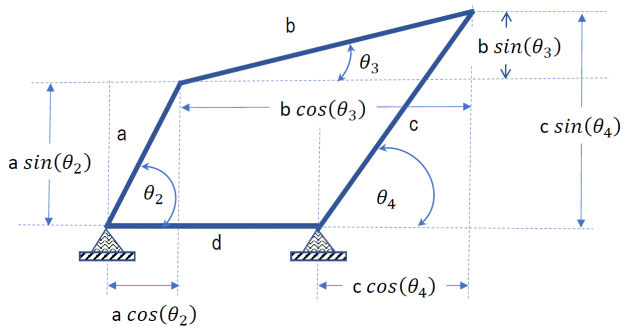


TABLE P7-1 Data for Problems 7-3, 7-4 and 7-11†

Row	Link 1	Link 2	Link 3	Link 4	θ_2	ω_2	α_2	R_{PO}	δ_3
a	6	2	7	9	30	10	0	6	30
b	7	9	3	8	85	-12	5	9	25
c	3	10	6	8	45	-15	-10	10	80
d	8	5	7	6	25	24	-4	5	45
e	8	5	8	6	75	-50	10	9	300
f	5	8	8	9	15	-45	50	10	120
g	6	8	8	9	25	100	18	4	300
h	20	10	10	10	50	-65	25	6	20
i	4	5	2	5	80	25	-25	9	80
j	20	10	5	10	33	25	-40	1	0
k	4	6	10	7	88	-80	30	10	330
l	9	7	10	7	60	-90	20	5	180
m	9	7	11	8	50	75	-5	10	90
n	9	7	11	6	120	15	-65	15	60

† Drawings of these linkages are in the PDF Problem Workbook folder on the DVD.

Source: Norton, R.L. Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines. 5th Edition. McGraw-Hill Co.



Inputs

```
a = 2;
b = 7;
c = 9;
d = 6;
theta2 = 30;
```

Initial Solution (Guess Values)

```
theta3 = 70;
theta4 = 60;

x = [theta3; theta4];
x = matlab_solver(x, theta2, a,b,c,d);
```

Iteration	Func-count	$ f(x) ^2$	Norm of step	First-order optimality	Trust-region radius
0	3	40.6723		0.85	1
1	6	38.4768	1	0.826	1
2	9	33.3065	2.5	0.765	2.5
3	12	22.4001	6.25	0.608	6.25
4	15	7.77361	15.625	0.213	15.6
5	18	0.70106	39.0625	0.112	39.1
6	21	0.00759546	13.2563	0.00726	97.7
7	24	2.20754e-06	1.71146	0.00017	97.7
8	27	6.4235e-14	0.0257659	2.84e-08	97.7

Equation solved.

fsolve completed because the vector of function values is near zero as measured by the value of the function tolerance, and the problem appears regular as measured by the gradient.

<stopping criteria details>

```
x = 2x1
    88.8372
   117.2861
fval = 2x1
10^-6 x
    0.2304
   -0.1057
```

% result at this point

```
theta3 = x(1);
theta4 = x(2);
```

```
disp([theta2, theta3, theta4])
```

```
30.0000    88.8372   117.2861
```

Velocity Analysis:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

```
w2 = 10;           % In rad/s
```

% Position vectors

```
r02A = [a*cosd(theta2) a*sind(theta2) 0]
```

```
r02A = 1x3
    1.7321    1.0000         0
```

```
rAB = [b*cosd(theta3) b*sind(theta3) 0]
```

```
rAB = 1x3
    0.1420    6.9986         0
```

```
r04B = [c*cosd(theta4) c*sind(theta4) 0]
```

```
r04B = 1x3
   -4.1259    7.9986         0
```

% w2 using unit vector k

```
k = [0 0 1];
VA = w2*cross(k,r02A)
```

```
VA = 1x3
  -10.0000   17.3205         0
```

% Given the x and y components of velocity vector at join B, you can use the Pythagoras' formula

