} = 27.6\ \text{kPa} \qquad \textbf{Ans.}$$

The minimum pressure occurs when $\theta = 90°$. Thus,

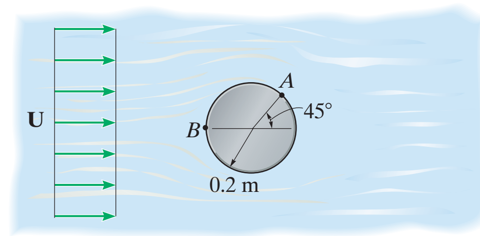
$$p_{\min} = 19.62\left(10^3\right)\ \text{Pa} + \frac{1}{2}\left(1000\ \text{kg/m}^3\right)(4\ \text{m/s})^2[1 - 4\sin^2 90°]$$

$$= -4.38\left(10^3\right)\ \text{Pa} = -4.38\ \text{kPa} \qquad \textbf{Ans.}$$

The negative sign indicates that suction occurs.

Ans:
$p_{\max} = 27.6$ kPa
$p_{\min} = -4.38$ kPa

7–79. Air flows around the cylinder such that the pressure, measured at A, is $p_A = -4$ kPa. Determine the velocity U of the flow if $\rho = 1.22\ \text{kg/m}^3$. Can this velocity be determined if instead the pressure at B is measured?



SOLUTION

We consider ideal fluid flow.
The pressure at a point removed from the cylinder is atmospheric. Thus, $p_0 = 0$.
At point A, $\theta = 45°$

$$p_A = p_0 + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$$

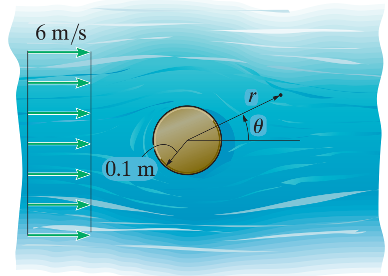
$$-4\left(10^3\right)\ \text{N/m}^2 = 0 + \frac{1}{2}\left(1.22\ \text{kg/m}^2\right)\left(U^2\right)\left(1 - 4\sin^2 45°\right)$$

$$U = 80.98\ \text{m/s} = 81.0\ \text{m/s}$$ **Ans.**

If the pressure is measured at B, the velocity of the uniform flow can be determined using the same equation with $\theta = 180°$.

Ans:
81.0 m/s; yes

***7–80.** The 200-mm-diameter cylinder is subjected to a uniform horizontal flow having a velocity of 6 m/s. At a distance far away from the cylinder, the pressure is 150 kPa. Plot the variations of the velocity and pressure along the radial line r, at $\theta = 90°$, and specify their values at $r = 0.1$ m, 0.2 m, 0.3 m, 0.4 m, and 0.5 m. Take $\rho = 1.5\ \text{Mg/m}^3$.



SOLUTION

We consider ideal fluid flow.

The r and θ components of velocity of the uniform flow around a cylinder can be determined using

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta = (6\ \text{m/s})\left[1 - \frac{(0.1\ \text{m})^2}{r^2}\right]\cos\theta = \left[\left(6 - \frac{0.06}{r^2}\right)\cos\theta\right]\text{m/s}$$

$$v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta = -(6\ \text{m/s})\left[1 + \frac{(0.1\ \text{m})^2}{r^2}\right]\sin\theta = \left[-\left(6 + \frac{0.06}{r^2}\right)\sin\theta\right]\text{m/s}$$

When $\theta = 90°$,

$$v_r = 0 \qquad v_\theta = -\left(6 + \frac{0.06}{r^2}\right)\text{m/s}$$

Thus, the magnitude of the velocity when $\theta = 90°$ is

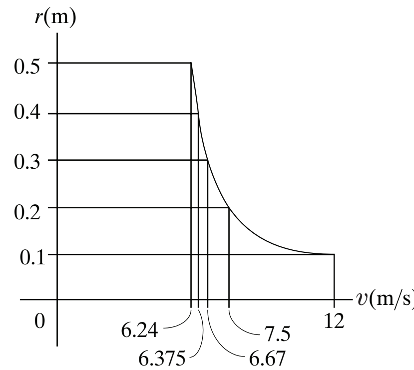
$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + \left[-\left(6 + \frac{0.06}{r^2}\right)\right]^2} = \left(6 + \frac{0.06}{r^2}\right)\text{m/s} \qquad \textbf{(1)}$$

Flow around a cylinder is irrotational since the potential function exists. Therefore, the Bernoulli equation is applicable. Neglecting the elevation terms,

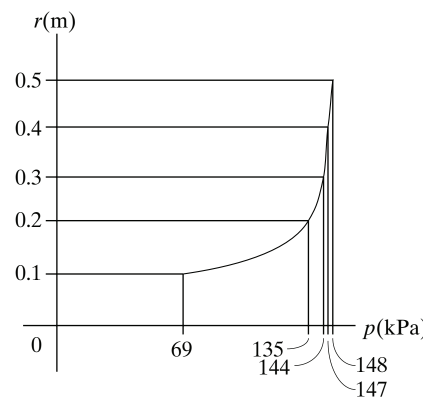
$$\frac{p_O}{\rho} + \frac{V_O^2}{2} = \frac{p}{\rho} + \frac{V^2}{2}$$

$$\frac{150(10^3)\frac{\text{N}}{\text{m}^2}}{1500\ \text{kg/m}^3} + \frac{(6\ \text{m/s})^2}{2} = \frac{p}{1500\ \text{kg/m}^3} + \frac{\left(6 + \frac{0.06}{r^2}\right)^2}{2}$$

$$p = \left[177(10^3) - 750\left(6 + \frac{0.06}{r^2}\right)^2\right]\text{Pa} \qquad \textbf{(2)}$$



(a)



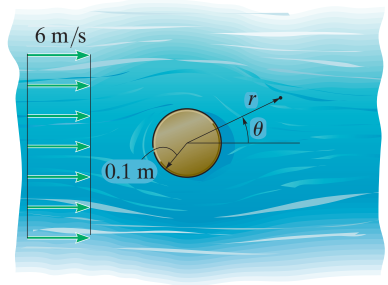
(b)

The values of V and P at $r = 0.1$ m, 0.2 m, 0.3 m, 0.4 m, and 0.5 m can be evaluated using Eqs. 1 and 2, respectively, and are tabulated below.

r(m)	0.1	0.2	0.3	0.4	0.5	
V(m/s) Eq. (1)	12	7.5	6.67	6.375	6.24	**Ans.**
p(kPa) Eq. (2)	69	135	144	147	148	

The plot of V vs. r and p vs. r are shown in Figs. a and b.

7–81. The 200-mm-diameter cylinder is subjected to a uniform flow having a velocity of 6 m/s. At a distance far away from the cylinder, the pressure is 150 kPa. Plot the variation of the velocity and pressure along the radial line r, at $\theta = 0°$, and specify their values at $r = 0.1$ m, 0.2 m, 0.3 m, 0.4 m, and 0.5 m. Take $\rho = 1.5\ \text{Mg/m}^3$.



SOLUTION

We consider ideal fluid flow.
The r and θ components of velocity of the uniform flow around a cylinder can be determined using

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta = (6\ \text{m/s})\left[1 - \frac{(0.1\ \text{m})^2}{r^2}\right]\cos\theta = \left[\left(6 - \frac{0.06}{r^2}\right)\cos\theta\right]\text{m/s}$$

$$v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta = -(6\ \text{m/s})\left[1 + \frac{(0.1\ \text{m})^2}{r^2}\right]\sin\theta = \left[-\left(6 + \frac{0.06}{r^2}\right)\sin\theta\right]\text{m/s}$$

When $\theta = 0°$,

$$v_r = \left(6 - \frac{0.06}{r^2}\right)\text{m/s} \qquad v_\theta = 0$$

Thus, the magnitude of the velocity when $\theta = 0°$ is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{\left(6 - \frac{0.06}{r^2}\right)^2 + 0^2} = \left(6 - \frac{0.06}{r^2}\right)\text{m/s} \qquad \textbf{(1)}$$

