

Radio-astronomy

Interferometry - introduction

**Action Fédératrice ALMA/NOEMA
Observatoire de Paris**

P. Salomé

Credits: F. Gueth, J. Pety, R. Neri, R. Moreno ...

References

Books

- « Interferometry and Synthesis in Radio Astronomy » Thompson, Moran, Swensson
- « Radio Astronomy » J.D. Kraus
- « Tools of Radio Astronomy » K. Rohlfs & T.L. Wilson

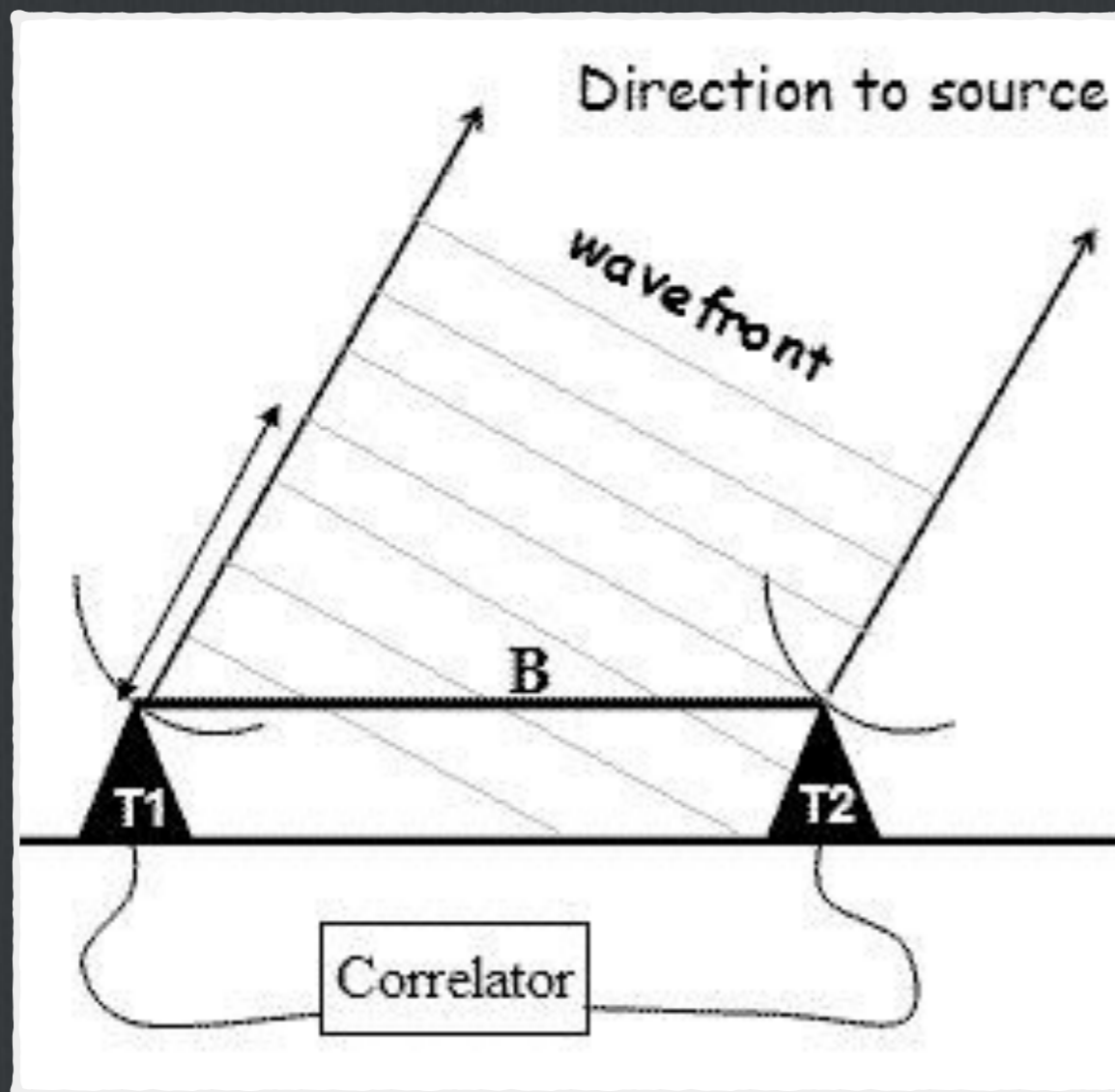
Proceedings and Talks

- NRAO and IRAM radio-interferometry summer schools (in particular by F. Gueth and J. Pety)
 - > <http://www.aoc.nrao.edu/events/synthesis/2012/lectures.shtml>
 - > <http://www.iram-institute.org/EN/content-page-182-7-67-182-0-0.html>

Credits

- Lectures : F. Gueth + J. Pety + R. Neri + R. Moreno + ...

Outline



- Interferometry principles

- Imaging & Calibration

- Tutorials

- Sensitivity

- Imaging simulation

- Proposal preparation

Principles

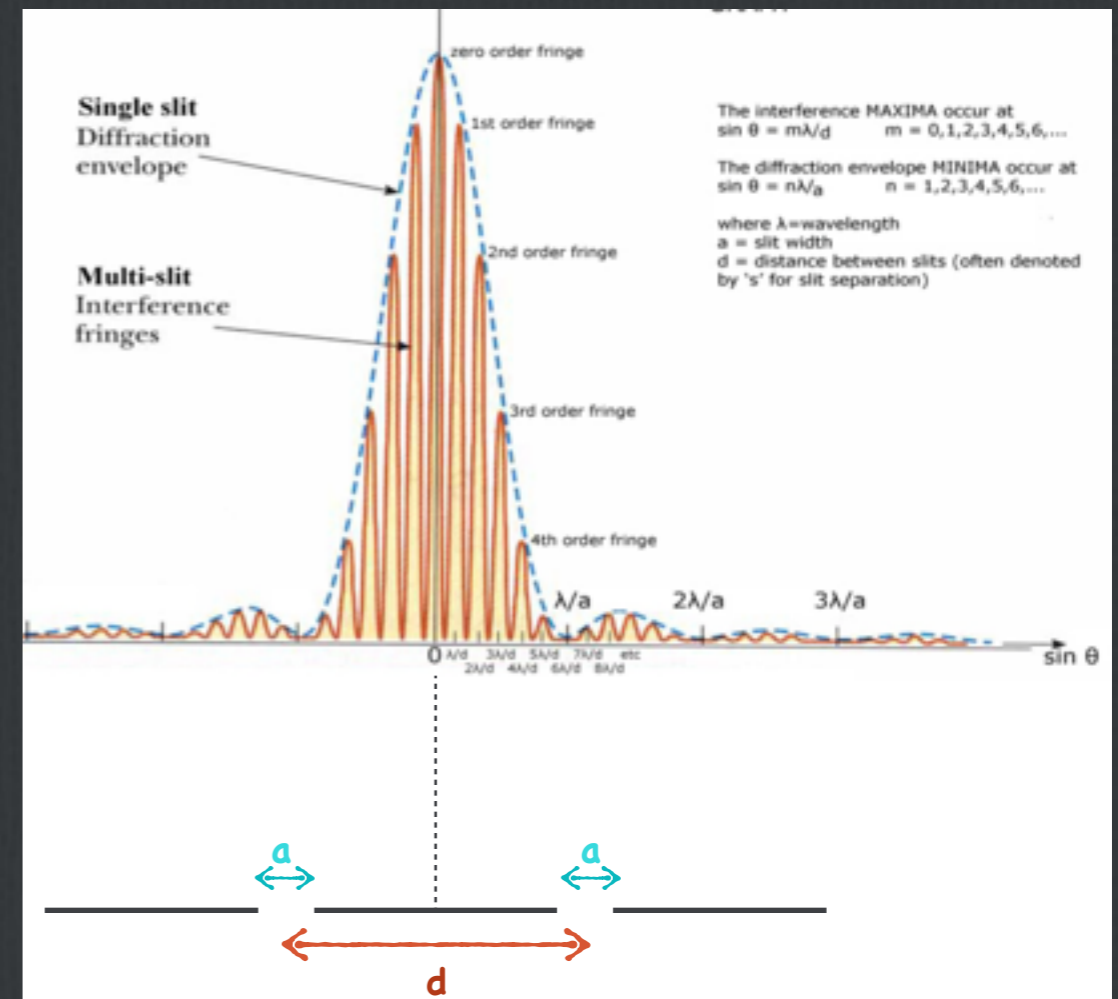
Principles

The Young's holes experiment

Diffraction pattern (related to the size of each hole) → the **primary beam** (defined by a the antenna diameter) : $\theta_{\text{prim}} \sim 1.2 \lambda/a$


Interferometric pattern (related to the distance between the holes) → the **synthesized beam** (define by the distance b between 2 antennas, also called the baseline)

$$\theta_{\text{synth}} \sim 1.2 \lambda/d$$



Principles

D = 50 m



- LMT/GMT 50m telescope


- IRAM-30m telescope

- Single dish > 50 m needs :

- High surface quality (efficiency):
 $\lambda/20 \sim 50\mu\text{m}$

- Excellent pointing accuracy
(wind / structure deformation):
HPBW/10

D = 30 m



30m telescope

Increase the spatial resolution
from λ/D to λ/B

Principles



- LMT/GMT 50m telescope

- IRAM-30m telescope

- Single dish > 50 m needs :

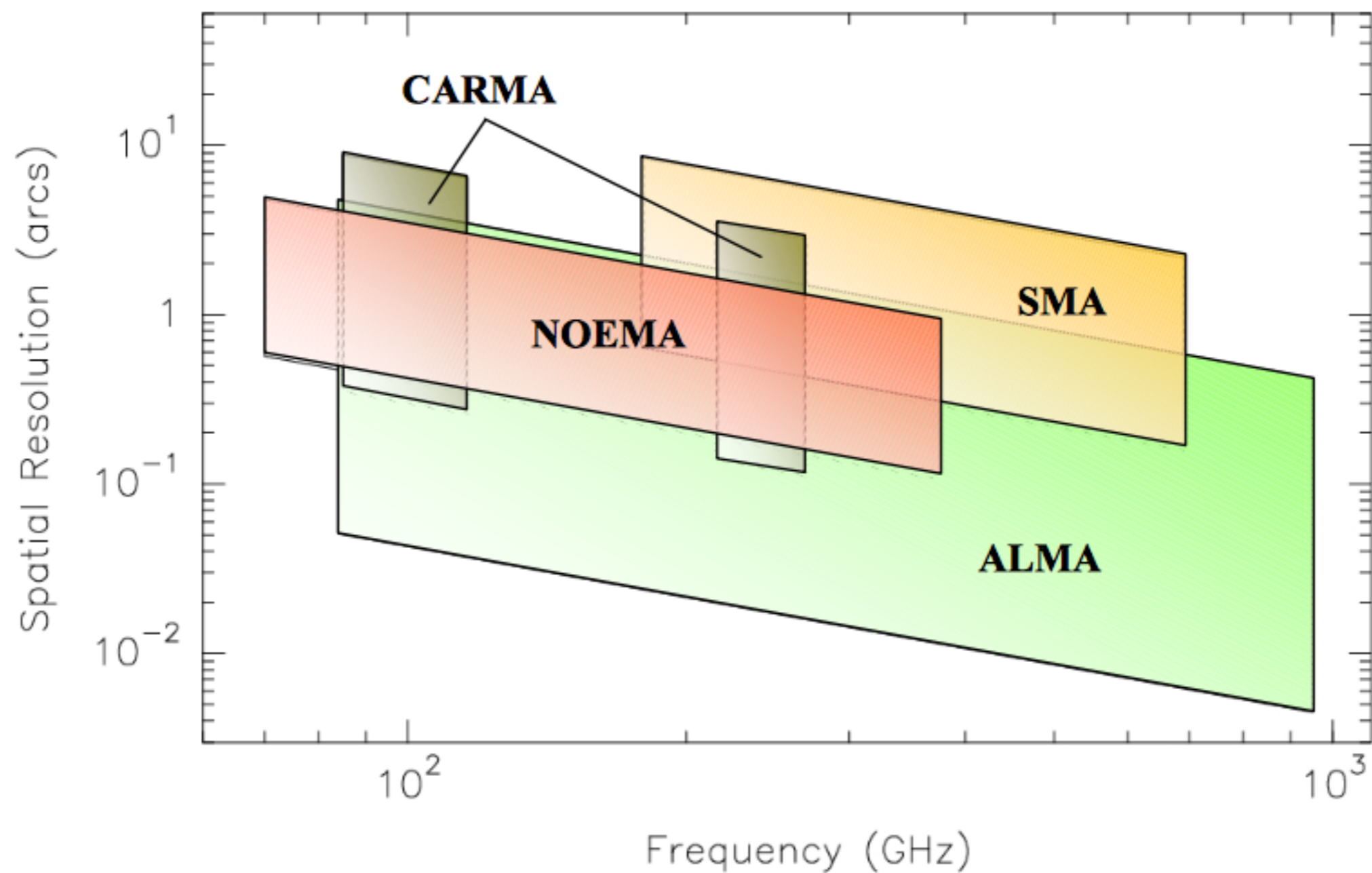
- High surface quality (efficiency):
 $\lambda/20 \sim 50\mu\text{m}$

