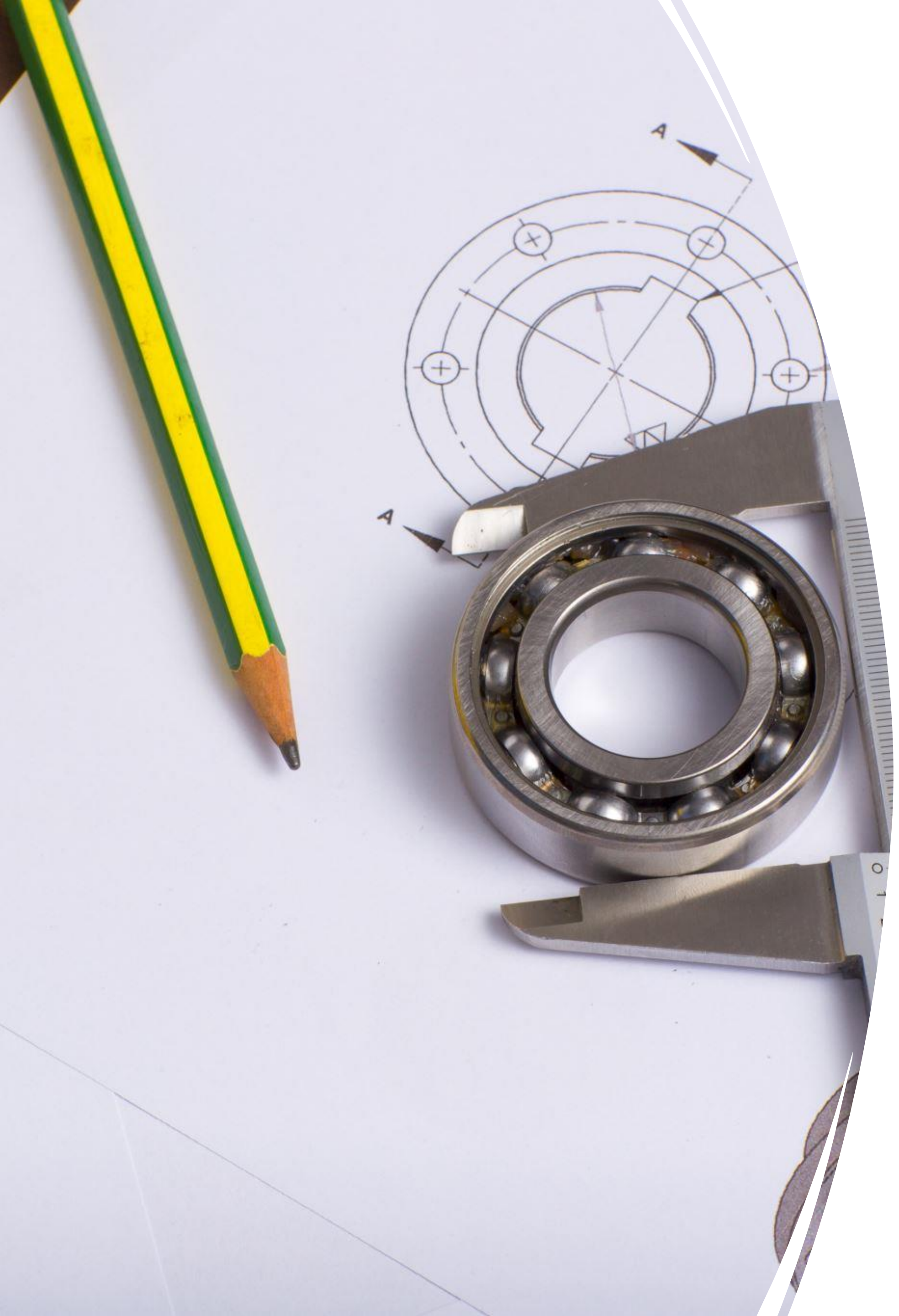
Gears and Gear Trains – Part #1

Gear Terminology. Gear Trains. Identifying the parameters of the design of transfer motion and the kinematics relations of Gear Trains.

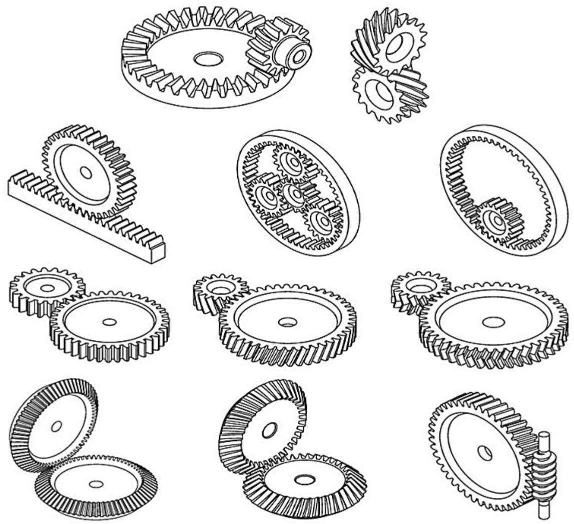


Learning Outcomes for Part #1

- Define the purpose of Geared Devices and the Types of Gears.
- Identifying the parameters of the design of transfer motion and the kinematics relations of mating gears.
- Designing gears for mating in a spur gear system.

Definition of Geared Devices

- Gears are toothed, mechanical transmission elements used to transfer motion and power between machine components.
- Geared devices can change the speed, torque, and direction of a power source. An advantage of gears is that the teeth of a gear prevent slippage.



Primary Purpose of Geared Devices

- Geared devices are employed to transform the speed and torque between the input and output shafts. Gears with different numbers of teeth can be used to achieve speed reduction (higher torque at the output) or speed increase (higher speed at the output).
- Geared devices enable the creation of compact and space-efficient mechanical systems.
- Planetary gears, for example, are known for their compact design and high-power density, making them suitable for various applications with limited space.

Driving Gear & Driven Gear Roles

- A driving gear role within a mechanical system is the component that imparts motion or force to the system, often connected to a power source such as a motor.
- On the other hand, a driven gear is the component that responds to the motion or force applied by the driving gear.
- Mesh gears change the direction of rotation of the output shaft. To avoid the reversed rotation, we use an *idle gear* between input shaft and output shaft.

Types of Gears – Spur Gears

- Spur Gears: have the teeth *parallel to the axis of rotation* as shown in Figure 8-1.
- It purposed is to transmit motion from one the shaft to another parallel shaft.
- They tend to be noisy in the meshing of the gears. For quiet operation, the spur gears are replaced by helical gears.
- Spur gears of different sizes produce a change in torque, creating a *mechanical advantage*, through their gear ratio, and thus may be considered a simple machine.

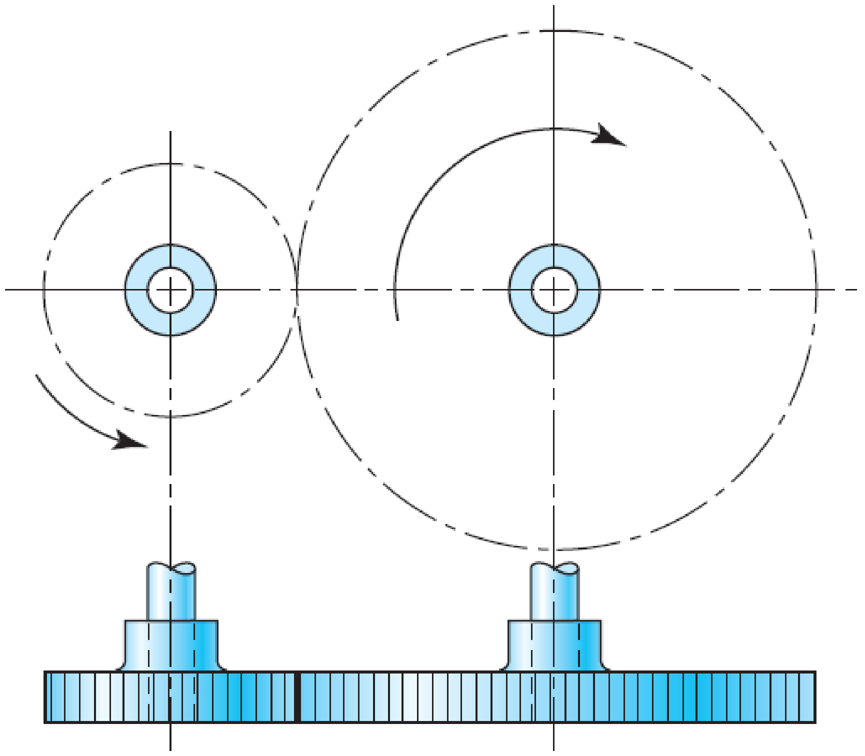


Figure 8-1 Spur Gears

Types of Gears – Helical Gears

- Helical Gears: have the teeth inclined to the axis of rotation as shown in Figure 8-2.
- Transmit motion between parallel or non-parallel shafts.
- They are not as noisy than spur gears because of the more gradual engagement of the teeth in meshing of the gears.

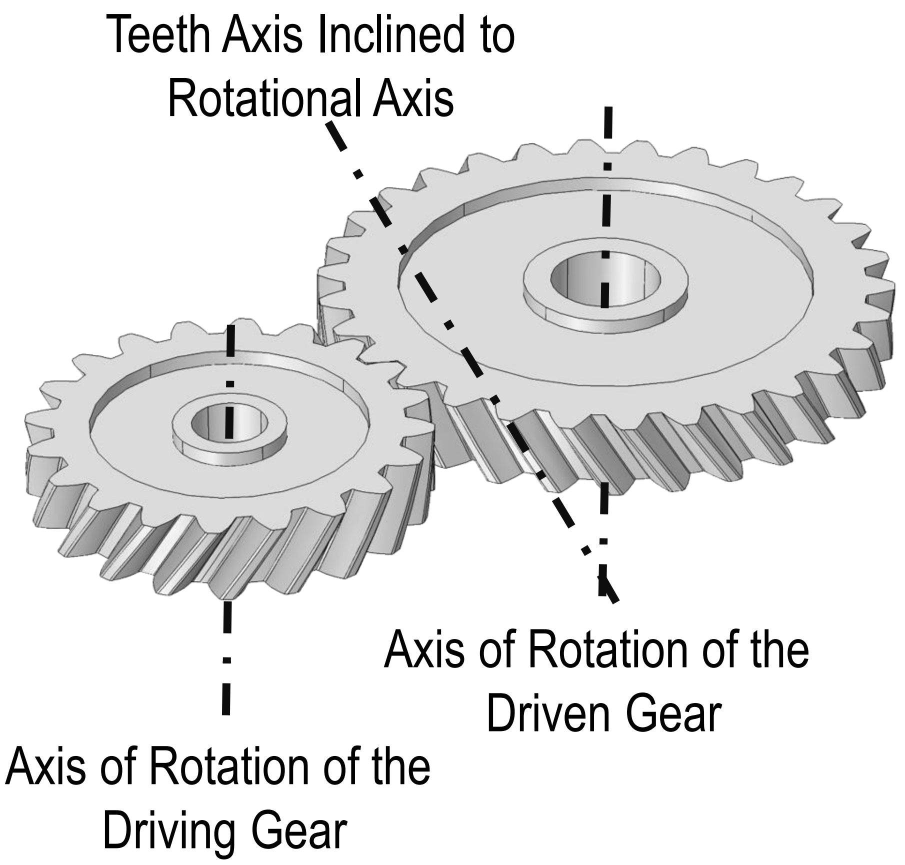


Figure 8-2 Helical Gears

Types of Gears – Bevel Gears

- Bevel Gears: have the teeth formed on conical surfaces.
- They are used to transmit motion between intersecting shafts as shown in Figure 8-3.

