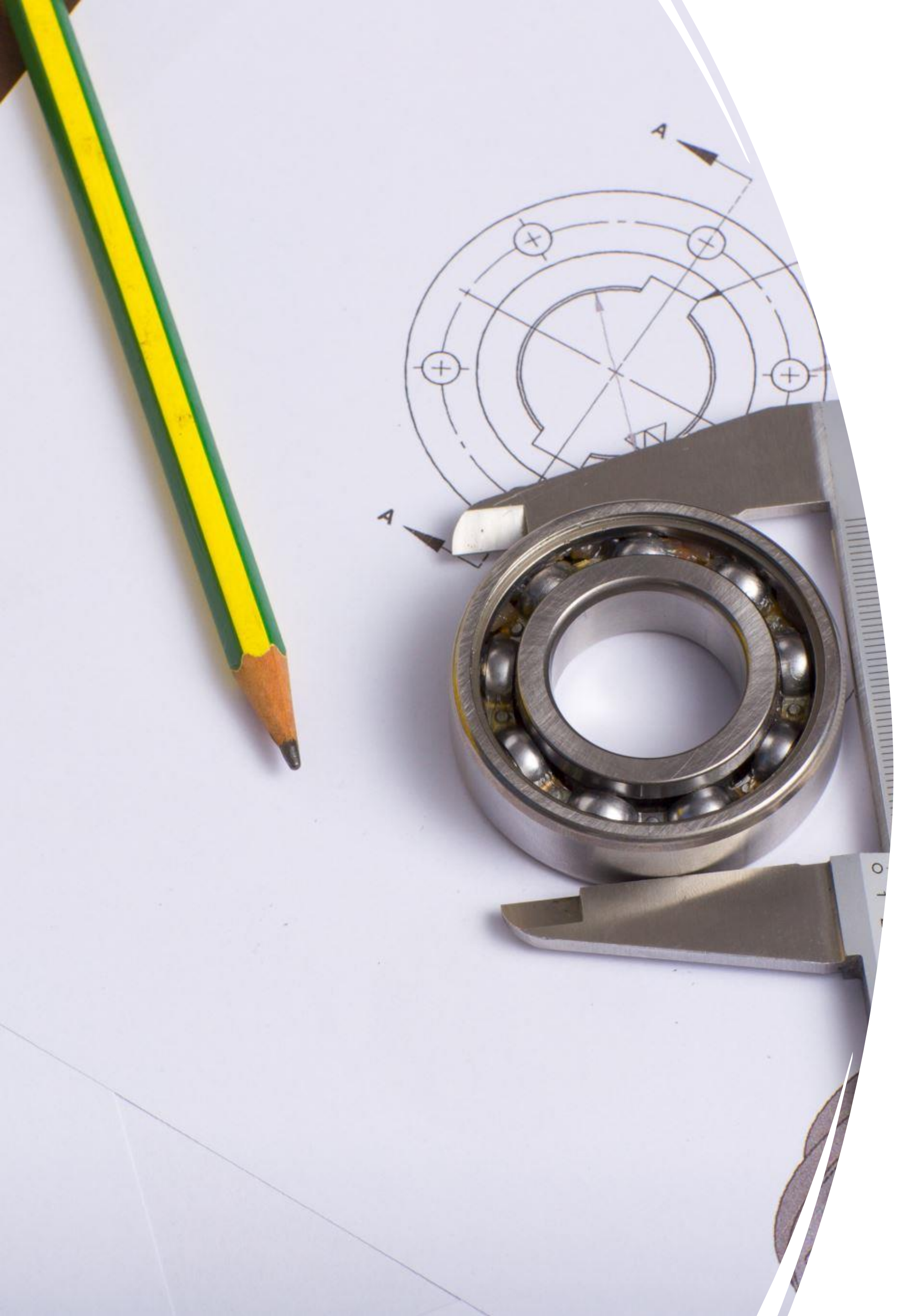
ACCELERATION ANALYSIS OF LINKAGES

Graphical & Analytical Analysis



Learning Outcomes

- Provide an overview of the analysis and design of four-bar mechanisms and the relationships between angular accelerations of the links in the mechanism based on a *closed-form solution.*
- Analyzing acceleration using a graphical approach utilizing acceleration polygon and vector algebra.
- Analyzing acceleration using an analytical approach utilizing closure-loop equations combined with the Newton-Raphson (N-R) technique.

Kinematics Design of Mechanisms - Analysis of Acceleration

- According to the Newton’s Second Law, forces are proportional to accelerations. Therefore, if we know the accelerations of every point on the linkage, we can calculate the forces that the linkage must resist.

- For example, in an automobile engine, particularly in the context of linkages, the focus is on understanding and optimizing the motion dynamics of engine components, such as pistons and connecting rods, which must operate efficiently at high speeds to ensure optimal engine performance and fuel combustion.

Kinematics Design of Mechanisms - Analysis of Acceleration

- Acceleration Vector: represents the *rate of change of the velocity vector* of a link. So, taking the derivative (rate of change) of the velocity equation with respect to time:

Velocity Vector Point B With Respect to Point A: $\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_{B/A}}$ $\rightarrow \frac{d\overrightarrow{v_B}}{dt} = \frac{d\overrightarrow{v_A}}{dt} + \frac{d\overrightarrow{v_{B/A}}}{dt}$ $\rightarrow \overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}}$

Where $a_{B/A}$ has two components: normal and tangential

$$\overrightarrow{a_{B/A}} = (a_{B/A})_n + (a_{B/A})_t$$

$$(a_{B/A})_n = -\omega^2 \times \overrightarrow{r_{B/A}}$$

$$\rightarrow \overrightarrow{a_B} = \overrightarrow{a_A} + (\vec{\alpha} \times \overrightarrow{r_{B/A}}) - \omega^2 \times \overrightarrow{r_{B/A}}$$

$$(a_{B/A})_t = \vec{\alpha} \times \overrightarrow{r_{B/A}}$$

Kinematics Design of Mechanisms – Acceleration Analysis

- The normal and tangential components of the acceleration of each link is shown in Figure #5(a):

