Energy and the first law of thermodynamics

Part II


Work from Expansion
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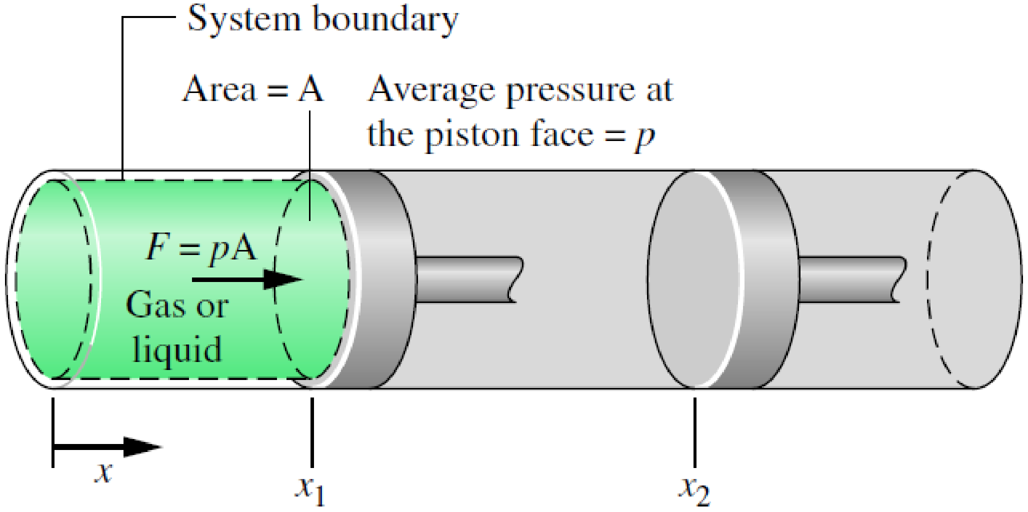
Lesson 2: Modeling Expansion or Compression Work

- Let us evaluate the work done by the closed system shown in Figure consisting of a gas (or liquid) contained in a piston-cylinder assembly as the gas expands. During the process, the gas pressure exerts a normal force on the piston.

- Let p denote the pressure acting at the interface between the gas and the piston. The force exerted by the gas on the piston is simply the product pA, where A is the area of the transversal section of the piston face. The work done by the system as the piston is displaced a distance dx is:

$$\delta W = pAdx$$



Lesson 2: Modeling Expansion or Compression Work

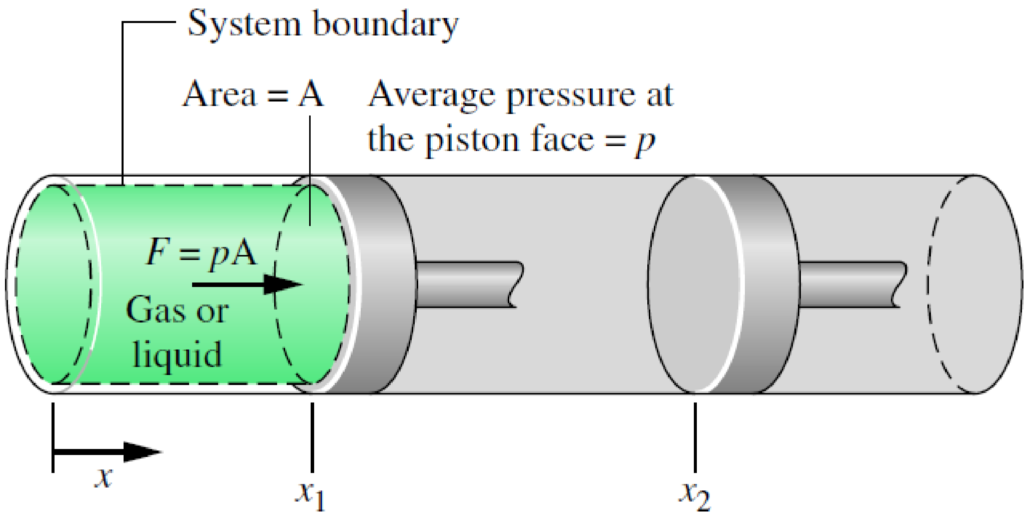
- The product Adx equals the change in volume of the system, dV.

- Thus, the work expression can be written as:

$$\delta W = pdV$$

- Since $dV > 0$ is positive when volume increases, the work at the moving boundary is positive when the gas expands. For a compression, $dV < 0$ is negative

- These signs agree with the previously stated sign convention for work.