% to find the resultant velocity:

```
mag_VA = round(sqrt(VA(1)^2+VA(2)^2),2)           % in/s
```

```
mag_VA =  
20
```

```
% You can calculate the magnitude of VA with norm() built-in function  
% in MATLAB by typing:  
Vel_mag_A = norm(VA,2) % Magnitude of velocity at join A.
```

```
Vel_mag_A =  
20
```

```
% Builtin MATLAB norm() function: norm of a vector  
is simply the  
% square root of the sum of each component squared.
```

```
% Next, the following two equations can be defined at join B. But, first,  
define the  
% symbolic variables, say, w3 & w4:  
syms w3 w4
```

```
% At join B:  
V_B1 = VA + w3*cross(k,rAB); % wrt coupler link -> Eq (1)  
V_B2 = w4*cross(k,r04B); % wrt rocker link -> Eq (2)  
  
eqn_vel = V_B1 - V_B2 == 0;  
[w3, w4] = vpasolve(eqn_vel,[w3 w4]) % rad/s
```

```
w3 = -5.9909661264503475173066209426324  
w4 = -3.9917352019590763262378163377905
```

```
w3 = round(w3, 1)
```

```
w3 = -6.0
```

```
w4 = round(w4, 1)
```

```
w4 = -4.0
```

```
VC = subs(V_B2,w4)
```

```
vc = (31.994233977336385521539341425523 16.503605430434102885328684351407 0)
```

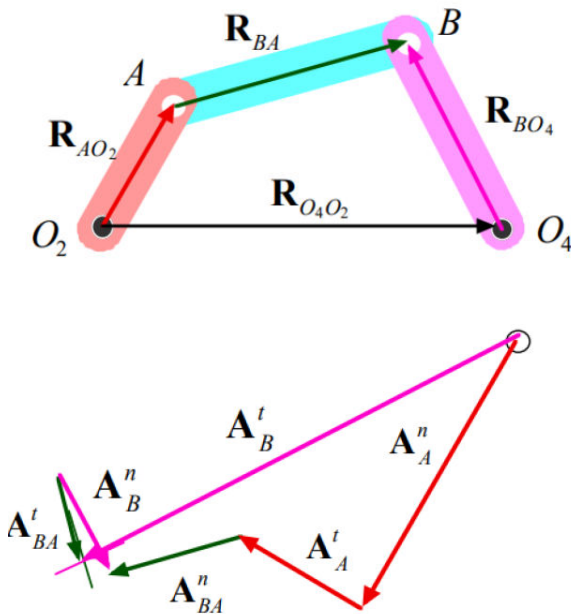
```
% Given the x and y components of velocity vector, you can use the  
Pythagoras' formula  
% to find the resultant velocity:  
mag_VC = round(sqrt(VC(1)^2+VC(2)^2),2)
```

```
mag_VC = 36.0
```

```
% Again, you can calculate the magnitude of VC with norm() built-in function
% in MATLAB:
Vel_mag_C = round(norm(VC,2),2) % Magnitude of velocity at point C
C
```

```
Vel_mag_C = 36.0
```

Acceleration Analysis:



Rotational Acceleration: α_3 and α_4

```
alpha2 = 0; % rad/s^2 % In rad/s^2, CCW
```

```
% Position vectors
```

```
rA02 = [a*cosd(theta2) a*sind(theta2) 0];
```

```
rBA = [b*cosd(theta3) b*sind(theta3) 0];
```

```
rB04 = [c*cosd(theta4) c*sind(theta4) 0];
```

```
A_An = -w2^2*rA02
```

```
A_An = 1×3
-173.2051 -100.0000 0
```

```
A_At = alpha2*cross(k,rA02)
```

```
A_At = 1×3
0 0 0
```

```
A_A = sqrt(A_An.^2 + A_At.^2)
```

```
A_A = 1×3
    173.2051    100.0000         0
```

```
% vnom => MATLAB norm with variable precision arithmetic.
acc_mag_A = norm(A_A,2) % Magnitude of acceleration at point B
```

```
acc_mag_A =
    200
```

```
syms alpha3 alpha4
A_A = -w2^2*rA02 + alpha2*cross(k,rA02)
```

```
A_A = 1×3
   -173.2051   -100.0000         0
```

```
A_B1 = A_A + alpha3*cross(k,rBA) - (w3)^2*rBA
```

```
A_B1 =

$$\left( -100 \sqrt{3} - 5.1137137608532415855577824004286 - \frac{3939838237903653 \alpha_3}{562949953421312} - 351.94810960110826 \right)$$

```

```
A_B2 = alpha4*cross(k,rB04) - (w4)^2*rB04
```

```
A_B2 =

$$\left( 66.014421721736411541314737405628 - \frac{4502788131823019 \alpha_4}{562949953421312} - 127.976935909345542086157365 \right)$$