- Excellent pointing accuracy
(wind / structure deformation):
HPBW/10



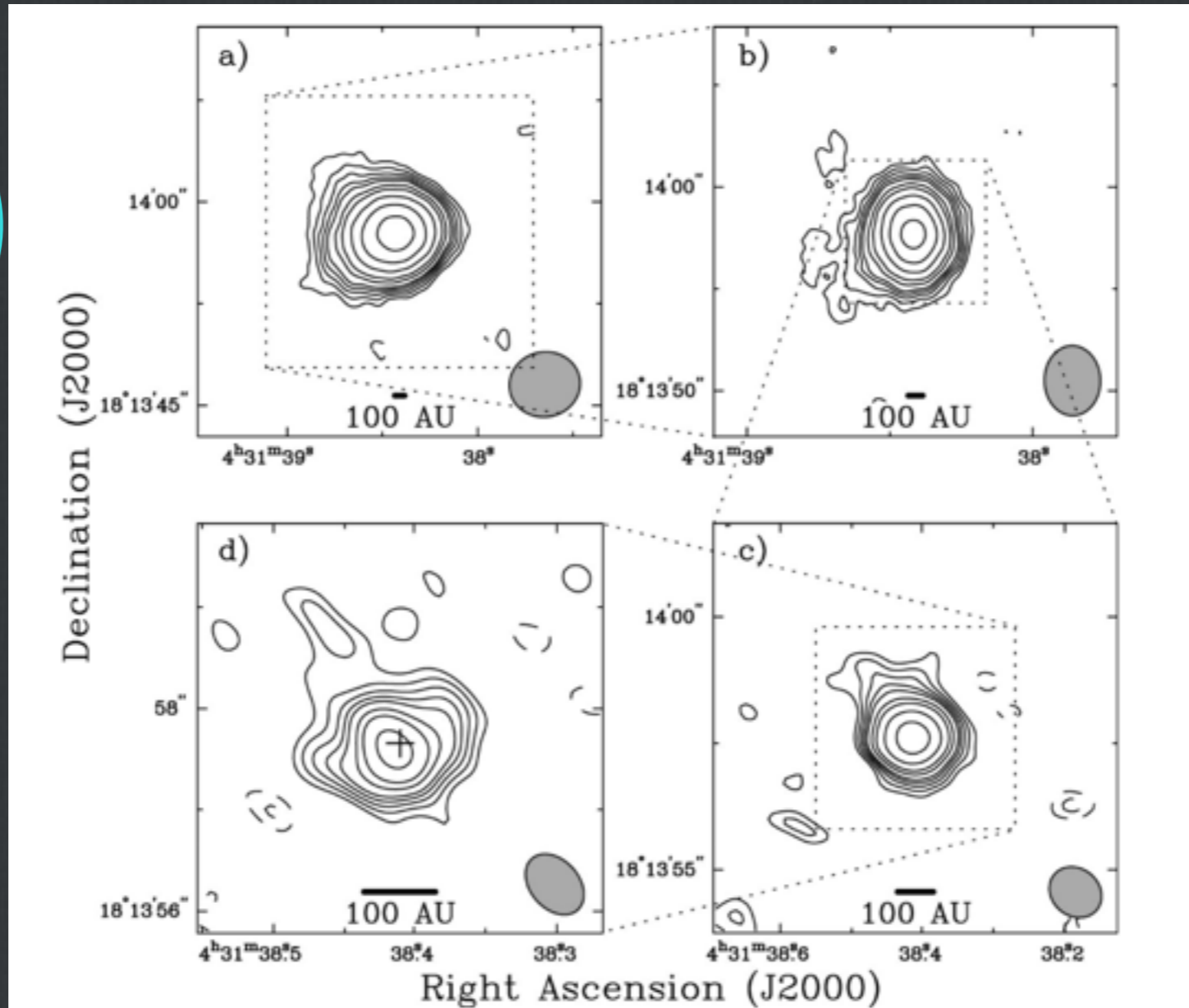
Increase the spatial resolution
from λ/D to λ/B

	Altitude (m)	N_{ANT}	Diameter (m)	Coll.Area (m ²)
IRAM PDBI	2550	6	15	1060
CARMA	2200	15	6/10	772
SMA+CSO+JCMT	4080	10	6/10/15	481
NMA	1340	6	10	471
IRAM NOEMA	2550	12	15	2120
ALMA	5060	50	12	5652



HL Tau

A planet-forming disc around a young star

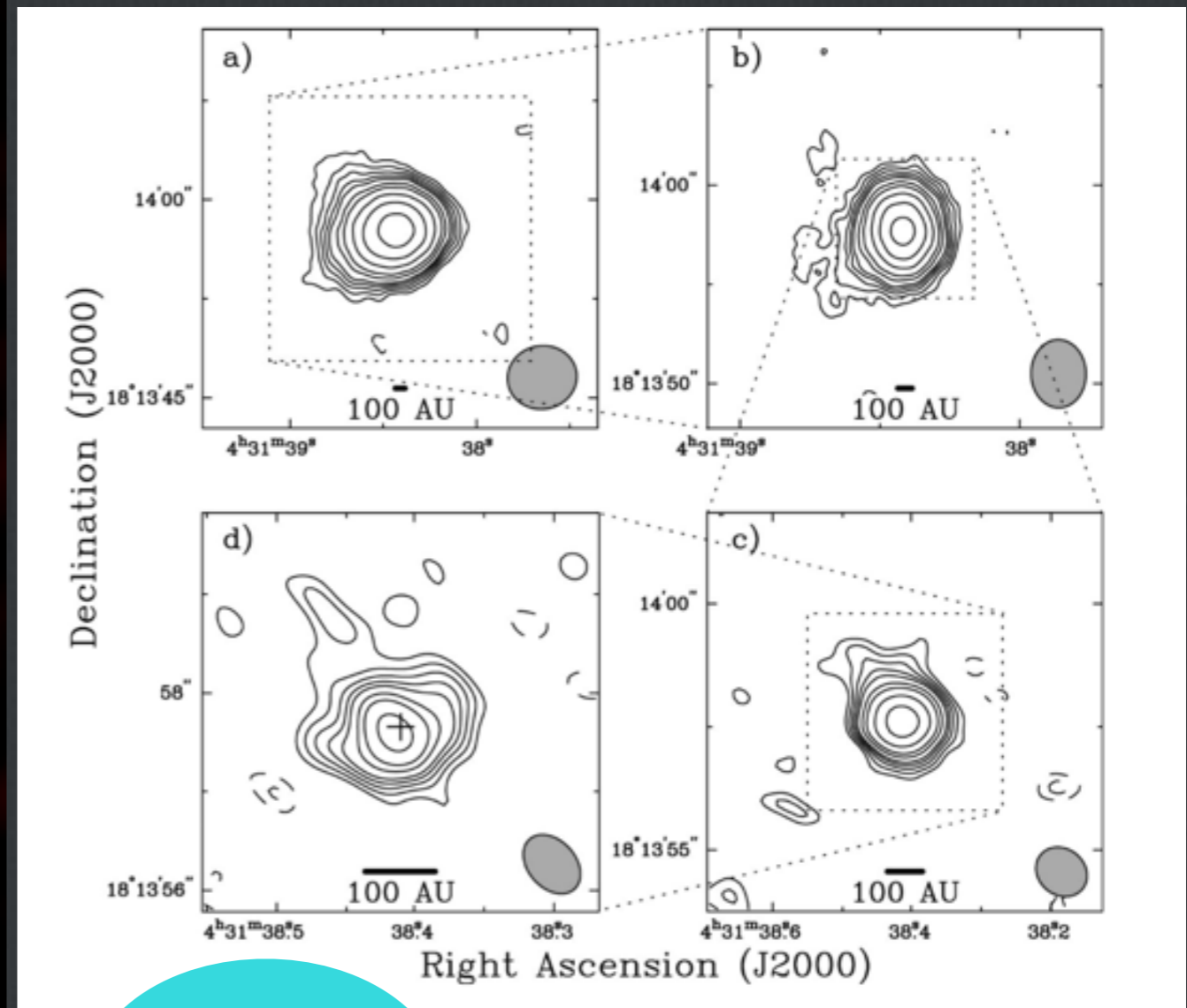
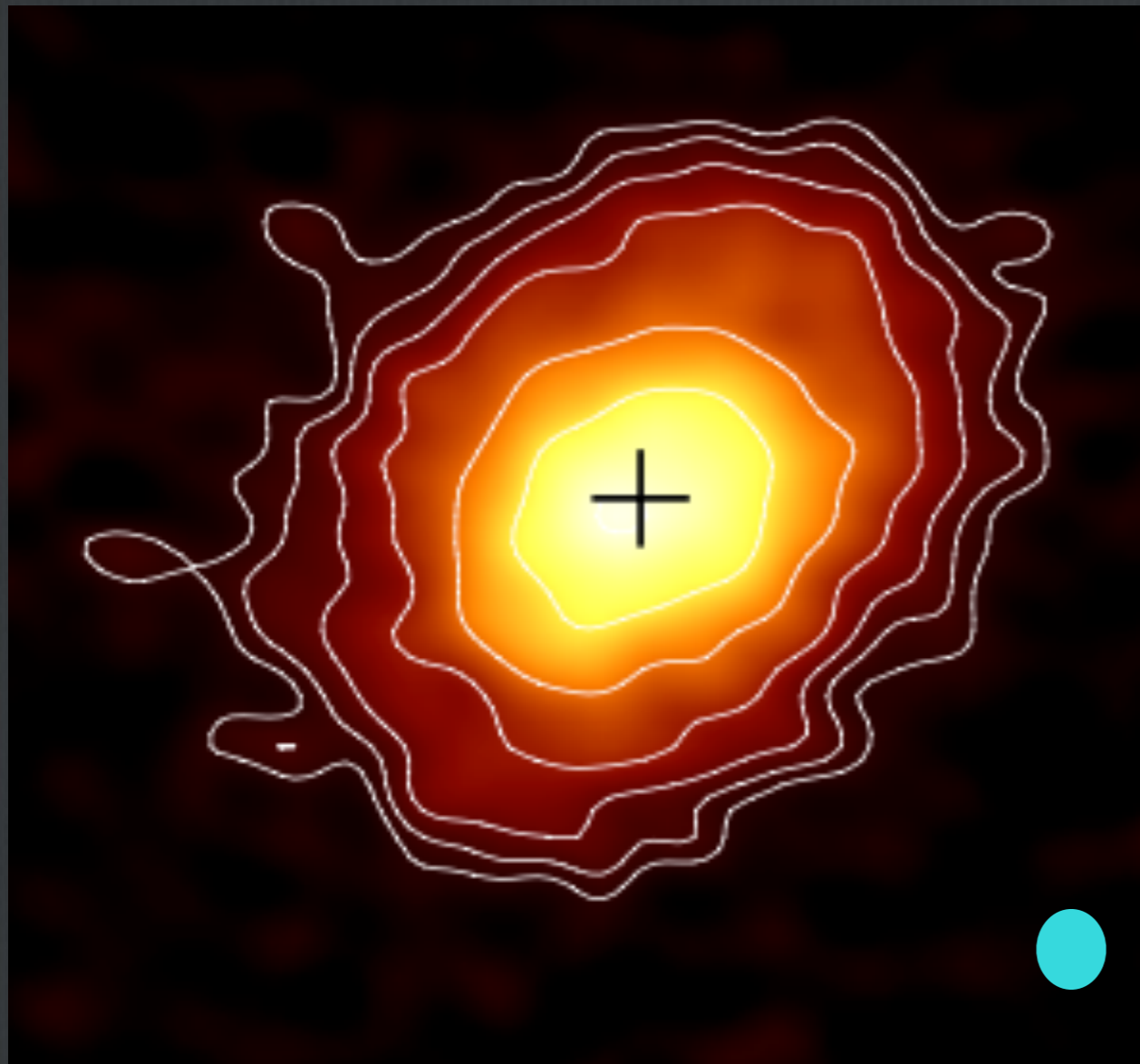


Looney et al (2000)
BIMA observations

FIG. 4.—HL Tauri maps of the $\lambda = 2.7$ mm continuum emission. All panels are contoured in steps of $(-4, -3, -2, 2, 3, 4, 5, 6, 8, 10, 14.14, 20, 28.28)$ times an rms noise of $2.9 \text{ mJy beam}^{-1}$. (a) $\sigma = 1.7 \text{ mJy beam}^{-1}$; beam is $5''.31 \times 4''.79$ P.A. = -81° . (b) $\sigma = 1.7 \text{ mJy beam}^{-1}$; beam is $3''.43 \times 2''.79$ P.A. = 1° . (c) $\sigma = 2.4 \text{ mJy beam}^{-1}$; beam is $1''.11 \times 0''.94$ P.A. = 53° . (d) $\sigma = 2.9 \text{ mJy beam}^{-1}$; beam is $0''.68 \times 0''.48$ P.A. = 43° . The cross in panel (d) is the $\lambda = 3.6$ cm peak from Rodríguez et al. 1994.

HL Tau

A planet-forming disc around a young star



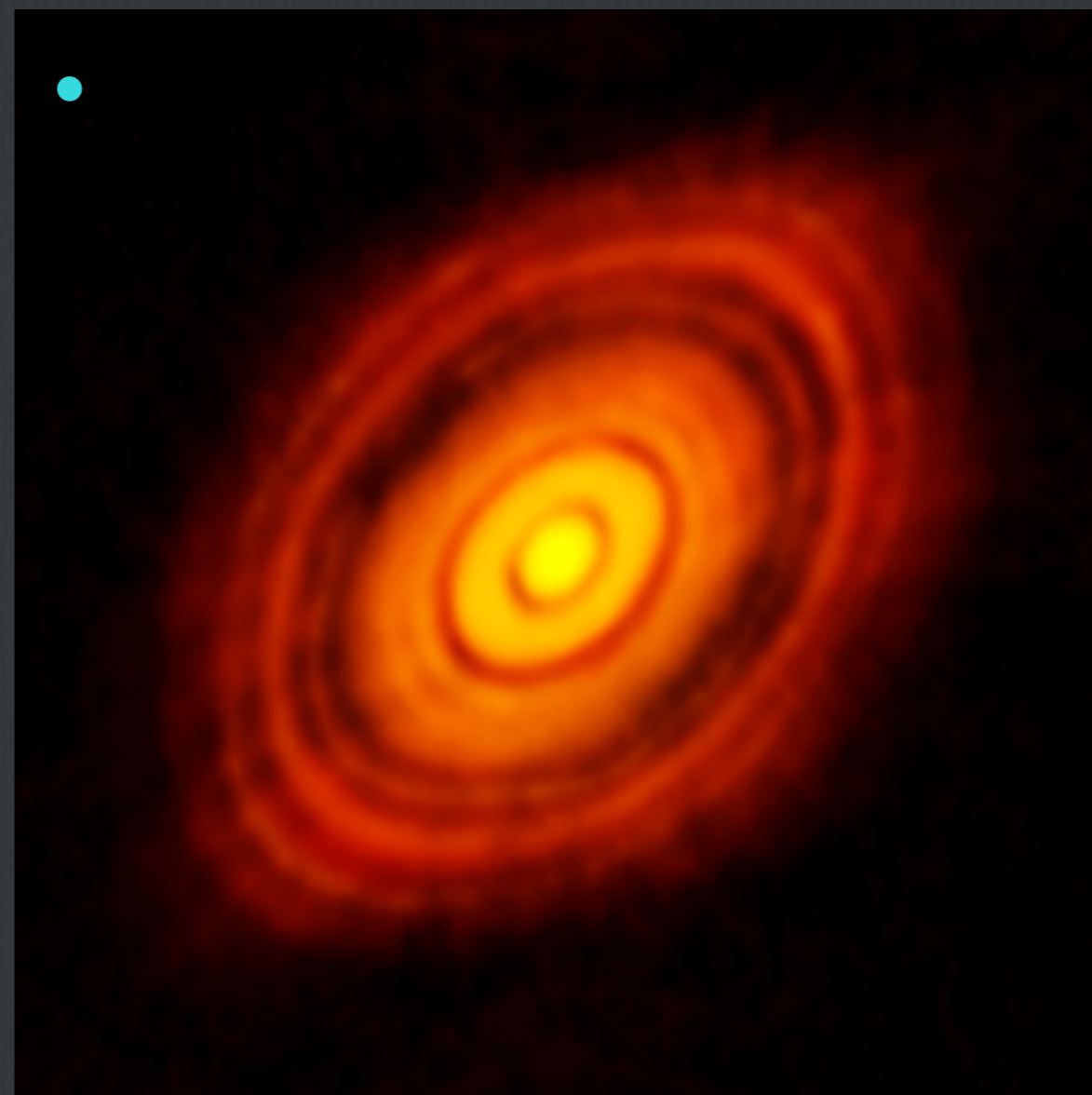
CARMA 2011: A,B,C configuration @ 230 GHz
→ 130 milli-arcsec. Kwon et al (2011)

