ME4932 Aircraft Performance & Design

Performance and Flight Mechanics(Chapter 17)

Source: Aircraft Design by Daniel P. Raymer

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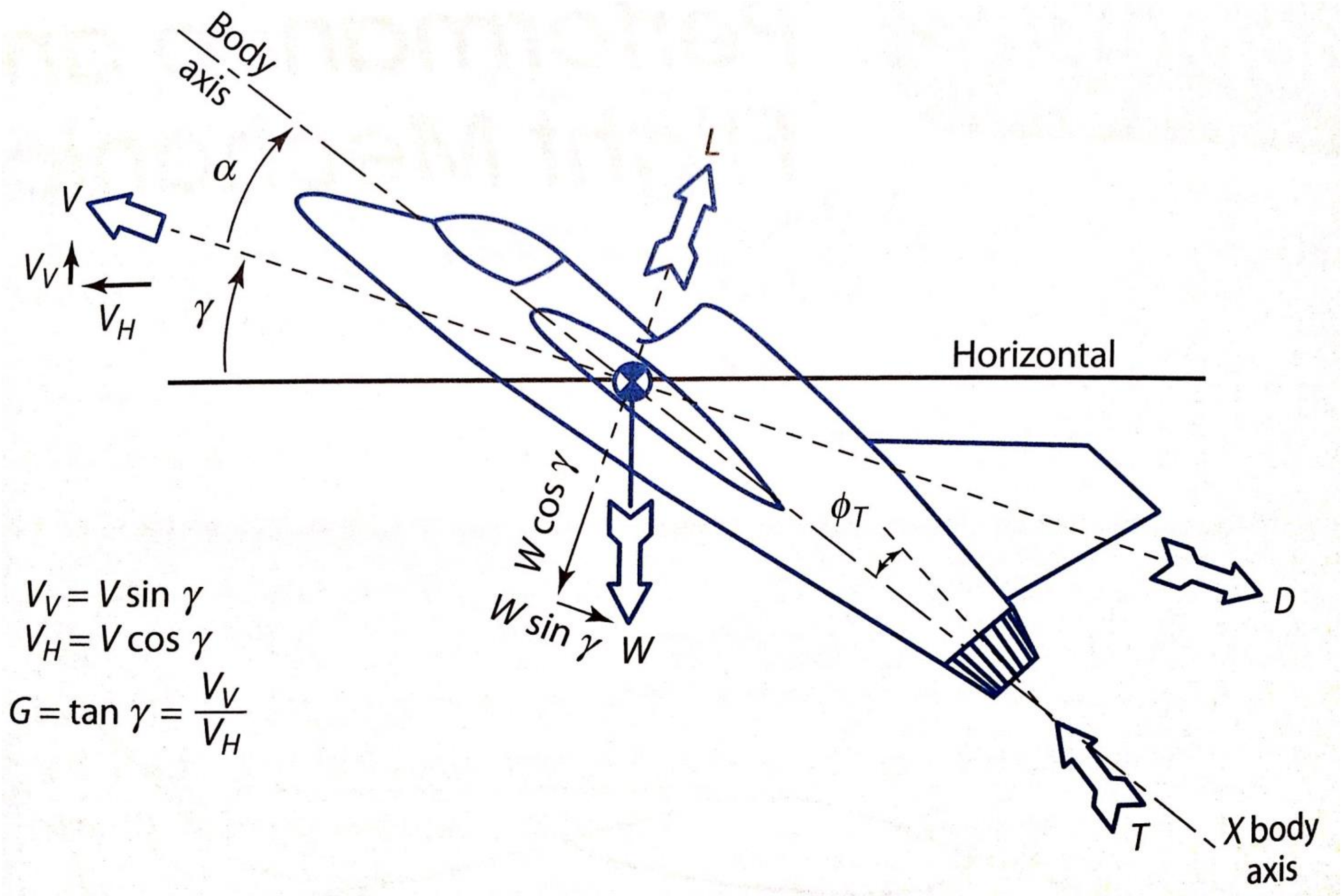
Flight Mechanics - Equations of Motion



$$\Sigma F_x = T\cos(\alpha + \phi_T) - D - W\sin\gamma$$

$$\Sigma F_z = T\sin(\alpha + \phi_T) + L - W\cos\gamma$$

$$\dot{W} = -CT$$

$$C = C_{\text{power}}\frac{V}{\eta_p} = C_{\text{bhp}}\frac{V}{550\ \eta_p}$$

$$T = P\eta_p/V = 550\ \text{bhp}\ \eta_p/V$$

Assuming thrust is <u>almost</u> aligned with V:

$$\Sigma F_x = T - D - W\sin\gamma$$

$$\Sigma F_z = L - W\cos\gamma$$

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Flight Mechanics - Steady Level Flight

Steady: Accelerations = 0
Level: Pitch angle = 0

$$T = D = qS(C_{D_0} + KC_L^2)$$

$$L = W = qSC_L$$

$$V = \sqrt{\frac{2}{\rho C_L}\left(\frac{W}{S}\right)}$$

$$\frac{T}{W} = \frac{1}{L/D} = \frac{qC_{D_0}}{(W/S)} + \left(\frac{W}{S}\right)\frac{K}{q}$$

The condition for minimum thrust to weight **required** is maximum L/D!

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Flight Mechanics / Performance

Minimum Thrust **Required** for Level Flight

$$\frac{\partial(T/W)}{\partial V} = \frac{\rho V C_{D_0}}{W/S} - \frac{W}{S}\frac{2K}{\frac{1}{2}\rho V^3} = 0$$

$$V_{\text{min thrust or drag}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{C_{D_0}}}}$$

$$C_{L\,\text{min thrust or drag}} = \sqrt{\frac{C_{D_0}}{K}}$$

$$D_{\text{min thrust or drag}} = qS\left[C_{D_0} + K\left(\sqrt{\frac{C_{D_0}}{K}}\right)^2\right] = qS(C_{D_0} + C_{D_0})$$

At maximum $\frac{L}{D}$,

$$C_D = 2C_{D_0}$$

$$C_{D_L} = C_{D_0}$$

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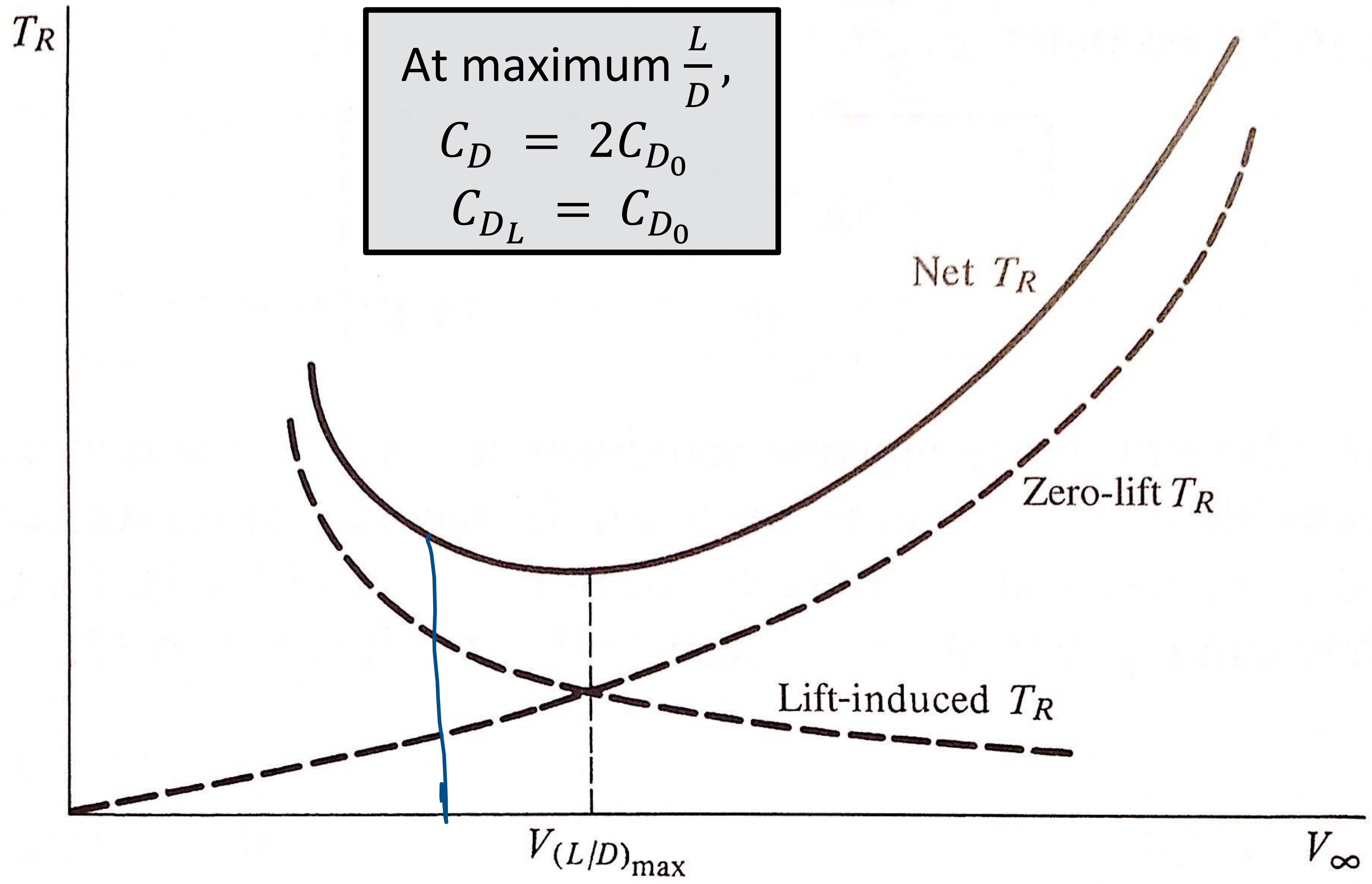
Flight Mechanics / Performance

Minimum Thrust **Required** for Level Flight



At maximum $\frac{L}{D}$,

$$C_D = 2C_{D_0}$$

$$C_{D_L} = C_{D_0}$$

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Flight Mechanics / Performance

Minimum Power **Required** for Level Flight

$$P = DV = qS(C_{D_0} + KC_L^2)V = \frac{1}{2}\rho V^3 S(C_{D_0} + KC_L^2)$$

$$P = \frac{1}{2}\rho V^3 S C_{D_0} + \frac{KW^2}{\frac{1}{2}\rho V S}$$

$$\frac{\partial P}{\partial V} = \frac{3}{2}\rho V^2 S C_{D_0} - \frac{KW^2}{\frac{1}{2}\rho V^2 S} = 0$$

$$V_{\substack{\text{min}\\\text{power}}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{3C_{D_0}}}}$$

$$C_{L\,\substack{\text{min}\\\text{power}}} = \sqrt{\frac{3C_{D_0}}{K}}$$

$$D_{\substack{\text{min}\\\text{power}}} = qS(C_{D_0} + 3C_{D_0})$$

At L/D for Minimum Power Required,

$$C_D = 4C_{D_0}$$

$$C_{D_L} = 3C_{D_0}$$

$$L/D = .866\,(L/D)_{Max}$$

$V = 0.76\,V$ for $(L/D)_{Max}$

$\frac{C_L}{C_D}^{3/2}$ is maximized!

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Flight Mechanics / Performance
Minimum Power Required for Level Flight

At L/D for Minimum Power Required,

$$C_D = 4C_{D_0}$$

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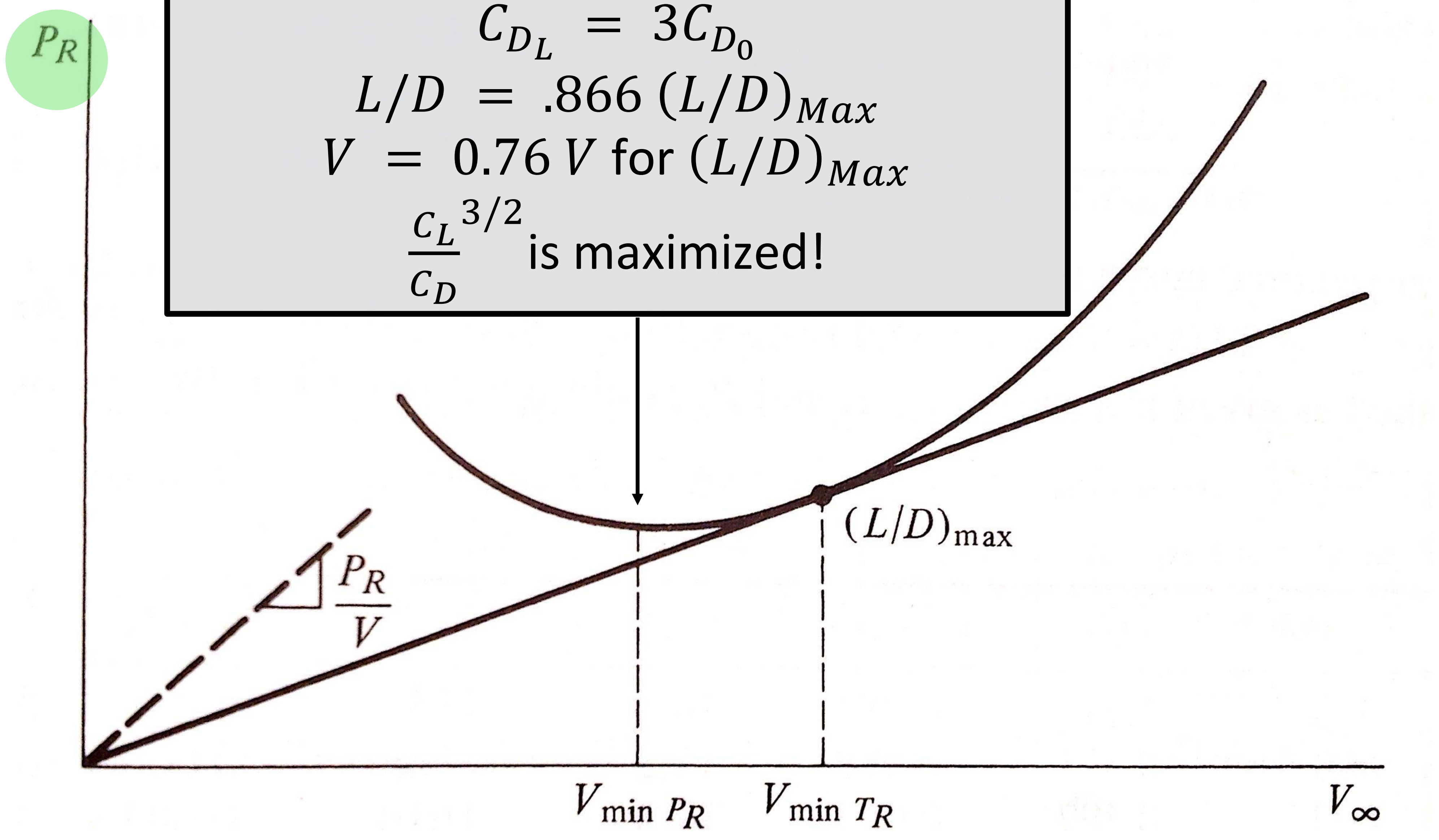
$$\frac{C_L^{3/2}}{C_D} \text{ is maximized!}$$



The parabolic drag polar approximation works o.k. for low Mach numbers and high aspect ratio wings at modest angles of attack.
In aircraft companies, optimization of flight conditions is done numerically by computer programs with accurate aerodynamic and propulsion data. These programs search for flight conditions that yield optimum results. However, for most first-order estimates, the parabolic drag polar will suffice.