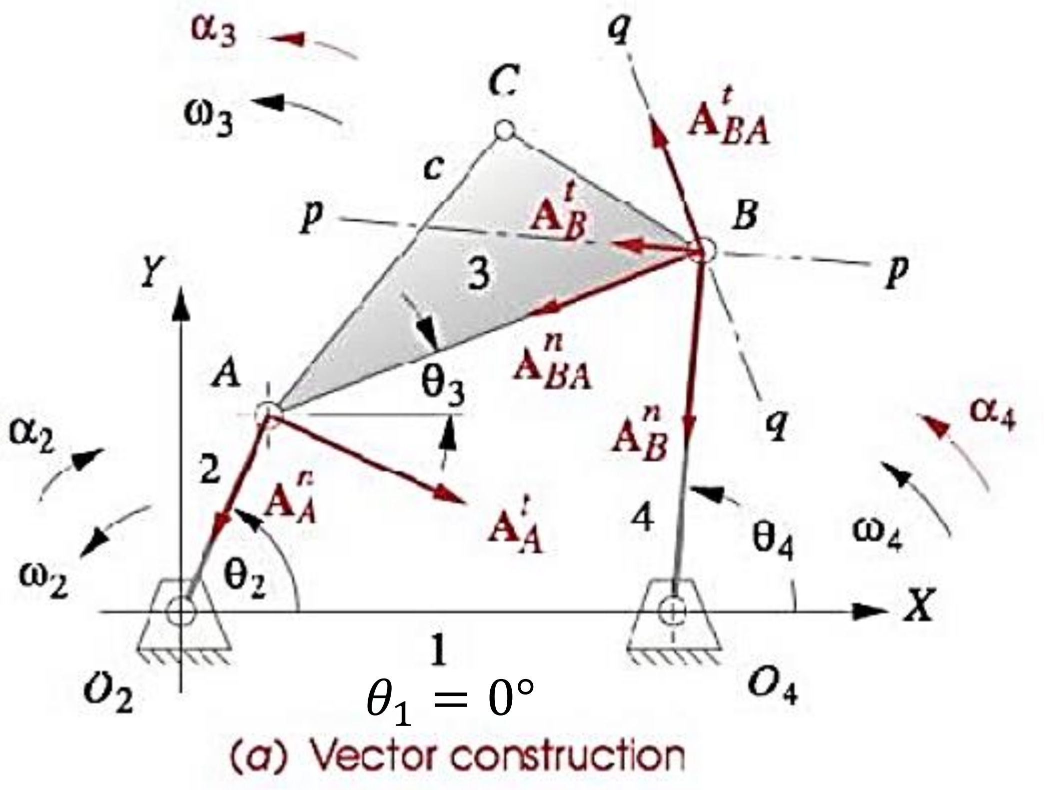


Figure #5(a). Fourbar Mechanism - Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

- Vector Algebra: Recall that we define all velocities in terms of components $(\hat{\imath}, \hat{\jmath}, \hat{k})$ and then, for 2-D plane, we equate the components to obtain two equations:

Velocity Vector Point B
With Respect to Point A:

$$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_{B/A}} \qquad \text{(a)}$$

$$\rightarrow \frac{d\overrightarrow{v_B}}{dt} = \frac{d\overrightarrow{v_A}}{dt} + \frac{d\overrightarrow{v_{B/A}}}{dt}$$

$$\rightarrow \overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}} \qquad \text{(b)}$$

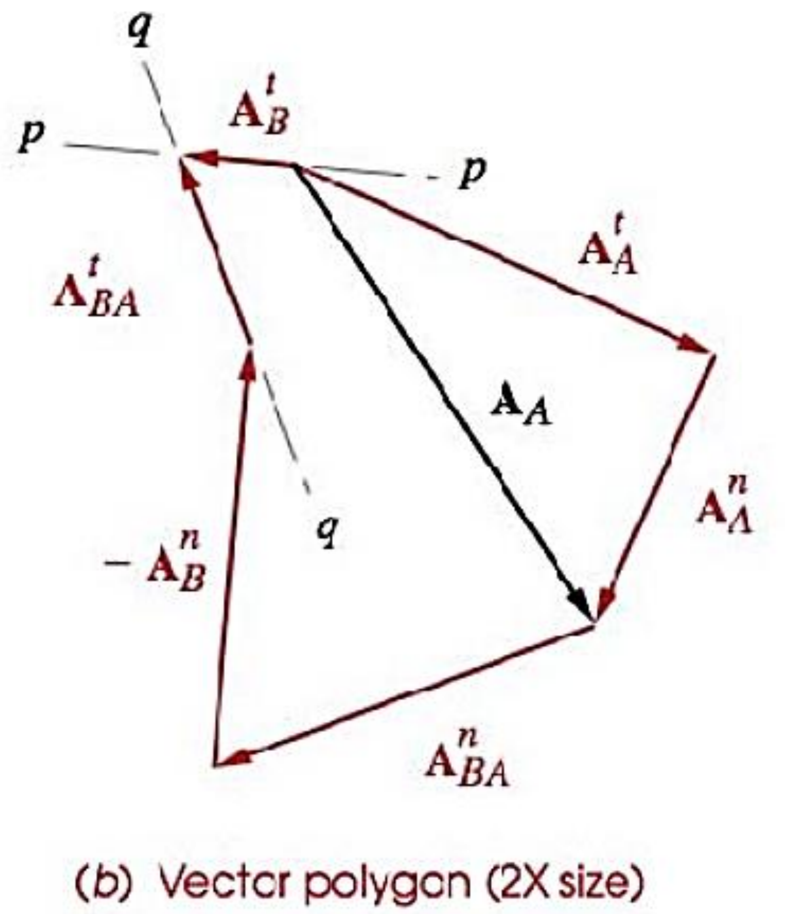


Figure #5(b). One Link of a Fourbar Mechanism. Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

- Vector Algebra (cont.):

