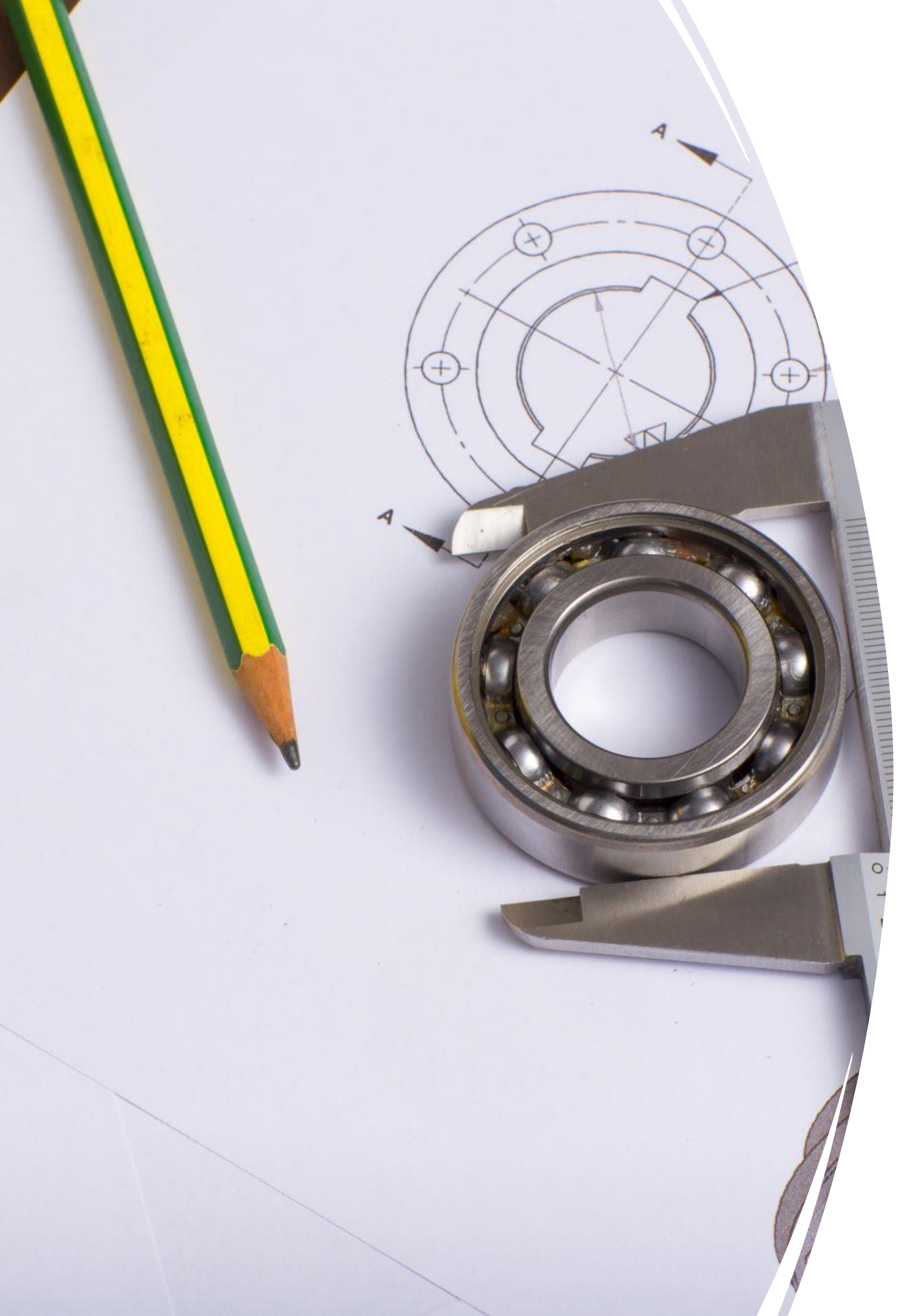
Cam-Follower Systems – Part #2

Cam-Follower Systems Using SVAJ Functions Polynomials (cont.). Geometrically construct Cam-Follower displacement diagrams. Construct Cam-Follower displacement diagrams in MATLAB®.



Learning Outcomes – Part #2

1. Describe the Cam-Follower Systems Using SVAJ Functions Polynomials (cont.).
2. Geometrically construct Cam-Follower displacement diagrams. Graphical Construction of a Displacement Diagram – An Example.
3. Construct Cam-Follower displacement diagrams in MATLAB®.

Cam-Follower Systems – SVAJ Functions Using Polynomials

- Normalized Cam Angle: $x = \frac{\theta}{\beta}$ where β is the interval period.
- Normalized follower displacement: $y = \frac{s}{h}$
- The Displacement (S), Velocity (V) and Acceleration (A) are:

$$s = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + \cdots$$

$$v = C_1 + 2 C_2 x + 3 C_3 x^2 + 4 C_4 x^3 + 5 C_5 x^4 + \cdots$$

$$a = 2 C_2 + 6 C_3 x + 12 C_4 x^2 + 20 C_5 x^3 + \cdots$$

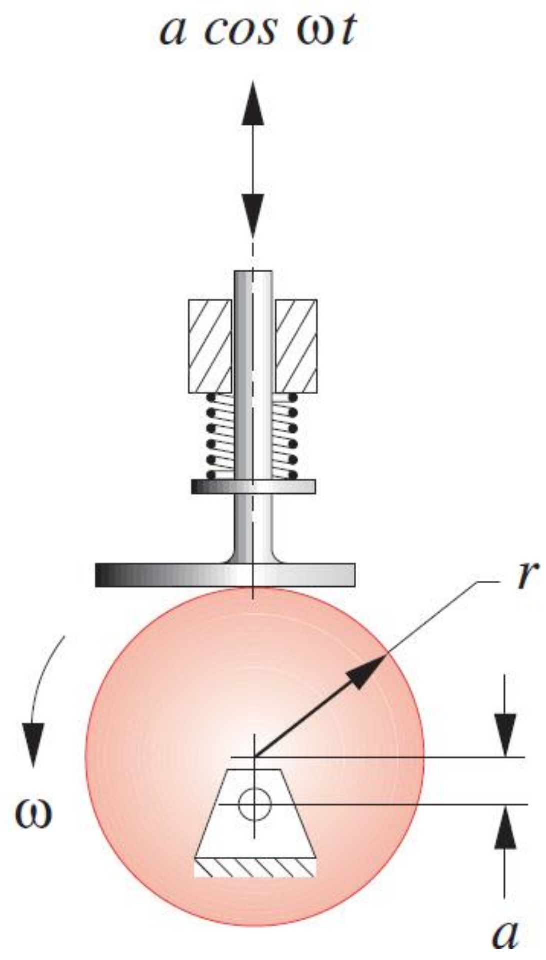
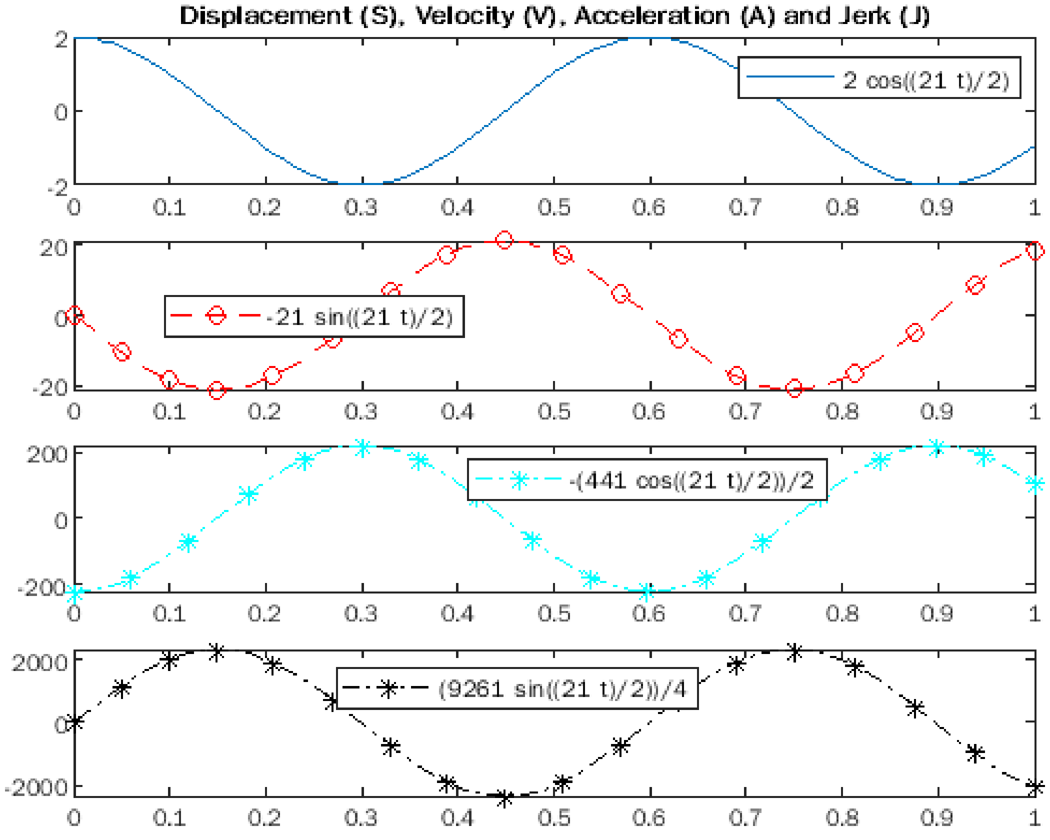