The flow around a cylinder is irrotational since the potential function exists. Therefore, the Bernoulli equation is applicable. Neglecting the elevation terms,

$$\frac{p_O}{\rho} + \frac{V_O^2}{2} = \frac{p}{\rho} + \frac{V^2}{2}$$

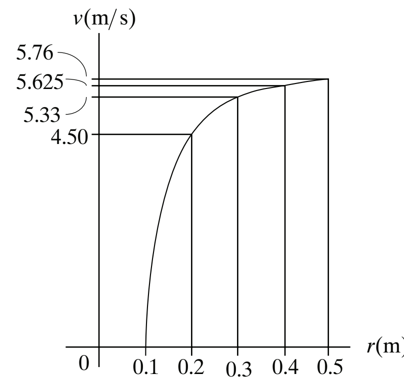
$$\frac{150(10^3)\dfrac{\text{N}}{\text{m}^2}}{1500\ \text{kg/m}^3} + \frac{(6\ \text{m/s})^2}{2} = \frac{p}{1500\ \text{kg/m}^3} + \frac{\left(6 - \dfrac{0.06}{r^2}\right)^2}{2}$$

$$p = \left[177(10^3) - 750\left(6 - \frac{0.06}{r^2}\right)^2\right]\text{Pa} \qquad \textbf{(2)}$$

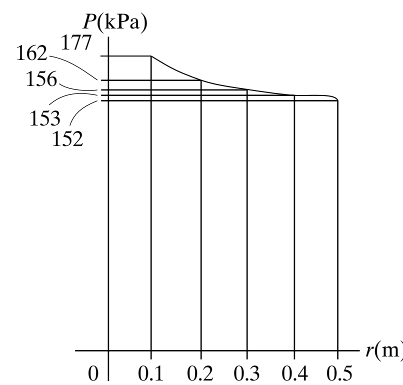
The values of V and P at $r = 0.1$ m, 0.2 m, 0.3 m, 0.4 m, and 0.5 m can be evaluated using Eqs. 1 and 2, respectively, and are tabulated below.

r(m)	0.1	0.2	0.3	0.4	0.5	
V(m/s)	0	4.50	5.33	5.625	5.76	**Ans.**
p(kPa)	177	162	156	153	152	

The plot of V vs. r and p vs. r are shown in Figs. a and b.



(a)

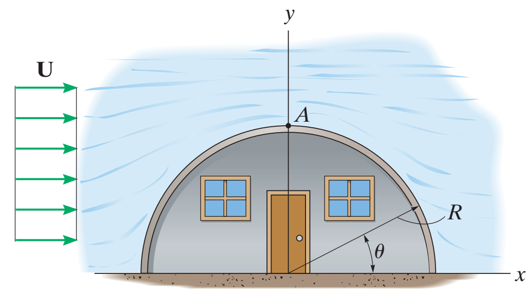


(b)

Ans:

r (m)	0.1	0.2	0.3	0.4	0.5
V (m/s)	0	4.50	5.33	5.625	5.76
p (kPa)	177	162	156	153	152

7–82. Air is flowing at $U = 30$ m/s past the model of a Quonset hut of radius $R = 3$ m. Find the velocity and absolute pressure distribution along the y axis for $3 \text{ m} \leq y \leq \infty$. The absolute pressure within the uniform flow is $p_0 = 100$ kPa. Take $\rho_a = 1.23 \text{ kg/m}^3$.



SOLUTION

We consider ideal fluid flow.
The velocity components of the flow around the building are

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta \qquad v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

Here $U = 30$ m/s and $a = 3$ m. Along the y axis $\theta = 90°$. Then

$$v_r = (30 \text{ m/s})\left[1 - \frac{(3 \text{ m})^2}{r^2}\right]\cos 90° = 0$$

$$v_\theta = -(30 \text{ m/s})\left[1 + \frac{(3 \text{ m})^2}{r^2}\right]\sin 90°$$

$$= -30\left(1 + \frac{9}{r^2}\right)\text{m/s}$$

Thus, the velocity distribution along the y axis is

$$V = v_\theta = 30\left(1 + \frac{9}{r^2}\right)\text{ m/s} \qquad \textbf{Ans.}$$

Since the flow is irrotational, Bernoulli's equation can be applied between two points on different streamlines. Here they are point O within the uniform flow and a point along the y axis,

$$\frac{p_0}{\rho_a} + \frac{V_O^2}{2} + gz_0 = \frac{p}{\rho_a} + \frac{V^2}{2} + gz$$

Since the density of air is small, the gravitational terms can be neglected. Here, $V_0 = U = 30$ m/s. Then

$$\frac{100(10^3)\text{ N/m}^2}{1.23 \text{ kg/m}^3} + \frac{(30 \text{ m/s})^2}{2} + 0 = \frac{p}{1.23 \text{ kg/m}^3} + \frac{\left[30\left(1 + \frac{9}{r^2}\right)\right]^2}{2} + 0$$

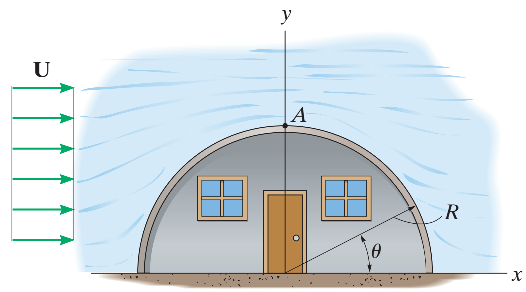
$$p = \left[100\,553.5 - 553.5\left(1 + \frac{9}{r^2}\right)^2\right]\text{ Pa} \qquad \textbf{Ans.}$$

Notice that at $r = \infty$,

$$p = 100\,553.5 - 553.5(1 + 0)^2$$
$$= 100(10^3)\text{ Pa} = p_0$$

Ans:
$V = 30\left(1 + \frac{9}{r^2}\right)$ m/s
$p = \left[100\,553.5 - 553.5\left(1 + \frac{9}{r^2}\right)^2\right]$ Pa

7–83. The Quonset hut of radius R is subjected to a uniform wind having a velocity U. Determine the resultant vertical force caused by the pressure that acts on the hut if it has a length L. The density of air is ρ.



SOLUTION

We consider ideal fluid flow.

For the flow around a cylinder, the pressure distribution on the boundary is described by

$$p = p_0 + \frac{1}{2}\rho U^2(1 - 4\sin^2\theta)$$

Here, p_0 is atmospheric pressure. Thus, the net pressure on the boundary is gauge pressure, which is

$$p_g = p - p_0 = \frac{1}{2}\rho U^2(1 - 4\sin^2\theta)$$

The force that the gauge pressure exerts on the differential area $dA = (Rd\,\theta)L = RLd\,\theta$ is

$$dF = p_g dA = \frac{1}{2}\rho U^2(1 - 4\sin^2\theta)(RLd\theta) = \frac{1}{2}\rho RLU^2(1 - 4\sin^2\theta)d\theta$$

Equating the forces along the y axis shown in Fig. a,

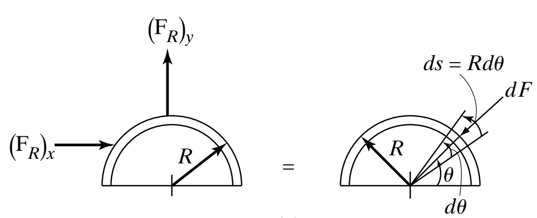
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -\int_0^{\pi} dFd\sin\theta$$

$$= -\int_0^{\pi}\frac{1}{2}\rho RLU^2(1 - 4\sin^2\theta)\sin\theta\,d\theta$$

$$= -\frac{1}{2}\rho RLU^2\int_0^{\pi}(\sin\theta - 4\sin^3\theta)d\theta$$

$$= -\frac{1}{2}\rho RLU^2\left[-\cos\theta - \left[-\frac{4}{3}\cos\theta(\sin^2\theta + 2)\right]\right]_0^{\pi}$$

$$= \frac{5}{3}\rho RLU^2$$ **Ans.**

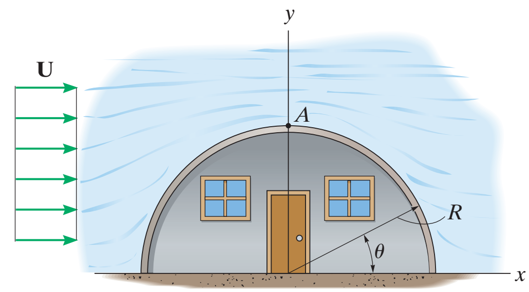


(a)

Ans:

$(F_R)_y = \frac{5}{3}\rho RLU^2$

***7–84.** The Quonset hut of radius R is subjected to a uniform wind having a velocity U. Determine the speed of the wind and the gage pressure at point A. The density of air is ρ.



SOLUTION

We consider ideal fluid flow.