Lesson 2: Modeling Expansion or Compression Work

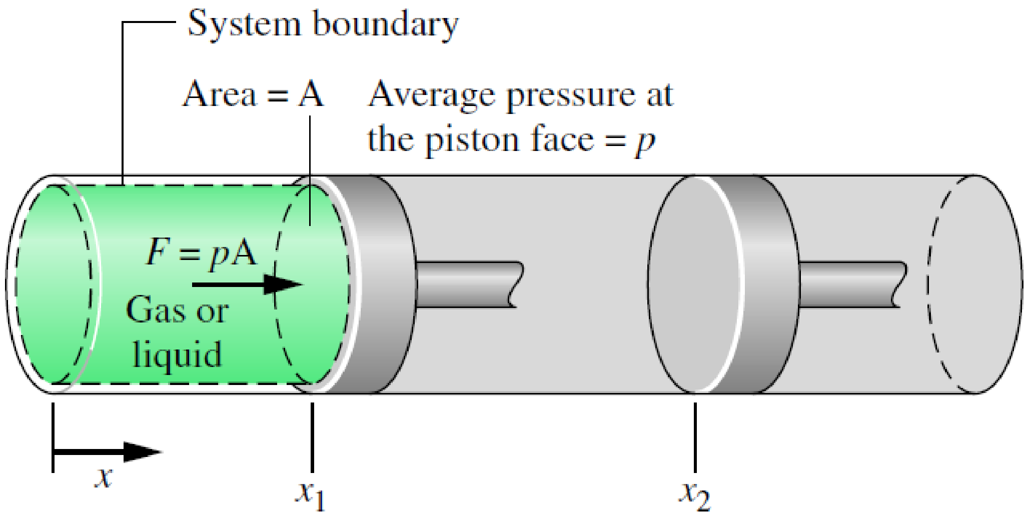
- For a change in volume from V_1 to V_2, the work is obtained by integrating:

$$W = \int_{V_1}^{V_2} p dV$$

- This equation applies to systems of any shape provided the pressure is uniform with position over the moving boundary.

- By free expansion, we mean expansion against zero opposing force. It occurs when $p = 0$. According to the last equation, $W = 0$ for each stage of the expansion. Hence, overall:

$$W = 0: \quad \text{Free expansion}$$

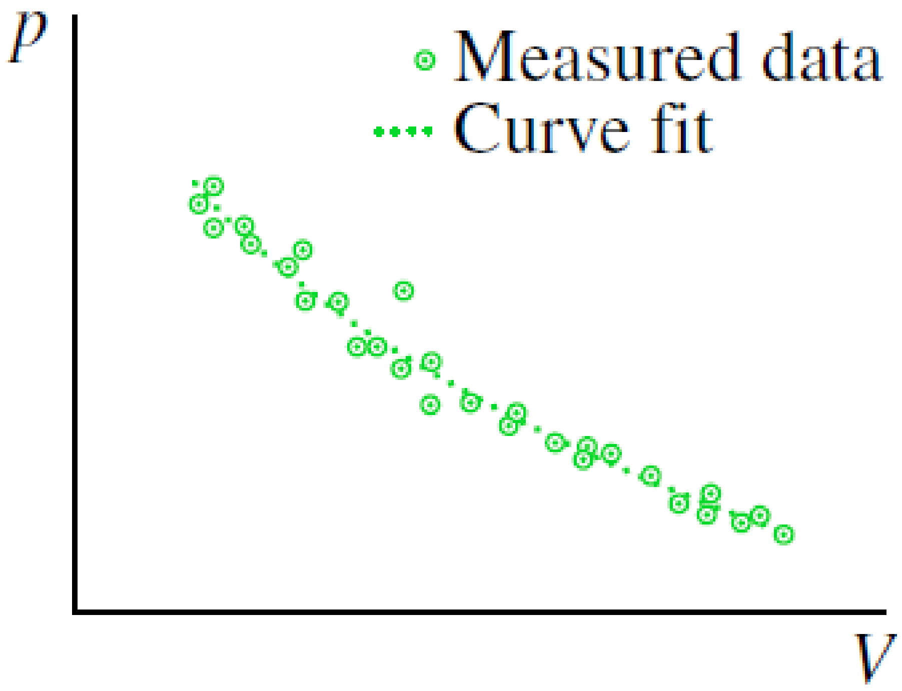


Subtheme 2.1: Expansion or Compression Process

- To calculate the Work, it is requires a relationship between the gas pressure at the moving boundary and the system volume, but this relationship may be difficult, or even impossible, to obtain for actual compressions and expansions
- In the cylinder of an automobile engine, for example, combustion and other nonequilibrium effects give rise to nonuniformities throughout the cylinder

Subtheme 2.1: Expansion or Compression Process

- Based on a curve fitted to the data could give a plausible estimate of the work

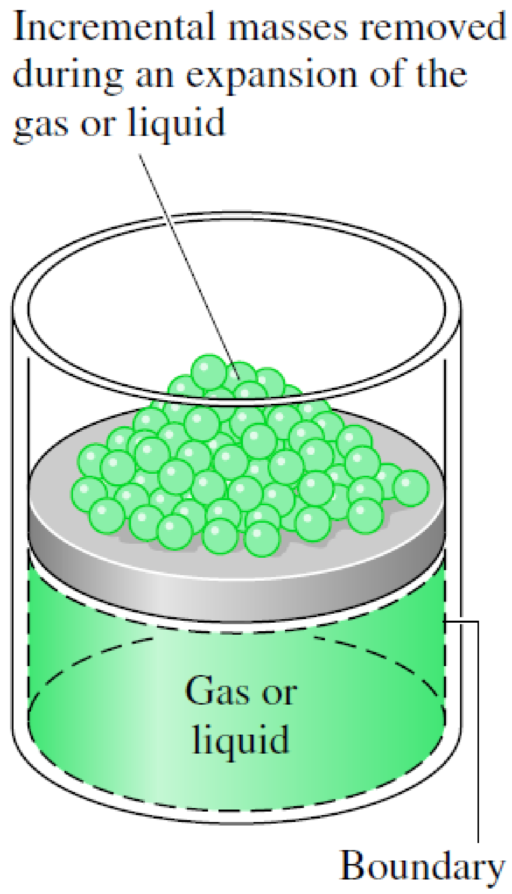


Subtheme 2.2: Quasiequilibrium Expansion or Compression Process

- A quasi-equilibrium process is one in which all states through which the system passes may be considered equilibrium states
- A particularly important aspect of the quasi-equilibrium process concept is that the values of the intensive properties are uniform throughout the system, or every phase present in the system, at each state visited
- With this approximation the Work equation can be applied to evaluate the work in quasi-equilibrium expansion or compression processes.