Figure 8-1(a). An oscillating Cam-Follower Effective Pin-Jointed 4-Bar Equivalent. Design of Machinery's Textbook by Robert L. Norton.

Cam-Follower Systems – SVAJ Functions Using Polynomials

- SVAJ Functions in MATLAB Online:



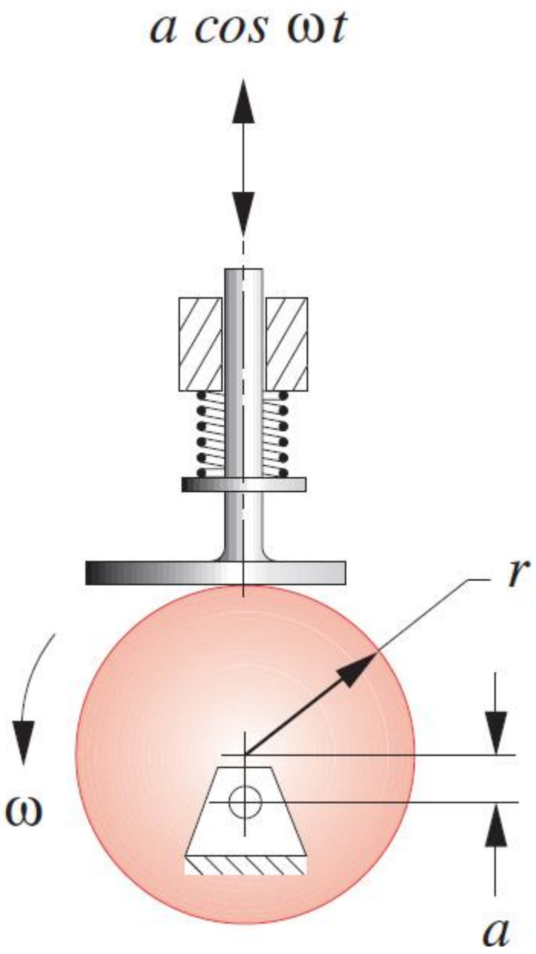


Figure 8-1(a). An oscillating Cam-Follower Effective Pin-Jointed Fourbar Equivalent.
Textbook: Design of Machinery by Robert L. Norton.

Cam-Follower Systems – Graphical Construction of a Displacement Diagram

- Graphical construction of a displacement diagram is an alternative method to generating a displacement diagram to scale. The construction using the constant acceleration motion scheme can be accomplished by referring to Figure (d) and using the following procedure:

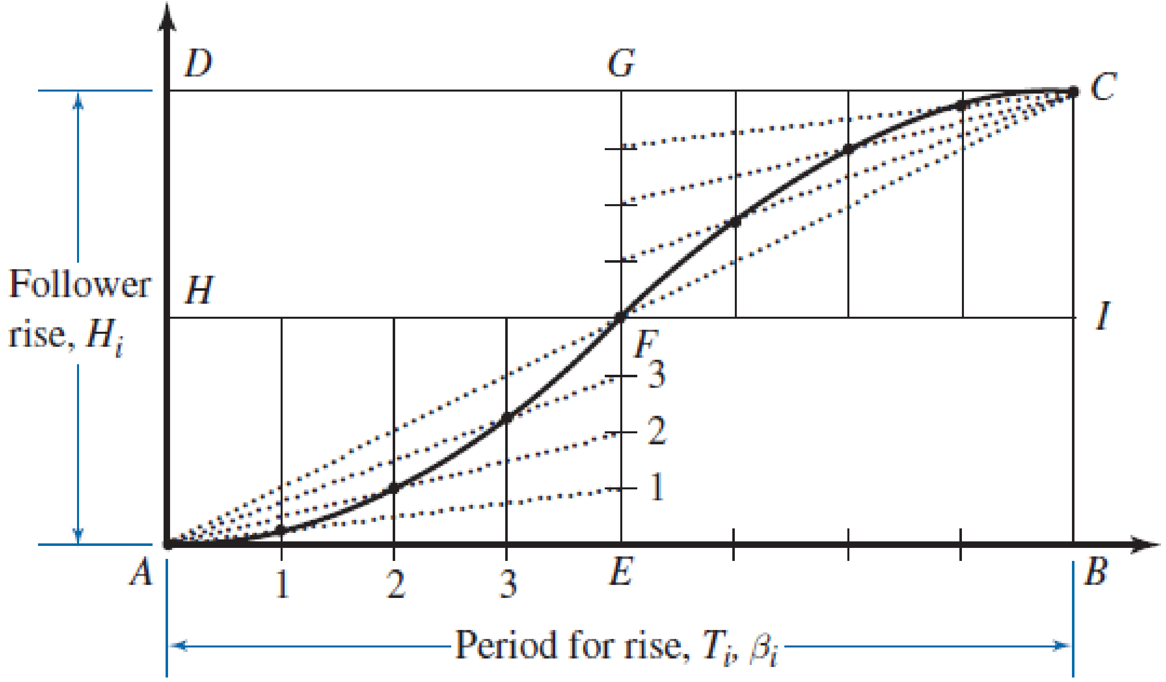


Figure (d)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems – Graphical Construction of a Displacement Diagram

1. Divide the follower rise (or fall) sequence into two halves. From Figure (d), AE represents the time (T_i) and vertical line (EF), the magnitude of rise for the first half of this motion scheme.
2. Divide both the horizontal and vertical axes of the quadrant $AEFH$ into equal parts.
3. Construct vertical lines from the horizontal divisions.
4. Construct straight lines from corner A to the vertical divisions.
5. Draw a smooth curve through the points of intersection of the vertical lines and the lines drawn from corner A.
6. Repeat steps 2 through 5 for the remaining half of the curve as shown in quadrant $FICG$ in Figure (d). A constant acceleration fall is constructed as a mirror image to Figure (b).