HL Tau

A planet-forming disc around a young star



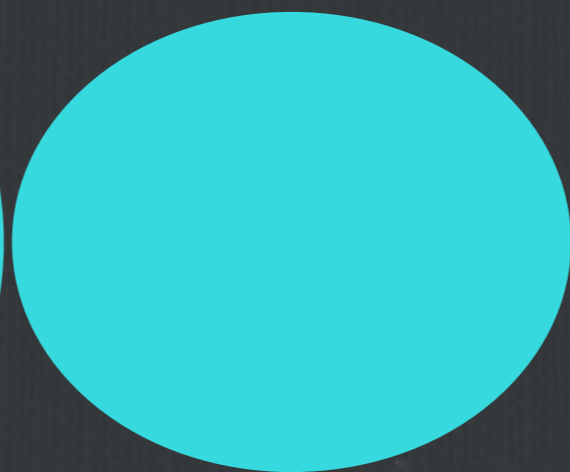
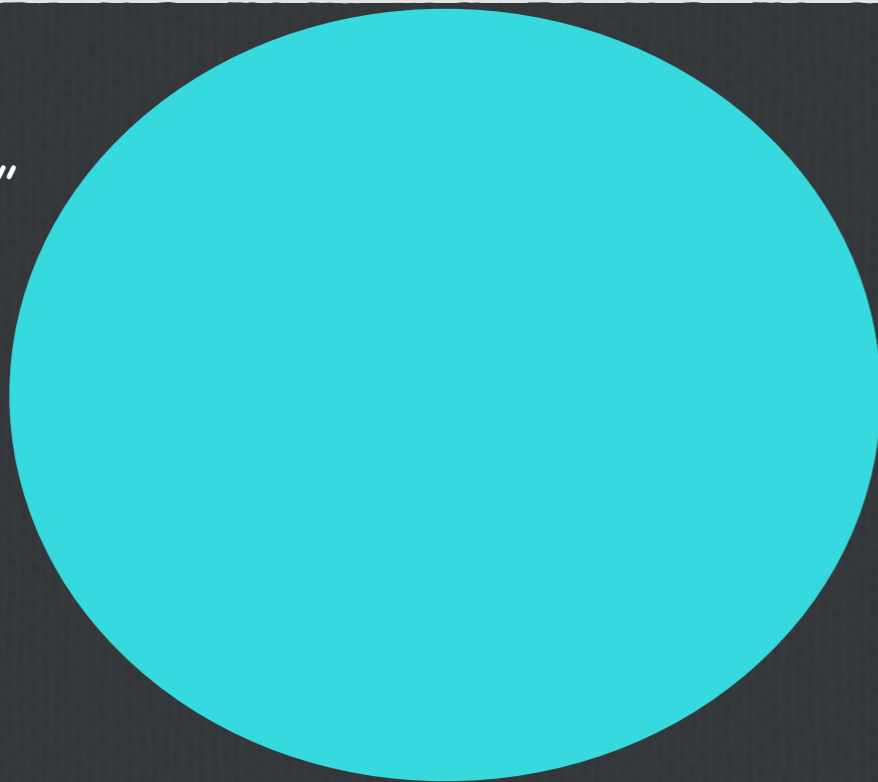
CARMA 2011: A,B,C configuration @ 230 GHz
→ 130 milli-arcsec. Kwon et al (2011)



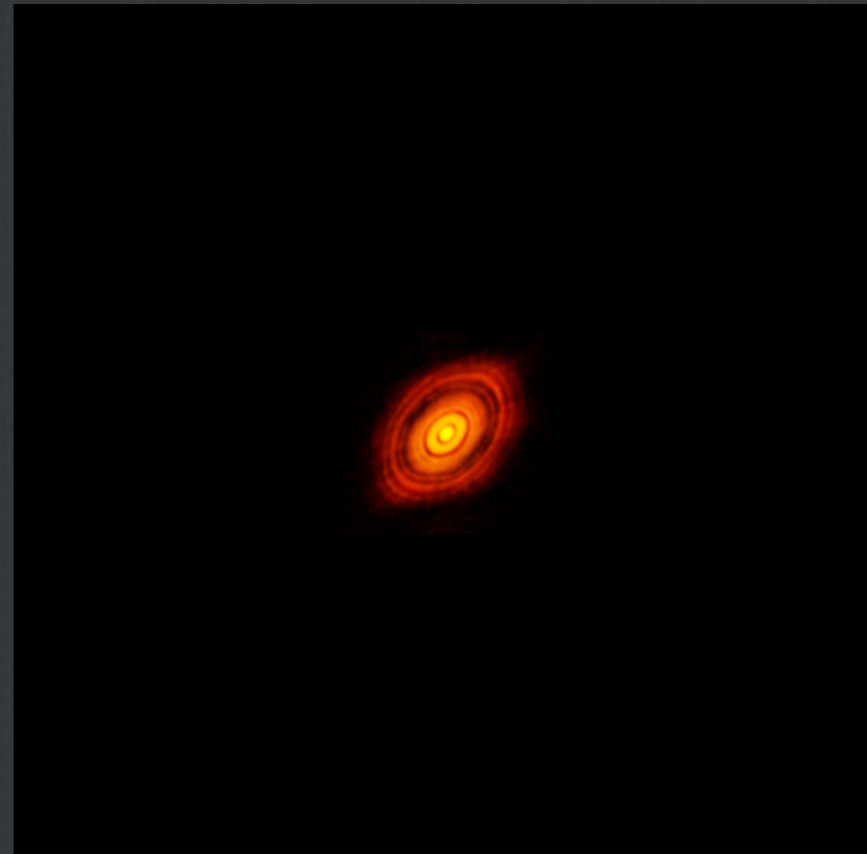
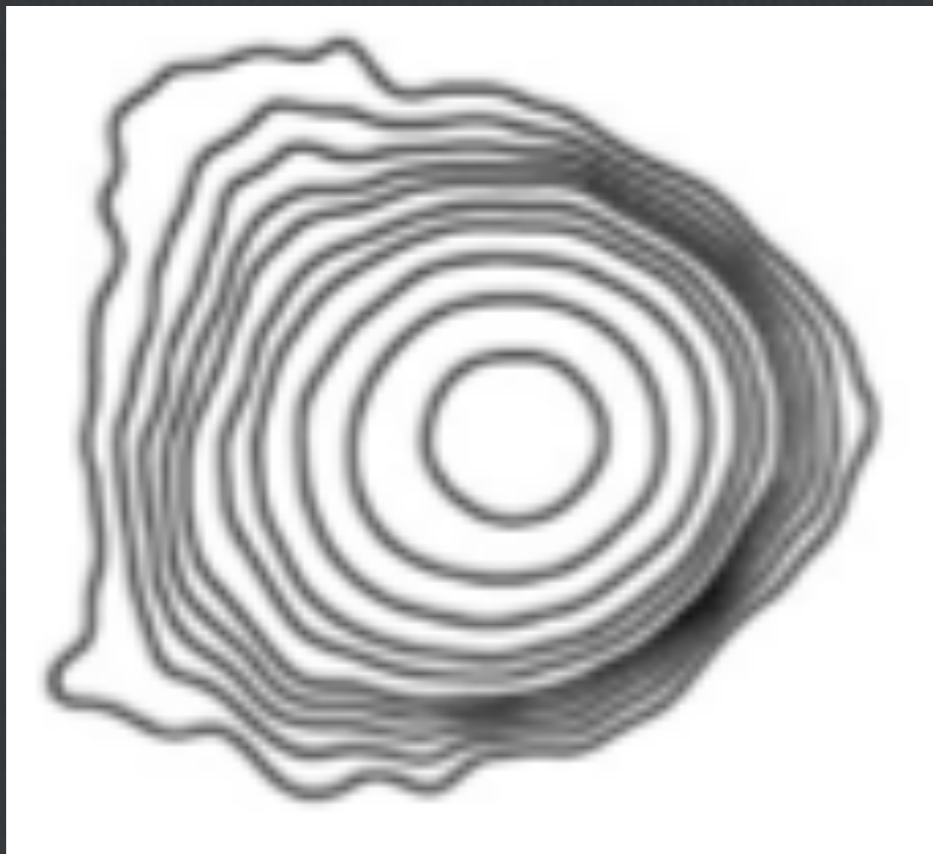
ALMA 2014: 15 km-baseline @ 233 GHz 4.5
hours → 35 milli-arcsec

HL Tau

5.3''



0.0035''



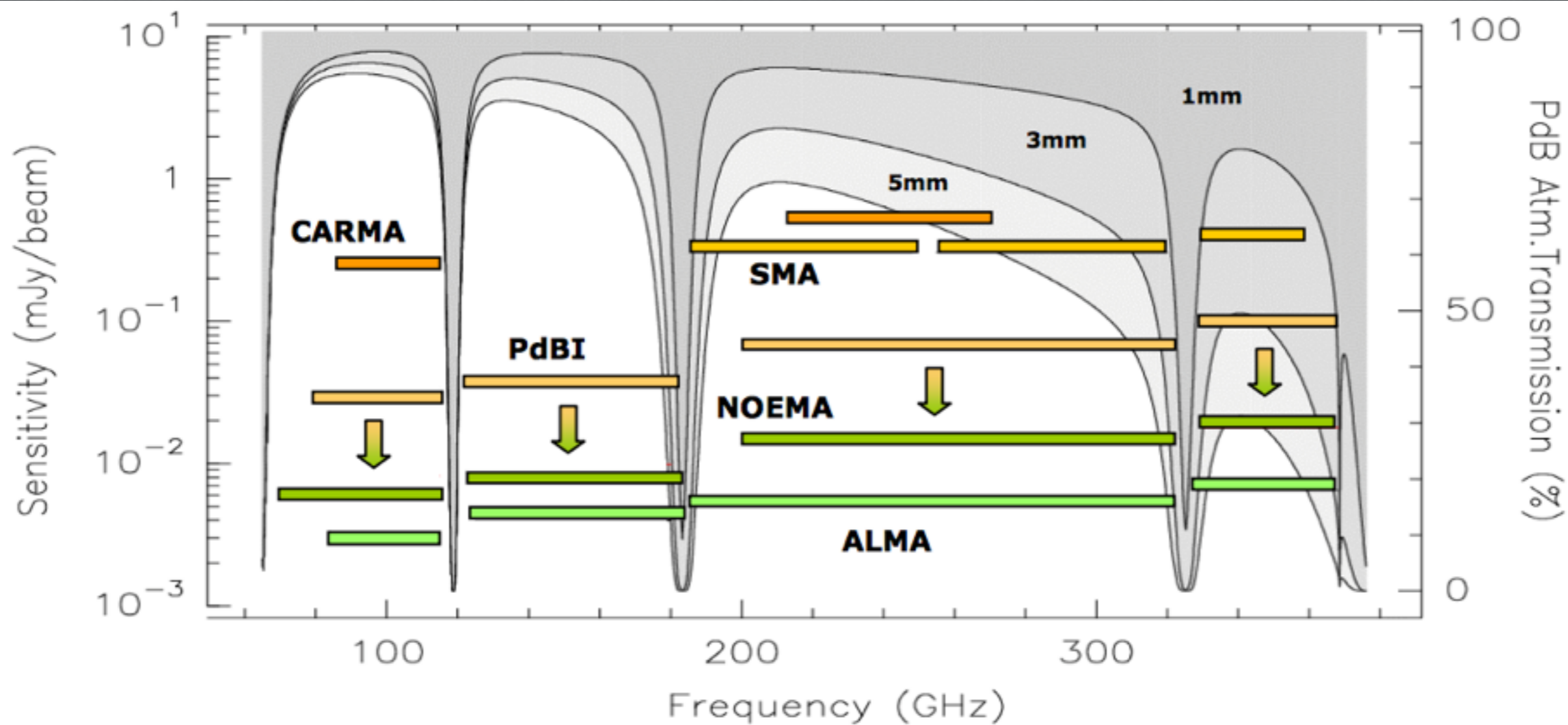
HL Tau

Good imaging capability needs :

- **if necessary : spatial resolution (and good weather)**
- **uv-coverage (sampling of the equivalent larger telescope area by the collection of smaller apertures)**
- **sensitivity**

	Altitude (m)	N_{ANT}	Diameter (m)	Coll.Area (m ²)
IRAM PdBI	2550	6	15	1060
CARMA	2200	15	6/10	772
SMA+CSO+JCMT	4080	10	6/10/15	481
NMA	1340	6	10	471