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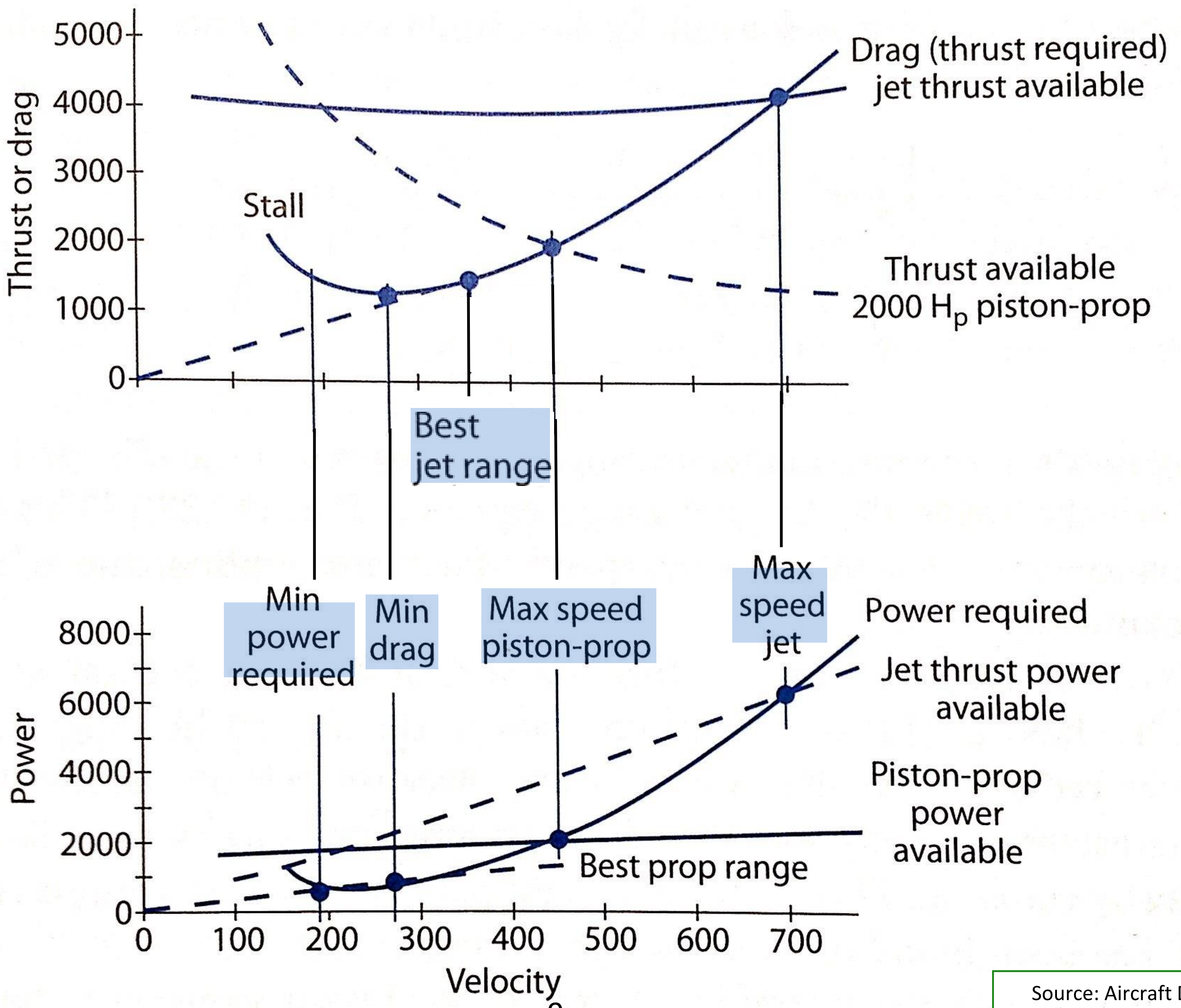
Minimum Thrust/Power **Required** for Level Flight



Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance
Range

- Range = Velocity x Time
- Time = Fuel Weight / Fuel Flow = Fuel Weight / $-CT_R$ (T_R =Thrust **Required** for Steady State)
- Range = Velocity x Fuel Weight / $-CT_R$ ➡ $$dR = \frac{V\,dW}{-CT_R}$$
- As the aircraft burns fuel, its weight diminishes, and so does the drag (thrust required) and therefore the rate of fuel burn!

$$\frac{dR}{dW} = \frac{V}{-CT_R} = \frac{V}{-CD} = \frac{V(L/D)}{-CW}$$

$$R = \int_{W_i}^{W_f} \frac{V(L/D)}{-CW}\,dW = \frac{V}{C}\frac{L}{D}\ell n\left(\frac{W_i}{W_f}\right)$$

Breguet Range Equation

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Flight Mechanics / Performance
Range

The Breguet range equation assumes that :

- $\left(\frac{V}{C} \cdot \frac{L}{D}\right)$ is approximately constant

- Constant L/D means constant C_L; therefore, the aircraft must climb as it becomes lighter.

- The segment of a mission or flight where Range is gained is called **Cruise**

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Flight Mechanics / Performance
Range Optimization - Jet

"Range Parameter:"

$$\frac{V}{C}\left(\frac{L}{D}\right) = \frac{V}{C}\left(\frac{C_L}{C_{D_0} + KC_L^2}\right) = \frac{2W/\rho VS}{CC_{D_0} + (4KW^2C)/(\rho^2 V^4 S^2)}$$

After setting its derivative w.r.t. V equal to 0:

$$V_{\substack{\text{best}\\\text{range}}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{3K}{C_{D_0}}}}$$

$$C_{L\,\substack{\text{best}\\\text{range}}} = \sqrt{\frac{C_{D_0}}{3K}}$$

$$D_{\substack{\text{best}\\\text{range}}} = qS\left(C_{D_0} + \frac{C_{D_0}}{3}\right)$$

At L/D for Best Jet Range,

$C_D = 4C_{D_0}/3$

$C_{D_L} = C_{D_0}/3$

$L/D = .866\,(L/D)_{Max}$

$V = 1.316\,V$ for $(L/D)_{Max}$

$\frac{C_L}{C_D}^{1/2}$ is maximized!

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Flight Mechanics / Performance
Range Optimization - Propeller

Recall:

$$C = C_{\text{power}} \frac{V}{\eta_p} = C_{\text{bhp}} \frac{V}{550 \;\; \eta_p}$$

Then:

$$R = \frac{\eta_p}{C_{\text{power}}} \frac{L}{D} \ell n \left(\frac{W_i}{W_f} \right) = \frac{550 \eta_p}{C_{\text{bhp}}} \frac{L}{D} \ell n \left(\frac{W_i}{W_f} \right)$$

$$V_{\text{min thrust or drag}} = \sqrt{\frac{2W}{\rho S} \sqrt{\frac{K}{C_{D_0}}}}$$

$$C_{L\,\text{min thrust or drag}} = \sqrt{\frac{C_{D_0}}{K}}$$

At maximum $\frac{L}{D}$,

$C_D \; = \; 2C_{D_0}$

$C_{D_L} \; = \; C_{D_0}$

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Flight Mechanics / Performance
Loiter Endurance Optimization-Jets

- Endurance = Fuel Weight / (Thrust x C)
- Notice, no velocity term...
- The segment of the mission or flight where the aircraft is spending time at a fixed location is called **Loiter.**

$$\frac{\mathrm{d}E}{\mathrm{d}W} = -\frac{1}{CT} = \frac{1}{-CW}\left(\frac{L}{D}\right)$$

$$E = \int_{W_i}^{W_f} \frac{1}{-CT}\mathrm{d}W = \int_{W_f}^{W_i} \frac{1}{CW}\left(\frac{L}{D}\right)\mathrm{d}W = \left(\frac{L}{D}\right)\left(\frac{1}{C}\right)\ell n\left(\frac{W_i}{W_f}\right)$$

To Maximize E, we maximize L/D!

$$V_{\text{min thrust or drag}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{C_{D_0}}}}$$

$$C_{L\,\text{min thrust or drag}} = \sqrt{\frac{C_{D_0}}{K}}$$

At maximum $\frac{L}{D}$,

$C_D = 2C_{D_0}$

$C_{D_L} = C_{D_0}$

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Flight Mechanics / Performance
Loiter Endurance Optimization-Propeller

Again:

$$C = C_{\text{power}} \frac{V}{\eta_p} = C_{\text{bhp}} \frac{V}{550 \ \eta_p}$$

Then:

$$E = \left(\frac{L}{D}\right)\left(\frac{\eta_p}{C_{\text{power}} V}\right) \ell n \left(\frac{W_i}{W_f}\right)$$

$$= \left(\frac{L}{D}\right)\left(\frac{550\eta_p}{C_{\text{bhp}} V}\right) \ell n \left(\frac{W_i}{W_f}\right)$$

$$\frac{\partial}{\partial V}\left(\frac{L}{DV}\right) = \frac{\partial}{\partial V}\left[\frac{2W/\rho V^3 S}{C_{D_0} + (4KW^2/\rho^2 V^4 S^2)}\right] = 0$$

At L/D for Minimum Power Required,

$$C_D = 4C_{D_0}$$

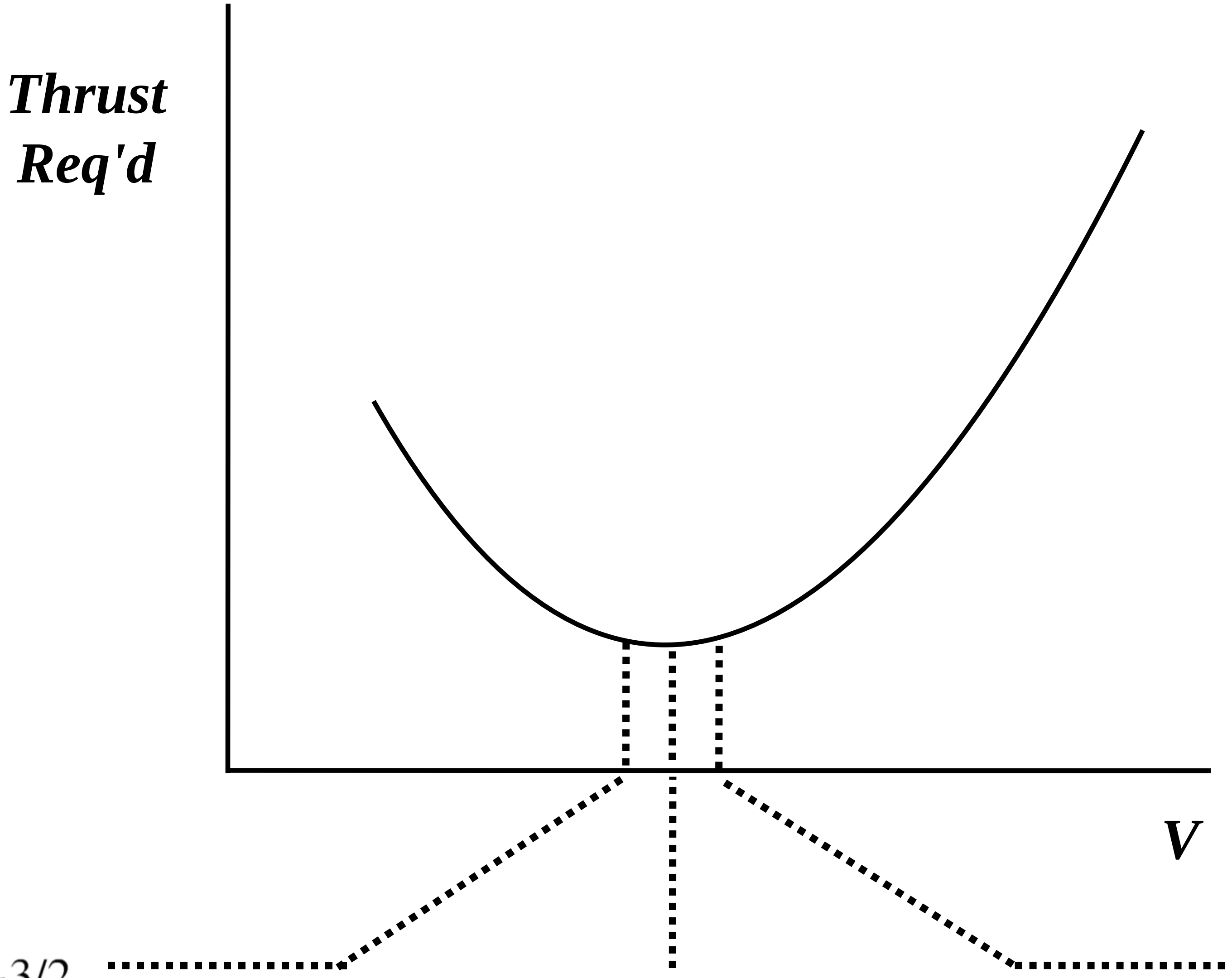
$$C_{D_L} = 3C_{D_0}$$

$$L/D = .866\,(L/D)_{Max}$$

$V = 0.76\,V$ for $(L/D)_{Max}$

$$V_{\substack{\text{min}\\ \text{power}}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{3C_{D_0}}}} \qquad C_{L\,\substack{\text{min}\\ \text{power}}} = \sqrt{\frac{3C_{D_0}}{K}} \qquad D_{\substack{\text{min}\\ \text{power}}} = qS(C_{D_0} + 3C_{D_0})$$

- Summary of Optimum Conditions for Range and Endurance



$V_{\left(\frac{C_L^{3/2}}{C_D}\right)_{max}}$ — Propeller **Endurance**

$V_{\left(\frac{C_L}{C_D}\right)_{max}}$ — Propeller **Range**, Jet **Endurance**

$V_{\left(\frac{C_L^{1/2}}{C_D}\right)_{max}}$ — Jet **Range**

- Summary of Optimum Conditions for Range and Endurance

	$V_{\left(\frac{C_L^{3/2}}{C_D}\right)_{max}}$	$V_{\left(\frac{C_L}{C_D}\right)_{max}}$	$V_{\left(\frac{C_L^{1/2}}{C_D}\right)_{max}}$
$C_{D_0}\, vs.\, C_{D_i}$	$KC_L^2 = 3C_{D_0}$	$KC_L^2 = C_{D_0}$	$KC_L^2 = \frac{1}{3}C_{D_0}$
C_D	$4C_{D_0}$	$2C_{D_0}$	$\frac{4}{3}C_{D_0}$
C_L	$\sqrt{\frac{3C_{D_0}}{K}}$	$\sqrt{\frac{C_{D_0}}{K}}$	$\sqrt{\frac{C_{D_0}}{3K}}$
V	$\sqrt{\frac{2\,W/S}{\rho\sqrt{3C_{D_0}/K}}}$	$\sqrt{\frac{2\,W/S}{\rho\sqrt{C_{D_0}/K}}}$	$\sqrt{\frac{2\,W/S}{\rho\sqrt{C_{D_0}/3K}}}$

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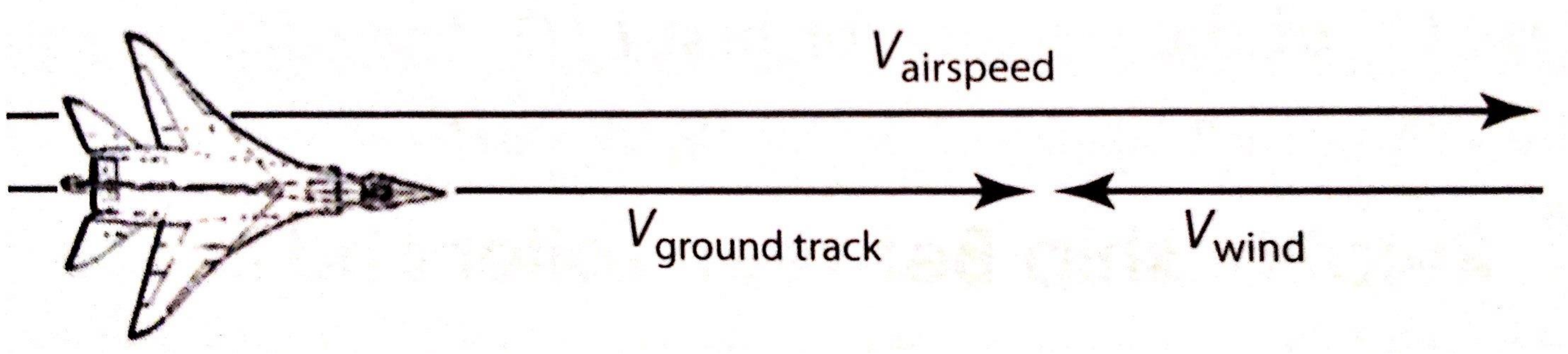
Flight Mechanics / Performance
Effects of Wind



- A headwind of X velocity will decrease the ground speed of the aircraft, thus reducing its range, and a tailwind will do the opposite, assuming in both situations it is flying at a given indicated airspeed .

- At the conceptual design level, if a mission requirement calls for a given range with a given headwind, just add the % cruise speed that the headwind amounts to the required range as an approximation when sizing the design.

