Homework 5 Antonio Perez

Phase	Motion	Displacement	Time (s)
T_1	Rise	2 in	1.2
T_2	Dwell	0 in	0.3
T_3	Fall	1 in	0.9
T_4	Dwell	0 in	0.6
T_5	Fall	1 in	0.9

Quintic Polynomial

(1.) Calculate the "Time for a full Cycle":

$\sum T_i = T_1 + T_2 + T_3 + T_4 + T_5$

$\sum T_{ciclo} = 1.2 + 0.3 + 0.9 + 0.6 + 0.9 = 3.9$ segundos.

(2.) Calculate the "Required Rotational Speed of the cam":

$\omega_{cam} = \frac{1 \text{ revolucion}}{T_{ciclo}} = \frac{1}{3.9 \text{ sec}} = 0.256 \frac{rev}{s}$

$RPM = 0.256 \frac{rev}{s} \cdot 60 = 15.38 \frac{rev}{min}$

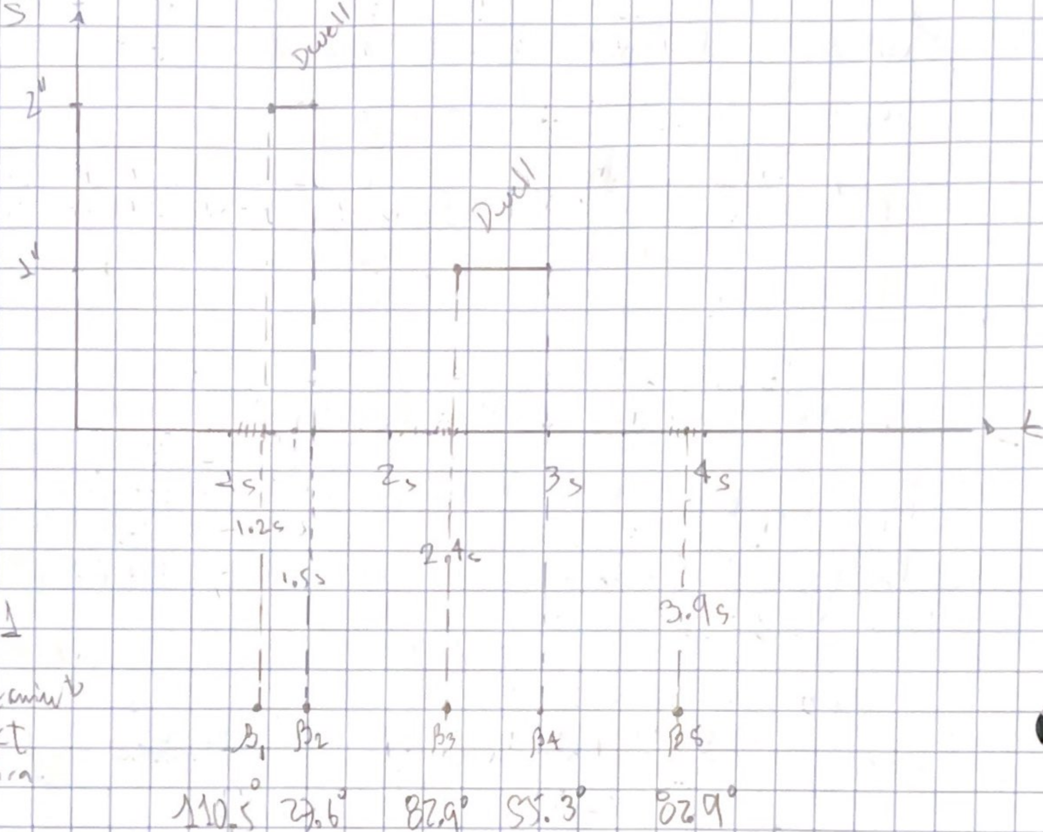
(3.) Cam Rotation for each follower Motion interval:

- $\beta_1 = \omega_{cam} \times T_1 = 0.256 \frac{rev}{s} * 1.2_s * 360° = 110.5°$
- $\beta_2 = \omega_{cam} \times T_2 = 0.256 \frac{rev}{s} * 0.3_s * 360° = 27.6°$
- $\beta_3 = \omega_{cam} \times T_3 = 0.256 \frac{rev}{s} * 0.9_s * 360° = 82.9°$
- $\beta_4 = \omega_{cam} \times T_4 = 0.256 \frac{rev}{s} * 0.6_s \times 360° = 55.3°$
- $\beta_5 = \omega_{cam} \times T_5 = 0.256 \frac{rev}{s} * 0.9_s \times 360° = 82.9°$

