Thermodynamics - HW7

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1. Energy Change Interpretation

If the change in energy of a closed system is known for a process between two end states, can you determine if the energy change was due to work, heat transfer, or some combination of work and heat transfer?

No, knowing only the change in internal energy ΔU of a closed system between two states is not sufficient to determine whether that change was caused by heat, by work, or by a combination of both.

According to the First Law of Thermodynamics in this convention:

$$\Delta U = Q - W$$

This equation means that the internal energy change is equal to the heat added to the system minus the work done by the system on its surroundings.

However, if only the value of ΔU is known, there are many possible combinations of heat and work that can produce the same energy change. For example:

- The energy change might be due entirely to heat, with no work done.
- It might be due to some heat input and some work done by the system.
- It might also be due to work done *on* the system (i.e., negative work) without any heat transfer.

Conclusion: Without additional information about the process (e.g., whether it is adiabatic, isochoric, or involves known heat/work interactions), it is impossible to determine the specific cause of the energy change based solely on ΔU.

2. Thermal Efficiency and Environmental Impact

Two power cycles each receive the same energy input Q_{in} and discharge energy Q_{out} to the same lake. If the cycles have different thermal efficiencies, which discharges the greater amount Q_{out}? Does this have any implications for the environment?

If both power cycles receive the same energy input Q_{in} but have different thermal efficiencies, then the cycle with the lower thermal efficiency will discharge a greater amount of energy as Q_{out} to the lake.

This is because thermal efficiency η is defined as:

$$\eta = \frac{W_{\text{net}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}}$$

Rearranging:

$$Q_{\text{out}} = Q_{\text{in}}(1 - \eta)$$

So, if Q_{in} is the same for both cycles, the one with lower efficiency will have a higher Q_{out}.

Environmental Implications:

A greater Q_{out} discharged into the lake means **more thermal pollution**, which can:

- Increase water temperature,
- Reduce dissolved oxygen levels,
- Disrupt aquatic ecosystems and biodiversity,
- Affect local water usage (e.g., agriculture, recreation).

Therefore, from an environmental perspective, **higher thermal efficiency** is not only better for energy conversion but also results in **less environmental impact**.

3. Energy Table Completion

Each line in the following table gives information about the process of a closed system. Every entry has the same energy units. Fill in the blank spaces in the table.

Process	Q	W	U_1	U_2	ΔU
a	+50	-20		+50	
b	+50	+20	+20		
c	-40			+60	+20
d		-90		+50	0
e	+50		+20		-100

Table 1: Energy Balance for Closed System Processes

$$\Delta U = Q - W \quad \text{and} \quad U_2 = U_1 + \Delta U$$

Process a: Given $Q = +50$, $W = -20$, $U_2 = +50$

$$\Delta U = Q - W = 50 - (-20) = 70, \quad U_1 = U_2 - \Delta U = 50 - 70 = -20$$

Process b: Given $Q = +50$, $W = +20$, $U_1 = +20$

$$\Delta U = Q - W = 50 - 20 = 30, \quad U_2 = U_1 + \Delta U = 20 + 30 = 50$$

Process c: Given $Q = -40$, $\Delta U = +20$, $U_2 = +60$

$$U_1 = U_2 - \Delta U = 60 - 20 = 40, \quad W = Q - \Delta U = -40 - 20 = -60$$

Process d: Given $W = -90$, $U_2 = +50$, $\Delta U = 0$

$$U_1 = U_2 = 50, \quad Q = \Delta U + W = 0 + (-90) = -90$$

Process e: Given $Q = +50$, $\Delta U = -100$, $U_1 = +20$

$$W = Q - \Delta U = 50 - (-100) = 150, \quad U_2 = U_1 + \Delta U = 20 - 100 = -80$$

Process	Q	W	U_1	U_2	ΔU
a	+50	-20	-20	+50	+70
b	+50	+20	+20	+50	+30
c	-40	-60	+40	+60	+20
d	-90	-90	+50	+50	0
e	+50	+150	+20	-80	-100

Table 2: Completed Energy Table for Closed System Processes

4. Final Specific Internal Energy

A closed system of mass 20 kg undergoes a process in which there is a heat transfer of 1000 kJ from the system to the surroundings. The work done on the system is 200 kJ. If the initial specific internal energy of the system is 300 kJ/kg, what is the final specific internal energy, in kJ/kg? Neglect changes in kinetic and potential energy.