Subtheme 2.2: Quasiequilibrium Expansion or Compression Process

- To consider how a gas might be expanded or compressed in a quasi-equilibrium fashion, refer to Figure
- As shown in the figure, the gas pressure is maintained uniform throughout by many small masses resting on the freely moving piston
- The system would eventually come to a new equilibrium state, where the pressure and all other intensive properties would again be uniform in value



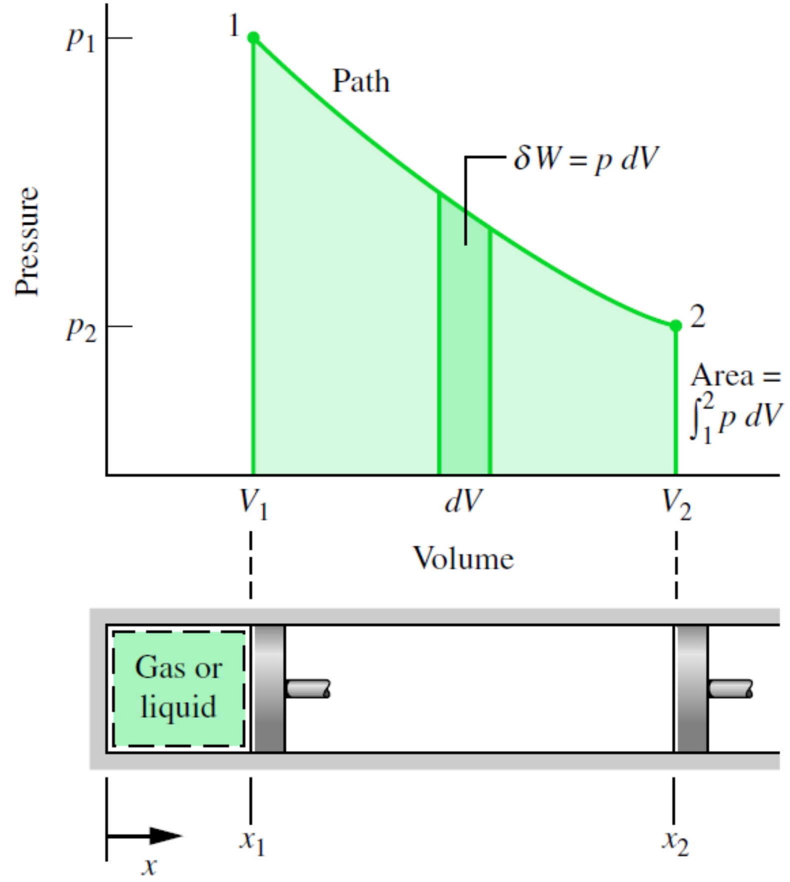
Subtheme 2.2: Quasiequilibrium Expansion or Compression Process

- For such idealized processes, the pressure p in the equation is the pressure of the entire quantity of gas undergoing the process and not just the pressure at the moving boundary.

Subtheme 2.2: Quasiequilibrium Expansion or Compression Process

- The relationship between pressure and volume may be graphical or analytical. Let us first consider a graphical relationship.

- A graphical relationship is shown in the pressure-volume diagram (p–V diagram). Initially, the piston face is at position x_1, and the gas pressure is p_1; after a quasi-equilibrium expansion process the piston face is at position x_2, and the pressure is reduced to p_2.