At point A, $r = R$ and $\theta = \dfrac{\pi}{2}$ rad. Thus,

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta = U\left(1 - \frac{R^2}{R^2}\right)\cos\frac{\pi}{2} = 0$$

$$v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta = -U\left(1 + \frac{R^2}{R^2}\right)\sin\frac{\pi}{2} = -2U$$

Thus, the magnitude of the velocity at point A is

$$V_A = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + (-2U)^2} = 2U \qquad \textbf{Ans.}$$

Here, $\mathbf{V}_A$ **is directed towards the positive *x* axis.**
For the flow around a cylinder, the pressure distribution on the boundary is described by

$$p = p_0 + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$$

Here, p_0 is atmospheric pressure. Thus, the net pressure on the boundary is gauge pressure, which is

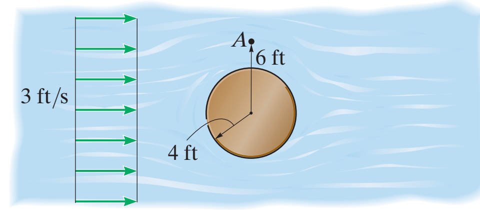
$$p_g = p - p_0 = \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$$

At point A, $\theta = \dfrac{\pi}{2}$ rad. Then,

$$\left(p_g\right)_A = \frac{1}{2}\rho U^2\left[1 - 4\sin^2\left(\frac{\pi}{2}\right)\right] = -\frac{3}{2}\rho U^2 \qquad \textbf{Ans.}$$

The negative sign indicates that suction occurs at A.

7–85. Water flows toward the circular column with a uniform speed of 3 ft/s. If the outer radius of the column is 4 ft, and the pressure within the uniform flow is 6 lb/in^2, determine the pressure at point A. Take $\rho_w = 1.94$ slug/ft^3.



SOLUTION

We consider ideal fluid flow.
The velocity components of the flow around the structure are

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta \qquad v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

Here $U = 3$ ft/s, and $a = 4$ ft. For point A, $r = 6$ ft and $\theta = 90°$. Then

$$v_r = (3\text{ ft/s})\left[1 - \frac{(4\text{ ft})^2}{(6\text{ ft})^2}\right]\cos 90° = 0$$

$$v_\theta = -(3\text{ ft/s})\left[1 + \frac{(4\text{ ft})^2}{(6\text{ ft})^2}\right]\sin 90° = -4.333\text{ ft/s}$$

Thus, the magnitude of the velocity at A is

$$V_A = v_\theta = 4.333\text{ ft/s}$$

Since the flow is irrotational, Bernoulli's equation can be applied between two points on different streamlines. Here, they are point O in the uniform flow and point A.

$$\frac{p_0}{\rho} + \frac{V_O^2}{2} + gz_0 = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

Since the flow is in the horizontal plane, $z_D = z_A = z$. Here, $V_0 = U = 3$ ft/s. Then

$$\frac{p_0}{\rho} + \frac{V_0^2}{2} + gz = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz$$

$$p_A = p_0 + \frac{\rho}{2}\left(V_0^2 - V_A^2\right)$$

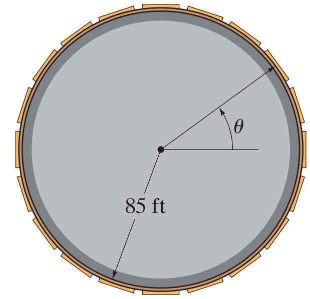
$$= \left(6\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)^2 + \frac{1.94\text{ slug/ft}^3}{2}\left[(3\text{ ft/s})^2 - (4.333\text{ ft/s})^2\right]$$

$$= \left(854.52\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)^2$$

$$= 5.93\text{ psi}$$ **Ans.**

Ans:
5.93 psi

7–86. The tall circular building is subjected to a uniform wind having a velocity of 150 ft/s. Determine the location θ of the window that is subjected to the smallest pressure. What is this pressure? Take $\rho_a = 0.00237\ \text{slug/ft}^3$.



SOLUTION

We consider ideal fluid flow.
This is a case of flow around a cylinder where the velocity components are

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta \qquad v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

On the surface of the building where $r = a = 85$ ft,

$$v_r = 0 \qquad v_\theta = -2U\sin\theta$$

Thus, the velocity of the wind on the surface of the building is

$$V = v_\theta = -2U\sin\theta$$

The minimum pressure occurs at the point where the magnitude of velocity is maximum, that is when

$$\sin\theta = 1 \quad \text{or} \quad \sin\theta = -1$$

Then $$\theta = 90° \quad \text{or} \quad 270°$$ **Ans.**

Therefore

$$V_{\max} = 2U = 2(150\ \text{ft/s}) = 300\ \text{ft/s}$$

Since the flow is irrotational, Bernoulli's equation can be applied between two points on different streamlines, such as between a point within the uniform flow and a point on the building. Since the flow occurs in the horizontal plane, the gravity term can be excluded.

$$\frac{p_0}{\rho} + \frac{V_0^{\,2}}{2} = \frac{p_{\min}}{\rho} + \frac{V_{\max}^2}{2}$$

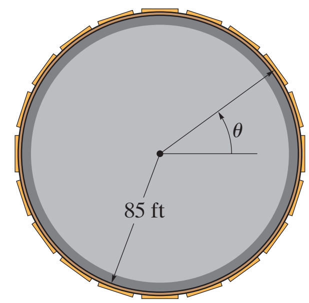
Here $p_0 = 0$ and $V_0 = U = 150$ ft/s. Then

$$0 + \frac{(150\ \text{ft/s})^2}{2} = \frac{p_{\min}}{0.00237\ \text{slug/ft}^3} + \frac{(300\ \text{ft/s})^2}{2}$$

$$p_{\min} = \left(-79.99\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)^2 = -0.555\ \text{psi}$$ **Ans.**

Ans:
$\theta = 90°$ or $270°$
$p_{\min} = -0.555$ psi

7–87. The tall circular building is subjected to a uniform wind having a velocity of 150 ft/s. Determine the pressure and the velocity of the wind on its walls at $\theta = 0°$, 90°, and 150°. Take $\rho_a = 0.00237\ \text{slug/ft}^3$.



SOLUTION

We consider ideal fluid flow.
This is a case of flow around a cylinder where the velocity components are

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta \qquad v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

On the surface of the building where $r = a = 85$ ft,

$$v_r = 0 \qquad v_\theta = -2U\sin\theta$$

Thus, the velocity of the wind on the surface of the building is

$$V = v_\theta = -2U\sin\theta$$

At $\theta = 0°$, 90° and 150°,

$$V|_{\theta=0°} = 2(150\ \text{ft/s})\sin 0° = 0 \qquad \textbf{Ans.}$$

$$V|_{\theta=90°} = 2(150\ \text{ft/s})\sin 90° = 300\ \text{ft/s} \qquad \textbf{Ans.}$$

$$V|_{\theta=150°} = 2(150\ \text{ft/s})\sin 150° = 150\ \text{ft/s} \qquad \textbf{Ans.}$$

Since the flow is irrotational, Bernoulli's equation can be applied between two points on different streamlines, such as between a point within the uniform flow and a point on the building. Since the flow occurs in the horizontal plane, the gravity term can be excluded.

$$\frac{p_0}{\rho} + \frac{V_0{}^2}{2} = \frac{p}{\rho} + \frac{V^2}{2}$$

$$p = p_0 + \frac{p}{2}\left(V_0^2 - V^2\right)$$

Here $p_0 = 0$ and $V_0 = 150$ ft/s. Then

$$P|_{\theta=0°} = 0 + \left(\frac{0.00237\ \text{slug/ft}^3}{2}\right)\left[(150\ \text{ft/s})^2 - 0\right]$$

$$= \left(26.66\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ft}}{12\ \text{in}}\right)^2 = 0.185\ \text{psi} \qquad \textbf{Ans.}$$

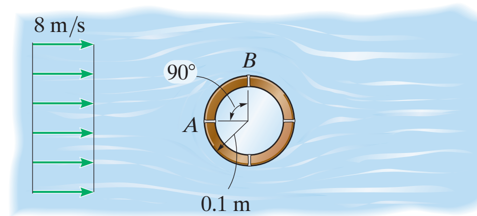
$$P|_{\theta=90°} = 0 + \left(\frac{0.00237\ \text{slug/ft}^3}{2}\right)\left[(150\ \text{ft/s})^2 - (300\ \text{ft/s})^2\right]$$

$$= \left(-79.99\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ft}}{12\ \text{in}}\right)^2 = -0.555\ \text{psi} \qquad \textbf{Ans.}$$

$$P_{\theta = 150°} = 0 + \left(\frac{0.00237\ \text{slug/ft}^3}{2}\right)\left[(150\ \text{ft/s})^2 - (150\ \text{ft/s})^2\right] = 0 \qquad \textbf{Ans.}$$

Ans:
$V|_{\theta=0°} = 0$
$V|_{\theta=90°} = 300$ ft/s
$V|_{\theta=150°} = 150$ ft/s
$p|_{\theta=0°} = 0.185$
$p|_{\theta=90°} = -0.555$ psi
$p|_{\theta=150°} = 0$

***7–88.** The pipe is built from four quarter segments that are glued together. If it is exposed to a uniform air flow having a velocity of 8 m/s, determine the resultant force the pressure exerts on the quarter segment AB per unit length of the pipe. Take $\rho = 1.22\ \text{kg/m}^3$.