IRAM NOEMA	2550	12	15	2120
ALMA	5060	50	12	5652

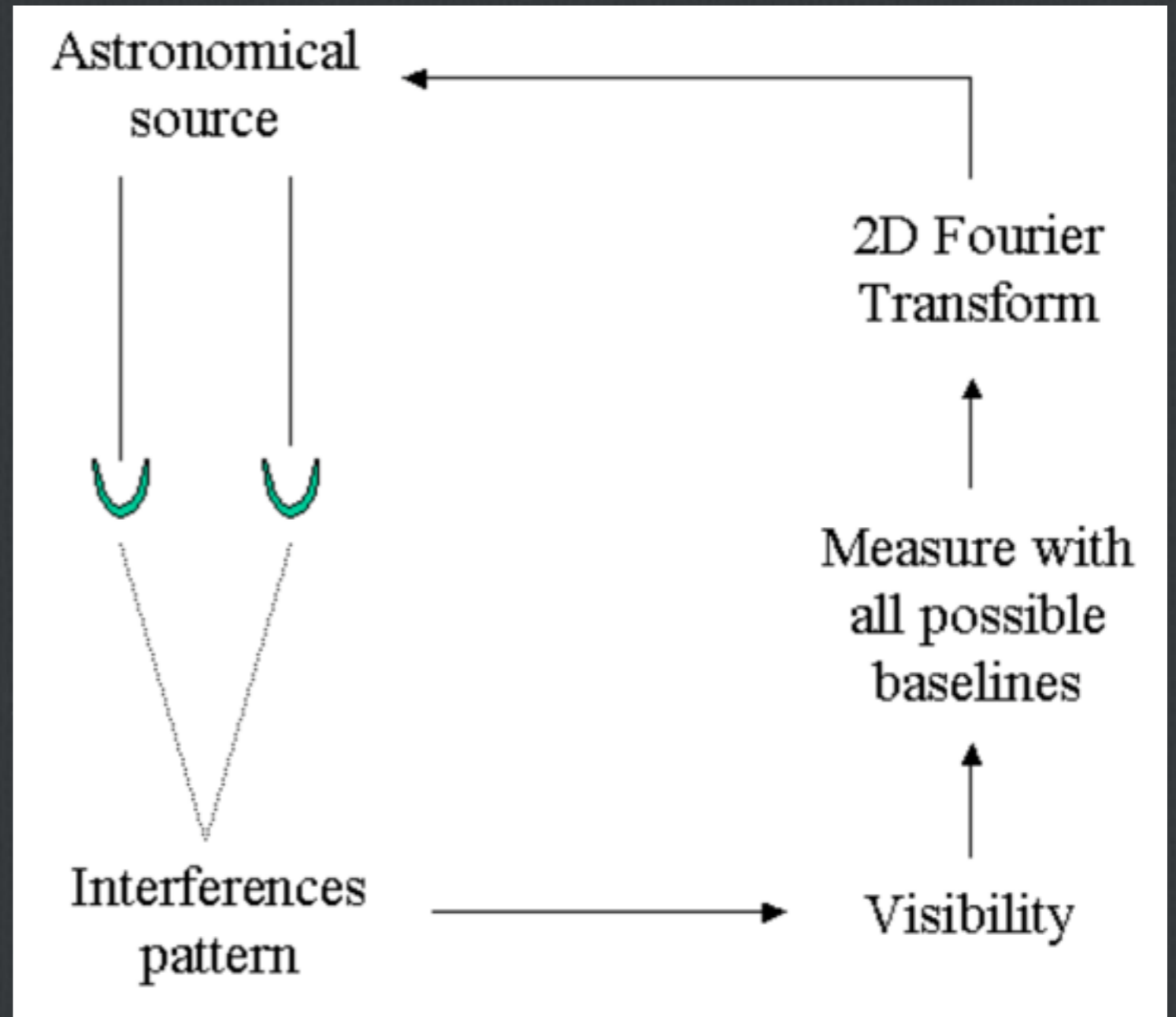


Principles : $V_v \leftrightarrow I_v(\sigma)$

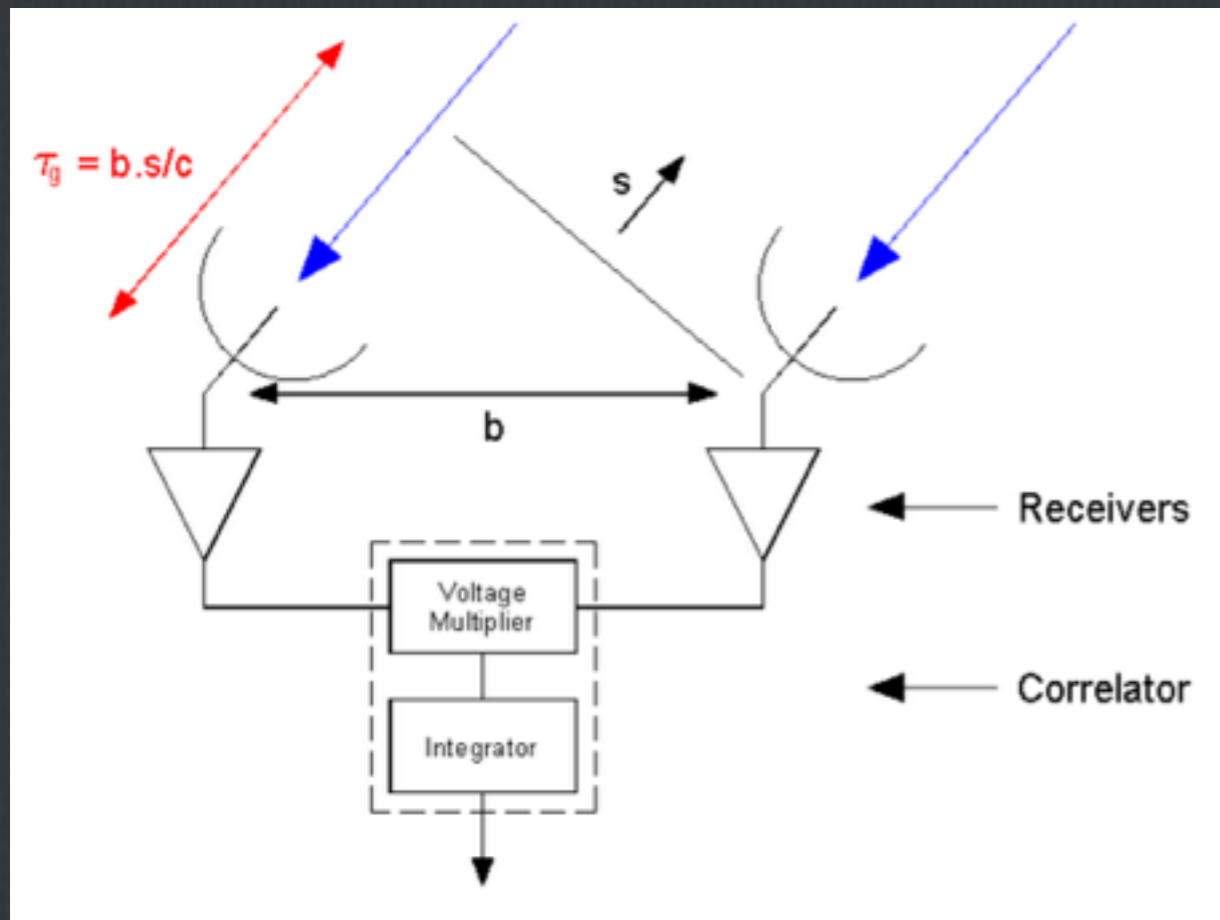
The van Cittert-Zernike theorem

The **Visibility** (the measured spatial coherence function) is the Fourier transform of the **source surface brightness spatial distribution**

$$V_v = \text{TF} [A(\sigma) I_v(\sigma)]$$



Principles : a point source



The heterodyne receivers measure the incoming electric field :

$$E \cos (2\pi\nu t)$$

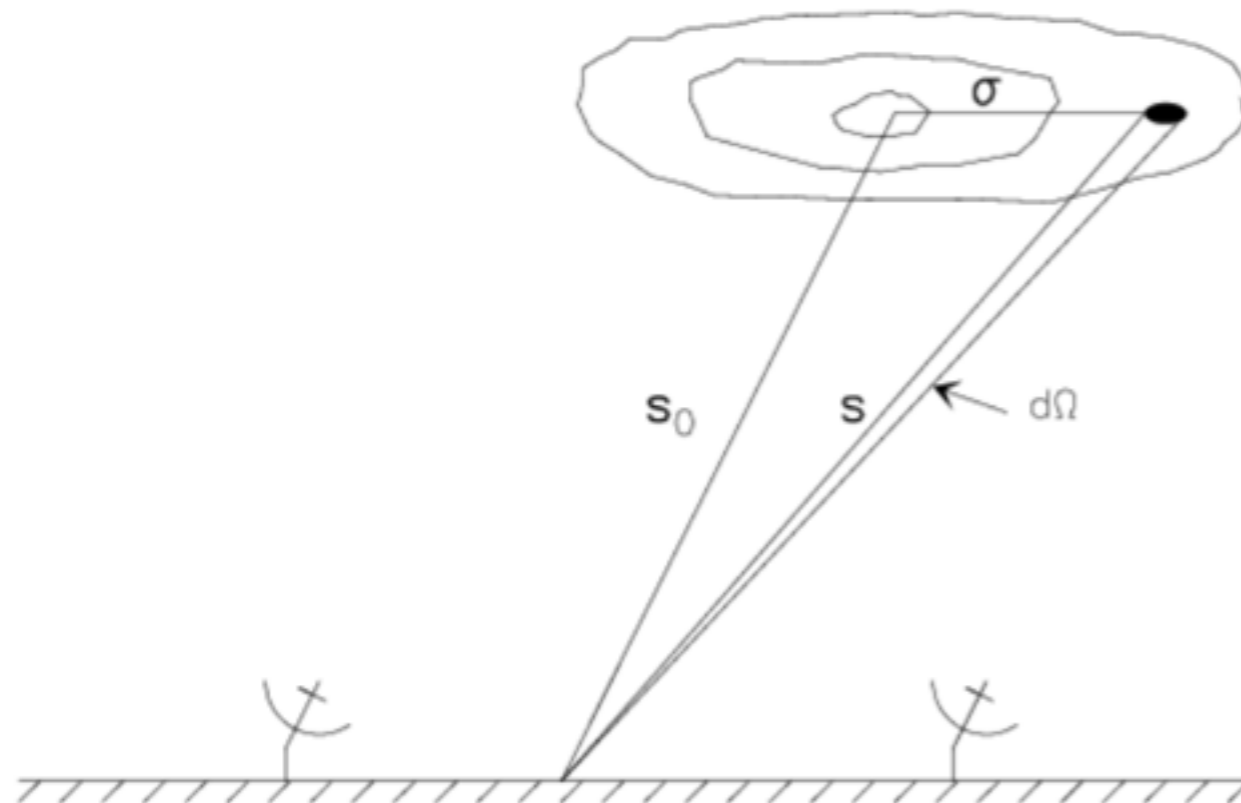
The correlator is a multiplier followed by a time integrator. It measures :

$$\begin{aligned} r(t) &= \langle E_1 \cos (2\pi\nu(t-\tau_g)).E_2 \cos (2\pi\nu t) \rangle \\ &= E_1 E_2 \cos (2\pi\nu\tau_g) \end{aligned}$$

with τ_g the time delay that corresponds to the geometrical delay for a coherent signal to reach each antenna :

$$\tau_g = (b.s)/c$$

Principles : an extended source



$$\mathbf{s} = \mathbf{s}_0 + \sigma$$

Power received from

$$d\Omega: A(\mathbf{s})I(\mathbf{s})d\Omega$$

$$A(\mathbf{s}) = \text{beam}$$

$$I(\mathbf{s}) = \text{source}$$

Correlator output: $r = E_1 E_2 \cos(2\pi\nu\tau_g)$

$$r = A(\mathbf{s})I(\mathbf{s})d\Omega \cos(2\pi\nu\tau_g(\mathbf{s}))$$

Principles : an extended source

$$\begin{aligned} R &= \int_{Sky} A(\mathbf{s}) I(\mathbf{s}) \cos(2\pi\nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega && \text{Measured correlator output} \\ &= \cos\left(2\pi\nu \frac{\mathbf{b} \cdot \mathbf{s}_0}{c}\right) \int_{Sky} A(\sigma) I(\sigma) \cos(2\pi\nu \mathbf{b} \cdot \sigma / c) d\Omega \\ &\quad - \sin\left(2\pi\nu \frac{\mathbf{b} \cdot \mathbf{s}_0}{c}\right) \int_{Sky} A(\sigma) I(\sigma) \sin(2\pi\nu \mathbf{b} \cdot \sigma / c) d\Omega \\ &= \cos\left(2\pi\nu \frac{\mathbf{b} \cdot \mathbf{s}_0}{c}\right) |V| \cos \varphi_V - \sin\left(2\pi\nu \frac{\mathbf{b} \cdot \mathbf{s}_0}{c}\right) |V| \sin \varphi_V \\ &= |V| \cos(2\pi\nu \tau_g - \varphi_V) \end{aligned}$$

need to correct from the delay

We want something that resembles a TF

$$V = |V| e^{i\varphi_V} = \int_{Sky} A(\sigma) I(\sigma) e^{-2i\pi\nu \mathbf{b} \cdot \sigma / c} d\Omega$$

Delay correction

- The geometrical delay varies slowly with the earth rotation at a rate of ($\nu \cdot d\tau_g/dt \approx \Omega_{\text{terre}} \cdot b \cdot \nu/c \sim 10$ Hz @ $b=300\text{m}$ and 100 GHz)
- Because the source is not monochromatic, the delay attenuates the fringes visibility

$$\begin{aligned} R &= \frac{1}{\Delta\nu} \int_{\nu_0 - \Delta\nu/2}^{\nu_0 + \Delta\nu/2} |V| \cos(2\pi\nu\tau_g - \varphi_V) d\nu \\ &= |V| \cos(2\pi\nu_0\tau_g - \varphi_V) \frac{\sin(\pi\Delta\nu\tau_g)}{\pi\Delta\nu\tau_g} \end{aligned}$$

It depends on the antenna positions, the source direction and the time

So the delay can be corrected

Delay correction

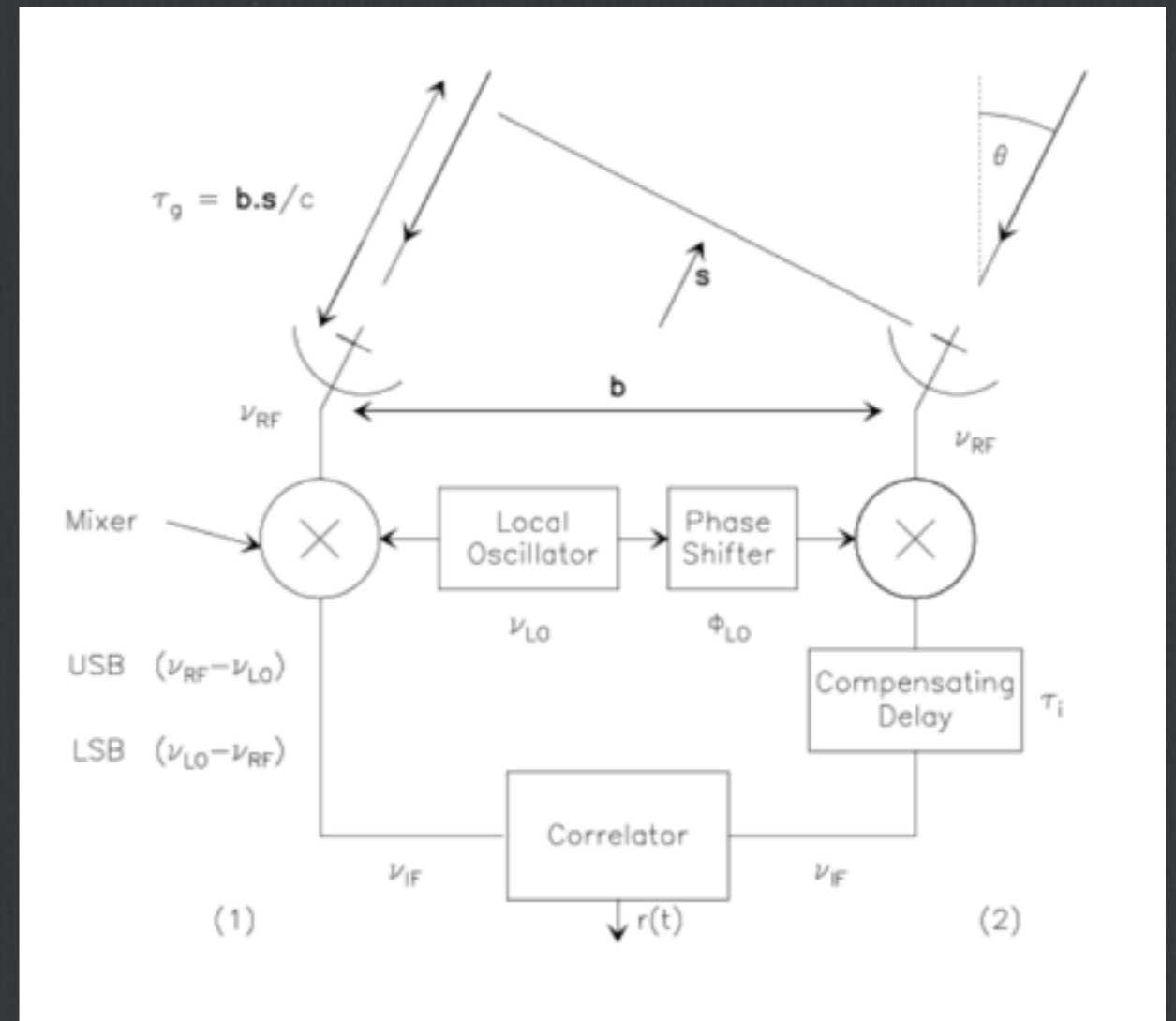
A compensating delay is introduced in one of the branch of the interferometer, on the IF signal

