

Polytechnic University of Puerto Rico – Orlando Campus  
**ME 3140 – Intermediate Fluid Mechanics**

# **Homework N-06**

## **One-Dimensional Isentropic Flow ( $\text{CO}_2$ )**

**Instructor:** Dr. Joaquín Valencia

**Estudiante:** Antonio Pérez

**ID:** 158655

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## Problema 1. One-Dimensional Isentropic Flow – Example 2 (CO<sub>2</sub>)

### One-Dimensional Isentropic Flow

#### Example 2

Carbon dioxide flows steadily through a varying cross-sectional area duct such as a nozzle shown in Figure E-2 at a mass flow rate of 3.00 kg/s. The carbon dioxide enters the duct at a pressure of 1400 kPa and 200°C with a low velocity, and it expands in the nozzle to an exit pressure of 200 kPa. The duct is designed so that the flow can be approximated as isentropic. Determine the density, velocity, flow area, and Mach number at each location along the duct that corresponds to an overall pressure drop of 200 kPa.

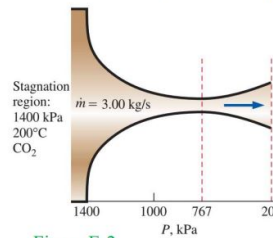


Figure E-2

Figura 1: Enunciado problema 2

#### Enunciado

Se tiene flujo unidimensional isentrópico de CO<sub>2</sub> (gas ideal, propiedades constantes a 200.000 °C) desde un reservorio hacia un duct/nozzle. En el reservorio:

$$p_{0,1} = 1400,000 \text{ kPa}, \quad T_{0,1} = 473,150 \text{ K}, \quad V_1 \approx 0$$

En una sección 2 aguas abajo, la presión estática es:

$$p_2 = 1200,000 \text{ kPa}$$

y el gasto másico es constante:

$$\dot{m} = 3,000 \text{ kg/s}$$

Propiedades (CO<sub>2</sub> a 200.000 °C):

$$R = 188,900 \text{ J/(kg K)}, \quad k = 1,288, \quad c_p = 843,900 \text{ J/(kg K)}$$

Calcular en la sección 2:  $T_2, \rho_1, \rho_2, V_2, c_2, M_2, A_2$ .

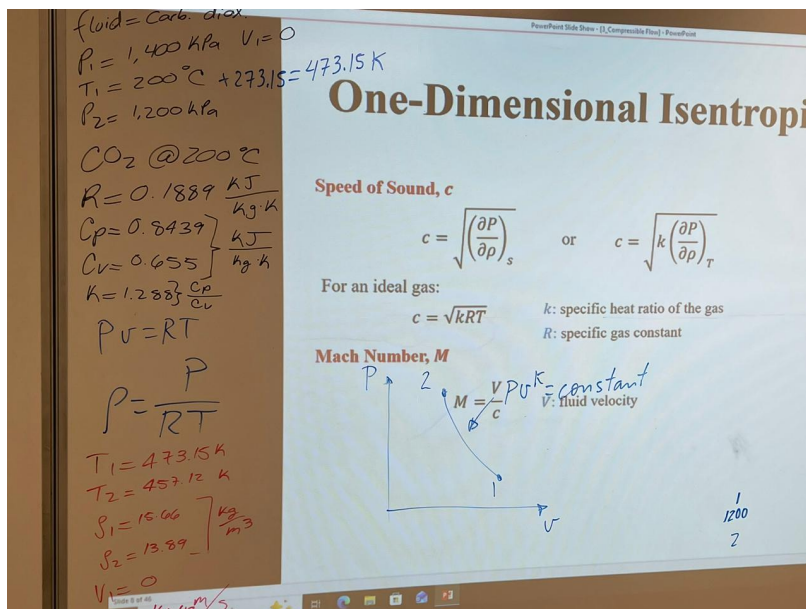


Figura 2: Definiciones: velocidad del sonido  $c$  y número de Mach  $M$ .

