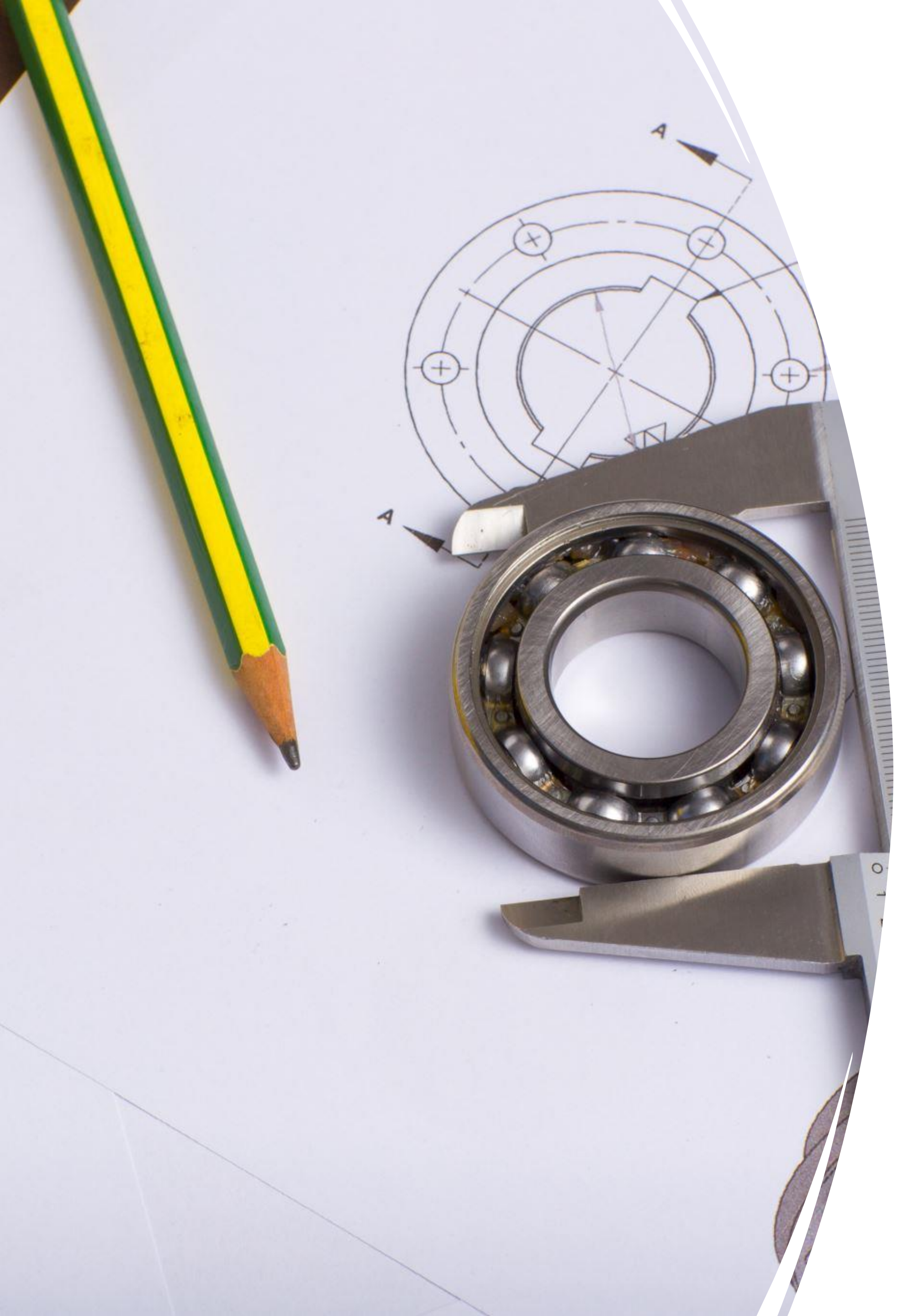
# Cam-Follower Systems – Part #1

Cam Terminology.  Common Types of Cam-Followers.  Analysis of Cam-Follower Motion.  Analytical Cam Design.



# Learning Outcomes – Part#1

1. Identify the different types of cams and cam followers.
2. Create a follower displacement diagram from prescribed follower motion criteria.
3. Understand the benefits of different follower motion schemes.
4. Use equations to construct cam-follower displacement diagrams.

# Cam and Follower Definition:

- Cam is a rotating machine component or element that gives reciprocating or oscillating motion to another element known as a Follower.
- The follower is the part that contacts the cam and converts its motion into linear or oscillating motion.

- Cam-Follower systems are used in the operation of machines such as: vehicle's engines, printing presses, shoe & textile machinery, gear-cutting machines, and screw machines.

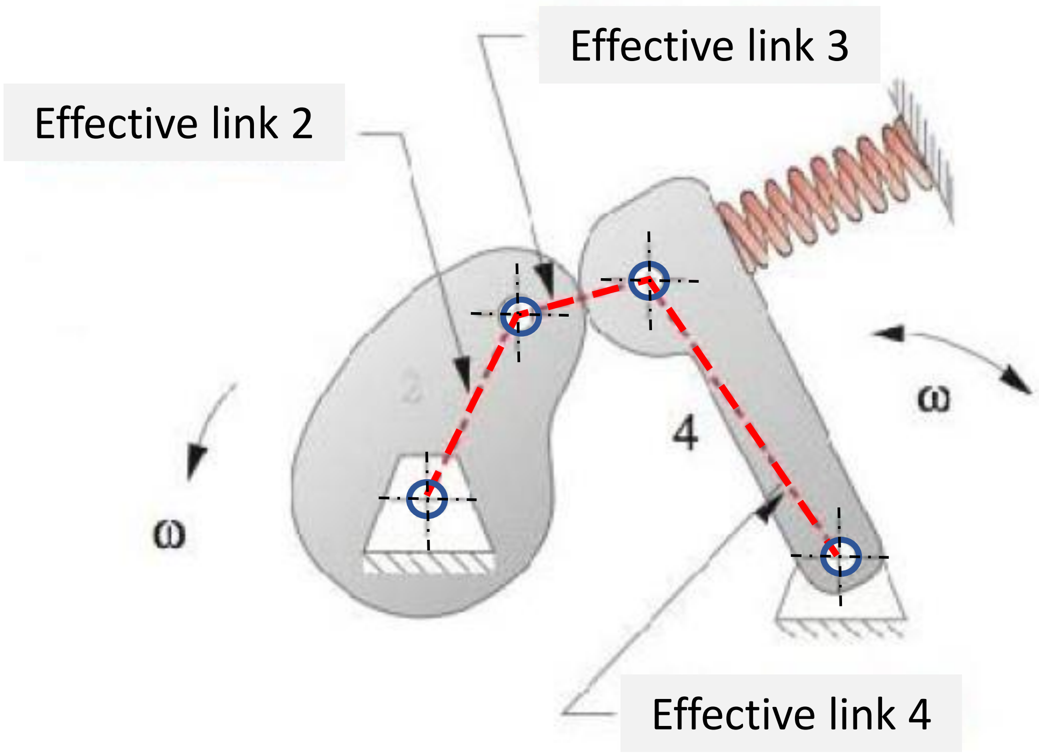



Figure 8-1. An oscillating Cam-Follower Effective Pin-Jointed Fourbar Equivalent.
Textbook: Design of Machinery by Robert L. Norton.

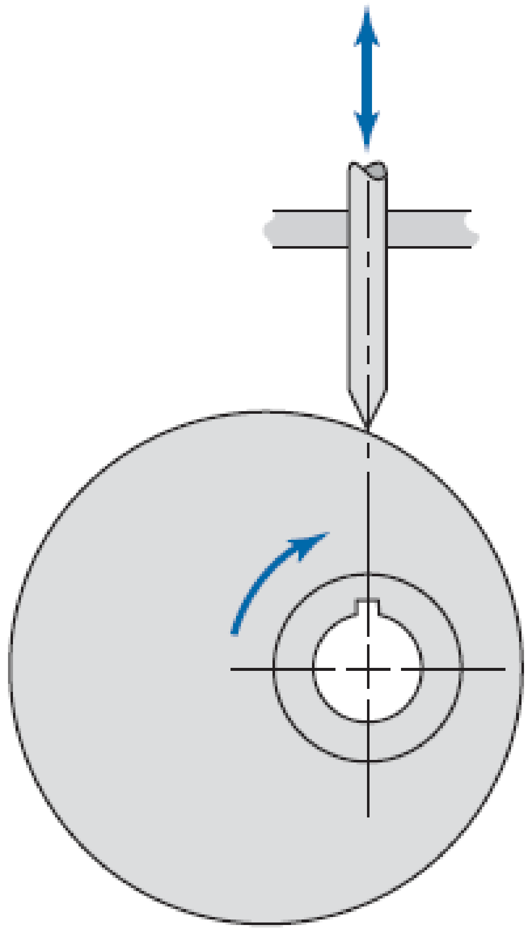
# Cylinders Crankshaft and Camshaft Stock:

Source: Camshaft and Crankshafts Explained. Low-Offset (low-offset.com). 2023.

# Types of Cams:

- The most common type of cam is the *radial cam* as shown in the figure:

- A radial cam is a two-dimensional (2D) body with the follower motion along its external surface of the cam.



Radial Cam-Follower System. Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Types of Cams:

- Another common type of cam is the *cylindrical cam* shown in Figure 8-3.
- In this figure, as the cylinder rotates, the follower moves in a direction parallel to the axis of the cylinder.