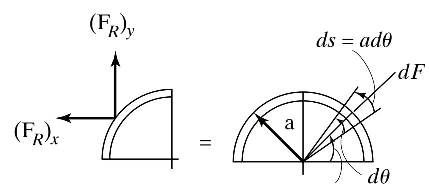
SOLUTION

We consider ideal fluid flow.
Here, the pressure at point O removed from the cylinder is atmospheric. Thus, the net pressure on the surface of the cylinder is the gauge pressure given by

$$p_g = p - p_0 = \frac{1}{2}\rho U^2(1 - 4\sin^2\theta)$$

The force that the gauge pressure exerts on the differential area $dA = (ad\theta)(1) = ad\theta$ is

$$dF = p_g dA = \frac{1}{2}\rho U^2(1 - 4\sin^2\theta)(ad\theta) = \frac{1}{2}\rho a U^2(1 - 4\sin^2\theta)d\theta$$



(a)

Equating the forces along the x and y axes shown in Fig. a,

$$\overset{+}{\leftarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = \int_{\pi/2}^{\pi} dF\cos\theta$$

$$= \int_{\pi/2}^{\pi} \frac{1}{2}\rho a U^2(1 - 4\sin^2\theta)\cos\theta d\theta$$

$$= \int_{\pi/2}^{\pi} \frac{1}{2}\rho a U^2(\cos\theta - 4\sin^2\theta\cos\theta)d\theta$$

$$= \frac{1}{2}\rho a U^2\left(\sin\theta - \frac{4}{3}\sin^3\theta\right)\Bigg|_{\pi/2}^{\pi}$$

$$= \frac{1}{6}\rho a U^2$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -\int_{\pi/2}^{\pi} dF\sin\theta$$

$$= -\int_{\pi/2}^{\pi} \frac{1}{2}\rho a U^2(1 - 4\sin^2\theta)\sin\theta d\theta$$

$$= -\int_{\pi/2}^{\pi} \frac{1}{2}\rho a U^2(\sin\theta - 4\sin^3\theta)d\theta$$

$$= -\frac{1}{2}\rho a U^2\left[-\cos\theta - \left[-\frac{4}{3}\cos\theta(\sin^2\theta + 2)\right]\right]\Bigg|_{\pi/2}^{\pi}$$

$$= \frac{5}{6}\rho a U^2$$

Here, $U = 8\ \text{m/s}$, $\rho = 1.22\ \text{kg/m}^3$, and $a = 0.1\ \text{m}$,

$$(F_R)_x = \frac{1}{6}(1.22\ \text{kg/m}^3)(0.1\ \text{m})(8\ \text{m/s})^2 = 1.3013\ \text{N/m}$$

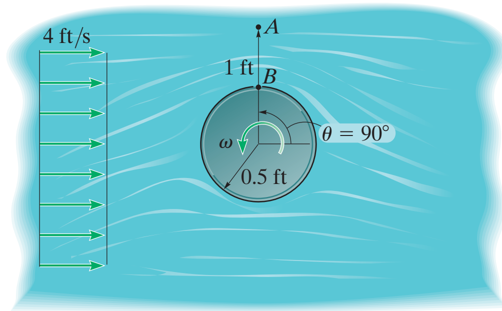
$$(F_R)_y = \frac{5}{6}(1.22\ \text{kg/m}^3)(0.1\text{m})(8\ \text{m/s})^2 = 6.507\ \text{N/m}$$

Thus, per unit length the magnitude of the resultant force on the quarter-segments is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(1.3013\ \text{N})^2 + (6.507\ \text{N})^2}$$

$$= 6.64\ \text{N/m outward (suction)} \qquad \textbf{Ans.}$$

7–89. The 1-ft-diameter cylinder is rotating at $\omega = 5$ rad/s while it is subjected to a uniform flow having a velocity of 4 ft/s. Determine the lift force on the cylinder per unit length. Take $\rho = 2.38\left(10^{-3}\right)$ slug/ft^3.



SOLUTION

For the corresponding free vortex, at $r = 0.5$ ft,

$$v_\theta = \omega r = (5\text{ rad/s})(0.5\text{ ft}) = 2.5\text{ ft/s}$$

$$v_r = 0$$

Thus, the circulation around the cylinder can be determined using

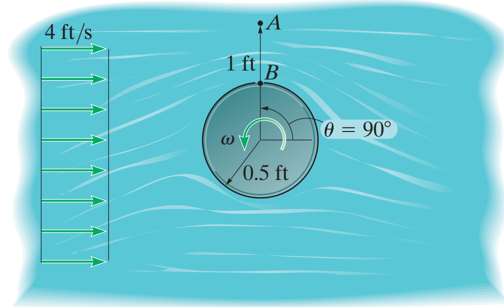
$$\Gamma = \oint \mathbf{V}\cdot d\mathbf{s} = \int_0^{2\pi} v_\theta r\, d\theta = \int_0^{2\pi} (2.5\text{ ft/s})(0.5\text{ ft})d\theta = 1.25\theta\big|_0^{2\pi} = 2.5\pi\text{ ft}^2/\text{s}$$

The uplift force on the cylinder can be determined using

$$F_y = \rho U\Gamma = \left[2.38(10^{-3})\text{ slug/ft}^3\right](4\text{ ft/s})(2.5\pi\text{ ft}^2/\text{s})$$
$$= 0.0748\text{ lb/ft}$$ **Ans.**

Ans:
0.0748 lb/ft

7–90. The 1-ft-diameter cylinder is rotating at $\omega = 8$ rad/s while it is subjected to a flow having a uniform horizontal velocity of 4 ft/s. If the pressure within the uniform flow is 80 lb/ft^2, determine the pressure on the surface B of the cylinder at $\theta = 90°$, and at A, where $r = 1$ ft, $\theta = 90°$. Also find the resultant force acting per unit length of the cylinder. Take $\rho = 1.94$ slug/ft^3.



SOLUTION

We consider ideal fluid flow.

For the corresponding free vortex, at $r = 0.5$ ft, $v_\theta = \omega r = (8\text{ rad/s})(0.5\text{ ft}) = 4$ ft/s. Thus, the circulation is

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s} = \int_0^{2\pi} v_0(r d\theta) = \int_0^{2\pi} 4(0.5\, d\theta) = 2(2\pi) = 4\pi \text{ ft}^2/\text{s}.$$

The velocity components of the flow around the cylinder are

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta \qquad v_\theta = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta + \frac{\Gamma}{2\pi r}$$

Here, $U = 4$ ft/s, $a = 0.5$ ft and $\theta = 90°$. For point A, $r_A = 1$ ft. Then

$$(v_r)_A = (4\text{ ft/s})\left[1 - \frac{(0.5\text{ ft})^2}{(1\text{ ft})^2}\right]\cos 90° = 0$$

$$(v_\theta)_A = -(4\text{ ft/s})\left[1 + \frac{(0.5\text{ ft})^2}{(1\text{ ft})^2}\right]\sin 90° + \frac{4\pi\text{ ft}^2/\text{s}}{2\pi(1\text{ ft})} = -3\text{ ft/s}$$

Thus, $V_A = (v_\theta)_A = 3$ ft/s.

For point B, on the surface, $(v_r)_B = 0$

$$(v_\theta)_B = -(4\text{ft/s})\left[1 + \frac{(0.5\text{ ft})^2}{(0.5\text{ ft})^2}\right]\sin 90° + \frac{4\pi\text{ ft}^2/\text{s}}{2\pi\,(0.5\text{ ft})} - \omega r = -4\text{ft/s}$$

Thus, $V_B = (v_\theta)_B = 4$ ft/s.