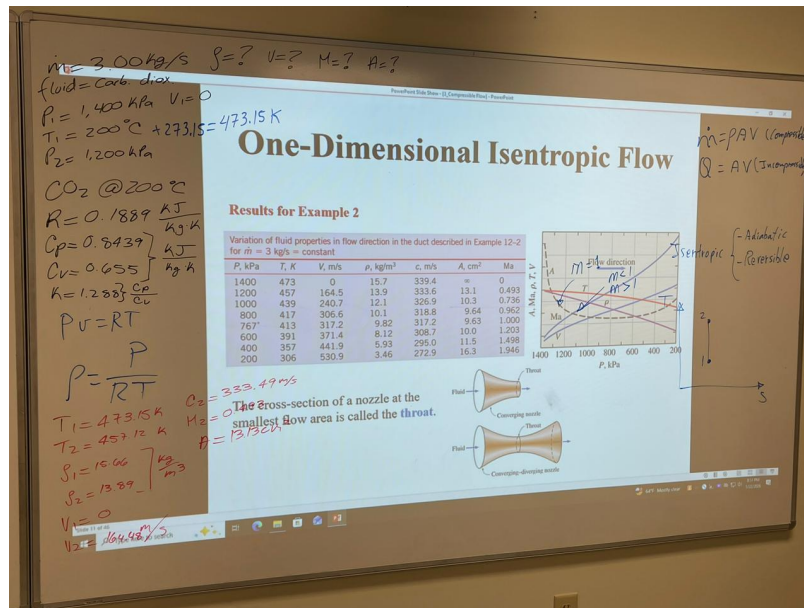


Figura 3: Contexto (Example 2): tabla/figura del ejemplo.

## Tabla de variables iniciales (P1)

| Variable  | Valor (SI)       | Descripción                          |
|-----------|------------------|--------------------------------------|
| $\dot{m}$ | 3.000 kg/s       | Gasto másico                         |
| $p_{0,1}$ | 1400.000 kPa     | Presión de estancamiento en 1        |
| $T_{0,1}$ | 473.150 K        | Temperatura de estancamiento en 1    |
| $p_2$     | 1200.000 kPa     | Presión estática en 2                |
| $R$       | 188.900 J/(kg K) | Constante del gas (CO <sub>2</sub> ) |
| $k$       | 1.288            | Razón de calores específicos         |
| $c_p$     | 843.900 J/(kg K) | Calor específico a $p$ cte           |

## Tabla de fórmulas (P1)

| ID | Fórmula                                                                    | Uso                            |
|----|----------------------------------------------------------------------------|--------------------------------|
| F1 | $\frac{T_2}{T_{0,1}} = \left( \frac{p_2}{p_{0,1}} \right)^{\frac{k-1}{k}}$ | Relación isentrópica $T$ - $p$ |
| F2 | $\rho = \frac{p}{RT}$                                                      | Gas ideal (densidad)           |
| F3 | $T_0 = T + \frac{V^2}{2c_p}$                                               | Estancamiento (energía)        |
| F4 | $c = \sqrt{kRT}$                                                           | Velocidad del sonido           |
| F5 | $M = \frac{V}{c}$                                                          | Número de Mach                 |
| F6 | $\dot{m} = \rho AV$                                                        | Continuidad (compresible)      |

## Desarrollo (P1)

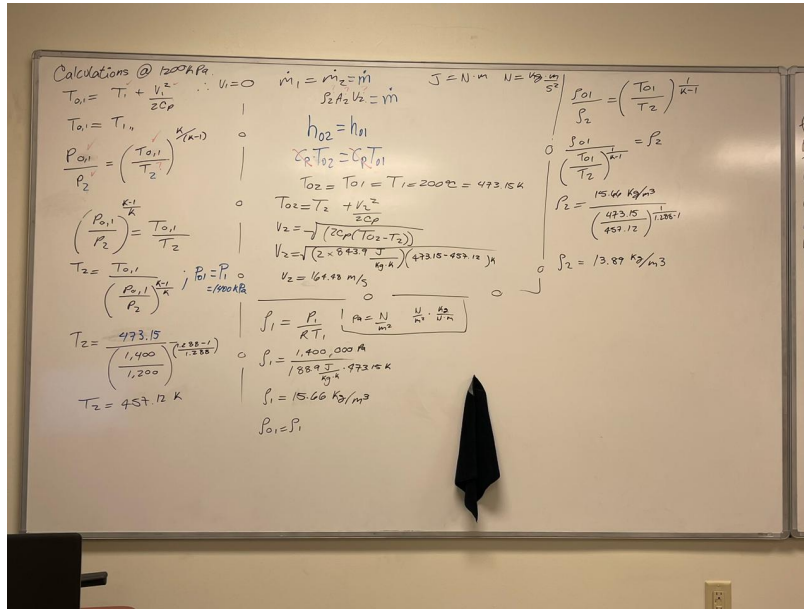


Figura 4: Relación isentrópica y densidades.