Source: Aircraft Design by Daniel P. Raymer

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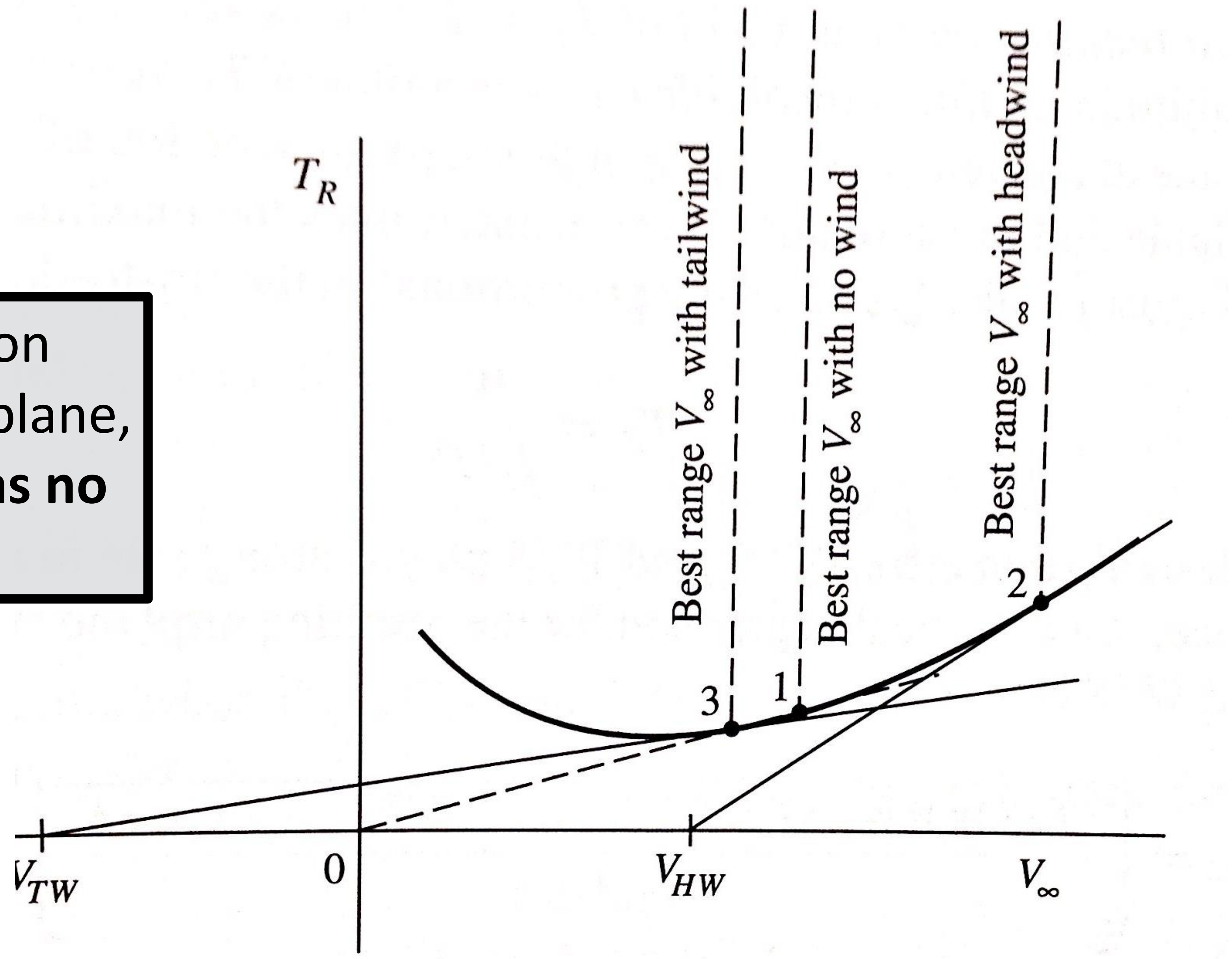
Flight Mechanics / Performance
Effects of Wind

For a jet plot thrust required on the y-axis. For a propeller airplane, plot power required. **Wind has no effect on loiter endurance**



Source: Aircraft Performance and Design by John D. Anderson

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Flight Mechanics / Performance
Steady Climbing

$$T = D + W \sin\gamma$$

$$L = W \cos\gamma$$

$$\gamma = \sin^{-1}\left(\frac{T-D}{W}\right) = \sin^{-1}\left(\frac{T}{W} - \frac{\cos\gamma}{L/D}\right) \cong \sin^{-1}\left(\frac{T}{W} - \frac{1}{L/D}\right)$$

$$V_v = V \sin\gamma = V\left(\frac{T-D}{W}\right) \cong V\left(\frac{T}{W} - \frac{1}{L/D}\right)$$

$$V = \sqrt{\frac{2}{\rho C_L}\left(\frac{W}{S}\right)\cos\gamma}$$

For cruise, $T/W = 1/(L/D)$; **no climb!**

$$T/W = \frac{\cos\gamma}{L/D} + \sin\gamma \cong \frac{1}{L/D} + \sin\gamma = \frac{1}{L/D} + \frac{V_v}{V}$$

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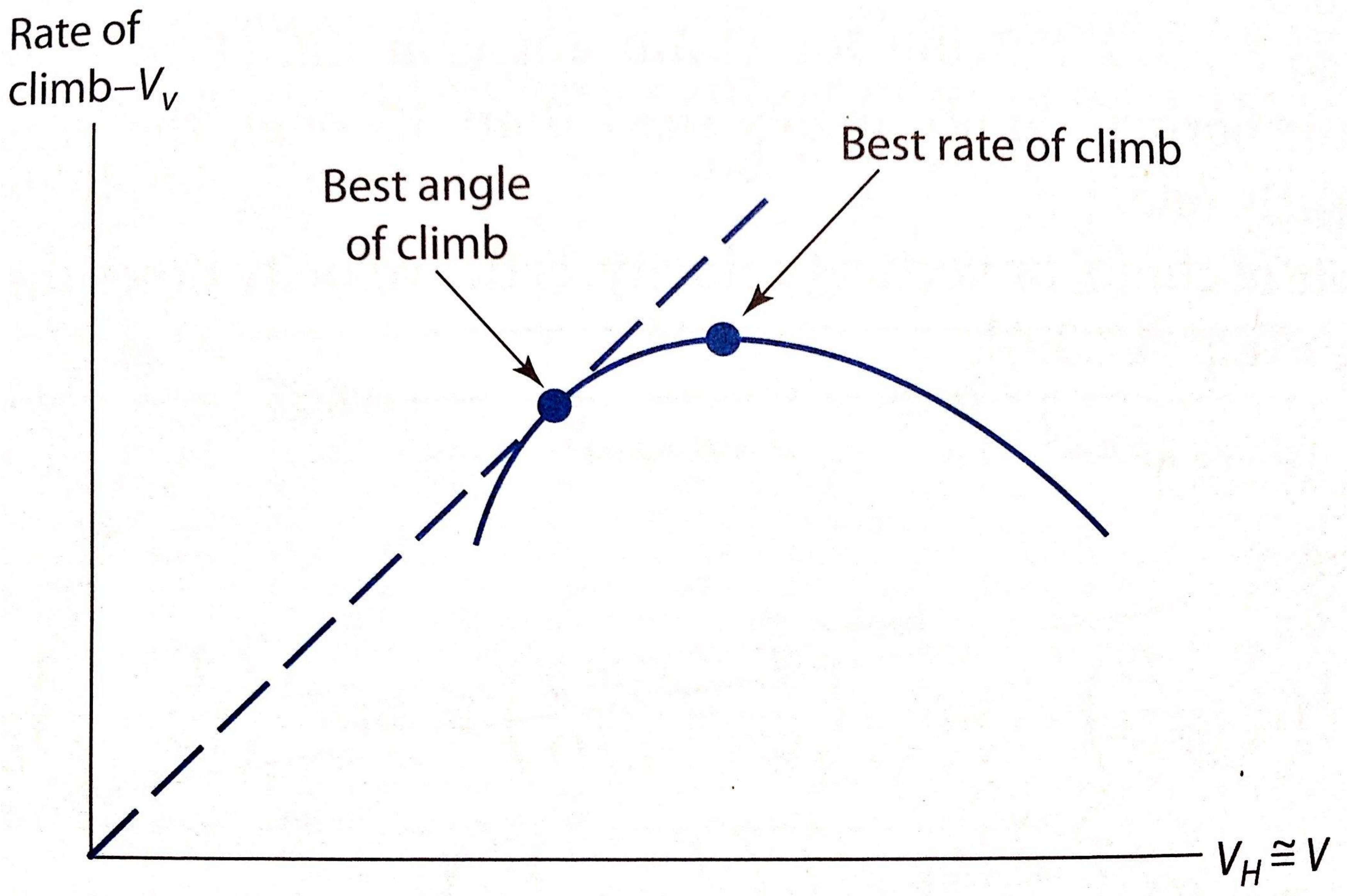
Flight Mechanics / Performance
Best Climbs

$$V_\nu = V\left(\frac{T-D}{W}\right)$$



Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance
Best Climbs - Jets

- Best climb angle: Thrust is essentially constant with velocity, so the climb angle equation suggests maximum L/D. Therefore to maximize climb angle:

$$\gamma \cong \sin^{-1}\left(\frac{T}{W} - \boxed{\frac{1}{L/D}}\right) \quad \Rightarrow \quad V_{\text{min thrust or drag}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{C_{D_0}}}}$$

- Best climb rate:

$$V_v = V\left(\frac{T-D}{W}\right) = V\left(\frac{T}{W}\right) - \frac{\rho V^3 C_{D_0}}{2(W/S)} - \frac{2K}{\rho V}\left(\frac{W}{S}\right)$$

$$\frac{\partial V_v}{\partial V} = 0 = \frac{T}{W} - \frac{3\rho V^2 C_{D_0}}{2(W/S)} + \frac{2K}{\rho V^2}\left(\frac{W}{S}\right)$$

$$V = \sqrt{\frac{W/S}{3\rho C_{D_0}}\left[T/W + \sqrt{(T/W)^2 + 12 C_{D_0} K}\right]}$$

> Good at one altitude, must recalculate at various altitudes!

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Flight Mechanics / Performance
Best Climbs - Propeller

- Climb angle:

$$\gamma = \sin^{-1}\left[\frac{P\eta_p}{VW} - \frac{D}{W}\right] = \sin^{-1}\left[\frac{550\,\text{bhp}\ \eta_p}{VW} - \frac{D}{W}\right]$$

- Steepest climb: The results obtained by taking the derivative of this equation yield a velocity too low for the parabolic drag approximation. **Use the graphical method.**

- Climb rate: $$V_v = V\sin\gamma = \frac{P\,\eta_p}{W} - \frac{DV}{W} = \frac{550\,\text{bhp}\ \eta_p}{W} - \boxed{\frac{DV}{W}}$$

To maximize climb rate, just minimize power required!

$$V_{\substack{\min\\ \text{power}}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{3C_{D_0}}}} \qquad C_{L\,\substack{\min\\ \text{power}}} = \sqrt{\frac{3C_{D_0}}{K}} \qquad D_{\substack{\min\\ \text{power}}} = qS(C_{D_0} + 3C_{D_0})$$

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Flight Mechanics / Performance
Time and Fuel to Climb

$$dt = \frac{dh}{V_\nu}$$

$$dW_f = -CT\,dt$$

- During climb, everything changes; density, weight, thrust, SFC, etc. Better to conduct a piecewise integration since climb rate reduces linearly with altitude.

$$V_\nu = V_{\nu_i} - a(h_{i+1} - h_i)$$

$$a = \frac{V_{\nu_2} - V_{\nu_1}}{h_2 - h_1}$$

$$t_{t+1} - t_i = \frac{1}{a}\ell n\left(\frac{V_{\nu_i+1}}{V_{\nu_i}}\right)$$

$$\Delta W_{\text{fuel}} = (-CT)_{\text{average}}(t_{i+1} - t_i)$$

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Flight Mechanics / Performance
A Note on Airspeeds

- TAS = True airspeed is the actual distance/time that the aircraft travels with respect to the air mass (wind)

- Ground speed = TAS corrected for wind.

- EAS = Equivalent airspeed is the airspeed at sea level in the std. atmosphere at which the dynamic pressure is the same as the dynamic pressure the aircraft is subjected to at the true airspeed and altitude at which is flying at. In low speed flight (no compressibility), this is the same speed that would be shown by an airspeed indicator without error. EAS is useful in calculating aerodynamic loads, handling qualities, etc.

$$EAS = TAS\sqrt{\frac{\rho}{\rho_0}}$$

Flight Mechanics / Performance
A Note on Airspeeds

- The aircraft pitot-static probe measures static and total (stagnation) pressures. The dynamic pressure (and from that, velocity) are calculated using Bernoulli's equation and may include compressibility effects, position errors, etc.

- Compressibility error is introduced when going at speeds where air is compressed in the probe.

- Position error is introduced as the static pressure at the probe is different than freestream static pressure.

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Flight Mechanics / Performance
A Note on Airspeeds

- CAS = Calibrated airspeed is the speed displayed after corrected for position error.

- IAS = Indicated airspeed that includes a cockpit instrument correction. When IAS is corrected for position error, it becomes CAS.

$$EAS = TAS\sqrt{\frac{\rho}{\rho_0}}$$

$$\text{EAS} = \text{CAS}\sqrt{P/P_0}\left[\frac{(q_c/P+1)^{0.286}-1}{(q_c/P_0+1)^{0.286}-1}\right]^{0.5}$$

$$q_c = P([1+0.2M^2]^{3.5}-1)$$

- At sea level, and incompressible flight regimes (slow), CAS = EAS = TAS

- Mach number = TAS / Speed of Sound at the altitude flown.

Flight Mechanics / Performance
Load Factor Limits

- **Limit** load factor.- if it is exceeded, the aircraft will suffer permanent structural deformation (structural damage). If "n" is less than limit load factor, the structure may deflect but will return to its original state when "n" = 1.

- **Ultimate** load factor.- if "n" > ultimate load factor, the structure will suffer structural failure; parts of the aircraft will break.

- The V-n diagram shows aerodynamic and structural limitations of the aircraft. It is a plot of "n" vs. velocity.

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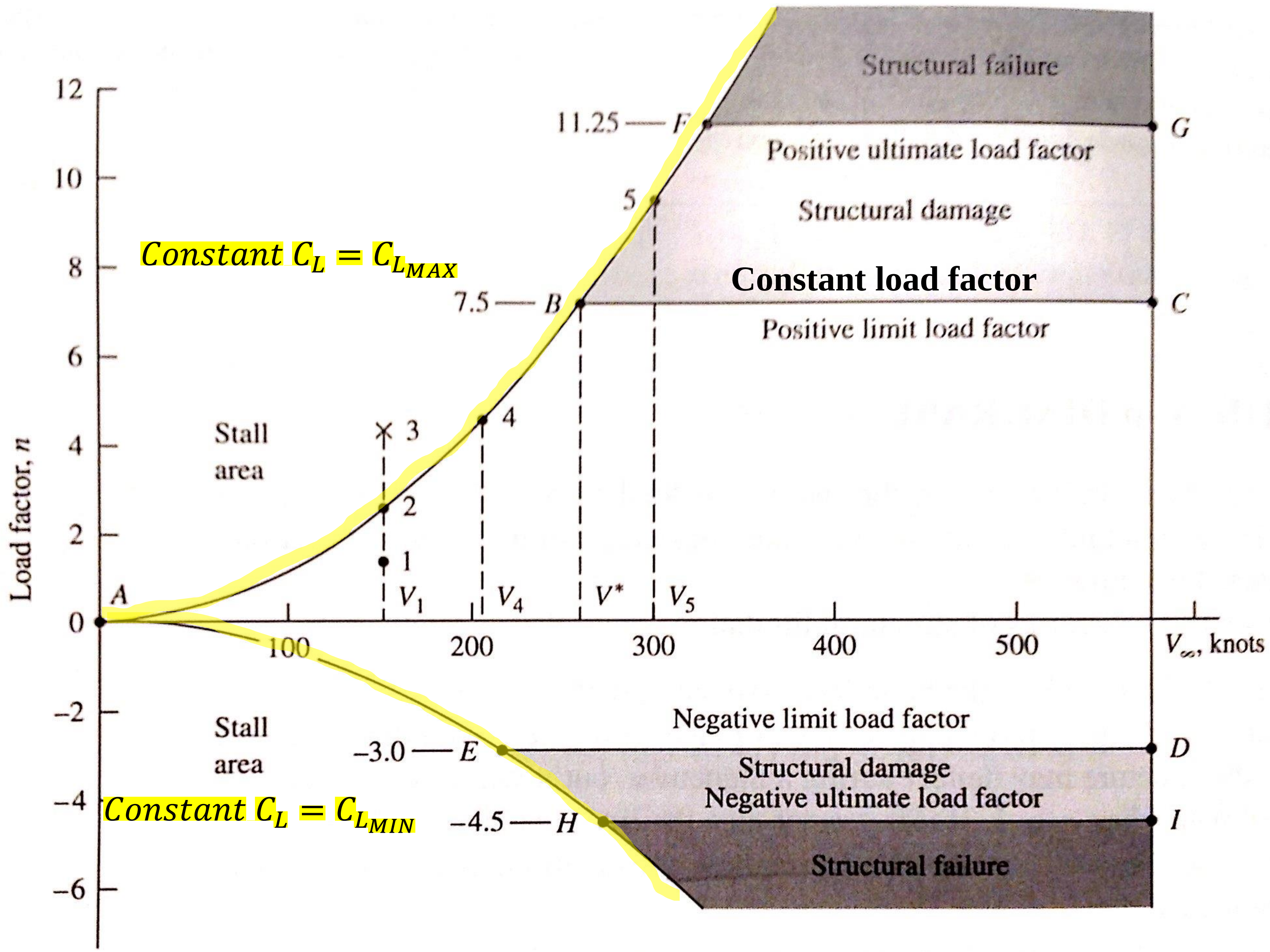
Flight Mechanics / Performance
Load Factor Limits