Since the flow is irrotational, Bernoulli's equation can be applied between two points located on the different streamlines. Here these two points are one within the uniform flow and the other one is point A (or B)

$$\frac{p_0}{\rho} + \frac{v_0^2}{2} + gz_0 = \frac{p}{\rho} + \frac{v^2}{2} + gz$$

Since the flow occurs in the horizontal plane, $z_0 = z$

Here, $V_0 = U$, Thus

$$p = p_0 + \frac{\rho}{2}(U^2 - V^2)$$

Here, $U = 4$ ft/s; $p_0 = 80$ lb/ft^2. At point A,

$$p_A = 80\text{ lb/ft}^2 + \frac{1.94\text{ slug/ft}^3}{2}\left[(4\text{ ft/s})^2 - (3\text{ ft/s})^2\right]$$

$$= 86.8\text{ lb/ft}^2 \qquad \textbf{Ans.}$$

At point B,

$$p_B = 80\text{ lb/ft}^2 + \frac{1.94\text{ slug/ft}^3}{2}\left[(4\text{ft/s})^2 - (4\text{ft/s})^2\right]$$

$$= 80\text{ lb/ft}^2 \qquad \textbf{Ans.}$$

The resultant force acting on the cylinder is

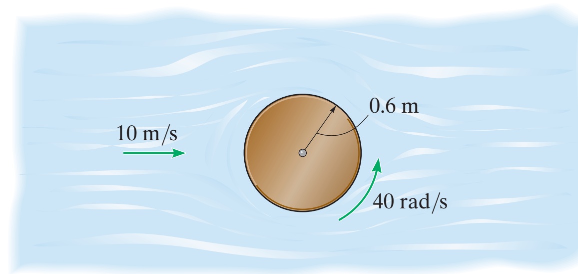
$$F_y = -\rho U \Gamma = -(1.94\text{ slug/ft}^3)(4\text{ ft/s})(4\pi\text{ ft}^2/\text{s})$$

$$= -97.5\text{ lb/ft}$$

$$= 97.5\text{ lb/ft}\downarrow$$

Ans:
$p_A = 86.8$ lb/ft^2
$p_B = 80$ lb/ft^2
$F_y = 97.5$ lb/ft

7–91. The cylinder rotates counterclockwise at 40 rad/s. If the uniform velocity of the air is 10 m/s, and the pressure within the uniform flow is 300 Pa, determine the maximum and minimum pressure on the surface of the cylinder. Also, what is the lift force on the cylinder? Take $\rho_a = 1.20\ \text{kg/m}^3$.



SOLUTION

We consider ideal fluid flow.
For the corresponding free vortex at $r = 0.6$ m, $v_\theta = \omega r = (40\ \text{rad/s})(0.6\ \text{m}) = 24\ \text{m/s}$. Thus, the circulation is

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s} = \int_0^{2\pi} v_\theta(r\,d\theta) = \int_0^{2\pi} 24(0.6 d\theta) = 28.8\pi\ \text{m}^2/\text{s}$$

Since $\Gamma > 4\pi Ua = 4\pi(10\ \text{m/s})(0.6\ \text{m}) = 24\pi\ \text{m}^2/\text{s}$, the stagnation point will not be on the surface of the wheel. The pressure at a point on the surface is

$$p = p_0 + \frac{1}{2}\rho U^2\left[1 - \left(-2\sin\theta + \frac{\Gamma}{2\pi Ua}\right)^2\right]$$

Since $\Gamma > 4\pi Ua$, the term $-2\sin\theta + \dfrac{\Gamma}{2\pi\ va}$ is the smallest when $\theta = 90°$, which yields the maximum pressure. Thus,

$$p_{\max} = p_0 + \frac{1}{2}\rho U^2\left[1 - \left(-2 + \frac{\Gamma}{2\pi Ua}\right)^2\right]$$

$$= 300\ \text{Pa} + \frac{1}{2}(1.20\ \text{kg/m}^3)(10\ \text{m/s})^2\left\{1 - \left[-2 + \frac{28.8\pi\ \text{m}^2/\text{s}}{2\pi(10\ \text{m/s})(0.6\ \text{m})}\right]^2\right\}$$

$$= 350\ \text{Pa} \qquad \textbf{Ans.}$$

Also, we notice that the minimum pressure occurs at a point where $\theta = 90°$. Then

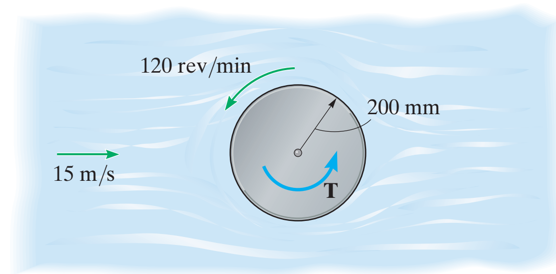
$$p_{\min} = p_0 + \frac{1}{2}\rho U^2\left[1 - \left(2 + \frac{\Gamma}{2\pi Ua}\right)^2\right]$$

$$= 300\ \text{Pa} + \frac{1}{2}(1.20\ \text{kg/m}^3)(10\ \text{m/s})^2\left\{1 - \left[2 + \frac{28.8\pi\ \text{m}^2/\text{s}}{2\pi(10\ \text{m/s})(0.6\ \text{m})}\right]^2\right\}$$

$$= -802\ \text{Pa} \qquad \textbf{Ans.}$$

Ans:
$p_{\max} = 350$ Pa
$p_{\min} = -802$ Pa

***7–92.** A torque **T** is applied to the cylinder, causing it to rotate counterclockwise with a constant angular velocity of 120 rev/min. If the wind is blowing at a constant speed of 15 m/s, determine the location of the stagnation points on the surface of the cylinder, and find the maximum pressure. The pressure within the uniform flow is 400 Pa. Take $\rho_a = 1.20\ \text{kg/m}^3$.



SOLUTION

We consider ideal fluid flow.
For the corresponding free vortex at $r = 0.2$ m, $v_\theta = \omega r$
$= \left[\left(120\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{m}}{60\ \text{s}}\right)\right](0.2\ \text{m}) = 0.8\pi\ \text{m/s}$. Thus, the circulation of this free vortex is

$$\Gamma = \oint \mathbf{V}\cdot d\mathbf{s} = \int_0^{2\pi} v_\theta(r\,d\theta) = \int_0^{2\pi}(0.8\ \pi)(0.2\ d\theta) = 0.32\pi^2\ \text{m}^2/\text{s}$$

Since $\Gamma < 4\pi Ua = 4\pi(15\ \text{m/s})(0.2\ \text{m}) = 12\pi\ \text{m}^2/\text{s}$, there exist two stagnation points on the surface. The location of these two points can be found using

$$\sin\theta = \frac{\Gamma}{4\pi Ua} = \frac{0.32\pi^2\ \text{m}^2/\text{s}}{4\pi(15\ \text{m/s})(0.2\ \text{m})} = 0.08378$$

$$\theta = 4.806^\circ = 4.81^\circ \text{ or } \theta = 175.19^\circ = 176^\circ \qquad \textbf{Ans.}$$

The maximum pressure occurs at the stagnation point where $V = 0$. Since the flow is irrotational, Bernoulli's equation can be applied between two points on different streamlines, such as one within the uniform flow and the other one at the stagnation point. Here, the gravity term can be neglected since the flow involves air which has a low density.

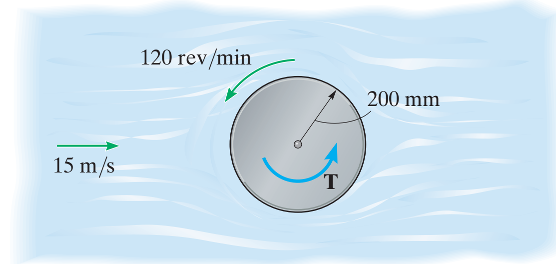
$$\frac{p_0}{\rho_a} + \frac{V_0^{\,2}}{2} = \frac{p}{\rho_a} + \frac{V^2}{2}$$

Here $V_0 = U = 15$ m/s, $p_0 = 400$ Pa, $p = p_{\max}$ and $V = 0$. Then

$$\frac{400\ \text{N/m}^2}{1.20\ \text{kg/m}^3} + \frac{(15\ \text{m/s})^2}{2} = \frac{p_{\max}}{1.20\ \text{kg/m}^3} + 0$$

$$p_{\max} = 535\ \text{Pa} \qquad \textbf{Ans.}$$

7–93. A torque **T** is applied to the cylinder, causing it to rotate counterclockwise with a constant angular velocity of 120 rev/min. If the wind is blowing at a constant speed of 15 m/s, determine the lift per unit length on the cylinder and the minimum pressure on the cylinder. The pressure within the uniform flow is 400 Pa. Take $\rho_a = 1.20\ \text{kg/m}^3$.