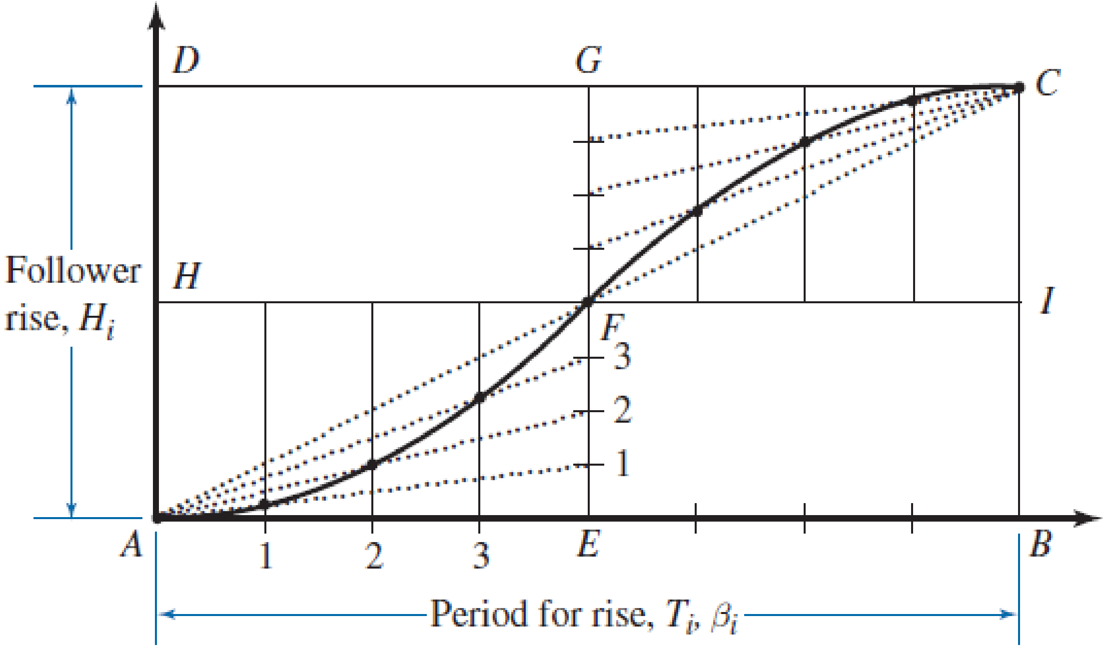


Figure (d)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Graphical Construction of a Displacement Diagram – An Example

- Example 7-2. A cam is to be designed for an automated part loader as shown in Figure (e). Using the motion equations, construct a chart that tabulates follower displacement versus time and cam rotation. Also plot this data when the prescribed motion for this application is as follows:

1. Rise 50 mm in 1.5 s using the constant velocity scheme.
2. Return in 2.0 s using the cycloidal motion scheme.
3. Dwell 0.75 s.
4. Repeat the sequence.

Cam-Follower Systems – Graphical Construction of a Displacement Diagram

- The Cam-Follower kinematics is discussed for the displacement diagram where the follower movements are designed for a series of motion versus time and cam rotation; that is: rise, fall, dwell.
- The prescribed motion are:
  - Cam-Follower Kinematics for Constant Acceleration Motion (*)
  - Cam-Follower Kinematics for Harmonic Motion and,
  - Cam-Follower Kinematics for Cycloidal Motion

(*) Infinity jerk

TABLE 9.2 Cam Follower Kinematics for Constant Acceleration Motion

	Rise	Fall
For $0 < t < 0.5\,T$ $(0 < \phi < 0.5\,\beta)$:		
Displacement:	$\Delta R_i = H_0 + 2H_i\left(\frac{t_i}{T_i}\right)^2 = H_0 + 2H_i\left(\frac{\phi_i}{\beta_i}\right)^2$	$\Delta R_j = H_F + H_j - 2H_j\left(\frac{t_j}{T_j}\right)^2 = H_F + H_j - 2H_j\left(\frac{\phi_j}{\beta_j}\right)^2$
Velocity:	$v_i = \frac{4H_i t_i}{T_i^2} = \frac{4H_i\omega\phi_i}{\beta_i^2}$	$v_j = \frac{-4H_j t_j}{T_j^2} = \frac{-4H_j\omega\phi_j}{\beta_j^2}$
Acceleration:	$a_i = \frac{4H_i}{T_i^2} = \frac{4H_i\omega^2}{\beta_i^2}$	$a_j = \frac{-4H_j}{T_j^2} = \frac{-4H_j\omega^2}{\beta_j^2}$
For $0.5\,T < t < T$ $(0.5\,\beta < \phi < \beta)$:		
Displacement:	$\Delta R_i = H_0 + H_i - 2H_i\left(1 - \frac{t_i}{T_i}\right)^2 = H_0 + H_i + 2H_i\left(1 - \frac{\phi_i}{\beta_i}\right)^2$	$\Delta R_j = H_F + 2H_j\left(1 - \frac{t_j}{T_j}\right)^2 = H_F + 2H_j\left(1 - \frac{\phi_j}{\beta_j}\right)^2$
Velocity:	$v_i = \frac{4H_i}{T_i}\left(1 - \frac{t_i}{T_i}\right) = \frac{4H_i\omega}{\beta_i}\left(1 - \frac{\phi_i}{\beta_i}\right)$	$v_j = \frac{-4H_j}{T_j}\left(1 - \frac{t_j}{T_j}\right) = \frac{-4H_j\omega}{\beta_j}\left(1 - \frac{\phi_j}{\beta_j}\right)$
Acceleration:	$a_i = \frac{-4H_i}{T_i^2} = \frac{-4H_i\omega^2}{\beta_i^2}$	$a_j = \frac{4H_j}{T_j^2} = \frac{4H_j\omega^2}{\beta_j^2}$

