Thermodynamics - HW3

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Question 1

Given a column of mercury (Hg) with a density of 13,600 kg/m^3 in a manometer, calculate the pressure difference it indicates when the mercury column height difference is 760 mm. Assume gravitational acceleration is 9.81 m/s^2. Give your result in Pascal.

Solution

The pressure difference in a manometer is determined using the hydrostatic pressure formula:

$$\Delta P = \rho g h \tag{1}$$

where:

- $\rho = 13600$ kg/m^3 (density of mercury)
- $g = 9.81$ m/s^2 (gravitational acceleration)
- $h = 760$ mm $= 0.760$ m (height difference of mercury column)

Substituting the values:

$$\Delta P = (13600)(9.81)(0.760) \tag{2}$$

$$\Delta P = 101325.6\,\text{Pa} \tag{3}$$

Conclusion

The pressure difference indicated by the mercury column is **101,325.6 Pa** or approximately **101.3 kPa**. This value corresponds closely to the standard atmospheric pressure at sea level.

Question 2

Convert the results in question 2 into atm unit. How does this value compare to the standard atmospheric pressure value?

Conversion to Atmospheres

To convert the pressure from Pascals to atmospheres (atm), we use the conversion factor:

$$1 \text{ atm} = 101325 \text{ Pa} \tag{4}$$

Using the previously calculated pressure:

$$\text{Pressure in atm} = \frac{101325.6 \text{ Pa}}{101325 \text{ Pa/atm}} \tag{5}$$

$$= 1.000006 \approx 1.00 \text{ atm} \tag{6}$$

Comparison to Standard Atmospheric Pressure

The calculated pressure of 1.00 atm is essentially equal to the standard atmospheric pressure, confirming that a 760 mm mercury column represents normal atmospheric conditions at sea level.

Question 3

Describe how a constant-volume gas thermometer and a Bourdon tube gauge work for measuring temperature and pressure, respectively. What are the key components of each device, and what principle do they operate on?

Constant-Volume Gas Thermometer

A **constant-volume gas thermometer** measures temperature by utilizing the relationship between pressure and temperature in a gas at constant volume, based on the **ideal gas law**:

$$P = nRT/V \tag{7}$$

where:

- P is the pressure of the gas,
- n is the amount of gas (constant),
- R is the universal gas constant,

- T is the temperature,
- V is the volume (kept constant).

Since volume and the amount of gas remain constant, any change in temperature directly affects the pressure, allowing temperature measurement.

Key Components

- A bulb filled with a gas (e.g., helium or nitrogen). - A pressure-measuring device (manometer or pressure gauge). - A fixed-volume chamber to maintain constant gas volume.

Operating Principle

When the thermometer is placed in contact with a system, the gas inside adjusts its pressure according to the system's temperature. By measuring the pressure, the temperature can be determined using a calibrated scale.

Bourdon Tube Gauge

A **Bourdon tube gauge** is a mechanical device used to measure pressure by converting it into mechanical displacement.

Key Components

- A curved, hollow, flexible metal tube (Bourdon tube). - A pointer and dial mechanism. - Gears and linkages that amplify movement.

Operating Principle

- When pressure is applied inside the Bourdon tube, it tends to straighten out due to the force exerted by the fluid. - This movement is transmitted through gears and linkages to rotate a pointer on a calibrated dial, indicating the pressure. - The deflection is proportional to the applied pressure, allowing accurate pressure readings.

Summary

Both the constant-volume gas thermometer and the Bourdon tube gauge rely on fundamental thermodynamic principles to measure temperature and pressure, respectively. The gas thermometer operates on the ideal gas law, while the Bourdon gauge utilizes mechanical deformation under pressure to provide accurate measurements.

Question 4

Explain the Zeroth Law of Thermodynamics in the context of thermal equilibrium. How can it be used to determine if two bodies are at the same temperature without direct contact?

Zeroth Law of Thermodynamics

The Zeroth Law of Thermodynamics states that:

> *"If two systems are each in thermal equilibrium with a third system, then they are in thermal equilibrium with each other."*

This implies that if we have three bodies or systems A, B, and C, and if:

- A is in thermal equilibrium with C.
- B is in thermal equilibrium with C.

Then, A and B are in thermal equilibrium with each other, even if they have never been in direct contact.