### Paso 1. Temperatura $T_2$ (F1)

$$T_2 = T_{0,1} \left( \frac{p_2}{p_{0,1}} \right)^{\frac{k-1}{k}} = (473,150 \text{ K}) \left( \frac{1200}{1400} \right)^{\frac{1,288-1}{1,288}} = 457,120 \text{ K}$$

$$T_2 = 457,120 \text{ K}$$

### Paso 2. Densidades $\rho_1$ y $\rho_2$ (F2)

$$\rho_1 = \frac{1,400 \times 10^6 \text{ Pa}}{(188,900 \text{ J/(kg K)})(473,150 \text{ K})} = 15,660 \text{ kg/m}^3$$

$$\rho_2 = \frac{1,200 \times 10^6 \text{ Pa}}{(188,900 \text{ J/(kg K)})(457,120 \text{ K})} = 13,890 \text{ kg/m}^3$$

$$\rho_1 = 15,660 \text{ kg/m}^3 \quad \rho_2 = 13,890 \text{ kg/m}^3$$

### Paso 3. Velocidad $V_2$ (F3)

Como  $V_1 \approx 0$ , entonces  $T_{0,2} = T_{0,1}$ .

$$V_2 = \sqrt{2c_p(T_{0,1} - T_2)} = \sqrt{2(843,900 \text{ J/(kg K)})(473,150 \text{ K} - 457,120 \text{ K})} = 164,480 \text{ m/s}$$

$$V_2 = 164,480 \text{ m/s}$$

Calculations @ 1200 kg/s

$$c_2 = \sqrt{kRT_2} \quad M = \frac{V}{c}$$

$$c_2 = \sqrt{1.288 \times 1889 \frac{J}{kg \cdot K} \times 457.12 K}$$

$$c_2 = 333.49 \text{ m/s}$$

$$M_2 = \frac{V_2}{c_2}$$

$$M_2 = \frac{164.48 \text{ m/s}}{333.49 \text{ m/s}}$$

$$M_2 = 0.493$$

$$\dot{m} = \rho_2 A_2 V_2$$

$$A_2 = \frac{\dot{m}}{\rho_2 V_2}$$

$$A_2 = \frac{3.00 \text{ kg/s}}{13.890 \frac{kg}{m^3} \times 164.48 \frac{m}{s}}$$

$$A_2 = 1.312 \times 10^{-3} \text{ m}^2 \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^2$$

$$A_2 = 13.13 \text{ cm}^2$$

Figura 5: Cálculo de  $c_2$ ,  $M_2$  y  $A_2$ .

#### Paso 4. Velocidad del sonido $c_2$ y Mach $M_2$ (F4–F5)

$$c_2 = \sqrt{kRT_2} = \sqrt{(1.288)(188,900 \text{ J/(kg K)})(457,120 \text{ K})} = 333,490 \text{ m/s}$$

$$M_2 = \frac{V_2}{c_2} = \frac{164,480 \text{ m/s}}{333,490 \text{ m/s}} = 0,493$$

$$c_2 = 333,490 \text{ m/s} \quad M_2 = 0,493$$

#### Paso 5. Área $A_2$ (F6)

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{3,000 \text{ kg/s}}{(13,890 \text{ kg/m}^3)(164,480 \text{ m/s})} = 1,312 \times 10^{-3} \text{ m}^2 \approx 13,130 \text{ cm}^2$$

$$A_2 = 1,312 \times 10^{-3} \text{ m}^2 \approx 13,130 \text{ cm}^2$$

## Problema 2. One-Dimensional Isentropic Flow – Example 3 (condición crítica)

### Enunciado

Usando  $p_0 = 1400,000 \text{ kPa}$ ,  $T_0 = 473,150 \text{ K}$  y  $k = 1,288$  para  $\text{CO}_2$ , determinar la **temperatura crítica**  $T$  y la **presión crítica**  $p$  en la garganta cuando el flujo alcanza condición sónica ( $M = 1$ ).

