Velocidad Vectorial

Antonio Perez

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$\circledast \; |\vec{V_A}| = \vec{V_{O_2}} + \vec{\omega_2} \times \vec{R_{AO_2}}$ — $V_{O_2} = 0$ (estatico)

$\vec{R_{AO_2}}$ ↦ Vector posicion de A con respecto O_2.

- $\vec{R_{AO_2}} = [a * \cos(\theta_2), a * \sin(\theta_2), 0]$
- $\vec{R_{AO_2}} = [2 * \cos(30°), 2 * \sin(30°), 0]$
- * $\vec{R_{AO_2}} = [1.732, 1, 0]$: Vector posicion desde el origen O_2 hasta el punto A.
- $\vec{R_{AB}} = [b \times \cos(\theta_3), b \times \sin(\theta_3), 0]$
- $\vec{R_{AB}} = [7 \times \cos(89°), 7 \times \sin(89°), 0]$
- * $\vec{R_{AB}} = [0.12215, 6.99895, 0]$: Vector posicion desde el origen A hasta el punto B
- $\vec{R_{O_4B}} = [c \times \cos(\theta_4), c \times \sin(\theta_4), 0]$
- $\vec{R_{O_4B}} = [9 \times \cos(117°), 9 \times \cos(117°), 0]$
- * $\vec{R_{O_4B}} = [-4.08591, 8.01909, 0]$
- * $|\vec{V_A}| = \vec{\omega_2} \times \vec{R_{AO_2}} \therefore |\vec{V_A}| = 10 \times [1.732, 1, 0]$
- $|\vec{V_A}| = \begin{vmatrix} i & j & k \\ 0 & 0 & 10 \\ 1.732 & 1 & 0 \end{vmatrix} = i(0\times0 - 10\times1.0) - j(0\times0 - 10\times1.732) + k(0\times1 - 0\times1.732)$
- $|\vec{V_A}| = i(-10) - j(-17.32) + k(0)$
- * $|\vec{V_A}| = [-10, 17.32, 0]$ unidades de longitud x Segundo

La magnitud o modulo de un vector $V = [V_x, V_y, V_z]$ se calcula como:

- $|V| = \sqrt{V_x^2 + V_y^2 + V_z^2} \therefore V_z = 0 \therefore |V| = \sqrt{V_x^2 + V_y^2}$
- $|V_A| = \sqrt{(-10)^2 + (-17.32)^2} = |V_A| = \sqrt{399.9824} = |V_A| \cong 20.0$
- $|V_A| \cong 20.$ unidades de longitud por segundo
- $k = [0, 0, 1]$: eje de rotacion z

$\circledast \; |V_{B1}| = \vec{V_A} + \vec{\omega_3} \times \vec{R_{AB}}$

$\circledast \; |V_{B2}| = \vec{\omega_4} \times R_{O_4B}$

- Producto Cruzado entre dos Vectores:

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \times \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

- Producto Vectorial Cruzado:
- $\omega_3 = [0, 0, \omega_3]$, $\omega_4 = [0, 0, \omega_4]$
- $\omega_3 \times R_{AB} = \begin{bmatrix} 0 \\ 0 \\ \omega_3 \end{bmatrix} \times \begin{bmatrix} X_{AB} \\ Y_{AB} \\ 0 \end{bmatrix} = \begin{bmatrix} -\omega_3 Y_{AB} \\ \omega_3 X_{AB} \\ 0 \end{bmatrix}$
- $\omega_4 \times R_{O_4B} = \begin{bmatrix} 0 \\ 0 \\ \omega_4 \end{bmatrix} \times \begin{bmatrix} x_{O_4B} \\ y_{O_4B} \\ 0 \end{bmatrix} = \begin{bmatrix} -\omega_4 Y_{O_4B} \\ \omega_4 X_{O_4B} \\ 0 \end{bmatrix}$

(*) . Ecuacion de igualdad:

- $V_A + \omega_3 \times R_{AB} = \omega_4 \times R_{O_4B}$
- $[-10, 17.32, 0] + [-\omega_3 * 6.99895, \omega_3 * 0.12215, 0] =$

  $[-10 - 6.99895\omega_3, 17.32 + 0.12215\omega_3, 0]$
- $[-\omega_4 * 8.01909, \omega_4 * (-4.08591), 0] = [-8.01909\omega_4, -4.08591\omega_4, 0]$