- Point 3 can never be reached (beyond stall)
- BC Positive n limit
- CD speed limit (max q, about $1.2V_{max}$(
- V^* is the corner velocity (maximum instantaneous turn rate and minimum turn radius)
- At $V > V^*$, the a/c can reach the limit n_{max} aerodynamically at $C_L < C_{L_{max}}$!

$$V^* = \sqrt{\frac{2n_{\max}}{\rho_\infty (C_L)_{\max}} \frac{W}{S}}$$

Source: Aircraft Performance and Design by John D. Anderson

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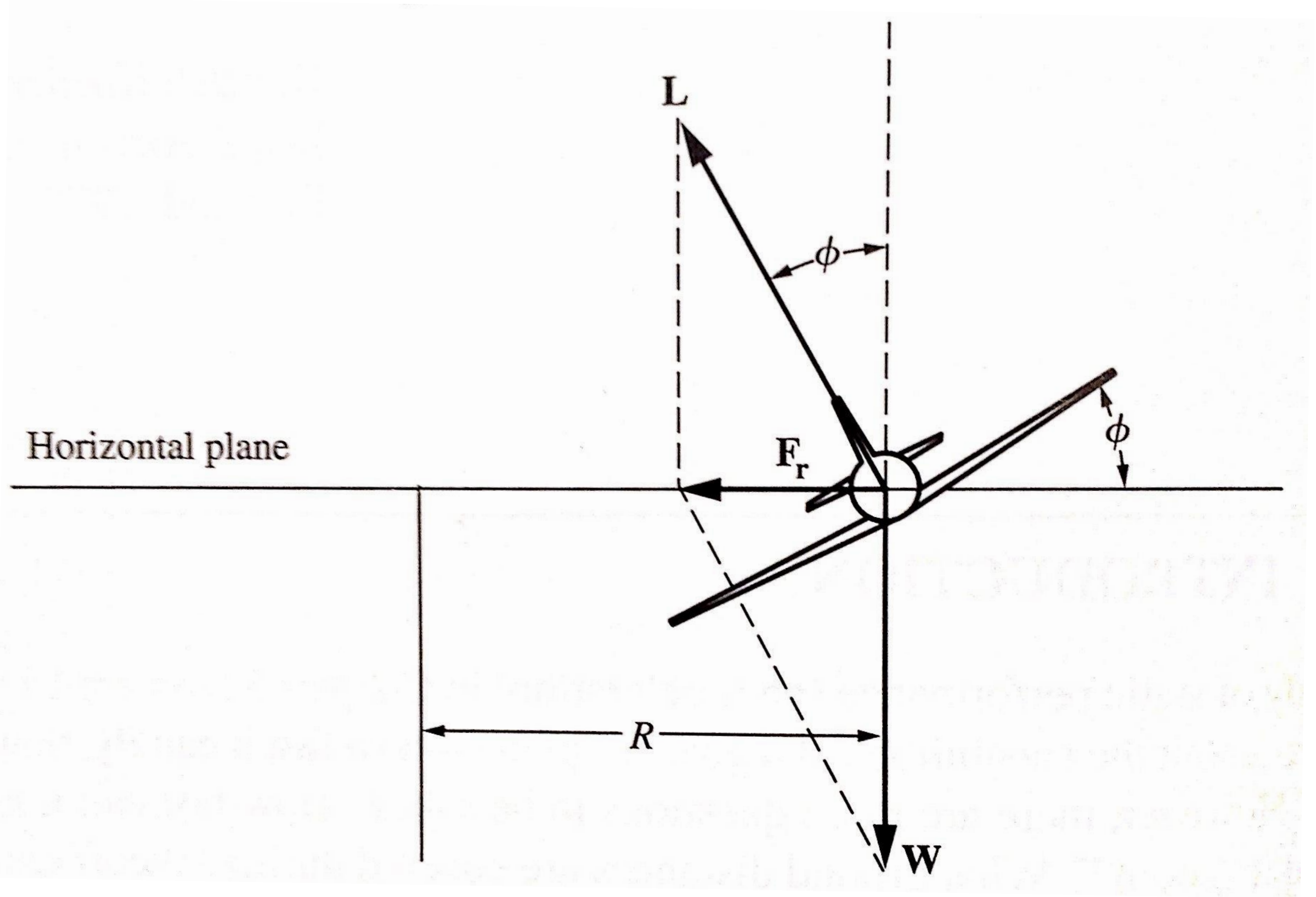
Flight Mechanics / Performance
Level Turning Flight

- $n = L / W$
- No change in altitude, so $L\cos(\phi) = W$
- The force causing the centripetal acceleration is $L\sin(\phi)$.
- To take ϕ out of the equations:



$$F_r^2 + W^2 = (\eta W)^2$$

$$F_r = W\sqrt{\eta^2 - 1}$$

$$mass = \frac{W}{g}$$

$$TurnRate = \frac{d\psi}{dt} = \frac{radial.\,acceleration}{V} = \frac{F_r/mass}{V}$$

$$TurnRate = \frac{d\psi}{dt} = \frac{g\sqrt{\eta^2 - 1}}{V}$$

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Flight Mechanics / Performance
Instantaneous Turn Rate

- In a maximum load factor turn, the pilot pulls the stick until aircraft reaches either $C_{L_{MAX}}$ or the limit load factor n_{MAX}, and the aircraft turns sharply and most probably loses airspeed if altitude is maintained.

- Maximum Instantaneous Turn Rate occurs at corner speed, where the stall limit and the structural limit meet (300-350 kts). However, a good technique would call for the pilot to pull a max-g (load factor) at a somewhat faster airspeed and naturally decelerate while holding the "n" until it hits $C_{L_{MAX}}$ and ends the maneuver at a somewhat slower speed, thus maximizing average turn rate.

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Flight Mechanics / Performance
Sustained Turn Rate

- In a sustained turn, the aircraft must not lose airspeed or altitude. $T = D;\ L = n \cdot W$

$$\text{Sustained } n = (T/W)(L/D)$$

$$\text{Sustained } n = \sqrt{\frac{q}{K(W/S)}\left(\frac{T}{W} - \frac{qC_{D_0}}{W/S}\right)}$$

- This is a performance item of interest in fighters. They also fly very fast; therefore, C_{D_0} **increases** with Mach, and with a high T/W, you are probably at a very high C_L and therefore higher K due to separation and Mach.

- For maximum sustained load factor, you want high T/W, low W/S, low K, maximum $(T/W)(L/D)$ flight conditions ($(L/D)_{MAX}$ for a jet).

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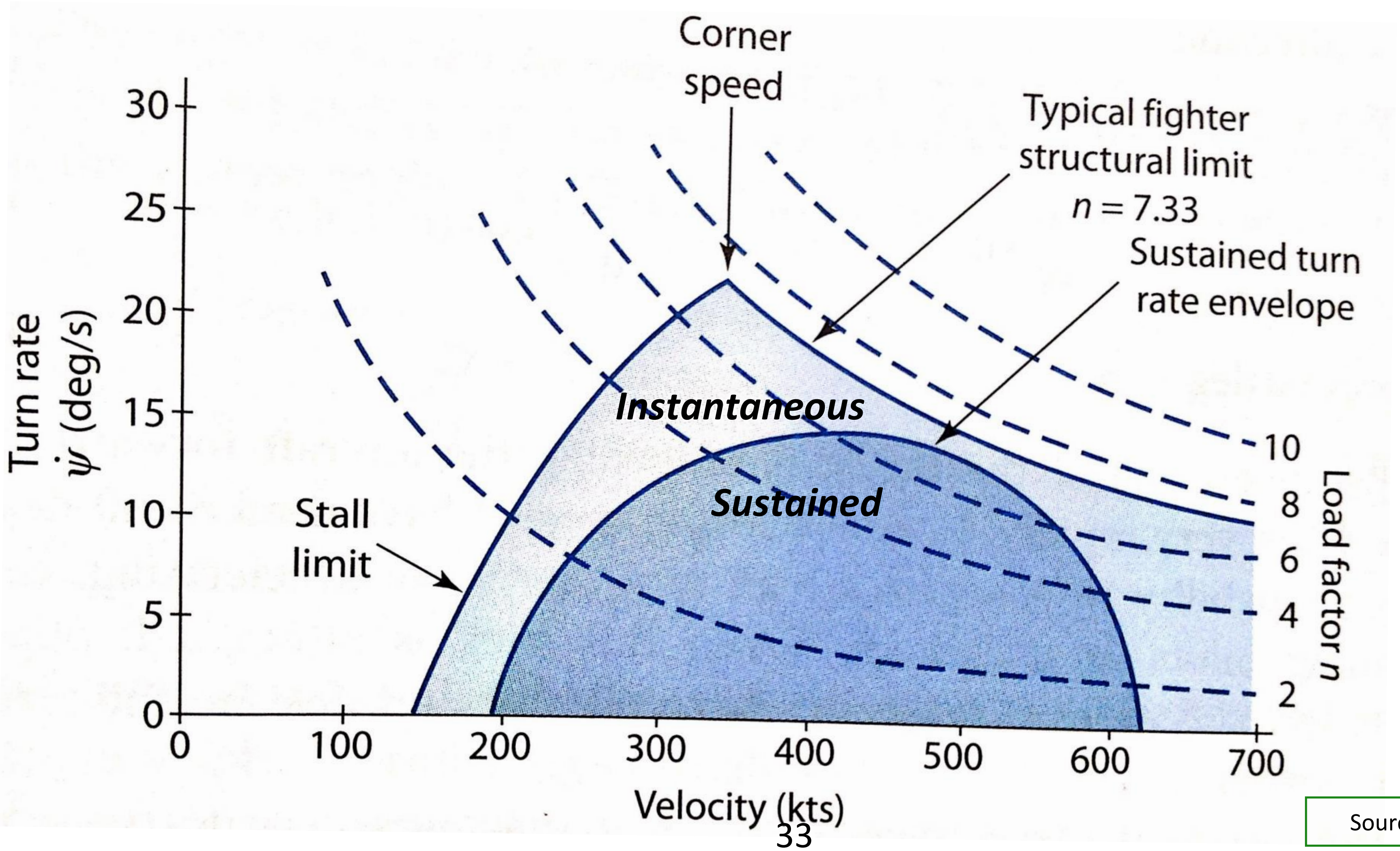
Flight Mechanics / Performance
Sustained Turn Rate

- But maximum sustained turn rate occurs at a slightly lower "n" because:

$$TurnRate = \frac{d\psi}{dt} = \frac{g\sqrt{\eta^2 - 1}}{V}$$



Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance
Gliding Flight

- Set T = 0:

$$D = W \sin \gamma$$
$$L = W \cos \gamma$$
$$\frac{L}{D} = \frac{W \cos \gamma}{W \sin \gamma} = \frac{1}{\tan \gamma} \cong \frac{1}{\gamma}$$

- Glide Ratio = (horizontal distance / vertical distance) = L/D

- To maximize glide range, must glide at $(L/D)_{MAX}$!

$$V_{\max L/D} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{C_{D_0}}}}$$

$$C_{L_{\max L/D}} = \sqrt{\frac{C_{D_0}}{K}}$$

$$\left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{C_{D_0}K}} = \frac{1}{2}\sqrt{\frac{\pi A e}{C_{D_0}}}$$

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Flight Mechanics / Performance
Gliding Flight

- To maximize time aloft, we look at sink rate:

$$V_v = V \sin\gamma = \sin\gamma \sqrt{\left(\frac{W}{S}\right)\frac{2\cos\gamma}{\rho C_L}}$$

$$\sin\gamma = \frac{D}{L}\cos\gamma = \frac{C_D}{C_L}\cos\gamma$$

$$V_v = \sqrt{\frac{W}{S}\frac{2\cos^3\gamma C_D^2}{\rho C_L^3}} \cong \sqrt{\frac{W}{S}\frac{2}{\rho(C_L^3/C_D^2)}}$$

$$\frac{\partial}{\partial C_L}\left(\frac{C_L^3}{C_D^2}\right) = \frac{\partial}{\partial C_L}\left[\frac{C_L^3}{(C_{D_0}+KC_L^2)^2}\right] = 0$$

$$C_{L_{\text{min sink}}} = \sqrt{\frac{3C_{D_0}}{K}}$$

$$V_{\text{min sink}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{3C_{D_0}}}}$$

$$\left(\frac{L}{D}\right)_{\text{min sink}} = \sqrt{\frac{3}{16KC_{D_0}}} = \sqrt{\frac{3\pi Ae}{16C_{D_0}}}$$

- Conditions for minimum power required! →

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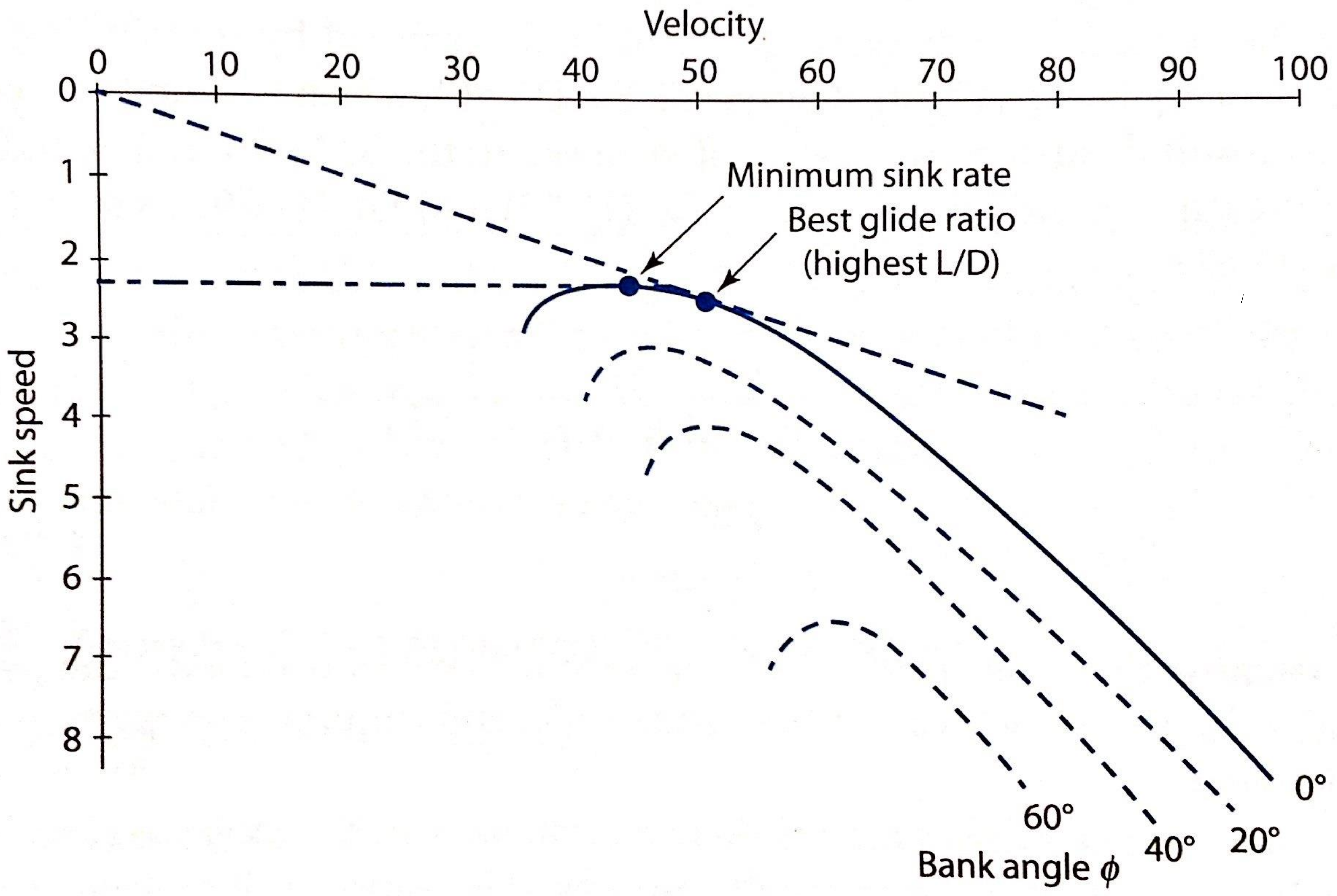
Flight Mechanics / Performance
Gliding Flight