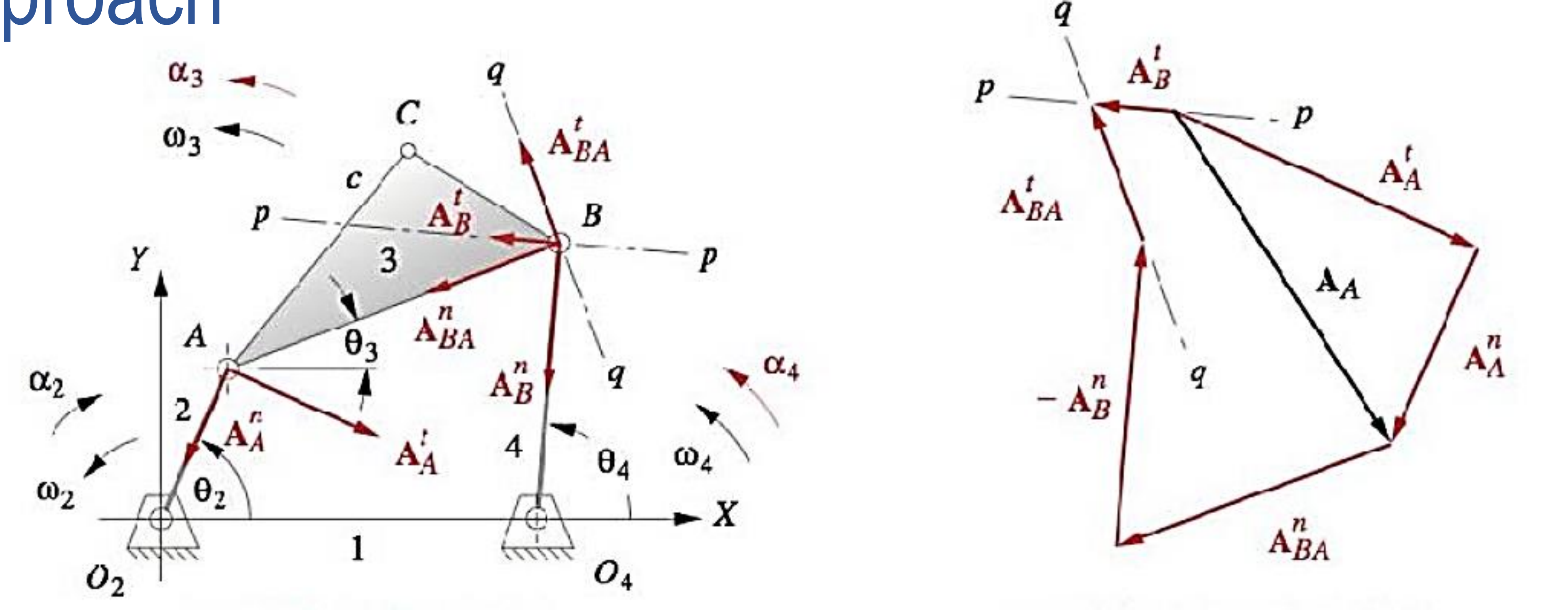
$$\left(a_{B/A}\right)_n = -\omega^2 \times \overrightarrow{r_{B/A}} \qquad \text{(c)}$$ [Direction is towards A]

$$\left(a_{B/A}\right)_t = \vec{\alpha} \times \overrightarrow{r_{B/A}} \qquad \text{(d)}$$ [Direction is $\perp$ to $\overrightarrow{r_{B/A}}$]

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \left(\vec{\alpha} \times \overrightarrow{r_{B/A}}\right) - \omega^2 \times \overrightarrow{r_{B/A}} \qquad \text{(e)}$$

$$\overrightarrow{a_{B/A}} = \left(a_{B/A}\right)_n + \left(a_{B/A}\right)_t \qquad \text{(f)}$$

Kinematics Design of Mechanisms – Graphical Approach



(a) Vector construction (b) Vector polygon (2X size)

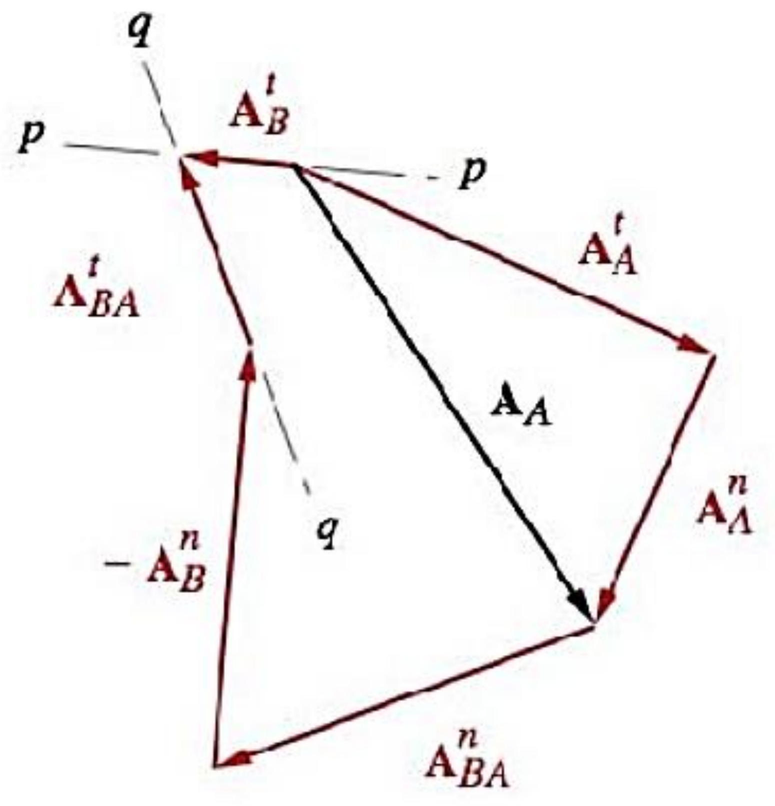
- Vector Algebra: From Figure #5(b), the magnitudes of acceleration for the crank are:

$$A_A^n = (O_2A)\omega_2^2 \qquad A_A^t = (O_2A)\alpha_2 \qquad \text{(g)}$$

Kinematics Design of Mechanisms – Graphical Approach

- Vector Algebra (cont.):
- A_A^n direction is towards A, and A_A^t direction is ⊥ to $\overrightarrow{r_{B/A}}$.
- Draw these magnitudes *at some convenient scale.*

- The sense of A_A^t is defined by that of α_2 according to RH rule.



(b) Vector polygon (2X size)

Figure #5(b). One Link of a Fourbar Mechanism. Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

- Vector Algebra (cont.):
- Move to the B point on link #4. Note that the directions of the normal and tangential components of acceleration of point B are predictable because link #4 is in pure rotation about O_4.
- Draw the construction line pp through point B perpendicular to BO_4 to represent the direction of A_B^t as shown in Figure #5(b).