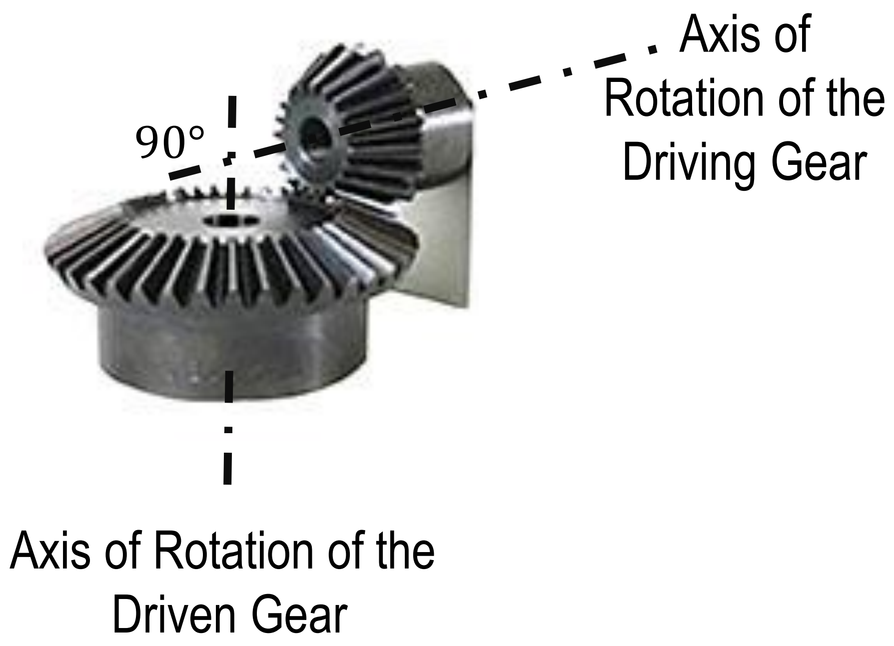


Figure 8-3 Bevel Gears

Types of Gears – Worm Gears

- Worm Gears: Worm gearsets are used to transmit rotary motion between non-parallel and non-intersecting shafts. As shown in Figure 8-4, the worm resembles a screw.

- The direction of rotation of the worm gear (aka the worm wheel) depends upon the direction of rotation of the worm and upon whether the worm teeth are cut right-hand or left-hand.

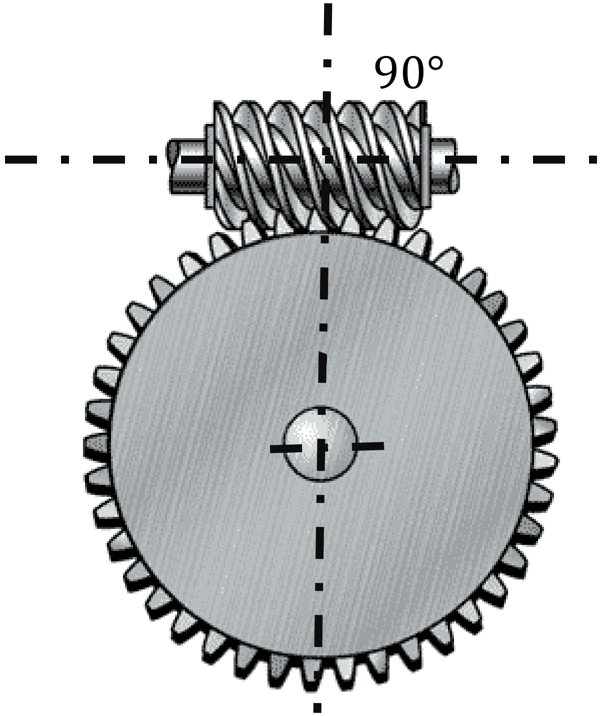


Figure 8-4 Worm Gears

Rack-and-Pinion Mechanism

- A rack-and-pinion mechanism is a type of linear motion system that converts rotational motion into linear motion or vice versa as shown in Figure 8-5:

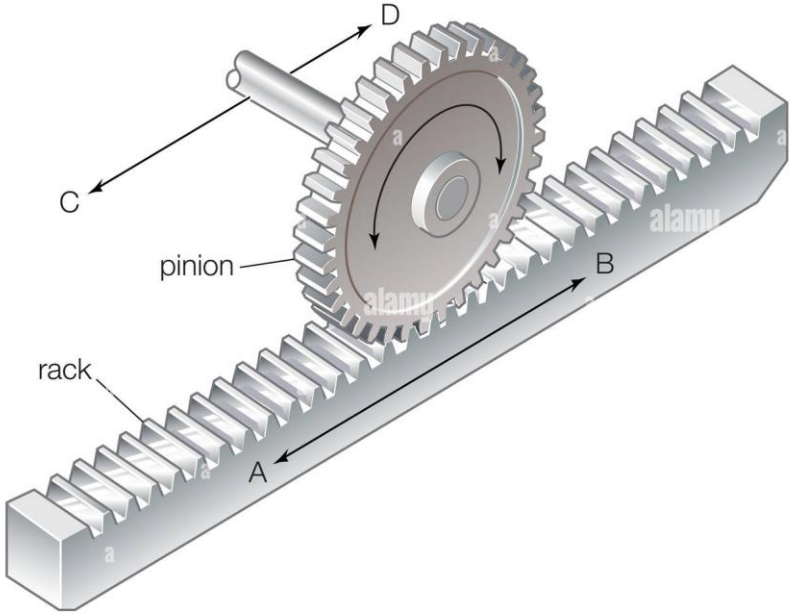


Figure 8-5. A Rack-and-Pinion Mechanism. Source: Alamy

Rack-and-Pinion Mechanism

- A rack-and-pinion mechanism consists of two primary components: a rack, which is a flat or toothed bar with linear motion, and a pinion, which is a gear attached to a rotating shaft.

- The pinion meshes with the teeth of the rack, and as the pinion rotates, it causes the rack to move linearly. In other words, the pinion acts as a revolute joint and rack acts as prismatic joint.

Designing Gears for Mating in a Spur Gear System

- When designing gears for mating in a spur gear system, several important parameters need consideration to ensure proper functioning and longevity of the gears. These are:

1. Module (Pitch Module):
    - The module represents the size of the gear teeth and is the ratio of the pitch diameter to the number of teeth. Matching modules ensure proper engagement.
2. Number of Teeth:
    - The number of teeth on each gear affects the gear ratio. Both gears in a mating pair should have compatible tooth counts for smooth operation.
3. Pitch Diameter:
    - The pitch diameter is crucial for determining the gear ratio and should be consistent between mating gears, ensuring reliable and efficient performance in various mechanical systems.

Designing Gears for Mating in a Spur Gear System

- 4. Pressure Angle:

  The pressure angle defines the shape of the gear teeth and should be the same for mating gears to ensure proper contact.

- 5. Helix Angle (if helical gears):
  - For helical gears, the helix angle determines the slant of the gear teeth. Matching helix angles are essential for proper engagement or mating.

- 6. Face Width:
  - The face width is the width of the gear teeth along the axis. Compatible face widths ensure proper load distribution.

Designing Gears for Mating in a Spur Gear System

- 7. Material and Hardness:
  - Gears should be made from materials with appropriate strength and wear resistance. Ensuring similar hardness between mating gears prevents premature wear.
- 8. Clearance and Backlash:

  Proper clearance or backlash between gears is essential to accommodate manufacturing tolerances, thermal expansion, and ensures smooth operation.
- 9. Center Distance:
  - The distance between the gear centers affects the gears' meshing and should be carefully determined for proper tooth engagement.

Designing Gears for Mating in a Spur Gear System

- 10. Alignment:
  - Proper alignment of mating gears is crucial to avoid misalignment issues that can lead to uneven wear and increased noise.
- 11. Tooth Profile (Involute):
  - Gears with involute tooth profiles ensure smooth and efficient power transmission. Matching tooth profiles are crucial for proper meshing. Nowadays, the involute is made by CAD software like PTC Creo, Solidworks, OperSCAD, etc.
- 12. Load Capacity:
  - Consider the loads the gears will experience in the application. Ensuring that both gears have adequate load capacity is crucial for durability.

Designing Gears for Mating in a Spur Gear System

- 13. Lubrication:
  - Proper lubrication is essential for reducing friction and wear. Compatibility in terms of lubrication requirements should be considered for mating gears.

- By carefully considering these parameters, you can design spur gears that effectively mate (or mesh) with each other, ensuring reliable and efficient performance in various mechanical systems.

Nomenclature of Spur Gear Teeth

- The *pitch circle* is a theoretical circle upon which all calculations are usually based; its diameter is the *pitch diameter.*

- The pitch circles of a pair of mating gears are tangent to each other. A *pinion* is the smaller of two mating gears. The larger is often called the *gear.*

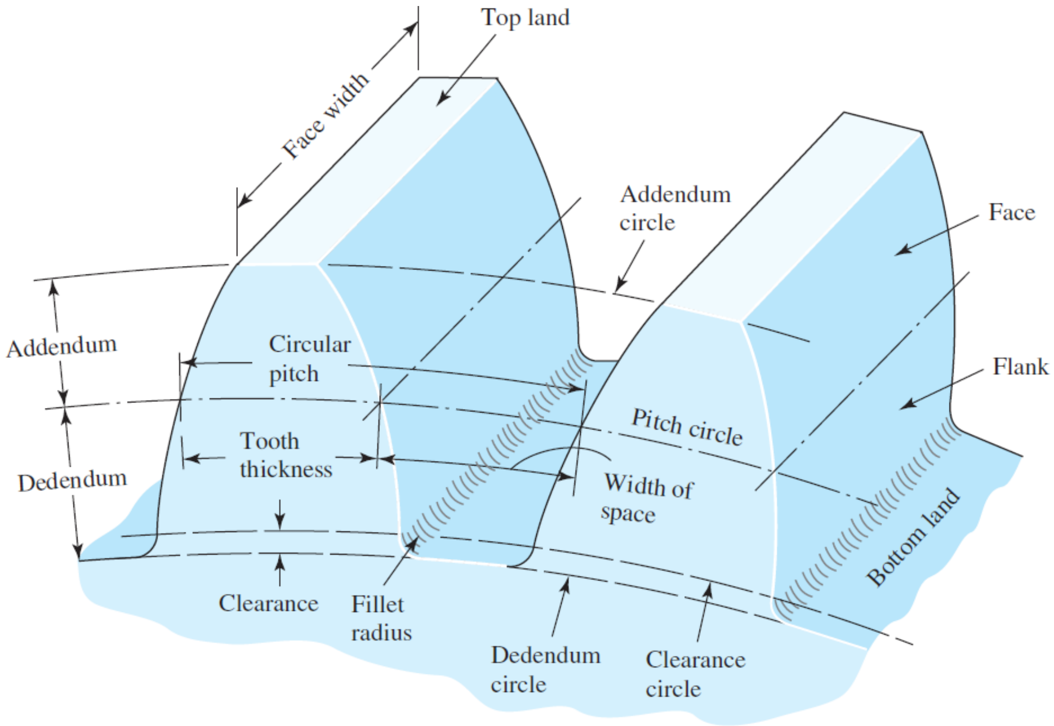


Figure 8-5 Nomenclature of Spur-Gear Teeth.
Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY.

