

Polytechnic University of Puerto Rico – Orlando Campus
ME 3140 – Intermediate Fluid Mechanics

Homework N-06

Instructor: Dr. Joaquín Valencia

Estudiante: Antonio Pérez

ID: 158655

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Problema. Turbina de reacción (Francis) – Example 2

Data

$r_2 = 2.5\text{ m}$
 $r_1 = 1.77\text{ m}$
 $b_2 = 0.914\text{ m}$
 $b_1 = 2.62\text{ m}$
 $\omega = 12.57\text{ rad/s}$
 $\alpha_2 = 33^\circ$
 $-10^\circ \leq \alpha_1 \leq 10^\circ$
 $Q = 599\text{ m}^3/\text{s}$
 $H_{\text{gross}} = 92.4\text{ m}$

a) $\beta_2 = ?$ $\beta_1 = ?$
 For $\alpha_1 = 10^\circ$
 $\dot{W}_{\text{shaft}} = ?$ $H = ?$

Reaction Turbines

Example 2

A retrofit Francis radial-flow hydroturbine is being designed to replace an old turbine in a hydroelectric dam. The new turbine must meet the following design restrictions in order to properly couple with the existing setup: The runner inlet radius is $r_1 = 8.20\text{ ft}$ (2.50 m) and its outlet radius is $r_2 = 5.80\text{ ft}$ (1.77 m). The runner blade widths are $b_2 = 3.00\text{ ft}$ (0.914 m) and $b_1 = 8.60\text{ ft}$ (2.62 m) at the inlet and outlet, respectively. The runner must rotate at $\dot{n} = 120\text{ rpm}$ ($\omega = 12.57\text{ rad/s}$) to turn the 60-Hz electric generator. The wicket gates turn the flow by angle $\alpha_2 = 33^\circ$ from radial at the runner inlet, and the flow at the runner outlet is to have angle α_1 between -10° and 10° from radial (Figure E-2) for proper flow through the draft tube. The volume flow rate at design conditions is $9.50 \times 10^6\text{ gpm}$ ($599\text{ m}^3/\text{s}$), and the gross head provided by the dam is $H_{\text{gross}} = 303\text{ ft}$ (92.4 m). (a) Calculate the inlet and outlet runner blade angles β_2 and β_1 , respectively, and predict the power output and required net head if irreversible losses are neglected for the case with $\alpha_1 = 10^\circ$ from radial (with-rotation swirl). (b) Repeat the calculations for the case with $\alpha_1 = 0^\circ$ from radial (no swirl). (c) Repeat the calculations for the case with $\alpha_1 = -10^\circ$ from radial (reverse swirl). Assume: $\eta_{\text{turbine}} = 100\%$

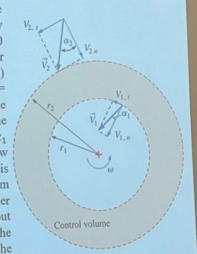


Figure E-2

Figura 1: Diapositiva: “Reaction Turbines – Example 2” (enunciado y esquema).

Data

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 $r_1 = 1.77\text{ m}$
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 $\alpha_2 = 33^\circ$
 $-10^\circ \leq \alpha_1 \leq 10^\circ$
 $Q = 599\text{ m}^3/\text{s}$
 $H_{\text{gross}} = 92.4\text{ m}$
 $\eta_{\text{turbine}} = 100\%$

a) $\beta_2 = ?$ $\beta_1 = ?$
 For $\alpha_1 = 10^\circ$
 $\dot{W}_{\text{shaft}} = ?$ $H = ?$

Reaction Turbines

Ideal power production, $\dot{W}_{\text{ideal}}$ (no irreversible losses anywhere in the system)

$$\dot{W}_{\text{ideal}} = \rho g Q H_{\text{gross}}$$

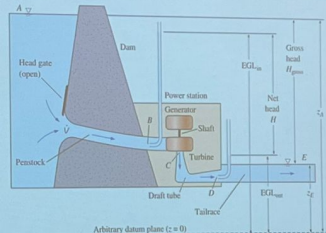
$$H_{\text{gross}} = z_A - z_E$$

Net head for a hydraulic turbine, H

$$H = EGL_{\text{in}} - EGL_{\text{out}}$$

EGL: energy grade line

Turbine efficiency, η_{turbine}

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{water hp}}} = \frac{bhp}{\rho g H Q}$$


Typical setup and terminology for a hydroelectric plant that utilizes a Francis turbine to generate electricity

Figura 2: Diapositiva: “Reaction Turbines – Example 2” (enunciado y esquema).