SOLUTION

We consider ideal fluid flow.
For the corresponding free vortex at $r = 0.2$ m, $v_\theta = \omega r$ $= \left[\left(120\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\right](0.2\ \text{m}) = 0.8\pi\ \text{m/s}$. Thus, the circulation of this free vortex is

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s} = \int_0^{2\pi} v_\theta (r\, d\theta) = \int_0^{2\pi} (0.8\pi)(0.2\, d\theta) = 0.32\pi^2\ \text{m}^2/\text{s}$$

The "lift" exerted on the cylinder can be determined from

$$F = \rho U \Gamma = \left(1.20\ \text{kg/m}^3\right)(15\ \text{m/s})\left(0.32\pi^2\ \text{m}^2/\text{s}\right)$$
$$= 5.68\ \text{kN/m} \qquad \textbf{Ans.}$$

The pressure at a point on the surface is

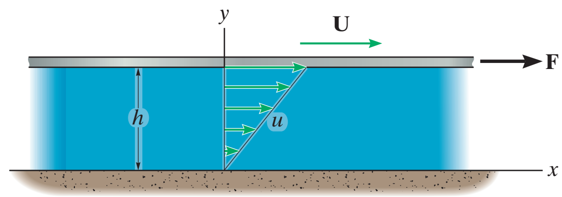
$$p = p_0 + \frac{1}{2}\rho U^2\left[1 - \left(-2\sin\theta + \frac{\Gamma}{2\pi U a}\right)^2\right]$$

We notice from this equation that p will be minimum at a point where $\theta = -90°$. Then

$$p = p_0 + \frac{1}{2}\rho_a U^2\left[1 - \left(-2\sin(-90°) + \frac{\Gamma}{2\pi U a}\right)^2\right]$$
$$= p_0 + \frac{1}{2}\rho_a U^2\left[1 - \left(2 + \frac{\Gamma}{2\pi U a}\right)^2\right]$$
$$= 400\ \text{Pa} + \frac{1}{2}\left(1.20\ \text{kg/m}^3\right)(15\ \text{m/s})^2\left\{1 - \left[2 + \frac{0.32\pi^2\ \text{m}^2/\text{s}}{2\pi(15\ \text{m/s})(0.2\ \text{m})}\right]^2\right\}$$
$$= -99.3\ \text{Pa} \qquad \textbf{Ans.}$$

Ans:
$F = 5.68\ \text{kN/m}$
$p = -99.3\ \text{Pa}$

7–94. Liquid is confined between a top plate having an area A and a fixed surface. A force $\mathbf{F}$ is applied to the plate and gives the plate a velocity $\mathbf{U}$. If this causes laminar flow, and the pressure does not vary, show that the Navier–Stokes and continuity equations indicate that the velocity distribution for this flow is defined by $u = U(y/h)$, and that the shear stress within the liquid is $\tau_{xy} = F/A$.



SOLUTION

Since the flow is steady and is only along the x axis then $v = w = 0$. Also, the liquid is incompressible. Thus, the continuity equation reduces to

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$0 + \rho\frac{\partial u}{\partial x} + 0 + 0 = 0$$

$$\frac{\partial u}{\partial x} = 0$$

Integrating this equation with respect to x

$$u = u(y)$$

Using this result, when the pressure p remains constant along x axis, then Navier-Stokes equation along x axis gives

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$0 + 0 + 0 + 0 = 0 - 0 + \mu\left(0 + \frac{\partial^2 u}{\partial y^2} + 0\right)$$

$$\mu\frac{\partial^2 u}{\partial y^2} = 0$$

Since u is a function of y only, $\frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dy^2}$. Integrating this equation with respect to y twice.

$$\frac{du}{dy} = C_1 \qquad \textbf{(1)}$$

And,

$$u = C_1 y + C_2 \qquad \textbf{(2)}$$

Applying the boundary condition, $u = 0$ at $y = 0$

$$0 = C_1(0) + C_2 \quad C_2 = 0$$

and $u = U$ at $y = h$,

$$U = C_1(h) \quad C_1 = \frac{U}{h}$$

Substituting these results into Eq. 1

$$u = \left(\frac{U}{h}\right)y \qquad \textbf{(Q.E.D)}$$

7–94. Continued

Applying

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

Here, $\dfrac{\partial v}{\partial x} = 0$ and from Eq. 1 $\dfrac{\partial u}{\partial y} = \dfrac{U}{h}$. Then

$$\tau_{xy} = \mu\left(\frac{U}{h} + 0\right) = \mu\left(\frac{U}{h}\right)$$

This shows that τ_{xy} is a constant between the liquid layers. Therefore, its value is equal to that at the bottom surface of the top plate which is

$$\tau_{xy} = \frac{F}{A} \qquad \textbf{(Q.E.D)}$$

Although not necessary, the Navier-Stokes equation along the y axis gives

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial \rho}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho(0 + 0 + 0 + 0) = \rho(-g) - \frac{\partial p}{\partial y} + \mu(0 + 0 + 0 + 0)$$

$$\frac{\partial \rho}{\partial y} = -\rho g$$

Integrating this equation with respect to y by realizing the pressure variation is along y axis only,

$$p = -\rho g y + C_3$$

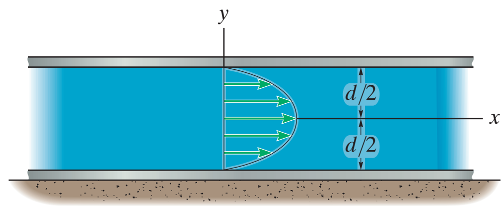
Applying the boundary condition $p = 0$ at $y = h$,

$$0 = -\rho g h + C_3 \quad C_3 = \rho g h$$

Then

$$p = \rho g(h - y)$$

7–95. The channel for a liquid is formed by two fixed plates. If laminar flow occurs between the plates, show that the Navier–Strokes and continuity equations reduce to $\partial^2 u/\partial y^2 = (1/\mu)\,\partial p/\partial x$ and $\partial p/\partial y = 0$. Integrate these equations to show that the velocity profile for the flow is $u = (1/(2\mu))\,(dp/dx)\left[y^2 - (d/2)^2\right]$. Neglect the effect of gravity.



SOLUTION

Since the flow is steady and is along the x axis only, then $v = w = 0$. Also, the liquid is incompressible. Thus, the continuity equation becomes

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$0 + \rho\frac{\partial u}{\partial x} + 0 + 0 = 0$$

$$\frac{\partial u}{\partial x} = 0$$

Integrating this equation with respect to x,

$$u = u(y)$$

Using this result, the Navier-Stokes equation along the x and y axes gives

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho(0 + 0 + 0 + 0) = 0 - \frac{\partial p}{\partial x} + \mu\left(0 + \frac{\partial^2 u}{\partial y^2} + 0\right)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} \qquad \textbf{(Q.E.D)} \qquad \textbf{(1)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho(0 + 0 + 0 + 0) = 0 - \frac{\partial p}{\partial y} + \mu(0 + 0 + 0)$$

$$\frac{\partial p}{\partial y} = 0 \qquad \textbf{(Q.E.D)} \qquad \textbf{(2)}$$

Integrating Eq. 2 with respect to y,

$$p = p(x)$$

Since u is a function of y only and p is a function of x only, then $\frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dy^2}$ and $\frac{\partial \rho}{\partial x} = \frac{d\rho}{dx}$. Eq. 1 becomes

$$\frac{d^2 u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

***7–95. Continued**

Integrating this equation twice with respect to y,

$$\frac{du}{dy} = \left(\frac{1}{\mu}\frac{dp}{dx}\right)y + C_1 \qquad \textbf{(3)}$$

$$u = \left(\frac{1}{2\mu}\frac{dp}{dx}\right)y^2 + C_1 y + C_2 \qquad \textbf{(4)}$$

Since u is maximum at $y = 0$. Then $\frac{du}{dy} = 0$ at $y = 0$. Using Eq. 3

$$0 = \left(\frac{1}{\mu}\frac{dp}{dx}\right)(0) + C_1 \qquad C_1 = 0$$

Also, $u = 0$ at $y = \frac{d}{2}$. Using Eq. 4 with $C_1 = 0$,

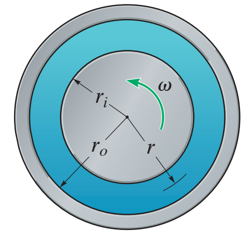
$$0 = \left(\frac{1}{2\mu}\frac{dp}{dx}\right)\left(\frac{d}{2}\right)^2 + 0 + C_2 \qquad C_2 = -\left(\frac{1}{2\mu}\frac{dp}{dx}\right)\left(\frac{d}{2}\right)^2$$

Substituting these results into Eq. 4,

$$u = \left(\frac{1}{2\mu}\frac{dp}{dx}\right)y^2 - \left(\frac{1}{2\mu}\frac{dp}{dx}\right)\left(\frac{d}{2}\right)^2$$

$$= \left(\frac{1}{2\mu}\frac{dp}{dx}\right)\left[y^2 - \left(\frac{d}{2}\right)^2\right] \qquad \textbf{(Q.E.D)}$$

***7–96.** Fluid having a density ρ and viscosity μ fills the space between the two cylinders. If the outer cylinder is fixed, and the inner one is rotating at ω, apply the Navier–Stokes equations to determine the velocity profile assuming laminar flow.