Nomenclature of Spur Gear Teeth

- The *circular pitch (p)* is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. So, the circular pitch is equal to the sum of the *tooth thickness* and the *width of space.*

- The *module (m)* is the ratio of the pitch diameter to the number of teeth. The customary unit of length used is the millimeter. The module is the index of tooth size in SI.

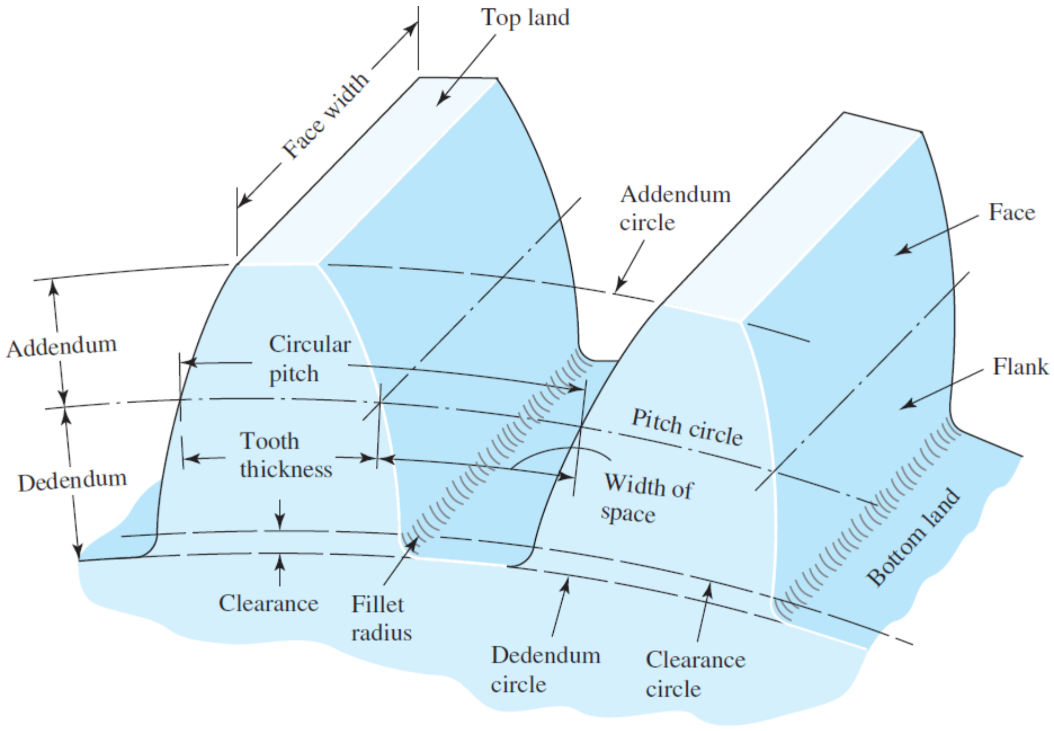


Figure 8-5 Nomenclature of Spur-Gear Teeth.
Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY.

Nomenclature of Spur Gear Teeth

- The *diametral pitch (P)* is the ratio of the number of teeth on the gear to the pitch diameter. So, it is the reciprocal of the module. Since diametral pitch is used only with U.S. units, it is expressed as teeth per inch.

- The *addendum (a)* is the radial distance between the *top land* and the pitch circle.

- The *dedendum (b)* is the radial distance from the *bottom land* to the pitch circle. The *whole depth* h_t is the sum of the addendum and the dedendum.

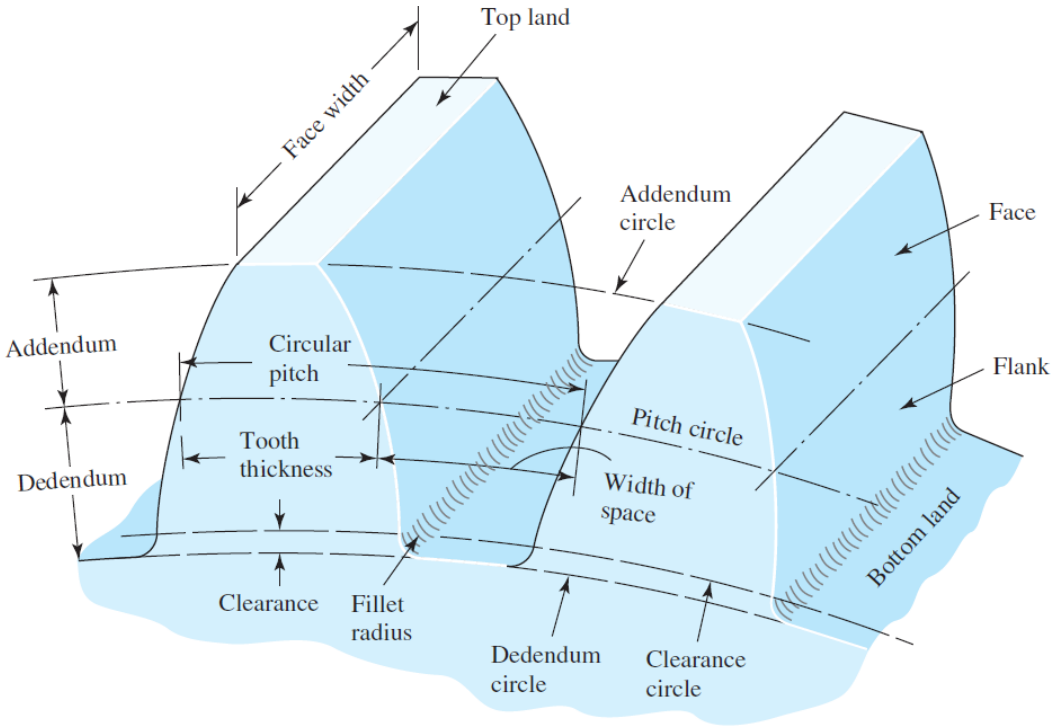


Figure 8-5 Nomenclature of Spur-Gear Teeth.
Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY.

Nomenclature of Spur Gear Teeth

- The *clearance circle* is a circle that is tangent to the addendum circle of the mating gear. The *clearance c* is the amount by which the dedendum in each gear exceeds the addendum of its mating gear as shown in Table 10-3 (next slide).

- The *backlash (B)* is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles. AGMA recommends backlash values of:

$$\frac{0.05}{P_d} < B_{\text{recommended}} < \frac{0.1}{P_d}$$

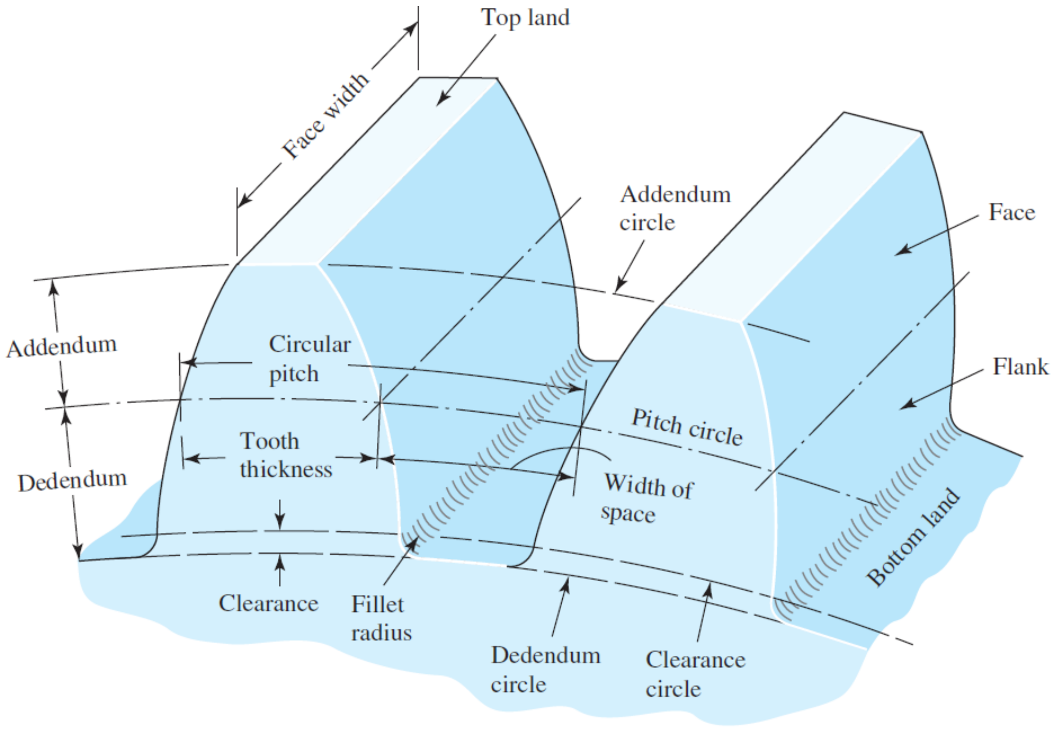


Figure 8-5 Nomenclature of Spur-Gear Teeth.
Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY.

TABLE 10.3 AGMA Full-Depth Gear Tooth Specifications

Tooth Feature Pressure angle, ϕ	Coarse Pitch ($P_d < 20$) $14\frac{1}{2}°$ or 20° or 25°	Fine Pitch ($P_d \geq 20$) 20°
Addendum, a	$\frac{1.000}{P_d}$	$\frac{1.000}{P_d}$
Dedendum, b	$\frac{1.250}{P_d}$	$0.002 + \frac{1.2}{P_d}$
Working depth, h_k	$\frac{2.000}{P_d}$	$\frac{2.000}{P_d}$
Whole depth, h_t	$\frac{2.250}{P_d}$	$0.002 + \frac{2.200}{P_d}$
Circular tooth thickness, t	$\frac{1.571}{P_d}$	$\frac{1.571}{P_d}$
Fillet radius, r_f	$\frac{0.300}{P_d}$	not standardized
Min. clearance, c	$\frac{0.250}{P_d}$	$0.002 + \frac{0.200}{P_d}$
Clearance (ground tooth), c	$\frac{0.350}{P_d}$	$0.002 + \frac{0.350}{P_d}$
Min top land width	$\frac{0.250}{P_d}$	not standardized
AGMA standard	201.02	207.04
Face width	$\frac{12}{P_d}$	$\frac{12}{P_d}$

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Formulas in Gearing:

- $P = \frac{N}{d}$ Eq. (8-1) where P := diametral pitch, in teeth per inch.
  N:= number of teeth
  d:= pitch diameter, in inches.