Thermal Equilibrium

Thermal equilibrium is defined as the condition under which two bodies in contact do not exchange energy as heat. When this state is achieved, their temperatures are equal. The temperature is the property that determines whether two objects would be in thermal equilibrium if brought into contact.

Determination of Temperature Without Direct Contact

The Zeroth Law allows temperature measurement by comparison with a third body, typically a thermometer.

Procedure

1. A thermometer C is brought into contact with object A until thermal equilibrium is reached. The temperature is recorded.
2. The same thermometer C is then brought into contact with object B until thermal equilibrium is reached, and the temperature is recorded.
3. If the thermometer shows the same reading for both A and B, it can be concluded that A and B are in thermal equilibrium with each other, even if they have never been in direct contact.

Justification

Using a thermometer as an intermediary body is valid based on the Zeroth Law, which ensures that the temperature readings are accurate indicators of thermal equilibrium between other bodies.

Practical Importance

The Zeroth Law is fundamental for constructing temperature scales and for the proper calibration of thermometers. Without this law, consistent and reproducible temperature measurement would be impossible.

Question 5

Convert 25°C to Fahrenheit (F) and Kelvin (K). Provide the formulas you use for the conversion.

Conversion from Celsius to Fahrenheit

The formula to convert from degrees Celsius (°C) to degrees Fahrenheit (°F) is:

$$F = \frac{9}{5}C + 32$$

Where:

- C is the temperature in degrees Celsius.

Substituting $C = 25^\circ\text{C}$:

$$F = \frac{9}{5}(25) + 32 = 45 + 32 = 77^\circ\text{F}$$

Result: $25^\circ\text{C} = 77^\circ\text{F}$

Conversion from Celsius to Kelvin

The formula to convert from degrees Celsius (°C) to Kelvin (K) is:

$$K = C + 273.15$$

Where:

- C is the temperature in degrees Celsius.

Substituting $C = 25^\circ\text{C}$:

$$K = 25 + 273.15 = 298.15\,\text{K}$$

Result: $25^\circ\text{C} = 298.15\,\text{K}$

Question 6

Using the Ideal Gas Law, calculate the volume occupied by one mole of an ideal gas at standard temperature and pressure (STP), where $R = 8.314$ J/(mol·K), $T = 273.15$ K, and $P = 101,325$ Pa.

Ideal Gas Law

The Ideal Gas Law is expressed by the equation:

$$PV = nRT$$

Where:

- P = Pressure in pascals (Pa)
- V = Volume in cubic meters (m^3)
- n = Number of moles
- R = Ideal gas constant (8.314 J/(mol·K))
- T = Temperature in Kelvin (K)

Given Data

- $n = 1$ mol (one mole of ideal gas)
- $R = 8.314$ J/(mol·K)
- $T = 273.15$ K (Standard Temperature)
- $P = 101,325$ Pa (Standard Pressure)

Calculation of Volume

Rearranging the Ideal Gas Law equation to solve for V:

$$V = \frac{nRT}{P}$$

Substituting the given values:

$$V = \frac{(1)(8.314)(273.15)}{101,325}$$

$$V = \frac{2270.52}{101,325} \approx 0.0224\,\mathrm{m}^3$$

Result

The volume occupied by one mole of an ideal gas at standard temperature and pressure (STP) is approximately:

$$\boldsymbol{V \approx 0.0224\,\mathrm{m}^3}$$

Relation Between R and k_B

The ideal gas constant R is related to the Boltzmann constant k_B by the equation:

$$R = N_A k_B$$

Where:

- N_A = Avogadro's number (6.022×10^{23} mol^{-1})
- k_B = Boltzmann constant (1.380649×10^{-23} J/K)

$$R = (6.022 \times 10^{23}) \times (1.380649 \times 10^{-23}) \approx 8.314\,\mathrm{J/mol{\cdot}K}$$

This relationship shows that R is essentially a scaled-up version of k_B when applied to a mole of particles rather than a single particle.

Question 7

Discuss the conditions under which a real gas behaves like an ideal gas. Why do deviations from ideal gas behavior occur at high pressures?