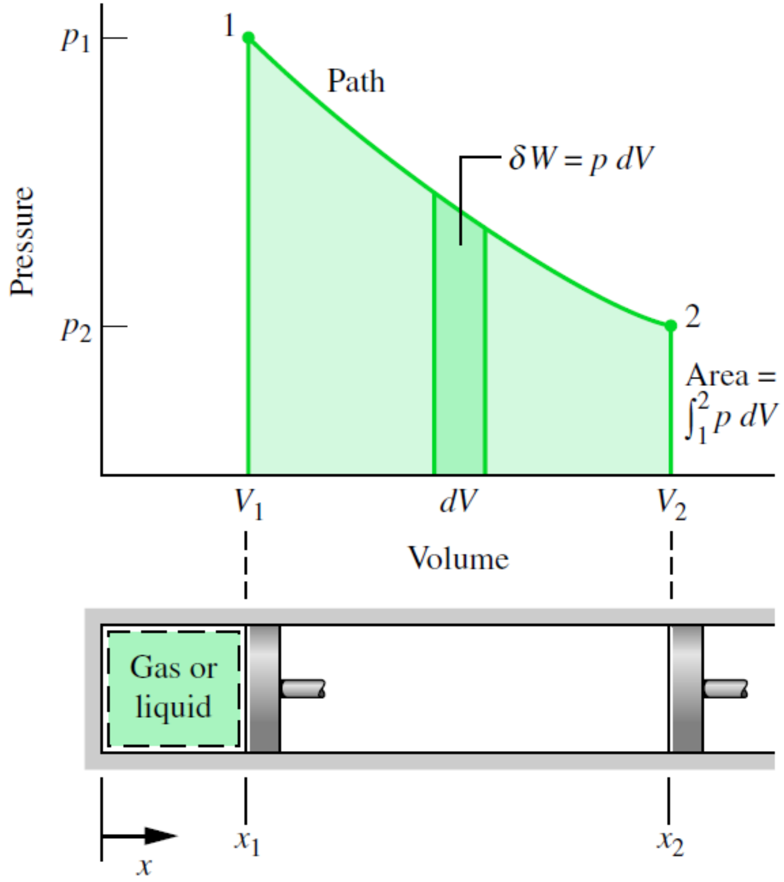


Subtheme 2.2: Quasiequilibrium Expansion or Compression Process

- At each intervening piston position, the uniform pressure throughout the gas is shown as a point on the diagram. The curve, or path, connecting states 1 and 2 on the diagram represents the equilibrium states through which the system has passed during the process.

- The work done by the gas on the piston during the expansion is given by Work Equation, which can be interpreted as the area under the curve of pressure versus volume. Thus, the shaded area in Figure is equal to the work for the process.

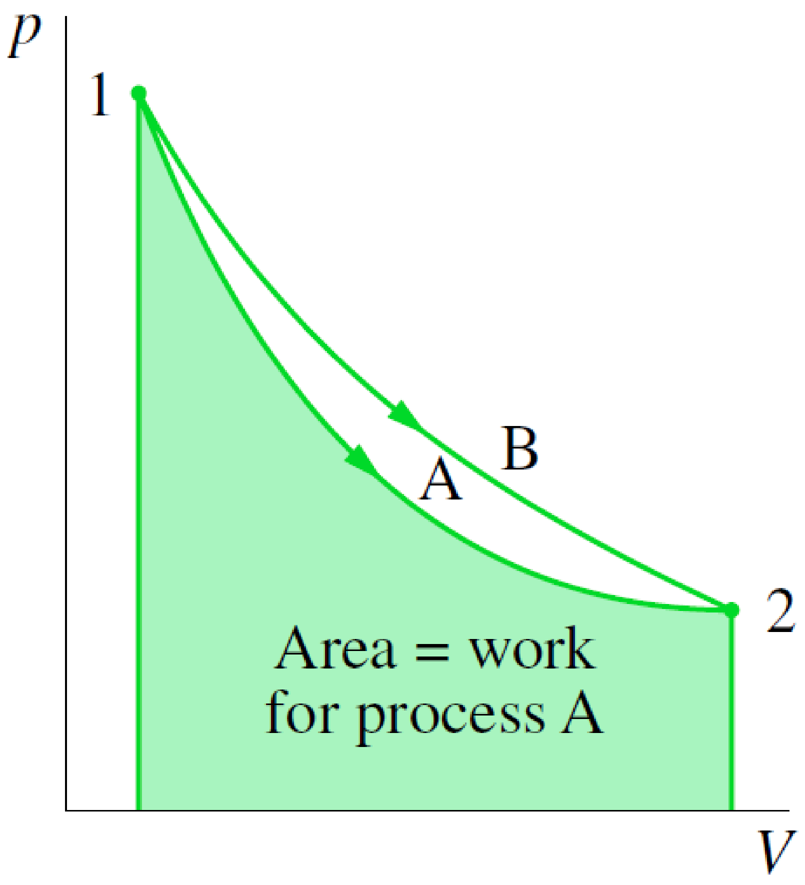
- Had the gas been compressed from 2 to 1 along the same path on the $p-V$ diagram, the magnitude of the work would be the same, but the sign would be negative, indicating that for the compression the energy transfer was from the piston to the gas.