Total ciclo:

$\sum \beta_i = 110.5° + 27.6° + 82.9° + 55.3° + 82.9° = 359.2° \cong 360°$

- $X = \frac{\theta_i}{\beta_i} = 1$
- $Y = \frac{S}{h}$ = lift (desplazamiento) / altura



- Formato clasico de polinomios normalizados:
  - $X \in [0, 1]$
  - $\beta_1 = T_1 = 1.2$ (s) (Rise)
  - ω es la velocidad angular de la leva (rad/s), en este sistema se cancelan en las ecuaciones homogeneas
  - Objetivo: calcular los coeficientes C_3, C_4, C_5 del Polinomio
  - Sistema de ecuaciones con condiciones de frontera:
    - evaluadas en $X = 1$ ($\beta = t = 1.2s$) ; $h = 2''$

- Ecuacion 1: $S(1) = C_3 + C_4 + C_5 = h$
- Ecuacion 2: $V(1) = 3C_3 + 4C_4 + 5C_5 = 0$
- Ecuacion 3: $A(1) = 6C_3 + 12C_4 + 20C_5 = 0$

$$\begin{cases} C_3 + C_4 + C_5 = 2 \\ 3C_3 + 4C_4 + 5C_5 = 0 \\ 6C_3 + 12C_4 + 20C_5 = 0 \end{cases}$$

• Resolviendo el sistema de 3x3 matricial: $A \cdot \vec{C} = \vec{b}$

└ Vector Result

└ Vector Incognita

└ matriz de coeficiente

$$\begin{array}{c} Ec1 \\ Ec2 \\ Ec3 \end{array} \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \cdot \begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

• Se aplica la eliminación gaussiana para obtener una matriz triangular superior; el objetivo es transformar la matriz en forma escalonada, donde los elementos por debajo de la diagonal sean 0.

ⓐ eliminar C_3 de la segunda y tercera fila.

• Restamos 3 veces la fila 1:

Fila 2 = Fila 2 − (3 x Fila 1) ∴

$3-(3\times1) = 0$

$4-(3\times1) = 1$

$5-(3\times1) = 2$

$0-(3\times2) = -6$

$$\begin{array}{ccc} & & \\ \end{array}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 6 & 12 & 20 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix}$$

$C_3 \quad C_4 \quad C_5$

ⓑ • Restamos 6 veces la fila 1:

fila 3 = fila 3 − (6 x fila 1) ∴

$6-(6\times1) = 0$

$12-(6\times1) = 6$

$20-(6\times1) = 14$

$0-(6\times2) = -12$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 6 & 14 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ -12 \end{bmatrix}$$

$C_3 \quad C_4 \quad C_5$

ⓒ • Restamos 6 veces la fila 2 de la fila 3:

fila 3 = fila 3 − (6 x fila 2)

$0-(6\times0) = 0$

$6-(6\times1) = 0$

$14-(6\times2) = 2$

$-12-(6\times-6) = 24$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ 24 \end{bmatrix}$$

$C_3 \quad C_4 \quad C_5$

• Sucesión regresiva para encontrar C_3, C_4, C_5. Comenzamos desde la última ecuación y vamos hacia arriba.

• Ecuación 3: $2C_5 = 24$ ∴ $\boxed{C_5 = 12}$ ✓

• Ecuación 2: $C_4 + 2C_5 = -6$ ∴ $\boxed{C_4 = -30}$ ✓

• Ecuación 1: $C_3 + C_4 + C_5 = 2$ ∴ $\boxed{C_3 = 20}$ ✓

$\boxed{\therefore S(x) = 20x^3 - 30x^4 + 12x^5}$ ✓ Posición $B_1 = 1.2 = t$

- Derivar la función de Velocidad $V(x)$

$$V(x) = \frac{ds}{dt} = \frac{ds}{dx} \cdot \frac{dx}{dt} = \frac{ds}{dx} \cdot \frac{1}{\beta_1}$$

$$S(x) = 20x^3 - 30x^4 + 12x^5$$ ✓

$$\frac{ds}{dx} = S'(x) = 60x^2 - 120x^3 + 60x^4 \quad \therefore$$

$$V(x) = \frac{1}{\beta_1}\left(60x^2 - 120x^3 + 60x^4\right)$$ ✓ (NO escalado)

$$V(x) = \frac{1}{1.2_{(1)}}\left(60x^2 - 120x^3 + 60x^4\right)$$

$$\boxed{V(x) = 50x^2 - 100x^3 + 50x^4 \ [in/s]}$$ ✓

- Derivar la función de aceleración $a(x)$

- $$a(x) = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot \frac{1}{\beta_1}$$

