Consider the block diagram of the control system with PID controller. Design constraint: If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

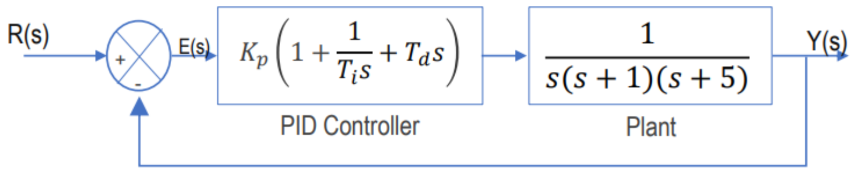


Figure 11-5 PID-Controlled System.

Since the plant has an integrator, we use the **second method of Z–N tuning rules**; the steps are:

**Step #1:** Evaluate closed-loop unit-step response at each 'K' that makes the system **marginally stable** so that sustained oscillation can be obtained by use of **Routh's stability criterion**.

As mentioned, Z-N method uses the **proportional control action only** to begin the procedure**.** In Figure 11-5, set = ∞ and = 0 in the PID controller block:

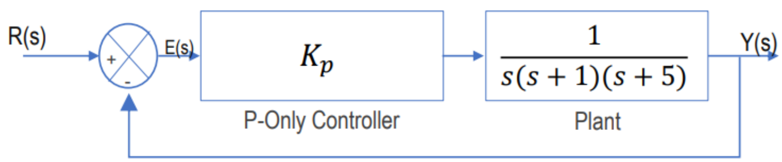


Figure 11-6 Proportional-Only Controlled Process System.

The closed-loop transfer function is:

$$\rightarrow \frac{Y(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5)+K_p}$$

Next, we increase the proportional gain, $K_p$, until the system's response oscillates at a constant amplitude. This means that the value of gain, , that makes the system **marginally stable** so that sustained oscillation occurs can be obtained by use of **Routh's stability criterion**.

**NOTE:** We developed a procedure of constructing the Routh's array in Module #5: Stability and Routh's Criterion.

The characteristic equation (CE) for the closed-loop system is:

$$s(s+1)(s+5)+K_p = 0$$

$$\rightarrow s^3+6s^2+5s+K_p = 0 \qquad \text{Eq. (11-7)}$$

Arranging the odd and even coefficients of the CE in tabulated form, the Routh's array is presented in Table #2:

**Table #2. Routh's Array.**

| | | | |
|---|---|---|---|
| Row 3 | $s^3$ | 1 | 5 |
| Row 2 | $s^2$ | 6 | $K_p$ |
| Row 1 | $s^1$ | $\frac{30 - K_p}{6}$ | 0 |
| Row 0 | $s^0$ | $K_p$ | 0 |

Examining the coefficients of the first column in Table 11-2, we find that sustained oscillation will occur if  = 30. Therefore, the critical gain is $K_{cr} = 30$.

**Note:** To verify this, you can run a simulation to see what of effect  has over the response for  = 5,10,15,20,25,30 in MATLAB®. Add the following commands to the live script window and run it:

```
clearvars;
s = tf('s');

% Test a list of values for K:
K = [5 10 15 20 25 30];

% open-loop transfer function:
TF = 1/(s*(s + 1)*(s + 5))

% Evaluate closed-loop unit-step response at each 'K' that makes
% the system marginally stable:

Gp = tf(1,[1 6 5]);          % transfer function of the plant
for i = 1:6
    figure(i);
    % Math trick: put the pole 's' of the transfer function
    % of the plant in transfer function of the proportional
    % controller:
    Gc = tf(K(i), [1 0]);    % proportional-only controller
    GCL = Gc*Gp/(1 + Gc*Gp); % closed-loop transfer function with
proportional-only regulation
    [y,t] = step(GCL,20);
    plot(t,y)
    grid on;
end
```

**Step #2.** From the constant amplitude response, we obtain the ultimate gain ($K_u$ or $K_{cr}$) and ultimate period ($T_u$ or $P_{cr}$):

You got sustained oscillations with  = 30; so, after substituting in Eq. (11-7), the characteristic equation becomes:

$$CE(s) = s^3 + 6s^2 + 5s + 30 = 0 \qquad \text{Eq. (11-8)}$$

To find the frequency of the sustained oscillation, we substitute  = into Eq. (11-8) as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0 \qquad \text{Eq. (11-9)}$$

From Eq. (11-9), the frequency of the sustained oscillation is: $(5 - \omega^2) = 0$; that is,  = $\sqrt{5}$ rad/s. So, the period of sustained oscillation is:

$$= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} \cong 2.8099 \qquad \text{Eq. (11-10)}$$

| Type of Controller | $K_p$ | $T_i$ | $T_d$ |
|---|---|---|---|
| P | $0.5K_{cr}$ | ∞ (or high value) | 0 |
| PI | $0.45K_{cr}$ | $\frac{P_{cr}}{1.2}$ | 0 |
| PID | $0.6K_{cr}$ | $0.5P_{cr}$ | $0.125P_{cr}$ |