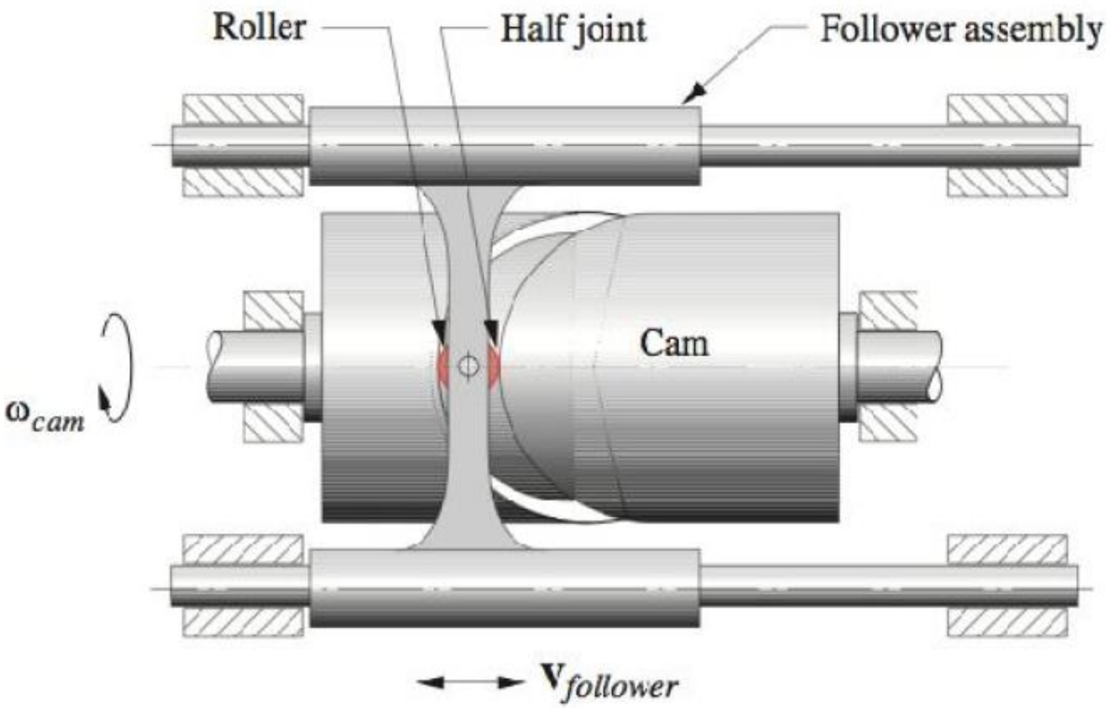
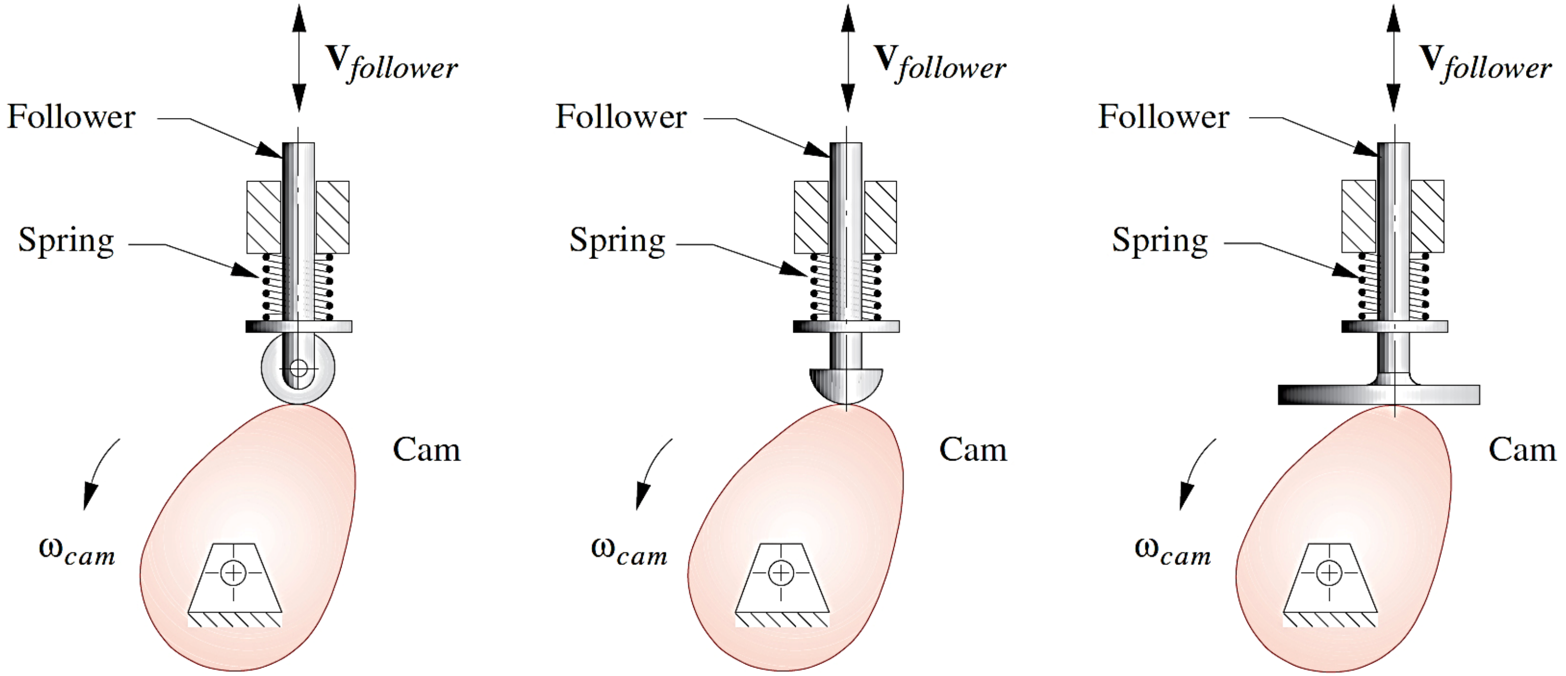



Figure 8-3. Cylindrical Cam-Follower Moving Along an Axis Parallel to the Axis of the Cam.
Textbook: Design of Machinery by Robert L. Norton.

# Types of Followers:

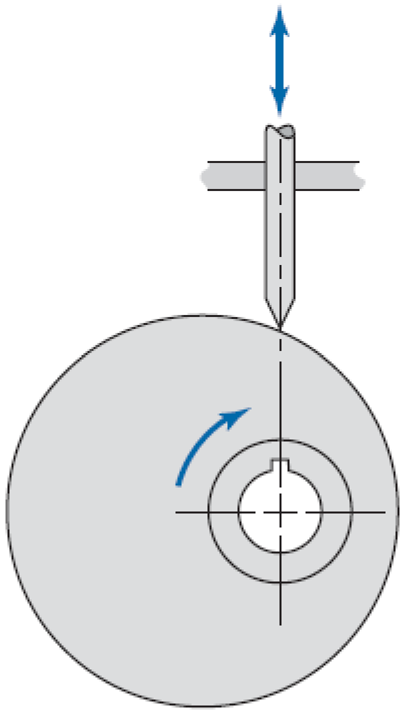
- There are four types of followers: roller, mushroom, flat, and knife-edge.






(a) Roller follower (b) Mushroom follower (c) Flat-faced follower

Figure 8-3. Three Common Types of Cam-Followers.
Textbook: Design of Machinery by Robert L. Norton.



(d) In-line, Knife-Edge follower*

*Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Cam Nomenclature

- Base Circle of a cam is the smallest circle centered on the cam rotation axis and tangent to the cam surface.

- Pitch Curve of a cam is a circle from the cam center through the pitch point (the pitch point is the point of contact between the cam and the follower when the follower is in its highest position).

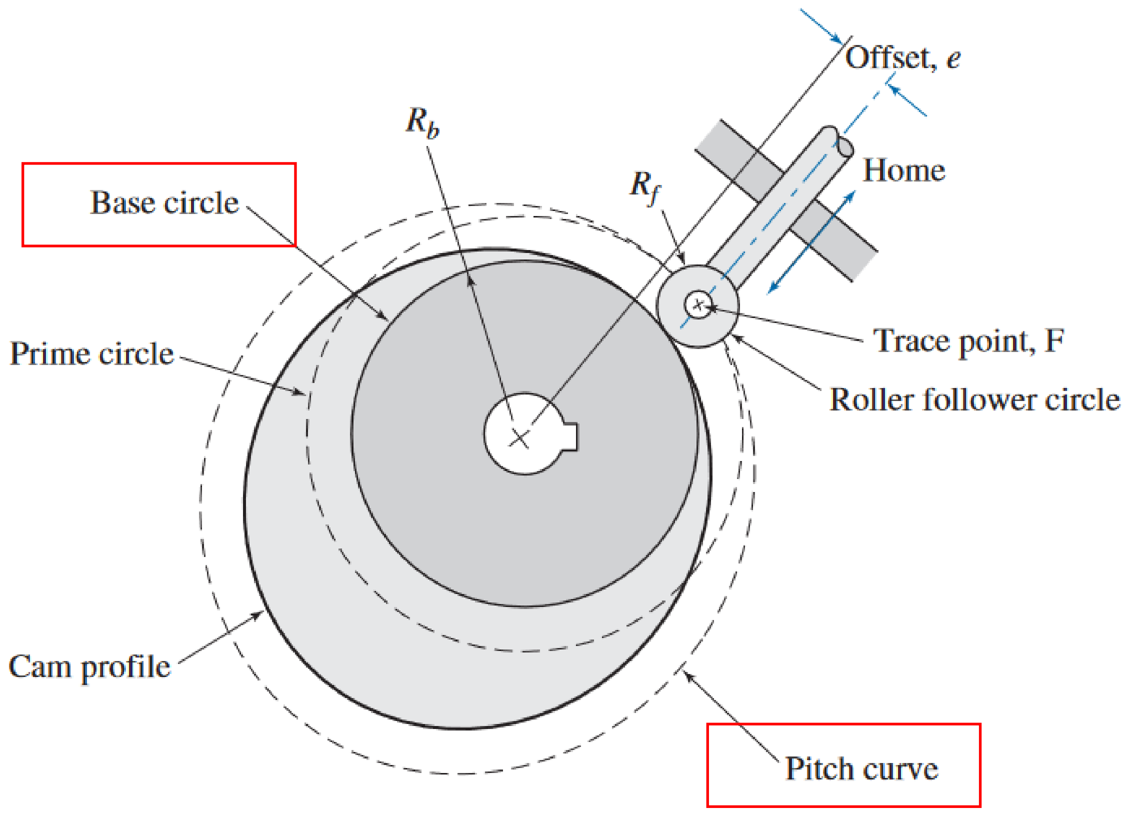



Figure (a). Cam Nomenclature.
Source: Myszka, D.H. Machines and Mechanisms Applied Kinematic Analysis. Prentice Hall. 4th Edition. University of Dayton. USA.

# Cam Nomenclature

- The pitch circle radius is used to calculate a cam of minimum size for a given pressure angle.

- The pressure angle is the angle between the direction of the follower motion and the normal to the pitch curve at the pitch point.

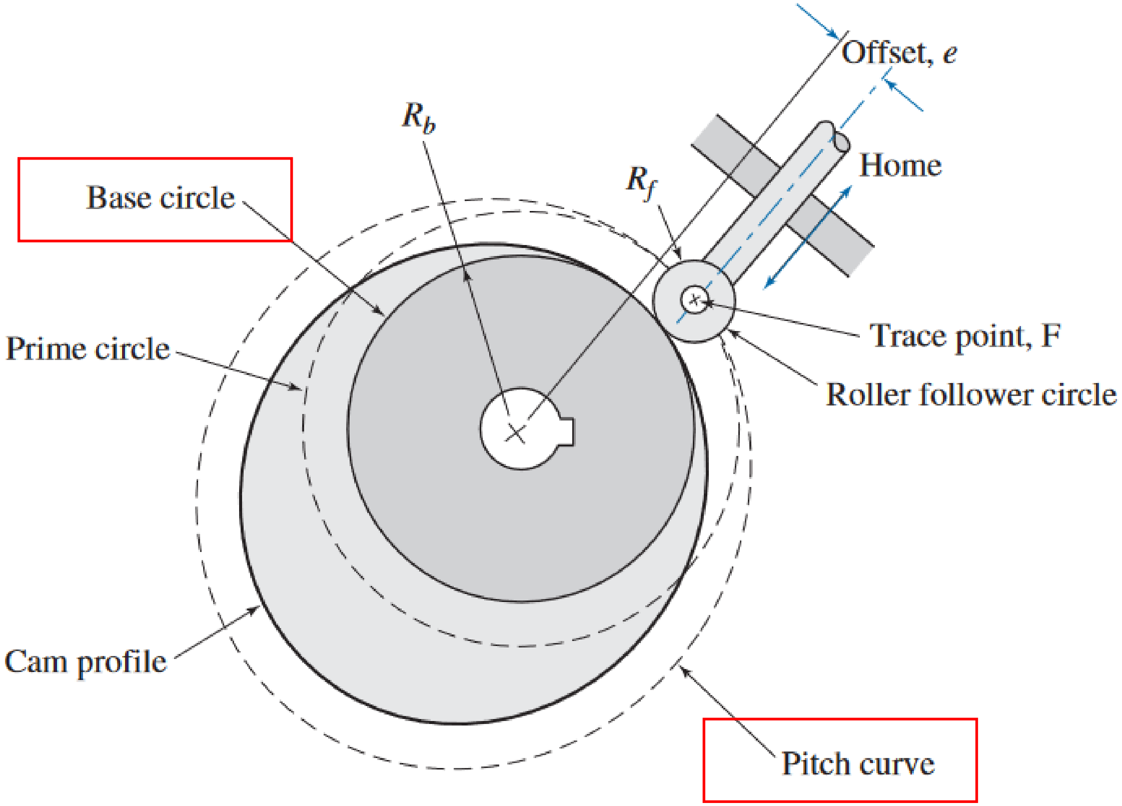



Figure (a). Cam Nomenclature.
Source: Myszka, D.H. Machines and Mechanisms Applied Kinematic Analysis. Prentice Hall. 4th Edition. University of Dayton. USA.

# Cam Nomenclature

- Trace Point: The trace point of a cam is a reference point on the follower that is used to generate the pitch curve, which is the contour of the cam surface.