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

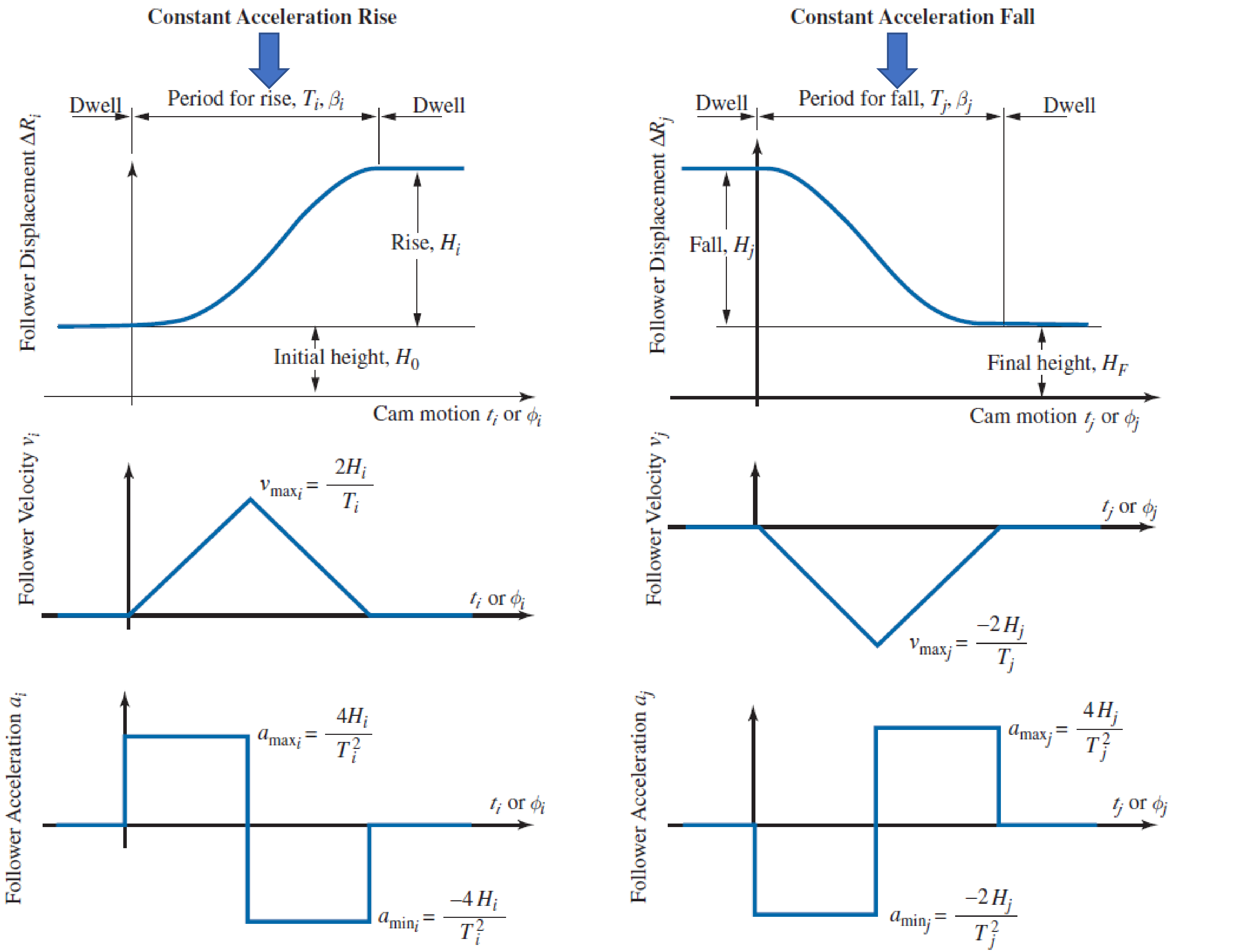


Figure (c) Constant Acceleration Motion Curves.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

TABLE 9.3 Cam Follower Kinematics for Harmonic Motion

	Rise	Fall
Displacement:	$\Delta R_i = H_0 + \frac{H_i}{2}\left[1 - \cos\left(\frac{\pi t_i}{T_i}\right)\right]$ $= H_0 + \frac{H_i}{2}\left[1 - \cos\left(\frac{\pi \phi_i}{\beta_i}\right)\right]$	$\Delta R_j = H_F + \frac{H_j}{2}\left[1 + \cos\left(\frac{\pi t_j}{T_j}\right)\right]$ $= H_F + \frac{H_j}{2}\left[1 - \cos\left(\frac{\pi \phi_j}{\beta_j}\right)\right]$
Velocity:	$v_i = \frac{\pi H_i}{2T_i}\left[\sin\left(\frac{\pi t_i}{T_i}\right)\right]$ $= \frac{\pi H_i \omega}{2\beta_i}\left[\sin\left(\frac{\pi \phi_i}{\beta_i}\right)\right]$	$v_j = \frac{-\pi H_j}{2T_j}\left[\sin\left(\frac{\pi t_j}{T_j}\right)\right]$ $= \frac{-\pi H_j \omega}{2\beta_j}\left[\sin\left(\frac{\pi \phi_j}{\beta_j}\right)\right]$
Acceleration:	$a_i = \frac{\pi^2 H_i}{2T_i^2}\left[\cos\left(\frac{\pi t_i}{T_i}\right)\right]$ $= \frac{\pi^2 H_i \omega^2}{2\beta_i^2}\left[\cos\left(\frac{\pi \phi_i}{\beta_i}\right)\right]$	$a_j = \frac{-\pi^2 H_i}{2T_i^2}\left[\cos\left(\frac{\pi t_j}{T_j}\right)\right]$ $= \frac{-\pi^2 H_j \omega^2}{2\beta_j^2}\left[\cos\left(\frac{\pi \phi_j}{\beta_j}\right)\right]$

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

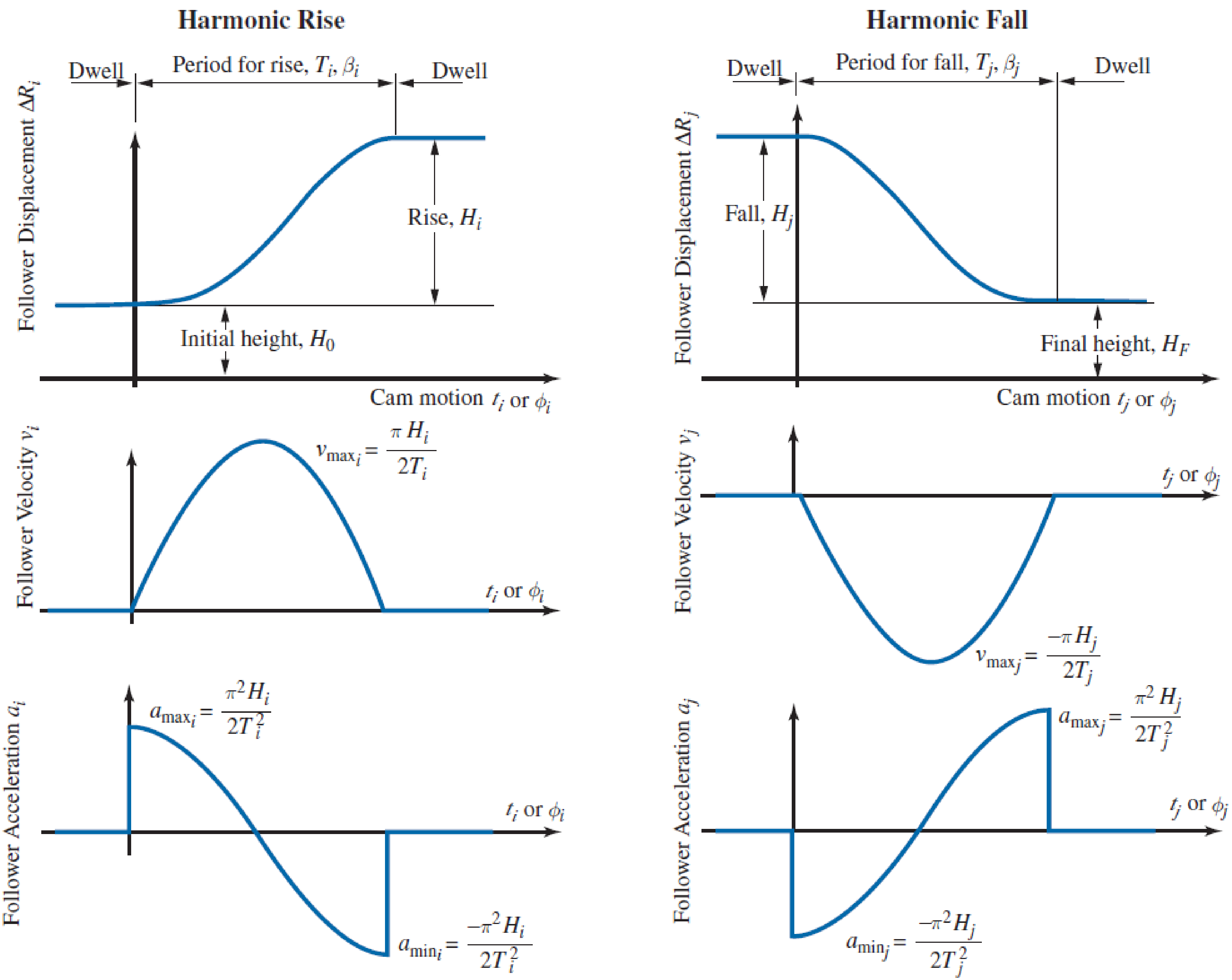


Figure (d) Harmonic Motion Curves.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

