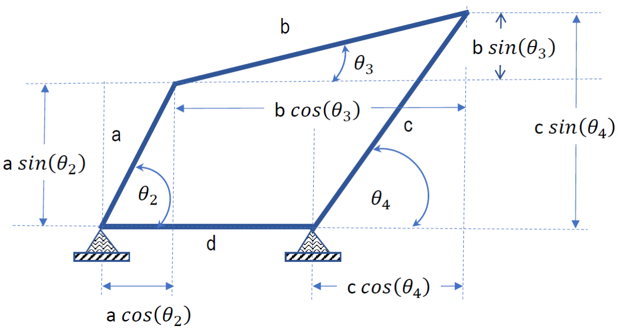
TABLE P7-1 Data for Problems 7-3, 7-4 and 7-11‡

Row	Link 1	Link 2	Link 3	Link 4	θ_2	ω_2	α_2	R_{pa}	δ_3
a	6	2	7	9	30	10	0	6	30
b	7	9	3	8	85	−12	5	9	25
c	3	10	6	8	45	−15	−10	10	80
d	8	5	7	6	25	24	−4	5	45
e	8	5	8	6	75	−50	10	9	300
f	5	8	8	9	15	−45	50	10	120
g	6	8	8	9	25	100	18	4	300
h	20	10	10	10	50	−65	25	6	20
i	4	5	2	5	80	25	−25	9	80
j	20	10	5	10	33	25	−40	1	0
k	4	6	10	7	88	−80	30	10	330
l	9	7	10	7	60	−90	20	5	180
m	9	7	11	8	50	75	−5	10	90
n	9	7	11	6	120	15	−65	15	60

‡ Drawings of these linkages are in the *PDF Problem Workbook* folder on the DVD.

Source: Norton, R.L. Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines. 5th Edition. McGraw-Hill Co.



Inputs

```
a = 2;
b = 7;
c = 9;
d = 6;
theta2 = 30;
```

Initial Solution (Guess Values)

```
theta3 = 70;
theta4 = 60;

x = [theta3; theta4];
x = matlab_solver(x, theta2, a,b,c,d);
```

```
                                         Norm of      First-order   Trust-region
 Iteration  Func-count     ||f(x)||^2        step      optimality         radius
     0          3           40.6723                         0.85              1
     1          6           38.4768             1          0.826              1
     2          9           33.3065           2.5          0.765            2.5
     3         12           22.4001          6.25          0.608           6.25
     4         15           7.77361        15.625          0.213           15.6
     5         18           0.70106       39.0625          0.112           39.1
     6         21        0.00759546       13.2563        0.00726           97.7
     7         24       2.20754e-06       1.71146        0.00017           97.7
     8         27        6.4235e-14     0.0257659       2.84e-08           97.7

Equation solved.
```

```
fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
x = 2×1
   88.8372
  117.2861
fval = 2×1
10^-6 ×
    0.2304
   -0.1057
```

```
% result at this point
theta3 = x(1);
theta4 = x(2);

disp([theta2, theta3, theta4])
```

```
   30.0000   88.8372  117.2861
```

Velocity Analysis:

$$\overrightarrow{v_B} = \overrightarrow{v_A} + \vec{\omega} \times \overrightarrow{r_{B/A}}$$

```
w2 = 10;            % In rad/s

% Position vectors
rO2A = [a*cosd(theta2) a*sind(theta2) 0]
```

```
rO2A = 1×3
    1.7321    1.0000         0
```

```
rAB = [b*cosd(theta3) b*sind(theta3) 0]
```

```
rAB = 1×3
    0.1420    6.9986         0
```

```
rO4B = [c*cosd(theta4) c*sind(theta4) 0]
```

```
rO4B = 1×3
   -4.1259    7.9986         0
```

```
% w2 using unit vector k
k = [0 0 1];
VA = w2*cross(k,rO2A)
```

```
VA = 1×3
  -10.0000   17.3205         0
```

```
% Given the x and y components of velocity vector at join B, you can use the Pythagoras' formula
% to find the resultant velocity:
mag_VA =  round(sqrt(VA(1)^2+VA(2)^2),2)        % in/s
```

```
mag_VA =
20
```

```
% You can calculate the magnitude of VA with norm() built-in function
% in MATLAB by typing:
Vel_mag_A = norm(VA,2)  % Magnitude of velocity at join A.
```

```
Vel_mag_A =
20
```

```
                        % Builtin MATLAB norm() function: norm of a vector
is simply the
                        % square root of the sum of each component squared.

% Next, the following two equations can be defined at join B. But, first,
define the
% symbolic variables, say, w3 & w4:
syms w3 w4

% At join B:
V_B1 = VA + w3*cross(k,rAB);   % wrt coupler link -> Eq (1)
V_B2 = w4*cross(k,rO4B);       % wrt rocker link -> Eq (2)

eqn_vel = V_B1 - V_B2 == 0;
[w3, w4] = vpasolve(eqn_vel,[w3 w4]) % rad/s
```

w3 = $-5.9909661264503475173066209426324$

w4 = $-3.9917352019590763262378163377905$

```
w3 = round(w3, 1)
```

w3 = -6.0

```
w4 = round(w4, 1)
```

w4 = -4.0

```
VC = subs(V_B2,w4)
```

VC = $(31.994233977336385521539341425523 \quad 16.503605430434102885328684351407 \quad 0)$