Enunciado

Una turbina Francis de flujo radial está siendo rediseñada para reemplazar una turbina antigua en una represa hidroeléctrica. La turbina nueva debe cumplir restricciones de diseño para acoplarse al sistema existente: el radio de entrada del rodete es $r_2 = 2,500$ m y el radio de salida es $r_1 = 1,770$ m. Los anchos del rodete son $b_2 = 0,914$ m en la entrada y $b_1 = 2,620$ m en la salida. El rodete gira a $n = 120,000$ rpm ($\omega = 12,570$ rad/s). Los álabes directrices (wicket gates) desvían el flujo un ángulo $\alpha_2 = 33^\circ$ medido desde la dirección radial en la entrada del rodete. A la salida, el flujo debe tener un ángulo α_1 entre -10° y 10° desde radial para asegurar buen flujo a través del tubo de aspiración. El caudal de diseño es $Q = 599,000$ m³/s y la carga bruta disponible es $H_{\text{gross}} = 92,400$ m.

- Calcular los ángulos de álabe del rodete β_2 (entrada) y β_1 (salida), y predecir la potencia y el head neto requerido, despreciando pérdidas irreversibles, para $\alpha_1 = +10^\circ$.
 - Repetir para $\alpha_1 = 0^\circ$.
 - Repetir para $\alpha_1 = -10^\circ$.
- Se asume $\eta_{\text{turbina}} = 100\%$.

Tabla de variables iniciales

Variable	Valor (SI)	Descripción
r_2	2,500 m	Radio del rodete en la entrada
r_1	1,770 m	Radio del rodete en la salida
b_2	0,914 m	Ancho del rodete en la entrada
b_1	2,620 m	Ancho del rodete en la salida
ω	12,570 rad/s	Velocidad angular
α_2	33°	Ángulo del flujo en la entrada (desde radial)
α_1	$\{-10^\circ, 0^\circ, +10^\circ\}$	Ángulo del flujo a la salida (desde radial)
Q	599,000 m ³ /s	Caudal volumétrico
ρ	998,200 kg/m ³	Densidad del agua (aprox.)
g	9,810 m/s ²	Gravedad
H_{gross}	92,400 m	Head bruto disponible (dato)
η_{turbina}	1,00	Turbina ideal (sin pérdidas)

Tabla de fórmulas (usadas en clase)

ID	Fórmula	Uso
F1	$Q = 2\pi r b V_n$	Continuidad en anillo: obtiene V_n en entrada/salida
F2	$\tan \alpha = \frac{V_t}{V_n}$	Relación ángulo absoluto (desde radial)
F3	$U = \omega r$	Velocidad periférica del rodete
F4	$\tan \beta = \frac{V_n}{U - V_t}$	Triángulo de velocidades $\rightarrow$ ángulos de álabe β
F5	$\dot{W}_{\text{shaft}} = \rho Q \omega (r_2 V_{2,*} - r_1 V_{1,t})$	Ecuación de Euler (forma usada en clase)
F6	$H = \frac{\dot{W}_{\text{shaft}}}{\rho g Q}$	Head neto ideal (si $\eta = 1$)

Desarrollo

Paso 1. Componentes normales por continuidad (F1)

$$V_{2,n} = \frac{Q}{2\pi r_2 b_2} = \frac{599}{2\pi(2,50)(0,914)} = 41,722 \text{ m/s}$$

$$V_{1,n} = \frac{Q}{2\pi r_1 b_1} = \frac{599}{2\pi(1,77)(2,62)} = 20,558 \text{ m/s}$$

$$V_{2,n} = 41,720 \text{ m/s} \quad V_{1,n} = 20,560 \text{ m/s}$$

Paso 2. Componente tangencial en la entrada usando α_2 (F2)

$$V_{2,t} = V_{2,n} \tan \alpha_2 = (41,722) \tan(33^\circ) = 27,094 \text{ m/s}$$

$$V_{2,t} = 27,090 \text{ m/s}$$

Paso 3. Velocidades periféricas (F3)

$$U_2 = \omega r_2 = (12,57)(2,50) = 31,425 \text{ m/s}, \quad U_1 = \omega r_1 = (12,57)(1,77) = 22,249 \text{ m/s}$$

$$U_2 = 31,430 \text{ m/s} \quad U_1 = 22,250 \text{ m/s}$$

Paso 4. Ángulo de álabe en la entrada β_2 (F4)

$$\tan \beta_2 = \frac{V_{2,n}}{U_2 - V_{2,t}} = \frac{41,722}{31,425 - 27,094} = 9,633 \Rightarrow \beta_2 = \arctan(9,633) = 84,070^\circ$$
$$\beta_2 = 84,070^\circ$$

(a) Caso $\alpha_1 = +10^\circ$ (with-rotation swirl)

Paso (a1). Componente tangencial a la salida $V_{1,t}$ (F2)

$$V_{1,t} = V_{1,n} \tan(10^\circ) = (20,558) \tan(10^\circ) = 3,625 \text{ m/s}$$
$$V_{1,t}(+10^\circ) = 3,620 \text{ m/s}$$