TABLE 9.4 Cam Follower Kinematics for Cycloidal Motion

	Rise	Fall
Displacement:	$\Delta R_i = H_0 + H_i\left[\frac{t_i}{T_i} - \frac{1}{2\pi}\sin\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= H_0 + H_i\left[\frac{\phi_i}{\beta_i} - \frac{1}{2\pi}\sin\left(\frac{2\pi \phi_i}{\beta_i}\right)\right]$	$\Delta R_j = H_F + H_j\left[1 - \frac{t_j}{T_j} + \frac{1}{2\pi}\sin\left(\frac{2\pi t_j}{T_j}\right)\right]$ $= H_F + H_j\left[\frac{\phi_j}{\beta_j} - \frac{1}{2\pi}\sin\left(\frac{2\pi \phi_j}{\beta_j}\right)\right]$
Velocity:	$v_i = \frac{H_i}{T_i}\left[1 - \cos\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= \frac{H_i\omega}{\beta_i}\left[1 - \cos\left(\frac{2\pi \phi_i}{\beta_i}\right)\right]$	$v_j = \frac{-H_j}{T_i}\left[1 - \cos\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= \frac{-H_j\omega}{\beta_j}\left[1 - \cos\left(\frac{2\pi \phi_j}{\beta_j}\right)\right]$
Acceleration:	$a_i = \frac{2\pi H_i}{T_i^2}\left[\sin\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= \frac{2\pi H_i\omega^2}{\beta_i^2}\left[\sin\left(\frac{2\pi \phi_i}{\beta_i}\right)\right]$	$a_j = \frac{-2\pi H_j}{T_j^2}\left[\sin\left(\frac{2\pi t_j}{T_j}\right)\right]$ $= \frac{-2\pi H_j\omega^2}{\beta_j^2}\left[\sin\left(\frac{2\pi \phi_j}{\beta_j}\right)\right]$

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

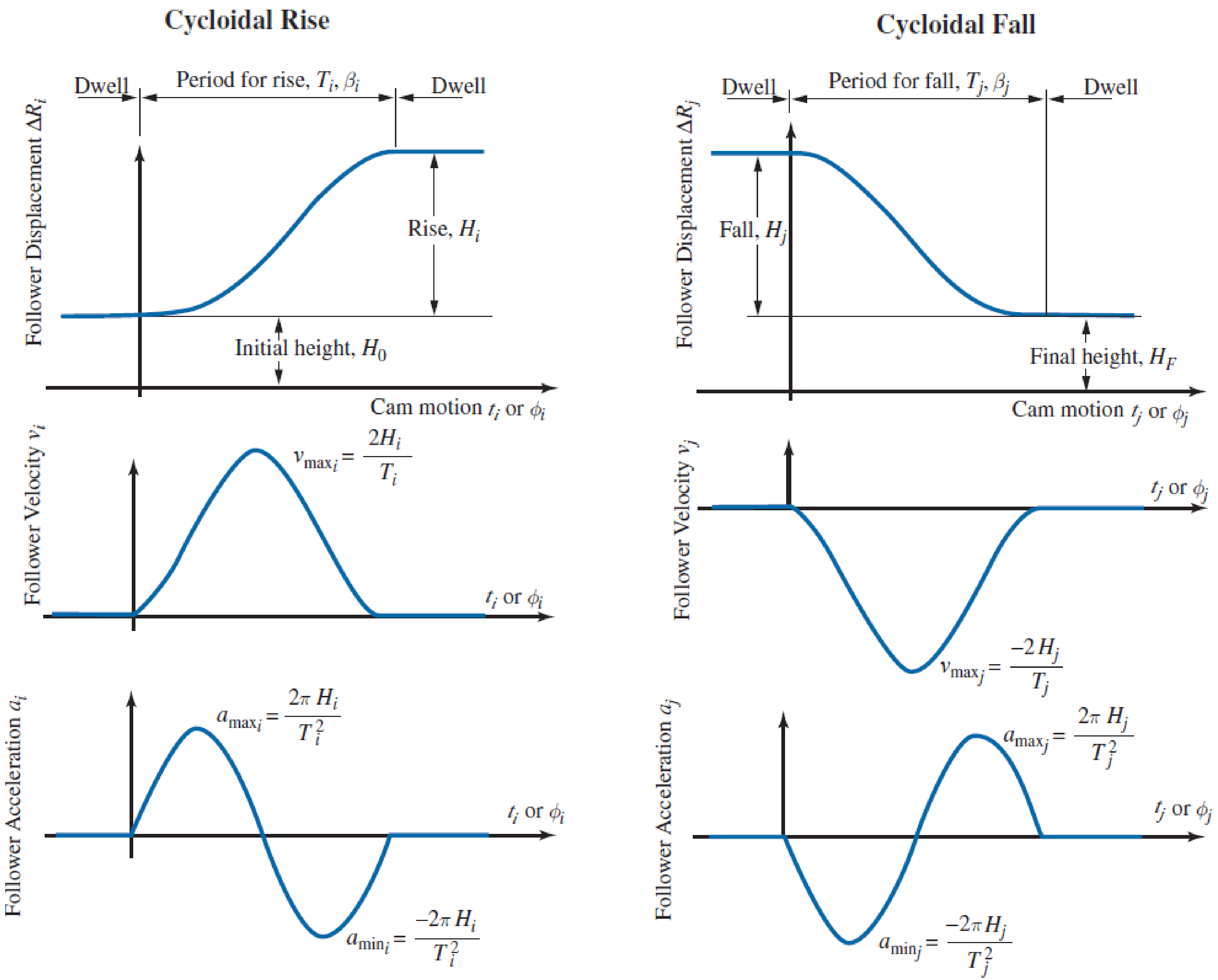


Figure (e) Cycloidal Motion Curves.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems – Graphical Construction of a Displacement Diagram