(*) Se iguala componente a componente (X y Y):

componente X:

$$-10 - 6.99895\omega_3 = -8.01909\omega_4$$

componente Y:

$$17.32 + 0.12215\omega_3 = -4.08591\omega_4$$

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$$\omega_4 = \frac{10 + 6.99895\omega_3}{8.01909}$$

- $17.32 + 0.12215\omega_3 = -4.08591\left(\dfrac{10 + 6.99895\omega_3}{8.01909}\right)$

(*) . $\omega_3 = -6.08$ rad/s ✓

(*) . $\omega_4 = \dfrac{10 + 6.99895(-6.08)}{8.01909} = -4.06$ rad/s ✓

- Entonces, ahora se calcula la Velocidad $|V_{B2}|$ (link C)

(*) $|V_{B_2}| = \omega_4 \times R_{O_4B} = -4.06 \times [-4.08591, 8.01909, 0]$

$\omega_4 \times R_{O_4B} = \begin{bmatrix} 0 \\ 0 \\ \omega_4 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\omega_4 \cdot y \\ \omega_4 \cdot x \\ 0 \end{bmatrix} = \begin{bmatrix} 32.585 \\ 16.593 \\ 0 \end{bmatrix}$

- $|V_{B_2}| = [32.585, 16.593, 0]$ ✓

(*) Siguiente se calcula la magnitud (modulo) del Vector de Velocidad C.

$|V_C| = \sqrt{(V_{Cx})^2 + (V_{Cy})^2} = |V_C| = \sqrt{(32.585)^2 + (16.593)^2}$

(*) $|V_C| \cong 36.57$ ✓ $= |V_{B_2}|$

- $|\vec{V}_{BA}| = \vec{\omega}_3 \times \vec{R}_{BA} = -6.08 \times \vec{R}_{BA}$
- $|\vec{V}_{BA}| = \vec{\omega}_3 \times b = -6.08 \times 7$

(*) $|\vec{V}_{BA}| = 42.56$ ✓

- $A_A^n + A_A^t + A_{BA}^n + A_{BA}^t - A_B^n - A_B^t = 0$

$$-\omega_2^2 R_{AO_2} + \alpha_2 \breve{R}_{AO_2} - \omega_3^2 R_{BA} + \alpha_3 \breve{R}_{BA} - (-\omega_4^2 R_{BO_4}) - \alpha_4 \breve{R}_{BO_4} = 0$$

$$\underbrace{-\omega_2^2 R_{AO_2} + \alpha_2 \breve{R}_{AO_2} - \omega_3^2 R_{BA} - (-\omega_4^2 R_{BO_4})}_{\text{Know}} + \underbrace{\alpha_3 \breve{R}_{BA} - \alpha_4 \breve{R}_{BO_4}}_{\text{UNKNOW}} = 0$$

- $|A_A^n = -\omega_2^2 \times R_{AO_2} = (10)^2 \cdot [-1.732, -1, 0] = [173.205, 100, 0]$

(*) $|A_A^n = [173.205, 100, 0]$: Acceleracion centripeta del punto A, dirigido hacia el punto O_2, y es causada exclusivamente por la rotacion del link 2.

↳ Acceleracion centripeta

- donde $\alpha_2 = 0$

(*) $|A_A^t| = \alpha_2 \times \breve{R}_{AO_2} = [0,0,0] : \underline{0}$ ✓

↳ Acceleracion tangencial

- $|\vec{A}_A = \vec{A}_A^n + \vec{A}_A^t = [173.205, 100, 0] + [0,0,0] = [173.205, 100, 0]$

↳ Acceleracion total del punto A

(*) $|\vec{A}_A^n| = \sqrt{(173.205)^2 + (100)^2} = \underline{200}$ ✓ $= (10)^2 \times 2 = 200$ ($\omega_2^2 * R_{AO_2}$)

(*) $|A_{BA}^n| = \omega_3^2 * R_{BA} = (-6.08)^2 * 7 = \underline{258.7648}$ ✓

(*) $|A_B^n| = \omega_4^2 * R_{BO_4} = (-4.06)^2 * 9 = \underline{148.3524}$ ✓

- Los terminos de aceleracion normal y tangencial se representan como escalares en una direccion proyectada. La forma escalar de esta ecuacion es:

- $\alpha_3 \cdot \breve{R}_{BA} - \alpha_4 \cdot R_{BO_4} = -(\underbrace{\omega_2^2 \cdot R_{AO_2}}_{200} + \underbrace{\alpha_2 \cdot R_{AO_2}}_{0} + \underbrace{\omega_3^2 \cdot R_{BA}}_{258.7648} - \underbrace{\omega_4^2 \cdot R_{BO_4}}_{148.3524})$

- $\alpha_3 \cdot \breve{R}_{BA} - \alpha_4 \breve{R}_{BO_4} = -(200 + 0 + 258.7648 - 148.3524)$
- $\alpha_3 \cdot \breve{R}_{BA} - \alpha_4 \breve{R}_{BO_4} = -310.4124$
- $|R_{BA}| = 7$ = link b
- $|R_{BO_4}| = 9$ = link c

(*) $7 \cdot \alpha_3 - 9 \cdot \alpha_4 = -310.4124$ (solo una ecuacion 2 incognitas); :/

Vamos a tomar como inicio la aceleracion total del punto A

- $\vec{A}_A = -\omega_2^2 \vec{R}_{AO_2} + \alpha_2 (k \times \vec{R}_{AO_2}) = \begin{bmatrix} -173.205 \\ -100 \\ 0 \end{bmatrix}$, razon porque $\alpha_2 = 0$.
- es negativo porque $\vec{R}_{AO_2}$, ya apunta hacia O_2, al multiplicar por $-\omega_2^2$, en realidad terminamos apuntando hacia afuera, es decir, hacia A, lo cual es incorrecto para una aceleracion centripeta, entonces:

$$\vec{R}_{O_2A} = [a\cos\theta_2, a\sin\theta_2] \therefore \vec{A}_A = -\omega_2^2 \cdot \vec{R}_{O_2A} = \begin{array}{cc} x & y \\ \end{array} [-173.205, -100]$$

- entonces:

(*) Se calcula la expresion simbolica para la aceleracion del punto B con respecto al eslabon coupler (link 3).

- $\vec{A}_{B1} = \vec{A}_A + \alpha_3 \cdot (\hat{k} \times \vec{R}_{BA}) - \omega_3^2 \cdot \vec{R}_{BA}$ ✓

(*) $$\vec{A}_{B1} = \begin{bmatrix} -173.205 + 0.488\alpha_3 + 4.516 \\ -100 - 27.996\alpha_3 + 258.725 \\ 0 \quad 0 \quad 0 \end{bmatrix}$$

(*) Se calcula la expresion simbolica de la aceleracion del punto B, con respecto al eslabon 4 (link 4 - rocker)

- $$\vec{A}_{B2} = \begin{bmatrix} 4.086\alpha_4 + 32.557 \\ -8.019\alpha_4 + 16.589 \\ 0 \quad 0 \end{bmatrix} = \alpha_4 \cdot (\hat{k} \times \vec{R}_{BO_4}) - \omega_4^2 \cdot \vec{R}_{BO_4}$$ ✓

- entonces:

$\vec{A}_{B1} - \vec{A}_{B2} = 0 \therefore \vec{A}_{B1} = \vec{A}_{B2}$

- Siguiente paso se igualan las componentes $X = X$ y $Y = Y$

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- Componente en X:

- $-\cancel{173.205} + 0.488\alpha_3 + \cancel{4.516} = 4.086\alpha_4 + \cancel{32.557}$

- $0.488\alpha_3 - 4.086\alpha_4 = 200.246 \therefore \alpha_3 = \dfrac{200.246 + 4.086\alpha_4}{0.488}$

- Componente y:

$-\cancel{100.0} - 27.996\alpha_3 + \cancel{258.725} = -8.019\alpha_4 + \cancel{16.589}$

$-27.996\alpha_3 + 8.019\alpha_4 = -142.136 \therefore \alpha_4 = \dfrac{-142.136 + 27.996\alpha_3}{8.019}$

- $\alpha_3 = \dfrac{200.246 + 4.086\left(\dfrac{-142.136 + 27.996\alpha_3}{8.019}\right)}{0.488}$

