ACCELERATION
Analysis of Linkages

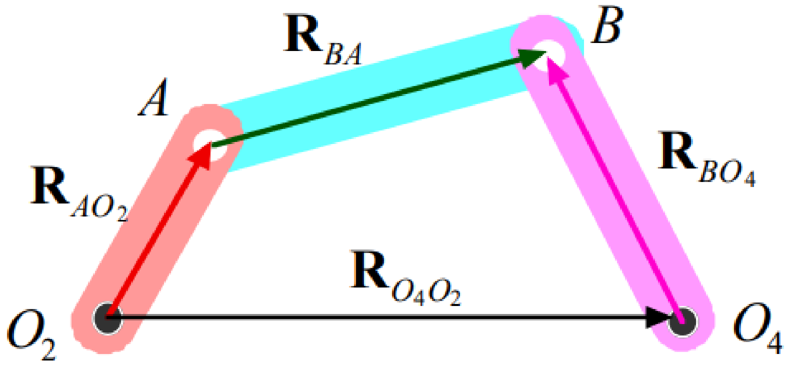
Acceleration Polygon

Acceleration Polygon:

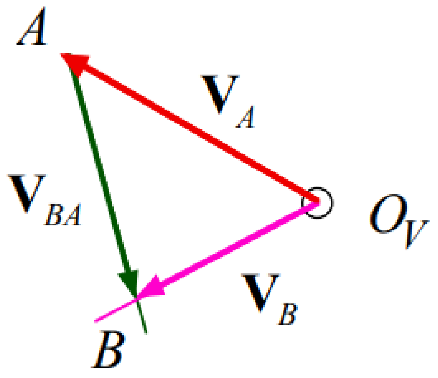
- As remarked, the acceleration polygon is a graphical representation in the study of four-bar mechanisms, depicting the relationships between angular accelerations of all links in the mechanism.

- Given a known configuration of a four-bar mechanism with the crank specified angular velocity (ω_2) and angular acceleration (α_2), we can construct the acceleration polygon and determine α_3 and α_4.

- Next, we establish the graphical approach for conducting acceleration analysis in four-bar mechanisms:



$$R_{AO_2} + R_{BA} - R_{BO_4} - R_{O_4O_2} = 0$$



$$V_A + V_{BA} - V_B = 0$$

Velocity Polygon

Acceleration Polygon:

The acceleration equation is obtained from the time derivative of the velocity equation as $\mathbf{A}_A + \mathbf{A}_{BA} = \mathbf{A}_B$. Since $\mathbf{R}_{AO_2}$, $\mathbf{R}_{BA}$, and $\mathbf{R}_{BO_2}$ are moving vectors with constant lengths, their acceleration vectors have normal and tangential components:

$$\boxed{\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}}$$

Or,

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \breve{\mathbf{R}}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4}) - \alpha_4 \breve{\mathbf{R}}_{BO_4} = \mathbf{0}$$

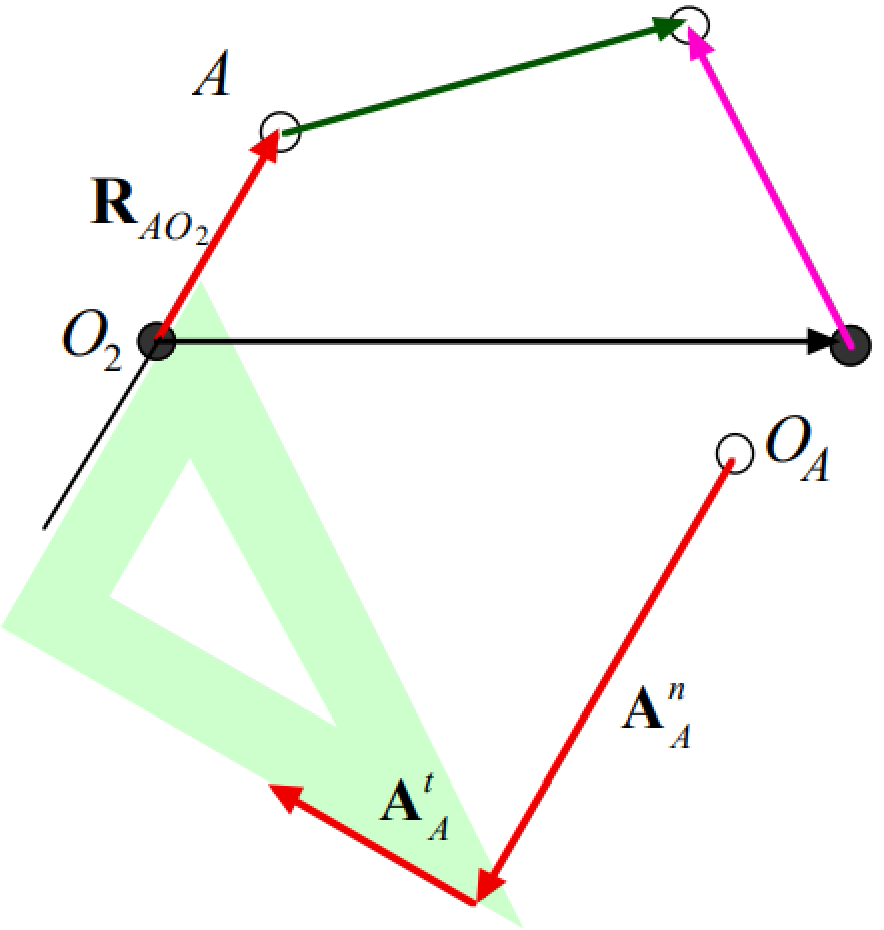
We note that since ω_2, ω_3, ω_4, and α_2 are known, $\mathbf{A}_A^n$, $\mathbf{A}_A^t$, $\mathbf{A}_{BA}^n$, and $\mathbf{A}_B^n$ can completely be constructed. The remaining components, $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B^t$, have known axes but unknown magnitudes. We rearrange the terms such that these unknown terms appear as the **last component** in the equation:

$$\boxed{\underbrace{-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4})}_{\text{All known these quantities (because the position and velocity analysis has been performed).}} + \underbrace{\alpha_3 \breve{\mathbf{R}}_{BA} - \alpha_4 \breve{\mathbf{R}}_{BO_4}}_{\alpha_3 \text{ and } \alpha_4 \text{ are to be determined (unknown quantities)}} = \mathbf{0}}$$

Procedure to Draw the Acceleration Polygon:

1. Select a point in a convenient position as the reference for zero acceleration. Name this point O_A (origin of accelerations).
2. Compute the magnitude of $\mathbf{A}_A^n$ as $R_{AO_2}\omega_2^2$. From O_V construct vector $\mathbf{A}_A^n$ in the opposite direction of $\mathbf{R}_{AO_2}$.
3. Compute the magnitude of $\mathbf{A}_A^t$ as $R_{AO_2}\alpha_2$. The direction of $\mathbf{A}_A^t$ is determined by rotating $\mathbf{R}_{AO_2}$ 90^{o} in the direction of α_2. Add this vector to $\mathbf{A}_A^n$

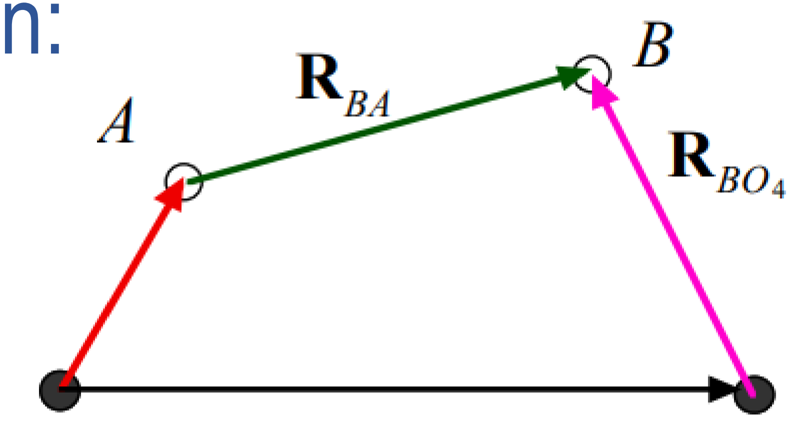
Note that the sum of $\mathbf{A}_A^n$ and $\mathbf{A}_A^t$ is $\mathbf{A}_A$.