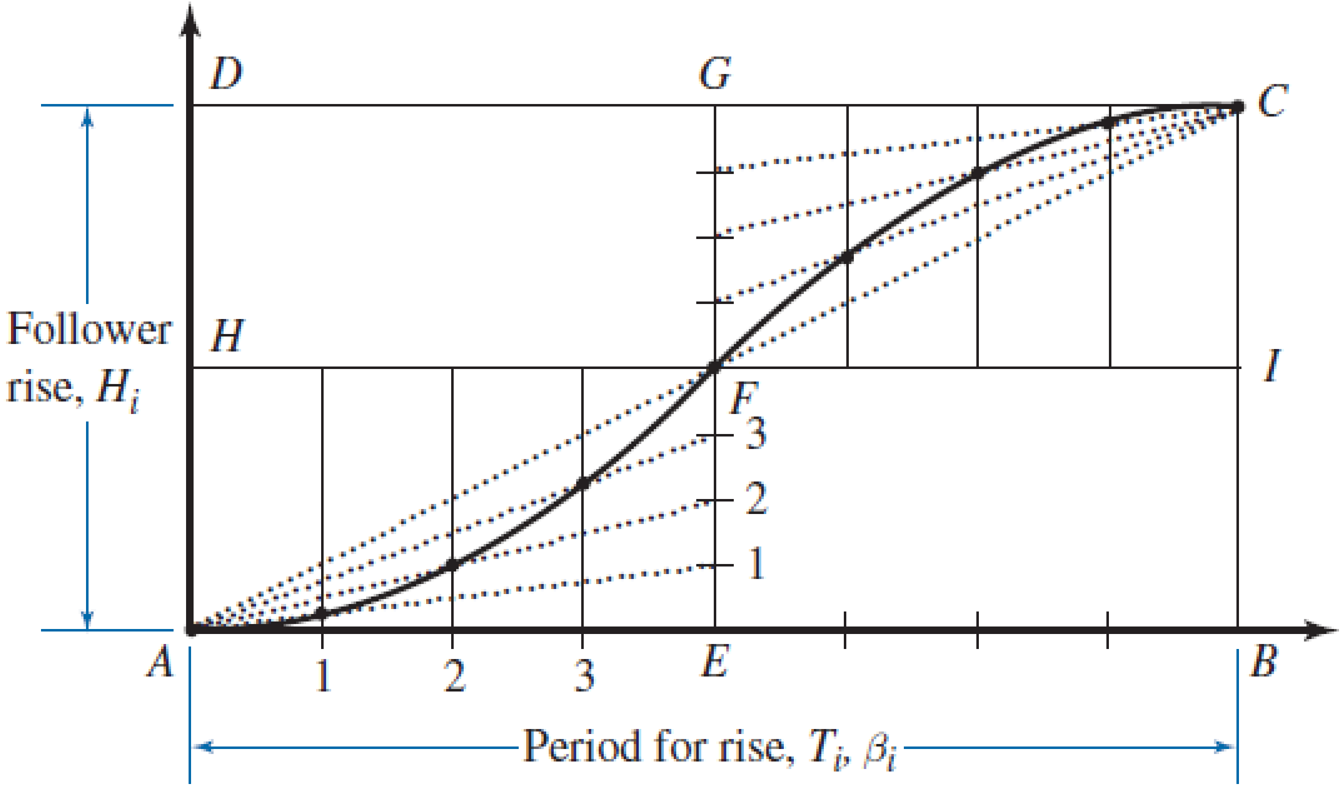


Figure (f) Construction of a Constant Acceleration Displacement Diagram.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems – Graphical Construction of a Displacement Diagram for Harmonic Motion

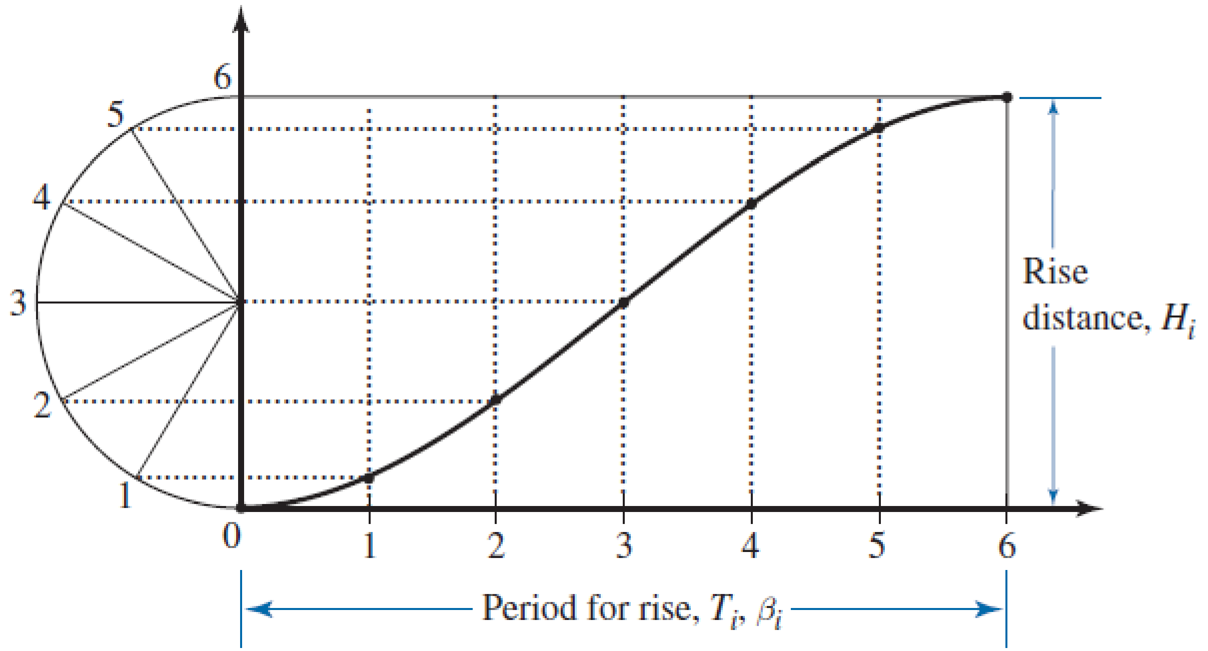


Figure (g) Construction of a Harmonic Displacement Diagram.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems – Graphical Construction of a Displacement Diagram for Cycloidal Motion