Conditions for Ideal Gas Behavior

A real gas behaves like an ideal gas under the following conditions:

1. **Low Pressure:** At low pressures, gas molecules are sufficiently separated, and intermolecular forces are negligible. The particles behave as point masses without interaction between them.
2. **High Temperature:** At high temperatures, the average kinetic energy of the molecules is much greater than the energy of intermolecular interactions. Therefore, collisions are essentially elastic, and attractive or repulsive forces are negligible.
3. **Low Density Gases:** When the density is low, molecules are far apart, reducing interactions between them.
4. **Monoatomic Gases:** Gases such as helium or argon, which are monoatomic and exhibit weak interactions, approximate ideal gas behavior better than diatomic or polyatomic gases.

Deviations from Ideal Gas Behavior at High Pressures

At **high pressures**, a real gas deviates significantly from ideal gas behavior due to:

1. **Molecular Volume:** At high pressures, molecules are compressed closer together. Therefore, the volume occupied by the molecules cannot be neglected as assumed in the ideal gas model.
2. **Intermolecular Forces:** When molecules are very close, attractive forces (such as Van der Waals forces) or repulsive forces become significant. At high pressures, the proximity of the molecules increases the likelihood of these forces affecting the gas behavior.
3. **Condensation:** Under certain conditions, if the pressure is sufficiently high and the temperature is low, the gas may condense into a liquid, which is a behavior completely deviated from the ideal gas model.

Summary

In summary, a real gas behaves like an ideal gas when **molecular interactions are insignificant** and the volume of the molecules is small compared to the total volume of the gas. This typically occurs at **low pressure and high temperature**. Deviations mainly occur due to **intermolecular forces and finite molecular volume**, especially at **high pressures**.

Question 8

Consider 1 mole of an ideal gas at three different temperatures (273 K, 298 K, and 323 K). Calculate the pressure for each temperature at volumes ranging from 0.01 m^3 to 0.10 m^3 in increments of 0.01 m^3. Tabulate your results and plot isotherms by setting the volume on the x-axis and pressure on the y-axis. Explain your results.

Ideal Gas Law and Pressure Calculation

The Ideal Gas Law is given by:

$$PV = nRT$$

Rearranging to solve for pressure (P):

$$P = \frac{nRT}{V}$$

Where:

- P = Pressure in pascals (Pa)

- V = Volume in cubic meters (m^3)
- n = Number of moles (1 mol in this case)
- R = Ideal gas constant (8.314 J/(mol·K))
- T = Temperature in Kelvin (K)

Calculation and Tabulation

The following table shows the calculated pressures for three temperatures: 273 K, 298 K, and 323 K. Volumes range from 0.01 m^3 to 0.10 m^3 in increments of 0.01 m^3.

Volume (m^3)	**Pressure at 273 K (Pa)**	**Pressure at 298 K (Pa)**	**Pressure at 323 K (Pa)**
0.01	226972.20	247757.20	268542.20
0.02	113486.10	123878.60	134271.10
0.03	75657.40	82585.73	89514.07
0.04	56743.05	61939.30	67135.55
0.05	45394.44	49551.44	53708.44
0.06	37828.70	41292.87	44757.04
0.07	32424.60	35393.89	38363.17
0.08	28371.52	30969.65	33567.78
0.09	25219.13	27528.58	29838.02
0.10	22697.22	24775.72	26854.22

Table 1: Pressure values calculated using the Ideal Gas Law for different volumes and temperatures.

Graphical Representation

The plot below shows the isotherms for the three temperatures: 273 K, 298 K, and 323 K.