- For example, in a roller follower, the trace point is at the center of the roller as shown in Figure (a):

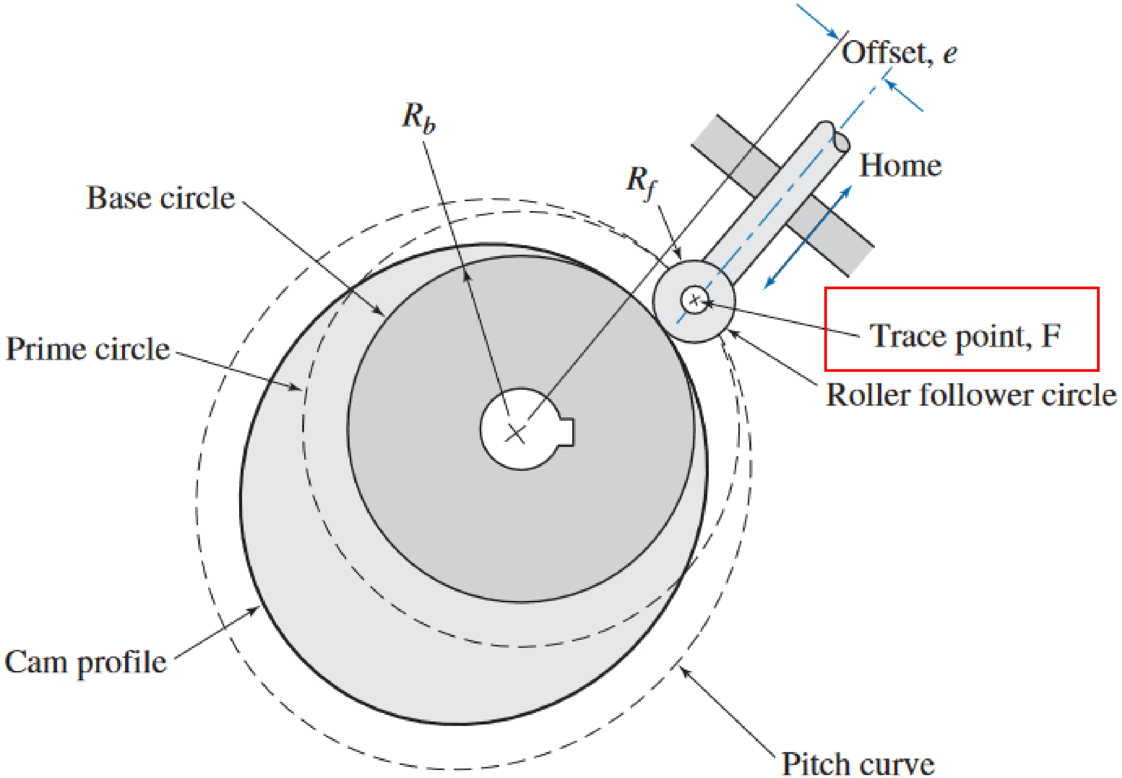



Figure (a). Cam Nomenclature.
Source: Myszka, D.H. Machines and Mechanisms Applied Kinematic Analysis. Prentice Hall. 4th Edition. University of Dayton. USA.

# Cam Nomenclature

- Prime Circle: is a circle drawn through the trace point of the follower while the cam is at its home position.

- The *home position of the cam* is the orientation that corresponds to the 0° reference position on a displacement diagram (further details later).

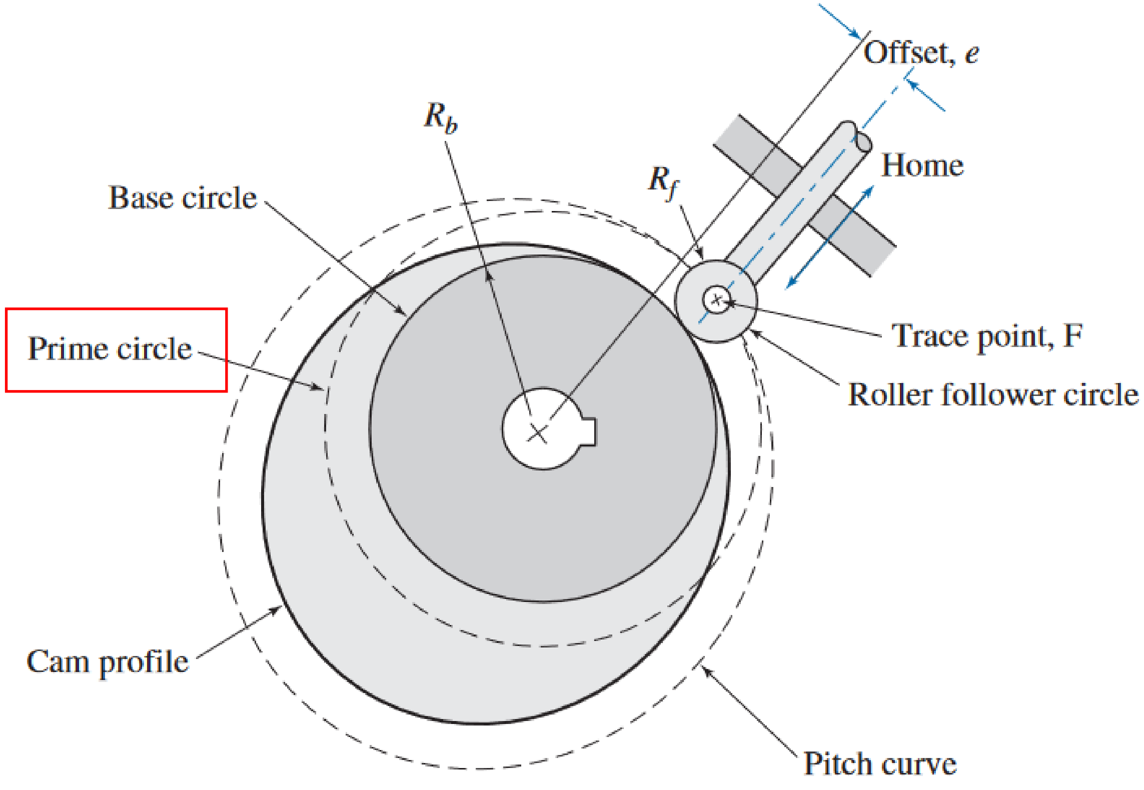



Figure (a). Cam Nomenclature.
Source: Myszka, D.H. Machines and Mechanisms Applied Kinematic Analysis. Prentice Hall. 4th Edition. University of Dayton. USA.

# Cam-Follower Systems - Types of Follower Motion

- The cam's motions can be considered to some extent as alternatives to motions obtained from linkages.
- For instance, an oscillating cam-follower has an effective pin-jointed fourbar linkage as shown in Figure 8-1.

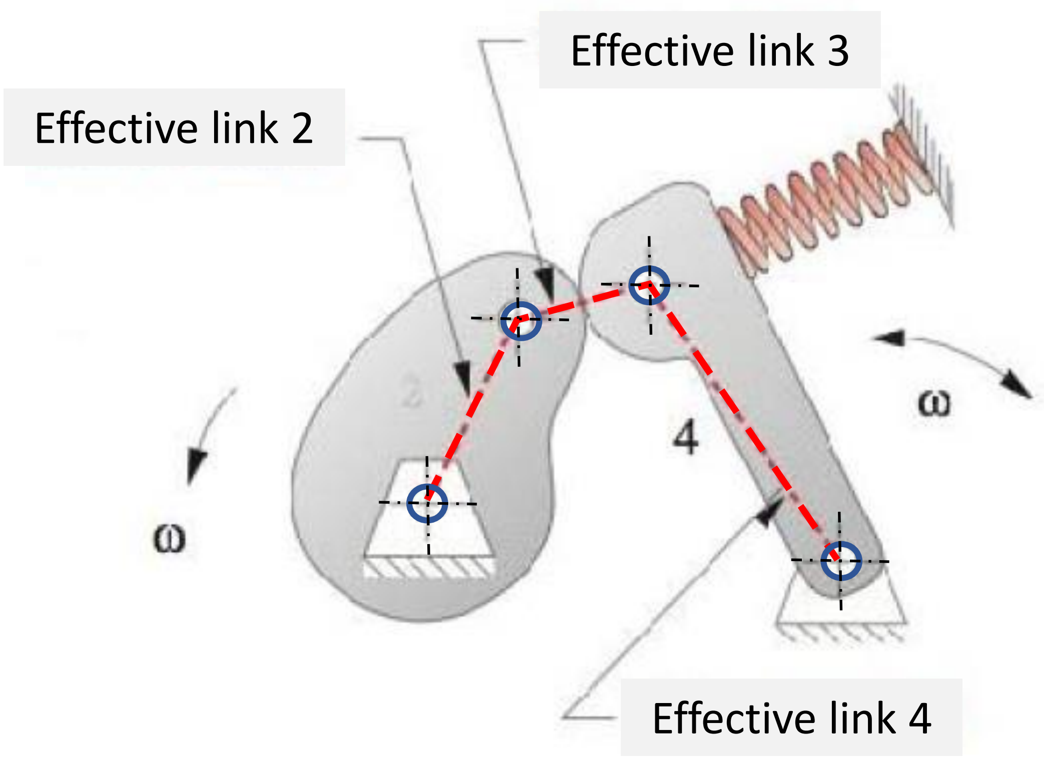



Figure 8-1. An oscillating Cam-Follower Effective Pin-Jointed Fourbar Equivalent.
Textbook: Design of Machinery by Robert L. Norton.

# Cam-Follower Systems - Generation of a Cam Profile

- To design the shape of the cam surface designer needs to know how the follower moves in response to the cam rotation.
- The *motion program* is a set of specifications that define the follower's displacement, velocity, and acceleration at different angles of the cam rotation.
- The motion program determines the cam profile, which is the contour of the cam surface (or cam profile).

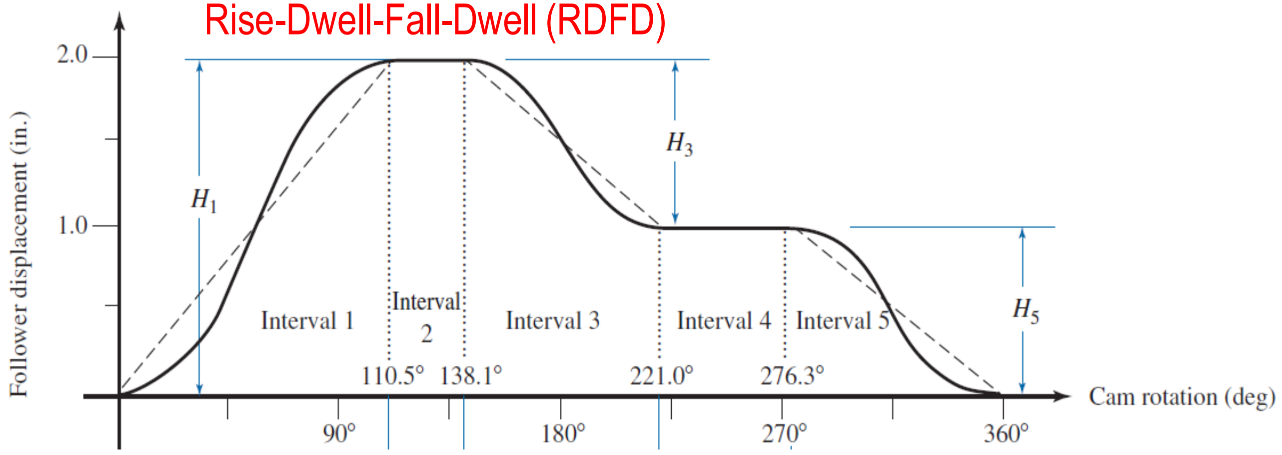
# Cam-Follower Systems – Motion Program

- The type of motion programs are:
  - Rise-Fall (RF)
  - Rise-Fall-Dwell (RFD), and
  - Rise-Dwell-Fall-Dwell (RDFD)

***Note***: A "dwell" is defined as "no output motion for a specified period of input motion".

- The motion programs define how many dwells are present in the full cycle of motion: either none dwells (RF), one dwell (RFD), or more than one dwells (RDFD).

Rise-Dwell-Fall-Dwell (RDFD)



# Follower Displacement Diagram:

- The position of the follower from a rest position in relation to time or the rotary angle of the cam. ***Note:*** Usually, it is the zero position when the follower contacts with the *base circle* of the cam.
- A plot showing the follower displacement versus time is termed a *Follower Displacement Diagram*.
- In the *follower displacement diagram, the f*ollower motion is typically expressed by the symbol $\beta$ (beta) at different angles of the cam rotation.