- $\alpha_3 = 21.196$ rad/s^2. ✓

- $\alpha_4 = 5.862$ rad/s^2. ✓

- ahora, calcular magnitudes:

- $\vec{A}_{B_1} = \vec{A}_A + \alpha_3 . (\hat{k} \times \vec{R}_{BA}) - \omega_3^2 . \vec{R}_{BA}$
- $\vec{A}_A = -\omega_2^2 . \vec{R}_{O_2A}$
- $\omega_3 = -6.08$
- $\alpha_3 = 21.196$
- $\vec{R}_{BA} = 7$
- Computando métricas:

$|\vec{A}_{B_1}| = \sqrt{x^2 + y^2}$

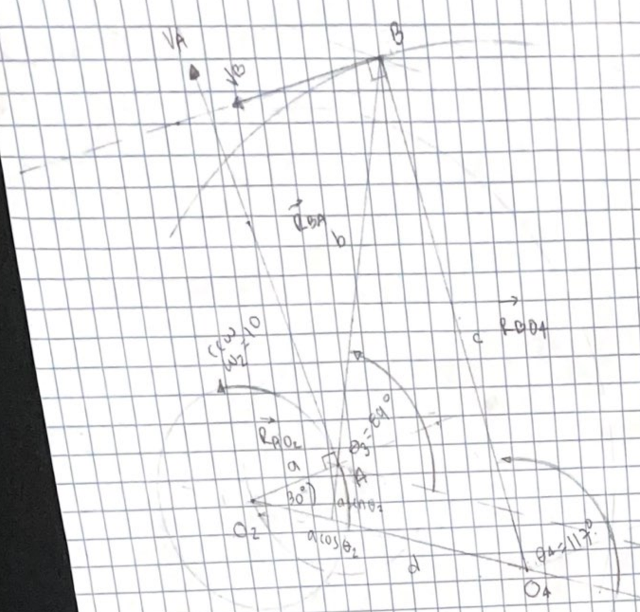
$|\vec{A}_{B_1}| = 157.455$ unidades/s^2.

Escala 1/2 Pulgada = 1 (9)

$\vec{R}_{AO_2}$ $a = 2$
$\vec{R}_{BA}$ $b = 7$
R_{BO_4} $c = 9$
$\vec{R}_{O_2O_4}$ $d = 6$

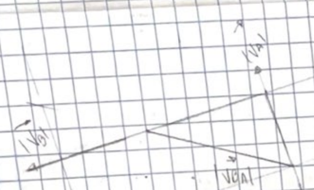
$\theta_2 = 30$ $\theta_3 = 80^\circ$ $\theta_4 = 117^\circ$
$\omega_2 = 10$ rad/s
$\alpha_2 = 0$
- $\alpha_3 = ?$
- $\alpha_4 = ?$
- $\omega_3 = -6.08$ rad/s
- $\omega_4 = -4.06$ rad/s

$|V_{B_2}| = |V_B| = 36.57$
$|V_A| = 20$



escala Velocidad
1 cm = 10

- $\vec{\omega}_2 \times a$ $|\vec{V}_A| = 20$
- $\vec{\omega}_4 \times c$ $|V_B| = 36.57$
- $\vec{\omega}_3 \times b$ $|\vec{V}_{BA}| = 42.56$



- $(\omega_2)^2 * \vec{R}_{AO_2}$ $|\vec{A}_A^n| = 200 = 200$
- ✓ $(\omega_3)^2 * \vec{R}_{BA}$ $|A_{BA}^n| = 258.7648 \cong 258$
- ✓ Negativo $(\omega_4)^2 \vec{R}_{BO_4}$ $|A_B^n| = 148.3524 \cong 144$
- ✓ $0 = (\alpha_2 * \vec{R}_{AO_2})$ $|A_A^t| = 0$

Escala Aceleraciones

1 pulgada = 100

$26.1^\circ \cong$ ✓ $\alpha_3 = 21.196$ rad/s^2

$33.39^\circ \neq$ ✓ $\alpha_4 = 5.862$ rad/s^2

$|A_{BA}^t| = 5.1''$ ∴ $1'' = 100$, $5.0625'' = x$

$501.62 \cong |A_{BA}^t| = \underline{506.25} \cong 501.62$