Paso (a2). Ángulo de álabe a la salida β_1 (F4)

$$\tan \beta_1 = \frac{V_{1,n}}{U_1 - V_{1,t}} = \frac{20,558}{22,249 - 3,625} = 1,104 \Rightarrow \beta_1 = \arctan(1,104) = 47,830^\circ$$
$$\beta_1(+10^\circ) = 47,830^\circ$$

Paso (a3). Potencia al eje (Euler) y head neto ideal (F5–F6)

Siguiendo la forma aplicada en clase (pizarra):

$$\dot{W}_{\text{shaft}} = \rho Q \omega (r_2 V_{2,n} - r_1 V_{1,t})$$
$$\dot{W}_{\text{shaft}} = (998,2)(599)(12,57) [(2,50)(41,722) - (1,77)(3,625)] = 7,357 \times 10^8 \text{ W}$$
$$\dot{W}_{\text{shaft}}(+10^\circ) = 7,360 \times 10^8 \text{ W} = 735,700 \text{ MW}$$
$$H = \frac{\dot{W}_{\text{shaft}}}{\rho g Q} = \frac{7,3572 \times 10^8}{(998,2)(9,81)(599)} = 125,430 \text{ m}$$
$$H(+10^\circ) = 125,430 \text{ m}$$

(b) Caso $\alpha_1 = 0^\circ$ (no swirl)

Paso (b1). $V_{1,t}$

$$V_{1,t} = V_{1,n} \tan(0^\circ) = 0 \Rightarrow V_{1,t}(0^\circ) = 0,000 \text{ m/s}$$

Paso (b2). β_1

$$\tan \beta_1 = \frac{V_{1,n}}{U_1 - 0} = \frac{20,558}{22,249} = 0,9246 \Rightarrow \beta_1 = \arctan(0,9246) = 42,740^\circ$$
$$\beta_1(0^\circ) = 42,740^\circ$$

Paso (b3). $\dot{W}_{\text{shaft}}$ y H

$$\dot{W}_{\text{shaft}} = \rho Q \omega (r_2 V_{2,n} - r_1 V_{1,t}) = (998,2)(599)(12,57) [(2,50)(41,722) - 0] = 7,839 \times 10^8 \text{ W}$$
$$\dot{W}_{\text{shaft}}(0^\circ) = 7,840 \times 10^8 \text{ W} = 783,900 \text{ MW}$$
$$H = \frac{7,8394 \times 10^8}{(998,2)(9,81)(599)} = 133,650 \text{ m}$$
$$H(0^\circ) = 133,650 \text{ m}$$

(c) Caso $\alpha_1 = -10^\circ$ (reverse swirl)

Paso (c1). $V_{1,t}$

$$V_{1,t} = V_{1,n} \tan(-10^\circ) = -(20,558) \tan(10^\circ) = -3,625 \text{ m/s}$$
$$V_{1,t}(-10^\circ) = -3,620 \text{ m/s}$$

Paso (c2). β_1

$$\tan \beta_1 = \frac{V_{1,n}}{U_1 - V_{1,t}} = \frac{20,558}{22,249 - (-3,625)} = 0,7947 \Rightarrow \beta_1 = \arctan(0,7947) = 38,470^\circ$$

$$\beta_1(-10^\circ) = 38,470^\circ$$

Paso (c3). $\dot{W}_{\text{shaft}}$ y H

$$\dot{W}_{\text{shaft}} = (998,2)(599)(12,57) [(2,50)(41,722) - (1,77)(-3,625)] = 8,322 \times 10^8 \text{ W}$$

$$\dot{W}_{\text{shaft}}(-10^\circ) = 8,320 \times 10^8 \text{ W} = 832,200 \text{ MW}$$

$$H = \frac{8,3217 \times 10^8}{(998,2)(9,81)(599)} = 141,870 \text{ m}$$

$$H(-10^\circ) = 141,870 \text{ m}$$

Resumen final

Caso	$V_{1,t}$ (m/s)	β_1 ($^\circ$)	$\dot{W}_{\text{shaft}}$ (MW)	H (m)
$\alpha_1 = +10^\circ$	+3,62	47,83	735,7	125,43
$\alpha_1 = 0^\circ$	0	42,74	783,9	133,65
$\alpha_1 = -10^\circ$	-3,62	38,47	832,2	141,87

$$\beta_2 = 84,070^\circ \text{ (mismo en los tres casos)}$$

Comentario final (consistencia física)

Bajo el modelo ideal usado, al reducir $V_{1,t}$ (hacerlo cero o negativo), el término de Euler aumenta, lo cual incrementa la potencia ideal al eje y el head neto ideal requerido.