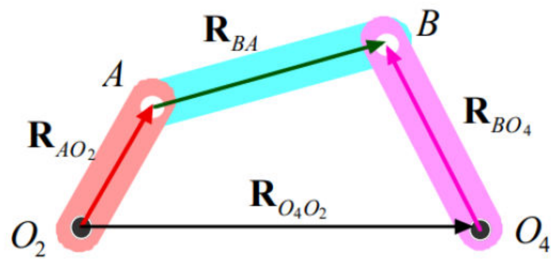
```
% Given the x and y components of velocity vector, you can use the
Pythagoras' formula
% to find the resultant velocity:
mag_VC =  round(sqrt(VC(1)^2+VC(2)^2),2)
```

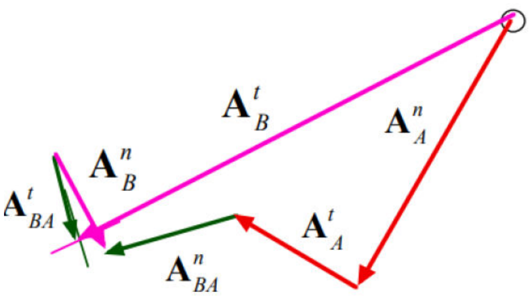
mag_VC = 36.0

```
% Again, you can calculate the magnitude of VC with norm() built-in function
% in MATLAB:
Vel_mag_C = round(norm(VC,2),2)             % Magnitude of velocity at point
C
```

```
Vel_mag_C = 36.0
```

Acceleration Analysis:





Rotational Acceleration: α_3 and α_4

```
alpha2 = 0;  % rad/s^2           % In rad/s2, CCW

% Position vectors
rAO2 = [a*cosd(theta2) a*sind(theta2) 0];
rBA =  [b*cosd(theta3) b*sind(theta3) 0];
rBO4 = [c*cosd(theta4) c*sind(theta4) 0];

A_An = -w2^2*rAO2
```

```
A_An = 1×3
 -173.2051 -100.0000         0
```

```
A_At = alpha2*cross(k,rAO2)
```

```
A_At = 1×3
     0     0     0
```

```
A_A = sqrt(A_An.^2 + A_At.^2)
```

```
A_A = 1×3
  173.2051  100.0000         0
```

```
% vnom => MATLAB norm with variable precision arithmetic.
acc_mag_A = vnorm(A_A,2)  % Magnitude of acceleration at point A
```

acc_mag_A =
200

```
syms alpha3 alpha4
A_A = -w2^2*rAO2 + alpha2*cross(k,rAO2)
```

A_A = 1×3
-173.2051 -100.0000 0

```
A_B1 = A_A + alpha3*cross(k,rBA) - (w3)^2*rBA    % with respect to coupler
```

A_B1 =

$$\left(-100\ \sqrt{3} - 5.1137137608532415855577824004286 - \frac{3939838237903653\ \alpha_3}{562949953421312} \quad -351.94810960110826\right.$$

```
A_B2 = alpha4*cross(k,rBO4) - (w4)^2*rBO4        % % with respect to rocker
```

A_B2 =

$$\left(66.014421721736411541314737405628 - \frac{4502788131823019\ \alpha_4}{562949953421312} \quad -127.97693590934554208615736\right.$$

```
eqn_ABB = A_B1 - A_B2 == 0;

% vpasolve => MATLAB solver with variable precision arithmetic.
% R = vpasolve(eqn_ABB,[alpha3; alpha4]);

[alpha3, alpha4] = vpasolve(eqn_ABB,[alpha3, alpha4])  % rad/sec^2
```

alpha3 = $26.101687028206136417485845465192$

alpha4 = $53.385544753910192164375737527304$

```
% Coupler's rotational acceleration
alpha3 = R.alpha3;
alpha3 = round(alpha3,2) % rad/s^2
```

alpha3 = 26.1

```
% Rocker's rotational acceleration
alpha4 = R.alpha4;
alpha4 = round(alpha4,2) % rad/s^2
```

alpha4 = 53.39

```
% Magnitude of acceleration at point C:
%
% For coupler:
A_B1 = subs(A_B1, alpha3)
```