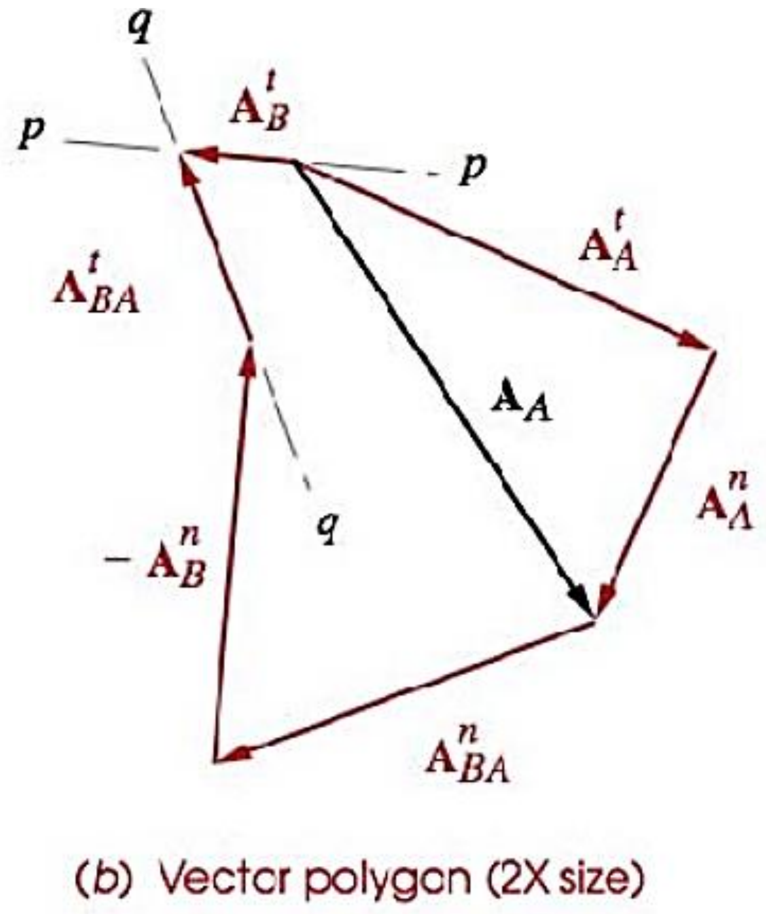


Figure #5(b). One Link of a Fourbar Mechanism. Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

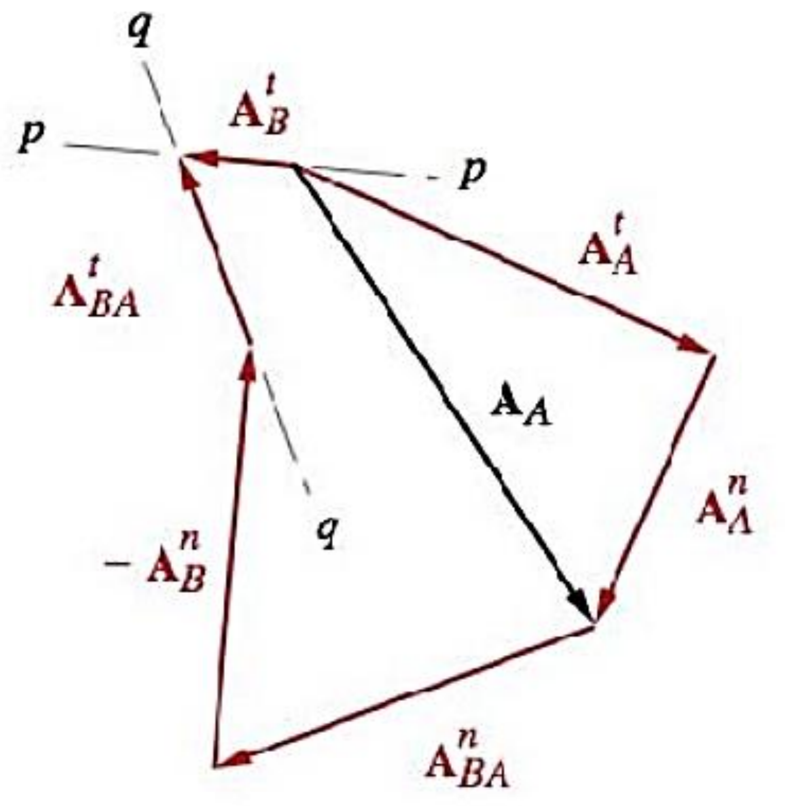
- Vector Algebra (cont.):
- Write the acceleration vector as:

$$A_B = A_A + A_{BA} \qquad \text{(h)}$$

- Substitute the normal and tangential components for each term:

$$(A_B^t + A_B^n) = (A_A^t + A_A^n) + (A_{BA}^t + A_{BA}^n) \qquad \text{(i)}$$

- There are potentially 12 unknowns in Eq. (i). We must know 10 of them to solve it.



(b) Vector polygon (2X size)

Figure #5(b). One Link of a Fourbar Mechanism. Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

- Vector Algebra (cont.):
- The term A_{BA} represents the acceleration difference of B with respect to A. This has two components.
- The normal component A_{BA}^{n} is directed along the line BA.
- The tangential component A_{BA}^{t} must be perpendicular to the line BA.

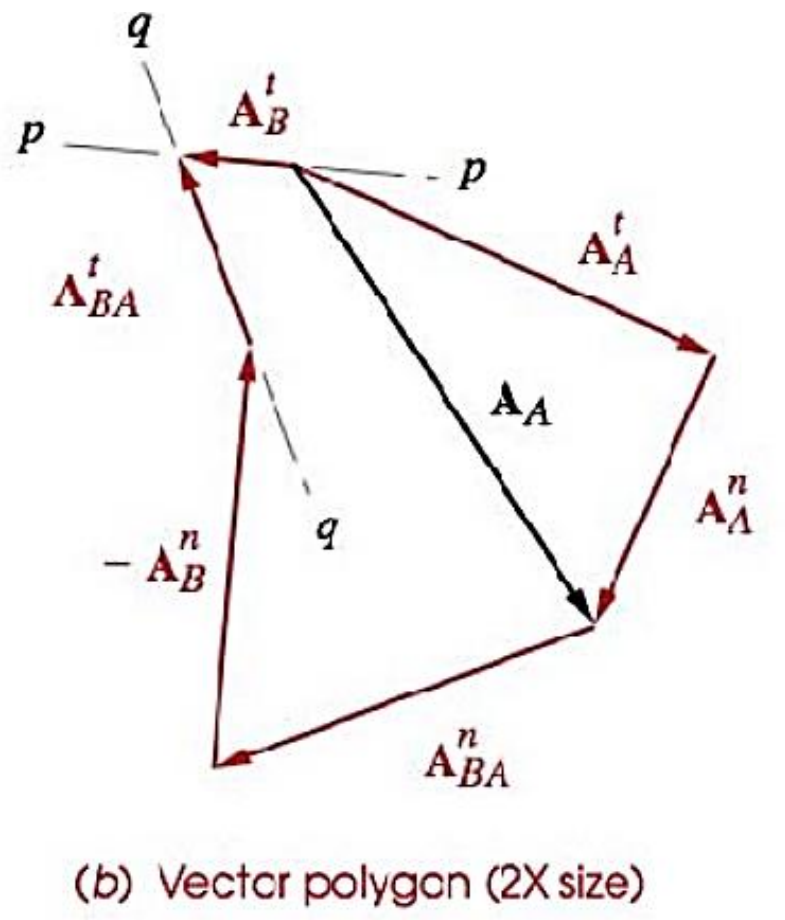


Figure #5(b). One Link of a Fourbar Mechanism. Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

- Vector Algebra (cont.):
- Now, the vector Eq. (i) can be solved graphically by drawing a vector diagram as shown in Figure #5(b). The strategy is to first draw all vectors for which we know both magnitude and direction.