- $m = \frac{d}{N}$ Eq. (8-2) where m:= module, in mm.
  d:= pitch diameter, in mm.

- $p = \frac{\pi d}{N} = \pi m$ Eq. (8-3) where p:= circular pitch.

- $pP = \pi$ Eq. (8-4)

- Center distance = C = $\frac{d_1}{2} + \frac{d_2}{2}$ Eq. (8-5) where d_1 and d_2 are the pitch diameters of the pinion and gear, respectively

Design of a Spur Gear – An Example

- Suppose we wish to design a *speed reducer* such that the input speed is 1800 ${}^{rev}/_{min}$ and the output speed is 1200 ${}^{rev}/_{min}$. This is a ratio of 3:2 (that is, ${}^{1800}/_{1200}$). [The gear pitch diameters would be in the same ratio].

- For example, a 4-in pinion driving a 6-in gear. In practice, the various dimensions found in gearing are always based on the pitch circles.

- Now, suppose we specify that an 18-tooth pinion is to mesh with a 30-tooth *gear* and that the diametral pitch of the gearset is to be 2 teeth per inch.

Design of a Spur Gear - An Example

- The pitch diameters of the pinion and gear are, respectively:

$$d_1 = \frac{N_1}{P} = \frac{18}{2} = 9'' \qquad d_2 = \frac{N_2}{P} = \frac{30}{2} = 15''$$

- The center distance of the mating gears is:

$$center\ distance\ (O_1 - O_2) = \frac{d_1}{2} + \frac{d_2}{2} = 12''$$

- With this information, construct the pitch circles of radii r_1 and r_2 as shown in Figure 8-10. [The circles are tangent at P, the pitch point].

Important: Identify the mating gears as driver (≡pinion) and driven (≡ gear)

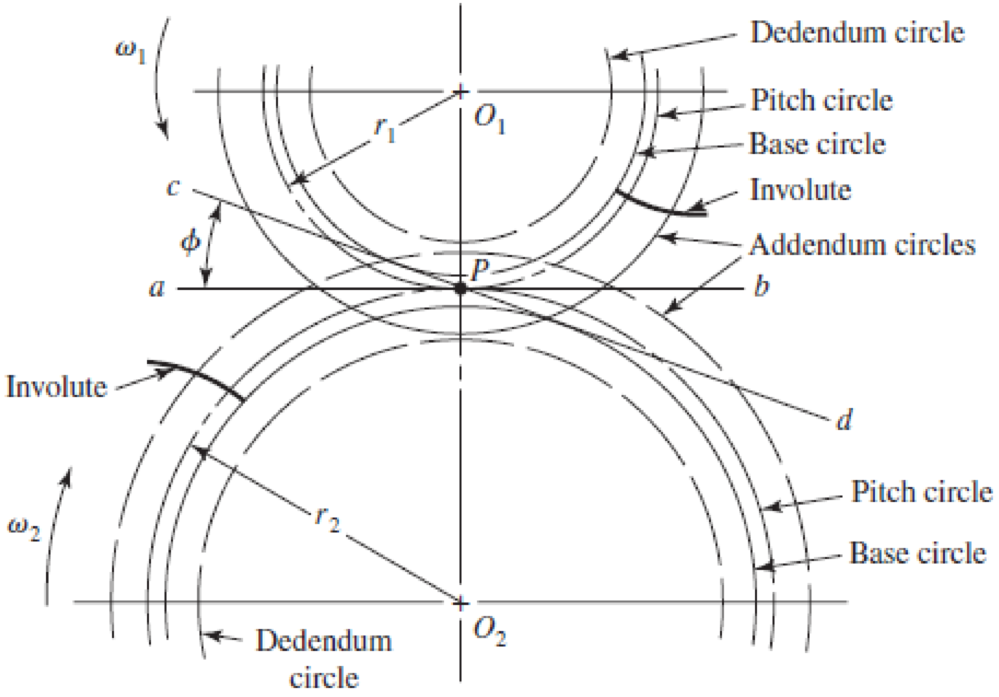


Figure 8-10 Circles of a Gear Layout.

Design of a Spur Gear - An Example

- When two gears are in mesh, their pitch circles roll on one another without slipping.
- We designate the pitch radii as r_1 and r_2 and the angular velocities as ω_1 and ω_2, respectively.
- Then, the pitch-line velocity is:

$$V = |r_1\omega_1| = |r_2\omega_2| \quad \rightarrow \left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} \qquad \text{Eq. (8-5)}$$

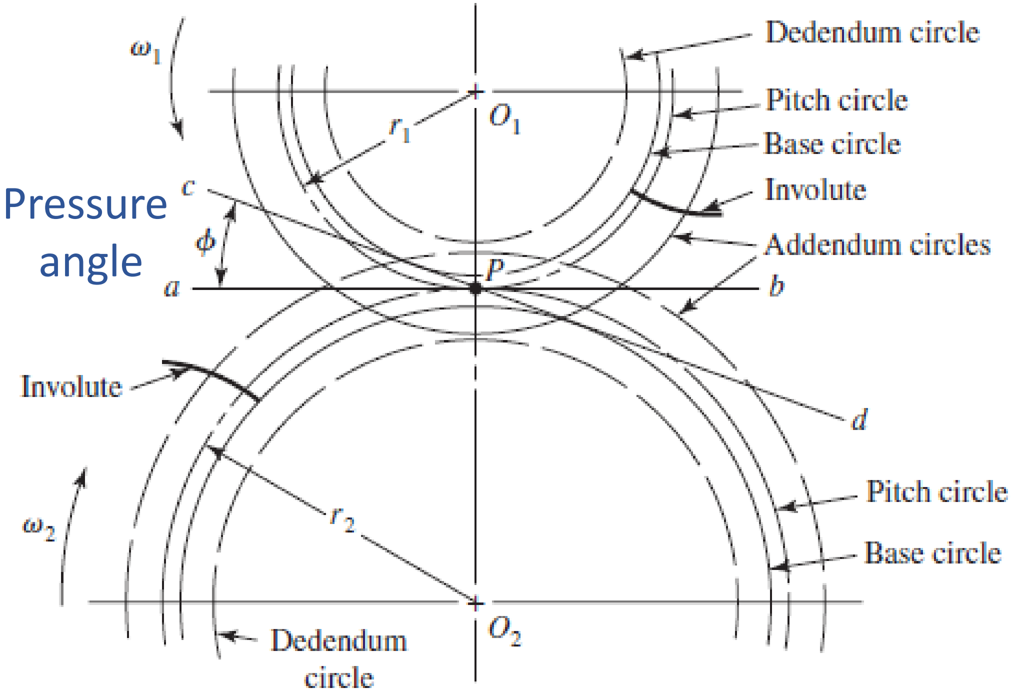


Figure 8-10 Circles of a Gear Layout (repeated).

Design of a Spur Gear - An Example

- Next, draw line ab. [This line is the common tangent through P].

- Identifying gear 1 as the driver, which rotating counterclockwise (CCW), we draw a line cd through point *P* at an angle ϕ to the common tangent ab.

- The line cd has three names, all of which are in general use. It is called the *pressure line,* the *generating line,* and the *line of action*. It represents the direction in which the resultant force acts between the gears.

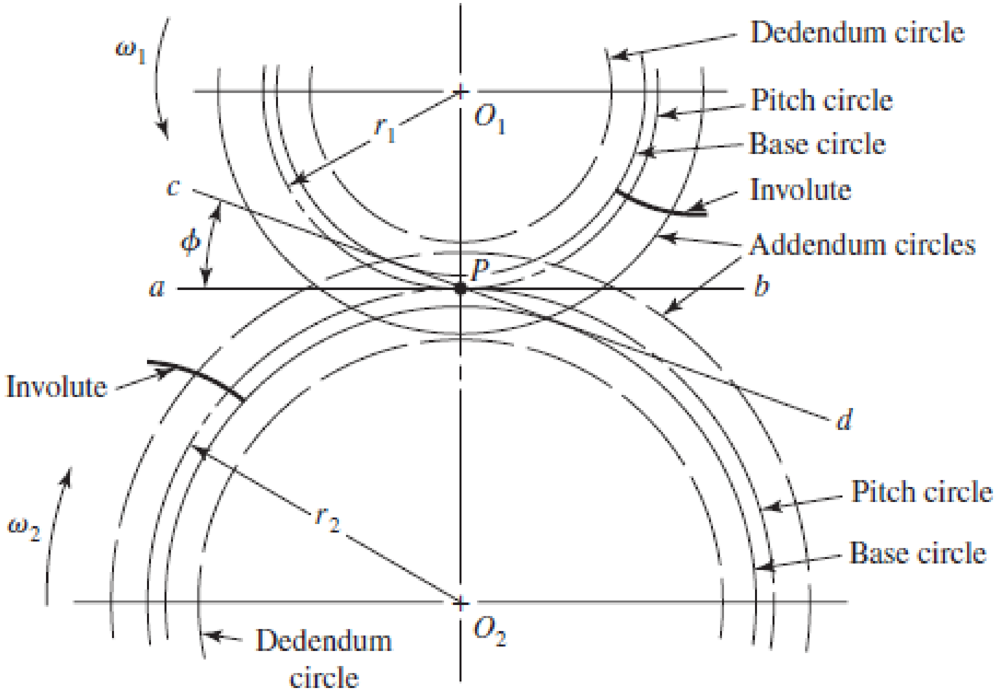


Figure 1-10 Circles of a Gear Layout (repeated).

Design of a Spur Gear - An Example

- The angle ϕ is called the *pressure angle,* and it usually has values in the range of 20° (or 25°) though 14.5°.

- Next, on each gear, draw a circle tangent to the pressure line. These circles are the *base circles* as illustrated in Figure 8-11.

- As shown in Fig. 8-11, the radius of the base circle is:

$$r_b = r\cos(\phi) \qquad \text{Eq. (8-6)}$$

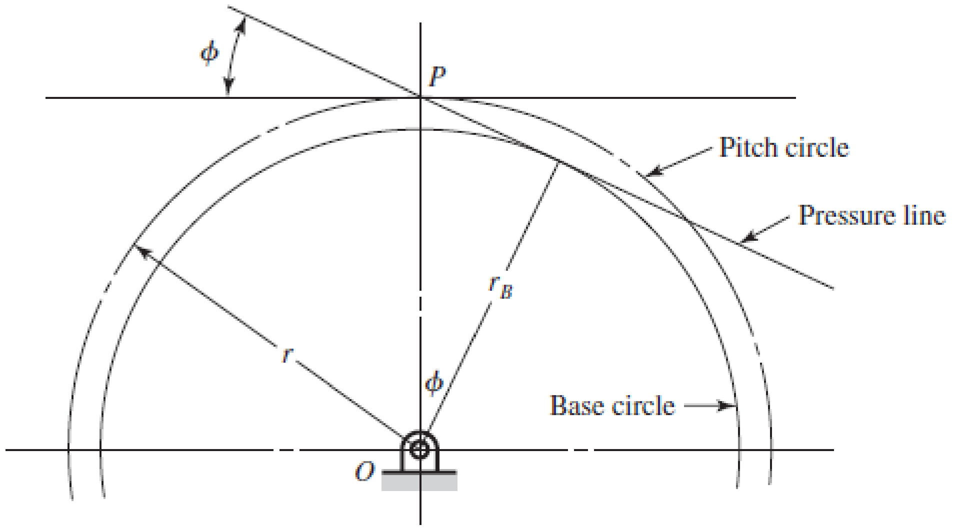


Figure 8-11 Pressure Angle ϕ and the Pitch Circle Radius.