- $$V(x) = \frac{1}{\beta} \cdot s'(x) \quad \therefore \quad a(x) = \frac{1}{\beta_1^2} \cdot s''(x)$$

$$s''(x) = \frac{d^2s}{dx^2} = 120x - 360x^2 + 240x^3$$

$$a(x) = \frac{1}{\beta_1^2} \cdot \left(120x - 360x^2 + 240x^3\right)$$ ✓ (NO escalado),

$$a(x) = \frac{1}{(1.2_{(1)})^2} \cdot \left(120x - 360x^2 + 240x^3\right)$$

$$\boxed{a(x) = 83.33x - 250x^2 + 166.67x^3 \ [in/s^2]}$$ ✓

Fase 1 (Rise) ($t = 1.2s$)

- $S(x) = 20x^3 - 30x^4 + 12x^5$
- $V(x) = 50x^2 - 100x^3 + 50x^4$
- $a(x) = 83.33x - 250x^2 + 166.67x^3$

Escalando a 1.2 (s)

Fase 2 (dwell)

- Sistema de ecuaciones con condiciones de frontera: evaluados en: $x = 1$ ($\beta = t = 0.3s$), $h = 2''$
- movimiento en reposo.
- Posición inicio: $S = 2''$
- Posición final. $S = 2''$
- Velocidad y aceleración: ambas en cero, durante todo el intervalo.

- Desplazamiento: $s(t) = 2$ constante
- Velocidad: $V(t) = \frac{ds}{dt} = 0$
- Aceleración: $a(t) = \frac{dv}{dt} = 0$

Fase 3 (fall) (0.9s)

- Movimiento: Caída de $1''$
- $\beta_3 = 0.9\,s$
- $S(0) = 2$ in
- $S(0.9) = 1$ in
- $V(0) = V(0.9) = 0$
- $h = -1$ in

- Ecuación 1: $S(1) = C_3 + C_4 + C_5 = h$
- Ecuación 2: $V(1) = 3C_3 + 4C_4 + 5C_5 = 0$
- Ecuación 3: $a(1) = 6C_3 + 12C_4 + 20C_5 = 0$

$$\begin{cases} C_3 + C_4 + C_5 = -1 \\ 3C_3 + 4C_4 + 5C_5 = 0 \\ 6C_3 + 12C_4 + 20C_5 = 0 \end{cases}$$

- Resolviendo el sistema 3x3 matricial $A \cdot \vec{C} = \vec{b}$, tenemos:

$$\begin{array}{c} \text{Ecu 1} \\ \text{Ecu 2} \\ \text{Ecu 3} \end{array} \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \cdot \begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

- Eliminación Gaussiana:

(a) eliminar C_3 de la segunda y tercera fila:
fila 2 − (3 × fila 1)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 6 & 14 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 6 \end{bmatrix}$$

- $3-(3\times1)=0$
- $4-(3\times1)=1$
- $5-(3\times1)=2$
- $0-(3\times(-1))=3$

(b) Restamos 6 veces la fila 1:
fila 3 − (6 × fila 2)

$$\begin{array}{ccc} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} & \begin{bmatrix} -1 \\ 3 \\ -12 \end{bmatrix} \\ C_3 \quad C_4 \quad C_5 & \end{array}$$

- $0-(6\times0)=0$
- $6-(6\times1)=0$
- $14-(6\times2)=2$
- $6-(6\times3)=-12$

. Sucesión regresiva para encontrar C_1, C_2, C_3.
Comensamos desde la última ecuación hasta la primera:

- $2C_5=-12 \therefore \boxed{C_5=-6}$ ✓ ecuación (3)
- $1C_4+2C_5=3 \therefore \boxed{C_4=15}$ ✓ ecuación (2)
- $C_3+C_4+C_5=-1 \therefore \boxed{C_3=-10}$ ✓ ecuación (1)