Source: Aircraft Design by Daniel P. Raymer

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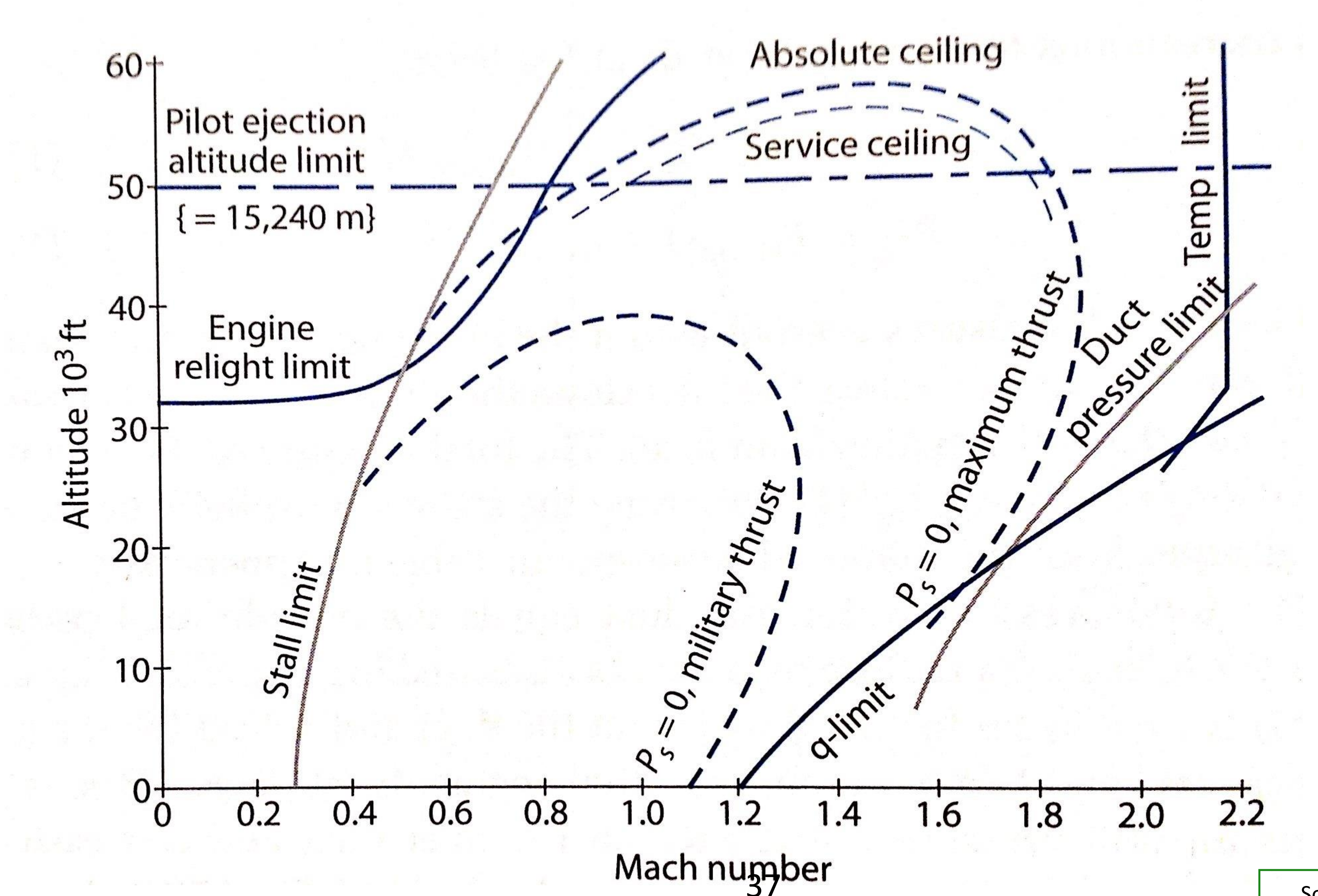
Flight Mechanics / Performance

The Flight Envelope

- Maps the combination of altitudes and speeds (velocity or Mach) that the aircraft can fly at a fixed load factor and throttle setting. Usually shown at a weight of interest, like combat weight.



$$P_s = 0$$
$$n = 1$$

Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance
Energy Maneuverability

- "Exchange the potential energy of altitude for the kinetic energy and or turn rate". And vice versa...

$$E = Wh + \frac{1}{2}\left(\frac{W}{g}\right)V^2$$

Accelerate!

At zero velocity, this is just altitude!

$$h_e = \frac{E}{W} = h + \frac{1}{2g}V^2$$

Specific power!

$$P_{s_{used}} = \frac{dh_e}{dt} = \boxed{\frac{dh}{dt}} + \boxed{\frac{V}{g}\frac{dV}{dt}}$$

Climb!

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Flight Mechanics / Performance
Energy Maneuverability

$$P = V(T - D)$$

$$P_s = \frac{V(T - D)}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt}$$

$$P_s = V\left[\frac{T}{W} - \frac{qC_{D_0}}{W/S} - n^2\frac{K}{q}\frac{W}{S}\right]$$

- The higher the "n" the lower the P_s.
- P_s has same units as $ROC = ft/\sec$.
- P_s @ "n" = 1 is the same ROC the pilot could obtain if he or she wished to climb. On the other hand, (gPs / V) would be the highest acceleration if he wished to increase V at constant altitude.

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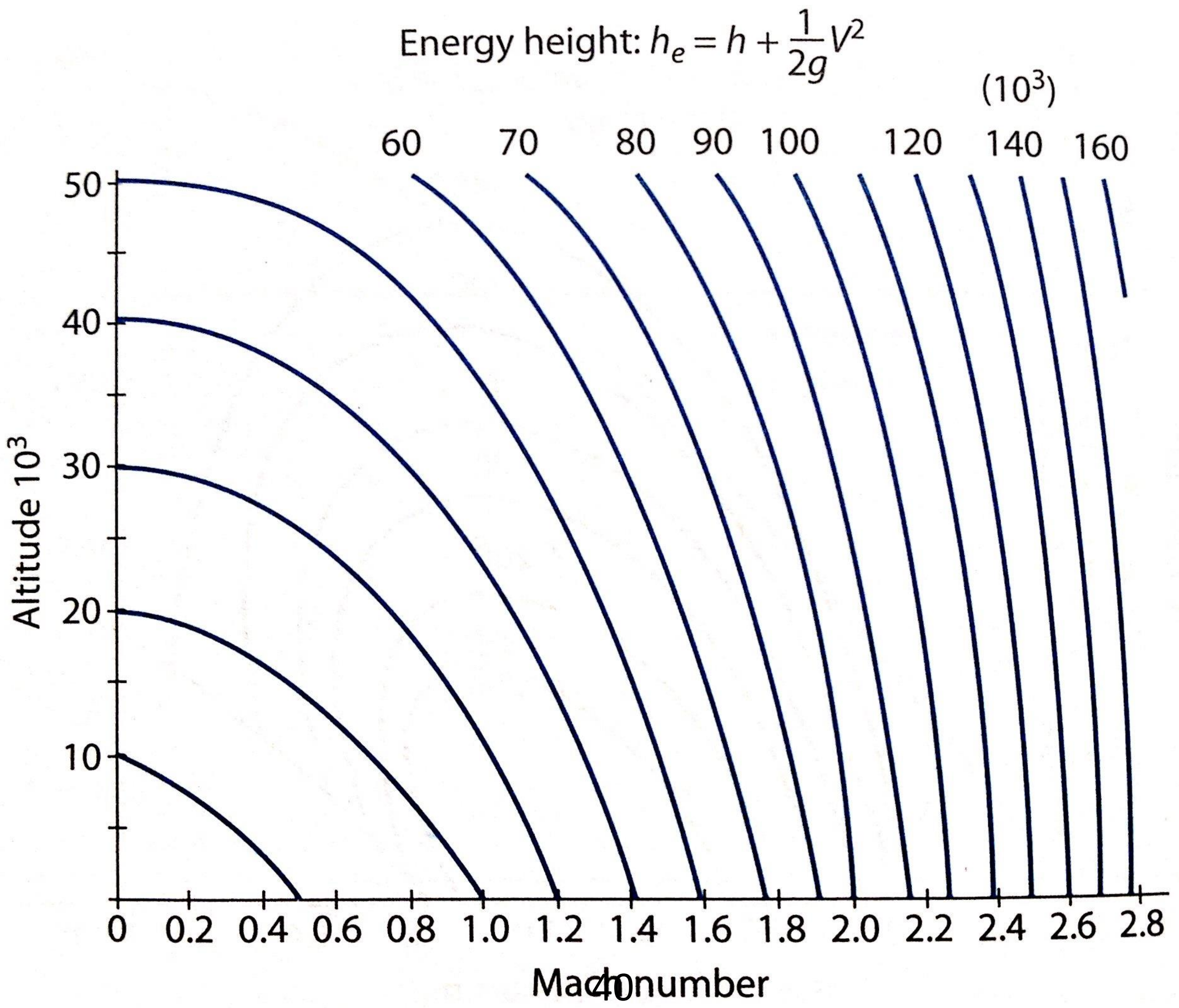
Flight Mechanics / Performance
Energy Maneuverability

- If an aircraft has $P_s = 0$, it can be flying level and steady or it can zoom along an energy line, trading altitude for velocity or viceversa. Only when $P_s > 0$ can the aircraft cross energy lines!



Source: Aircraft Design by Daniel P. Raymer

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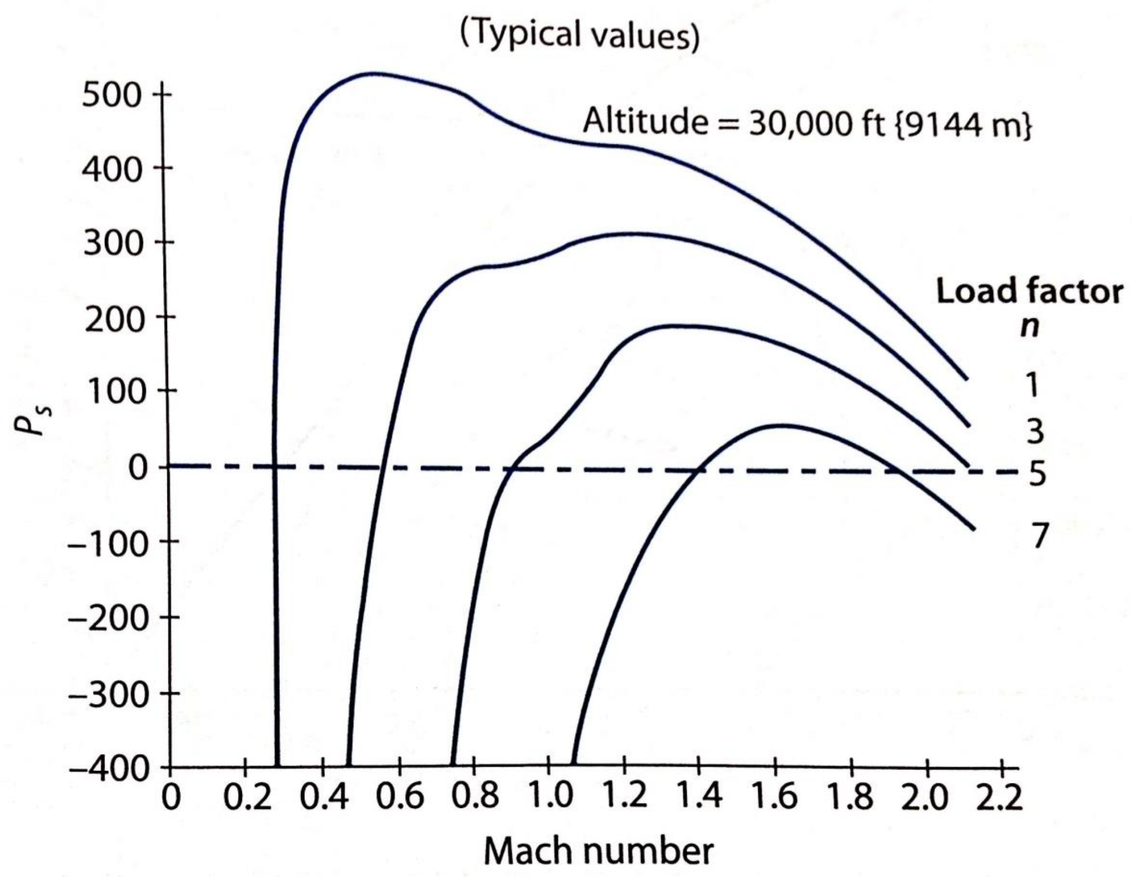
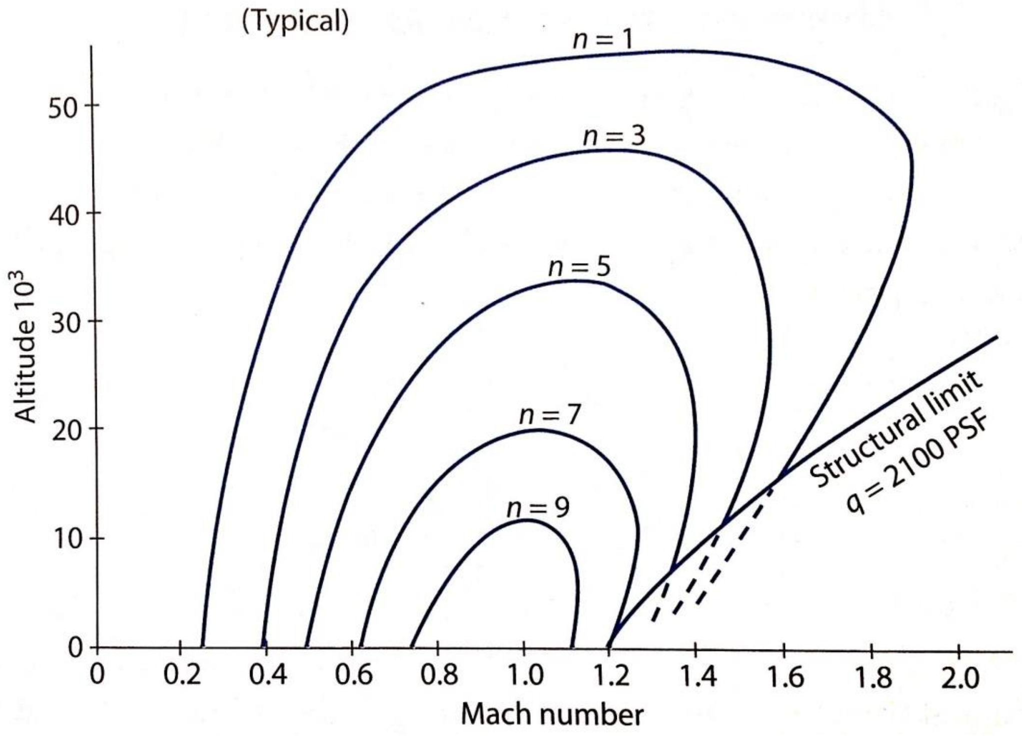
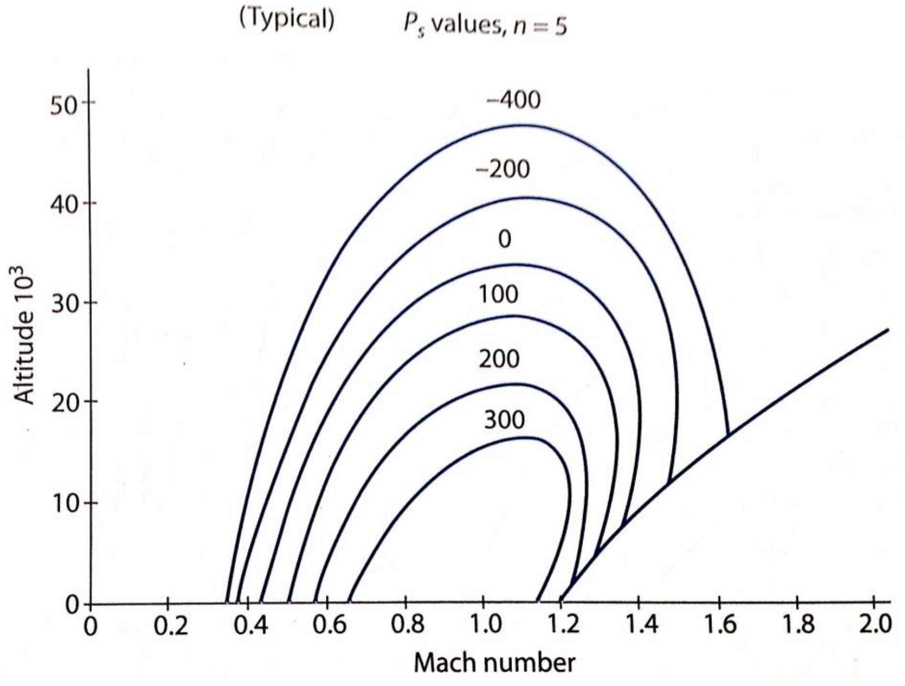
Flight Mechanics / Performance