Subtheme 2.2: Quasiequilibrium Expansion or Compression Process

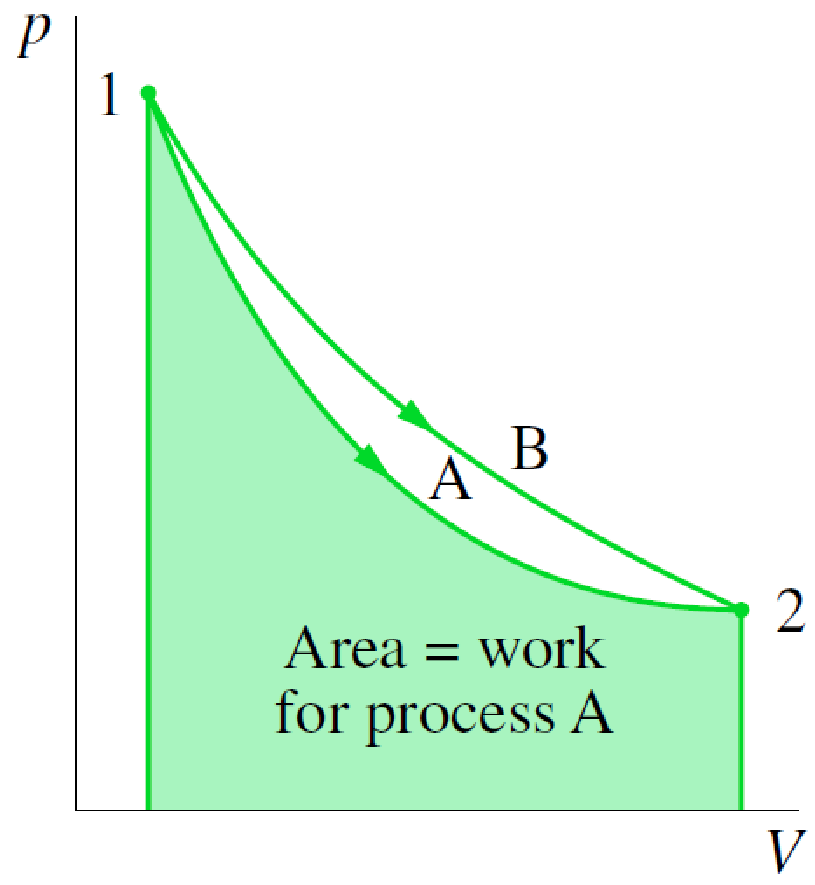
- The area interpretation of work in a quasi-equilibrium expansion or compression process allows a simple demonstration of the idea that work depends on the process. This can be brought out by referring to Figure. Suppose the gas in a piston-cylinder assembly goes from an initial equilibrium state 1 to a final equilibrium state 2 along two different paths, labeled A and B in Figure.

- Since the area beneath each path represents the work for that process, the work depends on the details of the process as defined by the curve and not just on the end states. Work is not a property; the value of work depends on the nature of the process between the initial and end states.



Subtheme 2.2: Quasiequilibrium Expansion or Compression Process

- The relationship between pressure and volume during an expansion or compression process also can be described analytically. An example is provided by the expression $pV^n = constant$, where the value of n is a constant for the process. A quasi-equilibrium process described by such an expression is called a polytropic process. Additional analytical forms for the pressure-volume relationship also may be considered.



Lesson 3: Broadening Our Understanding of Energy

- We consider the total energy of a system, which includes kinetic energy, gravitational potential energy, and other forms of energy
- When a gas initially at an equilibrium state in a closed, insulated vessel is stirred vigorously and allowed to come to a final equilibrium state, the energy of the gas is increased in the process
- In this case, the change in system energy cannot be attributed to changes in the system's overall kinetic or gravitational potential energy
- In engineering thermodynamics, the change in the total energy of a system is considered to be made up of three macroscopic contributions

Lesson 3: Broadening Our Understanding of Energy

- The change in the total energy of a system is: The identification of internal energy as a macroscopic form of energy is a significant step in the present development, for it sets the concept of energy in thermodynamics apart from that of mechanics
- The internal energy is an extensive property of the system, as is the total energy.

Lesson 3: Broadening Our Understanding of Energy

- Internal energy is represented by the symbol U, and the change in internal energy in a process is

$$U_2 - U_1.$$

- The specific internal energy is symbolized by u or $\bar{u}$, respectively, depending on whether it is expressed on a unit mass or per mole basis. The change in the total energy of a system is:

$$E_2 - E_1 = (KE_2 - UKE_1) + (PE_2 - UPE_1) + (U_2 - U_1)$$

$$\Delta E = \Delta KE + \Delta PE + \Delta U$$

- The identification of internal energy as a macroscopic form of energy is a significant step in the present development, for it sets the concept of energy in thermodynamics apart from that of mechanics.