Design of a Spur Gear - An Example

- The addendum a and dedendum b distances for standard interchangeable teeth are $^1/_P$ and $^{1.25}/_P$, respectively:

$$a = \frac{1}{P} = \frac{1}{2} = 0.5''$$

$$b = \frac{1.25}{P} = \frac{1.25}{2} = 0.625''$$

- Using these distances, draw the addendum and dedendum circles on the pinion and on the gear.

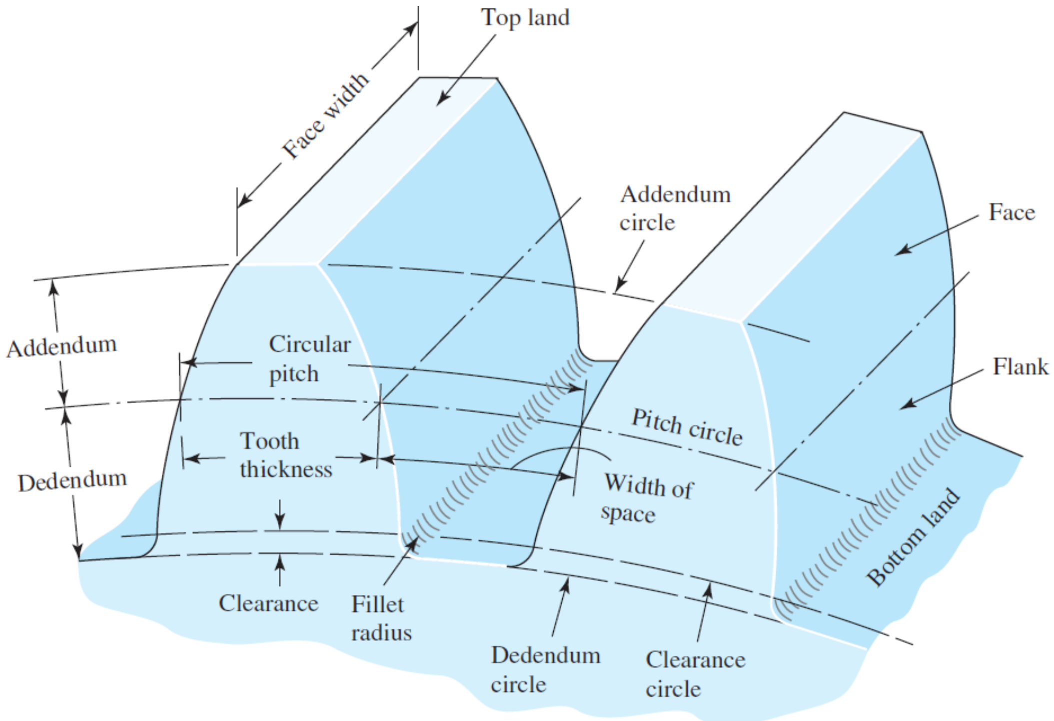


Figure 8-5 Nomenclature of Spur-Gear Teeth.
Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY.

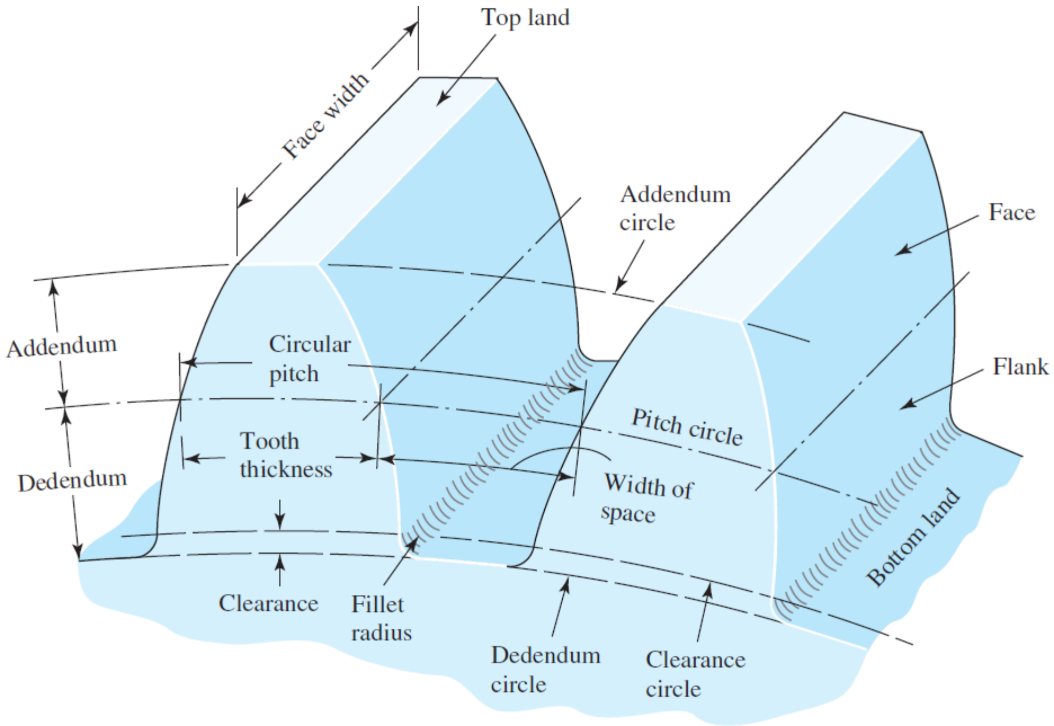
Design of a Spur Gear - An Example

- To draw a tooth, we must know the tooth thickness. So, from Eq. (8-4), the circular pitch is:

$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.57''$$

- Then, tooth thickness is:

$$\rightarrow t = \frac{p}{2} = \frac{1.57}{2} = 0.785''$$



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Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY.

Design of a Spur Gear - An Example

- Next, we generate an involute on each base circle as previously described and as shown in Figure 8-10.   This involute is to be used for one side of a gear tooth.

- In the past, before CAD software, we would sketch a template curve representing the involute profile. A reproduction of the template used is shown in Figure 8-12. Today, we use PTC Creo®, Solidworks®, or OpenSCAD).

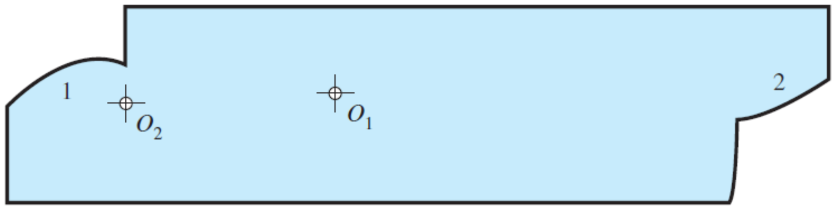

Figure 8-12 A Template for Drawing Gear Teeth. Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY

Tooth Profile of an Involute Gear - Requirements

- Requirements for tooth profile must be satisfactorily met:

1. There is no involute interference.
2. There is no fillet interference.
3. That there is ample overlap of tooth action.
4. That a suitable pressure angle has been selected.
5. That excessive slippage is avoided.

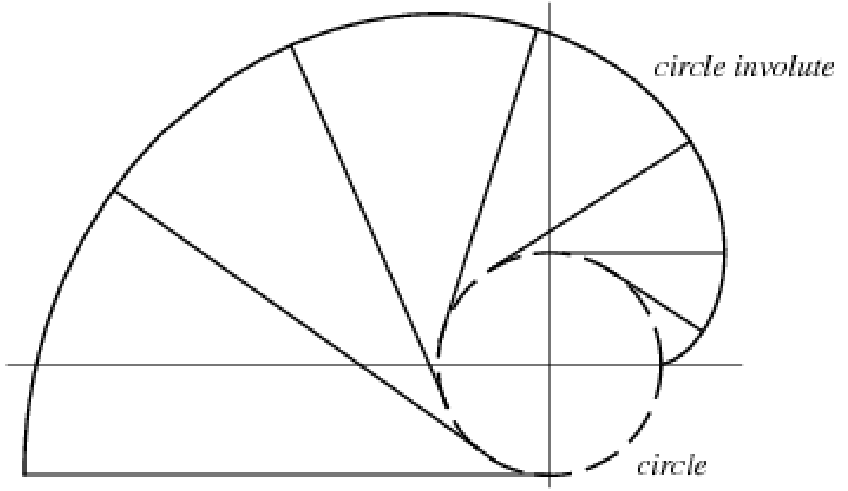

Figure 8-8 Involute of a Circle

Involute Profile

- The instant center of velocity between the two bodies remains *stationary* between the grounded instant centers as shown in Figure 8-9:

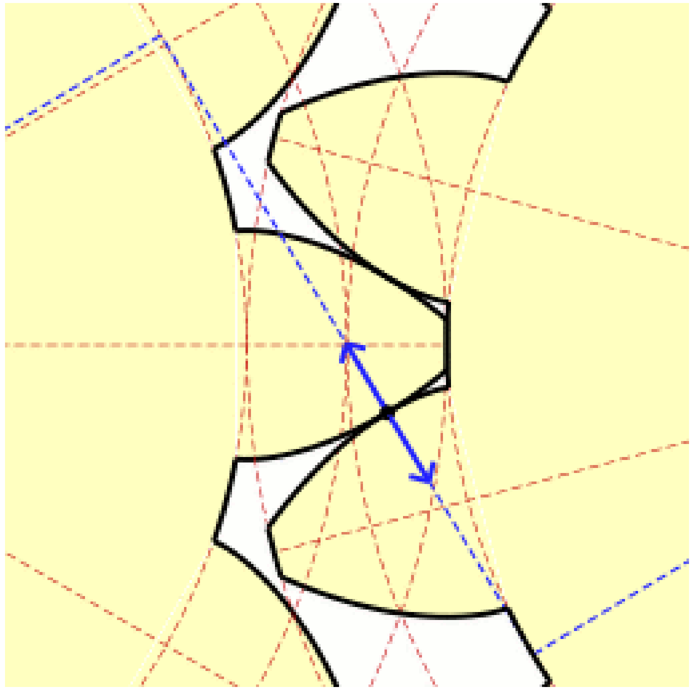

Figure 8-9 Involute Profile Animation.