- After fringe stopping, the correlator measures

$$R = |V| \cos(-\varphi_V)$$

- A second correlator is necessary, with a signal phase shifted by $\pi/2$

$$R_i = |V| \sin(-\varphi_V)$$



...it measures the complex visibility (amplitude and phase) for each baseline

uv-plane

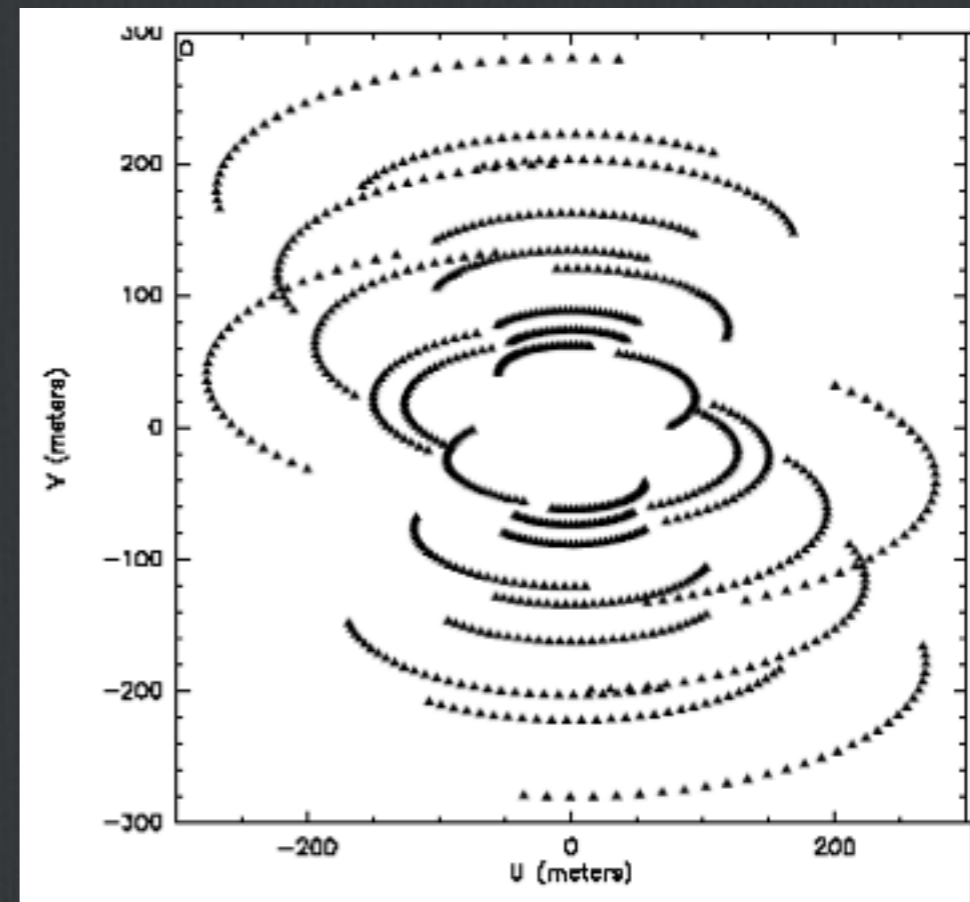
- The interferometer measures the complex visibility for each baseline
- (u, v) is the 2-antenna vector (baseline) projected on the plane perpendicular to the sourced direction : the uv-plane
- (u, v) are also called spatial frequencies
- Earth rotation (super-synthesis)

Spatial frequencies

$$V(u, v) = \int_{\text{Sky}} A(l, m) I(l, m) e^{-2i\pi\nu(ul+vm)} d\Omega$$

Weights

For small field of view, V is the 2D FT of the sky brightness distribution multiplied by $A(l, m)$ the primary beam



Measurements = uv plane sampling x visibilities

Imaging

The measurements by the interferometer are

$$V(u,v) = \iint A(x,y) I_{\text{source}}(x,y) e^{-2i\pi(ux+vy)} dx dy = \text{FT}\{B_{\text{primary}} \cdot I_{\text{source}}\}$$

To determine the source brightness distribution, one must compute the inverse Fourier transform FT^{-1}

$$I_{\text{meas}}(x,y) = \iint S(u,v) V(u,v) e^{2i\pi(ux+vy)} du dv = \text{FT}^{-1}\{S \cdot V\}$$

but because of the limited sampling function (uv-coverage), the measurements are discrete (need to be gridded for computation)

- $S(u,v)=1$ at (u, v) points where visibilities are measured and $S(u,v)=0$ elsewhere
- One defines the dirty beam as $B_{\text{dirty}} = 2\text{D FT}^{-1}\{S\}$: the FT of the uv plane coverage i.e. the PSF of the observations

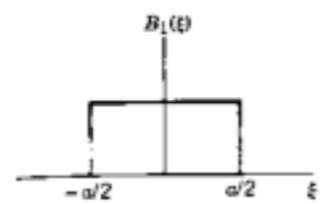
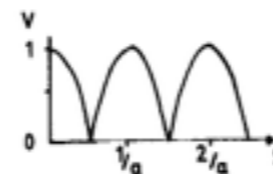
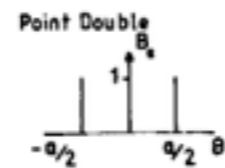
$$I_{\text{meas}} = \text{FT}^{-1}\{S \cdot V\} = \text{FT}^{-1}\{S\} * \text{FT}^{-1}\{V\} = \text{FT}^{-1}\{S\} * \text{FT}^{-1}\{\text{FT}\{B_{\text{primary}} \cdot I_{\text{source}}\}\}$$

\Leftrightarrow

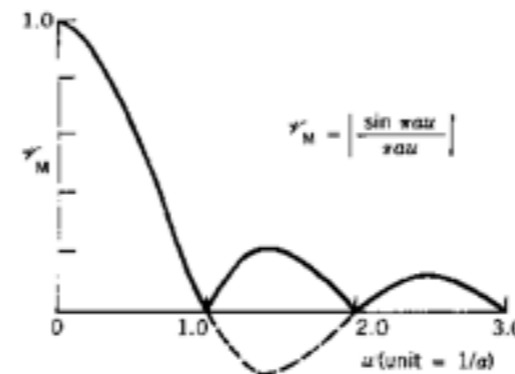
$$I_{\text{meas}} = B_{\text{dirty}} * (B_{\text{primary}} \cdot I_{\text{source}})$$

measure I_{meas} and B_{dirty} \rightarrow do a deconvolution (a clean) \rightarrow divide by B_{primary} \rightarrow get I_{source}

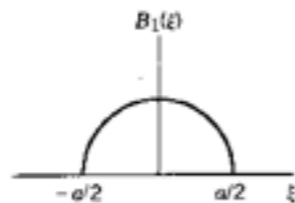
1D Fourier transform



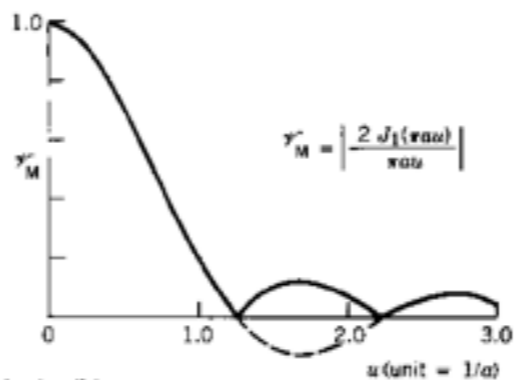
$$B_1(\xi) = \begin{cases} 1.0; & -a/2 < \xi < a/2 \\ 0; & \xi \leq -a/2, \xi \geq a/2 \end{cases}$$



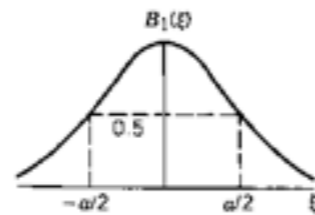
Rectangular model



$$B_1(\xi) = \begin{cases} \sqrt{1 - (2\xi/a)^2}; & -a/2 < \xi < a/2 \\ 0; & \xi \leq -a/2, \xi \geq a/2 \end{cases}$$

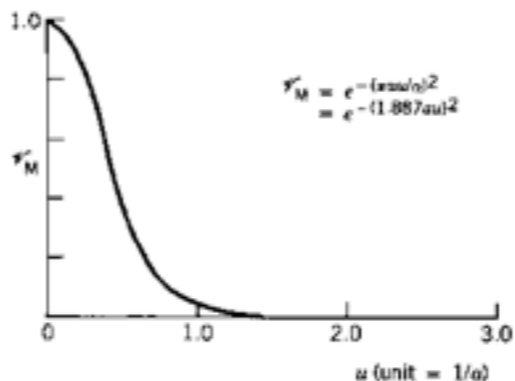


Uniform circular disk



$$B_1(\xi) = e^{-\alpha^2 \xi^2 / a^2} = e^{-11.665 \xi^2 / a^2}$$

$$\alpha = 2\sqrt{\ln 2}$$

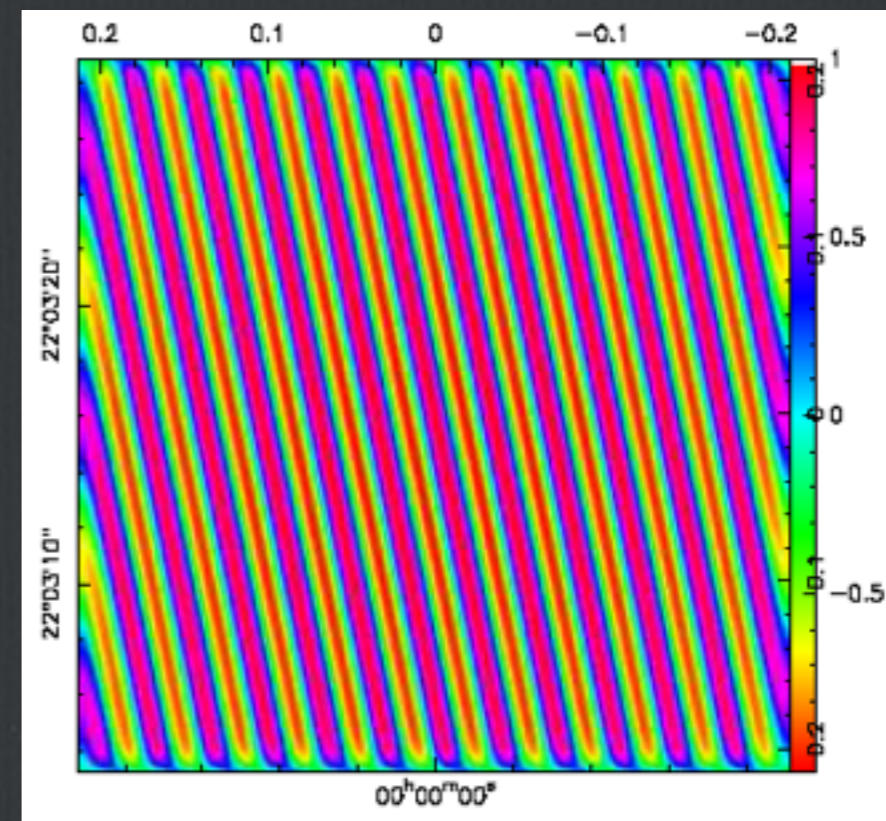
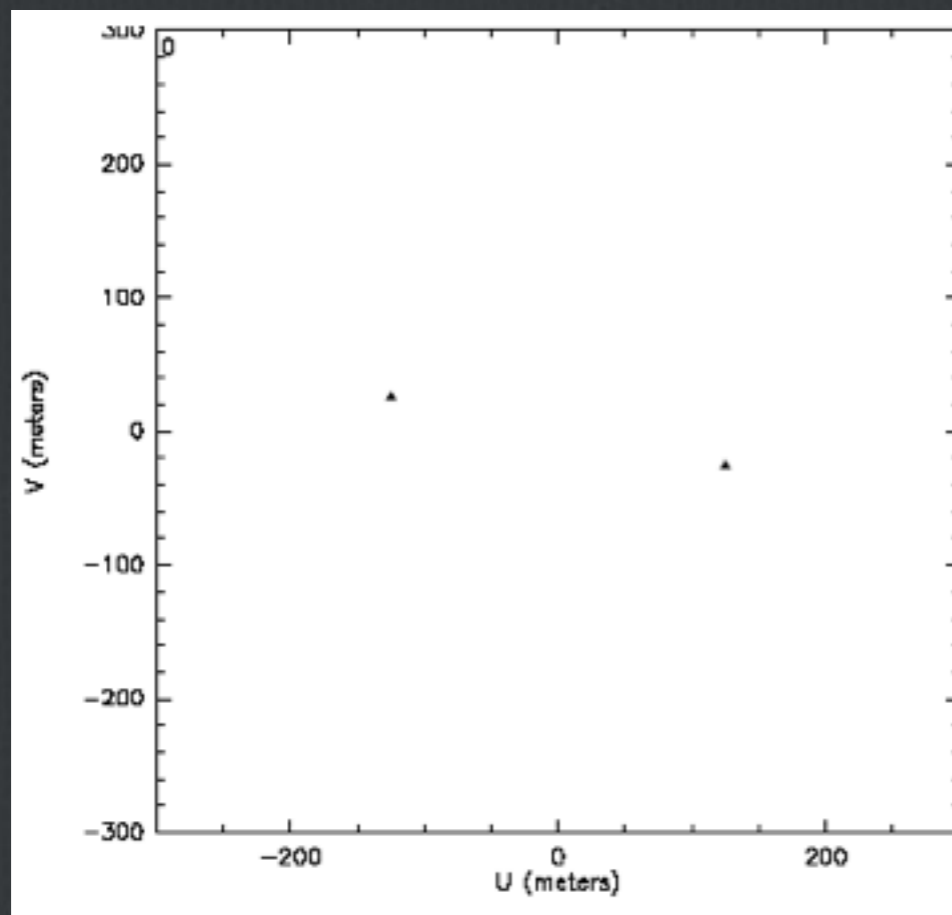