# Follower Displacement Diagram:

- The time elapsed during an interval (at different angles of the cam rotation) is designated $T$.
- The amount of follower rise, or fall, during an interval is designated by $h$.
- The relationship between cam rotation and time for an arbitrary interval, $i$, is:

$$\beta_i = \omega_{cam} \times T_i \qquad \text{Eq. (8-1)}$$

# Follower Displacement Diagram:

- Eq. (8-1) can also be used to determine the required speed of the cam, by observing the time consumed during one cycle; that is:

$$\omega_{cam} = \frac{1\,rev}{\sum T_i} \qquad \text{Eq. (8-2)}$$

- Where $\sum T_i$ is the total time for the follower motion intervals that comprise one cycle of the cam rotation.

# Follower Displacement Diagram:  Procedure

- First, we define the motion program of the follower which describe the follower's displacement.
- Second, once the prescribed or desired follower motion of has been defined through a displacement diagram, the actual shape of the cam can be designed.
- An example of prescribed follower motion is presented in next slide:

# Follower Displacement Diagram:

- Example 7-1.
- A cam is to be used for a platform that will repeatedly lift boxes from a lower conveyor to an upper conveyor.  This machine is shown in Figure (b):

  a. Plot the *displacement follower diagram* and
  b. Determine the cam rotation for each follower motion interval for the sequence of the follower motion as follows:

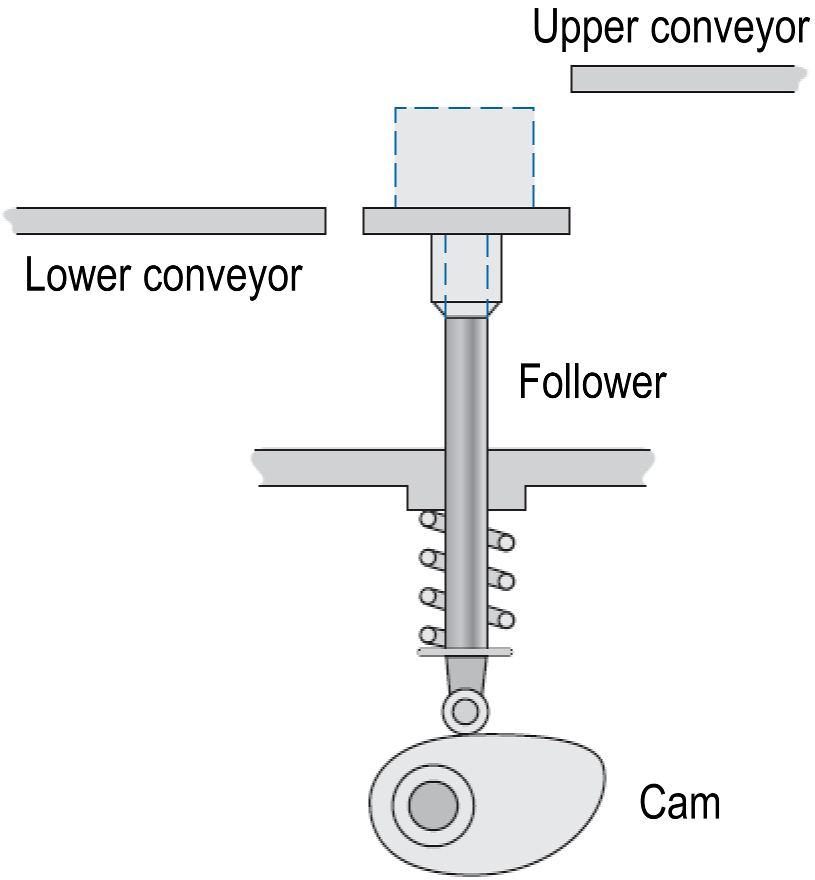



Figure (b)

Source:  Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Follower Displacement Diagram:

- Example 7-1 (cont.)
- The required follower motion sequence is as follows:

1. Rise 2 in. in 1.2 s.
2. Dwell for 0.3 s.
3. Fall 1 in. in 0.9 s.
4. Dwell 0.6 s.
5. Fall 1 in. in 0.9 s.

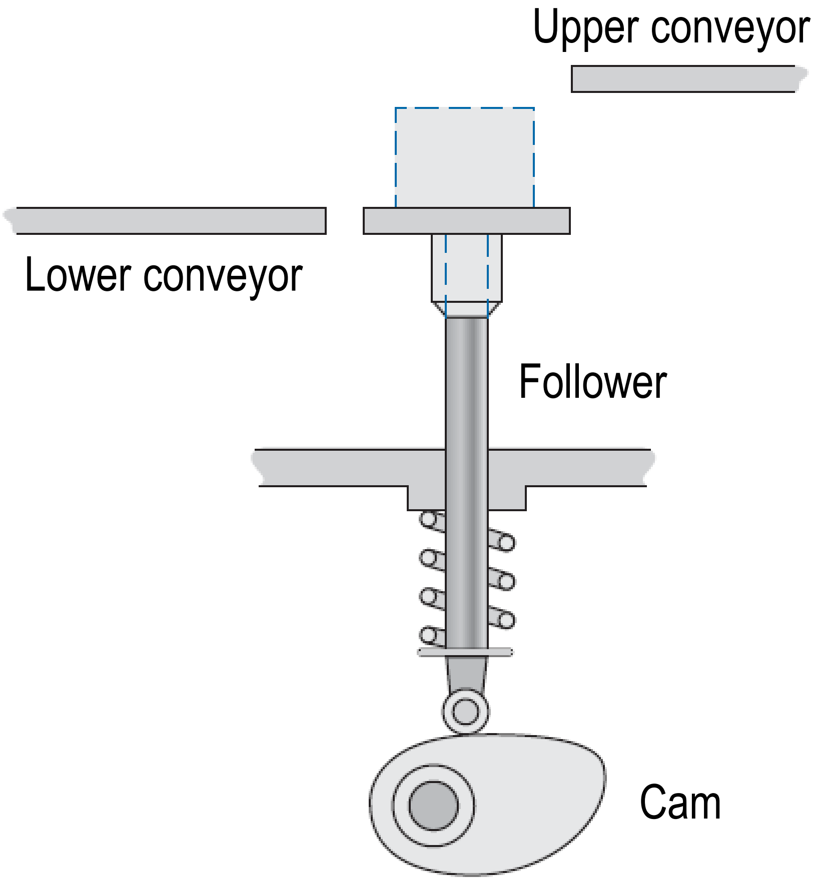



Figure (b)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Follower Displacement Diagram:

- Example 7-1 (cont.)
- (a) Plot a displacement diagram and determine the required speed of the cam for this sequence:
- Solution.

1. Calculate the “Time for a Full Cycle” (or complete rotation):

$$\sum T_i = T_1 + T_2 + T_3 + T_4 + T_5$$

$$= (1.2 + 0.3 + 0.9 + 0.6 + 0.9) = 3.9\ seconds$$

2. From Eq. (8-2), calculate the “Required Rotational Speed of the Cam”:

$$\omega_{cam} = \frac{1\ rev}{\sum T_i} = \frac{1\ rev}{3.9} = 0.256\frac{rev}{s}\left(\frac{60}{1\ min}\right) = 15.38\ rpm$$

# Follower Displacement Diagram:

- Example 7-1 (cont.)
- Solution.

3. Determine the Cam Rotation for Each Follower Motion Interval:

$$\beta_1 = \omega_{cam} \times T_1 = (0.256\, rev/s)(1.2\, s)$$
$$= 0.307\, rev \times \left(\frac{360°}{1\, rev}\right) = 110.5°$$

$$\beta_2 = \omega_{cam} \times T_2 = (0.256\, rev/s)(0.3\, s)$$
$$= 0.077\, rev \times \left(\frac{360°}{1\, rev}\right) = 27.6°$$

# Follower Displacement Diagram:

- Solution (cont.).

3. Determine the Cam Rotation for Each Follower Motion Interval:

$$\beta_3 = \omega_{cam} \times T_3 = (0.256\, rev/s)(0.9\, s)$$
$$= 0.230\, rev \times \left(\frac{360°}{1\, rev}\right) = 82.9°$$

$$\beta_4 = \omega_{cam} \times T_4 = (0.256\, rev/s)(0.6\, s)$$
$$= 0.154\, rev \times \left(\frac{360°}{1\, rev}\right) = 55.3°$$

$$\beta_5 = \omega_{cam} \times T_5 = (0.256\, rev/s)(0.9\, s)$$
$$= 0.230\, rev \times \left(\frac{360°}{1\, rev}\right) = 82.9°$$

# Follower Displacement Diagram:

- Plot the Displacement Diagram: The resulting displacement diagram with both cam angle and time displayed on the horizontal axis is shown in Figure (c ). Notice that a curved displacement profile was constructed during the rise and fall sequences.

$$\beta_1 = 110.5°$$

$$\beta_2 = 27.6°$$

$$\beta_3 = 82.9°$$

$$\beta_4 = 55.3°$$

$$\beta_5 = 82.9°$$

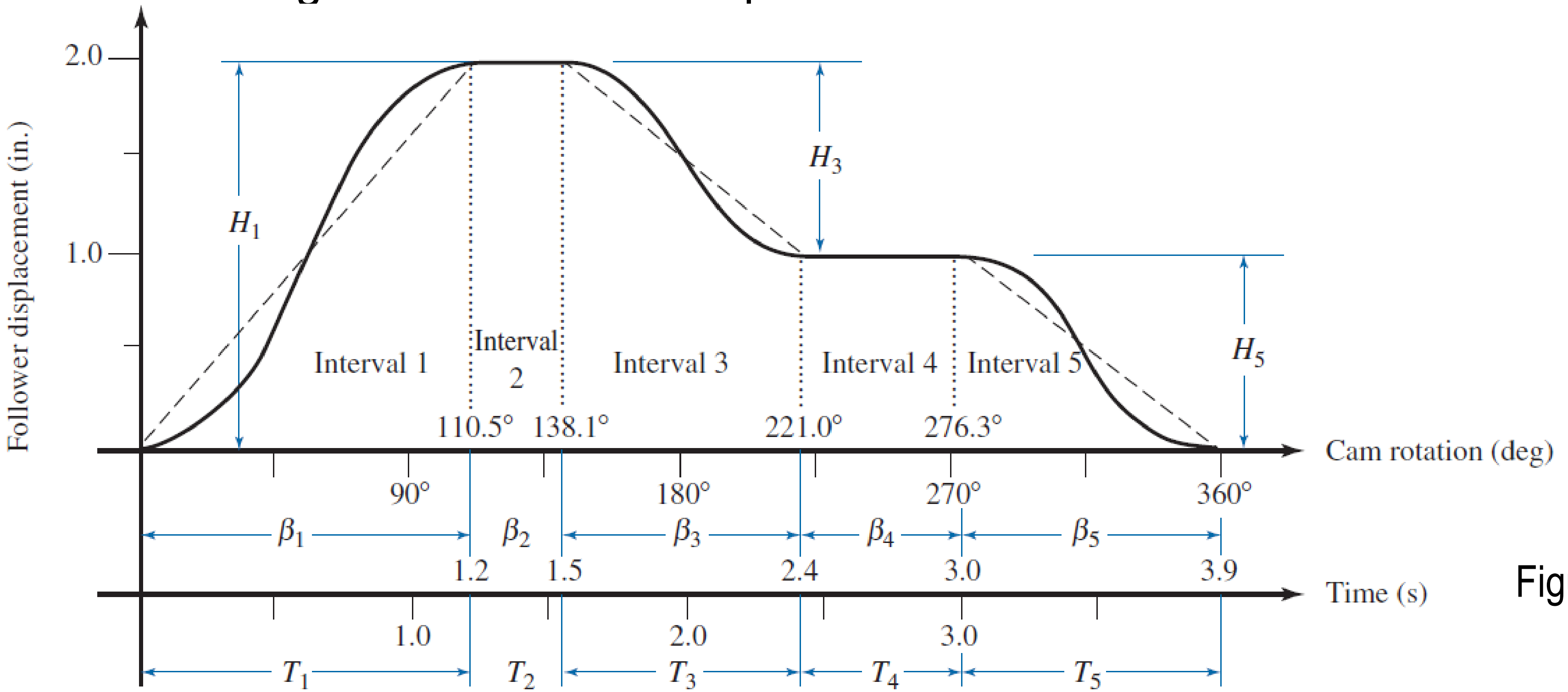



Figure (c)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# The Fundamental Law of Cam Design

- The Fundamental Law of Cam Design:
- The cam-follower function must have continuous velocity and acceleration across the entire interval, thus making the jerk finite.
- Therefore, any cam designed for operation at other than very low speeds must be designed with the following constraints:

# The Fundamental Law of Cam Design - Constraints

- The motion curve must be continuous and the first and second derivatives (corresponding to the velocity and acceleration of the follower) must be finite even at the transition points.
- That means that for the cam-follower motion system depends upon the shape of the cam.
- ***Note:*** Almost always the cam is the driver, and the follower is the driven member.