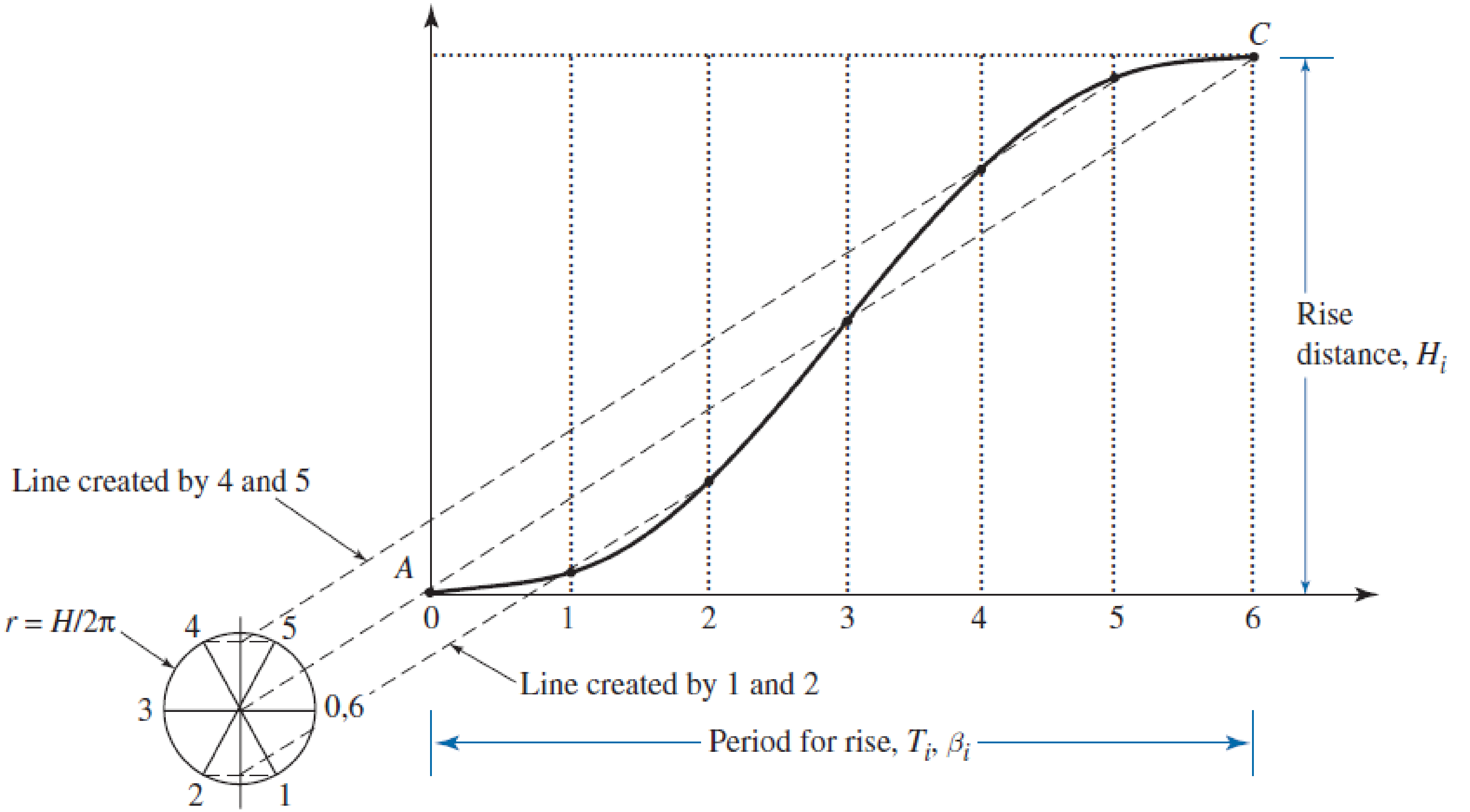


Figure (h) Construction of a Cycloidal Displacement Diagram.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems – Graphical Construction of a Displacement Diagram

1. Draw the displacement diagram according to the prescribed follower motion sequence. The X axis is drawn to scale and the exact dimensions are used for the y −axis.
2. Then, plot the cam by drawing diameter circle to represent the bore for the camshaft, and another circle to represent the base circle (i.e., the nearest the follower approaches to the center of rotation).
3. Draw radial lines θ apart from the cam center, and number them in the reverse direction to the cam rotation.
4. Plot the y-ordinates from the cam graph along each of the radial lines in turn, measuring from the base circle. Where rapid changes in direction occur, or where there is uncertainty regarding the position of the profile, more points can be plotted at 10° or 15° intervals.
5. Draw the best curve through the points to give the required cam profile (in Part #3).

Graphical Construction of a Displacement Diagram – An Example

- Example 7-2. A cam is to be designed for an automated part loader as shown in Figure (e). Using the motion equations, construct a chart that tabulates follower displacement versus time and cam rotation. Also plot this data when the prescribed motion for this application is as follows:

1. Rise 50 mm in 1.5 s using the constant velocity scheme.
2. Return in 2.0 s using the cycloidal motion scheme.
3. Dwell 0.75 s.
4. Repeat the sequence.

Graphical Construction of a Displacement Diagram – An Example

- Solution.

1. Calculate the "Time for a Full Cycle":

$$\sum T_i = T_1 + T_2 + T_3$$

$$= (1.5 + 2.0 + 0.75) = 4.25\ seconds$$

2. From Eq. (8-2), calculate the "Required Rotational Speed of the Cam":

$$\omega_{cam} = \frac{1\ rev}{\sum T_i} = \frac{1\ rev}{4.25\ s} = 0.235\frac{rev}{s}\left(\frac{60}{1\ min}\right)$$

$$= 14.12\ rpm$$

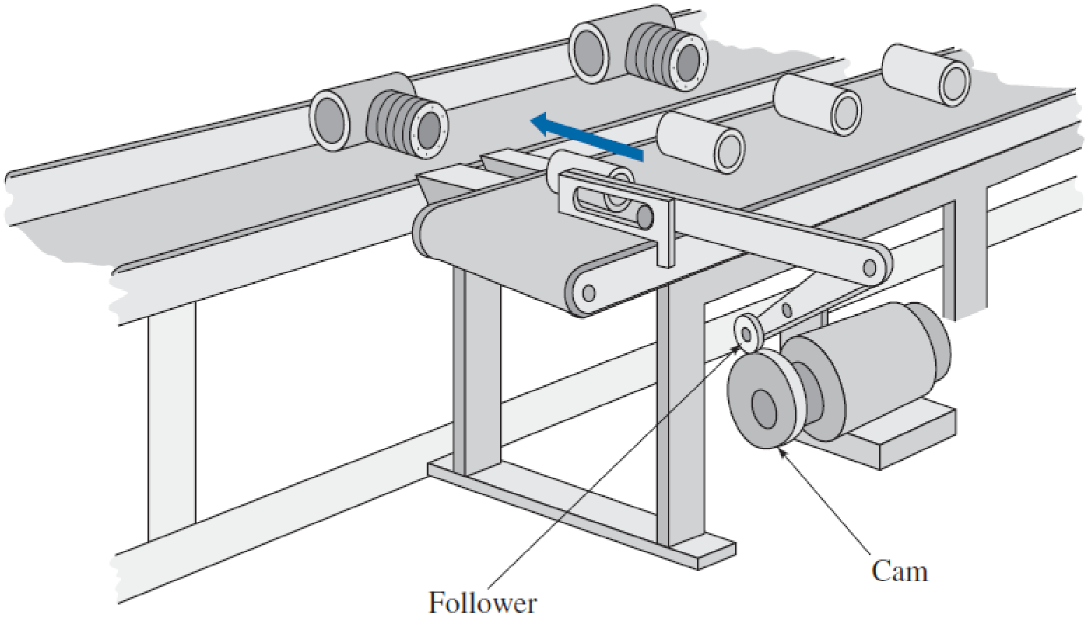


Figure (e)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems - Prescribed Follower Motion

- Solution (cont.).

3. Determine the Cam Rotation for Each Follower Motion Interval:

$$\beta_1 = \omega_{cam} \times T_1 = (0.235\ rev/s)(1.5\ s)$$
$$= 0.353\ rev \times \left(\frac{360°}{1\ rev}\right) = 127°$$

$$\beta_2 = \omega_{cam} \times T_2 = (0.235\ rev/s)(2.0\ s)$$
$$= 0.470\ rev \times \left(\frac{360°}{1\ rev}\right) = 169.3°$$

$$\beta_3 = \omega_{cam} \times T_3 = (0.235\ rev/s)(0.75\ s)$$
$$= 0.177\ rev \times \left(\frac{360°}{1\ rev}\right) = 63.7°$$