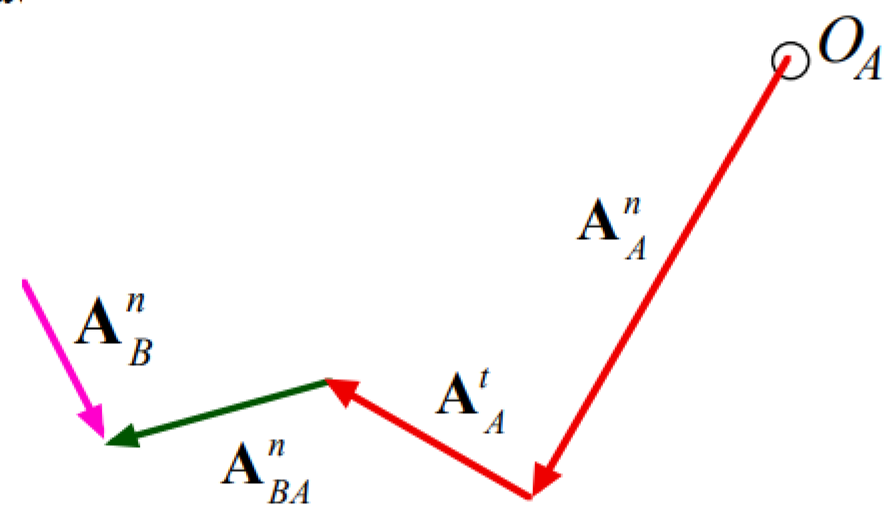


Procedure to Draw the Acceleration Polygon:

4. Compute the magnitude of $\mathbf{A}^n_{BA}$ as $R_{BA}\omega_3^2$. Add this vector in the opposite direction of $\mathbf{R}_{BA}$ to the other two vectors.

5. Compute the magnitude of $\mathbf{A}^n_{B}$ as $R_B\omega_4^2$. Note that $\mathbf{A}^n_{B}$ is in the opposite direction of $\mathbf{R}_{BO_4}$. Since $\mathbf{A}^n_{B}$ itself appears with a negative sign in the acceleration equation, it should be added to the other vectors in the diagram as shown; i.e., head-to-tail.