# Cam Profile Design: Remarks

- The cam motion function must be continuous through the first and second derivatives of displacement across the entire interval (360 degrees).

- SVAJ (Displacement (S), Velocity (V) and Acceleration (A), and Jerk (J) Motion Functions are mathematical expressions that will allow the follower to exhibit a desired motion.
- In this module, we describe several basic curves.  These are:

  - Constant Velocity Motion
  - Constant Acceleration Motion, and
  - Harmonic Motion.
  - Cycloidal Motion.

A file named “Cam Follower Kinematics Profiles” is available in BB.  Tables 9-1 to 9-4 present the mathematical expressions of the cam-follower kinematics.

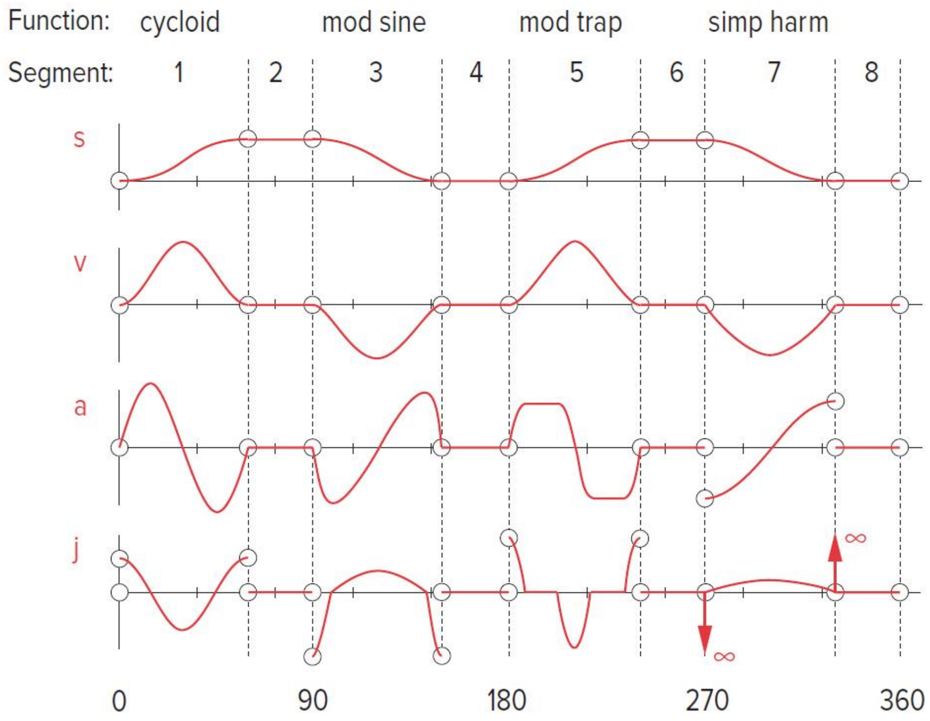
# Cam Profile Design: Remarks

- As the cam rotates, the follower is constrained to rise and fall, following plots of cam-follower SVAJ functions as shown Figure 8-6(a).



| Segment Number | Function Used | Start Angle | End Angle | Delta Angle |
|---|---|---|---|---|
| 1 | Cycloid rise | 0 | 60 | 60 |
| 2 | Dwell | 60 | 90 | 30 |
| 3 | ModSine fall | 90 | 150 | 60 |
| 4 | Dwell | 150 | 180 | 30 |
| 5 | ModTrap rise | 180 | 240 | 60 |
| 6 | Dwell | 240 | 270 | 30 |
| 7 | SimpHarm fall | 270 | 330 | 60 |
| 8 | Dwell | 330 | 360 | 30 |

(*a*) Cam program specifications




(*b*) Plots of cam-follower's *s v a j* diagrams

Figure 8-6. Three Common Types of Cam-Followers. Source: Design of Machinery's Textbook by Robert L. Norton.

# A Bad Cam Design

- Using the “Fundamental Law of Cam Design”, we apply the SVAJ motion program in terms of numbers and equations. As can be seen, constant velocity motion is not acceptable because the infinity jerk.



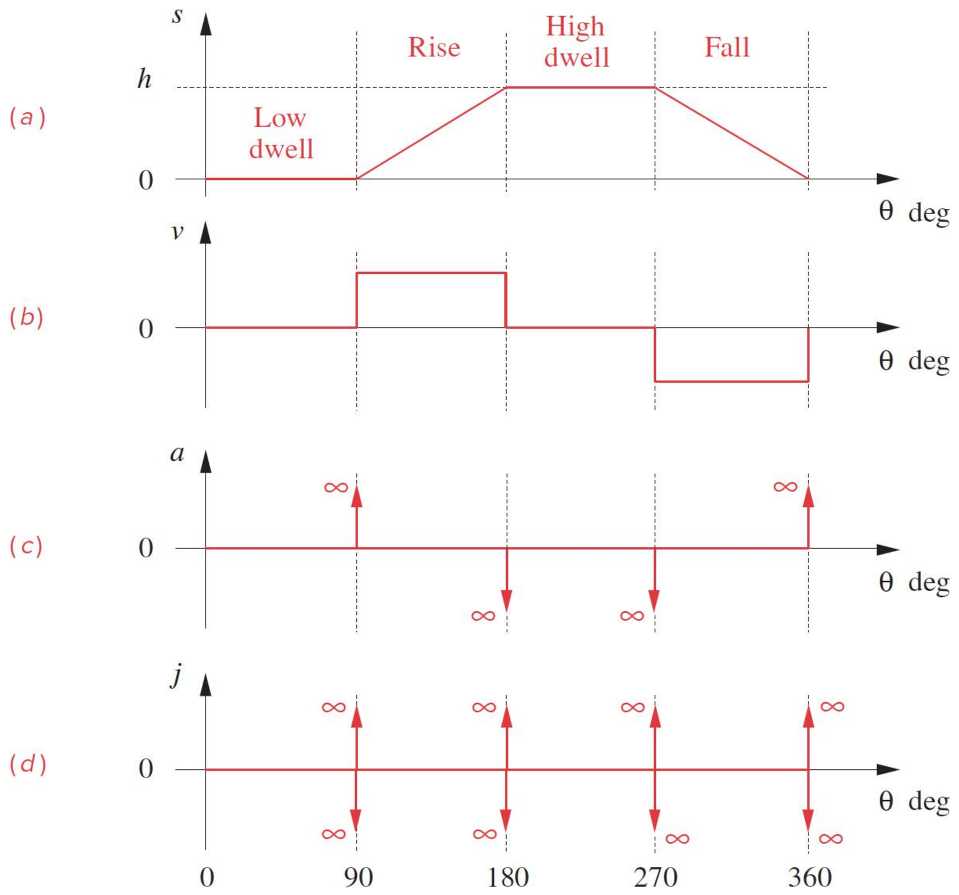



Figure 8-8. A Bad Cam Design. Source: Design of Machinery’s Textbook by Robert L. Norton.

# Cam-Follower Systems – A Cam Timing Diagram

- Constant Acceleration (Parabolic Displacement): Constant acceleration motion is also unacceptable; that is, motion described infinite Jerk.

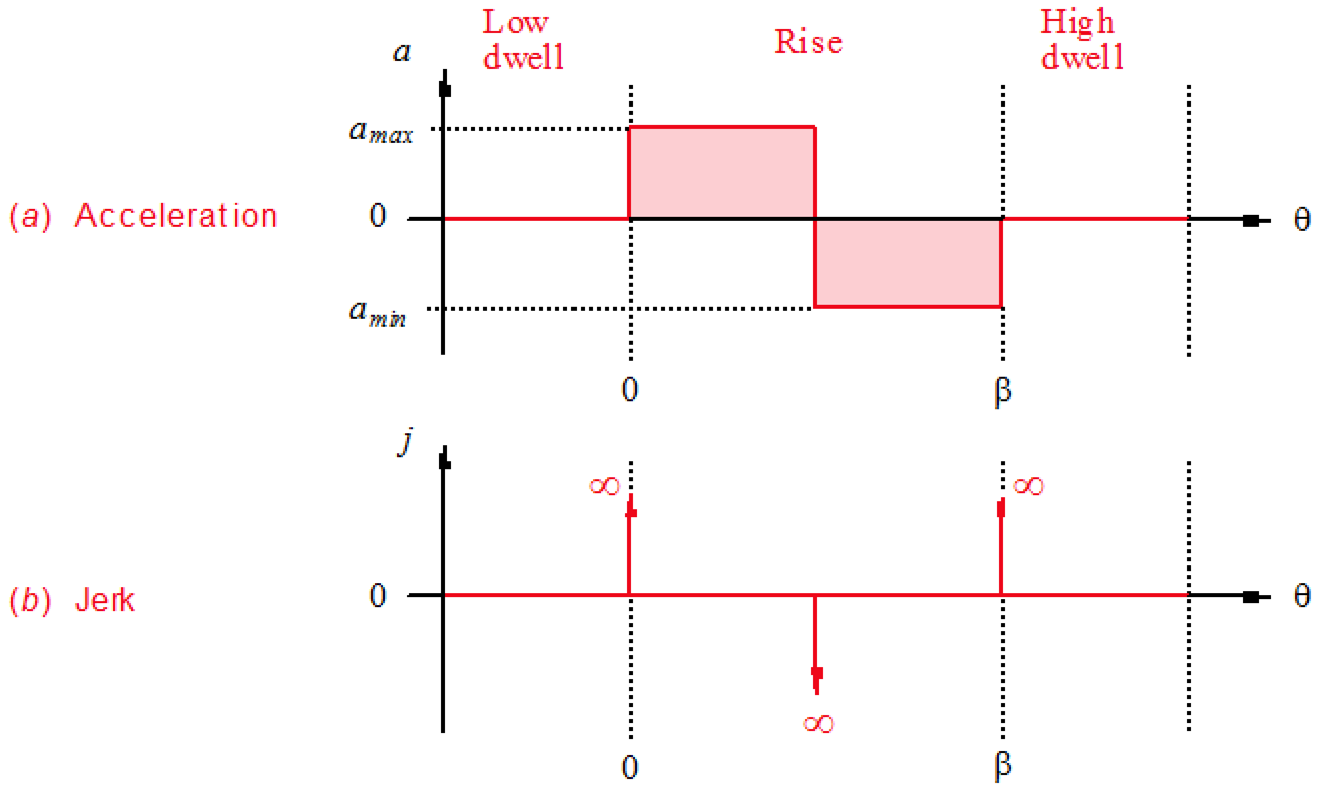



Figure 8-13. Constant Acceleration. Source: Design of Machinery's Textbook by Robert L. Norton.

# Cam-Follower Systems – A Cam Timing Diagram

- Harmonic Motion is acceptable because its continuous through the derivatives of displacement across the entire interval.