Design of a Spur Gear - An Example

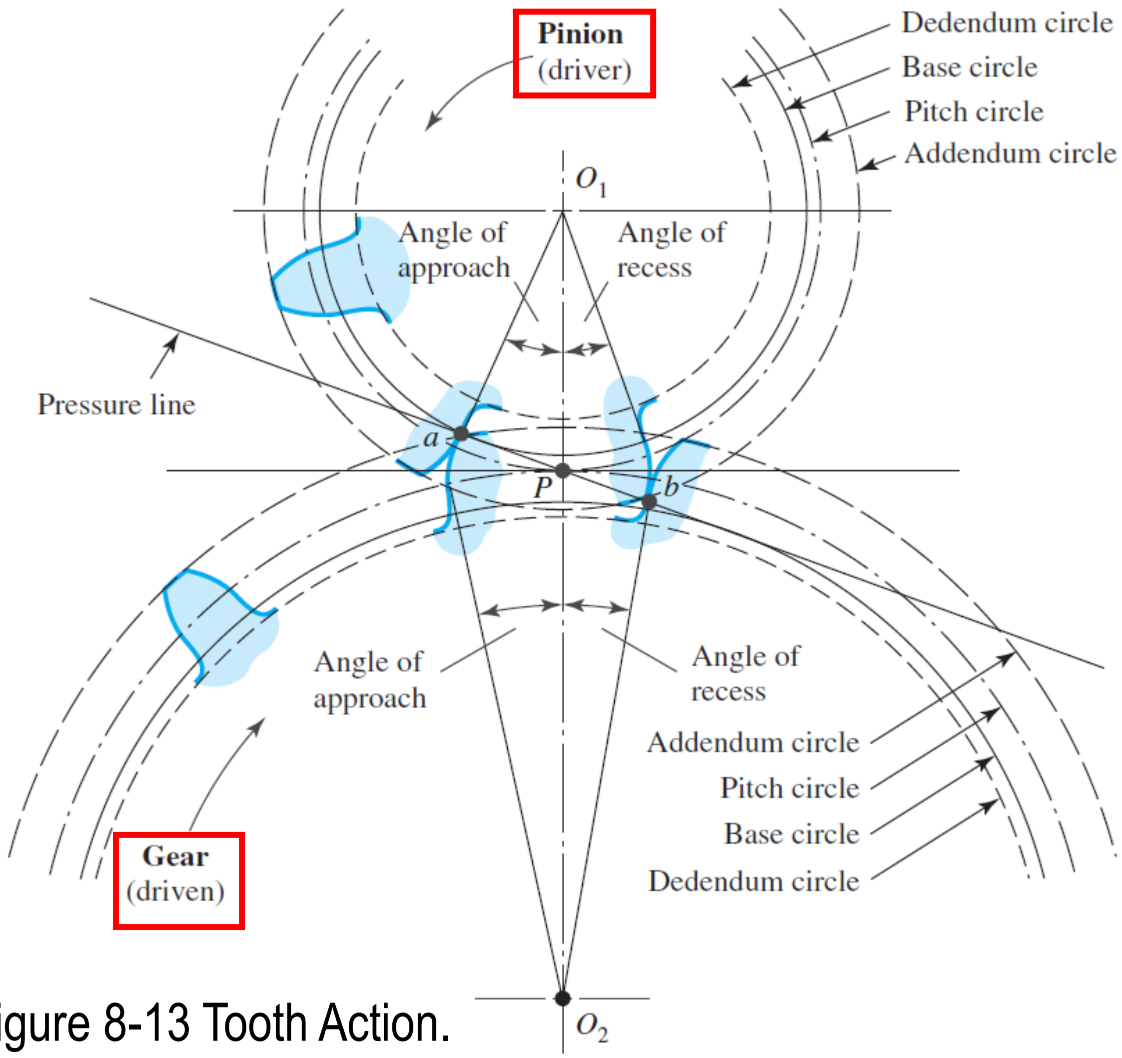

Figure 8-13 Tooth Action.

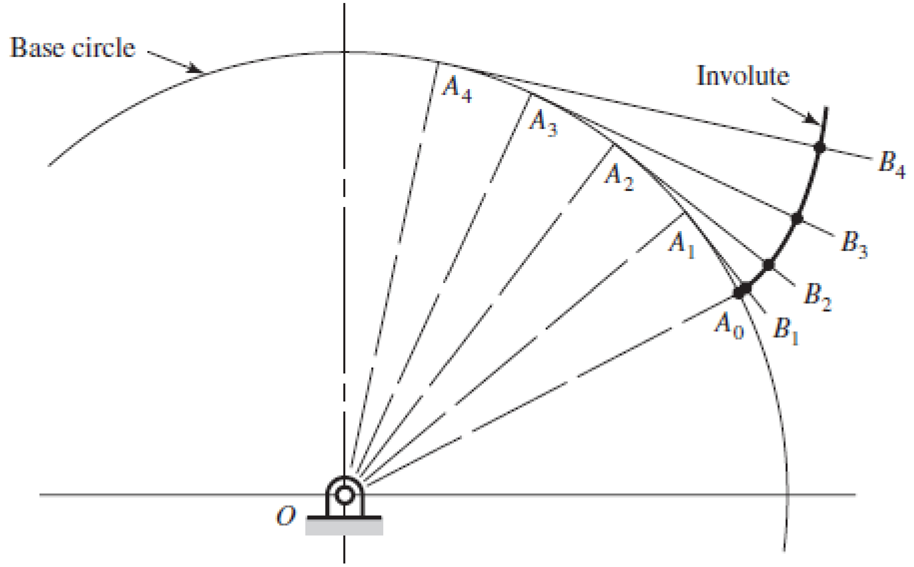

Figure 8-14 Design of an Involute Profile.

Figure 8-5 Nomenclature of Spur-Gear Teeth.

Source: Budynas, R.G. & Nisbett, J. K. (2015). Shigley's Mechanical Engineering Design. McGraw-Hill Education. 10th Edition. Penn Plaza, New York. NY.

3D Printed Design of a Plastic Spur Gear

- Student Activity: Design a basic involute profile of a spur gear in a CAD software (in SolidWorks, Creo, or Autodesk Fusion 360), draw the spur gear model and 3D print it. The parameters related to the design of a spur gears are:

1. Pitch diameter, p
2. Diametral pitch, P
3. Number of teeth, N
4. Module or size ratio (= pitch diameter/number of teeth), m
5. Pressure angle, ϕ
6. What is the backslash?
7. Root fillet thickness
8. What is the gear thickness?
9. What is the hole diameter?

Note: Watch the video at this link for the involute profile gear in SolidWorks: https://www.youtube.com/watch?v=TJhbSSSUYFw.

Gear Trains

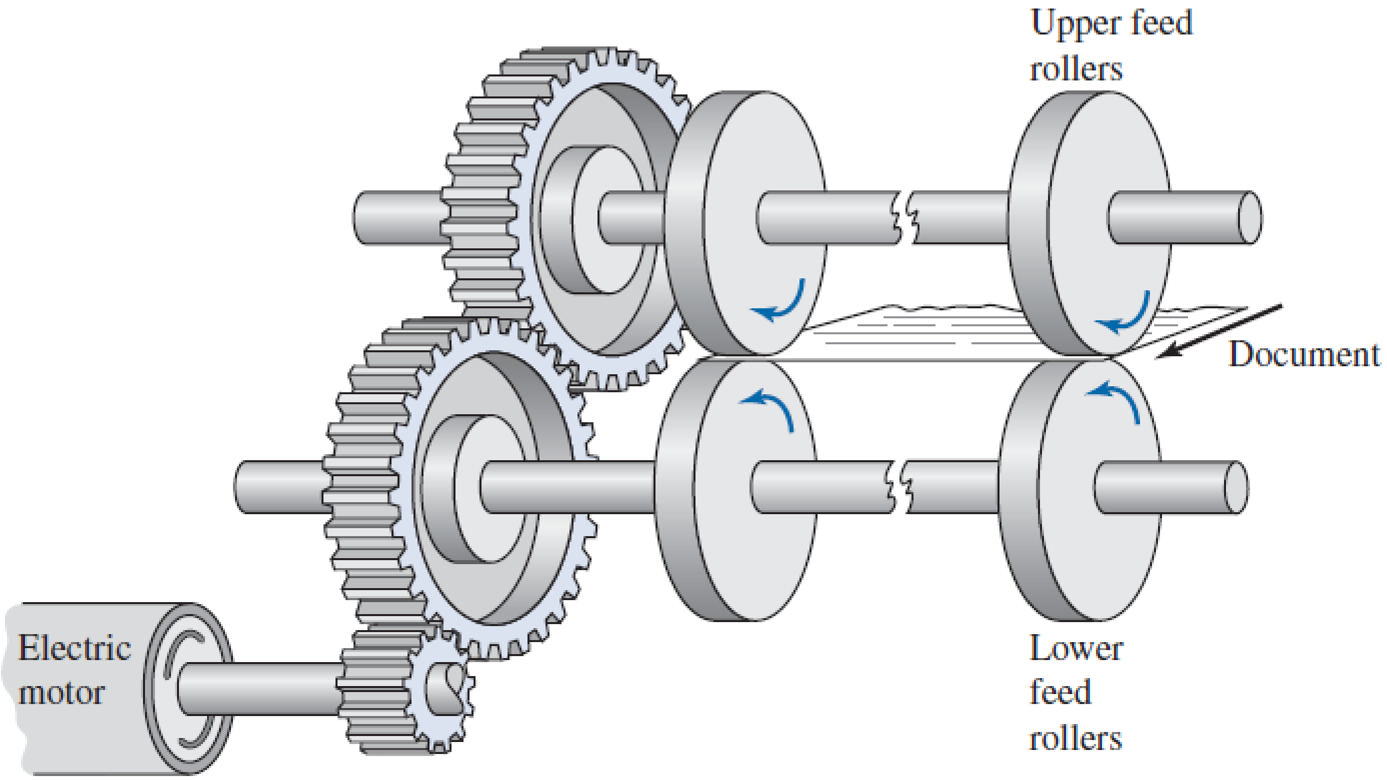

Figure 8-15 Gear Trains.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Gear Trains

- Combining Eq. (8-1) and Eq. (8-5), the velocity ratio, VR, is defined as:

$$VR = \frac{N_{gear}}{N_{pinion}} \qquad \text{Eq. (8-7)}$$

- The train value (TV) is defined as the ratio of the input velocity divided by the output velocity from the train. The formula to calculate the train value (TV) is:

$$TV = \frac{\omega_{in}}{\omega_{out}} = (VR_1)(VR_2)(VR_3)\cdots \qquad \text{Eq. (8-8)}$$

