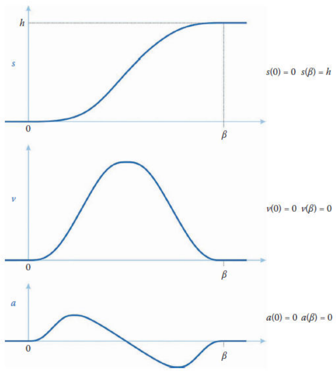
Design of a CAM Profile Using Quintic Polynomial (SVAJ)

We wish to find a rise function for the cam profile that is continuous in its first and second derivatives.

$$s(\lambda) = C_0 + C_1\left(\frac{\lambda}{\beta}\right) + C_2\left(\frac{\lambda}{\beta}\right)^2 + C_3\left(\frac{\lambda}{\beta}\right)^3 + \cdots + C_n\left(\frac{\lambda}{\beta}\right)^n$$



We assume that the cam starts at a displacement of zero and rises to a height, h, by the time the cam has rotated by an angle β. That is:

s(0) = 0 and s(β) = h

Since the velocity and acceleration are both zero during the dwell the first and second derivatives of the rise function must be zero at λ = 0 and λ = β. Therefore:

v(0) = 0 and v(β) = 0

a(0) = 0 and a(β) = 0

We will not specify any conditions on the jerk at this point, noting only that it must be finite since the acceleration is continuous. The following simple function for the pushrod rise, let us try a polynomial function:

$$s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5$$

$$v = \frac{1}{\beta}\left[C_1 + 2C_2\left(\frac{\theta}{\beta}\right) + 3C_3\left(\frac{\theta}{\beta}\right)^2 + 4C_4\left(\frac{\theta}{\beta}\right)^3 + 5C_5\left(\frac{\theta}{\beta}\right)^4\right]$$

$$a = \frac{1}{\beta^2}\left[2C_2 + 6C_3\left(\frac{\theta}{\beta}\right) + 12C_4\left(\frac{\theta}{\beta}\right)^2 + 20C_5\left(\frac{\theta}{\beta}\right)^3\right]$$

[Recall that x = theta/beta]

1. s(0) = 0, we find that **C0 = 0.**
2. By enforcing the boundary condition v(0) = 0 we find that **C1 is also zero.**
3. Enforcing the boundary condition a(0) = 0 **eliminates C2** from our polynomial.

Note that x = 0 when theta = 0 and x = 1 when theta = β. The revised boundary conditions are then:

s(0) = 0 s(1) = h

v(0) = 0 v(1) = 0

a(0) = 0 a(1) = 0

We must now enforce the remaining three boundary conditions that occur at x = 1. We are left with:

$$s(1) = C_3 + C_4 + C_5 = h$$
$$v(1) = (3C_3 + 4C_4 + 5C_5)\frac{\omega}{\beta} = 0$$
$$a(1) = (6C_3 + 12C_4 + 20C_5)\left(\frac{\omega}{\beta}\right)^2 = 0 \quad \text{Eq. (1)}$$

These are three linear equations with three unknowns, so it is easy to put them into matrix form for MATLAB® to solve. **Note that the factors of (ω/β) cancel out in the velocity and acceleration equations since we have zero on the right-hand side.**

The matrix equation is then:

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% matriz A:  is a matrix consisting of the coefficients of the
% variables in linear equations set in Eq. (1) above:
A = [1 1 1;
     3 4 5;
     6 12 20]

% vector b: is the column vector of constant terms:
syms h
b= [h; 0; 0]

% solving systems of linear equations, we get:
C = A\b   % where C stores the solution of the set of linear equations:
% C = inv(A)*b

% That is:
C3 = C(1);
C4 = C(2);
C5 = C(3);

% Enforcing the revised boundary conditions:
% s(0) = 0 s(1) = h
% v(0) = 0 v(1) = 0
% a(0) = 0 a(1) = 0
% Then, the coefficients C0, C1, and C2 are:
C0 = 0;
C1 = 0;
C2 = 0;
```

Now, we write the polynomials s, v, a, and j. But, first, we have to define a symbolic variable, x, to represent the variable $x = \frac{\theta}{\beta}$ in the polynomial expressions for s,v,a, and j; so, we have:

$$s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5$$

$$v = \frac{1}{\beta}\left[C_1 + 2C_2\left(\frac{\theta}{\beta}\right) + 3C_3\left(\frac{\theta}{\beta}\right)^2 + 4C_4\left(\frac{\theta}{\beta}\right)^3 + 5C_5\left(\frac{\theta}{\beta}\right)^4\right]$$

$$a = \frac{1}{\beta^2}\left[2C_2 + 6C_3\left(\frac{\theta}{\beta}\right) + 12C_4\left(\frac{\theta}{\beta}\right)^2 + 20C_5\left(\frac{\theta}{\beta}\right)^3\right]$$

```
syms x

% Change output display format of symbolic results.
% The value for 'PolynomialDisplayStyle'can be 'ascend', 'descend',
% or 'default'.

sympref('PolynomialDisplayStyle','ascend');

% s, v, a, and j polynomials are:
h = 1;
s = (C0 + C1.*(x) + C2.*(x)^2 + C3*(x)^3+ C4*(x)^4 + C5*(x)^5)
v = diff(s, x)
a = diff(v, x)
j = diff(a, x)
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