SOLUTION

Since the flow is steady and is along the transverse direction (θ axis) only, then $v_r = v_z = 0$. Also, the liquid is incompressible. Thus, the continuity equation reduces to

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(r\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

$$0 + 0 + \frac{\rho}{r}\frac{\partial v_\theta}{\partial_\theta} + 0 = 0$$

$$\frac{\partial v_\theta}{\partial_\theta} = 0$$

Integrating this equation with respect to θ,

$$v_\theta = v_\theta(r)$$

Using this result, the Navier-Stokes equations along the θ axis gives

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$\rho(0 + 0 + 0 + 0 + 0) = -0 + 0 + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} + 0 + 0 + 0\right]$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} = 0$$

However, it can be shown that

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial_r}(r v_\theta)\right] = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2}$$

Thus,

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right] = 0$$

7–96. Continued

Since $v_\theta = v_\theta(r)$, then the above equation can be written in the form of

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rv_\theta)\right] = 0$$

Integrating this equation with respect to r,

$$\frac{1}{r}\frac{d}{dr}(rv_\theta) = C_1$$

$$\frac{d}{dr}(rv_\theta) = C_1 r$$

Integrating again,

$$rv_\theta = C_1\left(\frac{r^2}{2}\right) + C_2$$

$$v_\theta = \frac{C_1}{2}r + \frac{C_2}{r} \qquad \textbf{(1)}$$

At $r = r_\theta$, $v_\theta = 0$. Then Eq. 1 gives

$$0 = \frac{C_1}{2}(r_0) + \frac{C_2}{r_0} \qquad \textbf{(2)}$$

At $r = r_i$, $v_\theta = \omega r_i$. Then Eq. 1 gives

$$\omega r_i = \frac{C_1}{2}r_i + \frac{C_2}{ri} \qquad \textbf{(3)}$$

Solving Eq. 2 and 3,

$$C_1 = -\frac{2\omega r_i^2}{r_0^2 - r_i^2} \qquad C_2 = \frac{\omega r_i^2 r_0^2}{r_0^2 - r_i^2}$$

Substituting these results into Eq. 1,

$$v_\theta = -\left(\frac{\omega r_i^2}{r_0^2 - r_i^2}\right)r + \left(\frac{\omega r_i^2 r_0^2}{r_0^2 - r_i^2}\right)\left(\frac{1}{r}\right)$$

$$v_\theta = \frac{\omega r_i^2}{r_0^2 - r_i^2}\left(\frac{r_0^2 - r^2}{r}\right) \qquad \textbf{Ans.}$$

7–97. A horizontal velocity field is defined by $u = 2\left(x^2 - y^2\right)$ ft/s and $v = (-4xy)$ ft/s. Show that these expressions satisfy the continuity equation. Using the Navier–Stokes equations, show that the pressure distribution is defined by $p = C - \rho V^2/2 - \rho g z$.

SOLUTION

For the continuity equation,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}$$

$$= 0 + 4\rho x + (-4\rho x) + 0$$

$$= 0 \text{ (satisfied)}$$

The Navier-Stokes equations along the x, y and z axes are

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left[0 + \left(2x^2 - 2y^2\right)(4x) + (-4xy)(-4y) + 0\right] = 0 - \frac{\partial p}{\partial x} + \mu(4 - 4 + 0)$$

$$\frac{\partial p}{\partial x} = -8\rho\left(x^3 + xy^2\right) \qquad \textbf{(1)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial \rho}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left[0 + \left(2x^2 - 2y^2\right)(-4y) + (-4xy)(-4x) + 0\right] = 0 - \frac{\partial \rho}{\partial y} + \mu(0 + 0 + 0)$$

$$\frac{\partial \rho}{\partial y} = -8\rho\left(y^3 + x^2 y\right) \qquad \textbf{(2)}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial \rho}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

$$\rho(0 + 0 + 0 + 0) = \rho(-g) - \frac{\partial \rho}{\partial z} + \mu(0 + 0 + 0)$$

$$\frac{\partial \rho}{\partial z} = -\rho g \qquad \textbf{(3)}$$

Integrating Eqs. 1 with respect to x,

$$p = -8\rho\left(\frac{x^4}{4} + \frac{x^2 y^2}{2}\right) + f(y) + g(z) \qquad \textbf{(4)}$$

Differentiate Eq. 4 with respect to y and equate to Eq. 2

$$\frac{\partial \rho}{\partial y} = -8\rho\left(x^2 y\right) + f^1(y) = -8\rho\left(y^3 + x^2 y\right)$$

$$f^1(y) = -8\rho y^3$$

7–97. Continued

Integrate this equation with respect to y,

$$f(y) = -2\rho y^4 + C_1 \qquad \textbf{(5)}$$

Differentiate Eq. 4 with respect to z and equal to Eq. 3

$$\frac{\partial \rho}{\partial z} = g^1(z) = -\rho g$$

Integrate this equation with respect to z,

$$g(z) = -\rho g z + C_2 \qquad \textbf{(6)}$$

Substitute Eq. 5 and 6 into 4,

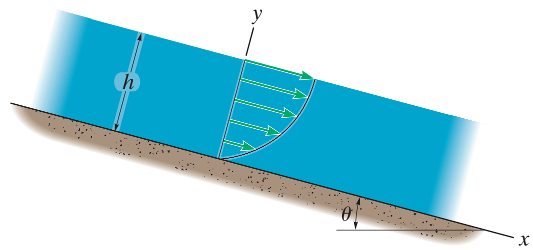
$$p = -8\rho\left(\frac{x^4}{4} + \frac{x^2y^2}{2}\right) - 2\rho y^4 - \rho g z + C$$

$$p = -2\rho\left(x^4 + y^4 + 2x^2y^2\right) - \rho g z + C$$

Since $\frac{1}{2}V^2 = \frac{1}{2}\left(u^2 + v^2\right) = \frac{1}{2}\left[\left(2x^2 - 2y^2\right)^2 + (-4xy)^2\right] = 2\left(x^4 + y^4 + 2x^2y^2\right)$, then the above equation becomes

$$p = C - \frac{1}{2}\rho V^2 - \rho g z \qquad \textbf{(Q.E.D)}$$

7–98. The sloped open channel has steady laminar flow at a depth h. Show that the Navier–Stokes equations reduce to $\partial^2 u/\partial y^2 = -(\rho\, g \sin\theta)/\mu$ and $\partial p/\partial y = -\rho\, g\cos\theta$. Integrate these equations to show that the velocity profile is $u = [(\rho\, g \sin\theta)/2\mu]\left(2hy - y^2\right)$ and the shear-stress distribution is $\tau_{xy} = \rho\, g \sin\theta\,(h - y)$.