Energy Maneuverability







Source: Aircraft Design by Daniel P. Raymer

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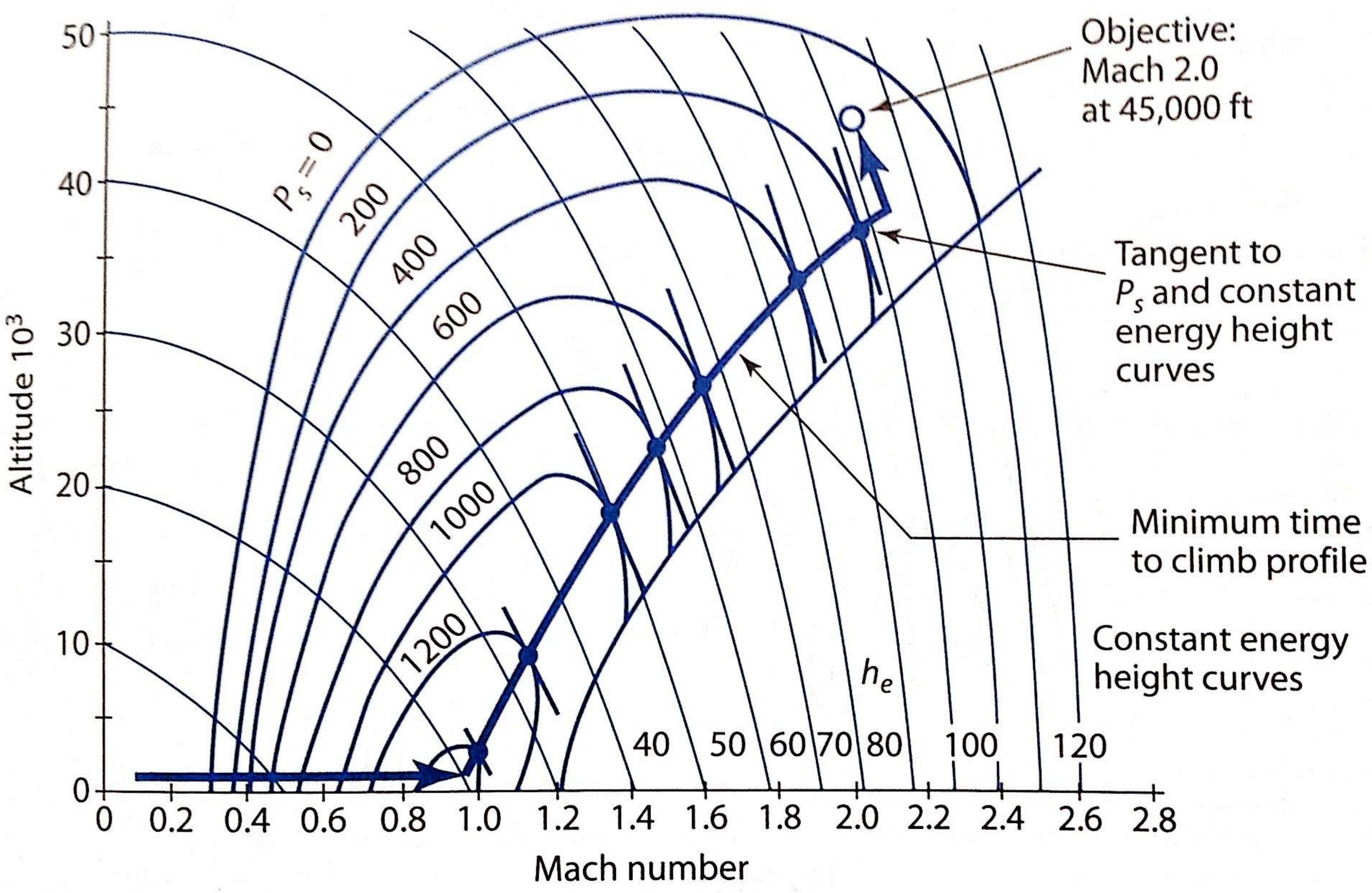
Flight Mechanics / Performance

Minimum Time to Climb

$$dt = \frac{dh_e}{P_s}$$

$$t_{1-2} = \int_{h_{e1}}^{h_{e2}} \frac{1}{P_s} dh_e$$



Source: Aircraft Design by Daniel P. Raymer

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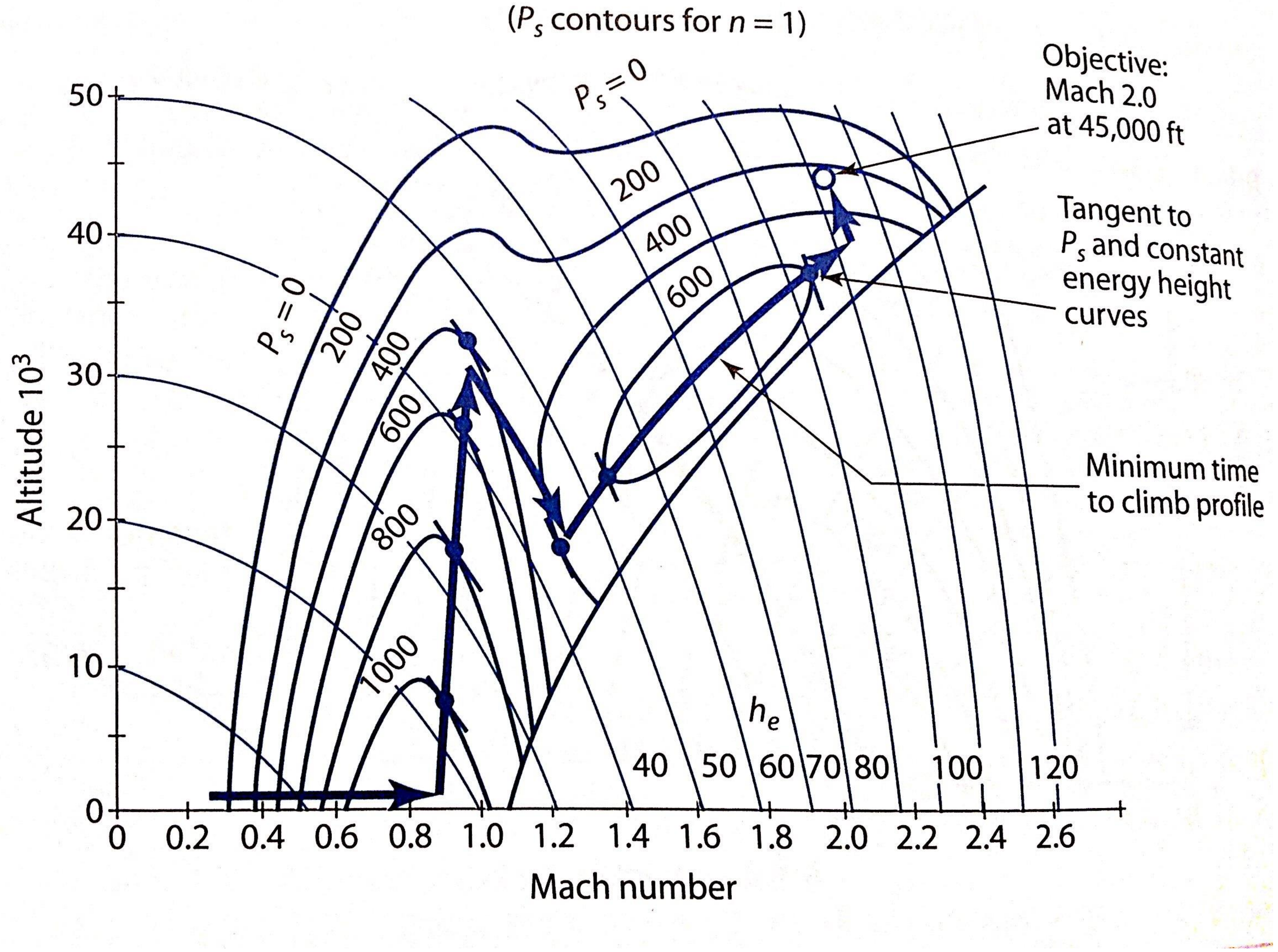
Flight Mechanics / Performance

Minimum Time to Climb



Source: Aircraft Design by Daniel P. Raymer

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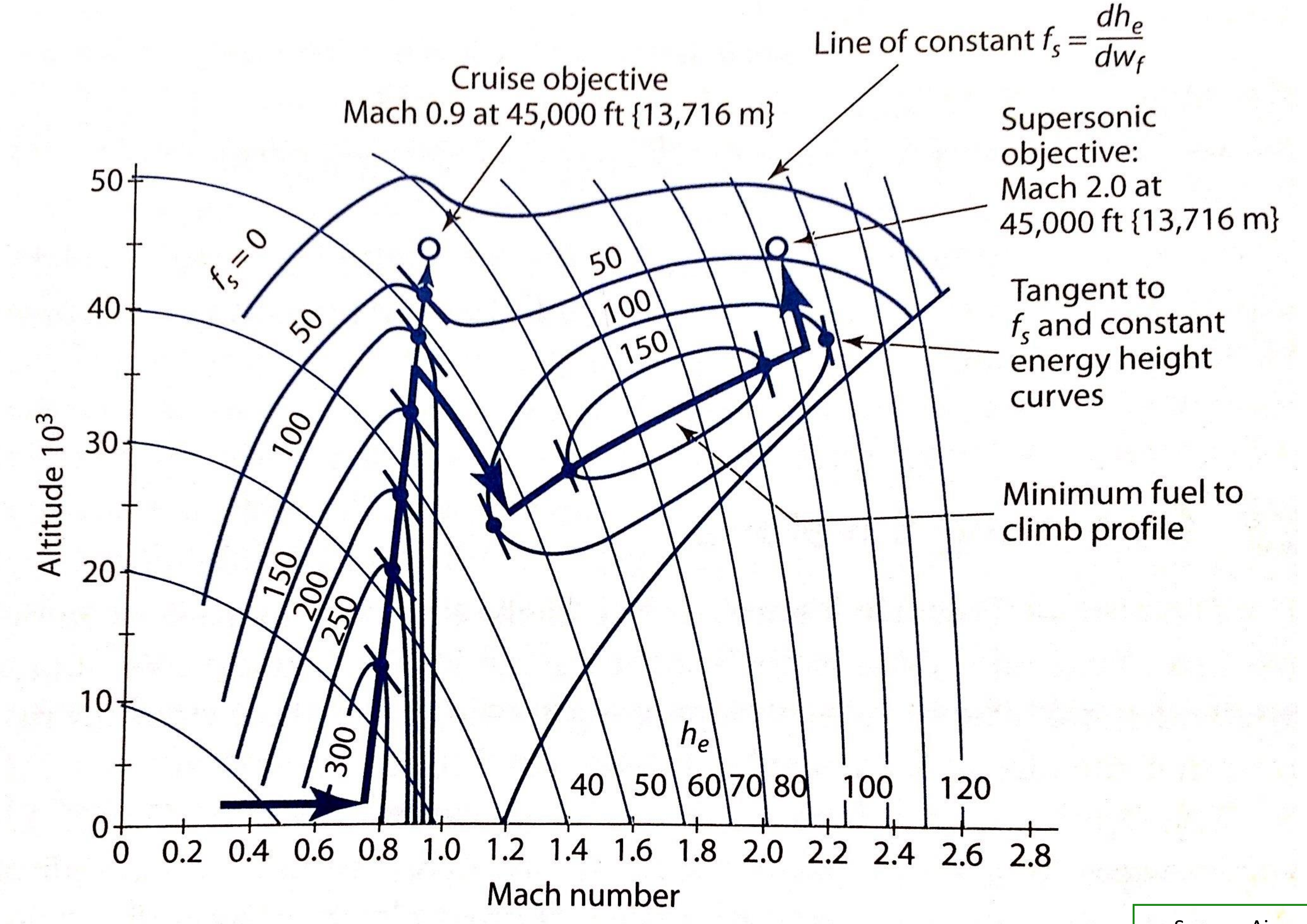
Flight Mechanics / Performance

Minimum Fuel to Climb

$$f_s = \frac{\mathrm{d}h_e}{\mathrm{d}W_f} = \frac{\mathrm{d}h_e/\mathrm{d}t}{\mathrm{d}W_f/\mathrm{d}t} = \frac{P_s}{CT}$$



Source: Aircraft Design by Daniel P. Raymer

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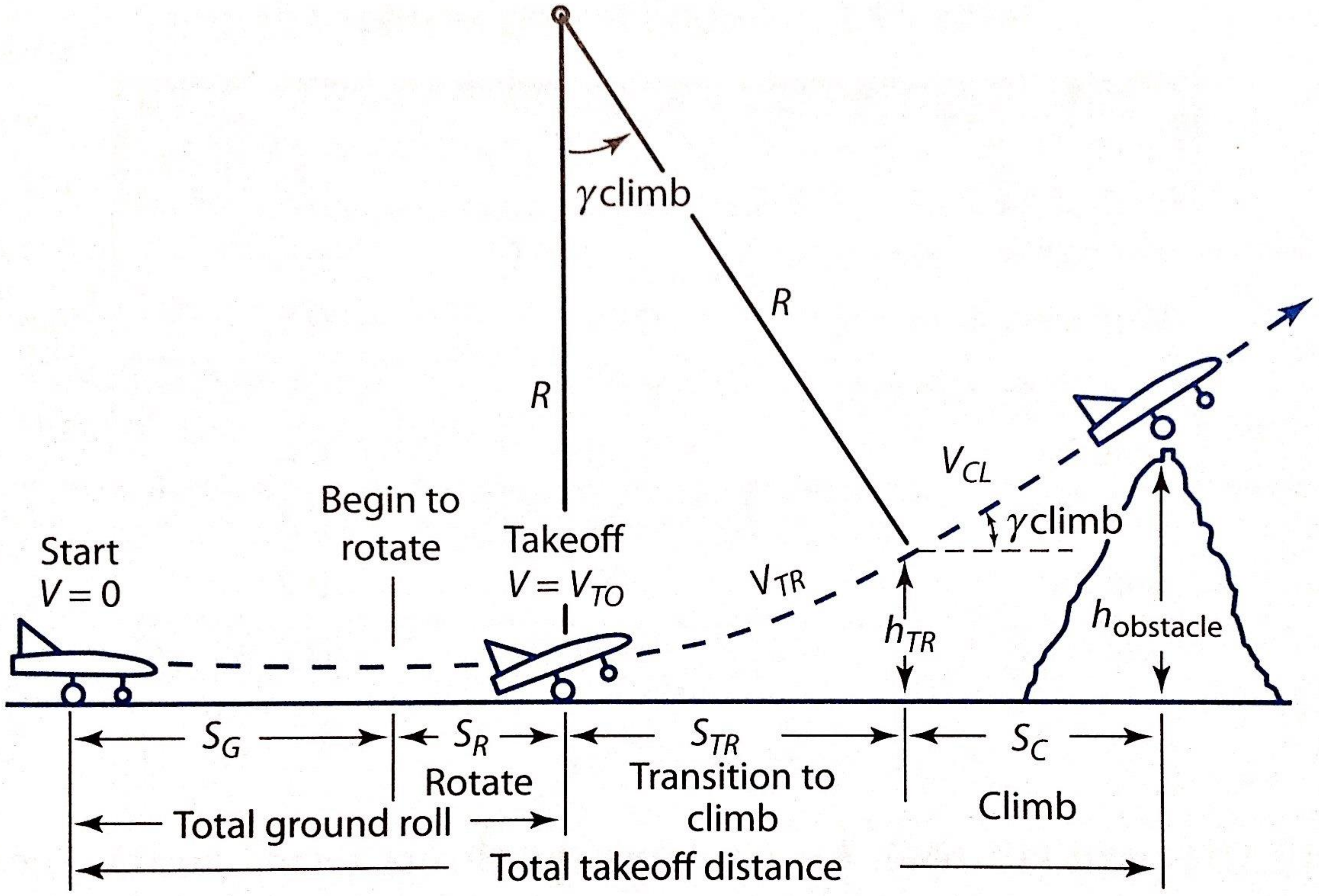
Flight Mechanics / Performance