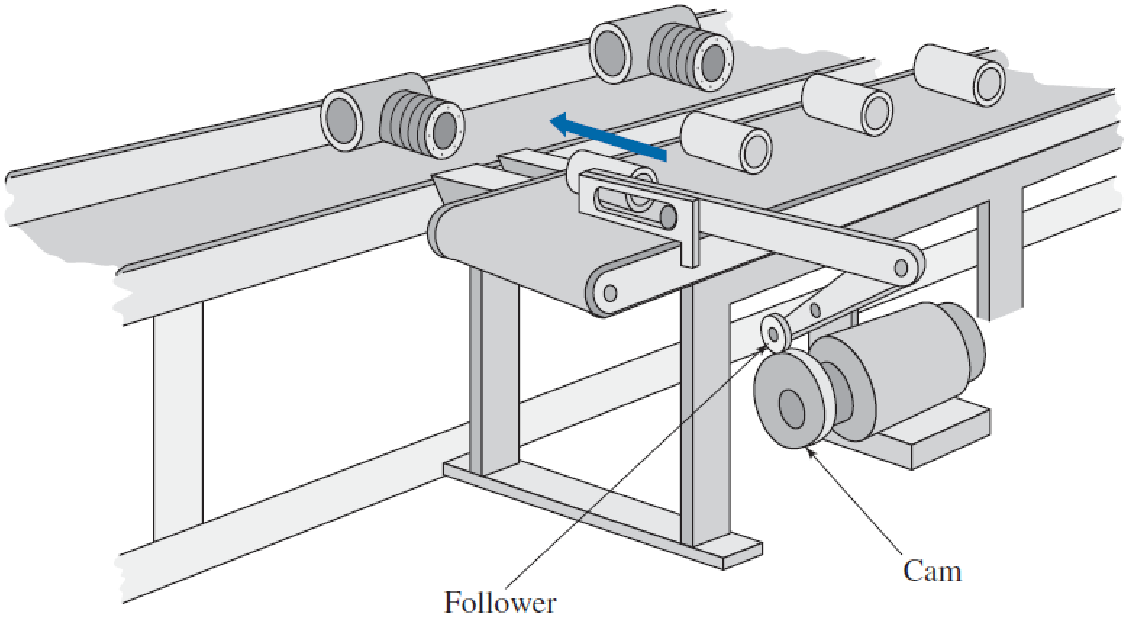


Figure (e)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems - Prescribed Follower Motion

- Solution (cont.).

3. Calculate the Displacement during Each Follower Motion Interval:
   - The first motion interval has $H_1 = 50\ mm$ and $T_1 = 1.5\ s$. For a constant velocity rise, the displacement equation is:

$$\Delta R_1 = \frac{H_1 t_1}{T_1} \qquad \text{(a)}$$

   - The second motion interval has $H_2 = 50\ mm$ and $T_2 = 2.0\ s$. For a cycloidal fall, the displacement equation is:

$$\Delta R_2 = H_2\left[1 - \left(\frac{t_2}{T_2}\right) + \frac{1}{2\pi} sin\left(\frac{2\pi t_2}{T_2}\right)\right] \qquad \text{(b)}$$

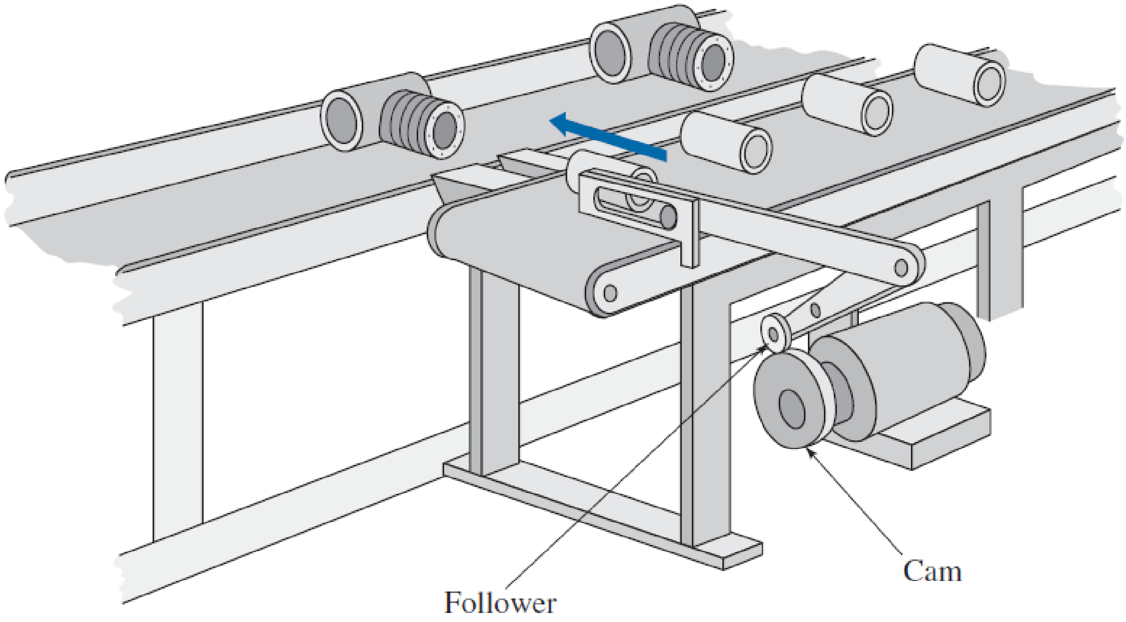


Figure (e)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Cam-Follower Systems - Prescribed Follower Motion

- Solution (cont.).

3. Calculate the Displacement during Each Follower Motion Interval:
   - And, the last motion interval is a dwell, where ΔR is constant. This dwell occurs at the retracted follower position; therefore:

$$\Delta R_3 = 0 \qquad \text{(c)}$$

- Eq. (a), (b), and (c), are substituted into a MATLAB Live Script online.

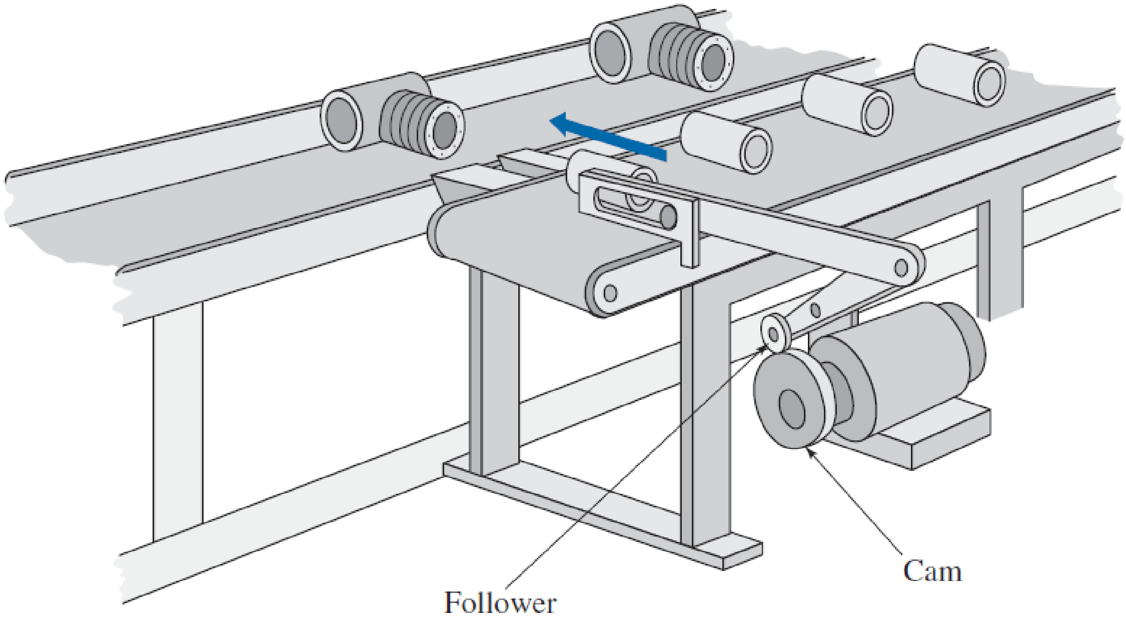


Figure (e)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

End of Part #2