$m = 3.00 \text{ kg/s}$   $\rho = ?$   $V = ?$   $M = ?$   $A = ?$   
 fluid = Carb. dioxide  
 $P = 1,400 \text{ kPa}$   $V = 0$   
 $T_1 = 200^\circ\text{C} + 273.15 = 473.15 \text{ K}$   
 $P_2 = 1,200 \text{ kPa}$   
 CO<sub>2</sub> @ 200°C  
 $R = 0.1889 \frac{\text{kJ}}{\text{kg K}}$   
 $C_p = 0.8439 \frac{\text{kJ}}{\text{kg K}}$   
 $C_v = 0.655 \frac{\text{kJ}}{\text{kg K}}$   
 $k = 1.288$   
 $PV = RT$   
 $P = \frac{P}{RT}$   
 $T_1 = 473.15 \text{ K}$   $C_2 = 333.49 \text{ m/s}$   
 $T_2 = 457.12 \text{ K}$   $M_2 = 0.493$   
 $\rho_1 = 15.06 \frac{\text{kg}}{\text{m}^3}$   $\rho = 13.13 \text{ kg/m}^3$   
 $\rho_2 = 13.39 \frac{\text{kg}}{\text{m}^3}$   
 $V_1 = 0$   
 $V_2 = 166.48 \text{ m/s}$

## One-Dimensional Isentropic Flow

**Example 3**  
Calculate the critical pressure and temperature of carbon dioxide for the flow conditions described in Example 2 (Figure E-3).

Figure E-3

Figura 6: Diapositiva: Example 3 (condición crítica).

$$\frac{T^*}{T_0} = \frac{2}{k+1}, \quad \frac{P^*}{P_0} = \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}}$$

$$T^* = \frac{2 T_0}{k+1}$$

$$T^* = \frac{2 \times 473.15 \text{ K}}{1.288 + 1}$$

$$T^* = 413.6 \text{ K}$$

$$P^* = P_0 \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}}$$

$$P^* = 1400000 \text{ Pa} \left( \frac{2}{1.288 + 1} \right)^{\frac{1.288}{1.288 - 1}}$$

$$P^* = 767 \text{ kPa}$$

Figura 7: Fórmulas para  $T$  y  $p$ .

### Tabla de variables iniciales (P2)

| Variable | Valor (SI)   | Descripción                  |
|----------|--------------|------------------------------|
| $P_0$    | 1400.000 kPa | Presión de estancamiento     |
| $T_0$    | 473.150 K    | Temperatura de estancamiento |
| $k$      | 1.288        | Razón de calores específicos |

## Tabla de fórmulas (P2)

| ID | Fórmula                                                      | Uso                             |
|----|--------------------------------------------------------------|---------------------------------|
| G1 | $\frac{T}{T_0} = \frac{2}{k+1}$                              | Temperatura crítica ( $M = 1$ ) |
| G2 | $\frac{p}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$ | Presión crítica ( $M = 1$ )     |

## Desarrollo (P2)

### Paso 1. Temperatura crítica $T$ (G1)

$$T = \frac{2T_0}{k+1} = \frac{2(473,150 \text{ K})}{1,288+1} = 413,590 \text{ K}$$

$$T = 413,600 \text{ K}$$

### Paso 2. Presión crítica $p$ (G2)

$$p = p_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = (1400,000 \text{ kPa}) \left(\frac{2}{2,288}\right)^{\frac{1,288}{0,288}} = 767,070 \text{ kPa}$$

$$p = 767,000 \text{ kPa} = 0,767 \text{ MPa}$$

## Interpretación física

La condición crítica ocurre cuando el flujo alcanza  $M = 1$  en la garganta. Si la presión aguas abajo disminuye hasta cumplir  $p_2 \leq p$ , el flujo se *estrangula* (choked) y el caudal máximo queda gobernado por la condición crítica.