(c) $\boxed{S(x)=-10x^3+15x^4-6x^5}$ ✓ Posición $\beta_3=0.9_s=t$

- Derivada de la función de Velocidad $V(x)$

- $V(x)=\dfrac{ds}{dt}=\dfrac{ds}{dx}\cdot\dfrac{1}{\beta_3}=\dfrac{1}{\beta_3}\cdot\left(-30x^2+60x^3-30x^4\right)$ ✓ [in/s]

- Derivada de la aceleración $a(x)$:

$a(x)=\dfrac{dv}{dt}=\dfrac{1}{\beta_3^2}\cdot\dfrac{d^2s}{dx^2}=\dfrac{1}{\beta_3^2}\cdot\left(-60x+180x^2-120x^3\right)$ ✓ [in/s²]

Con $\beta_3 = 0.9$

- $\frac{1}{\beta_3^1} = 1.1111 \Rightarrow \boxed{V(x) = -33.33x^2 + 66.67x^3 - 33.33x^4 \; [in/s]}$

- $\frac{1}{\beta_3^2} = \frac{1}{0.81} = 1.2346 \Rightarrow \boxed{a(x) = -74.08x + 222.22x^2 - 148.15x^3 \; [in/s^2]}$

- Fase 3. (Fall) ($t = 0.9\,s$)

- $S(x) = -10x^3 + 15x^4 - 6x^5$ in
- $V(x) = -30x^2 + 60x^3 - 30x^4$ in/s
- $a(x) = -60x + 180x^2 - 120x^3$ in/s²

NO escalado a 0.9 s.

- $S(x) = -10x^3 + 15x^4 - 6x^5$
- $V(x) = -33.3x^2 + 66.67x^3 - 33.33x^4$ in/s
- $a(x) = -74.08x + 222.22x^2 - 148.15x^3$ in/s²

Escalado a 0.9 s

—— // ——

Fase 4 (Dwell) ($t_4 = 0.6\,s$)

- Posicion constante: $S = 1$ in
- Velocidad: $V(t) = 0$
- Acceleracion $a(t) = 0$
- Duracion: $T_4 = 0.6\,s$

El t_4 simplemente mantiene $S = 1$ in, y se extiende en el tiempo hasta:

$$t_{final_{t_4}} = 1.2(T_1) + 0.3(T_2) + 0.9(T_3) + 0.6(T_4) = 3.0\,s$$

—— // ——

fase 5 (fall) ($t_5 = 0.9\,s$)