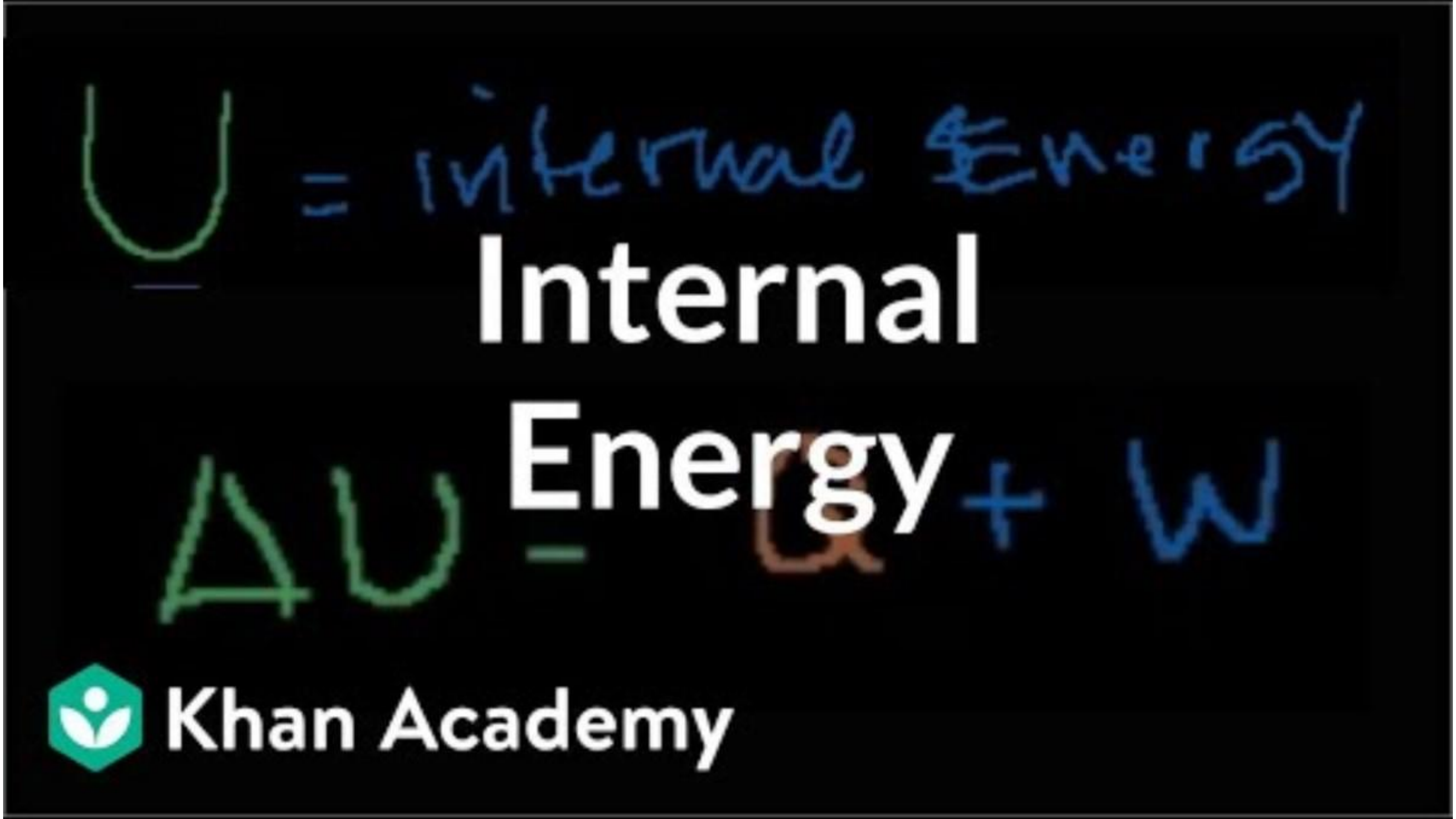
Lesson 3: Broadening Our Understanding of Energy

- To further expand our understanding of internal energy, consider a system we will often encounter in subsequent sections of the course, a system consisting of a gas contained in a tank. Let us develop a microscopic interpretation of internal energy by thinking of the energy attributed to the motions and configurations of the individual molecules, atoms, and subatomic particles making up the matter in the system.

- Gas molecules move about, encountering other molecules or the walls of the container. Part of the internal energy of the gas is the translational kinetic energy of the molecules. Other contributions to the internal energy include the kinetic energy due to rotation of the molecules relative to their centers of mass and the kinetic energy associated with vibrational motions within the molecules.

Lesson 3: Broadening Our Understanding of Energy

- Besides, energy is stored in the chemical bonds between the atoms that make up the molecules. Energy storage on the atomic level includes energy associated with electron orbital states, nuclear spin, and binding forces in the nucleus. In dense gases, liquids, and solids, intermolecular forces play an important role in affecting the internal energy.


U = internal Energy
Internal Energy
ΔU = Q + W
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Subtheme 3.1: Energy Transfer by Heat

- So far, we have considered only quantitatively those interactions between a system and its surroundings that can be classed as work
- Based on the experiment, beginning with the work of Joule in the early part of the nineteenth century, we know that energy transfers by heat are induced only as a result of a temperature difference between the system and its surroundings and occur only in the direction of decreasing temperature
- Because the underlying concept is so important in thermodynamics, this section is devoted to further consideration of energy transfer by heat

Subtheme 3.2: Sign Convention, Notation, and Heat Transfer Rate

- The symbol denotes an amount of energy transferred across the boundary of a system in a heat interaction with the system's surroundings
- The sign convention for heat transfer is the reverse of the one adopted for work, where a positive value for W signifies an energy transfer from the system to the surroundings
- These signs for heat and work are a legacy from engineers and scientists who were concerned mainly with steam engines and other devices that develop a work output from an energy input by heat transfer

Subtheme 3.2: Sign Convention, Notation, and Heat Transfer Rate

- The symbol Q denotes an amount of energy transferred across the boundary of a system in a heat interaction with the system's surroundings. Heat transferred into a system is taken to be positive, and heat transferred out from a system is taken as negative.

$$Q > 0: \text{ heat transfer to the system}$$
$$Q < 0: \text{ heat transfer from the system}$$

- This sign convention is used throughout the course. The sign convention for heat transfer is the reverse of the one adopted for work, where a positive value for W signifies an energy transfer from the system to the surroundings.

- These signs for heat and work are a legacy from engineers and scientists who were concerned mainly with steam engines and other devices that develop a work output from an energy input by heat transfer. For such applications, it was convenient to regard both the work developed and the energy input by heat transfer as positive quantities.

