Thermodynamics - HW4

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Question 1

An automobile has a mass of 1200 kg. What is its kinetic energy, in kJ, relative to the road when traveling at a velocity of 50 km/h? If the vehicle accelerates to 100 km/h, what is the change in kinetic energy, in kJ?

Kinetic Energy Formula

The kinetic energy (E_k) of an object in motion is calculated using the formula:

$$E_k = \frac{1}{2}mv^2$$

Where:

- E_k = Kinetic energy (Joules, J)
- m = Mass of the object (kg)
- v = Velocity of the object (m/s)

Unit Conversion

The given velocities are in km/h and need to be converted to m/s. This requires two steps:

1. **Converting kilometers to meters:**
   Since 1 kilometer = 1000 meters, we multiply the speed in km/h by 1000.
2. **Converting hours to seconds:**
   Since 1 hour = 3600 seconds, we divide the speed by 3600.

$$\text{Velocity in m/s} = \text{Velocity in km/h} \times \frac{1000}{3600}$$

Or, simplifying:

$$\text{Velocity in m/s} = \text{Velocity in km/h} \times 0.2778$$

Calculations:

$$v_1 = 50\,\text{km/h} = \frac{50 \times 1000}{3600} = 13.89\,\text{m/s}$$

$$v_2 = 100\,\text{km/h} = \frac{100 \times 1000}{3600} = 27.78\,\text{m/s}$$

Calculation of Initial Kinetic Energy

$$E_{k1} = \frac{1}{2}(1200)(13.89)^2$$

$$E_{k1} = 0.5 \times 1200 \times 192.45 = 115470\,\text{J} = 115.47\,\text{kJ}$$

Calculation of Final Kinetic Energy

$$E_{k2} = \frac{1}{2}(1200)(27.78)^2$$

$$E_{k2} = 0.5 \times 1200 \times 772.20 = 463320\,\text{J} = 463.32\,\text{kJ}$$

Change in Kinetic Energy

$$\Delta E_k = E_{k2} - E_{k1}$$

$$\Delta E_k = 463.32 - 115.47 = 347.85\,\text{kJ}$$

Explanation of the Conversion Process

The reason for multiplying by 1000 and dividing by 3600 is due to the need to convert units from km/h to m/s, which is the standard unit for velocity in the International System of Units (SI) when calculating kinetic energy.

- **Kilometers to meters:**
  Since 1 kilometer = 1000 meters, we multiply the speed by 1000.
- **Hours to seconds:**
  Since 1 hour = 3600 seconds, we divide the speed by 3600.

Thus, the full conversion is:

$$\text{Velocity in m/s} = \text{Velocity in km/h} \times \frac{1000}{3600}$$

Final Results

- Initial kinetic energy at 50 km/h: 115.47 kJ
- Final kinetic energy at 100 km/h: 463.32 kJ
- Change in kinetic energy: 347.85 kJ

Question 2

An object whose mass is 400 kg is located at an elevation of 25 m above the surface of the earth. For $g = 9.78\,\mathbf{m/s^2}$, determine the gravitational potential energy of the object, in kJ, relative to the surface of the earth.

Gravitational Potential Energy Formula

The **gravitational potential energy** (E_p) of an object in a gravitational field is calculated using the formula:

$$E_p = mgh$$

Where:

- E_p = Gravitational potential energy (Joules, J)
- m = Mass of the object (kg)
- g = Acceleration due to gravity ($9.78\,\mathrm{m/s^2}$)
- h = Height above the surface of the Earth (m)

Given Data

- Mass of the object, $m = 400$ kg
- Acceleration due to gravity, $g = 9.78$ m/s²
- Height above the surface, $h = 25$ m

Calculation of Gravitational Potential Energy

$$E_p = (400)(9.78)(25)$$

$$E_p = 400 \times 244.5 = 97800\,\mathrm{J}$$

$$E_p = 97.8\,\mathrm{kJ}$$

Final Result

The gravitational potential energy of the object is:

$$\boldsymbol{E_p = 97.8}\,\text{kJ}$$

Question 3

A system with a mass of 5 kg, initially moving horizontally with a velocity of 40 m/s, experiences a constant horizontal deceleration of 2 m/s² due to the action of a resultant force. As a result, the system comes to rest. Determine the time, in seconds, the force is applied and the amount of energy transfer by work, in kJ.

Given Data

- Mass of the system (m) = 5 kg
- Initial velocity (v_i) = 40 m/s
- Final velocity (v_f) = 0 m/s (system comes to rest)
- Deceleration (a) = -2 m/s² (negative because it opposes motion)

Calculation of Time Applied

Using the equation of motion:

$$v_f = v_i + at$$

Solving for t:

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{0 - 40}{-2} = \frac{-40}{-2} = 20\,\text{seconds}$$

Calculation of Work Done (Energy Transfer)

The work done by the force is calculated using the change in kinetic energy (ΔKE) of the system:

$$W = \Delta KE = KE_f - KE_i$$

Calculating initial kinetic energy:

$$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(5)(40)^2 = 2.5 \times 1600 = 4000\,\text{J}$$

Calculating final kinetic energy:

$$KE_f = \frac{1}{2}(5)(0)^2 = 0\,\text{J}$$

$$W = 0 - 4000 = -4000\,\text{J} = -4\,\text{kJ}$$

Interpretation of Negative Work

The negative sign indicates that energy was dissipated or removed from the system due to deceleration.