Gaussian model

Dirty beam

$$B_{\text{dirty}} = 2D \text{ FT}^{-1} \{S\}$$

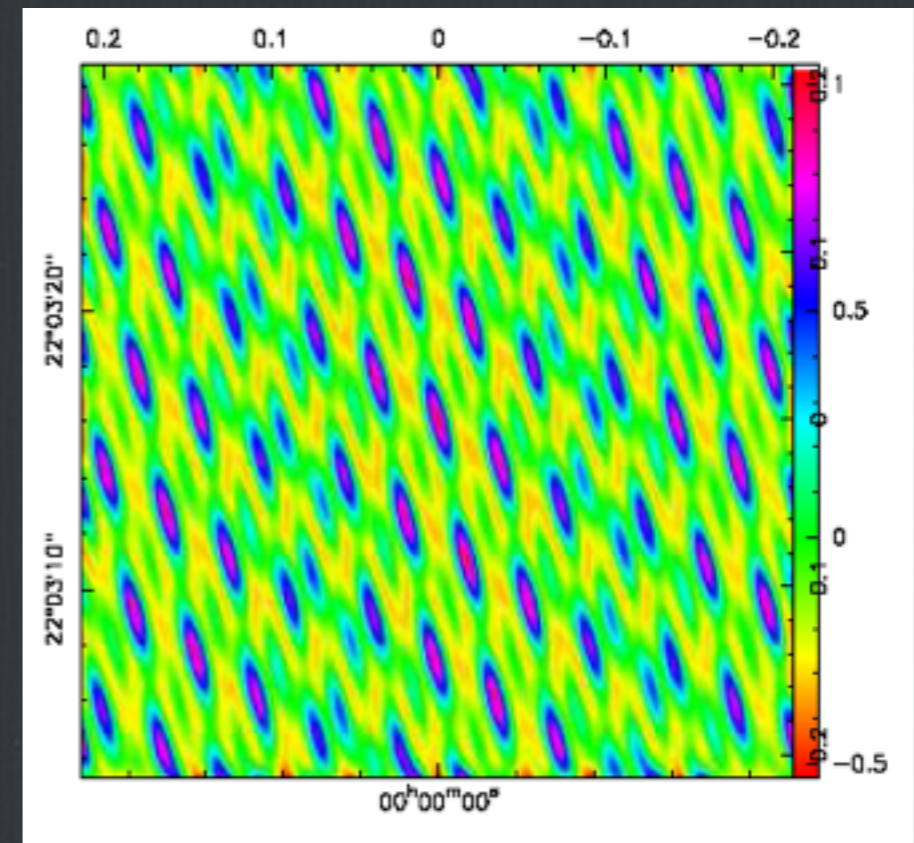
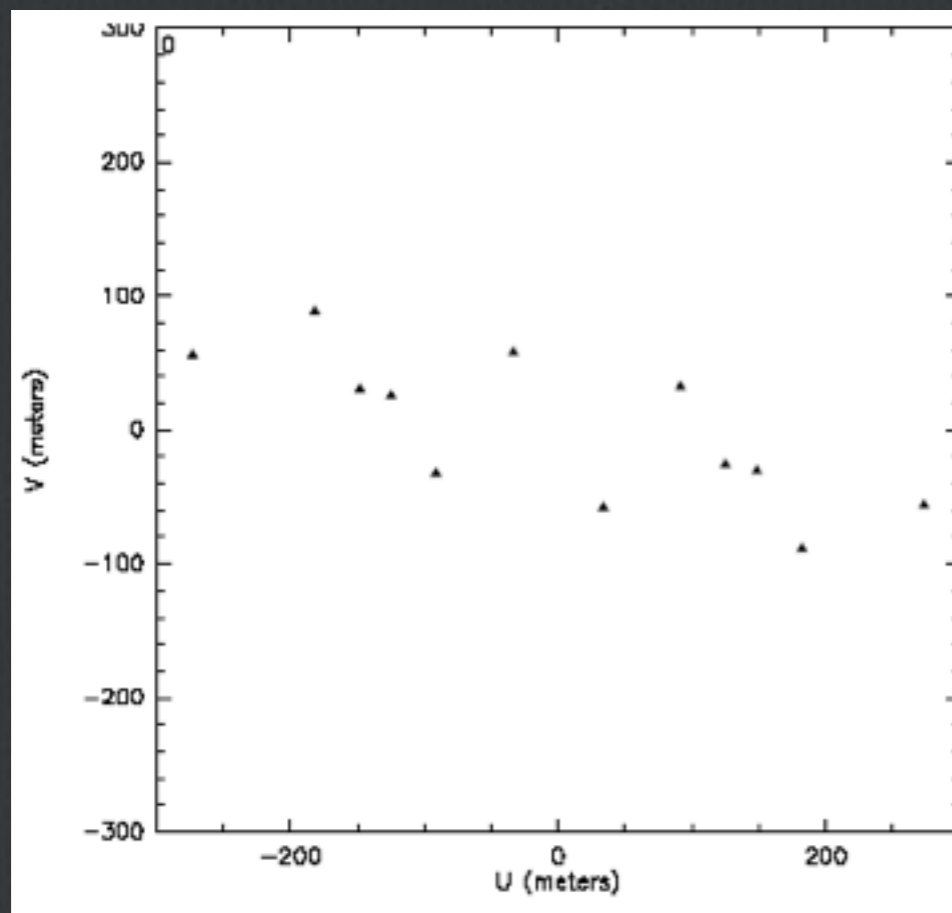
The FT of the uv plane coverage gives the dirty beam i.e. the PSF



Dirty beam

$$B_{\text{dirty}} = 2D \text{ FT}^{-1} \{S\}$$

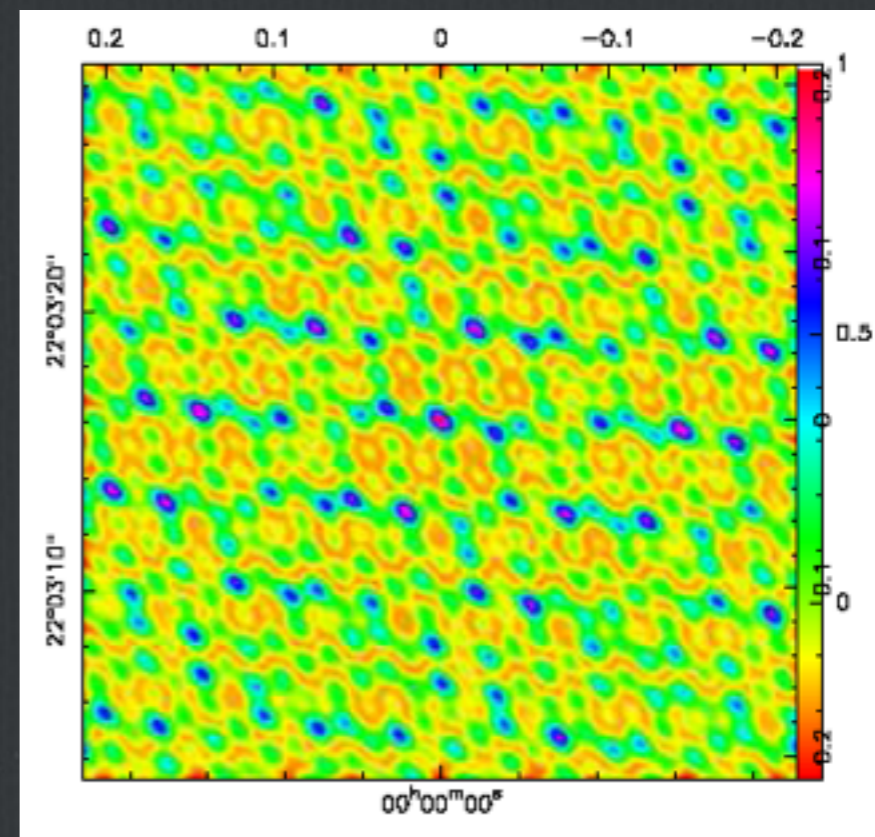
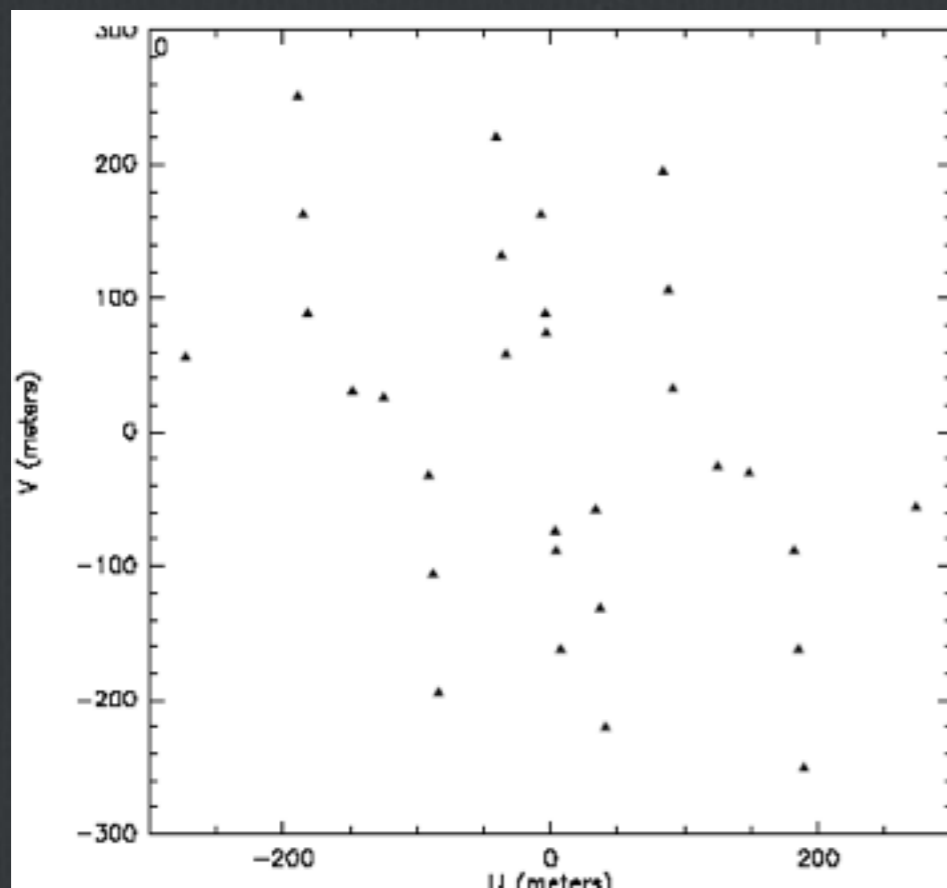
The FT of the uv plane coverage gives the dirty beam i.e. the PSF



Dirty beam

$$B_{\text{dirty}} = 2D \text{ FT}^{-1} \{S\}$$

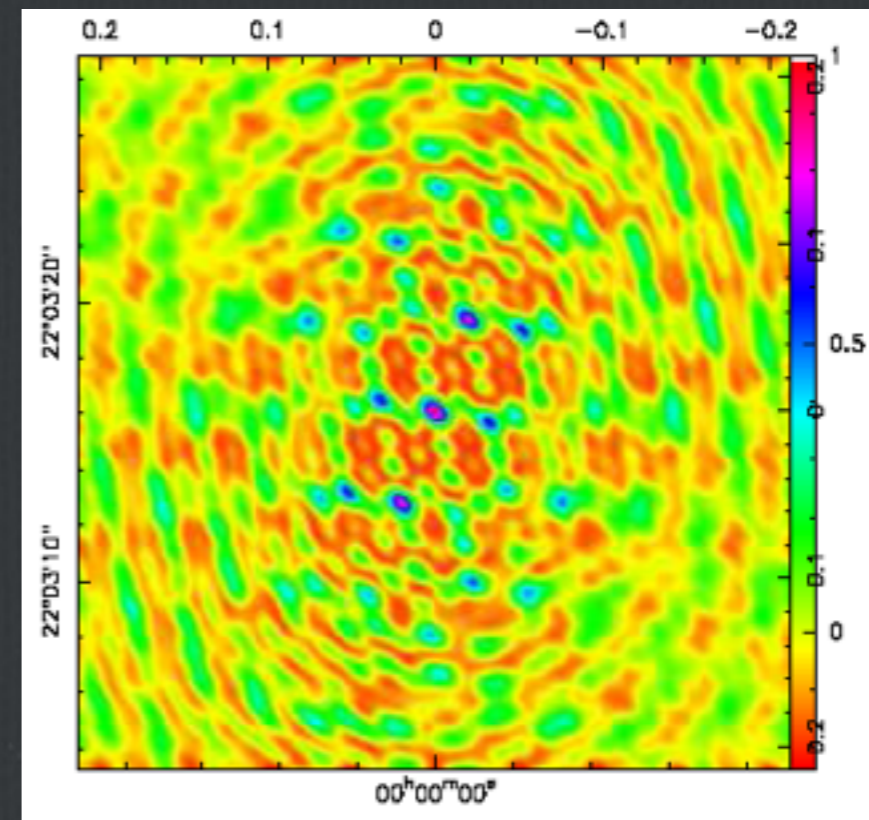
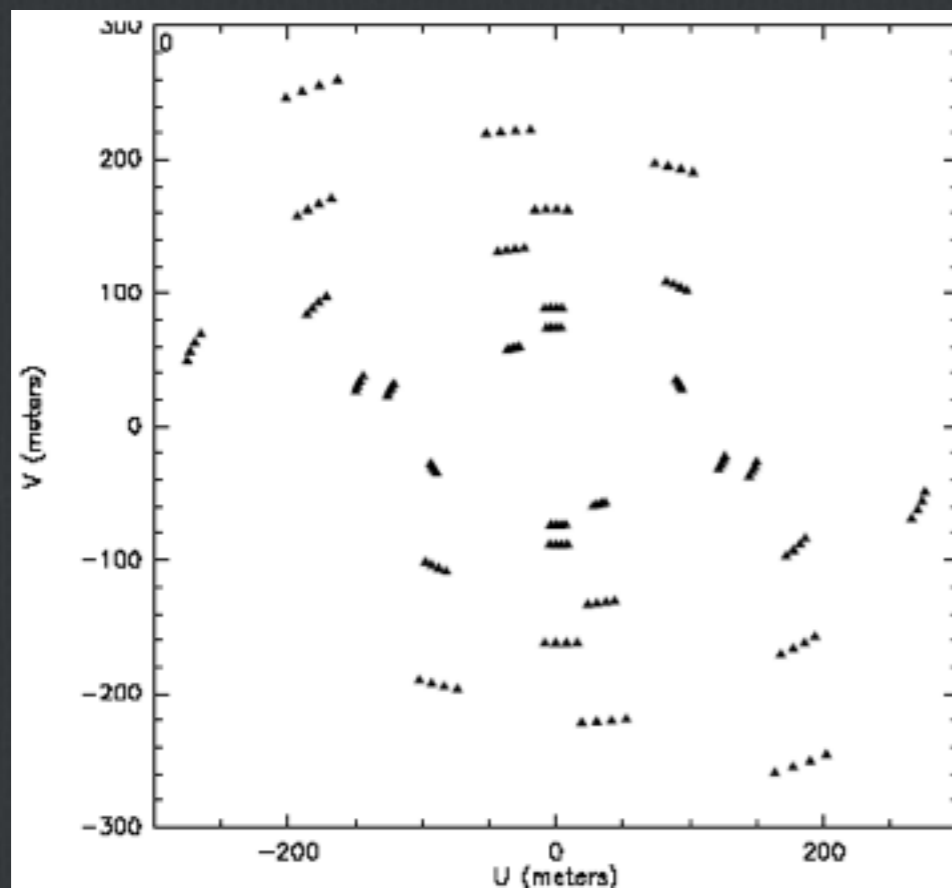
The FT of the uv plane coverage gives the dirty beam i.e. the PSF