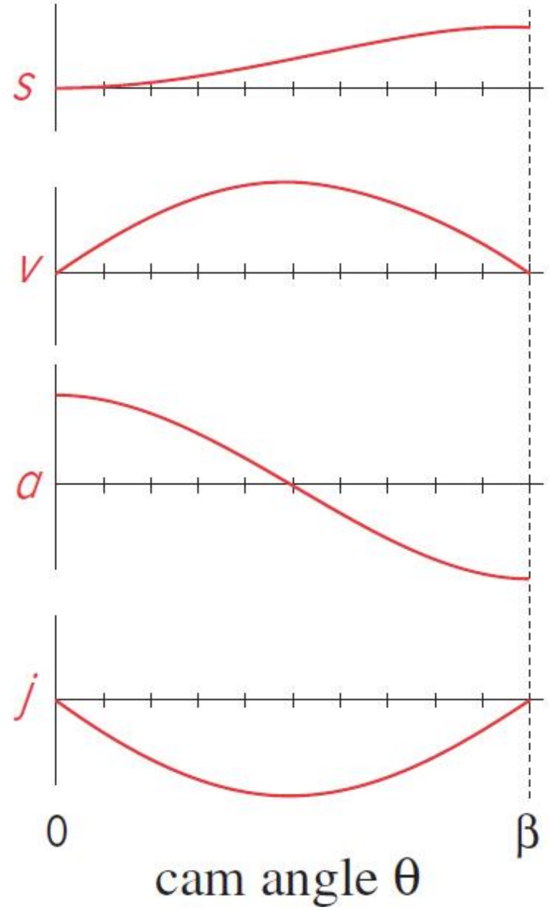
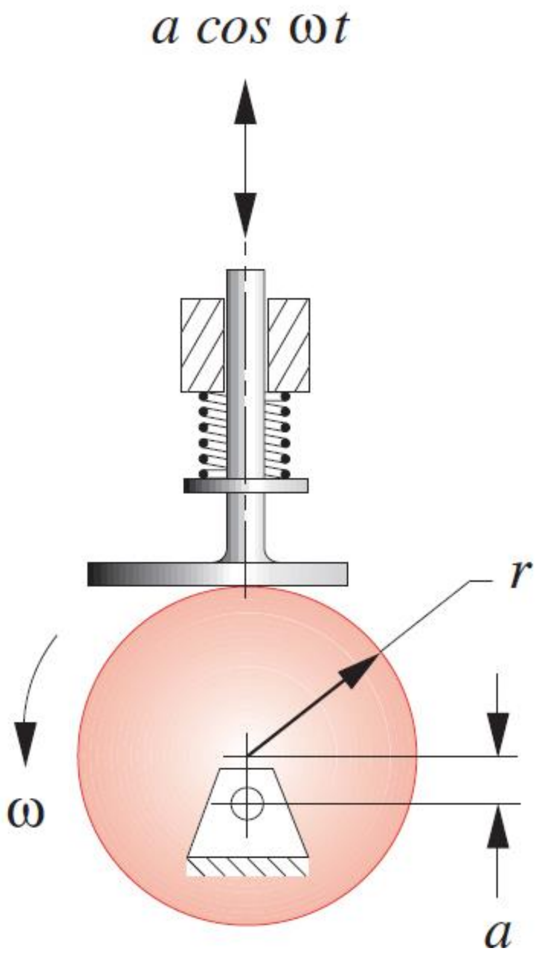
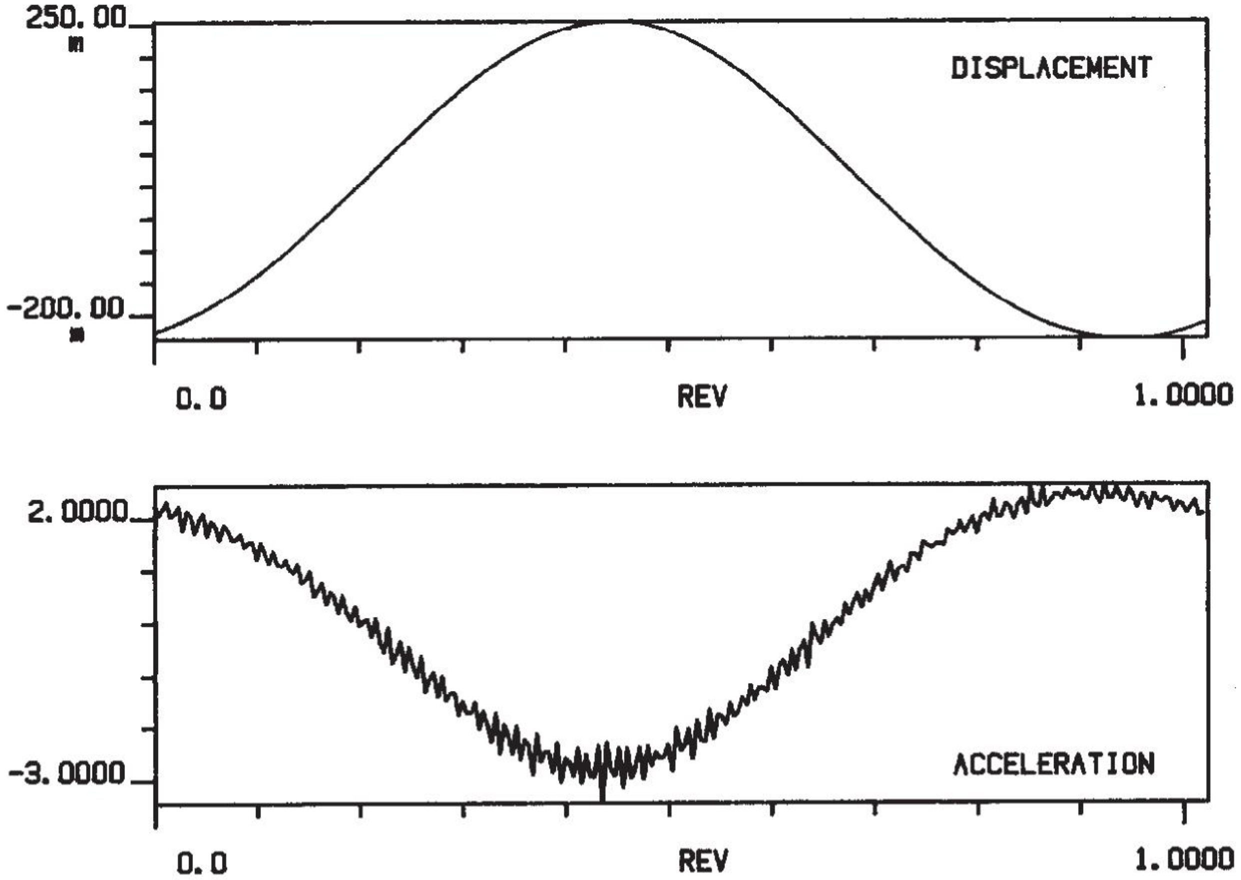



Figure 8-9. Harmonic Motion. Source: Design of Machinery's Textbook by Robert L. Norton.

# SVAJ Functions Using Polynomials:

- Plot of the Complete Solution is found in MATLAB Online.






Source: Design of Machinery's Textbook by Robert L. Norton.

# Cam-Follower Systems – SVAJ Functions Using Polynomials

- Normalized Cam Angle: $x = \dfrac{\theta}{\beta}$ where $\beta$ is the interval period.
- Normalized follower displacement: $y = \dfrac{s}{h}$
- The Displacement (S), Velocity (V) and Acceleration (A) are:

$$s = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + \cdots$$

$$v = C_1 + 2 C_2 x + 3 C_3 x^2 + 4 C_4 x^3 + 5 C_5 x^4 + \cdots$$

$$a = 2 C_2 + 6 C_3 x + 12 C_4 x^2 + 20 C_5 x^3 + \cdots$$

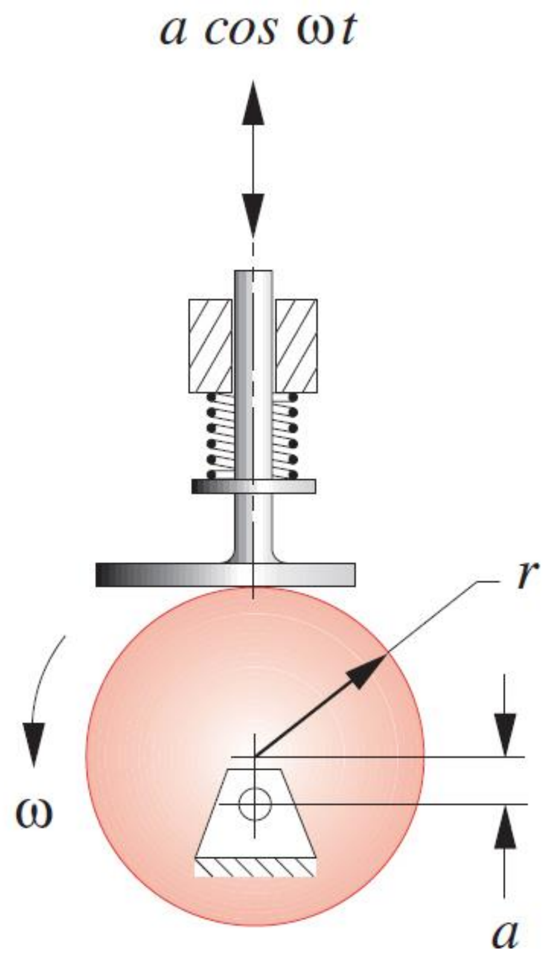
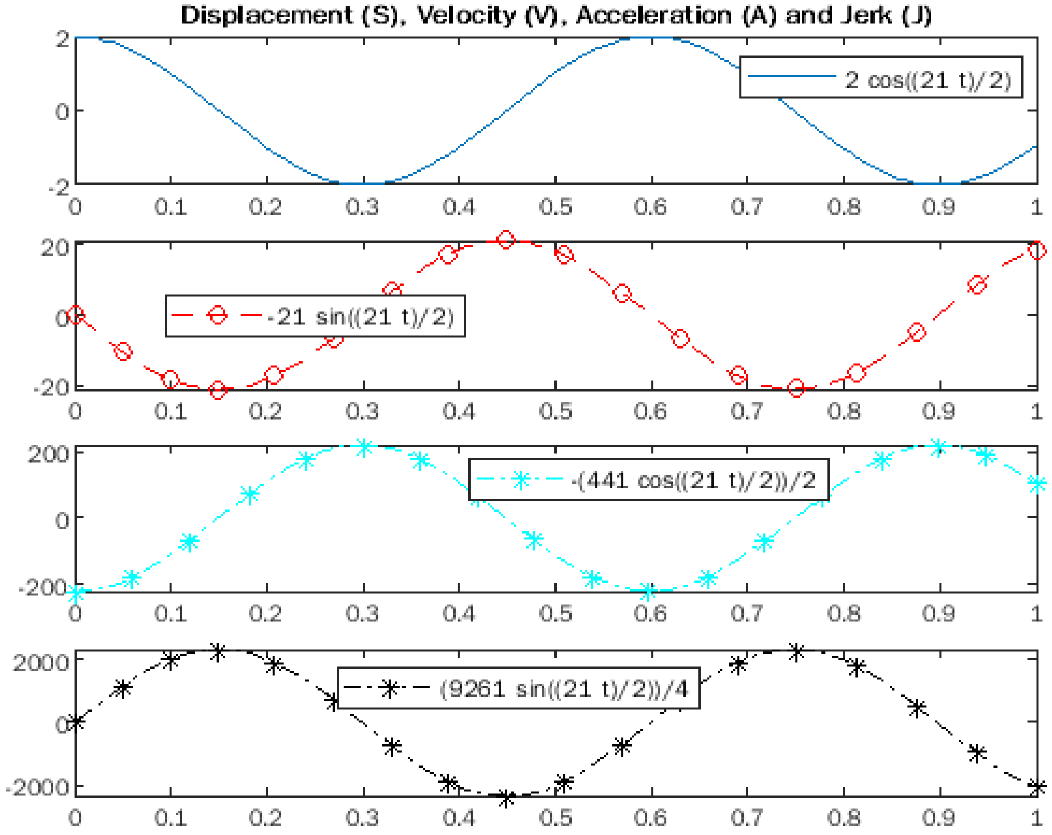



Figure 8-1(a). An oscillating Cam-Follower Effective Pin-Jointed 4-Bar Equivalent. Design of Machinery's Textbook by Robert L. Norton.