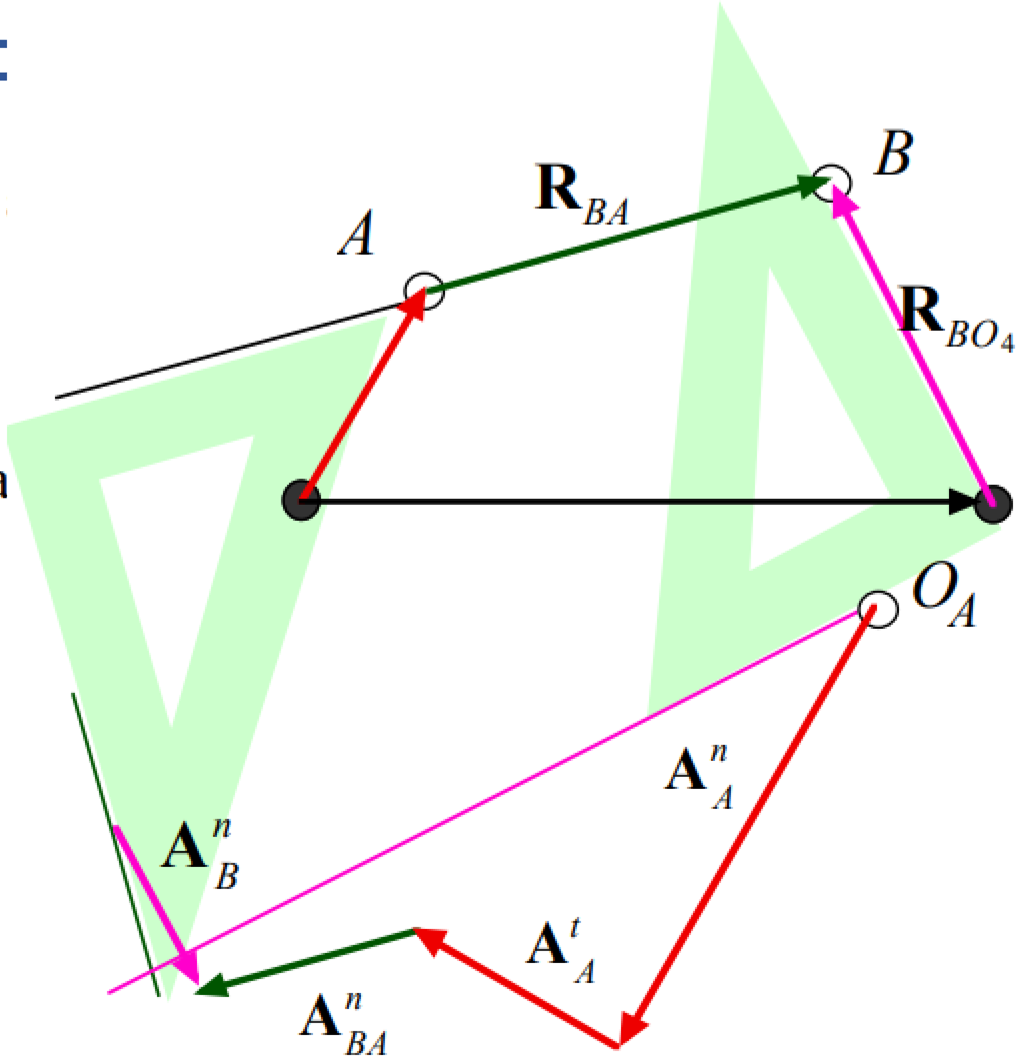


$$\boxed{\underbrace{-\omega_2^2\mathbf{R}_{AO_2} + \alpha_2\breve{\mathbf{R}}_{AO_2} - \omega_3^2\mathbf{R}_{BA} - (-\omega_4^2\mathbf{R}_{BO_4})}_{\text{All known these quantities (because the position and velocity analysis has been performed).}} + \underbrace{\alpha_3\breve{\mathbf{R}}_{BA} - \alpha_4\breve{\mathbf{R}}_{BO_4}}_{\alpha_3 \text{ and } \alpha_4 \text{ are to be determined (unknown quantities)}} = \mathbf{0}}$$

Procedure to Draw the Acceleration F

5. Since $\mathbf{A}_{BA}^{t}$ must be perpendicular to $\mathbf{R}_{BA}$, draw
line perpendicular to $\mathbf{R}_{BA}$ in anticipation of
adding $\mathbf{A}_{BA}^{t}$ to the diagram.

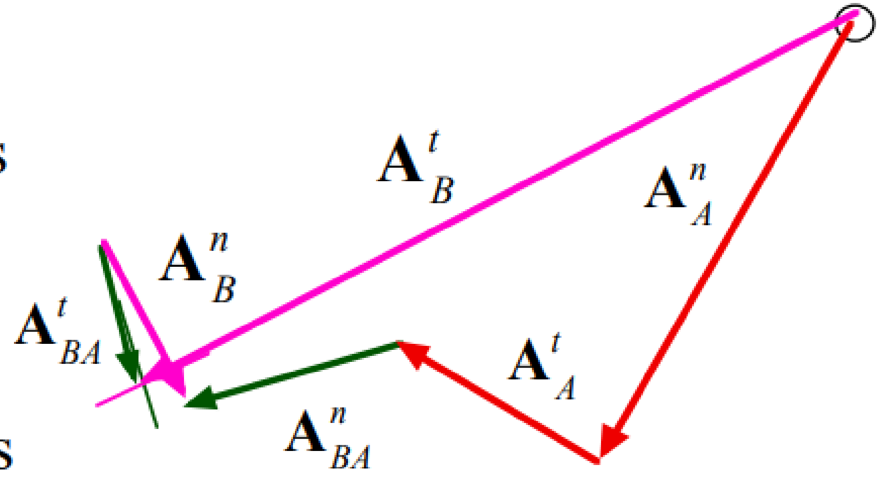
6. Since $\mathbf{A}_{B}^{t}$ must be perpendicular to $\mathbf{R}_{BO_4}$, draw a
line perpendicular to $\mathbf{R}_{BO_4}$ closing (completing)
the polygon.



$$\boxed{\underbrace{-\omega_2^2\mathbf{R}_{AO_2}+\alpha_2\breve{\mathbf{R}}_{AO_2}-\omega_3^2\mathbf{R}_{BA}-(-\omega_4^2\mathbf{R}_{BO_4})}_{\text{All known these quantities (because the position and velocity analysis has been performed).}}+\underbrace{\alpha_3\breve{\mathbf{R}}_{BA}-\alpha_4\breve{\mathbf{R}}_{BO_4}}_{\alpha_3 \text{ and } \alpha_4 \text{ are to be determined (unknown quantities)}}=\mathbf{0}}$$

Procedure to Draw the Acceleration Polygon:

8. Construct vectors $\mathbf{A}_{BA}^{t}$ and $\mathbf{A}_{B}^{t}$ on the polygon.
9. Determine the magnitude of $\mathbf{A}_{BA}^{t}$ from the polygon. Compute α_3 as $\alpha_3 = A_{BA}^{t} / R_{BA}$ (in this diagram it is CW).
10. Determine the magnitude of $\mathbf{A}_{B}^{t}$ from the polygon. Compute α_4 as $\alpha_4 = A_{B}^{t} / R_{BO_4}$ (in this diagram it is CCW).