Given:

- Mass: $m = 20\,\text{kg}$
- Heat transferred: $Q = -1000\,\text{kJ}$ (heat lost by the system)

- Work done on the system: $W = -200\,\text{kJ}$ (work input)
- Initial specific internal energy: $u_1 = 300\,\text{kJ/kg}$

Neglecting changes in kinetic and potential energy, the First Law of Thermodynamics states:

The total internal energy of a system is given by:

$$U = m \cdot u$$

where:

- m is the mass of the system (in kg),
- u is the specific internal energy (in kJ/kg),
- U is the total internal energy (in kJ).

$$\Delta U = Q - W \quad \text{and} \quad \Delta U = m(u_2 - u_1)$$

Solving for final specific internal energy:

$$u_2 = u_1 + \frac{Q - W}{m} = 300 + \frac{-1000 - (-200)}{20} = 300 + \frac{-800}{20} = 300 - 40 = 260\,\text{kJ/kg}$$

$$\boxed{u_2 = 260\,\text{kJ/kg}}$$

5. Polytropic Expansion and Heat Transfer

A gas expands in a piston-cylinder assembly from $p_1 = 8\,\text{bar}$, $V_1 = 0.02\,\text{m}^3$ to $p_2 = 2\,\text{bar}$ in a process during which the relation between pressure and volume is $pV^{1.2} = \text{constant}$. The mass of the gas is 0.25 kg. If the specific internal energy of the gas decreases by 55 kJ/kg during the process, determine the heat transfer, in kJ. Neglect kinetic and potential energy effects.

Given:

- $p_1 = 8\,\text{bar} = 800\,\text{kPa}$
- $p_2 = 2\,\text{bar} = 200\,\text{kPa}$
- $V_1 = 0.02\,\text{m}^3$
- $n = 1.2$

- $m = 0.25\,\mathrm{kg}$
- $\Delta u = -55\,\mathrm{kJ/kg} \Rightarrow \Delta U = m \cdot \Delta u = -13.75\,\mathrm{kJ}$

Step 1: Calculate V_2 using the polytropic relation

$$p_1 V_1^n = p_2 V_2^n \Rightarrow V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{1/n} = 0.02 \left(\frac{800}{200}\right)^{1/1.2} \approx 0.0635\,\mathrm{m}^3$$

Step 2: Calculate work W

$$W = \frac{p_2 V_2 - p_1 V_1}{1-n} = \frac{200 \cdot 0.0635 - 800 \cdot 0.02}{1 - 1.2} = \frac{12.70 - 16.00}{-0.2} = \frac{-3.30}{-0.2} = 16.5\,\mathrm{kJ}$$

Step 3: Apply the First Law of Thermodynamics

$$Q = \Delta U + W = -13.75 + 16.5 = \boxed{2.75\,\mathrm{kJ}}$$

6. Thermodynamic Cycle Table

The following table gives data, in kJ, for a system undergoing a thermodynamic cycle consisting of four processes in series. For the cycle, kinetic and potential energy effects can be neglected.

(a) Determine the missing table entries, each in kJ.

(b) Indicate whether the cycle is a power cycle or a refrigeration cycle.

Process	ΔU	Q	W
1–2	600	–	-600
2–3	–	–	-1300
3–4	-700	0	–
4–1	–	500	700

Step 1: Use the First Law of Thermodynamics for each process

$$Q = \Delta U + W \quad \Rightarrow \quad \Delta U = Q - W$$

We apply this relation to fill in the missing entries in the table.

Process 1–2:

$$\Delta U = 600\,\mathrm{kJ}, \quad W = -600\,\mathrm{kJ}$$

$$Q = \Delta U + W = 600 + (-600) = 0\,\text{kJ}$$

Process 3–4:

$$\Delta U = -700\,\text{kJ}, \quad Q = 0$$

$$W = Q - \Delta U = 0 - (-700) = 700\,\text{kJ}$$

Process 4–1:

$$Q = 500\,\text{kJ}, \quad W = 700\,\text{kJ}$$

$$\Delta U = Q - W = 500 - 700 = -200\,\text{kJ}$$

Now use the cyclic property: In a complete cycle, the change in internal energy is zero:

$$\sum \Delta U = 0 \Rightarrow \Delta U_{2-3} = -(\Delta U_{1-2} + \Delta U_{3-4} + \Delta U_{4-1}) = -(600 - 700 - 200) = +300\,\text{kJ}$$