SOLUTION

Since the flow is steady and is along the x axis only, then $v = w = 0$. Also, the liquid is incompressible. Thus, the continuity equation reduces to

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$0 + \rho\frac{\partial u}{\partial x} + 0 + 0 = 0$$

$$\frac{\partial u}{\partial x} = 0$$

Integrating this equation with respect to x,

$$u = u(y)$$

The Navier-Stokes equations along the x and y axes give

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho(0 + 0 + 0 + 0) = \rho g \sin\theta - 0 + \mu\left(0 + \frac{\partial^2 u}{\partial y^2} + 0\right)$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{\rho g \sin\theta}{\mu} \qquad \textbf{(Q.E.D)} \qquad \textbf{(1)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho(0 + 0 + 0 + 0) = \rho(-g\cos\theta) - \frac{\partial p}{\partial y} + \mu(0 + 0 + 0)$$

$$\frac{\partial \rho}{\partial y} = -\rho g \cos\theta \qquad \textbf{(Q.E.D)} \qquad \textbf{(2)}$$

Since $u = u(y)$, then $\dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial^2 u}{\partial y^2}$. Thus Eq. (1) becomes

$$\frac{d^2 u}{dy^2} = -\frac{\rho g \sin\theta}{\mu}$$

Integrating this equation with respect to y twice yields

$$\frac{du}{dy} = -\frac{\rho g \sin\theta}{\mu}y + C_1 \qquad \textbf{(3)}$$

$$u = -\frac{\rho g \sin\theta}{2\mu}y^2 + C_1 y + C_2 \qquad \textbf{(4)}$$

7–98. Continued

At $y = 0, u = 0$. Then, Eq. 4 gives

$$0 = -0 + 0 + C_2 \qquad C_2 = 0$$

At $y = h, \tau_{xy} = \mu\left(\frac{du}{dy}\right) = 0$. Then Eq. 3 gives

$$0 = -\frac{\rho g \sin\theta}{\mu}(h) + C_1 \qquad C_1 = \frac{\rho g h \sin\theta}{\mu}$$

Substituting these results into Eq 3 and 4

$$\frac{du}{dy} = -\frac{\rho g \sin\theta}{\mu}y + \frac{\rho g h \sin\theta}{\mu}$$

$$\frac{du}{dy} = \frac{\rho g \sin\theta}{\mu}(h - y)$$

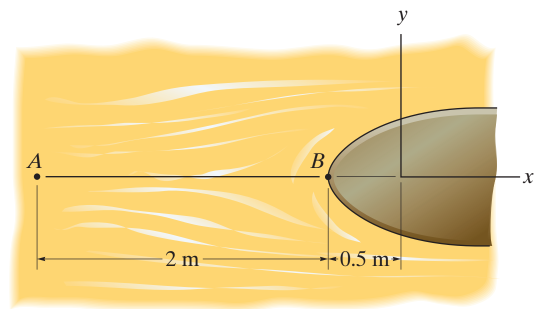
$$u = -\frac{\rho g \sin\theta}{2\mu}y^2 + \frac{\rho g h \sin\theta}{\mu}y$$

$$u = \frac{\rho g \sin\theta}{2\mu}\left(2hy - y^2\right) \qquad \textbf{(Q.E.D)}$$

The shear stress distribution is

$$\tau_{xy} = \mu\frac{du}{dy} = \rho g \sin\theta(h - y) \qquad \textbf{(Q.E.D)}$$

7–99. The laminar flow of a fluid has velocity components $u = 6x$ and $v = -6y$, where y is vertical. Use the Navier–Stokes equations to determine the pressure in the fluid, $p = p(x, y)$, if at point $(0, 0)$, $p = 0$. The density of the fluid is ρ.



SOLUTION

Since the flow is steady and the fluid is incompressible, the continuity equation is

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 + 6\rho + (-6\rho) + 0 = 0$$

is indeed satisfied. Writing the Navier-Stokes equation along the x and y axes gives

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial \rho}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho[0 + 6x(6) + 0 + 0] = 0 - \frac{\partial \rho}{\partial x} + 0$$

$$\frac{\partial \rho}{\partial x} = -36\rho x \qquad \textbf{(1)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial \rho}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho[0 + 0 + (-6y)(-6) + 0] = \rho(-g) - \frac{\partial \rho}{\partial y}$$

$$\frac{\partial \rho}{\partial y} = -\rho g - 36\rho y \qquad \textbf{(2)}$$

Integrating Eq. 1 with respect to x

$$\rho = -18\rho x^2 + f(y) \qquad \textbf{(3)}$$

Takes the partial derivative of Eq. 2 with respect to y and equate it to Eq. 2

$$\frac{\partial \rho}{\partial y} = f^1(y) = -\rho g - 36\rho y$$

Integrate this equation with respect to y,

$$f(y) = -\rho g y - 18\rho y^2 + C$$

Substitute this result into Eq. 3

$$p = -18\rho x^2 - pgy - 18\rho y^2 + C$$

$$p = -\rho\left(18x^2 + 18y^2 + gy\right) + C \qquad \textbf{(4)}$$

At point $(0, 0)$, $p = 0$. Then Eq. 4 gives

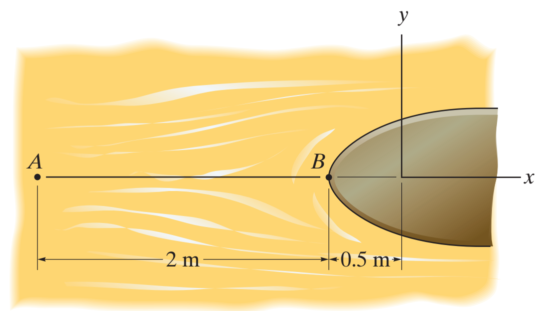
$$0 = \rho(0 + 0 + 0) + C \qquad C = 0$$

Thus, the pressure distribution is

$$p(x, y) = -\rho\left(18x^2 + 18y^2 + gy\right) \qquad \textbf{Ans.}$$

Ans:
$p = -\rho\left(18x^2 + 18y^2 + gy\right)$

***7–100.** The steady laminar flow of an ideal fluid towards the fixed surface has a velocity of $u = \left[10\left(1 + 1/\left(8x^3\right)\right)\right]$ m/s along the horizontal streamline AB. Use the Navier–Stokes equations and determine the variation of the pressure along this streamline, and plot it for $-2.5 \text{ m} \le x \le -0.5$ m. The pressure at A is 5 Pa, and the density of the fluid is $\rho = 1000 \text{ kg/m}^3$.



SOLUTION

We consider ideal fluid flow.
Since streamline AB is along the x axis, the velocity of the flow along this streamline will not have components along the y and z axes; ie, $v = w = 0$. Also, the fluid is ideal and the flow is steady. Writing the Navier-Stoke equation along x axis gives

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left[0 + 10\left(1 + \frac{1}{8x^3}\right)\left(-\frac{30}{8x^4}\right)\right] = 0 - \frac{\partial \rho}{\partial x} + 0\left(\frac{\partial^2 u}{\partial x^2} + 0 + 0\right)$$

$$\frac{\partial \rho}{\partial x} = \frac{75\rho}{2x^4}\left(1 + \frac{1}{8x^3}\right) = \frac{75\rho}{2x^4} + \frac{75\rho}{16x^7}$$

Integrating this equation with respect to x,

$$\rho(x) = -\frac{25\rho}{2x^3} - \frac{25\rho}{32x^6} + C$$

$$\rho(x) = -\frac{25\rho}{32x^6}\left(16x^3 + 1\right) + C$$

At $x = -2.5$ m, $p = 5$ kPa and with $\rho = 1000 \text{ kg/m}^3$. Then,

$$5 = -\frac{25}{32(-2.5)^6}\left[16(-2.5)^3 + 1\right] + C$$

$$C = 4.2032\rho$$

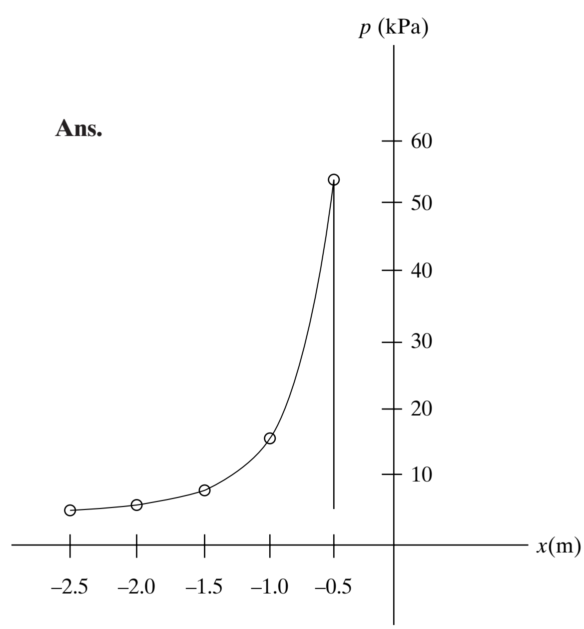
Thus, since $\rho = 1000 \text{ kg/m}^3$,

$$p(x) = \left[-\frac{25}{32x^6}\left(16x^3 + 1\right) + 4.2032\right] \text{kPa}$$

Ans.

The plot of this pressure distribution is shown in Fig. *a*

x(m)	−2.5	−2.0	−1.5	−1.0	−0.5
p(kPa)	5.0	5.75	7.84	15.92	54.20



(a)