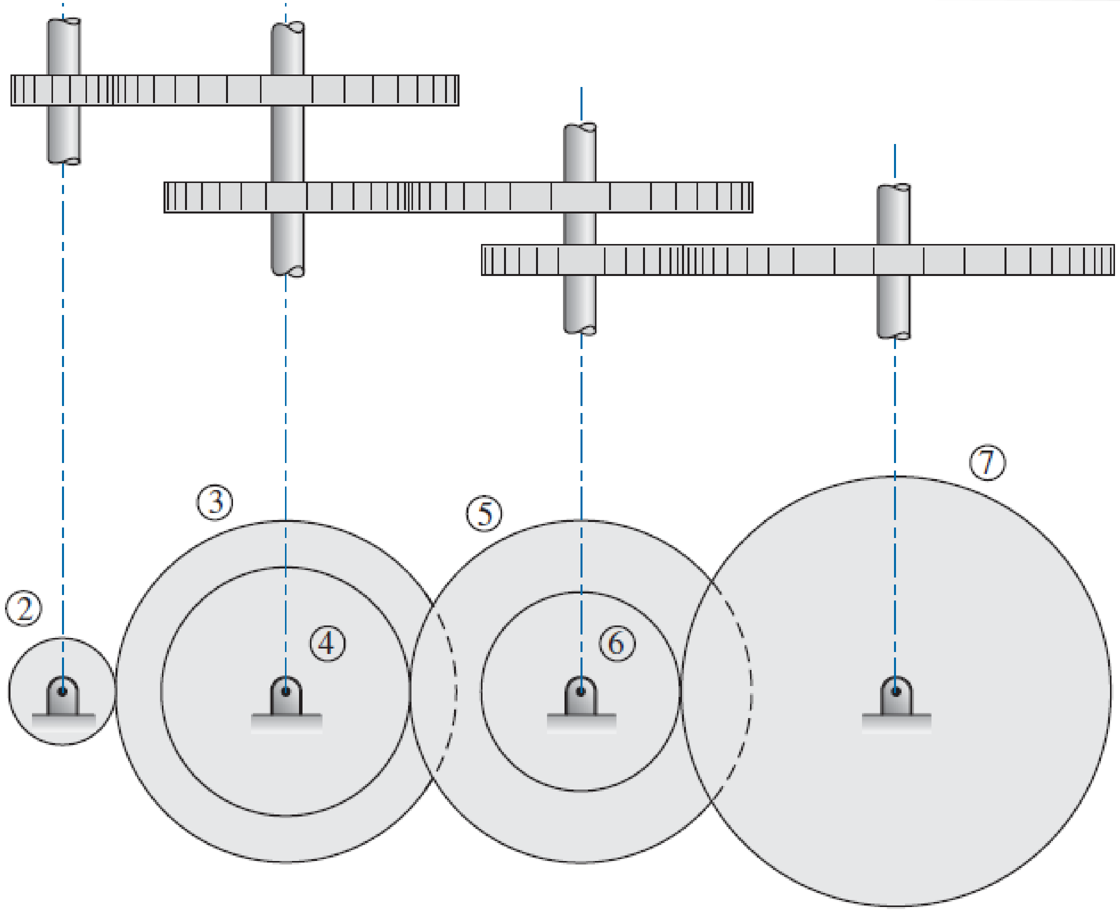

Figure 8-16 Gear Train.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Gear Trains

- For example, for gear train shown in Figure 8-16, it is computed in terms of the velocity ratio (VR) as:

$$TV = (VR_{2-3})(VR_{4-5})(VR_{6-7})$$

- The algebraic sign resulting from the multiplication of individual velocity ratios (VRs) determines the relative rotational direction of input and output shafts.

- ***Note:*** The positive values reveal that the input and output shafts rotate in the same direction, and negative values indicate opposite rotation.

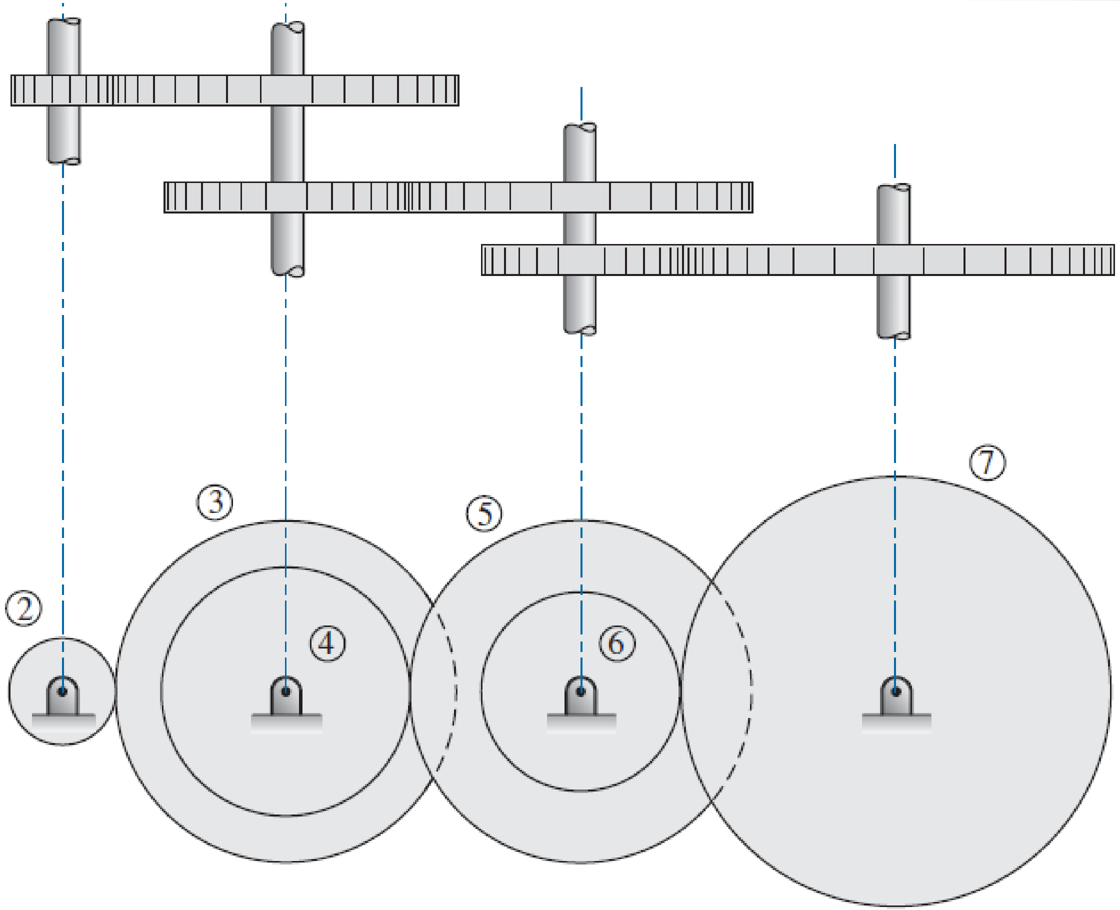

Figure 8-16 Gear Train.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Gear Trains – An Example

- Example 8-1. A gear train is shown in Figure 8-16 in previous slide. Determine the rotational velocity of gear 7 as gear 2 drives at 1800 rpm CCW. Also determine the distance between the shafts that carry gears 2 and 7.

- The gears have the following properties:

  - Gear 2: $N_2 = 12\ teeth$ and $P_d = 12$
  - Gear 3: $d_3 = 2.5\ in$
  - Gear 4: $N_4 = 15\ teeth$
  - Gear 5: $d_5 = 3.0\ in$ and $P_d = 10$
  - Gear 6: $d_6 = 1.5\ in$ and $P_d = 8$
  - Gear 7: $N_7 = 32\ teeth$

Gear Trains – An Example

- Solution:

1. Calculate Consistent Gear Dimensions:

$$d_2 = \frac{N_2}{P_d} = \frac{12}{12} = 1.0\ in$$

Gear 4 mates with gear 5 and must have an identical diametral pitch.

$$d_4 = \frac{N_4}{P_d} = \frac{15}{10} = 1.5\ in$$

And Gear 7 mates with gear 6 and must have an identical diametral pitch.

$$d_7 = \frac{N_7}{P_d} = \frac{32}{8} = 4.0\ in$$

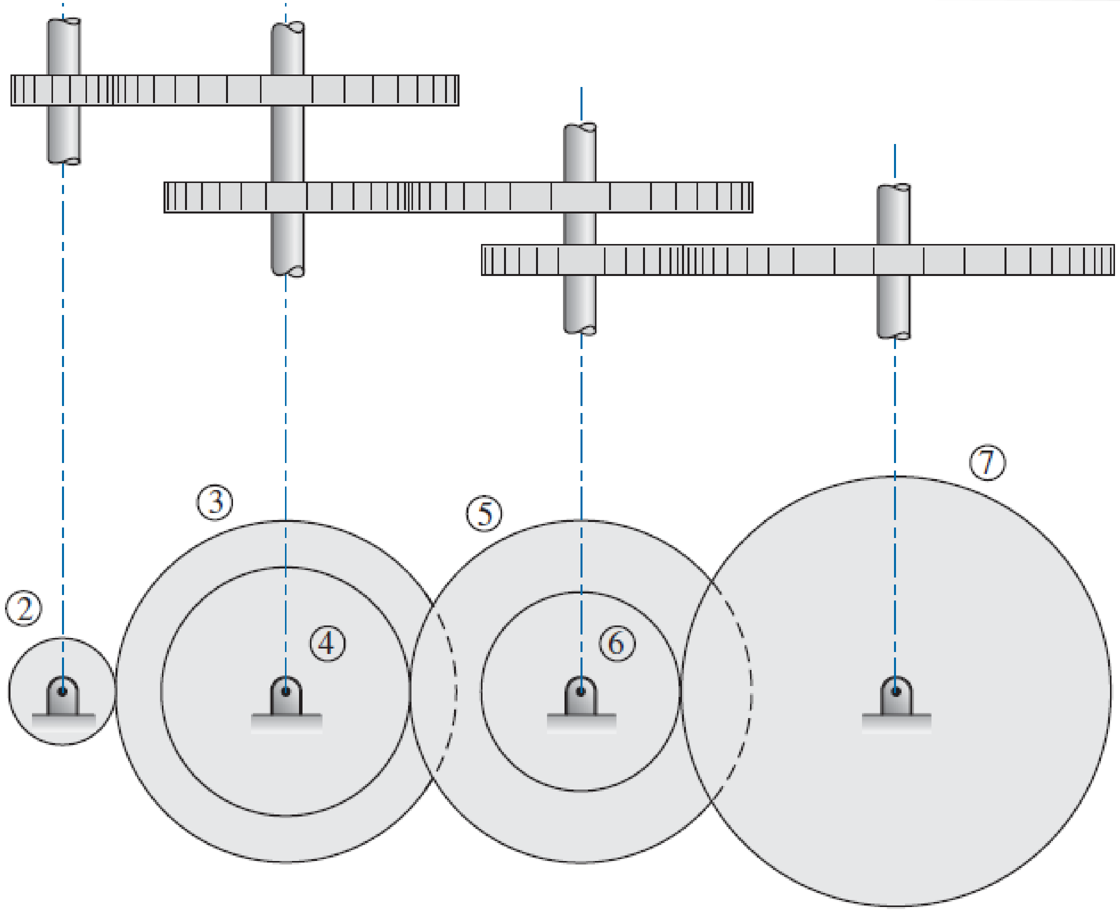

Figure 8-16 Gear Train.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Gear Trains – An Example

- Solution (cont.):

2. Calculate Velocities and Ratios:

   The train value can then be computed as:

$$TV = (VR_{2-3})(VR_{4-5})(VR_{6-7})$$

$$= \left(-\frac{d_3}{d_2}\right)\left(-\frac{d_5}{d_4}\right)\left(-\frac{d_7}{d_6}\right)$$

$$= \left(-\frac{2.5\ in}{1\ in}\right)\left(-\frac{3\ in}{1.5\ in}\right)\left(-\frac{4\ in}{1.5\ in}\right) = -13.33$$