Takeoff Analysis



Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance
Takeoff Analysis

$$E = \frac{1}{2} mV^2 = \frac{1}{2}\frac{W}{g}V^2 = FxD = \frac{W}{g} aS_g$$

$$S_g = \frac{V^2}{2a} = \int_{V_i}^{V_f} \frac{V}{a}\mathrm{d}V = \frac{1}{2}\int_{V_i}^{V_f}\frac{1}{a}\mathrm{d}(V^2)$$

$$a = \frac{g}{W}[T - D - \mu(W - L)]$$

$$= g\left[\left(\frac{T}{W} - \mu\right) + \frac{\rho}{2W/S}(-C_{D_0} - KC_L^2 + \mu C_L)V^2\right]$$

$$S_G = \frac{1}{2g}\int_{V_i}^{V_f}\frac{\mathrm{d}(V^2)}{K_T + K_A V^2} = \left(\frac{1}{2gK_A}\right)\ell n\left(\frac{K_T + K_A V_f^2}{K_T + K_A V_i^2}\right)$$

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Flight Mechanics / Performance
Takeoff Analysis

Surface	μ-typical values	
	Rolling (brakes off)	Brakes on
Dry concrete/asphalt	0.03–0.05	0.3–0.5
Wet concrete/asphalt	0.05	0.15–0.3
Icy concrete/asphalt	0.02	0.06–0.10
Hard turf	0.05	0.4
Firm dirt	0.04	0.3
Soft turf	0.07	0.2
Wet grass	0.08	0.2

- C_L for wing AOA ON GROUND + flap ΔC_L , Thrust @ $0.7V_{TO}$
- C_D for gear down and flaps; K corrected for ground effect
- $V_{TO} \geq 1.1V_{stall}$ (careful with tail-bump angle; may limit C_L !
- $\underbrace{V_{TO} \cdot 1\, sec.}_{\text{Small A/C}} < \; S_{ROT} < \underbrace{V_{TO} \cdot 3\, sec.}_{\text{Large A/C}}$

Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance

Takeoff Analysis

- Transition:
  - A/C follows approximately a circular arc and accelerates at $1.15V_{stall}$ and $C_L = 0.9\ C_{L_{MAX}}$

$$n = \frac{L}{W} = \frac{\frac{1}{2}\rho S(0.9\,C_{L_{\max}})(1.15\,V_{\text{stall}})^2}{\frac{1}{2}\rho S C_{L_{\max}} V_{\text{stall}}^2} = 1.2$$

$$n = 1.0 + \frac{V_{\text{TR}}^2}{Rg} = 1.2$$

$$R = \frac{V_{\text{TR}}^2}{g(n-1)} = \frac{V_{\text{TR}}^2}{0.2g}$$

$$\sin\gamma_{\text{climb}} = \frac{T-D}{W} \cong \frac{T}{W} - \frac{1}{L/D}$$

$$S_{\text{TR}} = R\sin\gamma_{\text{climb}} = R\left(\frac{T-D}{W}\right) \cong R\left(\frac{T}{W} - \frac{1}{L/D}\right)$$

$$h_{\text{TR}} = R(1-\cos\gamma_{\text{climb}})$$

If obstacle cleared before end of TR, use this for STR:

$$S_{\text{TR}} = \sqrt{R^2 - (R - h_{\text{obstacle}})^2}$$

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Flight Mechanics / Performance

Takeoff Analysis

- Climb to the obstacle height:
  - If obstacle height was reached during transition, $S_c = 0$.
  - Horizontal distance to clear obstacle (50 ft. for military % small A/C)

$$S_c = \frac{h_{\text{obstacle}} - h_{\text{TR}}}{\tan \gamma_{\text{climb}}}$$

- Balanced Field Length:
  - Total takeoff distance, including obstacle clearance when an engine fails @ decision speed V_1.
  - Decision speed is the speed at which upon an engine failure, the A/C can either brake to a halt or continue the takeoff in the **same total distance.**
  - If an engine fails at before V_1, the pilot can apply brakes and abort the takeoff.
  - At $V > V_1$, the pilot must continue the takeoff.

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Flight Mechanics / Performance
Takeoff Analysis

- Balanced Field Length (simple method):

$$\mathrm{BFL} = \frac{0.863}{1 + 2.3G}\left(\frac{W/S}{\rho g C_{L_{\mathrm{climb}}}} + h_{\mathrm{obstacle}}\right)\left(\frac{1}{T_{\mathrm{av}}/W - U} + 2.7\right) + \left(\frac{655}{\sqrt{\rho/\rho_{\mathrm{SL}}}}\right)$$

Jet:

$$T_{\mathrm{av}} = 0.75\ T_{\substack{\mathrm{takeoff}\\ \mathrm{static}}}\left[\frac{5 + \mathrm{BPR}}{4 + \mathrm{BPR}}\right]$$

Prop:

$$T_{\mathrm{av}} = 5.75\,\mathrm{bhp}\left[\frac{(\rho/\rho_{\mathrm{SL}})N_e D_p^2}{\mathrm{bhp}}\right]^{\frac{1}{3}}$$

where

BFL	= balanced field length (ft)
G	$= \gamma_{\mathrm{climb}} - \gamma_{\mathrm{min}}$
γ_{climb}	= arcsine $[(T-D)/W]$, 1-engine-out, climb speed
γ_{min}	= 0.024 2-engine; 0.027 3-engine; 0.030 4-engine
$C_{L_{\mathrm{climb}}}$	= C_L at climb speed (1.2 V_{stall})
h_{obstacle}	= 35 ft commercial, 50 ft military
U	= 0.01 $C_{L_{\mathrm{max}}}$ + 0.02 for flaps in takeoff position
BPR	= bypass ratio
bhp	= engine brake horsepower
N_e	= number of engines
D_p	= propeller diameter (ft)

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Flight Mechanics / Performance

Takeoff Analysis

- Balanced Field Length (more accurate method):

  - Takeoff should be simulated with engine failure at an assumed V_1 and continued until the obstacle heigh is reached. Note the total distance.

  - The takeoff must be simulated but aborted at the same V_1 (with 1 sec. delay of reaction time). Note the distance to a full stop.

  - V_1 should be iterated until:

total takeoff distance to the **obstacle**	=	total aborted takeoff distance to a **halt**.

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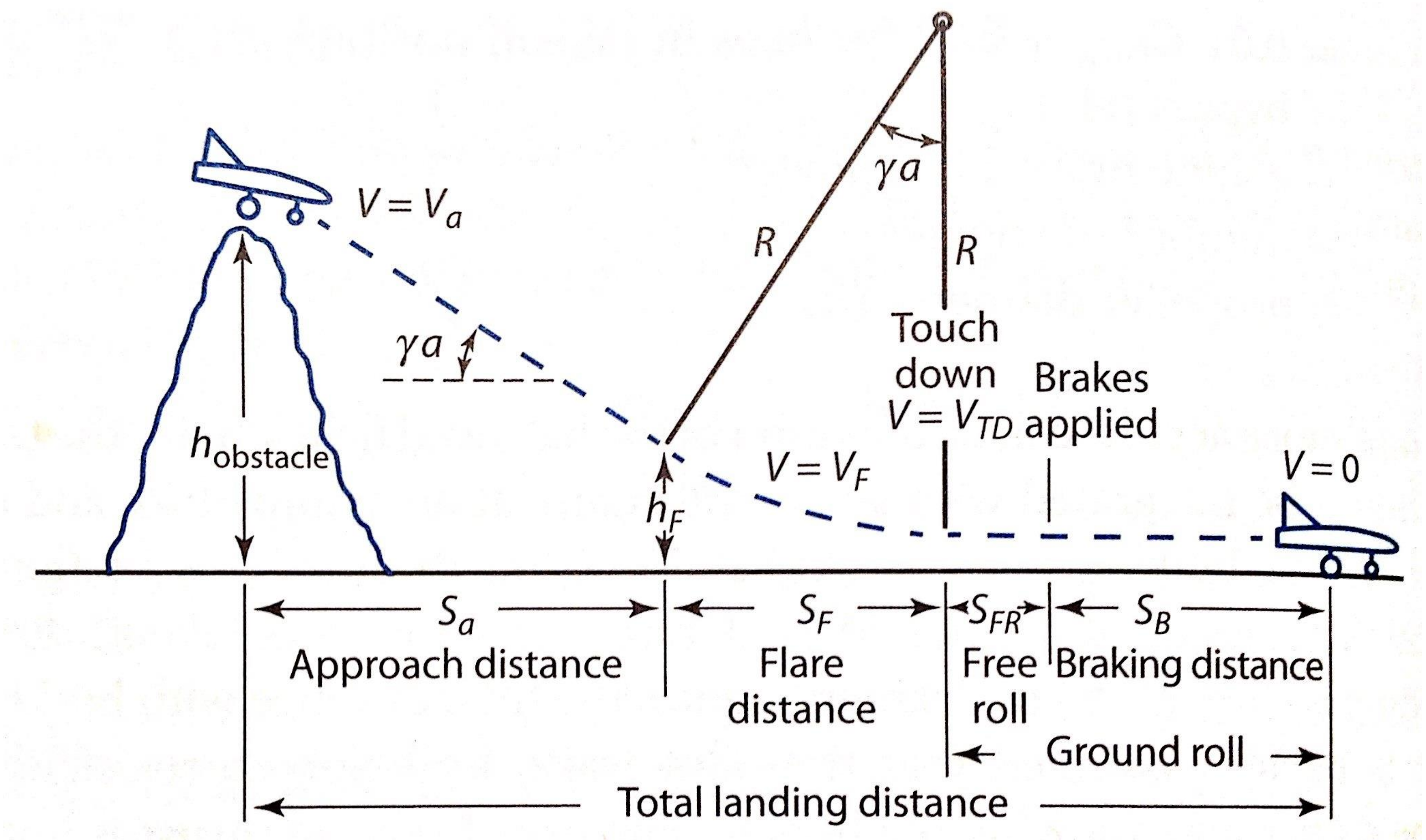
Flight Mechanics / Performance
Landing Analysis

- Landing weight is usually specified anywhere from W_0 to $.85W_0$.



Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance

Landing Analysis

- Approach:
  - Begins at the obstacle.
  - $V_{approach}$ = $1.3\ V_{stall}$ (1.2 for military)

$$\sin\gamma_{App} \doteq \frac{T-D}{W} \cong \frac{T}{W} - \frac{1}{L/D}$$

(Idle thrust/full flaps)

- For transports, approach angle <= 3 deg. (Thrust > Idle)
- Approach distance:

$$S_a = \frac{h_{\text{obstacle}} - h_f}{\tan\gamma_{App}}$$

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Flight Mechanics / Performance

Landing Analysis

- Flare:
  - The reverse of takeoff transition. A/C comes in at stable angle and brings nose up to touchdown and V_v approximately 0.
  - V_{TD} = $1.15\, V_{stall}$ 1.1)military)
  - V_f) average between $V_{approach}$ and V_{TD} = ($1.23\, V_{stall}$ 1.15)military)
  - Flare radius:

$$R_f = \frac{V_f^2}{g(n-1)} = \frac{V_f^2}{0.2g}$$

    - Flare height and horizontal distance covered during flare:

$$S_f = R_f \sin\gamma_{climb} = R_f\left(\frac{T-D}{W}\right) \cong R_f\left(\frac{T}{W} - \frac{1}{L/D}\right)$$

$$h_f = R_f(1 - \cos\gamma_{App})$$

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Flight Mechanics / Performance
Landing Analysis

- Ground roll:
  - Free roll : $V_{TD} \cdot 1\ sec. < \ S_{FR} < V_{TD} \cdot 3\ sec.$
  - Braking distance computed using same equation for takeoff ground roll (S_G), setting initial V to V_{TD}, and final V to 0.
  - Idle thrust,
  - Braking coefficient

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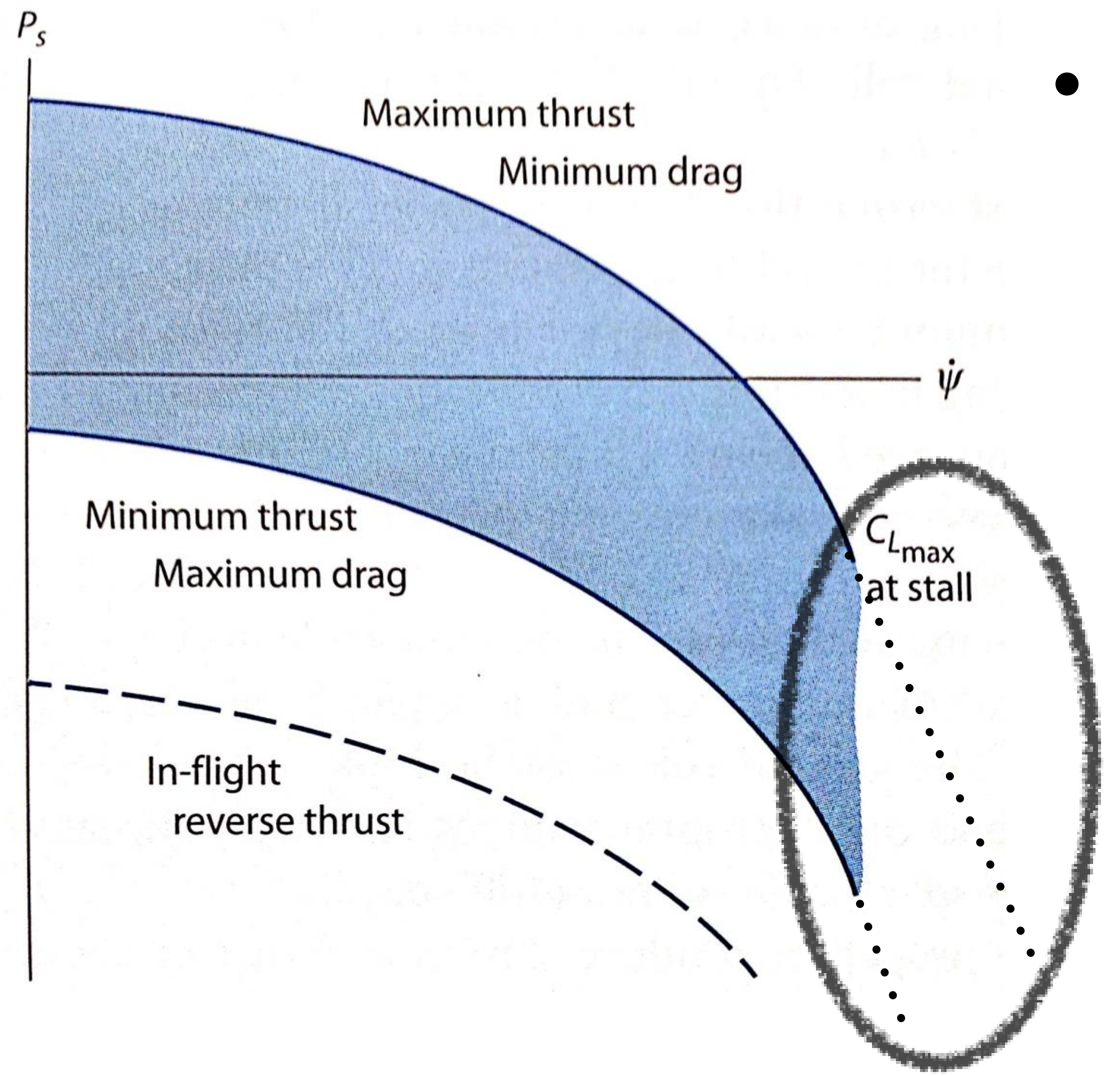
Flight Mechanics / Performance

Other Performance Measures of Merit

- Agility vs. Steady State
- Decoupled Energy Management (Potential and Kinetic Energies changed independently)
- Unpredictability



Post-stall area, potential for quick point & energy bleed (supermaneuver)