```

```
eqn_ABB = A_B1 - A_B2 == 0;
```

```
% vpasolve => MATLAB solver with variable precision arithmetic.
R = vpasolve(eqn_ABB,[alpha3; alpha4]);
```

```
% Coupler's rotational acceleration
alpha3 = R.alpha3;
alpha3 = round(alpha3,2) % rad/s^2
```

```
alpha3 = 26.1
```

```
% Rocker's rotational acceleration
alpha4 = R.alpha4;
alpha4 = round(alpha4,2) % rad/s^2
```

```
alpha4 = 53.39
```

```
% Magnitude of acceleration at point C:
%
% For coupler:
```

```
A_B1 = subs(A_B1, alpha3)
```

```
A_B1 = (-100*sqrt(3) - 187.77609322165673308990818668462 -348.24066712448966399440219987582 0)
```

```
Acc_mag_B1 = norm(A_B1, 2);  
Acc_mag_B1 = round(Acc_mag_B1, 2)
```

```
Acc_mag_B1 = 501.58
```

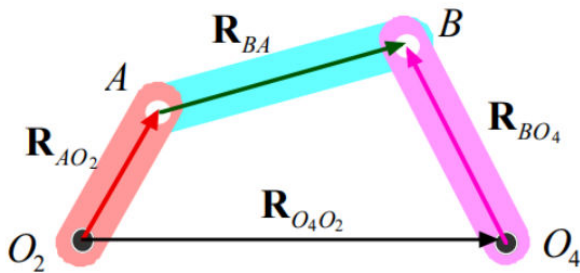
```
% For rocker:
```

```
A_B2 = subs(A_B2, alpha4);  
Acc_mag_B2 = norm(A_B2, 2);  
Acc_mag_B2 = round(Acc_mag_B2, 2)
```

```
Acc_mag_B2 = 501.62
```

```
% As expected A_C1 = A_C2
```

Normal and Tangential Components of Acceleration:



$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}$$

Or,

$$-\omega_2^2 \mathbf{r}_{AO_2} + \alpha_2 \check{\mathbf{r}}_{AO_2} - \omega_3^2 \mathbf{r}_{BA} + \alpha_3 \check{\mathbf{r}}_{BA} - (-\omega_4^2 \mathbf{r}_{BO_4}) - \alpha_4 \check{\mathbf{r}}_{BO_4} = \mathbf{0}$$

```
% Link's Acceleration: Normal and Tangential Components
```

```
% Acceleration of the crank:  
% Compute the magnitude of A_An:  
A_An = w2^2*norm(rA02,2)
```

```
A_An =  
200
```

```
% Compute the magnitude of A_At:  
A_At = alpha2*norm(rA02,2)
```

```
A_At =  
0
```

```
% Resultant Acceleration of the crank:  
A_A = sqrt(A_An^2 + A_At^2)
```

```
A_A =  
200
```

```
% Acceleration for the coupler:  
% Compute the magnitude of A_BAn:  
A_BAn = w3^2*norm(rBA,2)
```

```
A_BAn = 252.0
```

```
% Compute the magnitude of baAt:  
A_BAt = alpha3*norm(rBA,2)
```

```
A_BAt = 182.7
```

```
% Resultant Acceleration for the coupler:  
A_BA = round(sqrt(A_BAn^2 + A_BAt^2), 2)
```

```
A_BA = 311.26
```

```
% Acceleration for the rocker:  
% Compute the magnitude of A_Bn:  
A_Bn = w4^2*norm(rB04,2)
```

```
A_Bn = 144.0
```

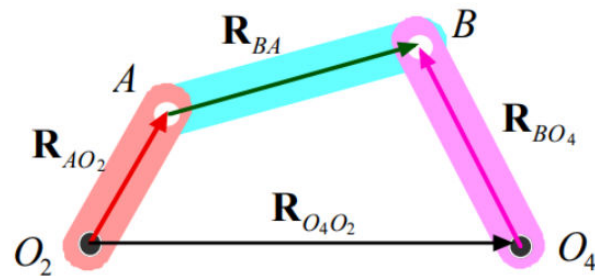
```
% Compute the magnitude of baAt:  
A_Bt = alpha4*norm(rB04,2)
```

```
A_Bt = 480.51
```

```
% Resultant Acceleration for the rocker:  
A_B = round(sqrt(A_Bn^2 + A_Bt^2),2)
```

```
A_B = 501.62
```

Normal and Tangencial Components of Acceleration:



$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}$$

Or,

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \ddot{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \ddot{\mathbf{R}}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4}) - \alpha_4 \ddot{\mathbf{R}}_{BO_4} = \mathbf{0}$$

```
% Link's Acceleration: Normal and Tangential Components
```

```
% Acceleration of the crank:
```

```
% Compute the magnitude of A_An:
```

```
m_rA02 = norm(rA02,2)
```

```
m_rA02 =  
2
```

```
A_An = -w2^2*rA02
```

```
A_An = 1×3  
-173.2051 -100.0000 0
```

```
mA_An = norm(A_An,2)
```

```
mA_An =  
200
```

```
%A_An = w2^2*sqrt((a*cosd(theta2))^2+(a*sind(theta2))^2+(0)^2)
```

```
% Compute the magnitude of A_At:
```

```
A_At = alpha2*cross(k,rA02)
```

```
A_At = 1×3  
0 0 0
```

```
mA_At = norm(A_At,2)
```

```
mA_At =  
0
```

```
% Resultant Acceleration of the crank:
```

```
A_A = sqrt(mA_An^2 + mA_At^2)
```

```
A_A =  
200
```

```
% Acceleration for the coupler:  
% Compute the magnitude of A_BAn:  
m_rBC = norm(rBA,2)
```

```
m_rBC =  
7
```

```
A_BAn = -(w3^2)*rBA
```

```
A_BAn = (-5.1137137608532415855577824004286 -251.94810960110826414393159211613 0)
```

```
mA_BAn = norm(A_BAn,2)
```

```
mA_BAn = 252.00000000000000411920109575326
```

```
% Compute the magnitude of baAt:  
A_BAt = alpha3*cross(k,rBA)
```

```
A_BAt = (-182.6623794608034915043504042842 3.7074424766186001495293922403107 0)
```

```
mA_BAt = norm(A_BAt,2)
```

```
mA_BAt = 182.70000000000000298642079442111
```

```
% Resultant Acceleration for the coupler:  
A_BA = sqrt(mA_BAn^2 + mA_BAt^2)
```

```
A_BA = 311.26080704129776956725875990209
```

```
% Acceleration for the rocker:  
% Compute the magnitude of A_Bn:  
m_rDC = norm(rB04,2)
```

```
m_rDC =  
9
```

```
A_Bn = -(w4^2)*rB04
```

```
A_Bn = (66.014421721736411541314737405628 -127.97693590934554208615736570209 0)
```

```
mA_Bn = norm(A_Bn,2)
```

```
mA_Bn = 144.0000000000000022814160894936
```

```
% Compute the magnitude of baAt:
```

```
A_Bt = alpha4*cross(k,rB04)
```

```
A_Bt = (-427.04303801249740574874635967717 -220.2818734827191882619246143804 0)
```

```
mA_Bt = norm(rB04,2)
```

```
mA_Bt =  
9
```

```
% Resultant Acceleration for the rocker:
```

```
A_B = round(sqrt(mA_Bn^2 + mA_Bt^2),2)
```

```
A_B = 144.28
```

$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}$$

Or,

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \tilde{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \tilde{\mathbf{R}}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4}) - \alpha_4 \tilde{\mathbf{R}}_{BO_4} = \mathbf{0}$$

```
% sum of acceleration (closure of the Acceleration polygon):
```

```
z = A_An + A_At + A_BAn + A_BAt - A_Bn - A_Bt
```

```
z = (173.25252306910426111752343558692 - 100 √3 0.018142267575066353679780206675787 0)
```

```
sumZ = norm(z,3)
```

```
sumZ =
```

```
(0.0000059713799258665899925807356331734 - (100 √3 - 173.25252306910426111752343558692)
```

```
% Acceleration of the crank:
```

```
A_An
```

```
A_An = 1×3  
-173.2051 -100.0000 0
```

```
A_At
```

```
A_At = 1×3  
0 0 0
```

```
% Acceleration for the coupler:
```

```
A_BAn
```

```
A_BAn = (-5.1137137608532415855577824004286 -251.94810960110826414393159211613 0)
```

A_BAt

A_BAt = (-182.6623794608034915043504042842 3.7074424766186001495293922403107 0)

% Acceleration for the rocker:

A_Bn

A_Bn = (66.014421721736411541314737405628 -127.97693590934554208615736570209 0)

A_Bt

A_Bt = (-427.04303801249740574874635967717 -220.2818734827191882619246143804 0)

% The summation of acceleration components must equal zero to
% satisfy the closure condition of the acceleration polygon:

z = A_An + A_At + A_BAn + A_BAt - A_Bn - A_Bt

z = (173.25252306910426111752343558692 - 100 $\sqrt{3}$ 0.018142267575066353679780206675787 0)

sumZ = norm(z,3)

sumZ =

(0.0000059713799258665899925807356331734 - (100 $\sqrt{3}$ - 173.25252306910426111752343558692)