# Cam-Follower Systems – SVAJ Functions Using Polynomials

- SVAJ Functions in MATLAB Online:



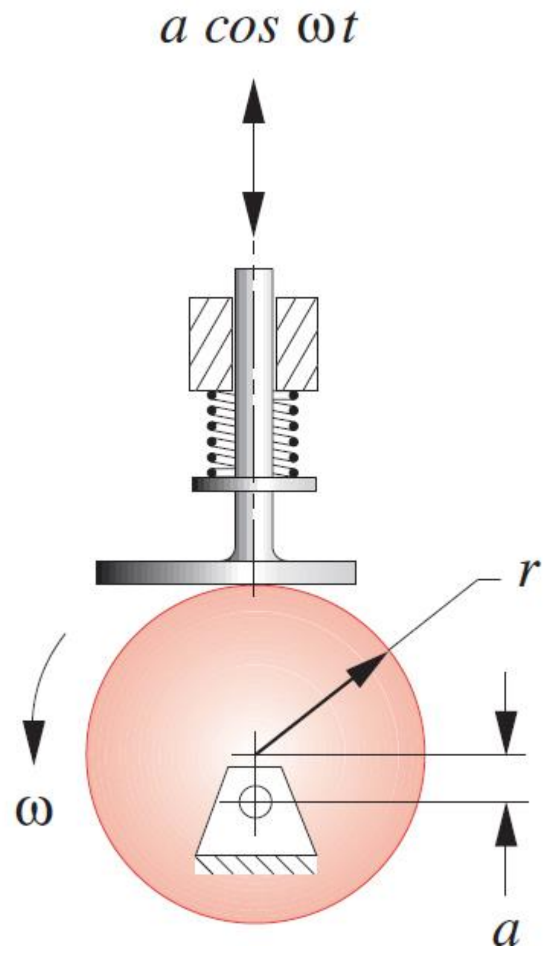




Figure 8-1(a). An oscillating Cam-Follower Effective Pin-Jointed Fourbar Equivalent.
Textbook: Design of Machinery by Robert L. Norton.

# Cam-Follower Systems – Graphical Construction of a Displacement Diagram

- Graphical construction of a displacement diagram is an alternative method to generating a displacement diagram to scale. The construction using the constant acceleration motion scheme can be accomplished by referring to Figure (d) and using the following procedure:

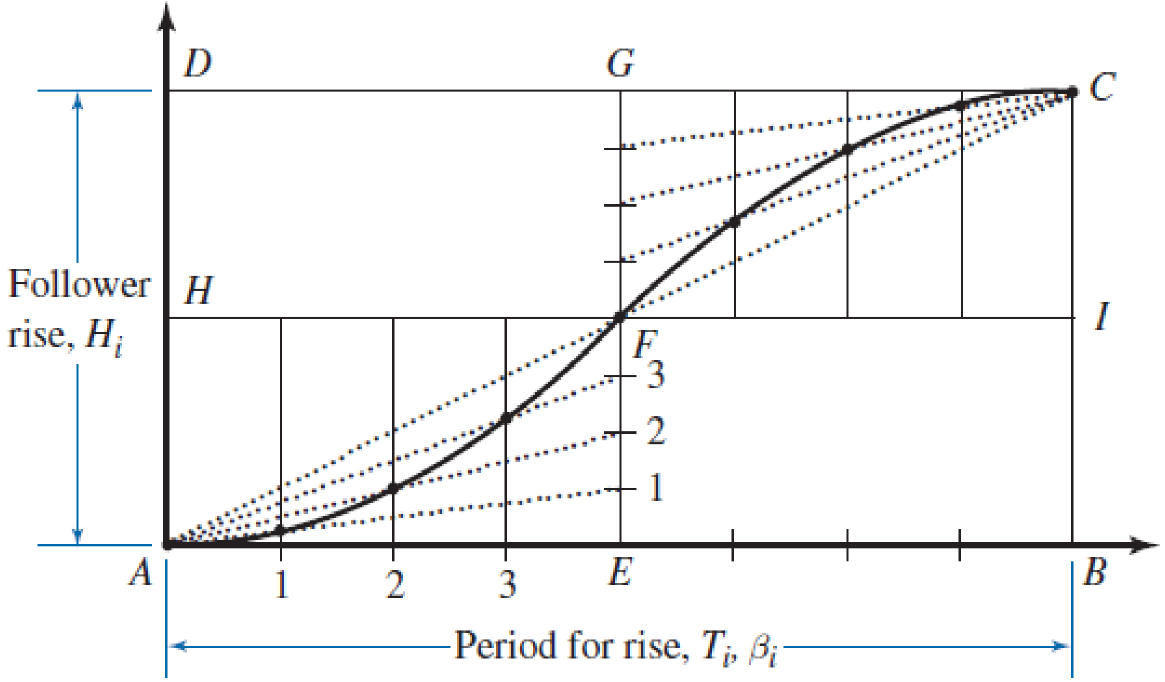



Figure (d)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Cam-Follower Systems – Graphical Construction of a Displacement Diagram

1. Divide the follower rise (or fall) sequence into two halves. From Figure (d), $AE$ represents the time $(T_i)$ and vertical line $(EF)$, the magnitude of rise for the first half of this motion scheme.
2. Divide both the horizontal and vertical axes of the quadrant $AEFH$ into equal parts.
3. Construct vertical lines from the horizontal divisions.
4. Construct straight lines from corner *A* to the vertical divisions.
5. Draw a smooth curve through the points of intersection of the vertical lines and the lines drawn from corner *A*.
6. Repeat steps 2 through 5 for the remaining half of the curve as shown in quadrant $FICG$ in Figure (d). A constant acceleration fall is constructed as a mirror image to Figure (b).

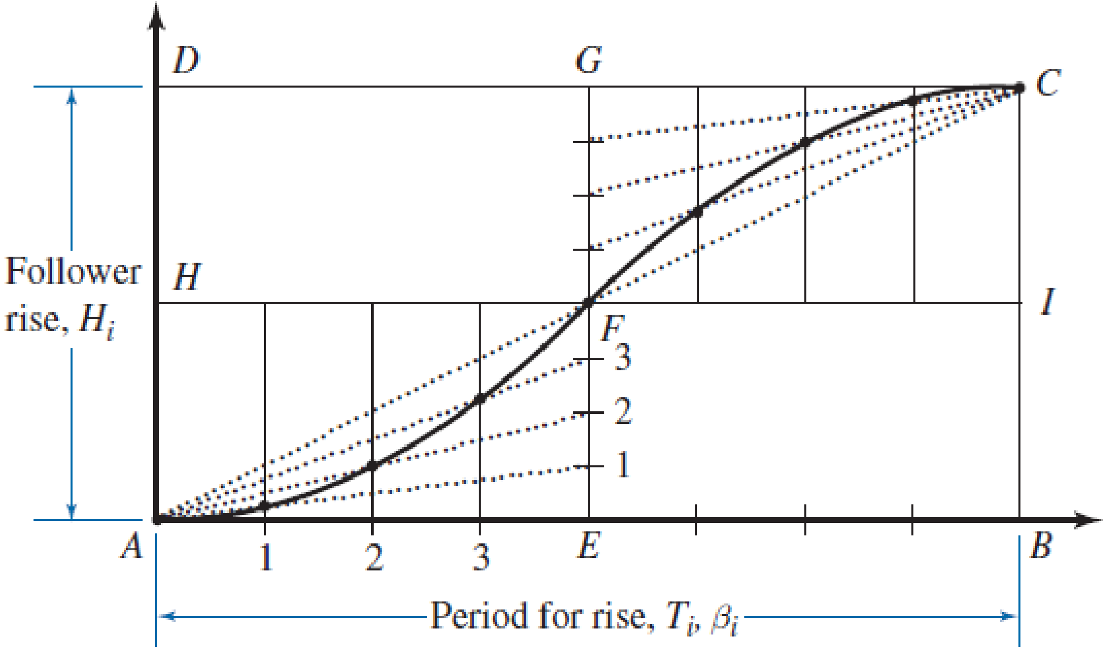



Figure (d)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Graphical Construction of a Displacement Diagram – An Example

- Example 7-2. A cam is to be designed for an automated part loader as shown in Figure (e). Using the motion equations, construct a chart that tabulates follower displacement versus time and cam rotation. Also plot this data when the prescribed motion for this application is as follows:

1. Rise 50 mm in 1.5 s using the constant velocity scheme.
2. Return in 2.0 s using the cycloidal motion scheme.
3. Dwell 0.75 s.
4. Repeat the sequence.

# Graphical Construction of a Displacement Diagram – An Example

- Solution.

1. Calculate the "Time for a Full Cycle":

$$\sum T_i = T_1 + T_2 + T_3$$

$$= (1.5 + 2.0 + 0.75) = 4.25\ seconds$$

2. From Eq. (8-2), calculate the "Required Rotationa Speed of the Cam":

$$\omega_{cam} = \frac{1\ rev}{\sum T_i} = \frac{1\ rev}{4.25\ s} = 0.235\frac{rev}{s}\left(\frac{60}{1\ min}\right)$$
$$= 14.12\ rpm$$

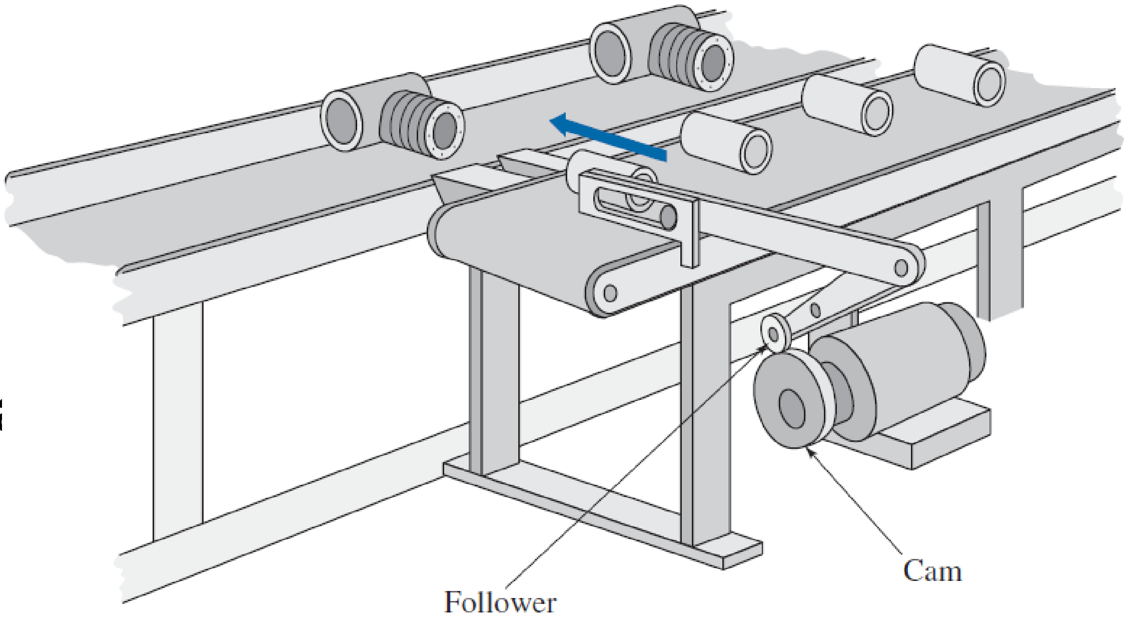



Figure (e)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Cam-Follower Systems - Prescribed Follower Motion

- Solution (cont.).