**Step #3.** Next, determine the proportional gain (Kp), integral time (Ti), and derivative time (Td). Refer to Table 11-1 for guidance and calculate the values as follows:

```
% From the constant amplitude response plot:
Kcr = 30
Pcr = 2.8099

% From Table 11-1:
Kp = 0.6*Kcr
Ti = 0.5*Pcr
Td = 0.125*Pcr

% Then, the Ki and Kd are obtained as follows:
Ki = Kp/Ti
Kd = Kp*Td
```

**Step #4.** Construct the PID controller, denoted as C(s), and generate a plot illustrating the closed-loop response with PID compensation using a unit-step input and unit-feedback. The corresponding block diagram is as follows:

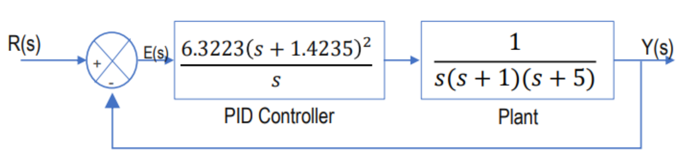


Figure 11-7 PID-Regulated Process System By Z-N Tuning Rules.

- So, applying the feedback formula, the closed-loop transfer function $\frac{Y(s)}{R(s)}$ is given by:

$$\frac{Y(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

```
% The numerator of the TF of the system with PID regulation is:
Kp = 6.3223;
num = Kp.*[1 18/Kp 12.811/Kp];

% The denominator of the TF of the system with PID regulation is:
den = [1 6 11.3223 18 12.811];

sys_cl = tf(num, den)  % closed-loop transfer function of the system
                       % with PID regulation

% Plot the response with PID regulation for an unit-step input:
step(sys_cl,20);
grid on;
xlabel('time');
ylabel('response, y(t)');
title('PID Regulated Response of the Plant')
stepinfo(sys_cl)
```

$$G_C(s) = K_P\left(1 + \frac{1}{T_i s} + T_D s\right) =$$

$$= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right)$$

$$= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}$$

```
figure
s = tf('s');
a = 4/Pcr % = 1.4235;

% The PID controller has a pole at the origin
% and double zero at s=-1.4235:

% The control system is:

Gc = (0.075*Kcr*Pcr)*(s+a)^2/s  % PID controller TF
Gp = 1/(s*(s+1)*(s+5))          % Plant TF

sys_ol = Gc*Gp                  % open-loop TF
```

```
% Plot the root locus of your open-loop system
rlocus(sys_ol)

% Determine the dominant poles of the open-loop system (CE = 0):
p = pole(sys_ol);
p_dom = p(real(p) < 0)

% rlocus(sys_ol)
% [K,POLES] = rlocfind(sys_ol)

% Plot the new response with PID regulation for an unit-step input:
sys_cl = feedback(sys_ol,1)
step(sys_cl,20);
grid on;
xlabel('time, in seconds');
ylabel('Transient response, y(t)');
title('PID Regulated Response of the Control System')
stepinfo(sys_cl)

% Observing the results of the stepinfo of the closed-loop
% response, the maximum overshoot 62%. It is excessive.
```

Fine Tuning:  The overshoot can be reduced by "fine tuning" the controller parameters.

**Increasing the proportional gain** in a control system tends to make the system respond more quickly, as the control signal increases proportionally for the same level of error.  However, this also leads to a greater overshoot, so system exceeds its target value in response to a change. While a faster response might be desirable in some cases, care must be taken not to make the system unstable. If the proportional gain is too large, the process variable will begin to oscillate. If the gain is increased further, the oscillations will become larger, and the system may become unstable and may even oscillate out of control.  Once the proportional gain has been set to obtain a desired fast response, the integral term can be increased to stop the oscillations.

**Increasing the integral term** reduces the steady-state error but increases overshoot. Therefore, a balance must be found to achieve the desired system performance.

**Increasing the derivative gain** in a control system has the effect of providing damping to the system, which helps to reduce overshoot. The derivative term is proportional to the rate of change of the error, so it responds to the speed at which the error is changing. This means that if the system starts to change rapidly, the derivative term will apply a large control signal to counteract this.  However, while increasing the derivative gain can help to reduce overshoot and improve stability, it can also slow down the system response.  If the derivative gain is too high, it can cause the system to become sluggish and degrade the response time. Furthermore, a high derivative gain can make the system more sensitive to noise, as it will react strongly to rapid changes, including those caused by noise.

**Therefore, when tuning a control system, it's important to find a balance between the proportional, integral, and derivative gains to achieve the desired system performance. Once the proportional and integral gains have been set to get the desired control system with minimal steady state error, the derivative term is increased until the loop is acceptably quick.**

$$G_C(s) = K_P\left(1 + \frac{1}{T_i s} + T_D s\right) =$$

$$= 0.6K_{\mathrm{cr}}\left(1 + \frac{1}{0.5P_{\mathrm{cr}}s} + 0.125P_{\mathrm{cr}}s\right)$$

$$= 0.075K_{\mathrm{cr}}P_{\mathrm{cr}}\frac{\left(s + \frac{4}{P_{\mathrm{cr}}}\right)^2}{s}$$