todo igual que la fase 3,

Mechanism Design

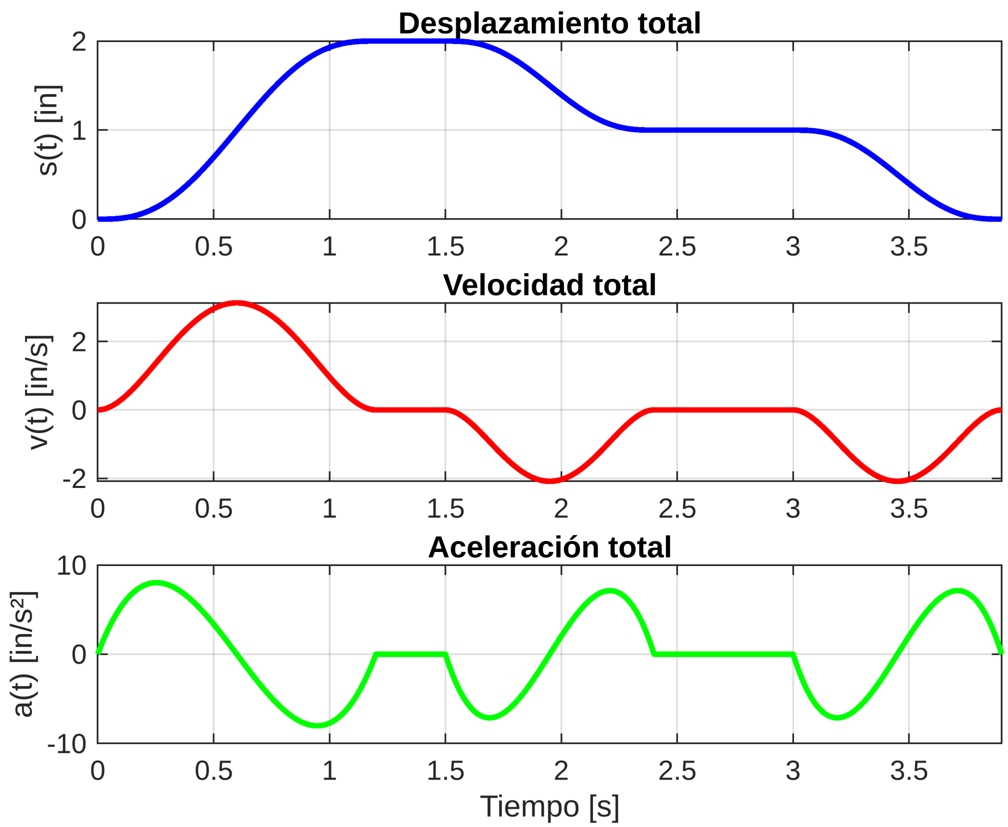
Homework 5

Antonio Perez

05 / 17 /2025

```
clc; clear; close all;

% ============================
% FASE T1 – SUBIDA (0 → 2 in)
% ============================
beta1 = 1.2;
x1 = linspace(0, 1, 1000);
t1 = x1 * beta1;
C3_T1 = 10; C4_T1 = -15; C5_T1 = 6;
h1 = 2;
s1 = h1 * (C3_T1*x1.^3 + C4_T1*x1.^4 + C5_T1*x1.^5);
v1 = h1 / beta1 * (3*C3_T1*x1.^2 + 4*C4_T1*x1.^3 + 5*C5_T1*x1.^4);
a1 = h1 / beta1^2 * (6*C3_T1*x1 + 12*C4_T1*x1.^2 + 20*C5_T1*x1.^3);

% ============================
% FASE T2 – REPOSO EN 2 in
% ============================
T2 = 0.3;
t2 = linspace(t1(end), t1(end) + T2, 300);
s2 = ones(size(t2)) * 2;
v2 = zeros(size(t2));
a2 = zeros(size(t2));

% ============================
% FASE T3 – CAÍDA (2 → 1 in)
% ============================
h3 = -1; beta3 = 0.9;
x3 = linspace(0, 1, 1000);
t3 = x3 * beta3 + t2(end);
C3_T3 = 10; C4_T3 = -15; C5_T3 = 6;
s3 = h3 * (C3_T3*x3.^3 + C4_T3*x3.^4 + C5_T3*x3.^5) + 2;
v3 = h3 / beta3 * (3*C3_T3*x3.^2 + 4*C4_T3*x3.^3 + 5*C5_T3*x3.^4);
a3 = h3 / beta3^2 * (6*C3_T3*x3 + 12*C4_T3*x3.^2 + 20*C5_T3*x3.^3);

% ============================
% FASE T4 – REPOSO EN 1 in
% ============================
```

```
T4 = 0.6;
t4 = linspace(t3(end), t3(end) + T4, 300);
s4 = ones(size(t4)) * 1;
v4 = zeros(size(t4));
a4 = zeros(size(t4));

% ============================
% FASE T5 – CAÍDA (1 → 0 in)
% ============================
h5 = -1; beta5 = 0.9;
x5 = linspace(0, 1, 1000);
t5 = x5 * beta5 + t4(end);
C3_T5 = 10; C4_T5 = -15; C5_T5 = 6;
s5 = h5 * (C3_T5*x5.^3 + C4_T5*x5.^4 + C5_T5*x5.^5) + 1;
v5 = h5 / beta5 * (3*C3_T5*x5.^2 + 4*C4_T5*x5.^3 + 5*C5_T5*x5.^4);
a5 = h5 / beta5^2 * (6*C3_T5*x5 + 12*C4_T5*x5.^2 + 20*C5_T5*x5.^3);

% ============================
% CONCATENACIÓN TOTAL
% ============================
t_total = [t1, t2, t3, t4, t5];
s_total = [s1, s2, s3, s4, s5];
v_total = [v1, v2, v3, v4, v5];
a_total = [a1, a2, a3, a4, a5];

% ============================
% GRÁFICAS COMPLETAS
% ============================
figure('Name', 'Perfil total: T1 + T2 + T3 + T4 + T5');

subplot(3,1,1)
plot(t_total, s_total, 'b', 'LineWidth', 2);
grid on; ylabel('s(t) [in]'); title('Desplazamiento total');
xlim([0 t_total(end)]);

subplot(3,1,2)
plot(t_total, v_total, 'r', 'LineWidth', 2);
grid on; ylabel('v(t) [in/s]'); title('Velocidad total');
xlim([0 t_total(end)]);

subplot(3,1,3)
plot(t_total, a_total, 'g', 'LineWidth', 2);
grid on; ylabel('a(t) [in/s²]'); xlabel('Tiempo [s]');
title('Aceleración total');