Subtheme 3.2: Sign Convention, Notation, and Heat Transfer Rate

- The value of a heat transfer depends on the details of a process and not just the end states. Thus, like work, heat is not a property, and its differential is written as δQ. The amount of energy transfer by heat for a process is given by the integral:

$$Q = \int_1^2 \delta Q$$

- where the limits mean “from state 1 to state 2” and do not refer to the values of heat at those states. As for work, the notion of “heat” at a state has no meaning, and the integral should never be evaluated as:

$$Q_2 - Q_1$$

Subtheme 3.2: Sign Convention, Notation, and Heat Transfer Rate

- The net rate of heat transfer is denoted by the amount of energy transfer by heat during a time and can be found by integrating from time t_1 to time t_2:

$$Q = \int_{t_1}^{t_2} \dot{Q}\, dt$$

- To perform the integration, it would be necessary to know how the rate of heat transfer varies with time. In some cases, it is convenient to use the heat flux, $\dot{q}$, which is the heat transfer rate per unit of the system surface area. The net rate of heat transfer, $\dot{Q}$, is related to the heat flux by the integral:

$$\dot{Q} = \int_{A} \dot{q}\, dA$$

where A represents the area on the boundary of the system where heat transfer occurs. The units for Q are the same as those introduced previously for W.

Subtheme 3.2: Sign Convention, Notation, and Heat Transfer Rate

- The net rate of heat transfer is denoted by the amount of energy transfer by heat during a time and can be found by integrating from time to time : To perform the integration, it would be necessary to know how the rate of heat transfer varies with time

Lesson 4: Practical Examples

- The gas is a closed system
- The moving boundary is the only work mode
- The expansion is a polytropic process


PV-diagrams
and Expansion
Work
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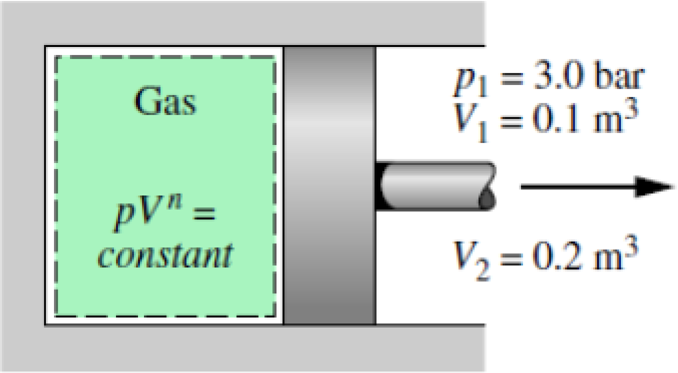
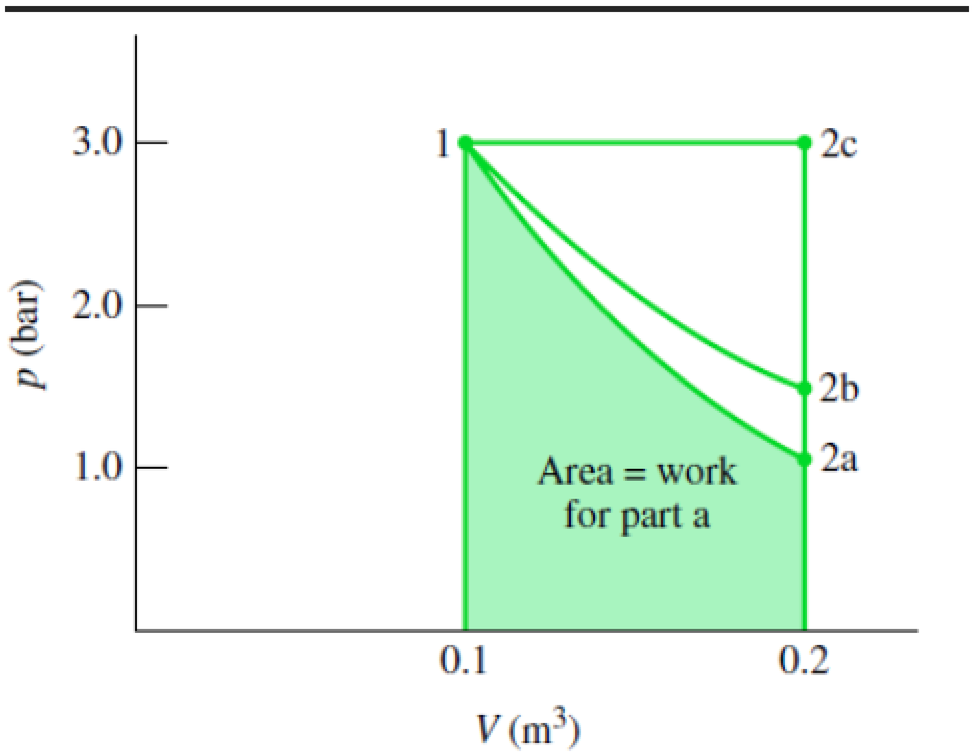
Lesson 4: Practical Examples

- A gas in a piston-cylinder assembly undergoes an expansion process for which the relationship between pressure and volume is given by $pV^n = constant$. The initial pressure is 3 bar, the initial volume is 0.1 m^3, and the final volume is 0.2 m^3. Determine the work for the process, in kJ, if (a) $n = 1.5$, (b) $n = 1.0$, and (c) $n = 0$.