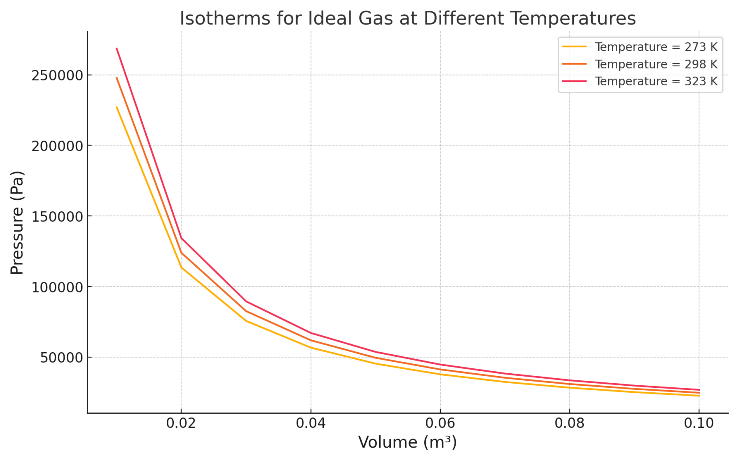

Figure 1: Isotherms for Ideal Gas at Different Temperatures

Explanation of Results

- As the volume increases, the pressure decreases for each temperature, consistent with Boyle's Law: $P \propto \frac{1}{V}$ when T is constant.
- Higher temperatures result in higher pressure for a given volume. This is because increased temperature means increased kinetic energy of molecules, thus exerting greater force per unit area.
- The curves (isotherms) are hyperbolic, which is characteristic of ideal gas behavior at constant temperature.

Question 9

Explain the difference between adiabatic and isothermal processes.

Adiabatic Process

An **adiabatic process** is one in which there is **no heat transfer** between the system and its surroundings. Any change in internal energy is solely due to the **work done by or on the system**.

- $Q = 0$ (no heat transfer).
- The internal energy changes due to the work done (W).
- If the gas is compressed rapidly, the temperature increases.
- If the gas expands rapidly, the temperature decreases.

- Example: Rapid compression in an internal combustion engine.

The relationship between pressure and volume for an adiabatic process in an ideal gas is given by:

$$PV^{\gamma} = \text{constant}$$

Where $\gamma = \frac{C_p}{C_v}$ is the heat capacity ratio.

Isothermal Process

An **isothermal process** is one in which the **temperature of the system remains constant** throughout the process. To maintain constant temperature, continuous heat exchange with the surroundings is required.

- $T = \text{constant}$.
- $\Delta U = 0$ for an ideal gas (no change in internal energy).
- Work done is balanced by heat transfer (Q).
- Example: Slow expansion or compression of an ideal gas in a piston with heat-conductive walls.

The relationship between pressure and volume for an isothermal process in an ideal gas is described by Boyle's Law:

$$PV = \text{constant}$$

Key Differences

Characteristic	**Adiabatic Process**	**Isothermal Process**
Heat Transfer (Q)	None ($Q = 0$)	Continuous ($Q \neq 0$)
Temperature (T)	Changes	Remains constant
Examples	Rapid compression in engines	Slow expansion in pistons
Relationship (PV)	$PV^{\gamma} = \text{constant}$	$PV = \text{constant}$

Table 2: Comparison between Adiabatic and Isothermal Processes

Question 10

Discuss how thermodynamics principles can be applied to develop sustainable energy solutions, such as in the design of solar thermal systems or geothermal heating systems. Describe the role of thermal efficiency and energy conservation play in these systems.

Application of Thermodynamics in Sustainable Energy

Thermodynamic principles are fundamental in designing and optimizing systems that harness renewable energy sources, such as solar and geothermal energy.

1. Solar Thermal Systems

- Solar thermal systems capture solar energy using **solar collectors** and convert it into heat for use in **heating or electricity generation**.
- Based on the **First Law of Thermodynamics**: The absorbed energy is converted into useful heat or lost through dissipation.
- The efficiency is measured by **thermal efficiency**, defined as:

$$\eta = \frac{\text{Useful Energy Output}}{\text{Incident Solar Energy}}$$

- Proper material design, insulation, and heat flow control are essential to maximize efficiency.

2. Geothermal Heating Systems

- Geothermal systems utilize **thermal energy stored within the Earth** for heating or electricity generation.
- They employ thermodynamic cycles such as the **Rankine Cycle** or **Reverse Refrigeration Cycle** to transfer heat.
- The **Second Law of Thermodynamics** helps identify losses and improve efficiency in these systems.
- By using renewable sources, they reduce dependence on fossil fuels and minimize emissions.

Thermal Efficiency and Energy Conservation

- **Thermal Efficiency:** Measures how effectively a system converts available energy into useful work or heat. Improving efficiency involves minimizing losses and maximizing the use of available energy.
- **Energy Conservation:** Ensures that energy extracted from natural sources is used efficiently, respecting the **First Law of Thermodynamics**.

Conclusion

Applying thermodynamic principles effectively enables the development of more efficient and sustainable technologies, crucial for addressing current energy challenges. Optimizing thermal efficiency and adhering to energy conservation principles are essential for the success of these technologies.