A_B1 = $\begin{pmatrix} -100\sqrt{3} - 187.77609322165673308990818668462 & -348.24066712448966399440219987582 & 0 \end{pmatrix}$

```
Acc_mag_B1 = norm(A_B1, 2);
Acc_mag_B1 = round(Acc_mag_B1, 2)
```

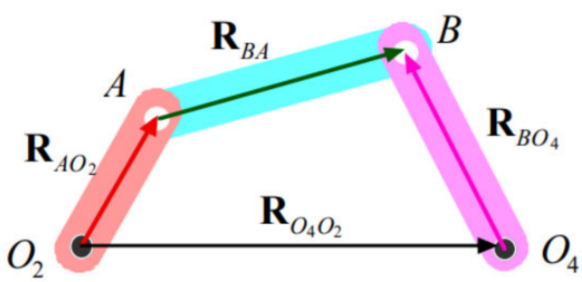
Acc_mag_B1 = 501.58

```


% For rocker:
A_B2 = subs(A_B2,alpha4);
Acc_mag_B2 = norm(A_B2, 2);
Acc_mag_B2 = round(Acc_mag_B2, 2)
```

Acc_mag_B2 = 501.62

```


% As expected A_C1 = A_C2
```

Normal and Tangencial Components of Acceleration:



$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}$$

Or,

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \breve{\mathbf{R}}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4}) - \alpha_4 \breve{\mathbf{R}}_{BO_4} = \mathbf{0}$$

```
% Link's Acceleration: Normal and Tangencial Components

% Acceleration of the crank, unit/time^2:
% Compute the magnitude of A_An:
A_An = w2^2*norm(rAO2,2)                    % unit/time^2
```

```
A_An =
200
```

```
% Compute the magnitude of A_At:
A_At = alpha2*norm(rAO2,2)                % unit/time^2
```

```
A_At =
0
```

```
% Resultant Acceleration of the crank:
A_A = sqrt(A_An^2 + A_At^2)               % unit/time^2
```

```
A_A =
200
```

```
% Acceleration for the coupler , unit/time^2:
% Compute the magnitude of A_BAn:
A_BAn = w3^2*norm(rBA,2)
```

A_BAn = 252.0

```
% Compute the magnitude of baAt:
A_BAt = alpha3*norm(rBA,2)
```

A_BAt = 182.7

```
% Resultant Acceleration for the coupler:
A_BA = round(sqrt(A_BAn^2 + A_BAt^2), 2)
```

A_BA = 311.26

```
% Acceleration for the rocker, unit/time^2:
% Compute the magnitude of A_Bn:
A_Bn = w4^2*norm(rBO4,2)
```

A_Bn = 144.0

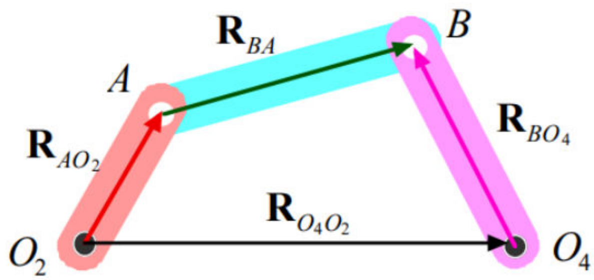
```
% Compute the magnitude of baAt:
A_Bt = alpha4*norm(rBO4,2)
```

A_Bt = 480.51

```
% Resultant Acceleration for the rocker:
A_B = round(sqrt(A_Bn^2 + A_Bt^2),2)
```

A_B = 501.62

Normal and Tangencial Components of Acceleration:



$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}$$

Or,

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \breve{\mathbf{R}}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4}) - \alpha_4 \breve{\mathbf{R}}_{BO_4} = \mathbf{0}$$

```
% Link's Acceleration: Normal and Tangencial Components

% Acceleration of the crank:
% Compute the magnitude of A_An:
m_rAO2 = norm(rAO2,2)
```

```
m_rAO2 =
2
```

```
A_An = -w2^2*rAO2
```

```
A_An = 1×3
 -173.2051 -100.0000         0
```

```
mA_An = norm(A_An,2)
```

```
mA_An =
200
```

```
%A_An = w2^2*sqrt((a*cosd(theta2))^2+(a*sind(theta2))^2+(0)^2)

% Compute the magnitude of A_At:
A_At = alpha2*cross(k,rAO2)
```

```
A_At = 1×3
     0     0     0
```

```
mA_At = norm(A_At,2)
```

```
mA_At =
0
```

```
% Resultant Acceleration of the crank:
A_A = sqrt(mA_An^2 + mA_At^2)
```

```
A_A =
200
```

```
% Acceleration for the coupler:
% Compute the magnitude of A_BAn:
m_rBC = norm(rBA,2)
```

```
m_rBC =
7
```

```
A_BAn = -(w3^2)*rBA
```