- ***Note:*** You might use drafting tools, MATLAB, Mathcad, or a CAD software.

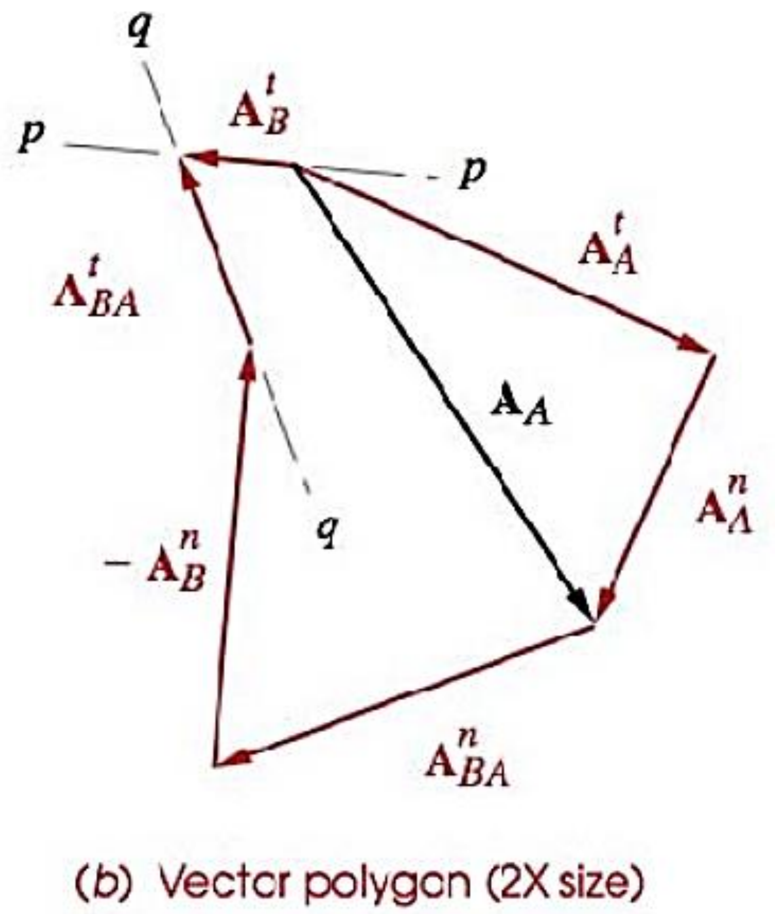


Figure #5(b). One Link of a Fourbar Mechanism.
Normal and Tangential Components of the Acceleration.
Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

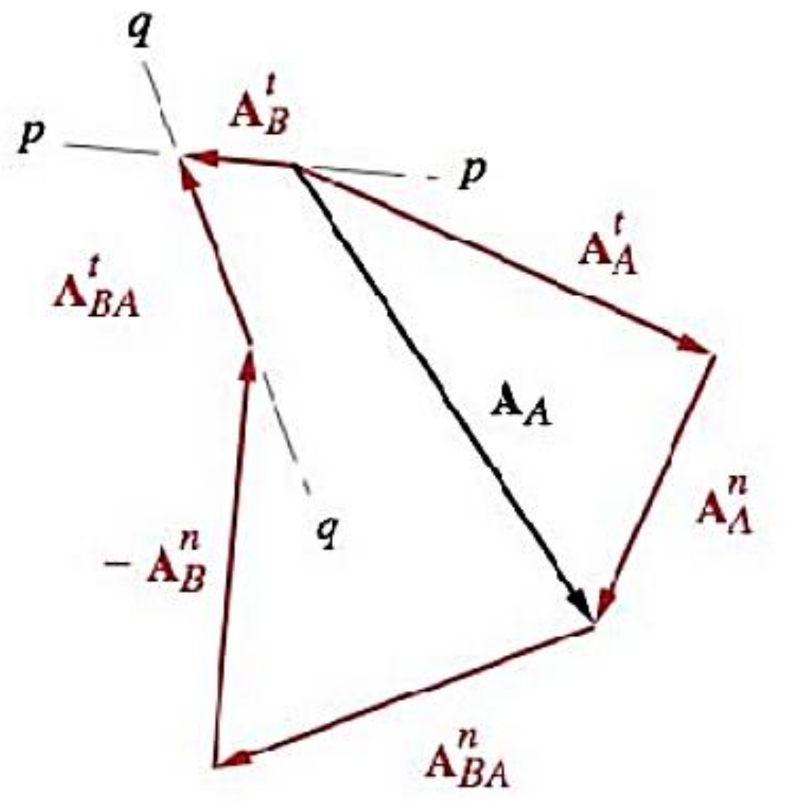
- Vector Algebra (cont.):
- The angular accelerations of links #3 and #4 can be calculated from equation Eq. (7-6), p. 353, in the textbook:

$$\alpha_3 = \frac{A_{BA}^t}{BA} \qquad \alpha_4 = \frac{A_B^t}{O_4B} \qquad \text{(j)}$$

- Finally, we solve for A_c using Eq. (7-4):

$$A_C = A_A + A_{CA}$$

$$(A_C^t + A_C^n) = (A_A^t + A_A^n) + (A_{CA}^t + A_{CA}^n) \qquad \text{(k)}$$



(b) Vector polygon (2X size)

Figure #5(b). One Link of a Fourbar Mechanism. Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach

- Vector Algebra (cont.):
- In this case, we can calculate the magnitude of A_C^t as we have already found α_3:

$$A_{CA}^t = c\alpha_3 \qquad \text{(l)}$$

- The magnitude of the component A_{CA}^n can be found from:

$$A_{CA}^n = c\omega_3^2 \qquad \text{(m)}$$

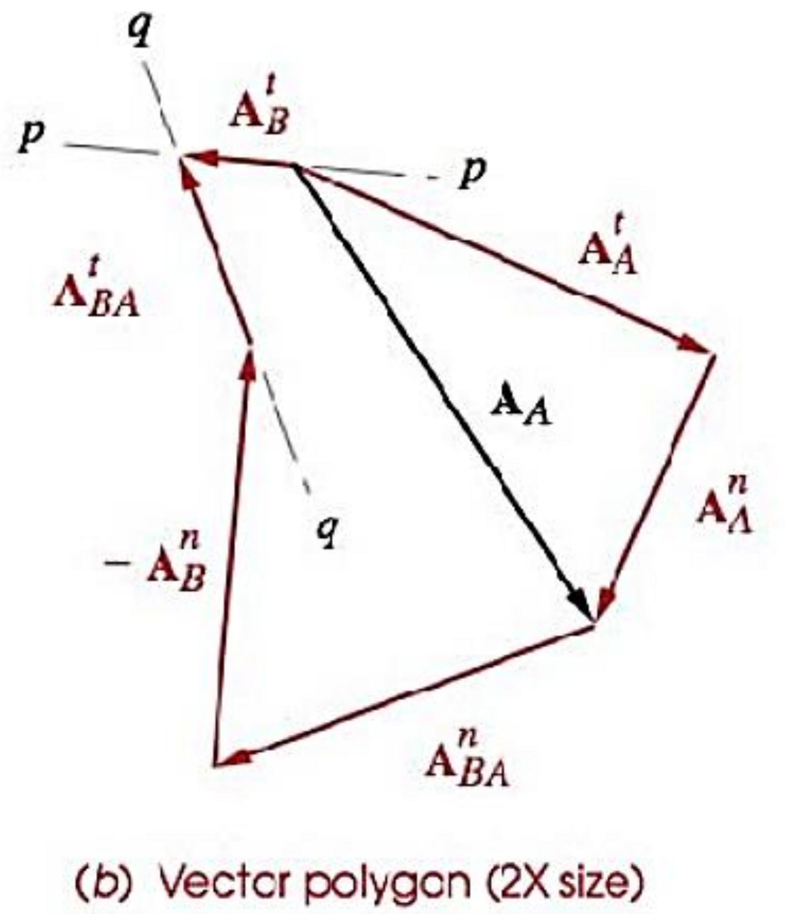
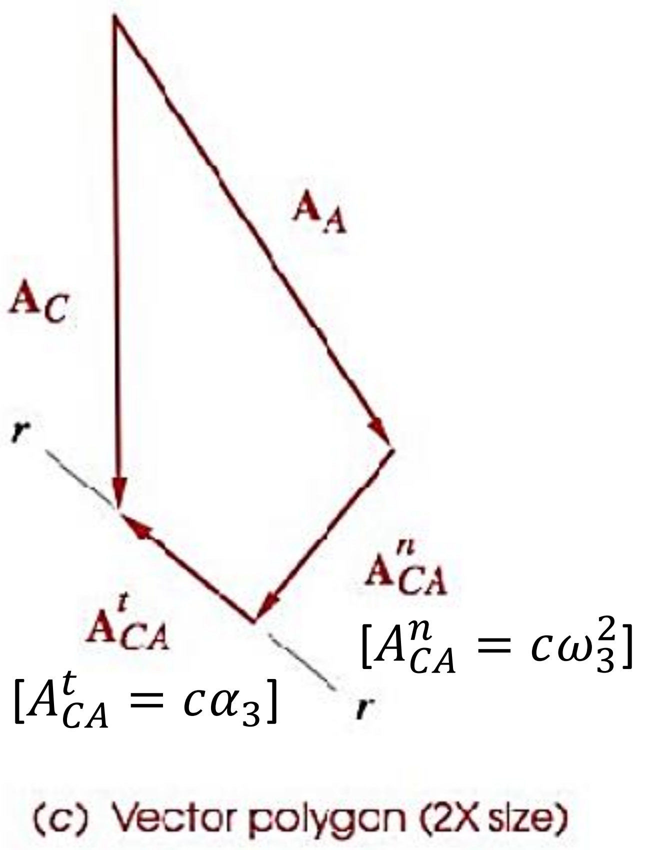


Figure #5(b). One Link of a Fourbar Mechanism. Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

Kinematics Design of Mechanisms – Graphical Approach



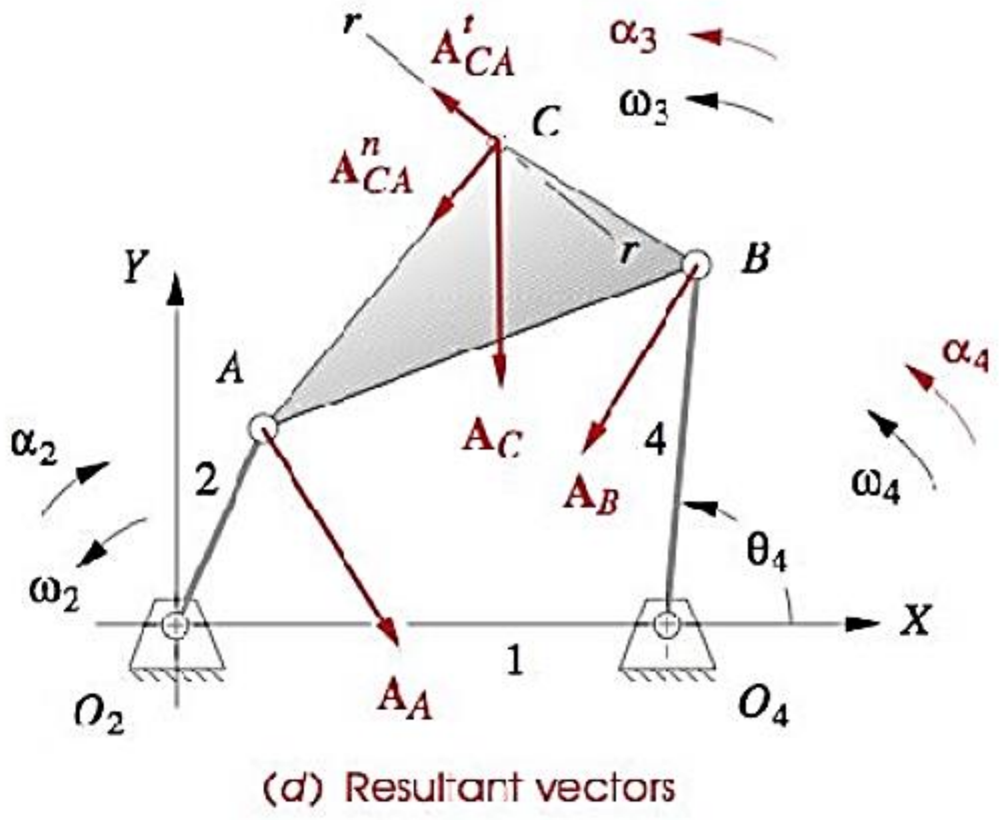


Figure #5(b). One Link of a Fourbar Mechanism.
Normal and Tangential Components of the Acceleration. Source: Norton, R.L. Design of Machinery's Textbook.