Dirty beam

$$B_{\text{dirty}} = 2D \text{ FT}^{-1} \{S\}$$

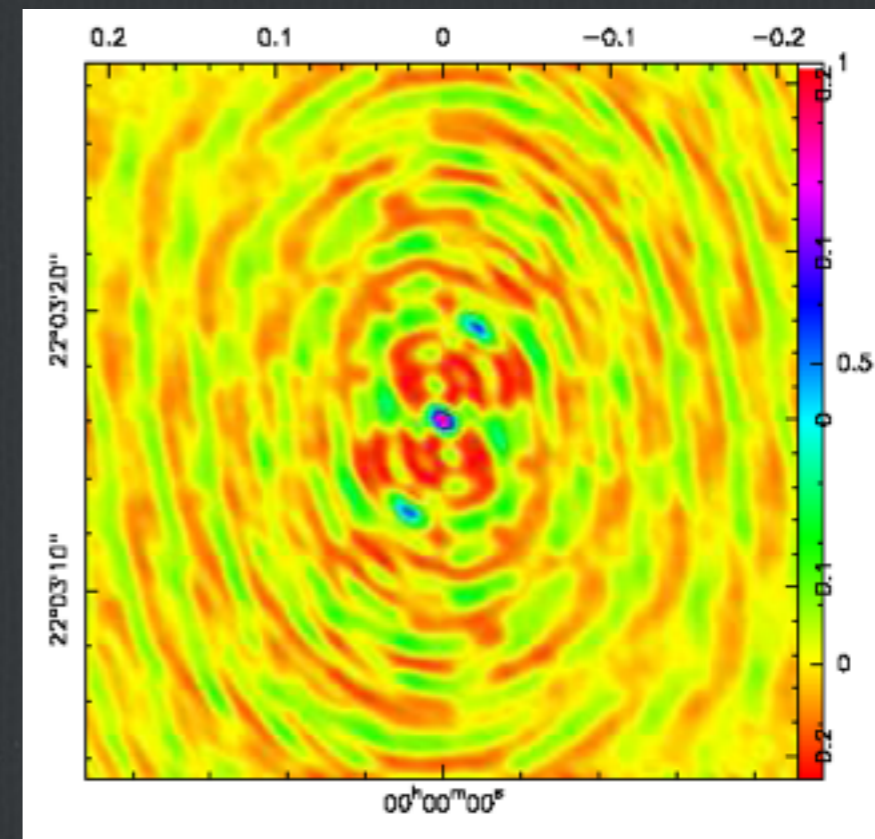
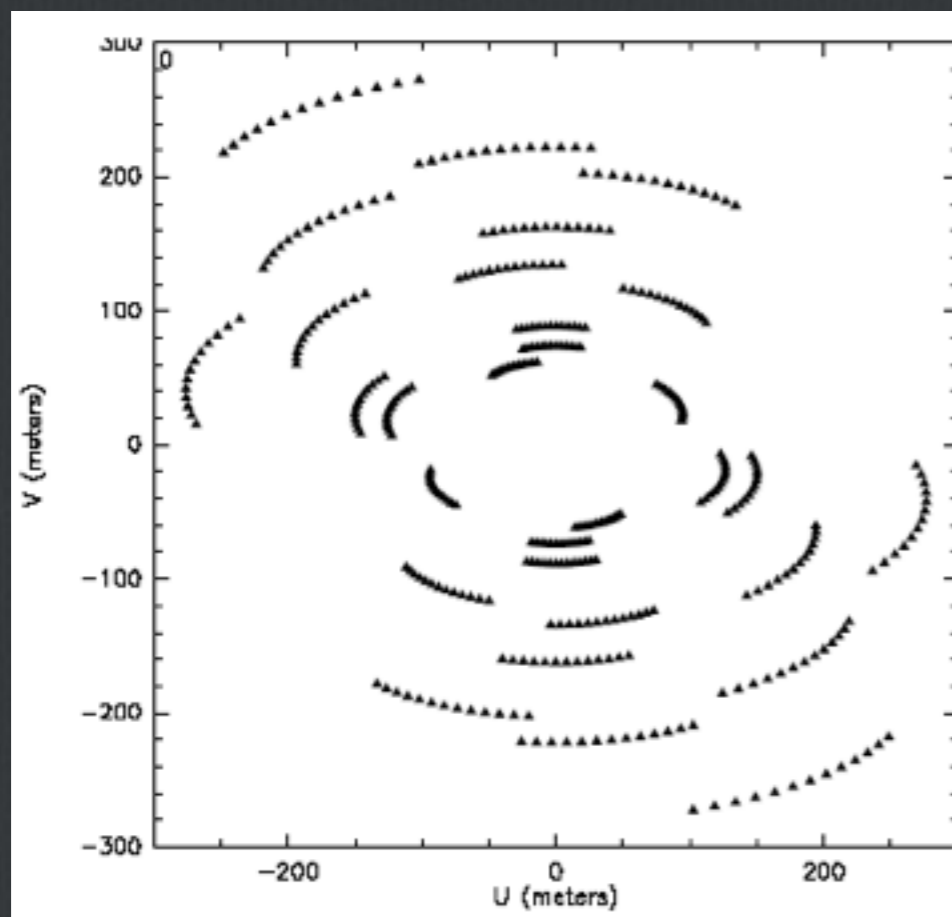
The FT of the uv plane coverage gives the dirty beam i.e. the PSF



Dirty beam

$$B_{\text{dirty}} = 2D \text{ FT}^{-1} \{S\}$$

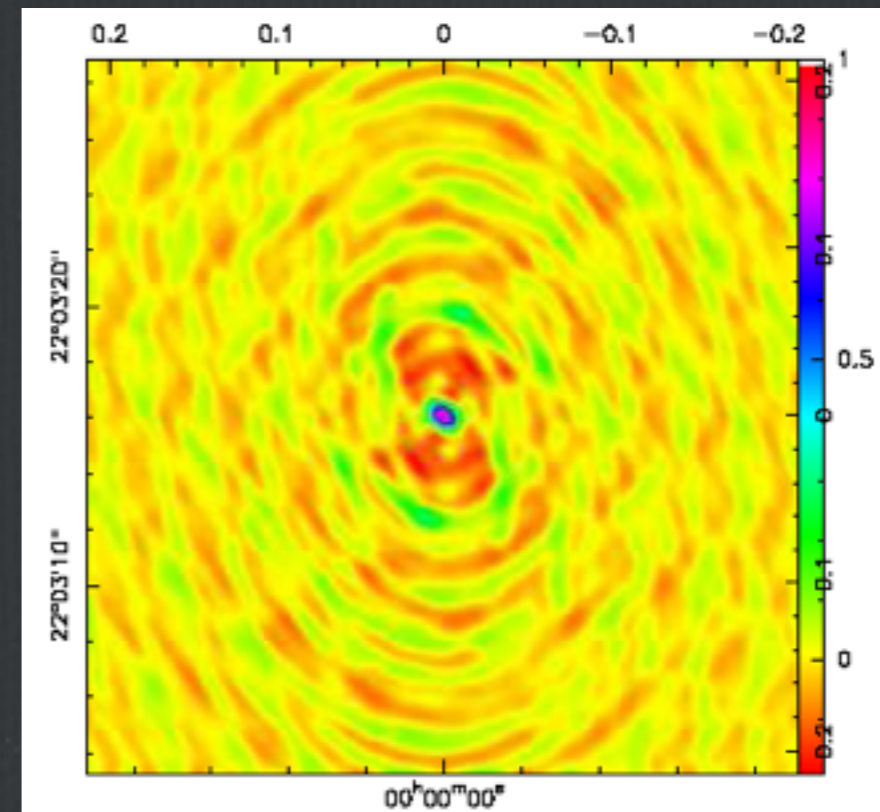
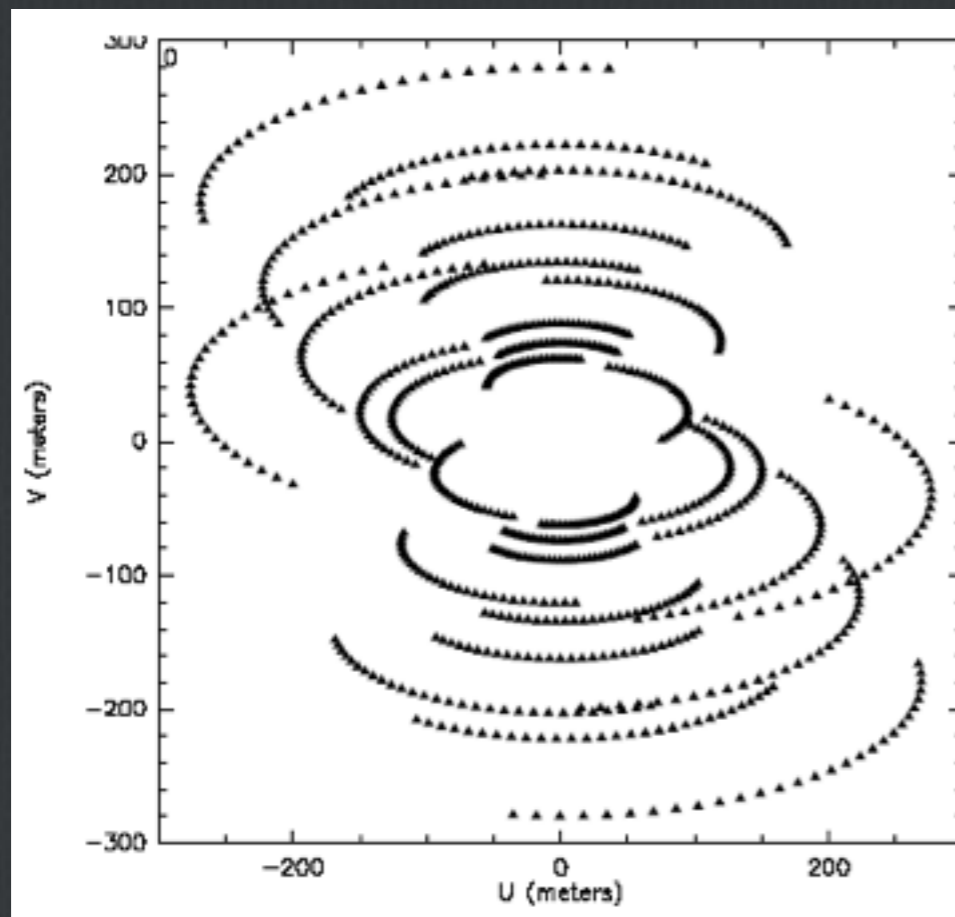
The FT of the uv plane coverage gives the dirty beam i.e. the PSF



Dirty beam

$$B_{\text{dirty}} = 2D \text{ FT}^{-1} \{S\}$$

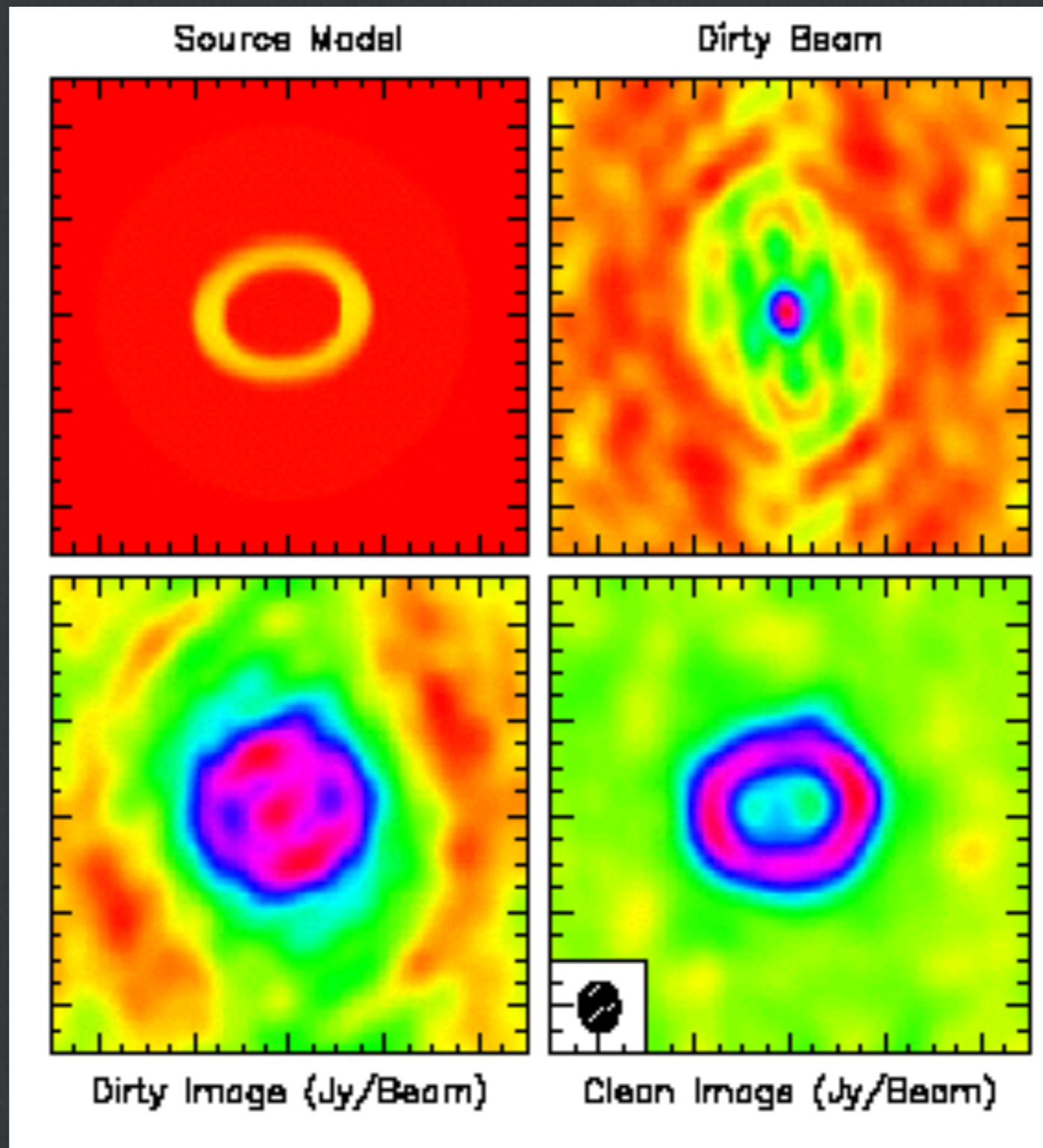
The FT of the uv plane coverage gives the dirty beam i.e. the PSF



Clean

- Look for clean components (emitting peaks in the map) in the dirty image
- Deconvolve from the dirty beam (side-lobes)
- Convolve the clean components by a clean beam (without the side-lobes) → clean image
- Different methods exists : adapted to source distribution kind

Clean



Real life : On-line calibrations

- Pointing
- Focus
- IF filters band pass

- Atmospheric calibration
- Antenna positions
- Delay

- Atmospheric phase correction

Real-time
calibrations

New values can be
entered off-line if
necessary

Uncorrected data
are also stored

Phase decorrelation

The atmosphere turbulence, the water vapor and temperature variations induce optical length variations, that means a adding a phase noise in the visibility : $\Delta \varphi = 2\pi \delta l / \lambda$

$$\Delta \varphi \propto \nu \quad \text{and} \quad \Delta \varphi \propto B^{0.5}$$