Then for Process 2–3:

$$\Delta U = 300\,\text{kJ}, \quad W = -1300\,\text{kJ}$$

$$Q = \Delta U + W = 300 + (-1300) = -1000\,\text{kJ}$$

Step 2: Completed Table

Table 3: Thermodynamic Cycle - Completed Energy Table

Process	ΔU (kJ)	Q (kJ)	W (kJ)
1–2	600	0	-600
2–3	300	-1000	-1300
3–4	-700	0	700
4–1	-200	500	700

Step 3: Determine the type of cycle

$$W_{\text{net}} = -600 - 1300 + 700 + 700 = -500\,\text{kJ}$$

Since the net work is negative, the system **requires work input**, meaning this is a:

$$\boxed{\text{Refrigeration Cycle}}$$

Characteristic	Power Cycle	Refrigeration Cycle
Net work	$W_{\text{net}} > 0$	$W_{\text{net}} < 0$
Main function	Produce work (energy output)	Remove heat (cooling)
Requires heat input	Yes, as energy source	Yes, but reversed heat flow
Typical usage	Engines, turbines	Refrigerators, A/C units
Energy flow	Converts heat to work	Uses work to transfer heat
Direction of heat flow	High to low temperature	Low to high temperature

Table 4: Power vs. Refrigeration Cycle

7. Three-Process Thermodynamic Cycle

A gas undergoes a thermodynamic cycle consisting of three processes:

- Process 1–2: compression with pV = constant, from $p_1 = 1\,\text{bar}$, $V_1 = 1.6\,\text{m}^3$ to $V_2 = 0.2\,\text{m}^3$, $U_2 - U_1 = 0$.
- Process 2–3: constant pressure to $V_3 = V_1$.
- Process 3–1: constant volume, $U_1 - U_3 = -3549\,\text{kJ}$.

There are no significant changes in kinetic or potential energy.

Determine the heat transfer and work for Process 2–3, in kJ. Indicate whether this is a power cycle or a refrigeration cycle.

Step 1: Use cyclic condition to find ΔU_{2-3}
Since the system completes a full cycle, the net change in internal energy is zero:

$$\Delta U_{1-2} + \Delta U_{2-3} + \Delta U_{3-1} = 0 \Rightarrow 0 + \Delta U_{2-3} + (-3549) = 0 \Rightarrow \Delta U_{2-3} = +3549\,\text{kJ}$$

Step 2: Find pressure for process 2–3 using pV = constant
From process 1–2:

$$p_1 V_1 = p_2 V_2 \Rightarrow p_2 = \frac{p_1 V_1}{V_2} = \frac{1.6}{0.2} = 8\,\text{bar} = 800\,\text{kPa}$$

Step 3: Calculate work during process 2–3 (constant pressure)

$$W_{2-3} = p(V_3 - V_2) = 800\,\text{kPa} \cdot (1.6 - 0.2)\,\text{m}^3 = 800 \cdot 1.4 = 1120\,\text{kJ}$$

Step 4: Apply the First Law to process 2–3

$$Q_{2-3} = \Delta U_{2-3} + W_{2-3} = 3549 + 1120 = \boxed{4669\,\text{kJ}}$$

Step 5: Analyze process 3–1
This is a constant volume process:

$$W_{3-1} = 0, \quad Q_{3-1} = \Delta U_{3-1} = -3549\,\mathrm{kJ}$$

Step 6: Net work of the cycle
Since $\Delta U = 0$ for the cycle:

$$Q_{\text{net}} = Q_{1-2} + Q_{2-3} + Q_{3-1} \Rightarrow W_{\text{net}} = Q_{\text{net}} = Q_{1-2} + 4669 - 3549 = Q_{1-2} + 1120$$

But for process 1–2, $\Delta U = 0 \Rightarrow Q_{1-2} = W_{1-2}$, so:

$$W_{\text{net}} = W_{1-2} + 1120$$

As the total work is positive, the system produces net work.

Conclusion: This is a

$$\boxed{\text{Power Cycle}}$$