An Example of the Graphical Approach:

- Example: Use an appropriate scale to draw the linkage configuration and terminology listed in Figure P7-1(a) to find the accelerations of points *A* and *B*. Then, calculate α_3 and α_4 and the acceleration of point *P* using the step-by-step procedure previously discussed.

- The link lengths, coupler point location, and the values of θ_2, ω_2, and α_2 for the same Fourbar mechanism as used for position and velocity analysis (see Modules #3 and #4) are redefined in Table P7-1 (which is the same as Table P6-1 in the textbook).

- For Table P7-1, ***row a***, presented in the next slide, draw the linkage to scale and graphically find the accelerations of points *A* and *B*.

Kinematics Design of Mechanisms: An Example of the Graphical Approach

- The general linkage configuration and terminology are shown in Figure P7-1.

TABLE P7-1 Data for Problems 7-3, 7-4 and 7-11‡

Row	Link 1	Link 2	Link 3	Link 4	θ_2	ω_2	α_2	R_{pa}	δ_3
a	6	2	7	9	30	10	0	6	30
b	7	9	3	8	85	−12	5	9	25
c	3	10	6	8	45	−15	−10	10	80
d	8	5	7	6	25	24	−4	5	45
e	8	5	8	6	75	−50	10	9	300
f	5	8	8	9	15	−45	50	10	120
g	6	8	8	9	25	100	18	4	300
h	20	10	10	10	50	−65	25	6	20
i	4	5	2	5	80	25	−25	9	80
j	20	10	5	10	33	25	−40	1	0
k	4	6	10	7	88	−80	30	10	330
l	9	7	10	7	60	−90	20	5	180
m	9	7	11	8	50	75	−5	10	90
n	9	7	11	6	120	15	−65	15	60

‡ Drawings of these linkages are in the *PDF Problem Workbook* folder on the DVD.

Source: Norton, R.L. Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines. 5th Edition. McGraw-Hill Co.

Kinematics Design of Mechanisms - Loop Closure Method

- Now, we analyze the accelerations of the Fourbar mechanism using the Loop Closure Method in MATLAB.

- A simulation of this mechanism is programmed in a "Live Script" in MATLAB Live Editor.

Kinematics Design of Mechanisms: An Example of the Analytical Approach using N-R Using MATLAB

- To solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4.

- From the N-R position solution below (MATLAB Live Script P7_3.mlx online); we get:

$$\theta_3 = 88.8403°$$

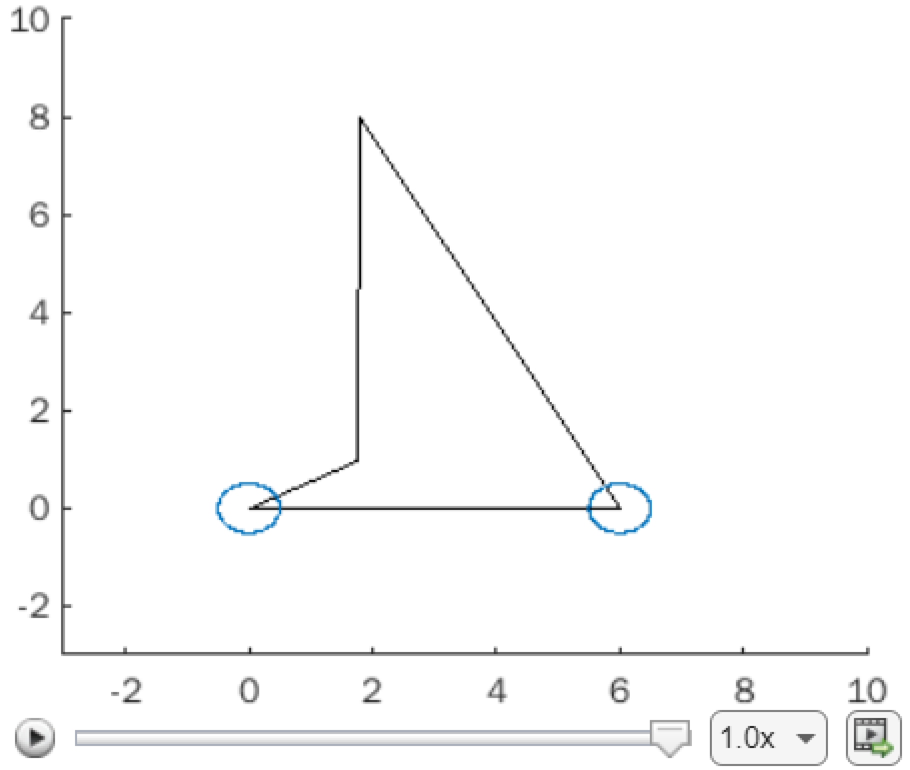
$$\theta_4 = 117.288°$$

$$\omega_3 = -5.98530\, rad/s$$

$$\omega_4 = -3.94925\, rad/s$$

$$\alpha_3 = 27.90980 \text{ rad}/s^2$$

$$\alpha_4 = 54.53483\text{rad}/s^2$$



Kinematics Design of Mechanisms – Acceleration Analysis of the Crank-Slider Mechanism