3. Determine the Cam Rotation for Each Follower M
Interval:

$$\beta_1 = \omega_{cam} \times T_1 = (0.235\, rev/s)(1.5\, s)$$
$$= 0.353\, rev \times \left(\frac{360°}{1\, rev}\right) = 127°$$

$$\beta_2 = \omega_{cam} \times T_2 = (0.235\, rev/s)(2.0\, s)$$
$$= 0.470\, rev \times \left(\frac{360°}{1\, rev}\right) = 169.3°$$

$$\beta_3 = \omega_{cam} \times T_3 = (0.235\, rev/s)(0.75\, s)$$
$$= 0.177\, rev \times \left(\frac{360°}{1\, rev}\right) = 63.7°$$

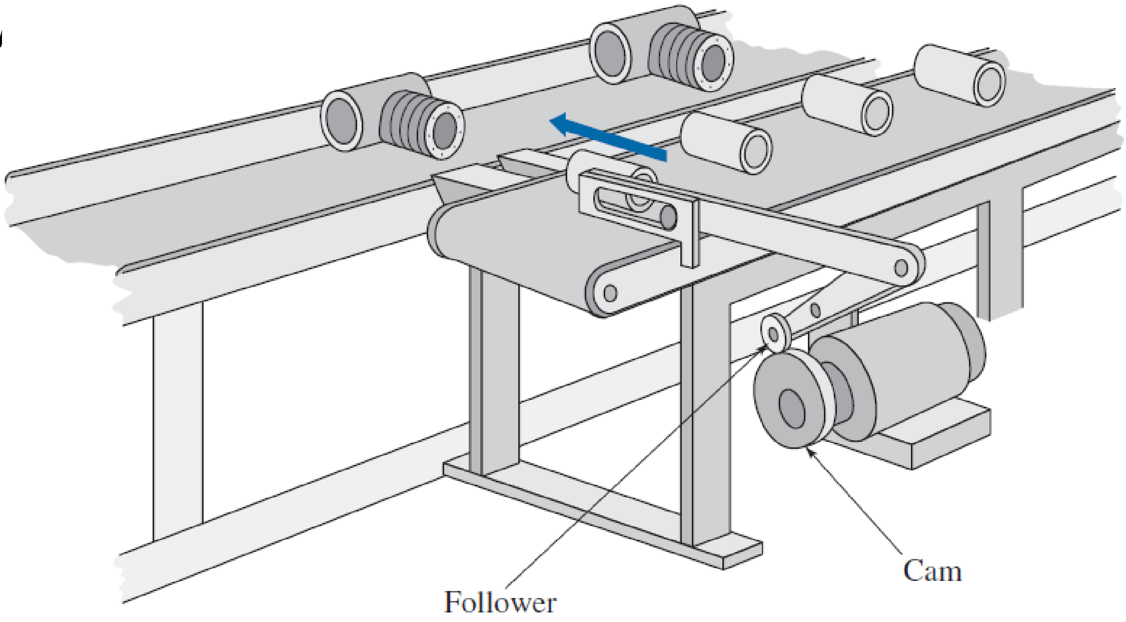



Figure (e)
Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Cam-Follower Systems - Prescribed Follower Motion

- Solution (cont.).

3. Calculate the Displacement during Each Follower Motion Interval:
   - The first motion interval has $H_1 = 50\ mm$ and $T_1 = 1.5\ s$. For a constant velocity rise, the displacement equation is:

$$\Delta R_1 = \frac{H_1 t_1}{T_1} \qquad \text{(a)}$$

   - The second motion interval has $H_2 = 50\ mm$ and $T_2 = 2.0\ s$. For a cycloidal fall, the displacement equation is:

$$\Delta R_2 = H_2 \left[1 - \left(\frac{t_2}{T_2}\right) + \frac{1}{2\pi} sin\left(\frac{2\pi t_2}{T_2}\right)\right] \qquad \text{(b)}$$

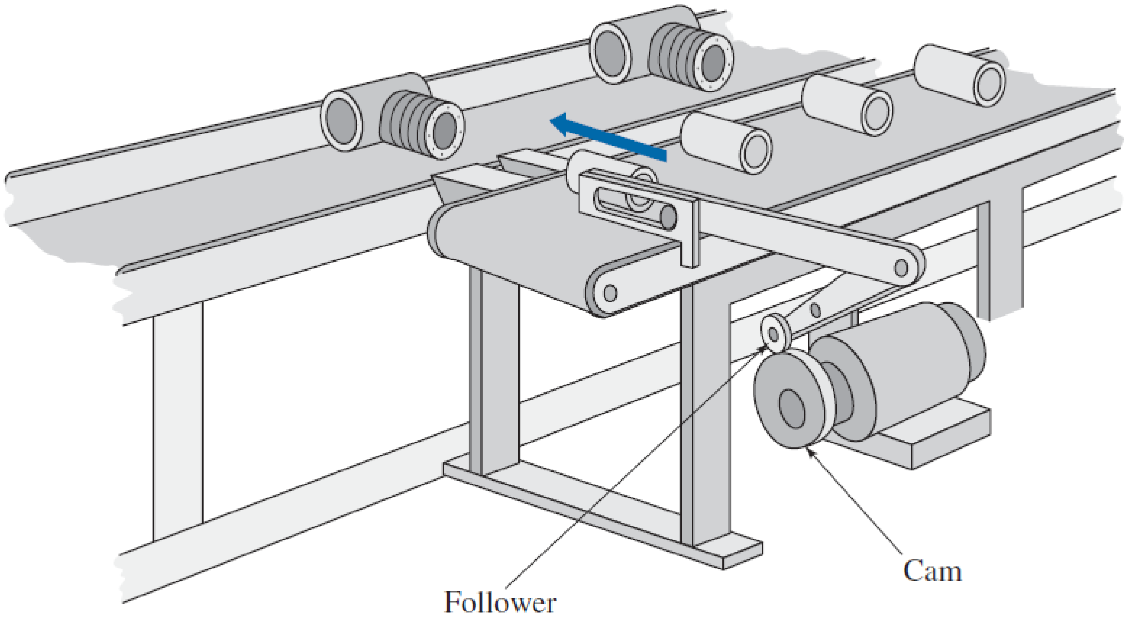



Figure (e)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Cam-Follower Systems - Prescribed Follower Motion

- Solution (cont.).

3. Calculate the Displacement during Each Follower Motion Interval:
   - And, the last motion interval is a dwell, where $\Delta R$ is constant. This dwell occurs at the retracted follower position; therefore:

$$\Delta R_3 = 0 \qquad \text{(c)}$$

- Eq. (a), (b), and (c ), are substituted into a MATLAB script as shown in Figure (f). This data is used to produce the plot in Figure (g).

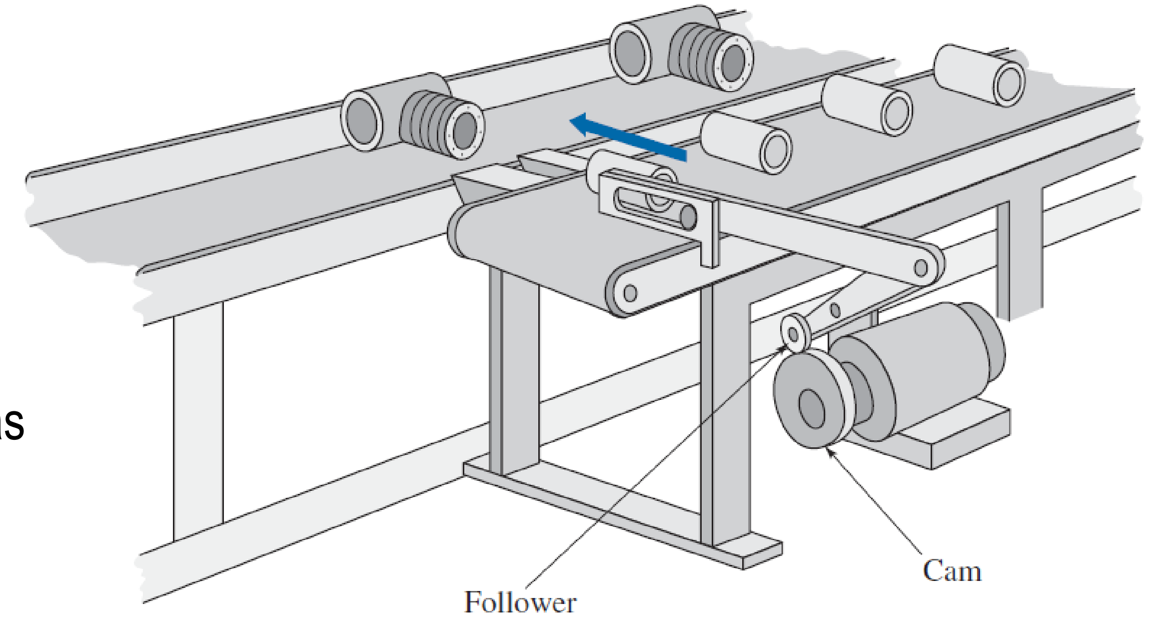



Figure (e)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# Cam-Follower Systems - Prescribed Follower Motion

- Plot the Displacement Diagram: In Part #2 of Module 7, we will construct a displacement diagram graphically (pen & paper) and with a procedure in MATLAB. For now, notice that the resulting displacement diagram with both cam angle and time displayed on the horizontal axis is shown in Figure (f). Notice that a curved displacement profile was constructed during the rise and fall sequences. Dynamic considerations dictate the actual shape of the rise and fall sections.

$\beta_1 = 110.5°$

$\beta_2 = 27.6°$

$\beta_3 = 82.9°$

$\beta_4 = 55.3°$

$\beta_5 = 82.9°$

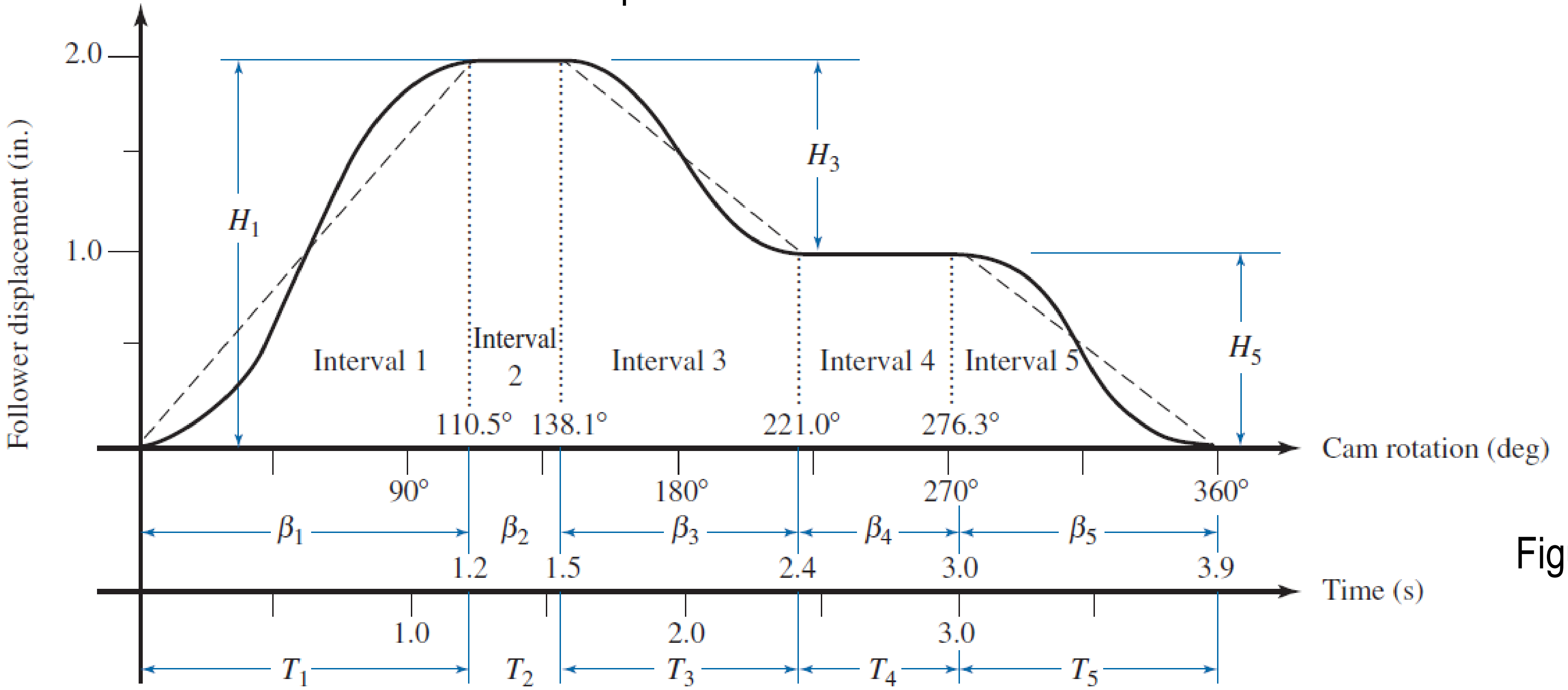



Figure (f)

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

# End of Part #1