A_BAn = $(-5.1137137608532415855577824004286 \quad -251.94810960110826414393159211613 \quad 0)$

```
mA_BAn = norm(A_BAn,2)
```

mA_BAn = $252.00000000000000411920109575326$

```
% Compute the magnitude of baAt:
A_BAt = alpha3*cross(k,rBA)
```

A_BAt = $(-182.66237946080349150435040428420 \quad 3.7074424766186001495293922403107 \quad 0)$

```
mA_BAt = norm(A_BAt,2)
```

mA_BAt = $182.70000000000000298642079442111$

```
% Resultant Acceleration for the coupler:
A_BA = sqrt(mA_BAn^2 + mA_BAt^2)
```

A_BA = $311.26080704129776956725875990209$

```
% Acceleration for the rocker:
% Compute the magnitude of A_Bn:
m_rDC = norm(rBO4,2)
```

```
m_rDC =
9
```

```
A_Bn = -(w4^2)*rBO4
```

A_Bn = $(66.014421721736411541314737405628 \quad -127.97693590934554208615736570209 \quad 0)$

```
mA_Bn = norm(A_Bn,2)
```

mA_Bn = $144.00000000000000228141608949360$

```
% Compute the magnitude of baAt:
A_Bt = alpha4*cross(k,rBO4)
```

A_Bt = $(-427.04303801249740574874635967717 \quad -220.28187348271918826192461438040 \quad 0)$

```
mA_Bt = norm(rBO4,2)
```

mA_Bt =
9

```
% Resultant Acceleration for the rocker:
A_B = round(sqrt(mA_Bn^2 + mA_Bt^2),2)
```

A_B = 144.28

$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^n - \mathbf{A}_B^t = \mathbf{0}$$

Or,

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \breve{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \breve{\mathbf{R}}_{BA} - (-\omega_4^2 \mathbf{R}_{BO_4}) - \alpha_4 \breve{\mathbf{R}}_{BO_4} = \mathbf{0}$$

```
% sum of acceleration (closure of the Acceleration polygon):
% z = A_An + A_At + A_BAn + A_BAt - A_Bn - A_Bt = 0
```

```
% Acceleration of the crank:
A_An
```

```
A_An = 1×3
 -173.2051 -100.0000         0
```

```
A_At
```

```
A_At = 1×3
     0     0     0
```

```
disp(["Normal Acceleration of the crank = ", string(A_An)])
```

```
    "Normal Acceleration of the crank = "    "-173.2051"    "-100"    "0"
```

```
disp(["Tangential Acceleration of the crank = ", string(A_At)])
```

```
    "Tangential Acceleration of the crank = "    "0"    "0"    "0"
```

```
% Acceleration for the coupler:
A_BAn
```

A_BAn = $(-5.1137137608532415855577824004286 \quad -251.94810960110826414393159211613 \quad 0)$

```
A_BAt
```

A_BAt = $(-182.66237946080349150435040428 42\quad 3.7074424766186001495293922403107\quad 0)$

```
disp(["Normal Acceleration of the coupler = ", string(A_BAn)])
```

    "Normal Acceleration of the coupler = "   "-5.1137137608532415855577824004286"   "-251.9481096011082

```
disp(["Tangential Acceleration of the coupler = ", string(A_BAt)])
```

    "Tangential Acceleration of the coupler = "   "-182.66237946080349150435040428 42"   "3.7074424766186

```

```

```
% Acceleration for the rocker:
A_Bn
```

A_Bn = $(66.014421721736411541314737405628\quad -127.97693590934554208615736570209\quad 0)$

```
A_Bt
```

A_Bt = $(-427.04303801249740574874635967717\quad -220.28187348271918826192461438 04\quad 0)$

```
disp(["Normal Acceleration of the rocker = ", string(A_Bn)])
```

    "Normal Acceleration of the rocker = "   "66.014421721736411541314737405628"   "-127.976935909345542

```
disp(["Tangential Acceleration of the rocker = ", string(A_Bt)])
```

    "Tangential Acceleration of the rocker = "   "-427.04303801249740574874635967717"   "-220.2818734827

```
% The summation of acceleration components must equal zero to
% satisfy the closure equation of the acceleration polygon:
z = A_An + A_At + A_BAn + A_BAt - A_Bn - A_Bt;

sumZ = vpa(norm(z,3)) % z =~ 0.048 -> Closed to zero, then you could said
that the closure equation is verified!
```

sumZ = $0.048310666495977592917854194489296$