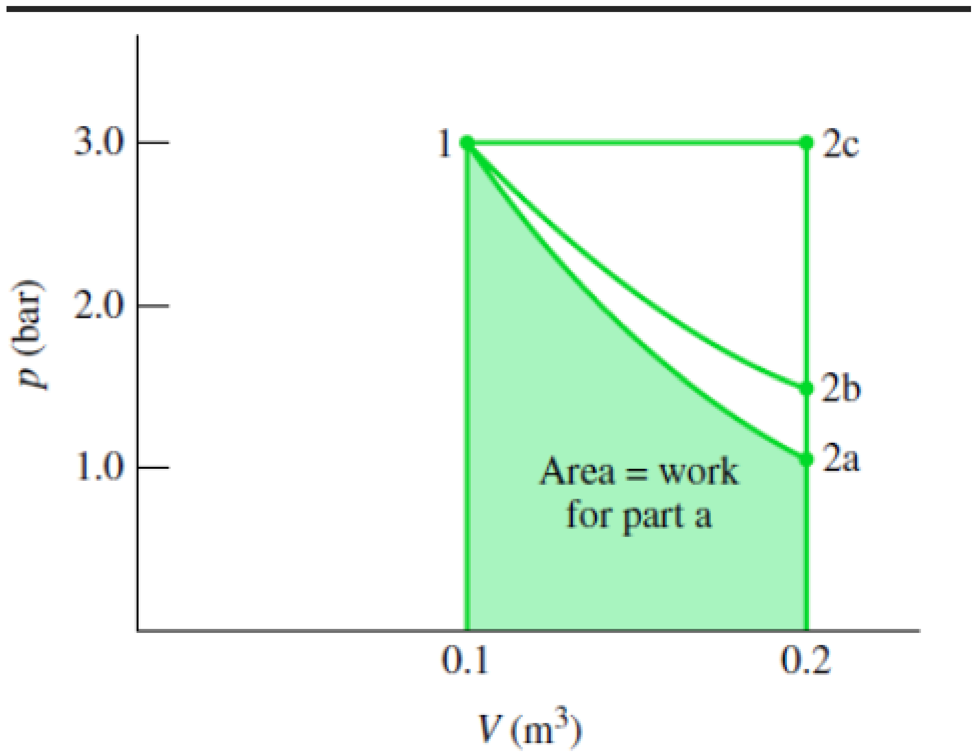


Lesson 4: Practical Examples

Assumptions:

- The gas is a closed system.
- The moving boundary is the only work mode.
- The expansion is a polytropic process.

Analysis: The required values for the work are obtained by the integration of Work Equation using the given pressure-volume relation.



Lesson 4: Practical Examples

(a) Introducing the relationship $p = constant/V^n$ into Eqn. 14 and performing the integration:

$$W = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \frac{constant}{V^n} dV$$

$$= \frac{(constant)V_2^{1-n} - (constant)V_1^{1-n}}{1-n}$$

The constant in this expression can be evaluated at either end state: $constant = p_1V_1^n = p_2V_2^n$. The work expression then becomes:

$$W = \frac{(p_2V_2^n)V_2^{1-n} - (p_1V_1^n)V_1^{1-n}}{1-n} = \frac{p_2V_2 - p_1V_1}{1-n}$$

This expression is valid for all values of n except $n = 1.0$. The case $n = 1.0$ is taken up in part (b).

Lesson 4: Practical Examples

To evaluate W, the pressure at state 2 is required. This can be found by using $p_1V_1^n = p_2V_2^n$ which on rearrangement yields:

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^n = (3\ bar)\left(\frac{0.1}{0.2}\right)^{1.5} = 1.06\ bar$$

Accordingly:

$$W = \left(\frac{(1.06\ bar)(0.2\ m^3) - (3)(0.1)}{1 - 1.5}\right)\left(\frac{10^5\ \frac{N}{m^2}}{1\ bar}\right)\left(\frac{1\ kJ}{10^3\ Nm}\right) = +17.6\ kJ$$

Lesson 4: Practical Examples

(b) For $n = 1.0$, the pressure-volume relationship is $pV = constant$ or $p = constant/V$. The work is:

$$W = constant \int_{V_1}^{V_2} \frac{dV}{V} = (constant) ln \frac{V_2}{V_1} = (p_1 V_1) ln \frac{V_2}{V_1}$$

Substituting values:

$$W = (3\ bar)(0.1\ m^3) \left(\frac{10 \frac{N}{m^2}}{1\ bar} \right) \left(\frac{1\ kJ}{10^3\ Nm} \right) ln \left(\frac{0.2}{0.1} \right) = +20.79\ kJ$$

(c) For $n = 0$, the pressure-volume relation reduces to $p = constant$, and the integral becomes $W = p(V_2 - V_1)$, which is a special case of the expression found in part (a). Substituting values and converting units as above, $W = +30\ kJ$.

Conclusion

- This module has centered on systems for which applied forces affect only their overall velocity and position
- To analyze such systems, the concepts of kinetic and potential energy alone do not suffice, nor does the rudimentary conservation of energy principle introduced in this lesson
- In thermodynamics, the concept of energy is broadened to account for other observed changes, and the principle of conservation of energy is extended to include a wide variety of ways in which systems interact with their surroundings