Source: Aircraft Design by Daniel P. Raymer

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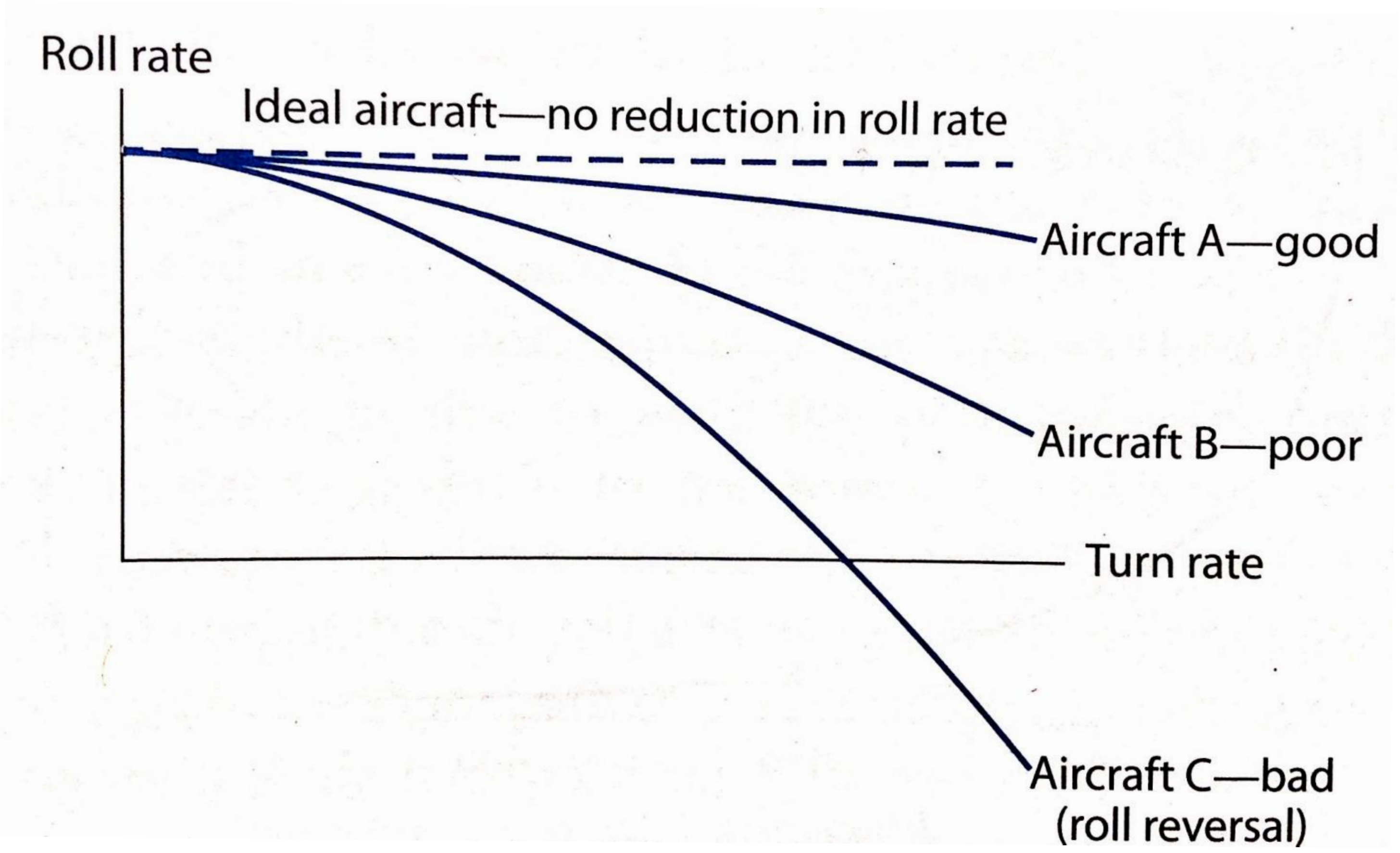
Flight Mechanics / Performance

Other Performance Measures of Merit

In the realm of stability and control, roll rate performance, usually not a problem at level flight, can be hurt at high load factors/turn rates rendering some combat aircraft at a disadvantage.



Source: Aircraft Design by Daniel P. Raymer

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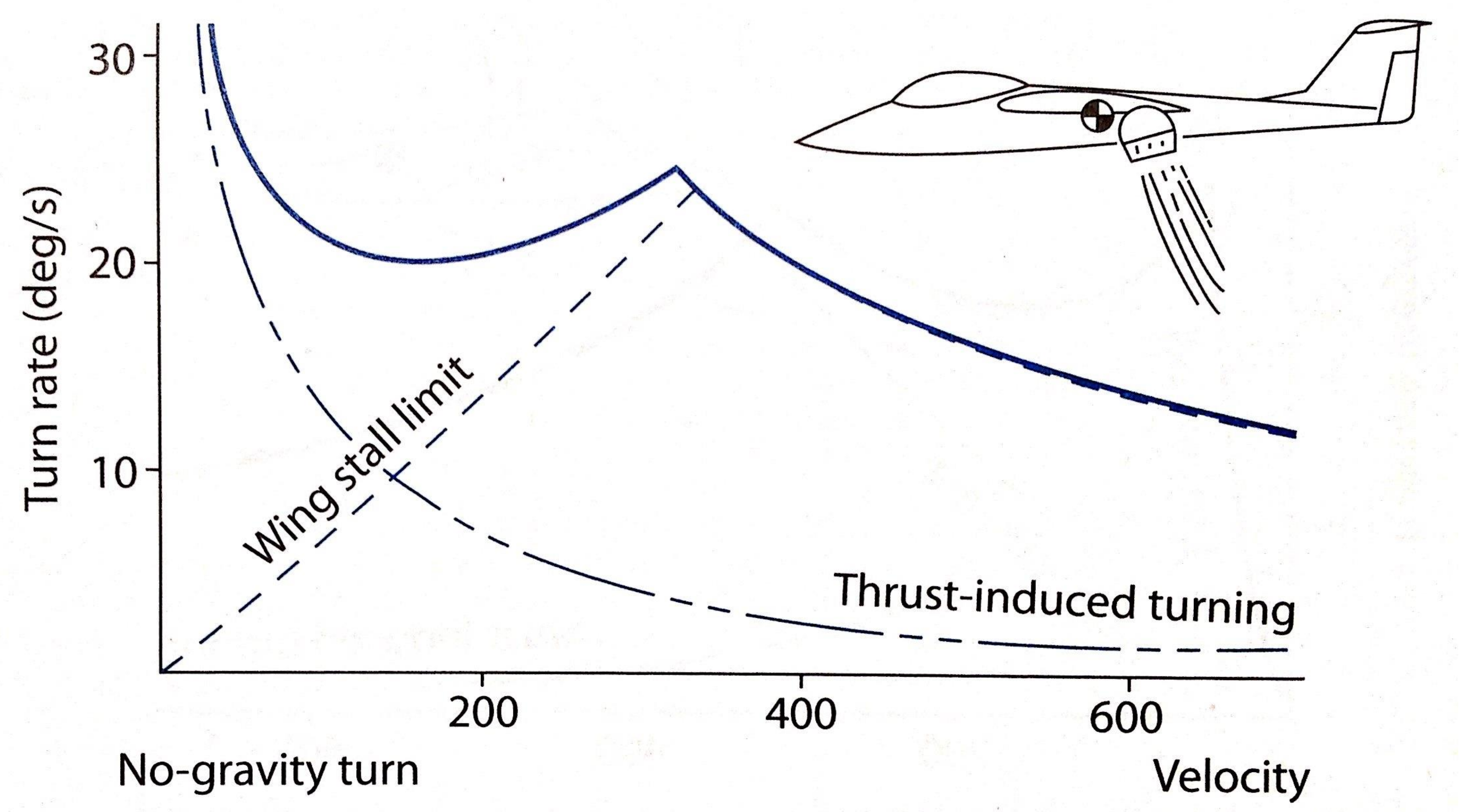
Flight Mechanics / Performance

Super-maneuver and Post-stall Maneuver

Thrust vectoring at c.g. (no net moment, a la Harrier)



Source: Aircraft Design by Daniel P. Raymer

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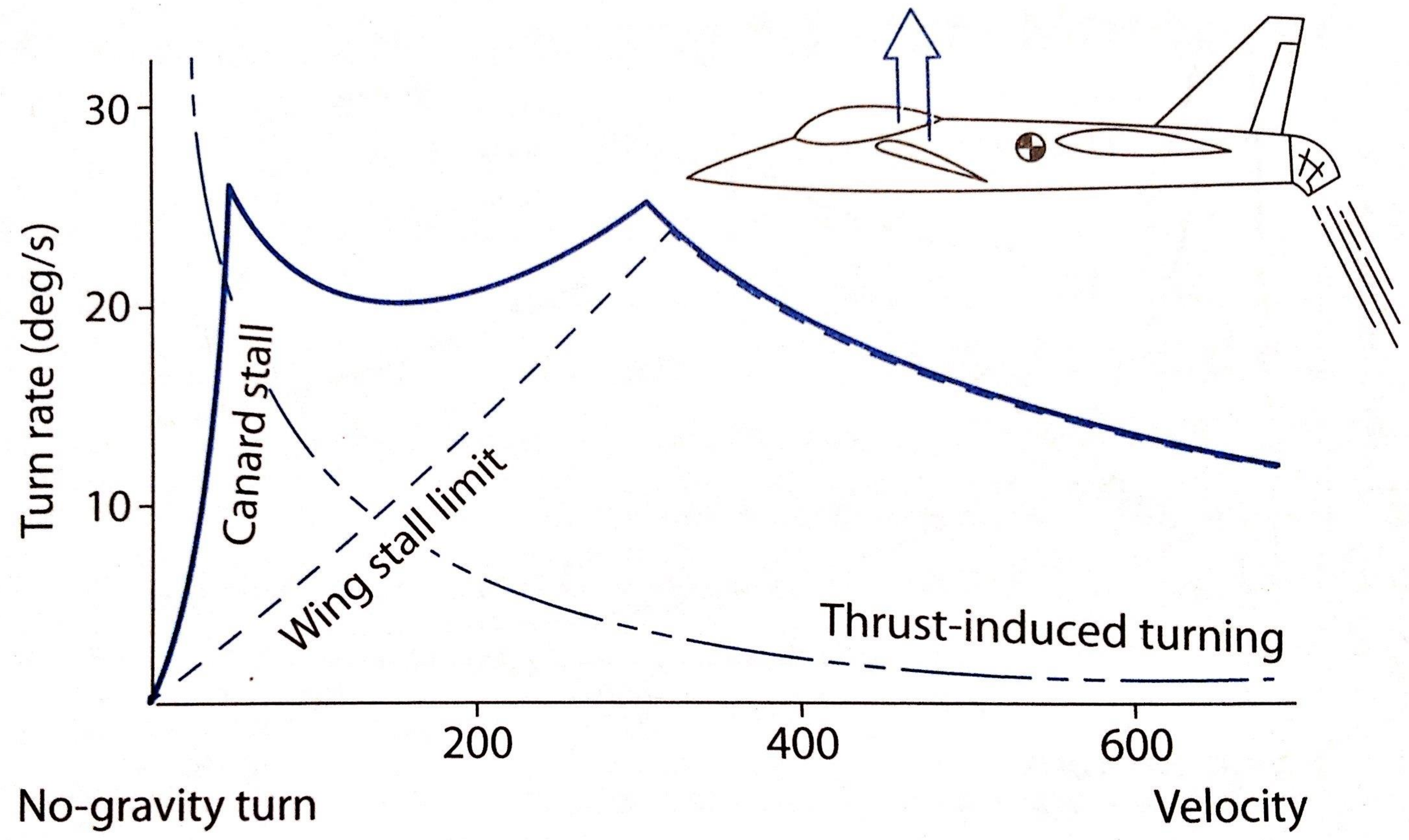
Flight Mechanics / Performance

Super-maneuver and Post-stall Maneuver

Thrust vectoring aft nozzle plus canard



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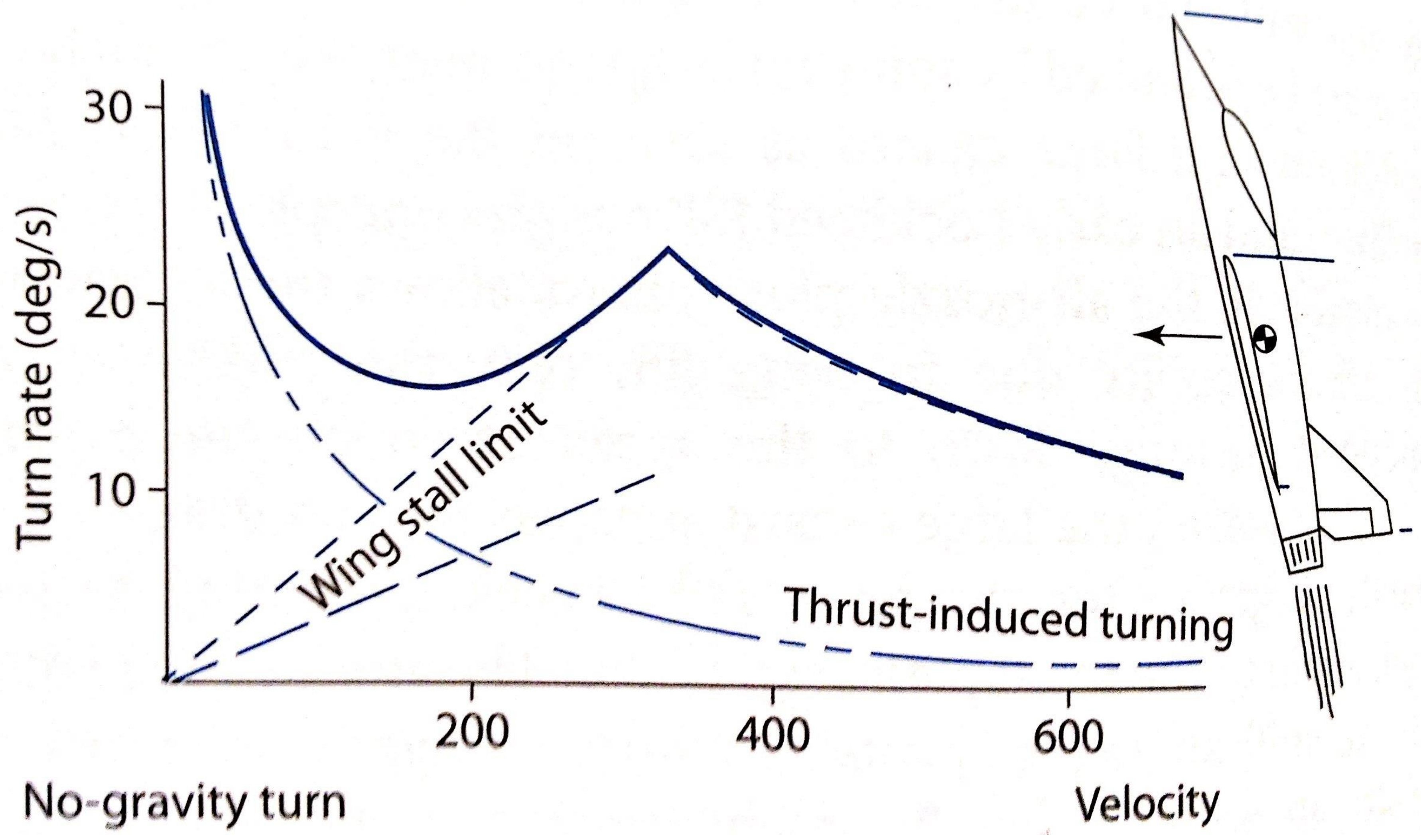
Flight Mechanics / Performance

Super-maneuver and Post-stall Maneuver

Fuselage Pointing



Source: Aircraft Design by Daniel P. Raymer

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Flight Mechanics / Performance

Flight Mechanics 3- DOF Simulations

$$\Sigma F_V = F_G cos\alpha - D - DD - Wsin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\Sigma F_{Horiz} = F_G sin\alpha sin\phi + Lsin\phi = \frac{W(Vcos\gamma)^2}{gR_H}$$

$$\Sigma F_{Vert} = Lcos\phi + F_G sin\alpha cos\phi - Wcos\gamma = \frac{WV^2}{gR_V}$$

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Flight Mechanics / Performance
Flight Mechanics 3- DOF Simulations

- Usually time-to-complete is the measure of merit
- 1-g Accelerations
- Sustained/Instantaneous level turns
- More complex maneuvers, usually changing altitude and velocity
    - Example: A-10 re-attack tank-busting maneuver
    - More control variables, difficult to optimize
    - Require numerical integration of 3-DOF equations of motion