The timescale of atmospheric phase fluctuations is ~ 30 s (short)

At PdB, for a 300m-baseline : $\delta l \sim 200\mu\text{m}$: $\Delta \varphi \sim 55^\circ$ @ 1.3mm

that corresponds to a radio-seeing of $\sim 0.3-1''$

Short time scale atmospheric phase fluctuations difficult to calibrate (WVR). It leads to a loss in the signal amplitude because of signal decorrelation (phase σ). The atm decorrelation efficiency is :

$$\eta_a = e^{-0.5\sigma^2} \sim 0.63 \text{ @ } 1.3\text{mm (63\% only)}$$

Real life : offline-calibrations

- **Bandpass** phase and amplitude vs freq
- **Phase** phase vs time
- **Amplitude** amplitude vs time
- **Flux** absolute flux scale

Calibration principles

- Calibrate only temporal or frequency effects, no dependence on (u,v)
- True visibility: $V_{ij}(\nu, t)$ (baseline ij)
- Observed visibility:

$$V_{\text{obs}ij}(\nu, t) = G_{ij}(\nu, t) V_{ij}(\nu, t) + \text{noise}$$

- G_{ij} = complex gain (amplitude & phase)
- Scalar description – no polarization

Calibration principles

- **Most of the effects are antenna-based**
 - Pointing, Focus, Antenna position, Atmosphere, Receivers noise, Receivers bandpass...
- **Gain decomposition:** $V_{obs_{ij}} = G_{ij} V_{ij} = g_i g_j V_{ij}$
- Baseline-based effect?
 - Correlator bandpass → real-time calibration
 - Time and frequency averaging → **decorrelation**

Calibration principles

- Observation of a **point source** of flux S :

$$V_{\text{obs}} = G_{ij} V \quad V = S \quad G_{ij} = V_{\text{obs}}/S$$

- Antenna –based gains: $g_i g_j = V_{\text{obs}}/S$

- Can solve for antenna gains:

$$(g_1)^2 = V_{\text{obs}_{12}} V_{\text{obs}_{31}} / S V_{\text{obs}_{32}}$$

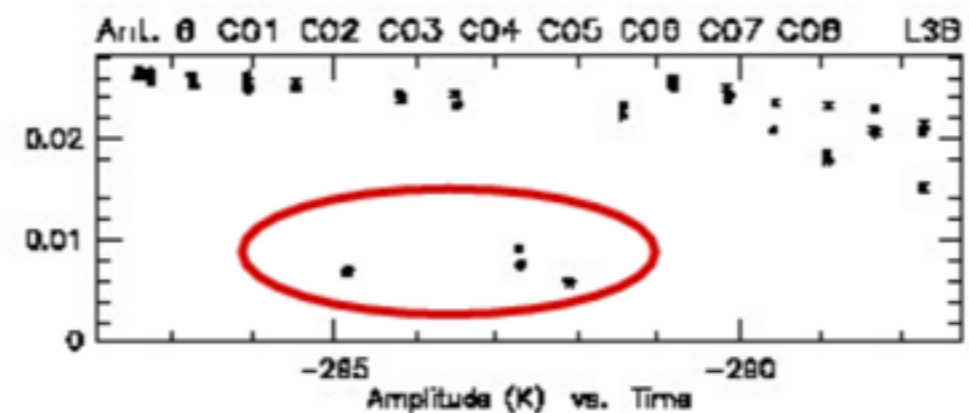
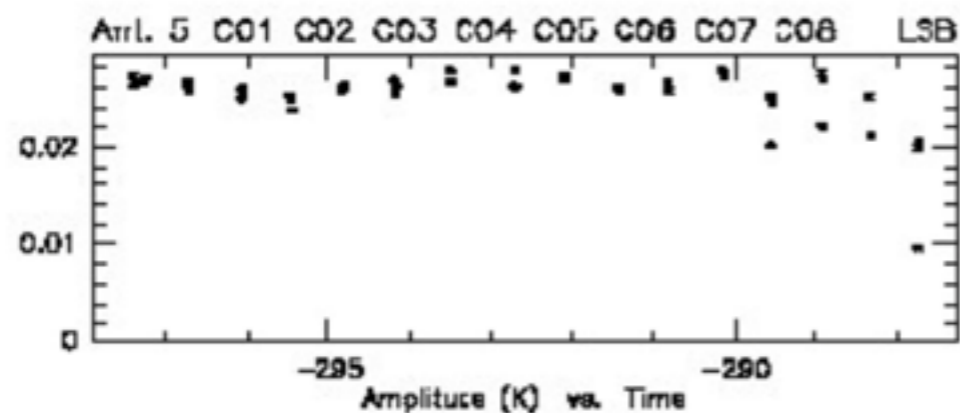
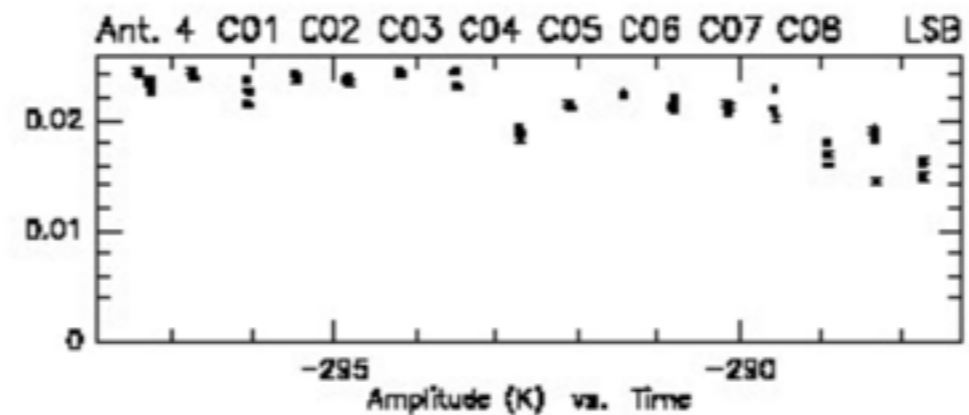
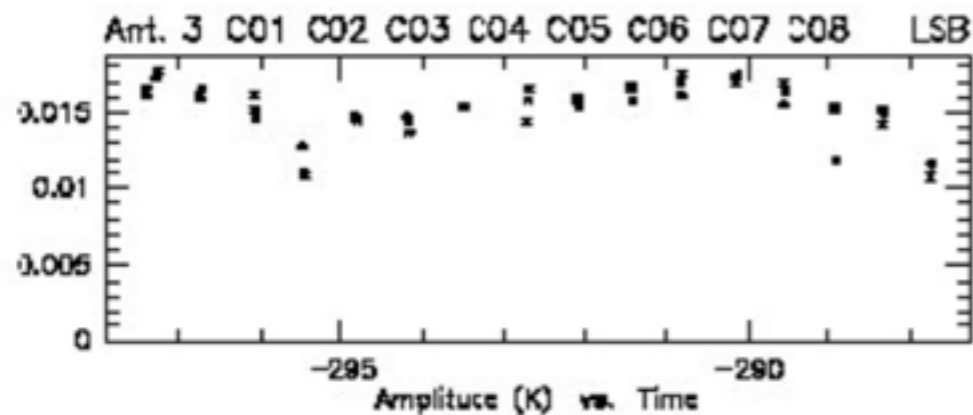
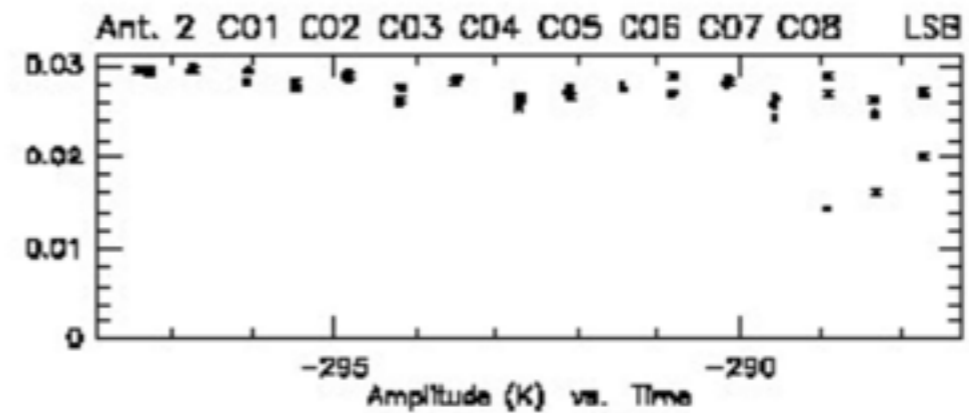
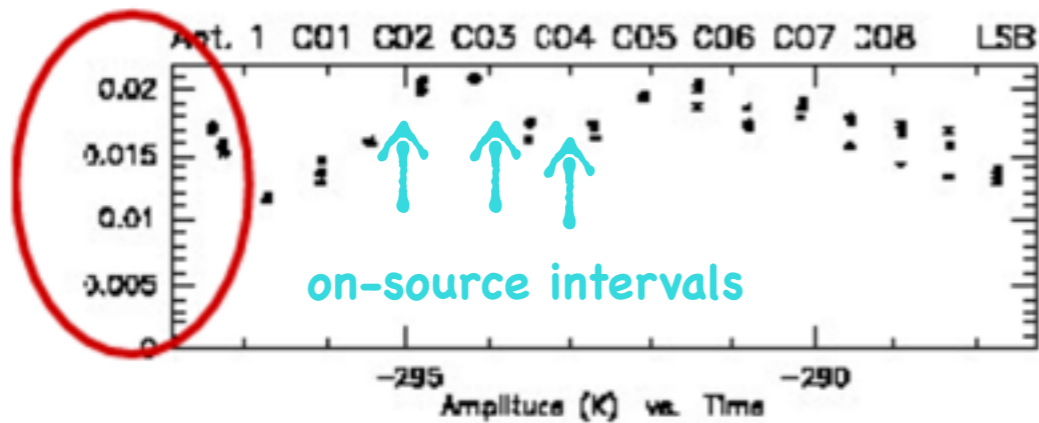
- Do it for all triangles and average

Example calibration : phase (gain)

RF: Uncal.
Am: Abs.
Ph: Abs.

CLIC - 04-OCT-2010 20:57:06 - gueth@dhcp-gueth W27E04E68N46N29E24 6Aq
A5F 12CO(2-1 230.538GHz B3 Q3(160,320,320,320)V Q3(160,320,320,320)H
(157 7275 P CORR)-(1116 B050 P CORR) 23-JAN-2010 14:33-00:16

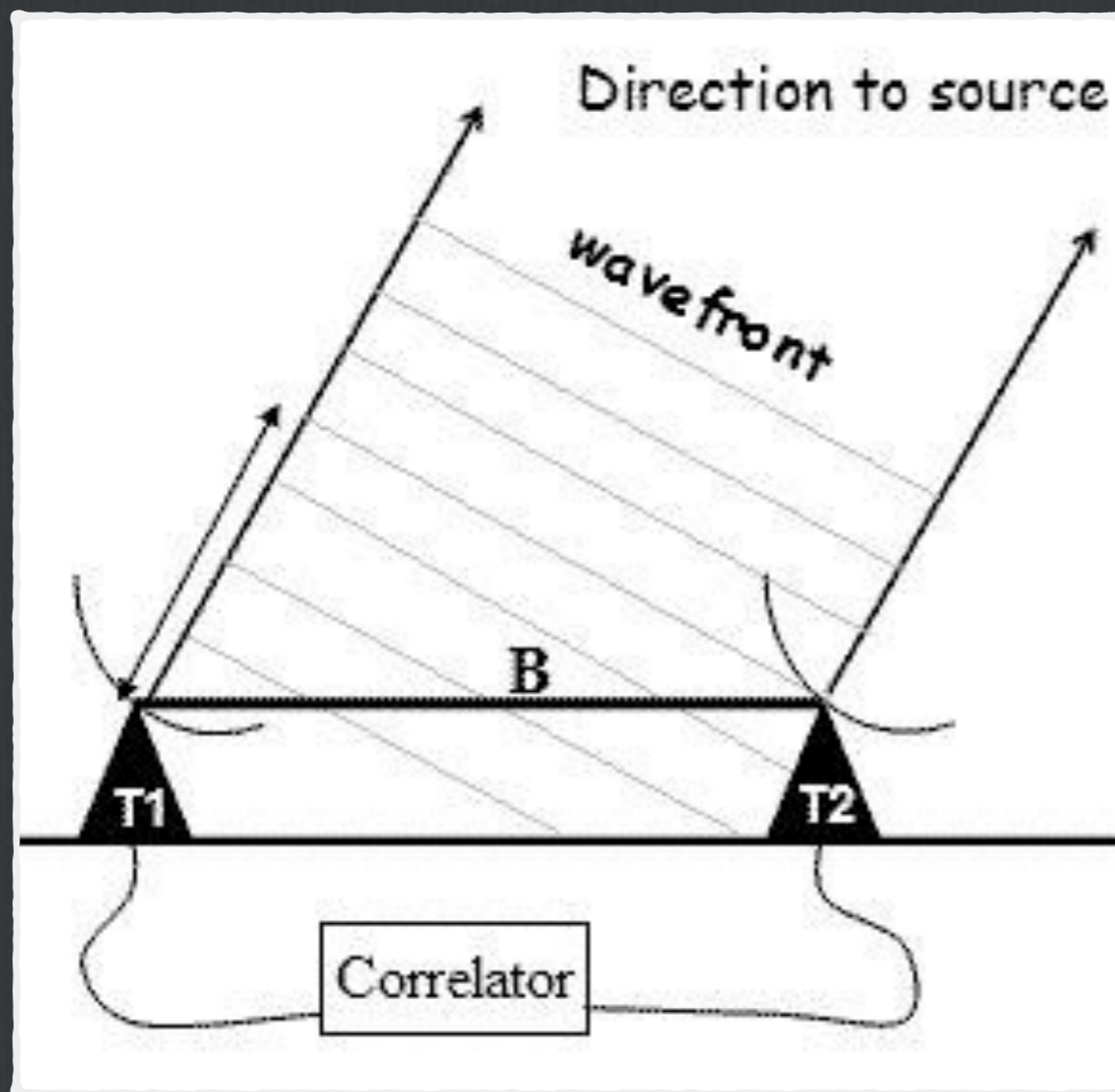
Scan Avg.



Softwares

- GILDAS / astro : prepare observations (source LST, uv_coverage, spectral configuration, calibrators) → ALMA/NOEMA ...
- GILDAS / clic : continuum and Line Interferometer Calibration) software → NOEMA
- GILDAS / mapping : uv-table to maps, deconvolution, analysis → ALMA / NOEMA ...
- ALMA-OT : proposal preparation
- CASA : Calibration (ALMA) + Imaging (ALMA / NOEMA)

Outline



- Interferometry principles

- Imaging & Calibration

- **Tutorials**

 - **Sensitivity**

 - **Imaging simulation**

 - **Proposal preparation**

Thank you