Final Results

- Time during which the force is applied: **20** seconds
- Work done (energy transferred): $\mathbf{-4}\,\text{kJ}$

Question 4

Consider 1 mole of an ideal gas undergoing an isothermal expansion. The initial state of the gas is characterized by a volume V_i and a pressure P_i. The final state after the expansion is characterized by a volume V_f and a pressure P_f. The temperature of the gas remains constant throughout the process and is equal to 25 Celsius degrees.

Given Data

- Number of moles (n) = 1 mol
- Temperature (T) = $25°\text{C} = 298.15\,\text{K}$
- Ideal gas constant (R) = 8.314 J/(mol·K)

Initial State

- Initial Volume (V_i) = 1.0 m^3
- Initial Pressure (P_i) = 10 atm

Final State

- Final Volume (V_f) = 5.0 m^3
- Final Pressure (P_f) = 2 atm

Isothermal Process

In an isothermal process, the temperature remains constant. For an ideal gas, the relationship between pressure and volume is described by **Boyle's Law**:

$$P_i V_i = P_f V_f$$

Checking if the relationship holds:

$$(10\,\text{atm})(1.0\,\text{m}^3) = (2\,\text{atm})(5.0\,\text{m}^3)$$

$$10 = 10 \quad \text{(The relationship holds)}$$

a) Calculation of Total Work Done

Isothermal Expansion of an Ideal Gas. Calculate the total work done by the gas. Use the ideal gas equation $P = \frac{nRT}{V}$ and replace P in the definition of work. Integrate this expression with V as the variable and the initial and final condition. Remember that n and R are constant and the process is isothermal.

Calculation of Work Done using Integration

The work done by an ideal gas during an isothermal process is calculated using:

$$W = \int_{V_i}^{V_f} P\,dV$$

From the ideal gas equation, we have:

$$P = \frac{nRT}{V}$$

Thus, the work becomes:

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Given Data:

- $n = 1$ mol
- $R = 8.314$ J/(mol·K)
- $T = 298.15$ K (25 °C)
- $V_i = 1.0$ m³

- $V_f = 5.0 \text{ m}^3$

$$W = (1)(8.314)(298.15)\ln\left(\frac{5.0}{1.0}\right)$$

Interpretation of $\ln\left(\frac{5}{1}\right)$: When we calculate $\ln\left(\frac{5}{1}\right)$, we are asking: **What power must e be raised to, in order to obtain 5?**

$$\ln\left(\frac{5}{1}\right) = \ln(5) \approx 1.6094$$

This means that:

$$e^{1.6094} \approx 5$$

In other words, if a process grows continuously following the base e, then to multiply its original value by 5, a proportional change of approximately 1.6094 is required. This value represents the magnitude of change in a continuous growth process or in a calculation involving work in an isothermal process.

$$W \approx 2478.47 \times 1.6094$$

$$W \approx 3987.28\,\text{J}$$

$$W \approx 3.99\,\text{kJ}$$

b) PV Curve

Use the following data to draw the PV curve and identify the area under the curve from V_i to V_f as the work done by the gas.

Volume (m³)	**Pressure (Pa)**
1.0	1013250
2.0	506625
3.0	337750
4.0	253312.5
5.0	202650

Table 1: Pressure vs Volume Data

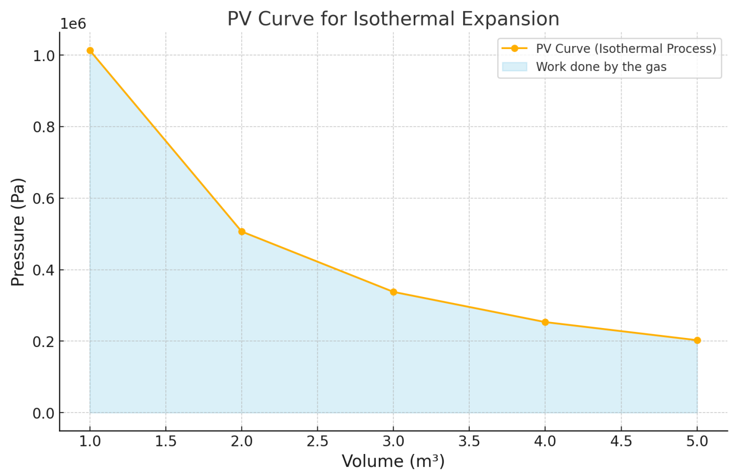

Figure 1: PV Curve for Isothermal Expansion

c) Numerical Approximation using Rectangle Area Method

Calculate the work done by the gas during this expansion using a numerical approximation (rectangle area) and graphically interpret the process on the PV diagram.

The rectangle approximation method calculates the work as:

$$W \approx \sum_{i=1}^{4} P_i \Delta V$$

$$\Delta V = 1.0\,\mathrm{m}^3$$

$$W \approx (1013250)(1) + (506625)(1) + (337750)(1) + (253312.5)(1) + (202650)(1)$$

$$W \approx 1013250 + 506625 + 337750 + 253312.5 + 202650$$

$$W \approx 2313587.5\,\mathrm{J}$$

$$W \approx 2313.59\,\mathrm{kJ}$$

Final Results

- **Exact Work Calculation (Integration):** $W \approx 3.99\,\mathrm{kJ}$
- **Numerical Approximation (Rectangle Method):** $W \approx 2313.59\,\mathrm{kJ}$