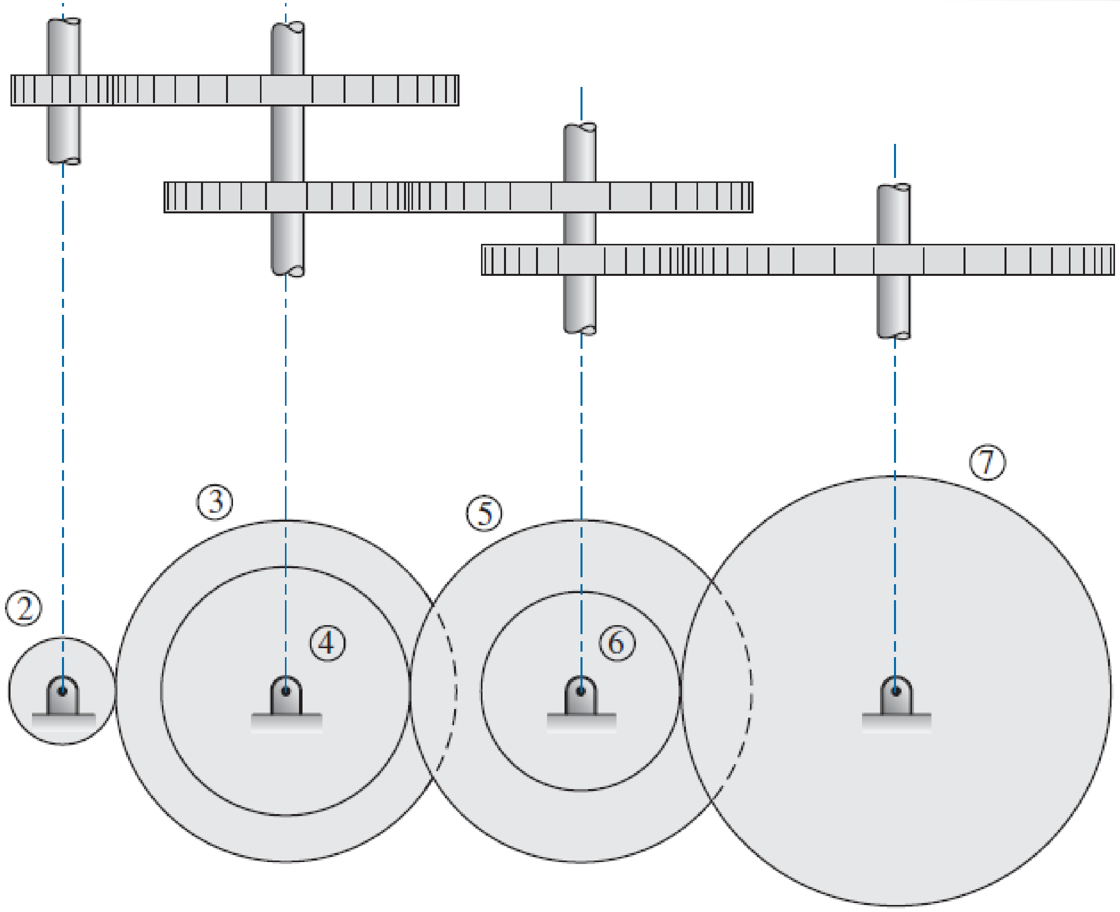

Figure 8-16 Gear Train.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Gear Trains – An Example

- Solution (cont.):

2. Calculate Velocities and Ratios (cont.):

The speed of gear 7 can be determined through this train value.

$$TV = \frac{\omega_2}{\omega_7}$$

$$\rightarrow \omega_7 = \frac{\omega_2}{TV} = \frac{1800\ rpm}{-13.33}$$

$$\therefore \omega_7 = -135\ rpm$$

or $135\ rpm, CW$

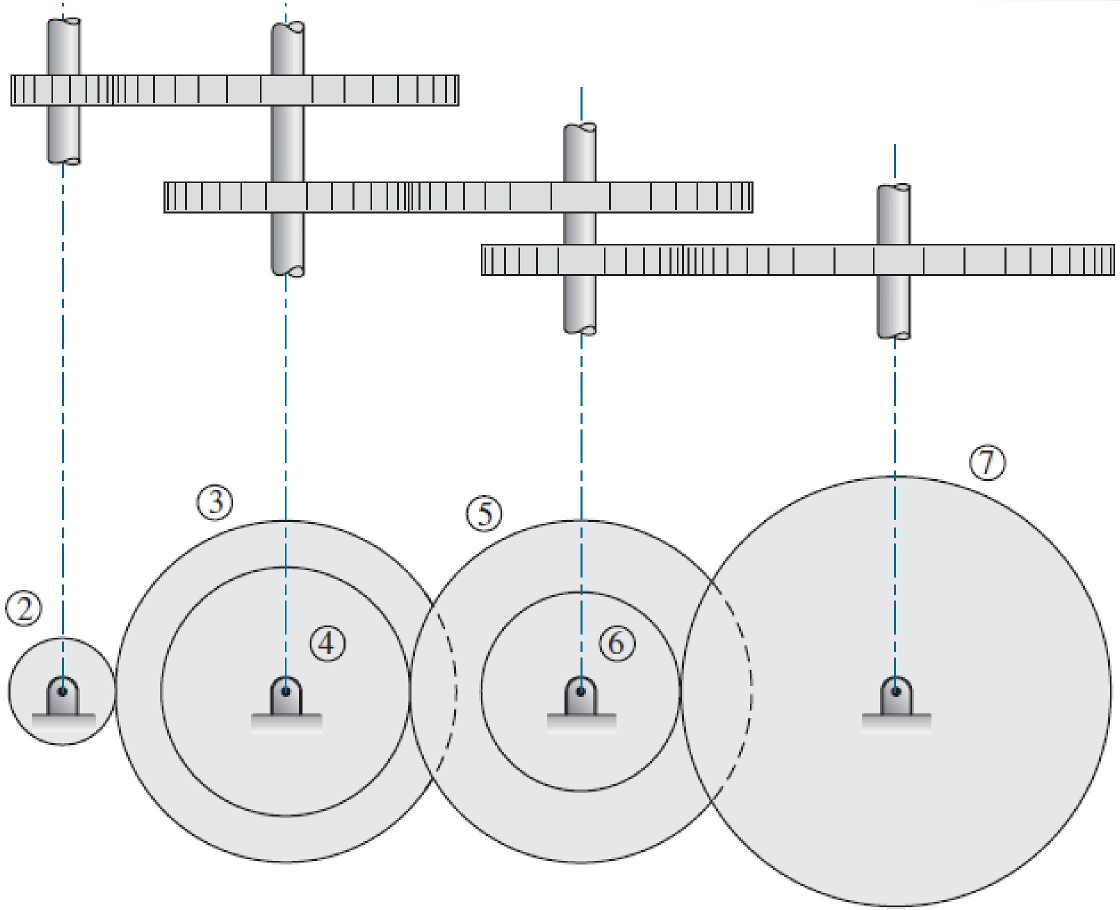

Figure 8-16 Gear Train.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

Gear Trains – An Example

- Solution (cont.):

3. Determine the distance between the shafts:

   As shown in Figure 8-16, the center distance between gears 2 and 7 can be determined by stacking the pitch radii from all gears between 2 and 7; that is:

$$C = r_2 + r_3 + r_4 + r_5 + r_6 + r_7 =$$

$$\left(\frac{1\ in}{2}\right) + \left(\frac{2.5\ in}{2}\right) + \left(\frac{1.5\ in}{2}\right) + \left(\frac{3\ in}{2}\right) + \left(\frac{1.5\ in}{2}\right) + \left(\frac{4\ in}{2}\right)$$

$$= 6.75\ \text{in.}$$

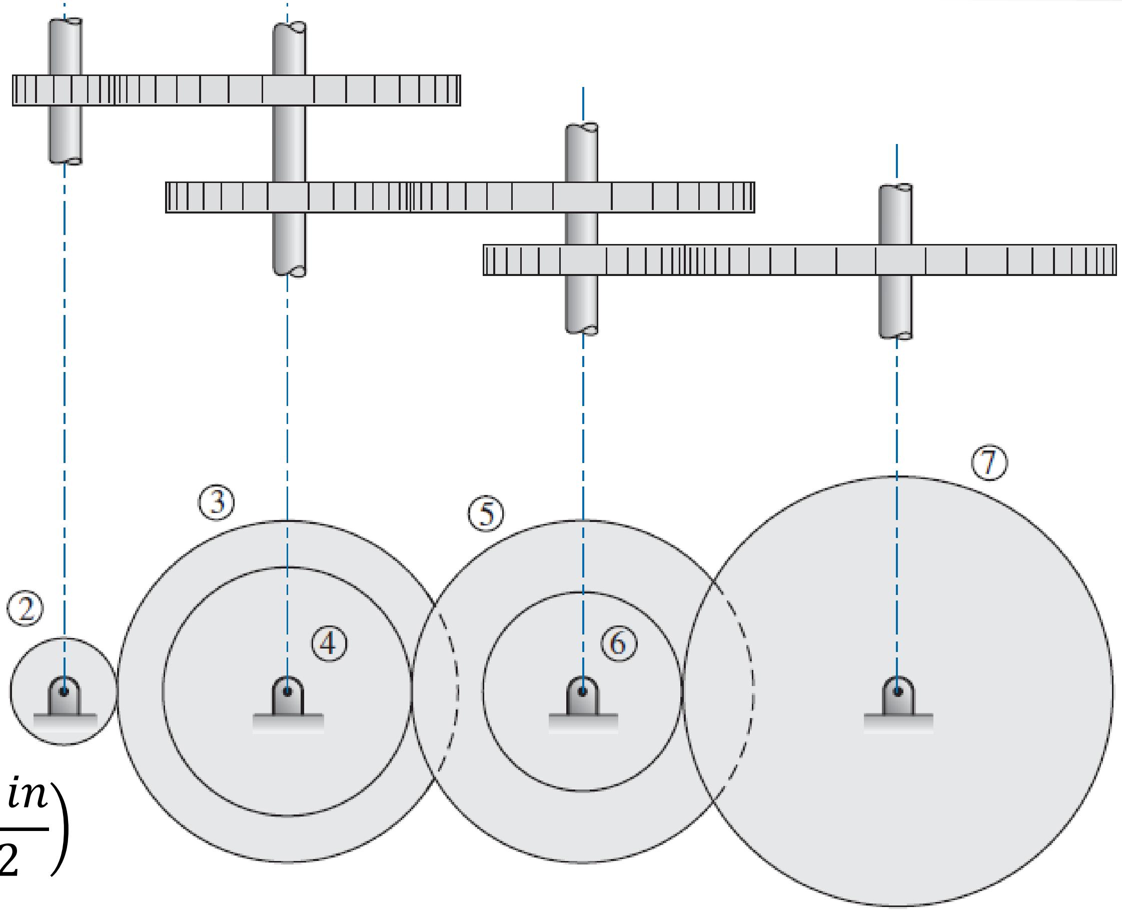

Figure 8-16 Gear Train.

Source: Myszka, D.H. Machines and Mech. Applied Kinematic Analysis. Prentice Hall. 4th Edition. USA.

End of Gears and Gear Trains – Part #1.