Procedure to Draw the Acceleration Polygon:

<u>Secondary equation(s)</u>

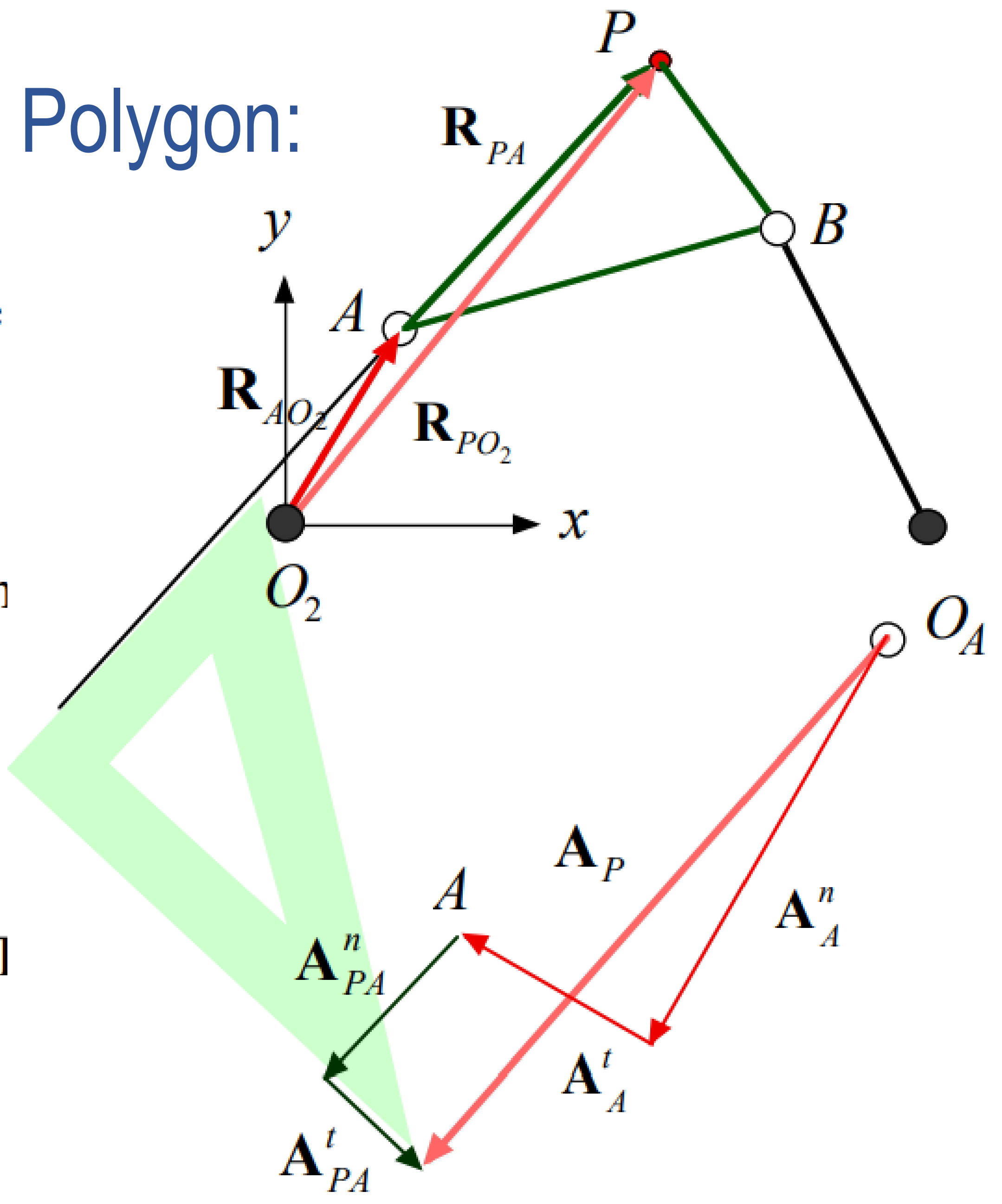
We can use the polygon method to determine acceleration of a coupler point, such as *P*. It is assumed that all the angular velocities and accelerations have already been determined.

For the position vector $\mathbf{R}_{PO_2} = \mathbf{R}_{AO_2} + \mathbf{R}_{PA}$, th acceleration expression becomes

$$\mathbf{A}_P = \mathbf{A}_A + \mathbf{A}_{PA} = \mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{PA}^n + \mathbf{A}_{PA}^t$$

$$= -\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{PA} + \alpha_3 \breve{\mathbf{R}}_{PA}$$

All four vectors can be constructed graphically. T vector sum is the acceleration of *P*.



The End of Module #7

Thank you,
Prof. Veras