xlim([0 t_total(end)]);
```




```
% ===========================
% IMPRESIÓN EN CONSOLA
% ===========================
fprintf('\n⊠ RESUMEN DETALLADO DE TODAS LAS FASES\n');
```

```
⊠ RESUMEN DETALLADO DE TODAS LAS FASES
```

```
fases = {'T1', 'T2', 'T3', 'T4', 'T5'};
h =     [h1,  0,  h3,  0,  h5];
beta =  [beta1, T2, beta3, T4, beta5];
C3 =    [C3_T1, 0, C3_T3, 0, C3_T5];
C4 =    [C4_T1, 0, C4_T3, 0, C4_T5];
C5 =    [C5_T1, 0, C5_T3, 0, C5_T5];
desc = {
    'subida de 0 → 2 in', ...
    'reposo en 2 in', ...
    'caída de 2 → 1 in', ...
    'reposo en 1 in', ...
    'caída de 1 → 0 in'};

for i = 1:5
    fprintf('\n⊠ FASE %s: %s\n', fases{i}, desc{i});
    fprintf('Duración: β = %.2f s\n', beta(i));
```

```
    if h(i) ~= 0
        C3r = h(i)*C3(i); C4r = h(i)*C4(i); C5r = h(i)*C5(i);
        v1 = 3*C3r / beta(i); v2 = 4*C4r / beta(i); v3 = 5*C5r / beta(i);
        a1 = 6*C3r / beta(i)^2; a2 = 12*C4r / beta(i)^2; a3 = 20*C5r /
beta(i)^2;

        fprintf('Desplazamiento: h = %.2f in\n', h(i));
        fprintf('s(x) = %.0f x^3 %+0.f x^4 %+0.f x^5\n', C3r, C4r, C5r);
        fprintf('v(x) = %.0f x^2 %+0.f x^3 %+0.f x^4\n', 3*C3r, 4*C4r,
5*C5r);
        fprintf('a(x) = %.0f x %+0.f x^2 %+0.f x^3\n', 6*C3r, 12*C4r,
20*C5r);
        fprintf('v(x) = %.2f x^2 %+6.2f x^3 %+6.2f x^4   [in/s]\n', v1, v2,
v3);
        fprintf('a(x) = %.2f x %+6.2f x^2 %+6.2f x^3     [in/s²]\n', a1, a2,
a3);
    else
        fprintf('s(x) = constante (%.1f in), v = 0, a = 0\n',
str2double(extractAfter(desc{i}, 'en ')));
    end
end
```

```
☒ FASE T1: subida de 0 → 2 in
Duración: β = 1.20 s
Desplazamiento: h = 2.00 in
s(x) = 20 x^3 -30 x^4 +12 x^5
v(x) = 60 x^2 -120 x^3 +60 x^4
a(x) = 120 x -360 x^2 +240 x^3
v(x) = 50.00 x^2 -100.00 x^3 +50.00 x^4   [in/s]
a(x) = 83.33 x -250.00 x^2 +166.67 x^3     [in/s²]
☒ FASE T2: reposo en 2 in
Duración: β = 0.30 s
s(x) = constante (NaN in), v = 0, a = 0
☒ FASE T3: caída de 2 → 1 in
Duración: β = 0.90 s
Desplazamiento: h = -1.00 in
s(x) = -10 x^3 +15 x^4 -6 x^5
v(x) = -30 x^2 +60 x^3 -30 x^4
a(x) = -60 x +180 x^2 -120 x^3
v(x) = -33.33 x^2 +66.67 x^3 -33.33 x^4   [in/s]
a(x) = -74.07 x +222.22 x^2 -148.15 x^3     [in/s²]
☒ FASE T4: reposo en 1 in
Duración: β = 0.60 s
s(x) = constante (NaN in), v = 0, a = 0
☒ FASE T5: caída de 1 → 0 in
Duración: β = 0.90 s
Desplazamiento: h = -1.00 in
s(x) = -10 x^3 +15 x^4 -6 x^5
v(x) = -30 x^2 +60 x^3 -30 x^4
a(x) = -60 x +180 x^2 -120 x^3
v(x) = -33.33 x^2 +66.67 x^3 -33.33 x^4   [in/s]
a(x) = -74.07 x +222.22 x^2 -148.15 x^3     [in/s²]
```