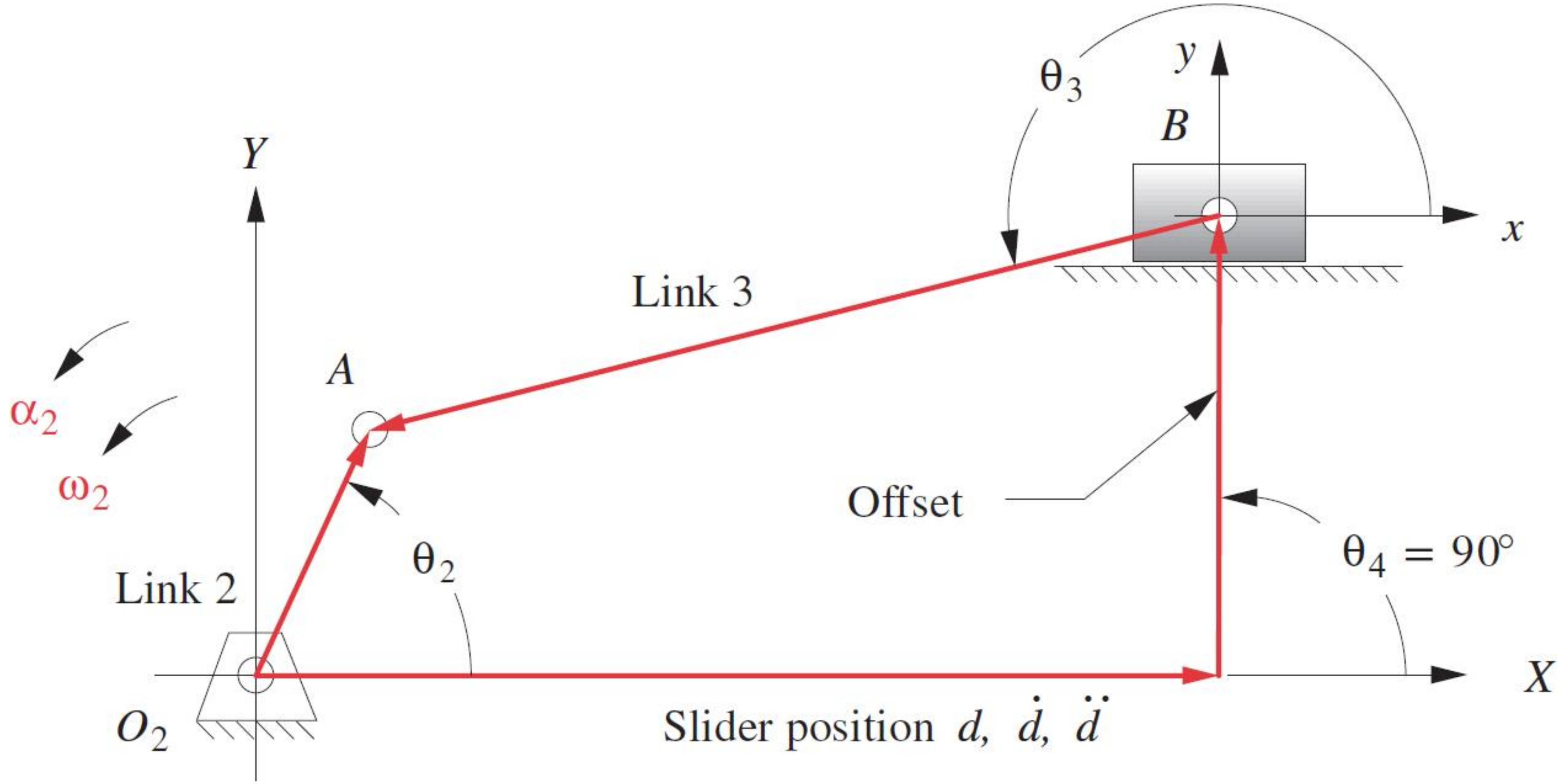


Figure #5(c). Crank-Slider Mechanism.

Source: Norton, R.L. Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines. 5th Edition. McGraw-Hill Co.

Kinematics Design of Mechanisms – Acceleration Analysis of the Crank-Slider Mechanism

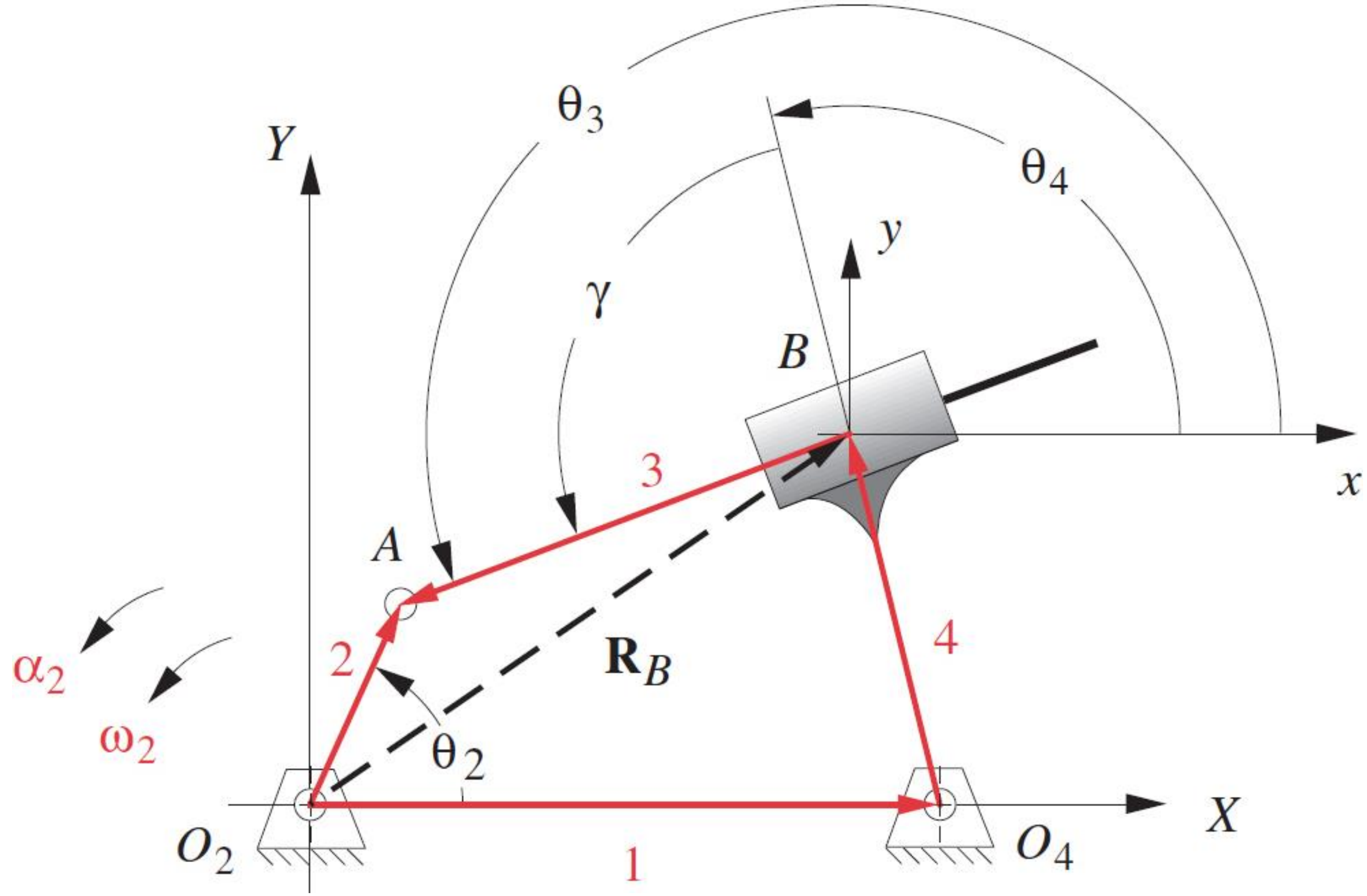


Figure #5(c). Crank-Slider Mechanism.

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